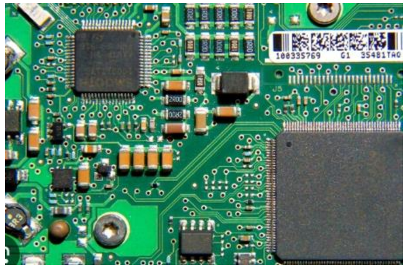
Circuit Analysis and Design

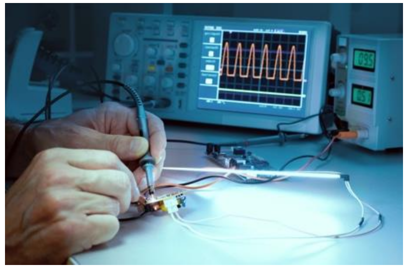
Academic Year 2025/2026 – Semester 1

Lecture 18 – RLC Filter Circuits

Masood Ur-Rehman, Muhammad Aslam and Ali Imran

{masood.urrehman, muhammad.aslam.2, ali.imran}@glasgow.ac.uk





Agenda

- Summarize previous lecture
- RLC series/parallel filter circuits
- Filter design

Review Previous Lecture

- Impedances of passive components of Filters:

$$\textit{Resistor}: Z_R = R, \qquad \textit{Inductor}: Z_L = j\omega L, \qquad \textit{Capacitor}: Z_C = \frac{1}{j\omega L}$$

- Analogue Filters are designed using R L and C.
- Filter: Device that passes certain frequencies and blocks others
- Mathematically, filters are explained by their transfer function as a function of frequency
- Transfer function $H(\omega)$ is ratio of a phasor output (voltage or current) to a phasor input (voltage or current).

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} \quad \textbf{OR} \quad H(\omega) = \frac{I_o(\omega)}{I_i(\omega)}$$

- Magnitude of $|H(\omega)|$, is magnitude response, Magnitude is also called the gain.
- Phase of the transfer function $\angle H(\omega)$, called the phase response

Second Order Filters/ RLC Filters

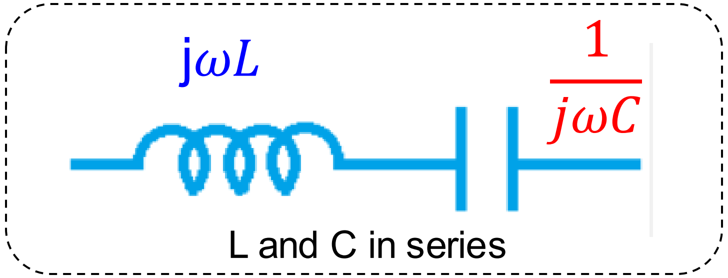
- Number of resonant elements (L or C) in a filter circuits determines the filter order.
  - An RLC circuit makes a second order filter.

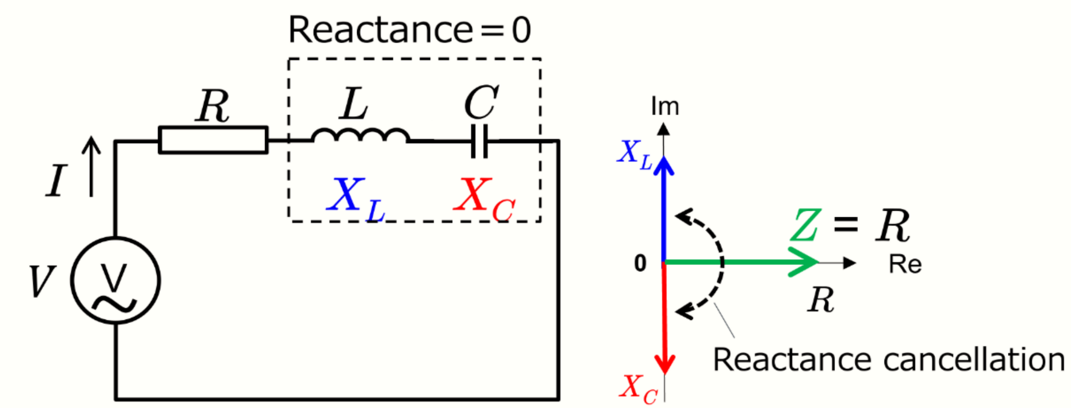
- Second order filters are combinations of R, L, and C in Series or Parallel configuration.

- RLC circuits can be configured in series or parallel configurations to design
  - Low pass Filter (LPF)
  - High pass Filter (HPF)
  - Band pass Filter (BPF)
  - Band stop Filter (BSF).

Equivalent Impedance of Series L and C at Resonance Frequency

- **Resonance Frequency, the frequency at which impedance of capacitor and inductor are equal in a circuit.**
  - $\omega L = \dfrac{1}{\omega C} \rightarrow$ $\boxed{\omega = \omega_o = \dfrac{1}{\sqrt{LC}} \rightarrow f_0 = \dfrac{1}{2\pi\sqrt{LC}}}$
- **Sum of these series connected impedance is zero at resonance frequency**
  - $Z_s = j\omega L + \dfrac{1}{j\omega C} = \dfrac{j1}{\sqrt{LC}}L - \dfrac{j\sqrt{LC}}{C} = \dfrac{j\sqrt{L}}{\sqrt{C}} - \dfrac{j\sqrt{L}}{\sqrt{C}} = 0$
- **Figure shows at resonance frequency, equivalent impedance of a series capacitor and inductor is zero**

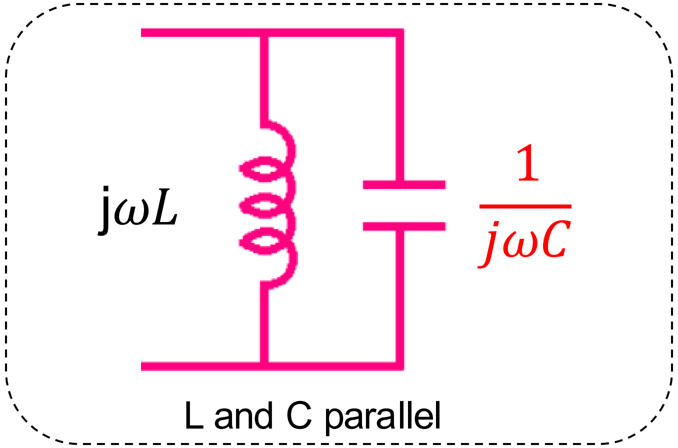


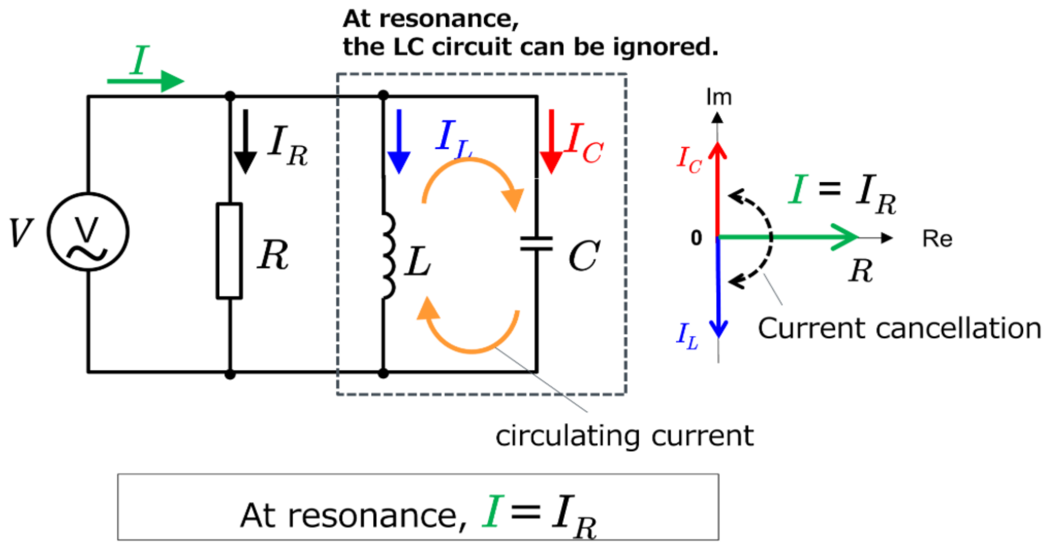


- **L and C in series behave as a short circuit line at resonance frequency**, impedance cancel each other and the circuit behaves purely resistive in RLC series circuit

Equivalent Impedance of Parallel L and C at Resonance Frequency

- **Resonance Frequency**
  - $\omega L = \frac{1}{\omega C} \rightarrow \omega = \omega_o = \frac{1}{\sqrt{LC}}$
- **When Capacitor and an Inductor are in parallel, at Resonance, parallel equivalent impedance is infinite**
  - $Z_P = \frac{(j\omega L)\left(\frac{1}{j\omega C}\right)}{\left(j\omega L + \frac{1}{j\omega C}\right)} = \frac{\left(\frac{jL}{\sqrt{LC}}\right)\left(-j\frac{\sqrt{LC}}{C}\right)}{\left(\frac{jL}{\sqrt{LC}} - j\frac{\sqrt{LC}}{C}\right)} = \frac{\frac{j\sqrt{L}}{\sqrt{C}}\left(-\frac{j\sqrt{L}}{\sqrt{C}}\right)}{\left(\frac{j\sqrt{L}}{\sqrt{C}} - \frac{j\sqrt{L}}{\sqrt{C}}\right)} = \infty$
- **At resonance frequency, L and C in parallel behave as an open circuit**





Quiz 1

To be shown and solved in CLASS

Series RLC LPF

- A series RLC circuit is shown in Figure 10.47.
- Application of the voltage divider rule yields

$$H(\omega)=\frac{\frac{1}{j\omega C}}{j\omega L+R+\frac{1}{j\omega C}}=\frac{1}{(j\omega)^2 LC+RCj\omega+1}=\frac{\frac{1}{LC}}{-\omega^2+\frac{R}{L}j\omega+\frac{1}{LC}}$$

- The magnitude and phase responses are given by

$$|H(\omega)|=\frac{\frac{1}{LC}}{\sqrt{\left(\frac{1}{LC}-\omega^2\right)^2+\left(\frac{R}{L}\omega\right)^2}},\quad \angle H(\omega)=-\tan^{-1}\left(\frac{\frac{R}{L}\omega}{\frac{1}{LC}-\omega^2}\right)$$

- The cutoff frequency is $\omega_0=\frac{1}{\sqrt{LC}}\quad rad/s$

- This is a second order LPF.

- Figure 10.48 shows $|H(\omega)|$ and $\angle H(\omega)$ for
- R = 2 kΩ, L = 50 mH, C = 5 nF → ω_0 = 63245.55 rad/s.

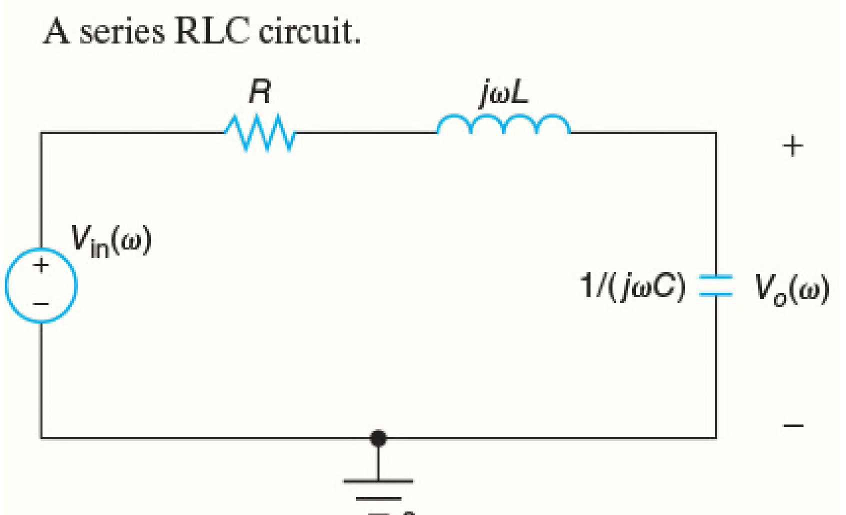


FIGURE 10.47

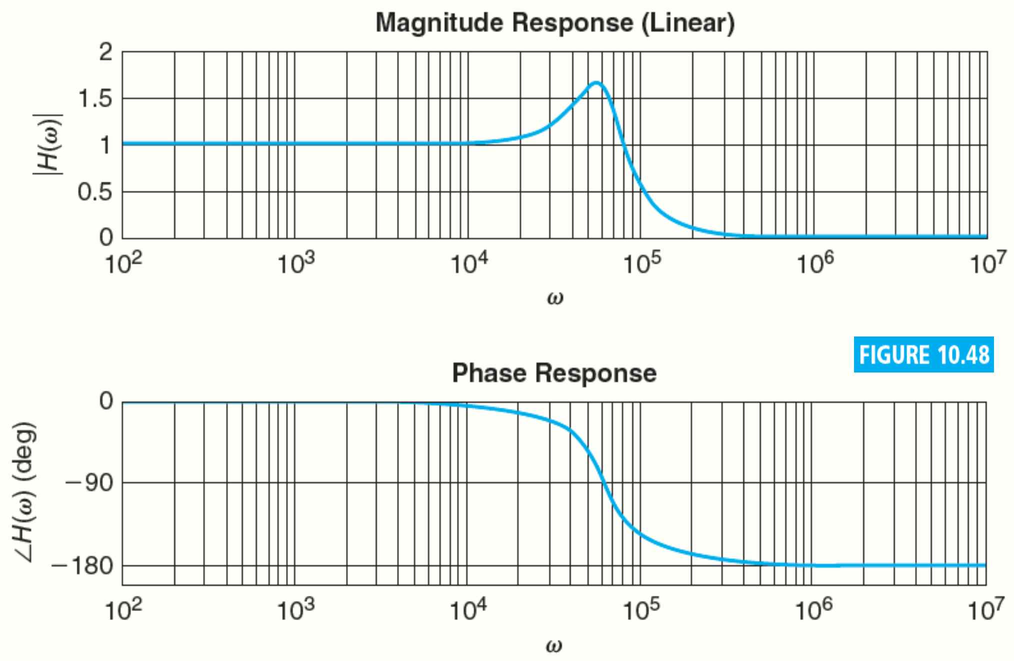


FIGURE 10.48

Series RLC HPF

- ❑ A series RCL circuit is shown in Figure 10.49.
- ❑ Application of the voltage divider rule yields

$$H(\omega)=\frac{j\omega L}{j\omega L+R+\dfrac{1}{j\omega C}}=\frac{(j\omega)^2 LC}{(j\omega)^2 LC+RCj\omega+1}=\frac{-\omega^2}{-\omega^2+\dfrac{R}{L}j\omega+\dfrac{1}{LC}}$$

- ❑ The magnitude and phase responses are given by

$$|H(\omega)|=\frac{\omega^2}{\sqrt{\left(\dfrac{1}{LC}-\omega^2\right)^2+\left(\dfrac{R}{L}\omega\right)^2}},\quad \angle H(\omega)=\pi-\tan^{-1}\left(\frac{\dfrac{R}{L}\omega}{\dfrac{1}{LC}-\omega^2}\right)$$

- ❑ The cutoff frequency is $$\omega_0=\frac{1}{\sqrt{LC}}\quad rad/s$$

- ❑ This is a **second order HPF.**

- ❑ Figure 10.50 shows $|H(\omega)|$ and $\angle H(\omega)$ for
  - R = 2 kΩ, L = 50 mH, C = 5 nF → ω_0 = 63245.55 rad/s.

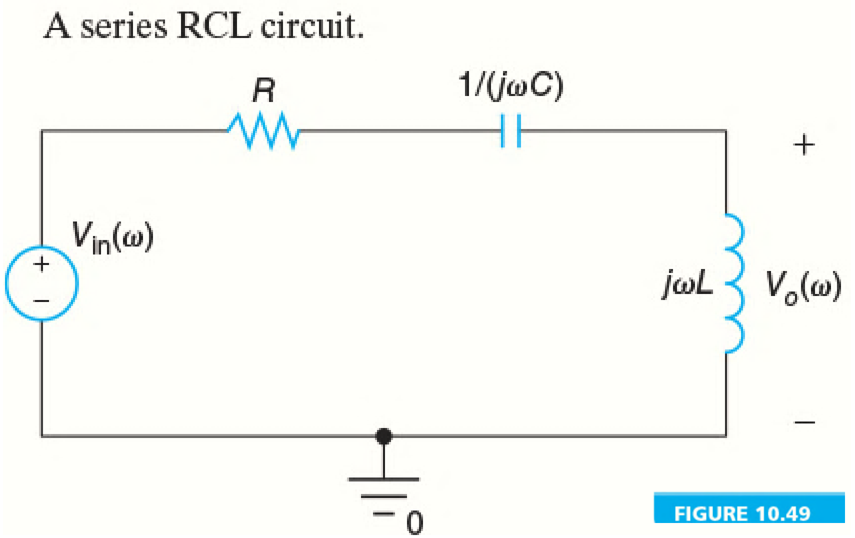


FIGURE 10.49

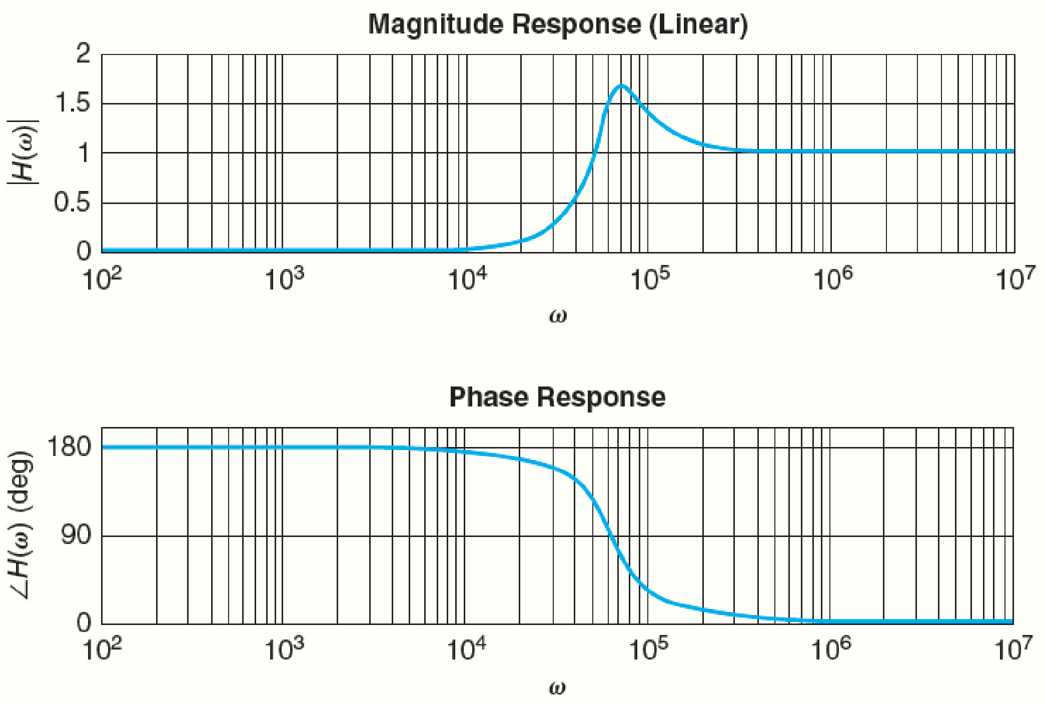


FIGURE 10.50

Series RLC BPF

- A series LCR circuit is shown in Figure 10.51.
- Application of the voltage divider rule yields

$$H(\omega)=\frac{R}{j\omega L+R+\dfrac{1}{j\omega C}}=\frac{RCj\omega}{(j\omega)^2 LC+RCj\omega+1}=\frac{\dfrac{R}{L}j\omega}{-\omega^2+\dfrac{R}{L}j\omega+\dfrac{1}{LC}}$$

- The magnitude and phase responses are given by

$$|H(\omega)|=\frac{\dfrac{R}{L}\omega}{\sqrt{\left(\dfrac{1}{LC}-\omega^2\right)^2+\left(\dfrac{R}{L}\omega\right)^2}},\quad \angle H(\omega)=\frac{\pi}{2}-\tan^{-1}\left(\frac{\dfrac{R}{L}\omega}{\dfrac{1}{LC}-\omega^2}\right)$$

- The resonant frequency is $\omega_0=\dfrac{1}{\sqrt{LC}}\quad rad/s$

- This is a **second order BPF.**

- Figure 10.52 shows $|H(\omega)|$ and $\angle H(\omega)$ for
  - *R = 2 kΩ, L = 50 mH, C = 5 nF →* ω_0 *= 63245.55 rad/s*.

A series LCR circuit.

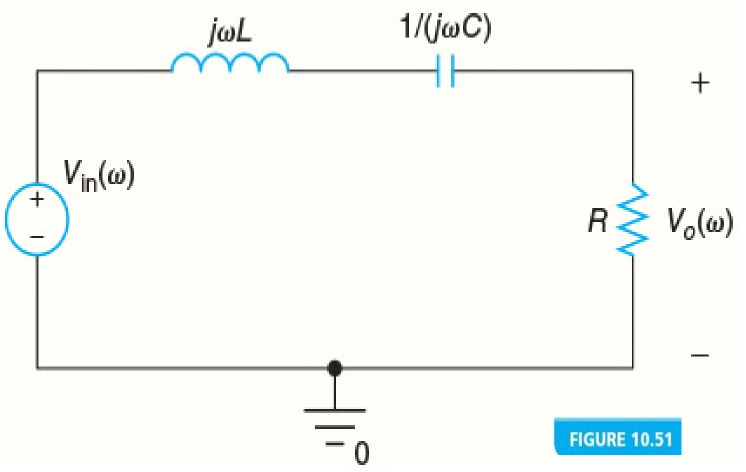


FIGURE 10.51

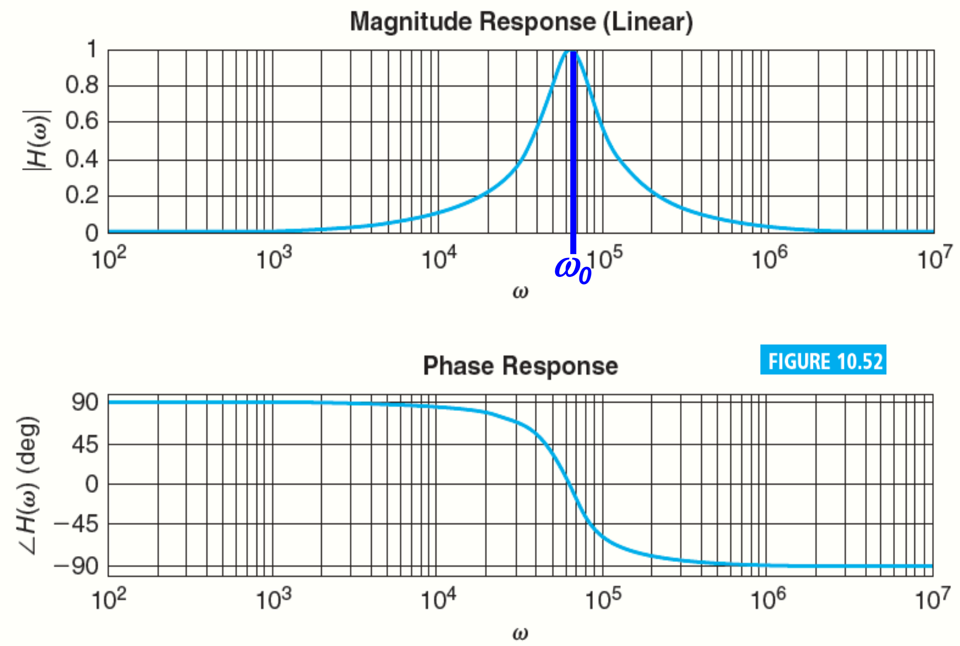


FIGURE 10.52

Series RLC BPF (Continued)

- **Lower *3-dB* cutoff frequency:**

$$\omega_1 = -\frac{R}{2L} + \frac{R}{2L}\sqrt{\frac{4L}{R^2C}+1}$$

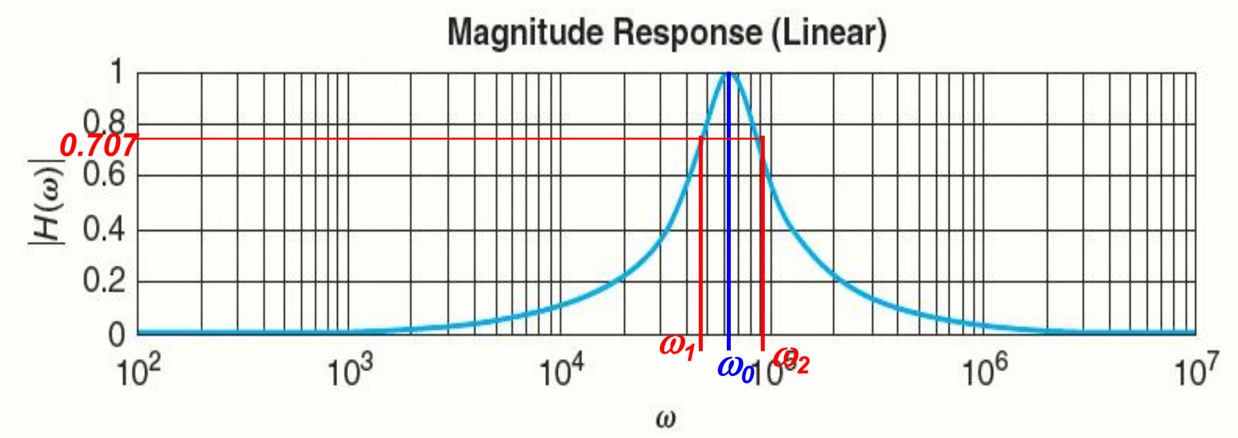
- **Upper *3-dB* cutoff frequency:**

$$\omega_2 = \frac{R}{2L} + \frac{R}{2L}\sqrt{\frac{4L}{R^2C}+1}$$

- ***3-dB* bandwidth:**

$$\omega_{3dB} = \omega_2 - \omega_1 = \frac{R}{L}$$

- For R = 2 $k\Omega$, L = 50 mH, C = 5 nF,
- ω_1 = 46,332.4958 rad/s,
- ω_2 = 86,332.4958 rad/s,
- $\omega_{3dB} = \omega_2 - \omega_1$ = 40,000 rad/s



- **Q is quality (selectivity) of filter which inversely proportional to bandwidth ω_{3dB}**

$$Q = \frac{\omega_0}{\omega_{3dB}} = \frac{L}{R}\omega_0$$

- **Cut off frequency in terms of Q**

$$\omega_1 = -\frac{\omega_0}{2Q} + \frac{\omega_0}{2Q}\sqrt{4Q^2+1}$$

$$\omega_2 = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q}\sqrt{4Q^2+1}$$

- *If Q>>1 then also*

$$\omega_1 \approx \omega_0 - \frac{\omega_{3dB}}{2} \qquad \omega_2 \approx \omega_0 + \frac{\omega_{3dB}}{2}$$

(only holds true if Q is very large)

Series RLC BSF

- A series RCL circuit is shown in Figure 10.53.
- Application of the voltage divider rule yields

$$H(\omega)=\frac{j\omega L+\frac{1}{j\omega C}}{j\omega L+R+\frac{1}{j\omega C}}=\frac{(j\omega)^2 LC+1}{(j\omega)^2 LC+RCj\omega+1}=\frac{-\omega^2+\frac{1}{LC}}{-\omega^2+\frac{R}{L}j\omega+\frac{1}{LC}}$$

- The magnitude and phase responses are given by

$$|H(\omega)|=\frac{\left|-\omega^2+\frac{1}{LC}\right|}{\sqrt{\left(\frac{1}{LC}-\omega^2\right)^2+\left(\frac{R}{L}\omega\right)^2}},\quad \angle H(\omega)=-\tan^{-1}\left(\frac{\frac{R}{L}\omega}{\frac{1}{LC}-\omega^2}\right)$$

- The resonant frequency is $\omega_0=\frac{1}{\sqrt{LC}}\quad rad/s$
- This is a **second order BSF**.
- Figure 10.54 shows $|H(\omega)|$ and $\angle H(\omega)$ for
  - $R = 2\ k\Omega$, $L = 50\ mH$, $C = 5\ nF$ → $\omega_0 = 63245.55\ rad/s$.

RCL circuit.

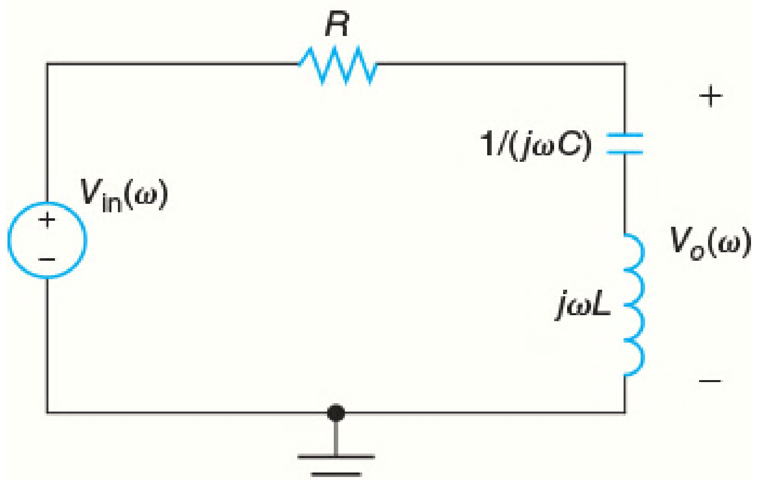


FIGURE 10.53

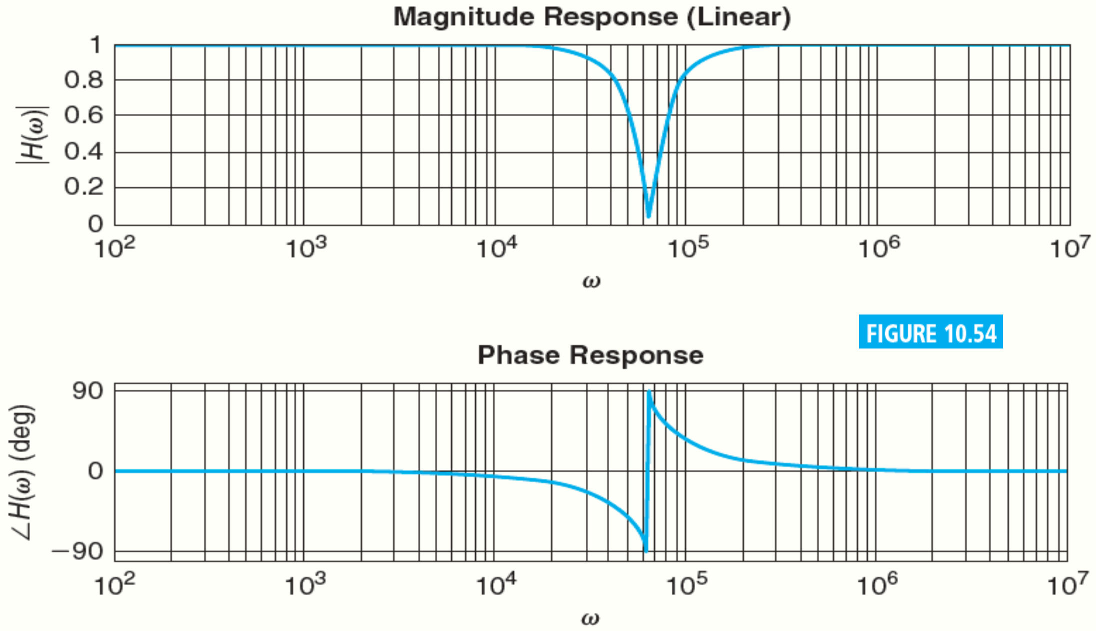


FIGURE 10.54

Series RLC BSF (Continued)

- **Lower *3-dB* cutoff frequency:**

$$\omega_1 = -\frac{R}{2L} + \frac{R}{2L}\sqrt{\frac{4L}{R^2C}+1}$$

- **Upper *3-dB* cutoff frequency:**

$$\omega_2 = \frac{R}{2L} + \frac{R}{2L}\sqrt{\frac{4L}{R^2C}+1}$$

- ***3-dB* bandwidth:**

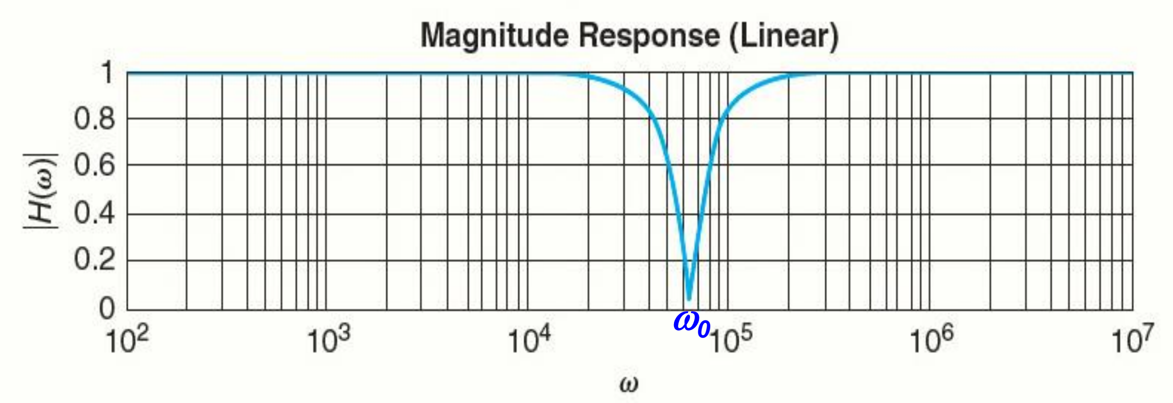
$$\omega_{3dB} = \omega_2 - \omega_1 = \frac{R}{L}$$

- For *R = 2 kΩ, L = 50 mH, C = 5 nF,*

  ω_1 = 46,332.4958 rad/s,

  ω_2 = 86,332.4958 rad/s,

  $\omega_{3dB} = \omega_2 - \omega_1$ = 40,000 rad/s



- **Q is quality (selectivity) of filter which inversely proportional to bandwidth ω_{3dB}**

$$Q = \frac{\omega_0}{\omega_{3dB}} = \frac{L}{R}\omega_0$$

- **Insert Q in cut-off frequency equations**

$$\omega_1 = -\frac{\omega_0}{2Q} + \frac{\omega_0}{2Q}\sqrt{4Q^2+1}$$

$$\omega_2 = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q}\sqrt{4Q^2+1}$$

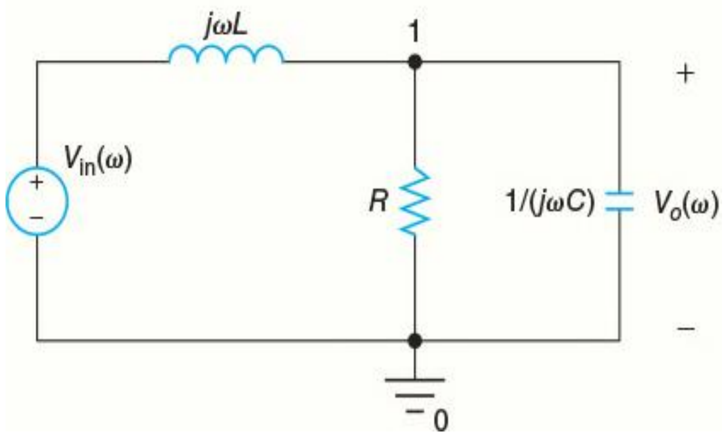
- *If Q>>1 then also*

$$\omega_1 \approx \omega_0 - \frac{\omega_{3dB}}{2} \qquad \omega_2 \approx \omega_0 + \frac{\omega_{3dB}}{2}$$

(only holds true if Q is very large)

Parallel RLC LPF

- A parallel LRC circuit is shown in Figure 10.55.
- Nodal analysis yields $\dfrac{V_o(\omega) - V_{in}(\omega)}{j\omega L} + \dfrac{V_o(\omega)}{R} + \dfrac{V_o(\omega)}{\frac{1}{j\omega C}} = 0$



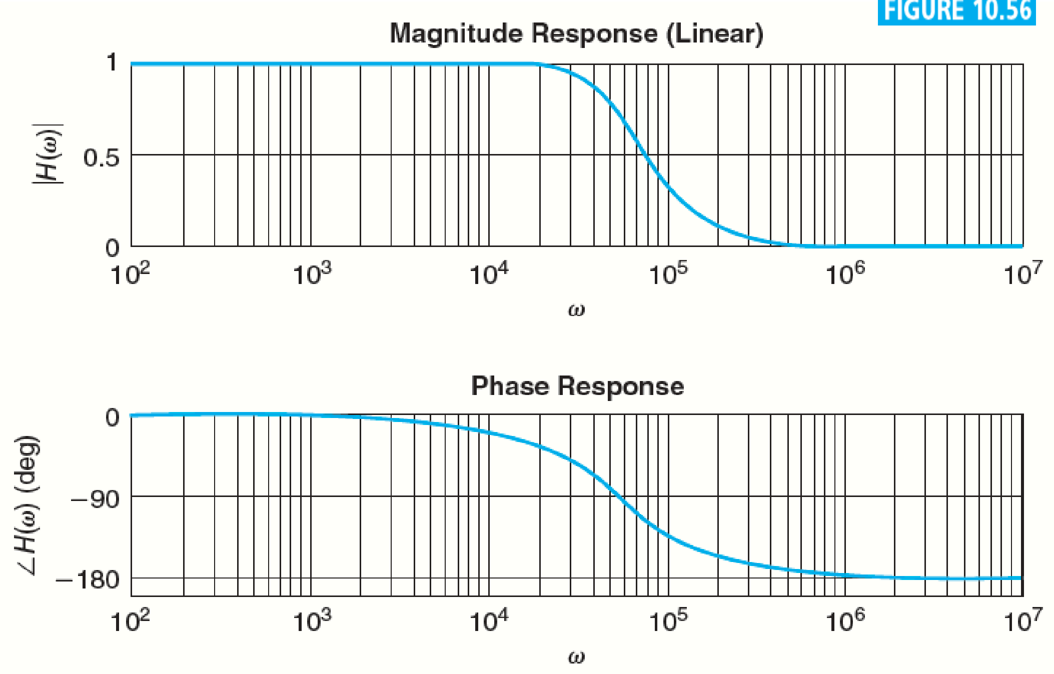
- Solving for $H(\omega) = \dfrac{V_o(\omega)}{V_{in}(\omega)}$ $\rightarrow H(\omega) = \dfrac{\frac{1}{j\omega L}}{j\omega C + \frac{1}{R} + \frac{1}{j\omega L}} = \dfrac{1}{(j\omega)^2 LC + \frac{L}{R} j\omega + 1} = \dfrac{\frac{1}{LC}}{-\omega^2 + \frac{1}{RC} j\omega + \frac{1}{LC}}$

- The magnitude and phase responses are given by

$$|H(\omega)| = \frac{\frac{1}{LC}}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{1}{RC}\omega\right)^2}}, \quad \angle H(\omega) = -\tan^{-1}\left(\frac{\frac{1}{RC}\omega}{\frac{1}{LC} - \omega^2}\right)$$

- The cutoff frequency is $\omega_0 = \dfrac{1}{\sqrt{LC}} \quad rad/s$

FIGURE 10.56



- This is a second order LPF.
- Figure 10.56 shows $|H(\omega)|$ and $\angle H(\omega)$ for
- *R = 2 kΩ, L = 50 mH, C = 5 nF → ω_0 = 63245.55 rad/s.*

Parallel RLC HPF

- A parallel CRL circuit is shown in Figure 10.57.
- Nodal analysis yields $$\frac{V_o(\omega)-V_{\text{in}}(\omega)}{\frac{1}{j\omega C}}+\frac{V_o(\omega)}{R}+\frac{V_o(\omega)}{j\omega L}=0$$
- Solving for $H(\omega)=\dfrac{V_o(\omega)}{V_{in}(\omega)}$ $\rightarrow$ $$H(\omega)=\frac{j\omega C}{j\omega C+\frac{1}{R}+\frac{1}{j\omega L}}=\frac{(j\omega)^2 LC}{(j\omega)^2 LC+\frac{L}{R}j\omega+1}=\frac{-\omega^2}{-\omega^2+\frac{1}{RC}j\omega+\frac{1}{LC}}$$
- The magnitude and phase responses are given by $$|H(\omega)|=\frac{\omega^2}{\sqrt{\left(\omega^2-\frac{1}{LC}\right)^2+\left(\frac{\omega}{RC}\right)^2}},\quad \angle H(\omega)=\pi-\tan^{-1}\left(\frac{\frac{\omega}{RC}}{\frac{1}{LC}-\omega^2}\right)$$
- The cutoff frequency is $$\omega_0=\frac{1}{\sqrt{LC}}\quad rad/s$$
- **This is a second order HPF.**
- Figure 10.58 shows $|H(\omega)|$ and $\angle H(\omega)$ for
- *R = 2 kΩ, L = 50 mH, C = 5 nF → ω_0 = 63245.55 rad/s.*

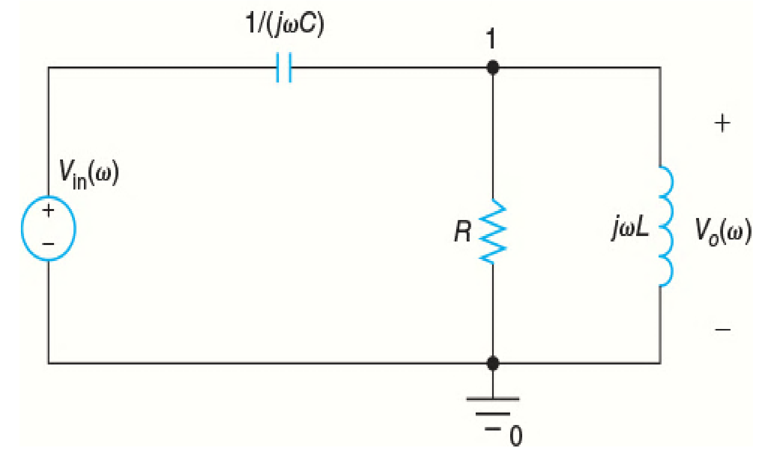
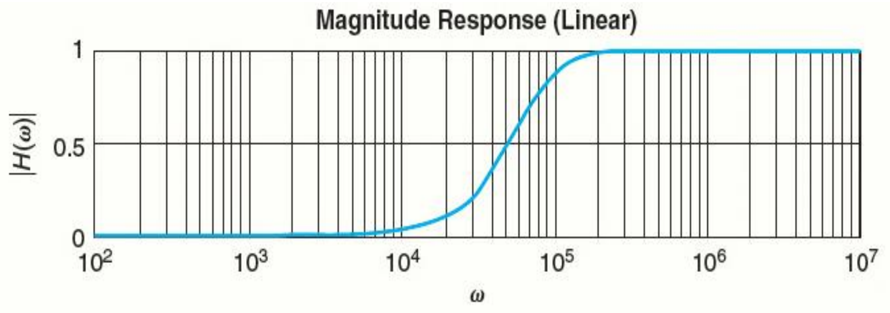
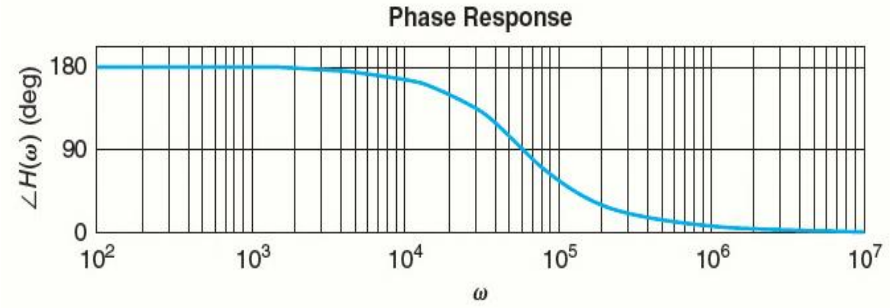


FIGURE 10.58





Parallel RLC BPF

- A parallel RCL circuit is shown in Figure 10.59.
- Nodal analysis yields → $H(\omega) = \dfrac{V_o(\omega)}{V_{in}(\omega)}$

$$H(\omega)=\frac{\frac{1}{R}}{j\omega C+\frac{1}{R}+\frac{1}{j\omega L}}=\frac{\frac{L}{R}j\omega}{(j\omega)^2 LC+\frac{L}{R}j\omega+1}=\frac{\frac{1}{RC}j\omega}{-\omega^2+\frac{1}{RC}j\omega+\frac{1}{LC}}$$

- The magnitude and phase responses are given by

$$|H(\omega)|=\frac{\frac{1}{RC}\omega}{\sqrt{\left(\omega^2-\frac{1}{LC}\right)^2+\left(\frac{1}{RC}\omega\right)^2}},\ \angle H(\omega)=\frac{\pi}{2}-\tan^{-1}\left(\frac{\frac{1}{RC}\omega}{\frac{1}{LC}-\omega^2}\right)$$

- The resonant frequency is: $\omega_0=\dfrac{1}{\sqrt{LC}}\quad rad/s$

- **This is a second order BPF.**
- Figure 10.60 shows $|H(\omega)|$ and $\angle H(\omega)$ for
  R = 2 kΩ, L = 50 mH, C = 5 nF → ω_0 = 63245.55 rad/s.

Parallel RCL circuit.

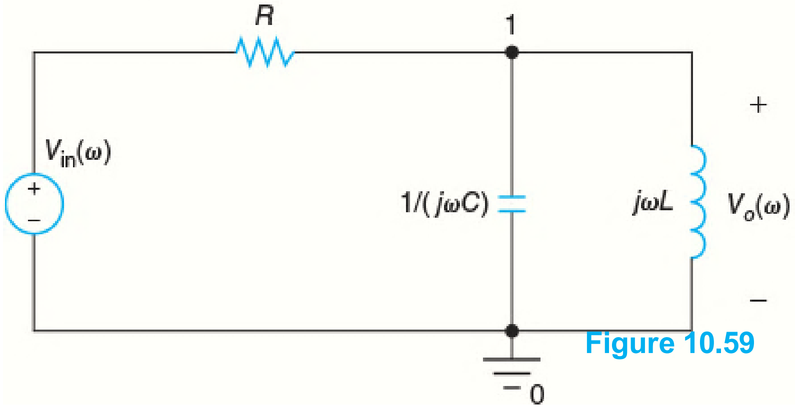


Figure 10.59

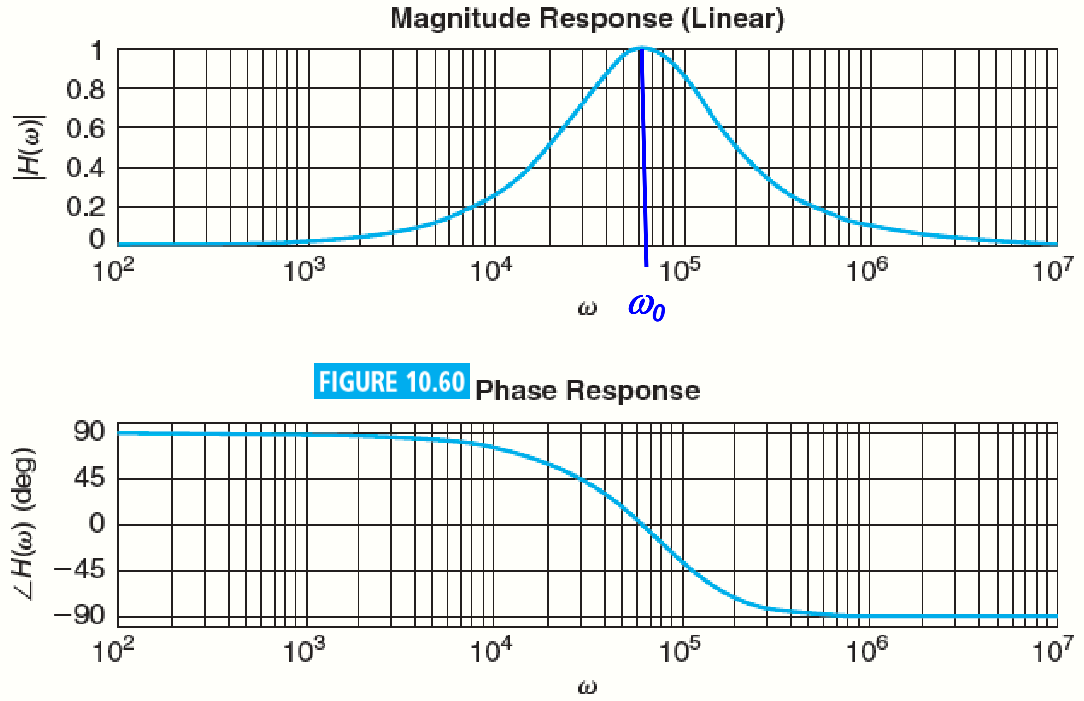


FIGURE 10.60

Parallel RLC BPF (Continued)

- **Lower *3-dB* cutoff frequency:**

$$\omega_1 = -\frac{1}{2RC} + \frac{1}{2RC}\sqrt{\frac{4R^2C}{L}+1}$$

- **Upper *3-dB* cutoff frequency:**

$$\omega_2 = \frac{1}{2RC} + \frac{1}{2RC}\sqrt{\frac{4R^2C}{L}+1}$$

- **3-dB bandwidth:**

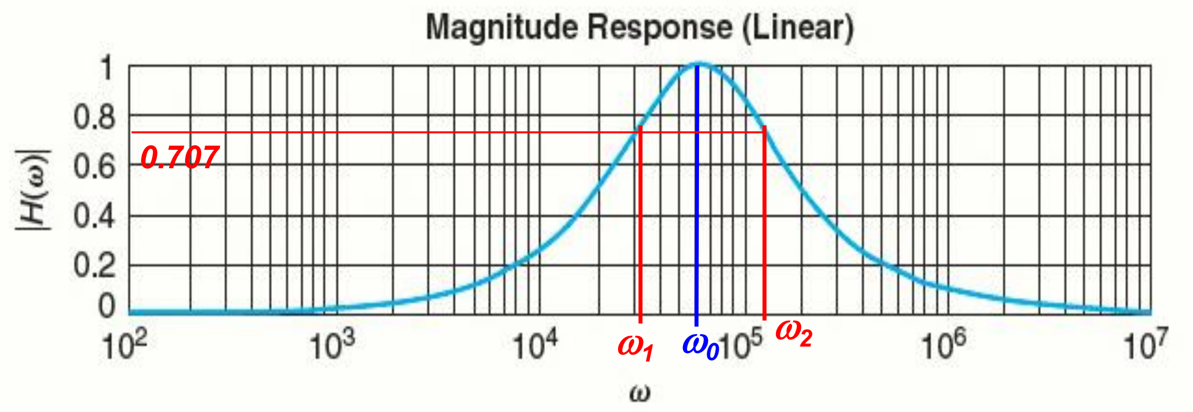
$$\omega_{3dB} = \omega_2 - \omega_1 = \frac{1}{RC}$$

- For R = 2 kΩ, L = 50 mH, C = 5 nF,

  ω_1 = 30,622.58 rad/s,

  ω_2 = 130,622.58 rad/s,

  $\omega_{3dB} = \omega_2 - \omega_1$ = 100,000 rad/s



- **Q is quality (selectivity) of filter which inversely proportional to bandwidth ω_{3dB}**

$$Q = \frac{\omega_0}{\omega_{3dB}} = RC\omega 0$$

- **Cut off frequencies in terms of Q**

$$\omega_1 = -\frac{\omega_0}{2Q} + \frac{\omega_0}{2Q}\sqrt{4Q^2+1}$$

$$\omega_2 = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q}\sqrt{4Q^2+1}$$

- If Q>>1 then also

$$\omega_1 \approx \omega_0 - \frac{\omega_{3dB}}{2} \qquad \omega_2 \approx \omega_0 + \frac{\omega_{3dB}}{2}$$

(Only holds true if Q is very large)

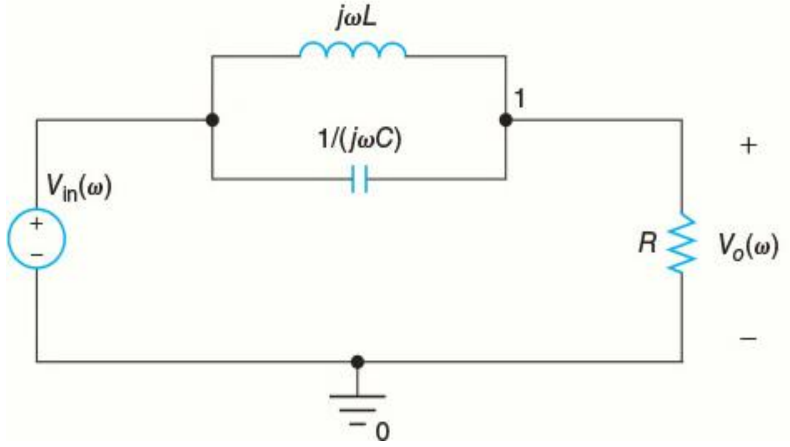
Parallel RLC BSF

- A parallel LCR circuit is shown in Figure 10.61.
- Nodal analysis yields → $H(\omega) = \dfrac{V_o(\omega)}{V_{in}(\omega)}$

$$H(\omega) = \frac{j\omega C + \dfrac{1}{j\omega L}}{j\omega C + \dfrac{1}{R} + \dfrac{1}{j\omega L}} = \frac{(j\omega)^2 LC + 1}{(j\omega)^2 LC + \dfrac{L}{R} j\omega + 1} = \frac{-\omega^2 + \dfrac{1}{LC}}{-\omega^2 + \dfrac{1}{RC} j\omega + \dfrac{1}{LC}}$$

Figure 10.61

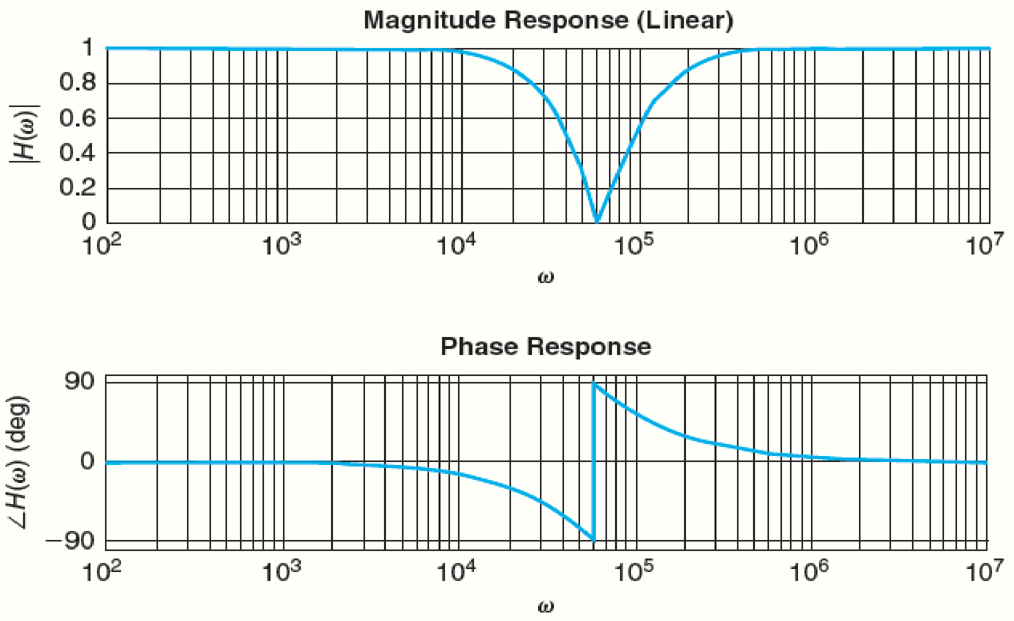
Parallel LCR circuit.



- The magnitude and phase responses are given by

$$|H(\omega)| = \frac{\left| -\omega^2 + \dfrac{1}{LC} \right|}{\sqrt{\left(\omega^2 - \dfrac{1}{LC}\right)^2 + \left(\dfrac{1}{RC}\omega\right)^2}}, \quad \angle H(\omega) = -\tan^{-1}\left(\frac{\dfrac{1}{RC}\omega}{\dfrac{1}{LC} - \omega^2} \right)$$

FIGURE 10.62



- The resonant frequency is $\omega_0 = \dfrac{1}{\sqrt{LC}} \quad rad/s$
- This is a second order BSF.
- Figure 10.62 shows $|H(\omega)|$ and $\angle H(\omega)$ for
  - R = 2 kΩ, L = 50 mH, C = 5 nF → ω_0 = 63245.55 rad/s.

Parallel RLC BSF (Continued)

- Lower *3-dB* cutoff frequency:

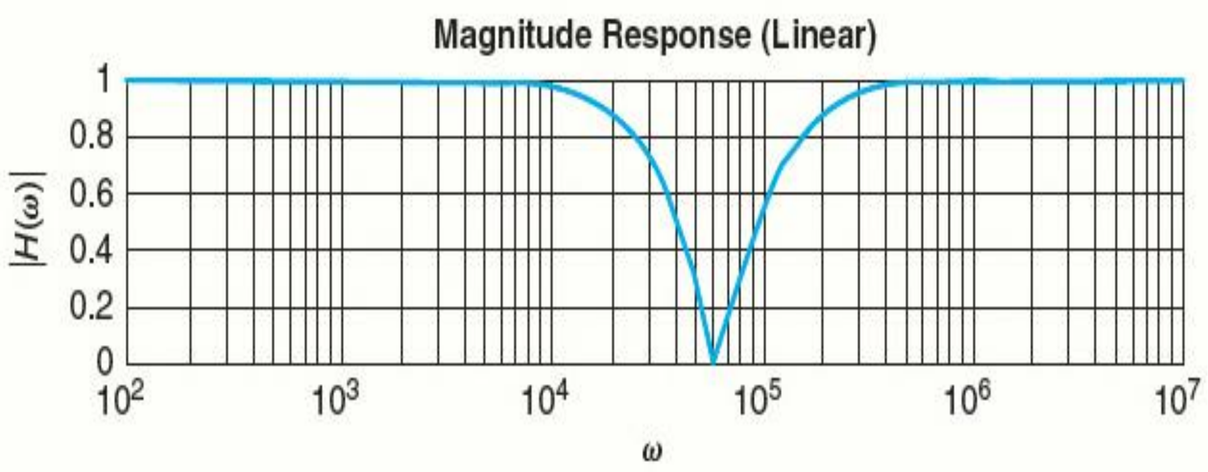
$$\omega_1 = -\frac{1}{2RC} + \frac{1}{2RC}\sqrt{\frac{4R^2C}{L}+1}$$

- Upper *3-dB* cutoff frequency:

$$\omega_2 = \frac{1}{2RC} + \frac{1}{2RC}\sqrt{\frac{4R^2C}{L}+1}$$

- *3-dB* bandwidth: $\omega_{3dB} = \omega_2 - \omega_1 = \dfrac{1}{RC}$

- For *R = 2 kΩ, L = 50 mH, C = 5 nF,*
  - ω_1 = *30,622.58 rad/s,*
  - ω_2 = *130,622.58 rad/s,*
  - $\omega_{3dB} = \omega_2 - \omega_1$ = *100,000 rad/s*



- **Q is quality (selectivity) of filter which inversely proportional to bandwidth ω_{3dB}** $Q = \dfrac{\omega_0}{\omega_{3dB}} = RC\omega 0$

- **Cut off frequencies in terms of Q**

$$\omega_1 = -\frac{\omega_0}{2Q} + \frac{\omega_0}{2Q}\sqrt{4Q^2+1}$$

$$\omega_2 = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q}\sqrt{4Q^2+1}$$

- If Q>>1 then also

$$\omega_1 \approx \omega_0 - \frac{\omega_{3dB}}{2} \qquad \omega_2 \approx \omega_0 + \frac{\omega_{3dB}}{2}$$

Holds only true if Q is very large

EXAMPLE 10.11

- Find the transfer function for the circuit shown in Figure 10.63, and state the type of filter (LPF, HPF, BPF, BSF).

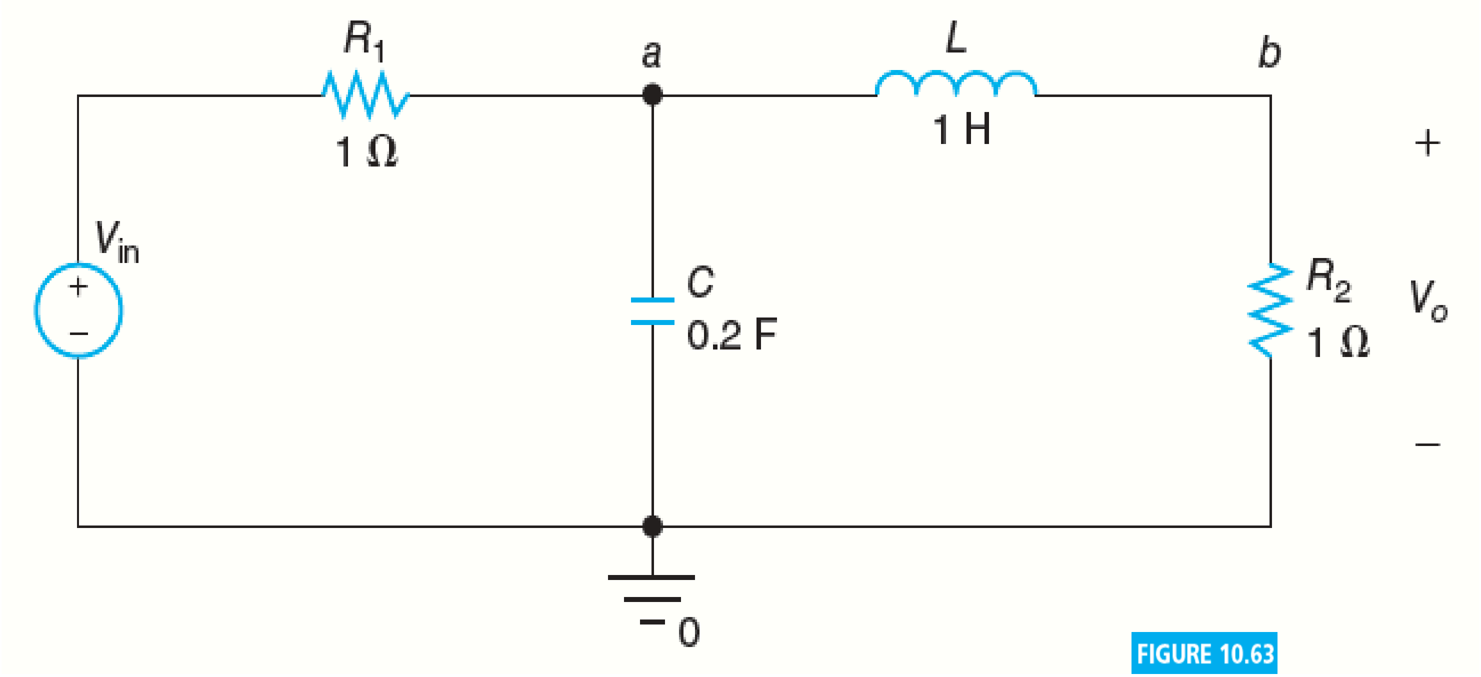


FIGURE 10.63

EXAMPLE 10.11

- **Find the transfer function for the circuit shown in Figure 10.63, and state the type of filter (LPF, HPF, BPF, BSF).**

- Node b: $$\frac{V_o - V_a}{j\omega \times 1} + \frac{V_o}{1} = 0 \Rightarrow V_o - V_a + j\omega V_o = 0 \Rightarrow V_a = (j\omega + 1)V_o \quad (1)$$

- Node a: $$\frac{V_a - V_{in}}{1} + V_a j\omega 0.2 + \frac{V_a - V_o}{j\omega \times 1} = 0 \Rightarrow j\omega V_a - j\omega V_{in} + (j\omega)^2 V_a 0.2 + V_a - V_o = 0 \quad (2)$$

- Substitute (1) into (2): $\left[(j\omega)^2 0.2 + j\omega + 1\right](j\omega + 1)V_o - V_o = j\omega V_{in}$
- Rearrangement yields $\left[(j\omega)^3 0.2 + (j\omega)^2 + j\omega + (j\omega)^2 0.2 + j\omega + 1\right]V_o - V_o = j\omega V_{in} \quad (3)$

- From Equation (3), we obtain

$$H(\omega) = \frac{V_o}{V_{in}} = \frac{j\omega}{(j\omega)^3 0.2 + (j\omega)^2 + j\omega + (j\omega)^2 0.2 + j\omega}$$

$$H(\omega) = \frac{1}{0.2(j\omega)^2 + 1.2 j\omega + 2} = \frac{5}{(j\omega)^2 + 6 j\omega + 10}$$

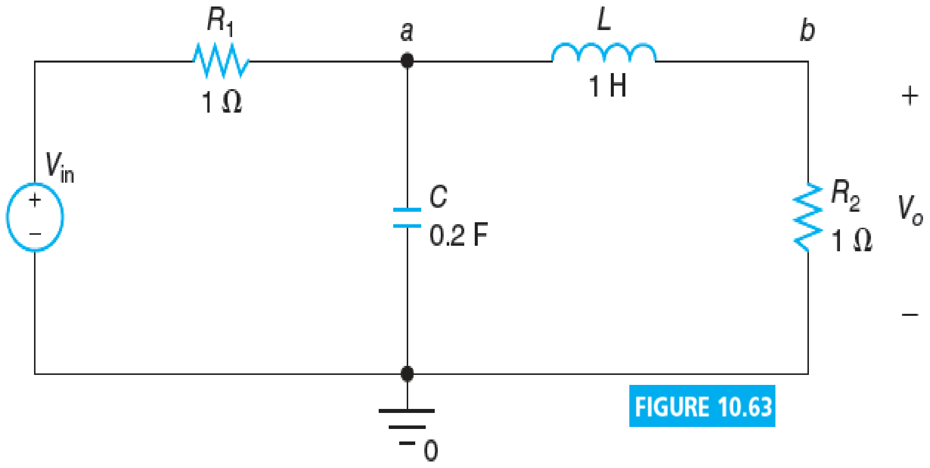


FIGURE 10.63

- *At ω = 0, $H(\omega)$ = 0.5. At ω =∞, $H(\omega)$ = 0.*
- **It is a LPF.**

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Filter Design

- The element values of a series RLC bandpass filter are $R = 5\ \Omega$, $L = 20\ mH$, and $C = 0.5\ \mu F$.
  - (a) Determine resonant frequency (ω_0), Q factor (Q), Bandwidth (B or ω_{3dB}), ω_1, and ω_2.
  - (b) Is it possible to double the magnitude of Q by changing the values of L and/or C, while keeping ω_0 and R unchanged?
- **Method 1:**

- Resonance frequency

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{20\times 10^{-3}\times 0.5\times 10^{-6}}} = 10^4 \text{ rad/s},$$

- Quality

$$Q = \frac{\omega_0 L}{R} = \frac{10^4\times 20\times 10^{-3}}{5} = 40,$$

- Bandwidth

$$B = \frac{R}{L} = \frac{5}{20\times 10^{-3}} = 250\, rad/s$$

OR

$$B = \frac{\omega_0}{Q} = \frac{10^4}{40} = 250 \text{ rad/s},$$

- Since Q >>1, we can compute:
  - Lower cut off frequency:

$$\omega_1 = \omega_0 - \frac{B}{2} = 10^4 - \frac{250}{2} = 9875 \text{ rad/s},$$

  - Upper cut off frequency:

$$\omega_2 = \omega_0 + \frac{B}{2} = 10^4 + \frac{250}{2} = 10125 \text{ rad/s}.$$

Filter Design (Continued)

- **Method 2 :**

- Upper and lower cut-off frequencies

$$\omega_1 = -\frac{R}{2L} + \frac{R}{2L}\sqrt{\frac{4L}{R^2C} + 1} = -\frac{5}{2\times20\times10^{-3}} + \frac{5}{2\times20\times10^{-3}}\sqrt{\frac{4\times20\times10^{-3}}{(5)^2\times0.5\times10^{-6}} + 1}$$

$$\omega_1 = -125 + 125\sqrt{6.4\times10^3 + 1} = 9{,}875\ rad/s$$

$$\omega_2 = \frac{R}{2L} + \frac{R}{2L}\sqrt{\frac{4L}{R^2C} + 1} = \frac{5}{2\times20\times10^{-3}} + \frac{5}{2\times20\times10^{-3}}\sqrt{\frac{4\times20\times10^{-3}}{(5)^2\times0.5\times10^{-6}} + 1}$$

$$\omega_2 = 125 + 125\sqrt{6.4\times10^3 + 1} = 10{,}126\ \mathrm{rad/s}$$

- Resonance frequency

$$\omega_o = \sqrt{\omega_1\omega_2} = \sqrt{9875\times10126} = 10{,}000\ rad/s$$

- Bandwidth

$$\mathrm{B} = \omega_2 - \omega_1 = 250\ rad/s$$

- Quality

$$\mathrm{Q} = \frac{\omega_o}{\mathrm{B}} = \frac{10{,}000}{250} = 40$$

Filter Design (Continued)

- Since,
$$Q = \frac{\omega_0 L}{R} \Rightarrow \frac{\omega_0}{R} = \frac{Q}{L}$$

- As ω_0 and R are need to be constants, doubling Q requires that L be doubled. But to keep ω_0 constant, C should be reduced to one half.

- Thus, the new set of element values are:
$$R = 5\,\Omega, \qquad L = 40\text{ mH}, \qquad \text{and } C = 0.25\,\mu\text{F}.$$

- The corresponding values of ω_0 and Q are:
$$\omega_0 = 10^4 \text{ rad/s (unchanged)}$$
$$Q = \frac{\omega_0 L}{R} = 80.$$

The Quality and Bandwidth Relation

Series Configuration			
Filter	**Bandwidth (BW)**	**Quality Factor (Q)**	**Relationship ($Q \cdot BW = \omega_0$)**
BPF	$BW = R / L$	$Q = \omega_0 L / R$	$Q \cdot BW = \omega_0$
BSF	$BW = R / L$	$Q = \omega_0 L / R$	$Q \cdot BW = \omega_0$
Parallel Configuration			
Filter	**Bandwidth (BW)**	**Quality Factor (Q)**	**Relationship ($Q \cdot BW = \omega_0$)**
BPF	$BW = 1 / RC$	$Q = \omega_0 RC$	$Q \cdot BW = \omega_0$
BSF	$BW = 1 / RC$	$Q = \omega_0 RC$	$Q \cdot BW = \omega_0$

- **Key Takeaways:**
  - In both series and parallel configurations, the relationship $Q \cdot BW = \omega_0$ holds.
  - The formulas for Q and BW depend on whether the circuit is in series or parallel.
  - For BPF and BSF, the quality factor determines the sharpness of the passband or stopband, while the bandwidth determines the width of the passband or stopband.

Quiz 2

To be shown and solved in CLASS

Summary

- The transfer function $H(\omega)$ is defined as the ratio of the output to input.
- A filter is a device that passes certain frequencies and blocks other frequencies.
- In practical filters, the gain in the passband cannot be one for all frequencies, and the gain in the stopband cannot be zero for all frequencies.
- A simple first order LPF can be implemented in RC circuit or LR circuit.
- A simple first order HPF can be implemented in CR circuit or RL circuit.
- The second order filters (LPF, HPF, BPF, BSF) can be implemented in series RLC circuit or parallel RLC circuit.
- What will we study next?