Physics II

Electromagnetism

Modern physics

Contents

- Electrostatics (Chapters 19-22)
- Magnetostatics (Chapters 25-26)
- Electromagnetic field (Chapters 27-29) *Electromagnetism*

- Special theory of relativity (Chapter 32)
- Early quantum theory (Chapter 33)
- Quantum mechanics (Chapter 34-35) *Modern physics*

Course grade

75% for the final exam,

15% for homework

10% for quizzes

No midterm exam!

Chapter 19

Electric Charge and Electric Field

▲ When a comb has been passed across your hair, it can attract paper scraps. Why does this happen?

▲ Why atoms and molecules can be held together to form liquids and solids?

▲ What really happens in an electric circuit?

▲ How do electric motors and generators works?

▲ And what is light?

Electromagnetism

☆**Electric Field**

☆**Magnetic Field**

☆**Electromagnetic Field**

☆**Gauss's law of EF**

☆**Gauss's law of MF**

☆**Ampère's law**

☆**Faraday's law**

☆**Maxwell's equations**

J. C. Maxwell

1831-1879

Then God said:

$$\begin{cases} \nabla \cdot \vec{E} = \rho_0 / \varepsilon_0 \\ \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{B} = \mu_0 \vec{J} + \varepsilon_0 \mu_0 \dfrac{\partial \vec{E}}{\partial t} \end{cases}$$

and there was light.

Light is a kind of electromagnetic wave!

Electric charge

1) **Two types:** positive & negative

Unlike charges attract; like charges repel.

2) **Quantized:** elementary charge $e = 1.6 \times 10^{-19} C$

3) **Law of conservation of electric charge:**

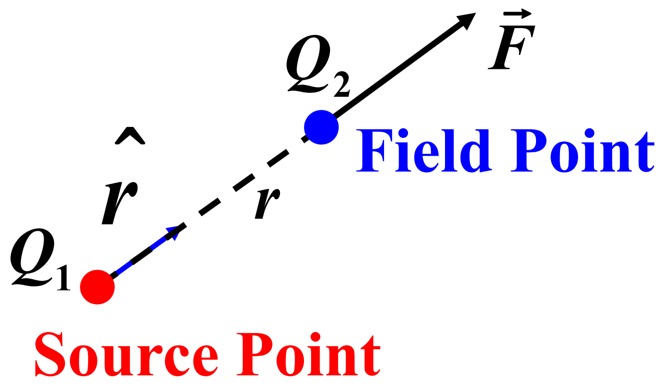
The net amount of electric charge produced in any process is zero.

▲ **Insulators, conductors, semiconductors**

Coulomb's law (1)

Electric forces between two point charges:

$$\boxed{\vec{F} = k\frac{Q_1 Q_2}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}\hat{r}}$$



where $\hat{r}$ is unit vector

in SI units: $k = 8.988\times10^{9} \approx 9.0\times10^{9}\, N\cdot m^2/C^2$

$$\varepsilon_0 = \frac{1}{4\pi k} = 8.85\times10^{-12}\, C^2/N\cdot m^2$$

ε_0 : **permittivity of free space**

Coulomb's law (2)

$$\vec{F} = k\frac{Q_1 Q_2}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}\hat{r}$$

1) It describes force at rest, in electrostatics

2) It is valid for two point charges

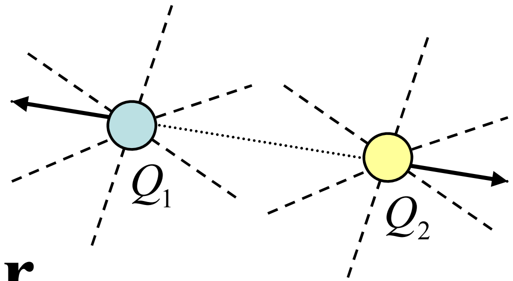
3) Principle of superposition:

If several charges are present, the net force on any one of them will be the vector sum of forces due to each of the others.

Electric field

Charges interact with each other by electric field

Charges $\xleftrightarrow[\textbf{interact on}]{\textbf{create}}$ **electric field**

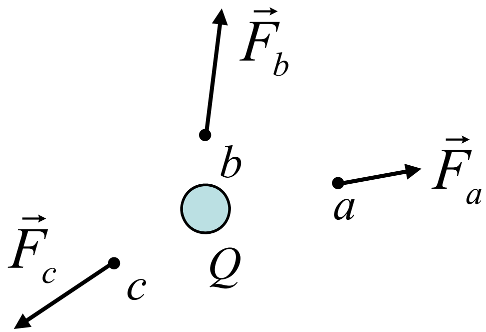


Field is a special form of matter

How to define it? **by using a test charge**

Electric field:

$$\vec{E} = \vec{F} / q \quad \text{or} \quad \vec{E} = \lim_{q \to 0} \vec{F} / q$$



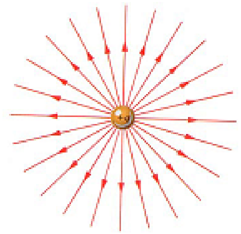
Properties of electric field

$$\vec{E} = \vec{F} / q \qquad\qquad \vec{F} = q\vec{E}$$

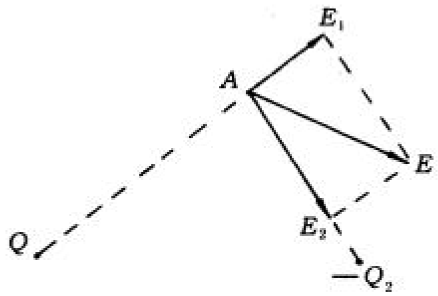
1) The field doesn't depend on the test charge *q*

2) It is a vector field, magnitude & direction

3) For a point charge $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}\hat{r}$



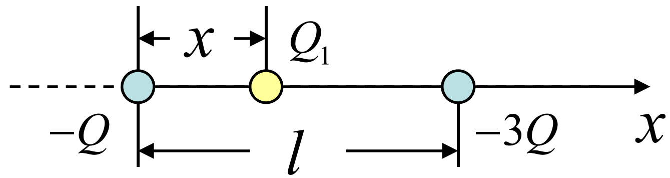
4) For many charges, total field



$$\vec{E} = \sum_i \vec{E}_i$$

Electric equilibrium

Example1: Two charges, −Q and −3Q, are a distance l apart. How can we place a third charge nearby to reach an equilibrium?



Solution: Position?

$$\frac{1}{4\pi\varepsilon_0}\frac{-Q}{x^2}-\frac{1}{4\pi\varepsilon_0}\frac{-3Q}{(l-x)^2}=0 \;\Rightarrow\; x=\frac{\sqrt{3}-1}{2}l=0.366l$$

What is the Charge?

$$-\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{x^2}-\frac{1}{4\pi\varepsilon_0}\frac{-3Q}{l^2}=0 \;\Rightarrow\; Q_1=\frac{6-3\sqrt{3}}{2}Q=0.402Q$$

Continuous charge distribution

The electric field can be calculated by integral

① Divide it into infinitesimal charges d*Q*

② Contribution from d*Q*: $d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2}\hat{r}$

③ Consider all the components $dE_x,\ dE_y,\ dE_z$

④ Finish the integration:

$$E_x = \int dE_x,\ E_y = \int dE_y,\ E_z = \int dE_z$$

A line of charge

Example2: **Charge is distributed uniformly along a line. Find the electric field at any given point *P* around it. (Charge per unit length is λ)**

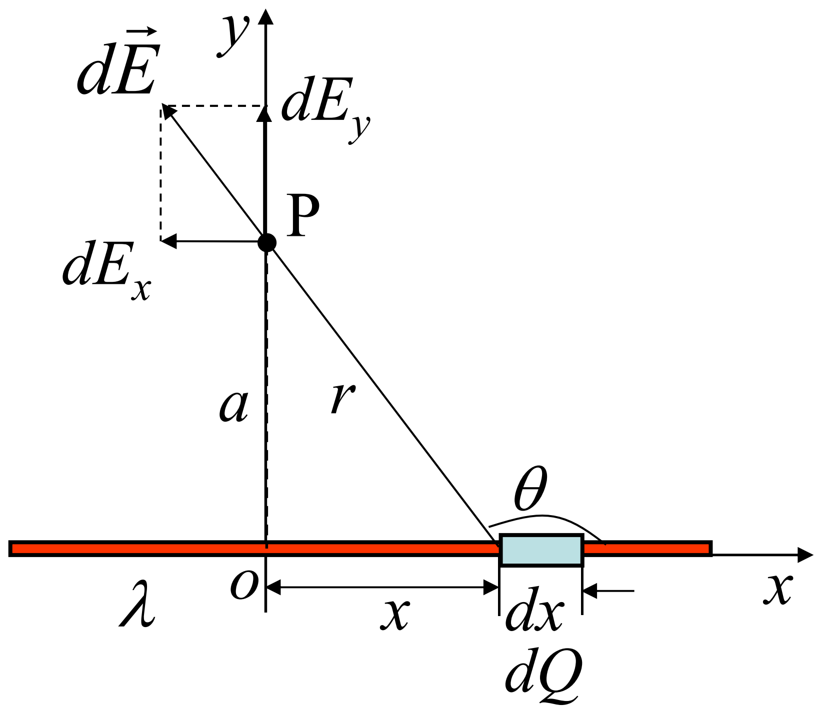
Solution: ① x-y axes

② $dQ = \lambda dx$

③ $dE = \dfrac{\lambda dx}{4\pi\varepsilon_0 r^2}$

④ $dE_x = dE \cdot \cos\theta$

$dE_y = dE \cdot \sin\theta$



⑤ $$E_x = \int \frac{\lambda dx}{4\pi\varepsilon_0 r^2}\cos\theta$$

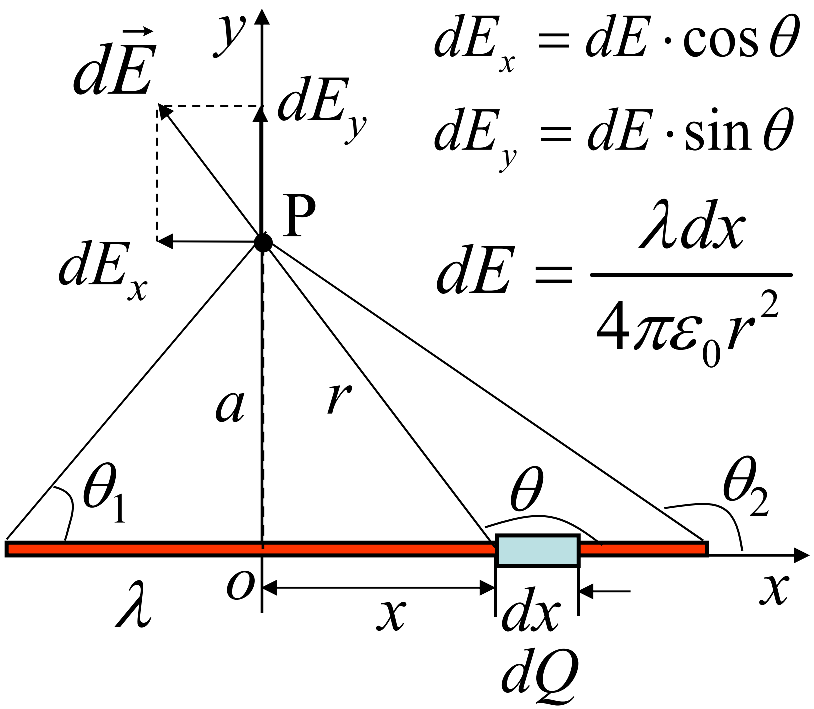
$$= \frac{\lambda}{4\pi\varepsilon_0 a}\int_{\theta_1}^{\theta_2}\cos\theta d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 a}(\sin\theta_2 - \sin\theta_1)$$

$$E_y = \int \frac{\lambda dx}{4\pi\varepsilon_0 r^2}\sin\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 a}\int_{\theta_1}^{\theta_2}\sin\theta d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 a}(\cos\theta_1 - \cos\theta_2)$$



$$dE_x = dE\cdot\cos\theta$$

$$dE_y = dE\cdot\sin\theta$$

$$dE = \frac{\lambda dx}{4\pi\varepsilon_0 r^2}$$

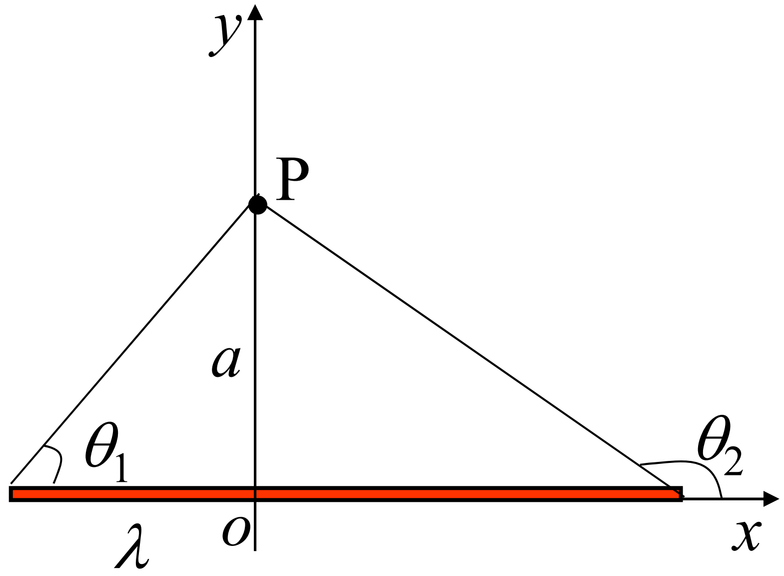
$r = a/\sin\theta$, $x = -a\cdot\cot\theta$,

$dx = ad\theta/\sin^2\theta$

$$\vec{E} = E_x\vec{i} + E_y\vec{j}$$

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 a}(\sin\theta_2 - \sin\theta_1)$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 a}(\cos\theta_1 - \cos\theta_2)$$

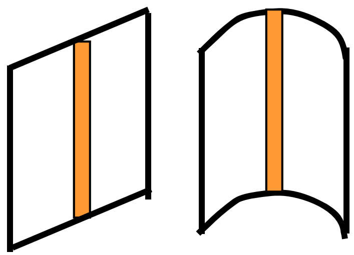


Discussion:

1) If it is very long or infinite, $\theta_1=0$, $\theta_2=\pi$

$$E_x = 0, \quad \boxed{E_y = \frac{\lambda}{2\pi\varepsilon_0 a}}$$ **→ useful result**

2) For surface distribution:



A plane of charge

Question: **Charge is distributed uniformly on an infinite plane. Find the electric field at any given point *P* around it. (Charge per unit area is *σ*)**

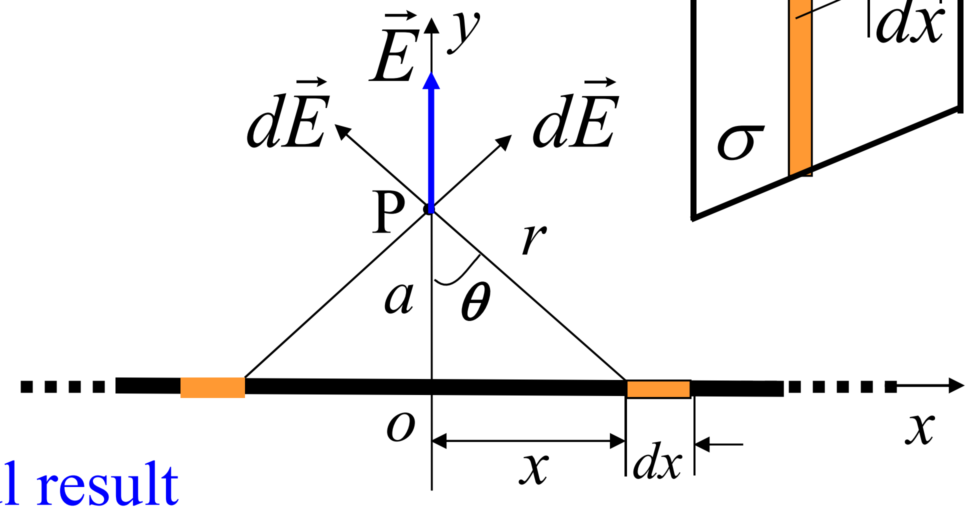
$$E = E_y = \int_{-\infty}^{\infty} \frac{\sigma dx}{2\pi\varepsilon_0 r} \cos\theta$$

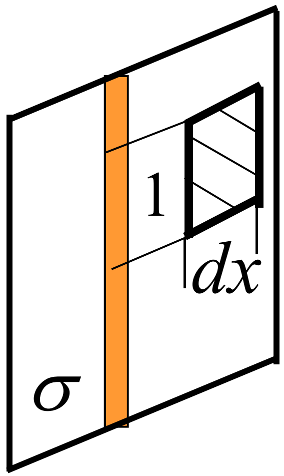
$$\lambda = \sigma \cdot dx$$

$$= \frac{\sigma}{2\pi\varepsilon_0} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta$$

$$= \frac{\sigma}{2\varepsilon_0}$$

$\longrightarrow$ useful result

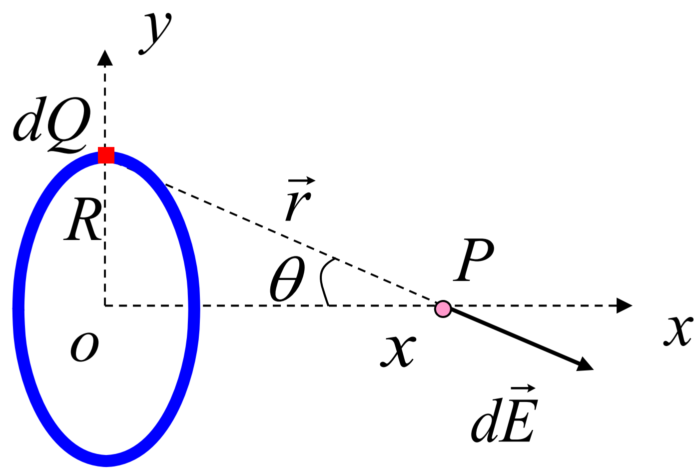




Uniformly charged ring

Example3: A thin ring of radius *R* holds a total charge *Q* distributed uniformly. Find the electric field at point *P* on the axis, *x* from its center.

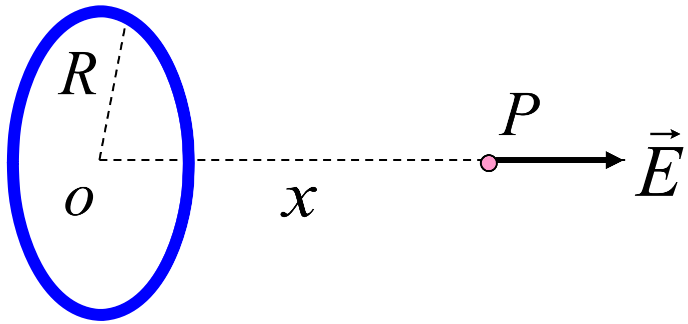
Solution: $$dE = \frac{dQ}{4\pi\varepsilon_0 r^2}$$



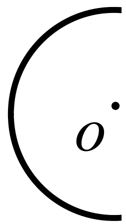
$$E = E_x = \int_{Ring} \frac{dQ}{4\pi\varepsilon_0 r^2}\cos\theta$$

$$= \frac{Q\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{Q\cdot x}{4\pi\varepsilon_0 (x^2+R^2)^{3/2}}$$

$$E = \frac{Q\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{Q\cdot x}{4\pi\varepsilon_0 (x^2+R^2)^{3/2}}$$



Discussion:



1) $x=0$ or $x>>R$, $E=?$

2) At what position along the axis, $E=E_{\max}$?

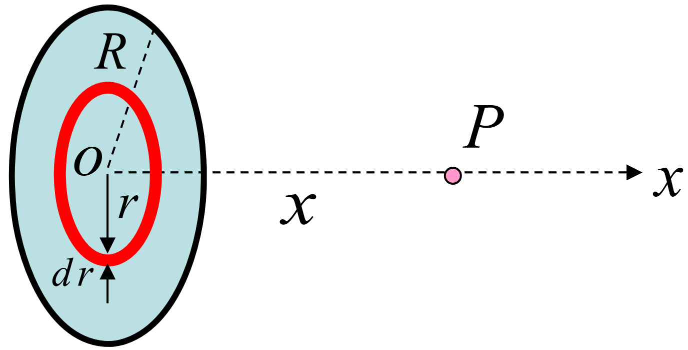
3) If there is a small gap in the circle, $E_o=?$

4) If there is only a semi-circle, $E_o=?$ $\boldsymbol{\frac{\lambda}{2\pi\varepsilon_0 R}}$

Uniformly charged disk

Example5: **Charge is distributed uniformly over a thin disk of radius *R*. Determine *E* at point *P* on the axis, *x* from its center. (Charge per unit area is σ)**

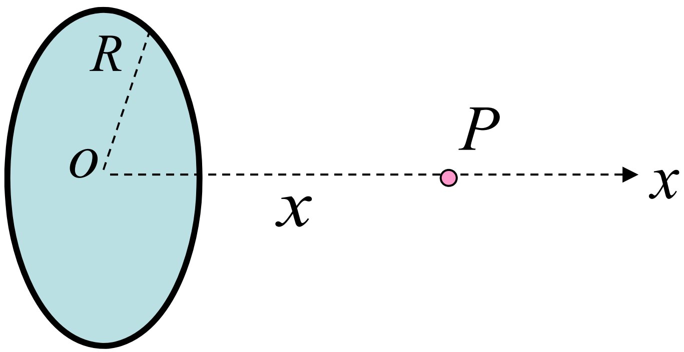
Solution: $dQ = \sigma 2\pi r dr$



$$dE = \frac{x dQ}{4\pi\varepsilon_0 (x^2 + r^2)^{3/2}}$$

$$E = \int_o^R \frac{\sigma \cdot x \cdot 2\pi r dr}{4\pi\varepsilon_0 (x^2 + r^2)^{3/2}} = \frac{\sigma}{2\varepsilon_0}[1 - \frac{x}{\sqrt{x^2 + R^2}}]$$

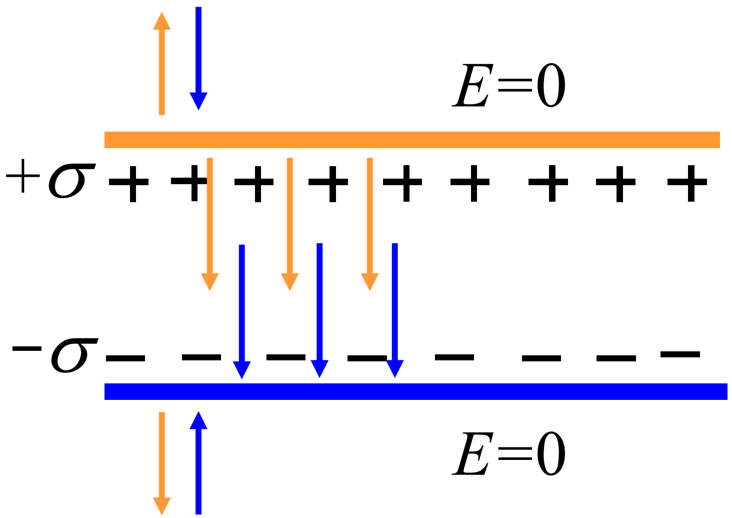
$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$



Discussion:

When $R \to \infty$: $\boxed{E = \frac{\sigma}{2\varepsilon_0}}$ infinite plane

Parallel-plate capacitor



$$E = \frac{\sigma}{\varepsilon_0}$$

Electric dipoles (1)

Combination of two equal charges of opposite sign is called an electric dipole.

Dipole moment: $\vec{p} = Q\vec{l} \quad (-Q \rightarrow +Q)$

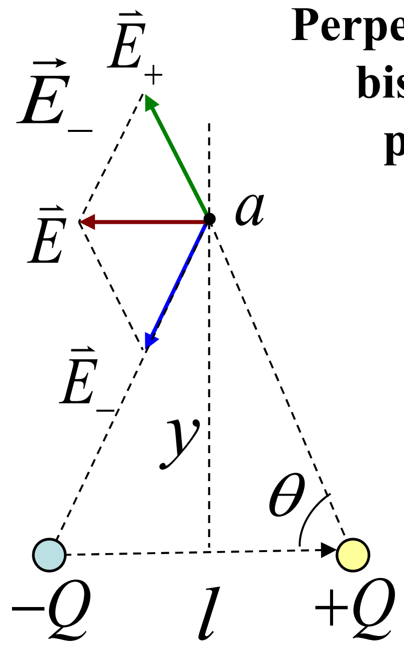
$$E_a = 2E_+ \cos\theta$$

$$= \frac{Q}{4\pi\varepsilon_0 (y^2 + \frac{l^2}{4})} \cdot \frac{l}{\sqrt{y^2 + \frac{l^2}{4}}}$$

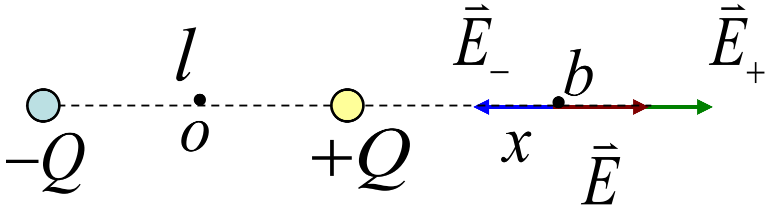
$$= \frac{p}{4\pi\varepsilon_0 (y^2 + \frac{l^2}{4})^{3/2}} \approx \frac{p}{4\pi\varepsilon_0 y^3}$$

$$(y \gg l)$$

$$\vec{E}_a = \vec{E}_+ + \vec{E}_-$$



Electric dipoles (2)



$$\vec{E}_b = \vec{E}_+ + \vec{E}_-$$

$$E_b = \frac{Q}{4\pi\varepsilon_0 (x - \frac{l}{2})^2} - \frac{Q}{4\pi\varepsilon_0 (x + \frac{l}{2})^2} = \frac{2px}{4\pi\varepsilon_0 (x^2 - \frac{l^2}{4})^2}$$

$$\approx \frac{2p}{4\pi\varepsilon_0 x^3} \quad (x \gg l) \qquad E_a \approx \frac{p}{4\pi\varepsilon_0 y^3} \quad (y \gg l)$$

Please Draw a Conclusion:

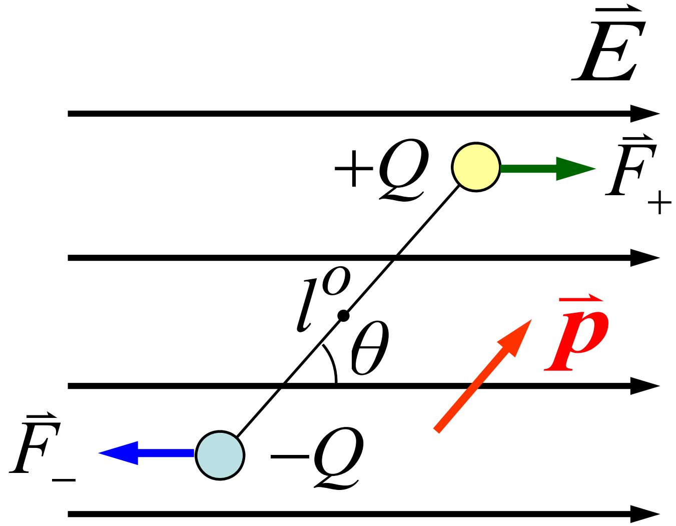
In which cases: $E \propto r^{-3} / r^{-2} / r^{-1} / r^0$?

Dipole in external field

1) The total force:

$$\vec{F} = \vec{F}_- + \vec{F}_+ = 0$$

2) Torque on the dipole:



$$\tau = F_+ \cdot \frac{1}{2} l \sin\theta + F_- \cdot \frac{1}{2} l \sin\theta$$

$$= QE \cdot l \sin\theta = pE \sin\theta \quad \textbf{or} \quad \boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

Discussion: what is the torque on the dipole when

$$\theta = 0,\ \pi/2,\ \pi\ ?$$

Vibrating charge

Thinking: **Negative charge $-Q$ is distributed on a ring uniformly. A positive charge q with mass m is placed from the center of ring a small distance x. Show that it will undergo SHM when released, and what is T?**

$$E = \frac{-Q \cdot x}{4\pi\varepsilon_0 (x^2 + R^2)^{3/2}}$$