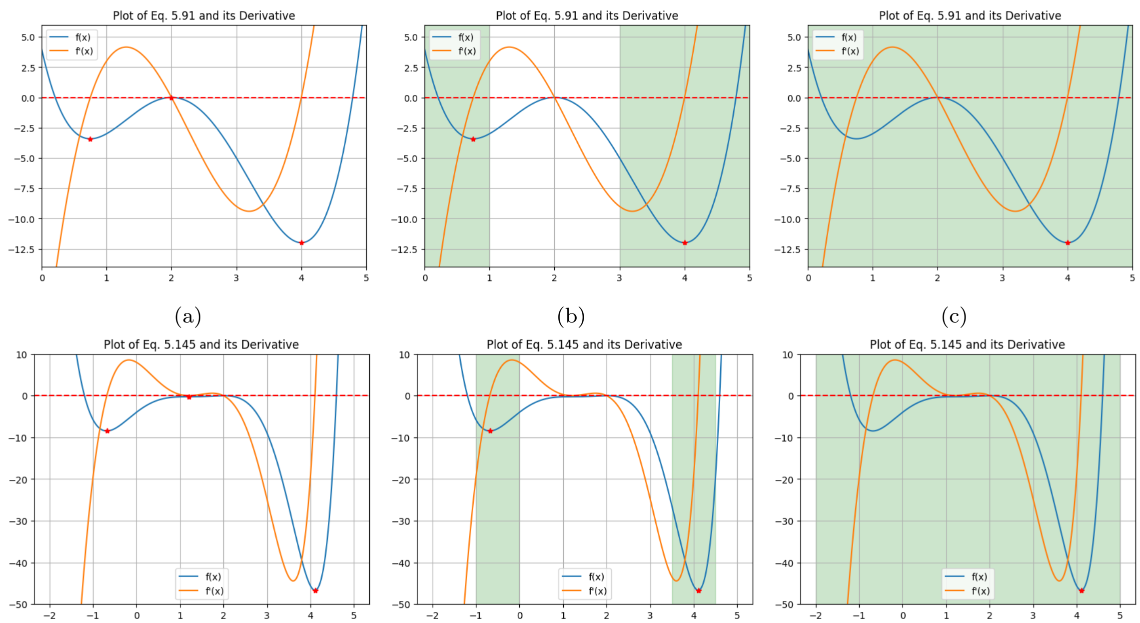

(a) (b) (c)

(d) (e) (f)

Figure 1: Eq. 5.91, Eq. 5.145, with derivatives and minima

The figures above show the graph of the two functions, along with their derivatives. The golden search algorithm returns the minima within some range (in the graph, I depicted the range x0 - x1 by shading it green). As you can see, depending on what range you set, the golden search algorithm gives different values - but for sure these values are the minimum within the range; and this is easily verified by the first derivative (f'(x) = 0 at local minima).

```
from math import sqrt
def golden(phi, x0, x1, kmax = 200, tol = 1.e-8):
    varphi = 0.5*(1 + sqrt(5))
    for k in range(1, kmax):
        x2 = x0 + (x1-x0)/(varphi + 1)
        x3 = x0 + (x1-x0)*varphi/(varphi+1)
        if phi(x3)<phi(x2):
            x0 = x2
        else:
            x1 = x3
        xnew = (x0 + x1)/2
        xdiff = abs(x1 - x0)
        if abs(xdiff)< tol:
            break
    else: xnew = None
    return xnew
```