Introduction

Covid-19 vaccines have rolled out recently. In this short article, we will estimate the efficacy of vaccines based on official data

Here's a summary:

Data from official website			
Developer	No of infected	Vaccines group	placebo group
Pfizer-Biontech	170	8	162
Moderna	95	5	90
AstraZeneca re1	37	3	34

One of the developer Pifzer-Biontech claim that they have 95 % efficacy against COVID-19 beginning 28 days after the first dose

However, another research team cast a doubt on the efficacy of the vaccine. They are not allowed to do another mass test because the time is ticking. Statisticians suggest that they can work with old data to estimate the unknown parameter with a suitable interval

Beta–Bernoulli Models

A beta distribution as an exponential family defined on the interval [0,1] with two positive shape parameters α and β

In Bayesian inference, it is conjugated with Bernoulli, binomial, negative binomial, and geometric distributions.

Conjugacy

By definition a prior $p(\theta)$ is conjugate to the likelihood function $p(\theta \mid x)$ when the posterior has the same function form with the prior.

In this case Bernoulli distribution as likelihood Beta distribution as prior and posterior

$$Beta \propto Bernoulli \times Beta = p(\theta \mid X) \propto \prod_{i=1}^{n} p(x_i \mid \theta)p(\theta)$$

x_i comes from Bernoulli distribution. Its either 1 or 0 with n number of trials, θ as a parameter which we want to estimate it is in a form of beta distribution

Beta distribution

In this case we will only use $Beta(1,1)$ as our prior which is equal to Uniform distribution

PDF:

$$\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)} \quad where \quad B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

Bernoulli distribution

A common discrete probability distribution of a random variable which takes the value 1 with probability θ and the value 0 with probability $(1-\theta)$

PMF:

$$\theta^k(1-\theta)^{1-k} \quad where \quad k \in \{0,1\} \quad and \quad 0 \leq \theta \leq 1$$

Parameter estimation

To estimate the unknown parameter θ which is also known as efficacy.

We will consider $Beta(1,1)$ as our prior. It means that the parameter from 0 to 1 interval's have equal probability. Bernoulli distribution will become our likelihood. It only has two outcomes vaccine group as 0 and placebo group as 1.

Generally, we can calculate the posterior with either equation or Metropolis-Hastings algorithm
Since Beta-Bernoulli have conjugacy properties we can hire the equation

The posterior distribution:

$$Beta(Placebo\ group + 1, Vaccine\ group + 1) = Beta(163, 9)$$

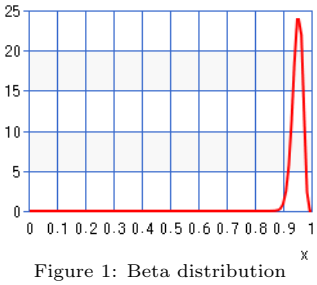

Figure 1: Beta distribution

Plot at keisan.casio.com
It show that we have 90% Confidence Interval to ensure that the efficacy of Pfizer-Biontech vaccine lie between 0.9171 and 0.9722

Meaning of efficacy

You have likely heard that Pfizer's COVID-19 vaccine efficacy is 95%, Moderna 's is 94% and Johnson & Johnson's is 66%. But what do these numbers actually mean?

The 95% efficacy actually means that vaccinated people had a 95% lower risk of getting COVID-19 compared with the control group participants, who weren't vaccinated.

In other words, vaccinated people in the Pfizer clinical trial were 20 times less likely than the control group to get COVID-19.

The concept of efficacy
https://www.livescience.com/covid-19-vaccine-efficacy-explained.html

Metropolis-Hastings
https://towardsdatascience.com/a-bayesian-model-for-estimating-the-effects-of-covid-19-vaccines-7e9cec99266b