Discrete Distributions

	Bernoulli with parameter p	Binomial with parameters n and p
p.f.	$f(x) = p^x(1-p)^{1-x}$, for $x = 0, 1$	$f(x) = \binom{n}{x} p^x(1-p)^{n-x}$, for $x = 0, \ldots, n$
Mean	p	np
Variance	$p(1-p)$	$np(1-p)$
m.g.f.	$\psi(t) = pe^t + 1 - p$	$\psi(t) = (pe^t + 1 - p)^n$

	Uniform on the integers $a, \ldots, b$	Hypergeometric with parameters A, B, and n
p.f.	$f(x) = \frac{1}{b-a+1}$, for $x = a, \ldots, b$	$f(x) = \frac{\binom{A}{x}\binom{B}{n-x}}{\binom{A+B}{n}}$, for $x = \max\{0, n-b\}, \ldots, \min\{n, A\}$
Mean	$\frac{b+a}{2}$	$\frac{nA}{A+B}$
Variance	$\frac{(b-a)(b-a+2)}{12}$	$\frac{nAB}{(A+B)^2}\frac{A+B-n}{A+B-1}$
m.g.f.	$\psi(t) = \frac{e^{(b+1)t} - e^{at}}{(e^t-1)(b-a+1)}$	Nothing simpler than $\psi(t) = \sum_x f(x)e^{tx}$

	Geometric with parameter p	Negative binomial with parameters r and p
p.f.	$f(x) = p(1-p)^x$, for $x = 0, 1, \ldots$	$f(x) = \binom{r+x-1}{x} p^r(1-p)^x$, for $x = 0, 1, \ldots$
Mean	$\frac{1-p}{p}$	$\frac{r(1-p)}{p}$
Variance	$\frac{1-p}{p^2}$	$\frac{r(1-p)}{p^2}$
m.g.f.	$\psi(t) = \frac{p}{1-(1-p)e^t}$, for $t < \log(1/[1-p])$	$\psi(t) = \left(\frac{p}{1-(1-p)e^t}\right)^r$, for $t < \log(1/[1-p])$

	Poisson with mean λ	Multinomial with parameters n and $(p_1, \ldots, p_k)$
p.f.	$f(x) = e^{-\lambda}\frac{\lambda^x}{x!}$, for $x = 0, 1, \ldots$	$f(x_1, \ldots, x_k) = \binom{n}{x_1, \ldots, x_k} p_1^{x_1} \cdots p_k^{x_k}$, for $x_1 + \cdots + x_k = n$ and all $x_i \ge 0$
Mean	λ	$E(X_i) = np_i$, for $i = 1, \ldots, k$
Variance	λ	$\text{Var}(X_i) = np_i(1-p_i)$, $\text{Cov}(X_i, X_j) = -np_ip_j$, for $i, j = 1, \ldots, k$
m.g.f.	$\psi(t) = e^{\lambda(e^t-1)}$	Multivariate m.g.f. can be defined, but is not defined in this text.

Continuous Distributions

	Beta with parameters α and β	Uniform on the interval $[a, b]$
p.d.f.	$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$, for $0 < x < 1$	$f(x) = \frac{1}{b-a}$, for $a < x < b$
Mean	$\frac{\alpha}{\alpha+\beta}$	$\frac{a+b}{2}$
Variance	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	$\frac{(b-a)^2}{12}$
m.g.f.	Not available in simple form	$\psi(t) = \frac{e^{-at}-e^{-bt}}{t(b-a)}$

	Exponential with parameter β	Gamma with parameters α and β
p.d.f.	$f(x) = \beta e^{-\beta x}$, for $x > 0$	$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$, for $x > 0$
Mean	$\frac{1}{\beta}$	$\frac{\alpha}{\beta}$
Variance	$\frac{1}{\beta^2}$	$\frac{\alpha}{\beta^2}$
m.g.f.	$\psi(t) = \frac{\beta}{\beta-t}$, for $t < \beta$	$\psi(t) = \left(\frac{\beta}{\beta-t}\right)^\alpha$, for $t < \beta$

	Normal with mean μ and variance σ^2	Bivariate normal with means μ_1 and μ_2, variances σ_1^2 and σ_2^2, and correlation ρ
p.d.f.	$f(x) = \frac{1}{(2\pi)^{1/2}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$	Formula is too large to print here. See Eq. (5.10.2) on page 338.
Mean	μ	$E(X_i) = \mu_i$, for $i = 1, 2$
Variance	σ^2	Covariance matrix: $\begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$
m.g.f.	$\psi(t) = \exp\left(\mu t + \frac{t^2\sigma^2}{2}\right)$	Bivariate m.g.f. can be defined, but is not defined in this text.

Continuous Distributions

	Lognormal with parameters μ and σ^2	F with m and n degrees of freedom
p.d.f.	$f(x) = \frac{1}{(2\pi)^{1/2}\sigma x} \exp\left(-\frac{(\log(x)-\mu)^2}{2\sigma^2}\right)$, for $x > 0$	$f(x) = \frac{\Gamma\left[\frac{1}{2}(m+n)\right] m^{m/2} n^{n/2}}{\Gamma\left(\frac{1}{2}m\right)\Gamma\left(\frac{1}{2}n\right)} \cdot \frac{x^{(m/2)-1}}{(mx+n)^{(m+n)/2}}$, for $x > 0$
Mean	$e^{\mu+\sigma^2/2}$	$\frac{n}{n-2}$, if $n > 2$
Variance	$e^{2\mu+\sigma^2}[e^{\sigma^2} - 1]$	$\frac{2n^2(m+n-2)}{m(n-2)^2(n-4)}$, if $n > 4$
m.g.f.	Not finite for $t > 0$	Not finite for $t > 0$

	t with m degrees of freedom	χ^2 with m degrees of freedom
p.d.f.	$f(x) = \frac{\Gamma\left(\frac{m+1}{2}\right)}{(m\pi)^{1/2}\Gamma\left(\frac{m}{2}\right)} \left(1 + \frac{x^2}{m}\right)^{-(m+1)/2}$	$f(x) = \frac{1}{2^{m/2}\Gamma(m/2)} x^{(m/2)-1} e^{-x/2}$, for $x > 0$
Mean	0, if $m > 1$	m
Variance	$\frac{m}{m-2}$, if $m > 2$	$2m$
m.g.f.	Not finite for $t \neq 0$	$\psi(t) = (1-2t)^{-m/2}$, for $t < 1/2$

	Cauchy centered at μ	Pareto with parameters x_0 and α_0
p.d.f.	$f(x) = \frac{1}{\pi(1+[x-\mu]^2)}$	$f(x) = \frac{\alpha x_0^\alpha}{x^{\alpha+1}}$, for $x > x_0$
Mean	Does not exist	$\frac{\alpha x_0}{\alpha-1}$, if $\alpha > 1$
Variance	Does not exist	$\frac{\alpha x_0^2}{(\alpha-1)^2(\alpha-2)}$, if $\alpha > 2$
m.g.f.	Not finite for $t \neq 0$	Not finite for $t > 0$