Chapter 32
Special Theory of Relativity

- Galilean-Newtonian Relativity
- *Michelson-Morley Experiment
- Postulates of the Special Theory of Relativity
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- Time Dilation
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- Lorentz Transformation
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At the end of the 19^{th} century, two puzzles that can not be explained by already known principles:

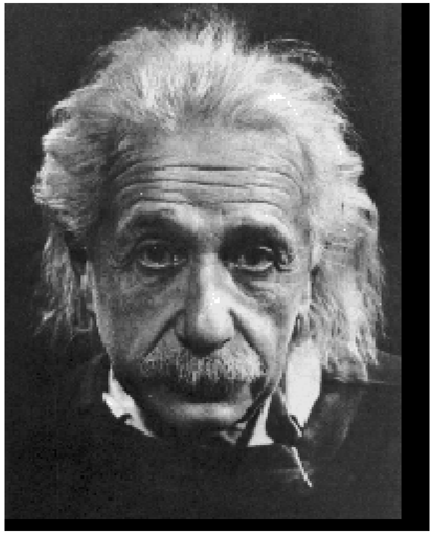
- the relative motion of the ether with respect to massive objects;
- Blackbody radiation.

These puzzles were solved by the introduction of two revolutionary new theories that changed our whole conception of nature:

- The special theory of relativity;
- Quantum theory.

Classical Physics: Physics already known at the end of 19^{th} century;

Modern Physics: New physics growing out of the great revolution at the turn of the 20^{th} century.



Albert Einstein
(1879 - 1955)

Albert Einstein: Annus Mirabilis 1905

- **Photoelectric effect**
- **Brownian motion**
- **Special theory of relativity**
- **Mass-energy equivalence**

Special theory of relativity:

- Inertial reference frames;
- Relationships between Space, Time and Motion.

§ 31-1 Galilean-Newtonian Relativity

1. Relativity principle

The basic laws of mechanics are the same in all inertia reference frames.

or: **All inertial reference frames are equivalent for the description of mechanical phenomena.**

Which means:

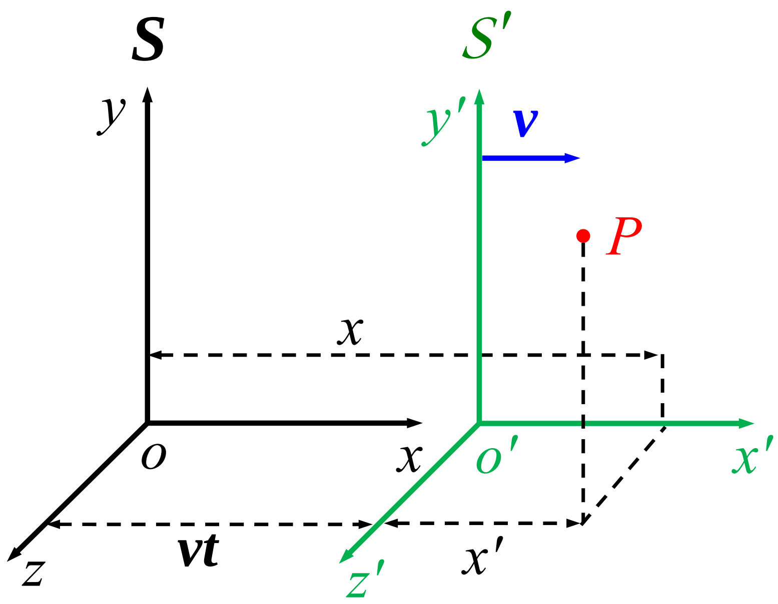
- No one inertial frame is special in any sense.
- There is no experiment to tell which frame is "really" at rest and which is moving.

1. Two inertial reference frames *S* and *S*’,

2. *S*’ moves in the *x* direction at constant speed *v* with respect to *S*;

3. The *x*' and *x* axes overlap one another;

4. The origins *O* and *O*' of the two frames are superimposed at time *t*=0.



For an object *P*, the relations between its motions in frames *S* and *S*’ are (Galilean transformation):

$$\begin{cases} x' = x - vt \\ y' = y \\ z' = z \\ t' = t \end{cases} \qquad \begin{cases} \upsilon_x' = \upsilon_x - v \\ \upsilon_y' = \upsilon_y \\ \upsilon_z' = \upsilon_z \end{cases} \qquad \begin{cases} a_x' = a_x \\ a_y' = a_y \\ a_z' = a_z \end{cases}$$

Mechanical laws have the same math form in *S* and *S*’:

$$\vec{F} = m\vec{a} = m\frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2}$$

$$\vec{F}' = m\vec{a}' = m\frac{\mathrm{d}^2\vec{r}'}{\mathrm{d}t^2}$$

Galilean- Newtonian relativity:

- The lengths of objects are the same in one reference frame as in another;
- Time passes at the same rate in different reference frames.
- Space and time intervals are considered to be absolute: their measurement does not change from one reference frame to another;
- The mass of an object, as well as all forces, are assumed to be unchanged by a change in inertial reference frame.

2. The issues encountered by Galilean transformation

(1) Maxwell's equations are not invariant under Galilean transformations

$$\text{Frame } S: \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \text{Frame } S': \quad \nabla' \times \vec{E} = -\frac{\partial \vec{B}}{\partial t'} + \left(\vec{u} \cdot \nabla'\right) \vec{B}$$

Is the principle of relativity invalid when applying to electromagnetic laws?

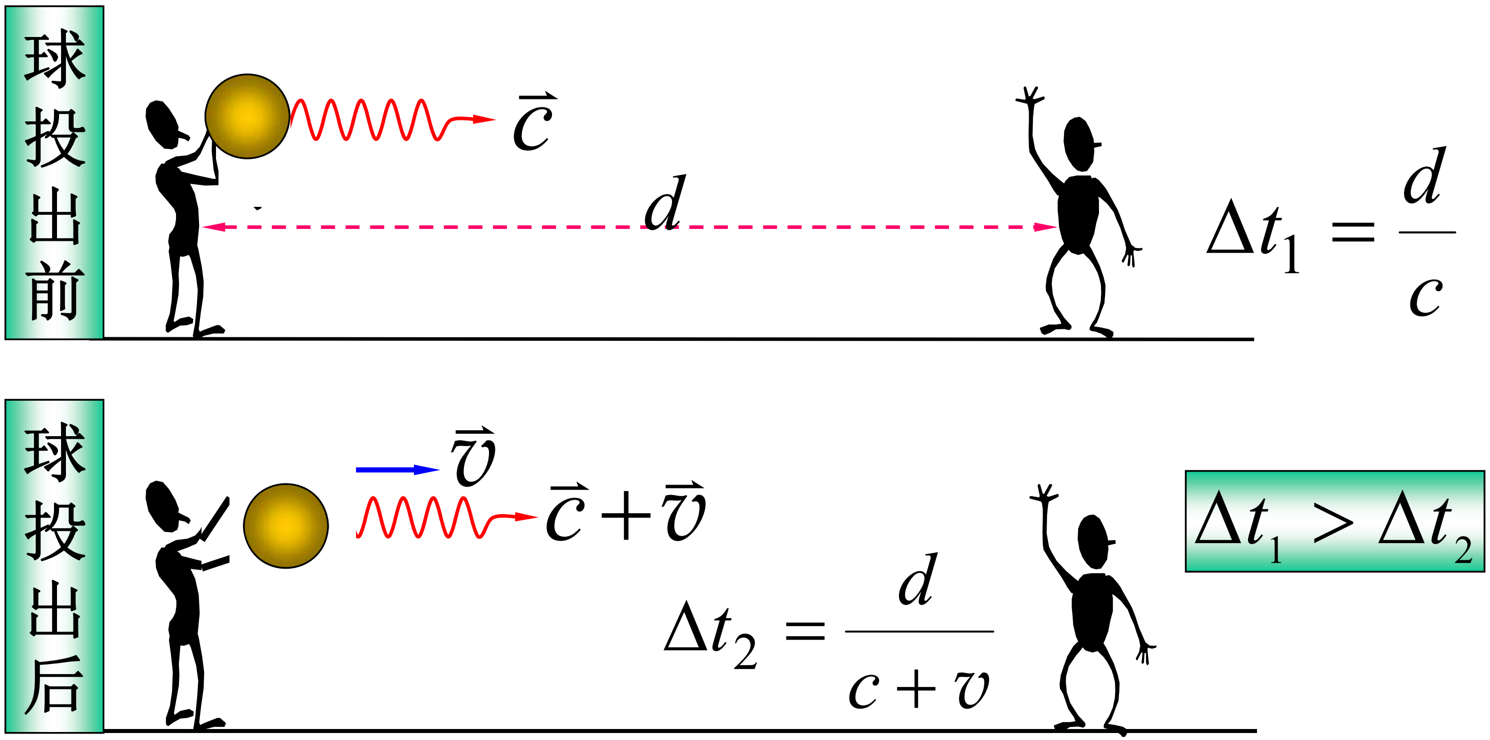
(2) Reference frame not specified for the speed of light

Maxwell's equations tell us the speed of light is:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \approx 3 \times 10^8 \text{ m/s}$$

But Maxwell's equations do not specify a reference frame.

(3) Violation of the principle of causality



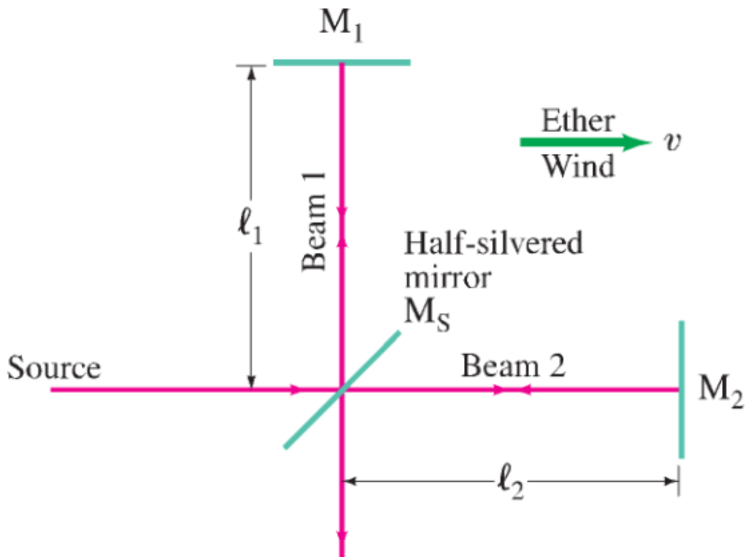
Ridiculous result: The observer sees the post-thrown ball before the pre-thrown ball.

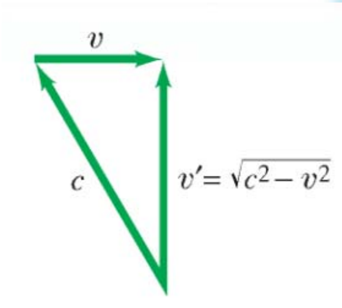
To avoid contradicting Galilean transformations, nineteenth-century physicists assumed:

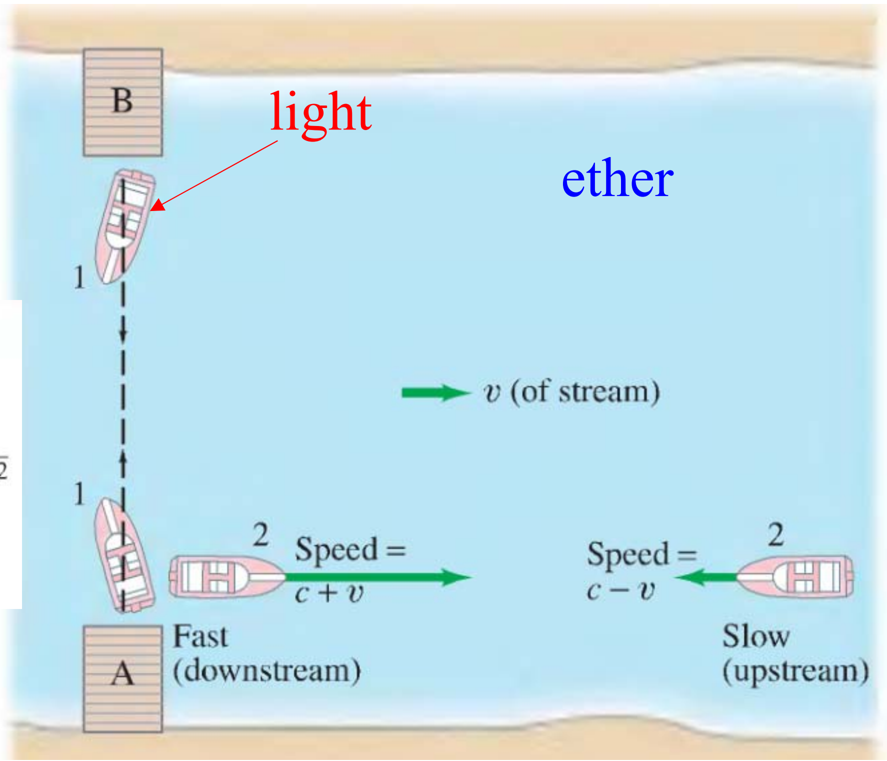
- Universe was filled with a substance called 'ether' (① has no mass; ② completely transparent; ③ offers no resistance to moving objects; ④ highly rigid material);
- Electromagnetic waves propagate through ether;
- Ether was chosen as the absolute stationary reference frame;
- The equations of the electromagnetic field only hold in the ether reference frame;
- The speed of electromagnetic waves is c in all directions within the ether reference frame.

§ 31-2 *Michelson-Morley Experiment

The Michelson-Morley experiment was designed to measure the speed of the ether.







$$\text{Beam 1:}\quad \Delta t_1 = \frac{2l_1}{v'} = \frac{2l_1}{\sqrt{c^2 - v^2}} \qquad \text{Beam 2:}\quad \Delta t_2 = \frac{l_2}{c+v} + \frac{l_2}{c-v}\Delta t = \frac{2l_2}{c(1 - v^2/c^2)}$$

Beam 2 will lag behind beam 1 by an amount:

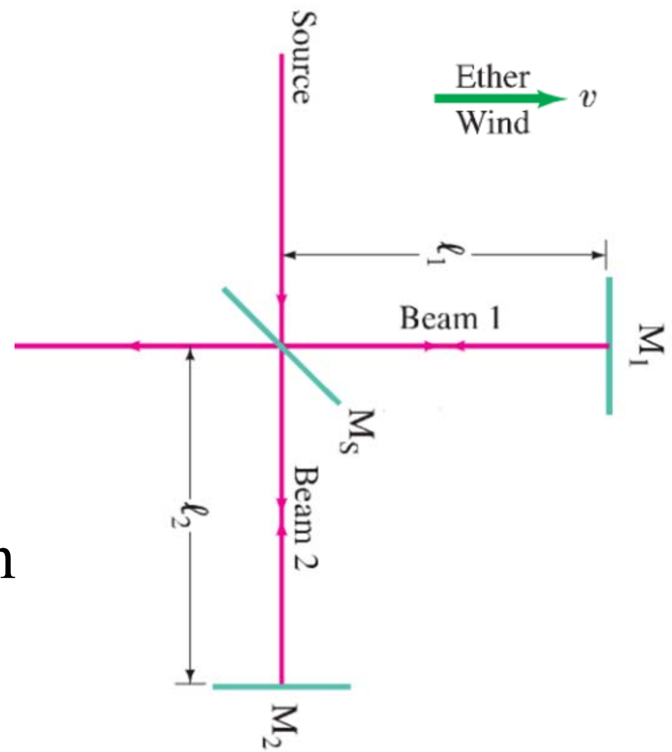
$$\Delta t = \Delta t_2 - \Delta t_1 = \frac{2l_2}{c(1-v^2/c^2)} - \frac{2l_1}{c\sqrt{1-v^2/c^2}}$$

Rotating the apparatus by 90°:

$$\Delta t' = \Delta t_2' - \Delta t_1' = \frac{2l_2}{c\sqrt{1-v^2/c^2}} - \frac{2l_1}{c(1-v^2/c^2)}$$

The number of fringes of the interference pattern will move is:

$$N = \frac{c(\Delta t - \Delta t')}{\lambda} = \frac{2(l_1 + l_2)}{\lambda}\left(\frac{1}{(1-v^2/c^2)} - \frac{1}{\sqrt{1-v^2/c^2}}\right)$$



They made observations day and night, and observed none significant fringe shift.

§ 31-3 Postulates of the Special Theory of Relativity

1. First postulate (the relativity principle):

The laws of physics have the same form in all inertial reference frames.

2. Second postulate (constancy of the speed of light):

Light propagates through empty space with a definite speed *c* independent of the speed of the source or observer.

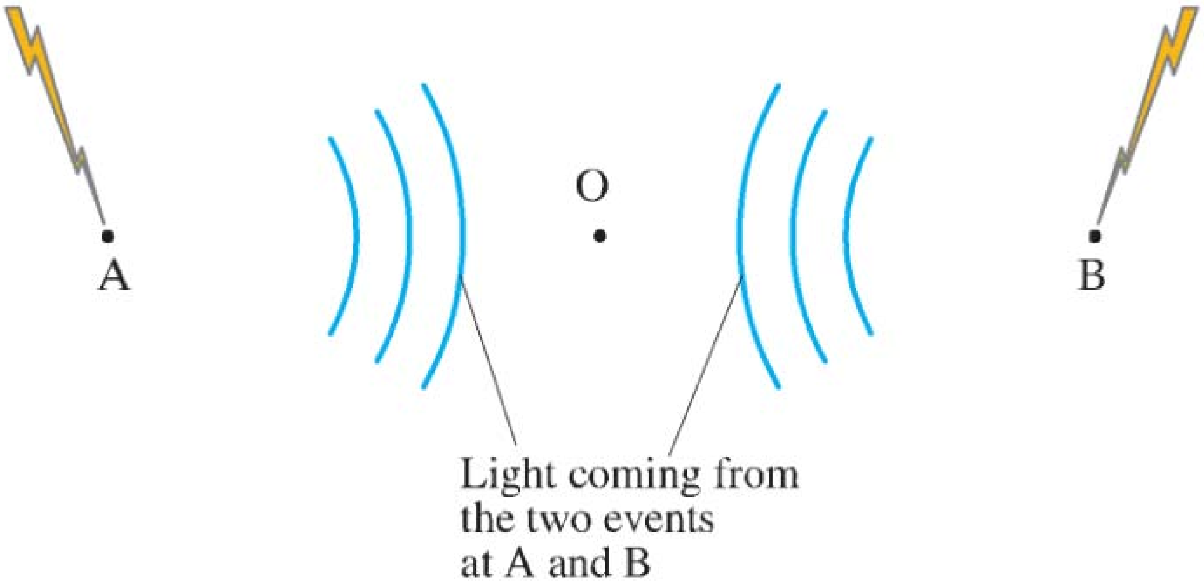
Note: These two postulates form the foundation of Einstein's special theory of relativity.

§ 31-4 Simultaneity

Event (x, y, z, t): something that happens at a particular place and at a particular time.

Events A and B:

A moment that lighting strikes at points A and B.



Observer O:

"Sees" the events when the light pulse sent by the events reaches O.

Events A_1A_2 and B_1B_2:

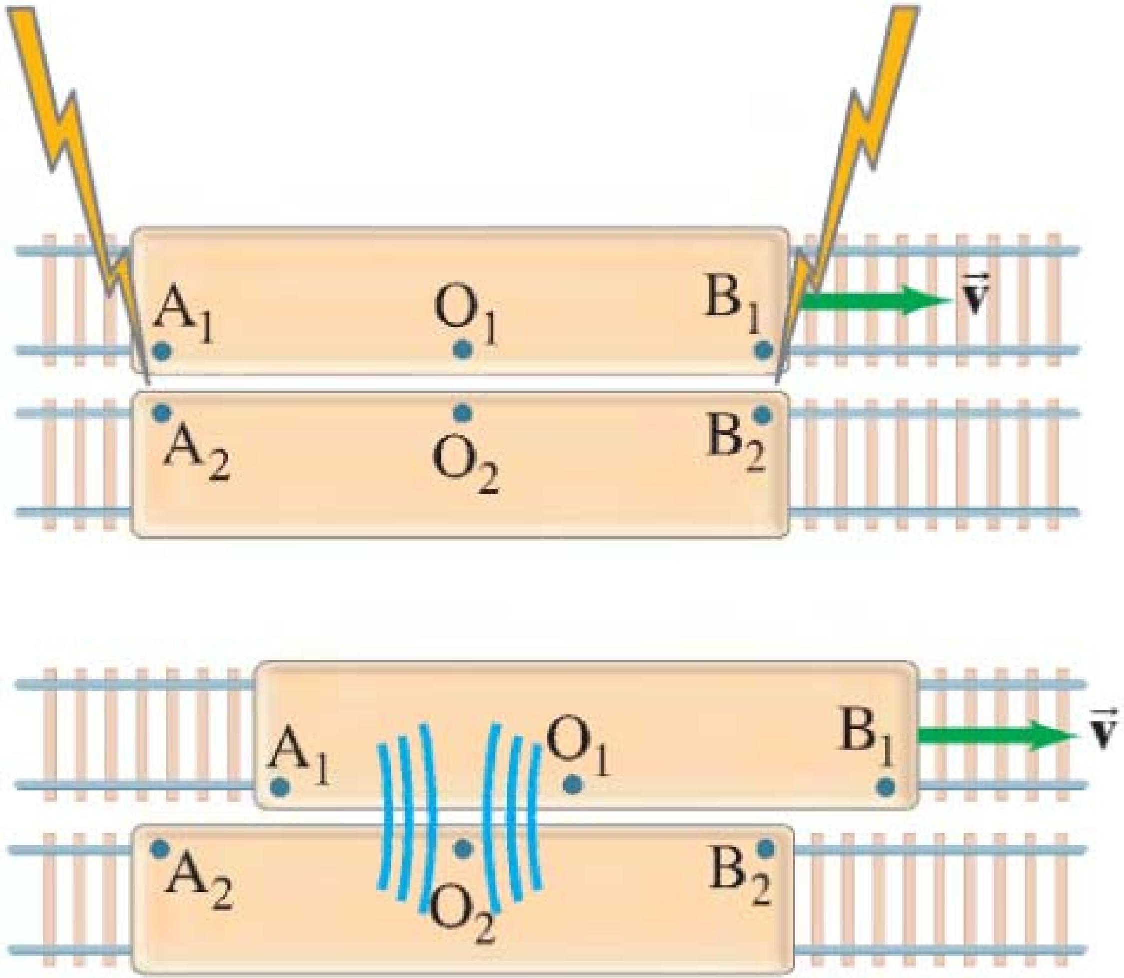
A moment that lighting strikes at points A_1, A_2 and B_1, B_2.

Observer O_1:

The two events are not simultaneous; event B_1B_2 precedes event A_1A_2.

Observer O_2:

The two events are simultaneous.



Conclusion:

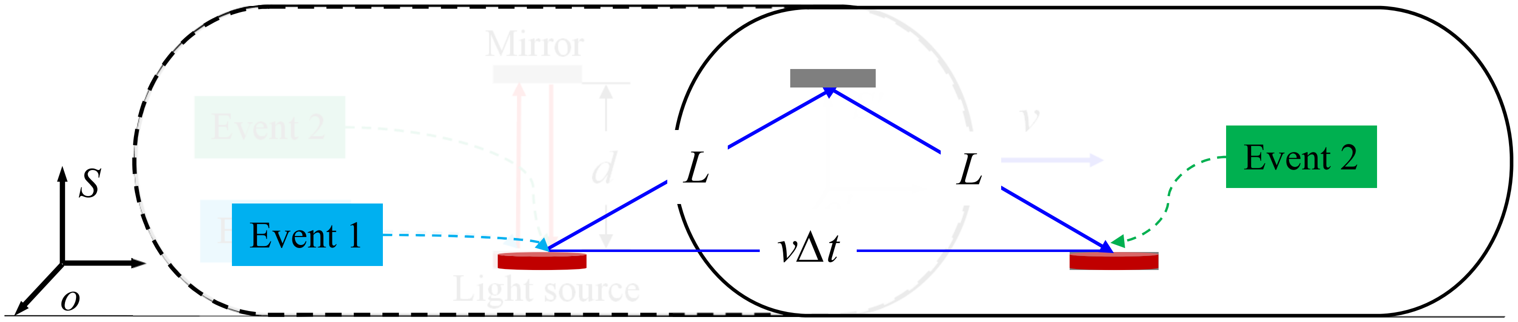
- Simultaneity of events depends on the observer;
- Simultaneity is not an absolute concept, but is relative;
- Time is no longer an absolute quantity, but rather related to motion.

You may ask: Which observer is right, O_1 or O_2 ?

Answer: according to relativity, they are both right. There is no "best" reference frame we can choose to determine which observer is right.

Give up commonsense notions about time!

§ 31-5 Time Dilation



1. Observer in S′

$$\Delta t_0 = \frac{2d}{c}$$

Event 1 and event 2 occurred at the same place.

2. Observer in S

$$\Delta t = \frac{2L}{c} = \frac{2\sqrt{d^2 + \left(v\Delta t/2\right)^2}}{c} \quad \Rightarrow \quad \Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

3. Time dilation

$$\boxed{\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2 / c^2}} = \frac{\Delta t_0}{\sqrt{1 - \beta^2}} = \gamma \Delta t_0}$$

Time dilation: Clocks moving relative to an observer are measured by that observer to run more slowly (as compared to clocks at rest).

1) Δt_0 is called Proper time (events occur at the same place);

2) Relativity factor $\gamma = 1/\sqrt{1 - v^2 / c^2}$

3) Relativistic effect & universality;

4) Space travel & Twin paradox.

*双生子佯谬(Twin paradox)

一对孪生兄弟 “明明”和“亮亮”，在他们20岁生日的时候，明明坐宇宙飞船去作一次星际旅游，飞船一去一回作匀速直线运动，速度为0.9998*c*。明明在天上过了一年，回到地球时，亮亮已多大年龄？如果亮亮在地球上过了一年，返回来的明明多大年龄？

（1） 对S′系(飞船)，固有时间： $\tau_0 = 1$年

对S系(地球)，膨胀时间： $\tau = \dfrac{\tau_0}{\sqrt{1-\beta^2}} = \dfrac{1\text{年}}{\sqrt{1-0.9998}} = 51\text{年}$ 亮亮71岁

（2） 对S系(地球)，固有时间： $\tau_0 = 1$年

对S′系(飞船)，膨胀时间： $\tau = \dfrac{\tau_0}{\sqrt{1-\beta^2}} = \dfrac{1\text{年}}{\sqrt{1-0.9998}} = 51\text{年}$ 明明71岁

?

这就是双生子佯谬，明明和亮亮到底是谁年轻呢？有些人用这来攻击相对论。其实不是相对论有问题，是人们不恰当地应用了相对论。相对论只适用于惯性系，飞船一去一回要加速和减速，不是惯性系，因此飞船上的明明得到的结论是不正确的。整个过程中地球是个惯性系（或近似惯性系），所以地球上亮亮的结论是正确的，因此，只有地球上亮亮的观点可信——亮亮年老的结论是正确的。所以不存在佯谬。

Lifetime of a moving muon：

Example 1: The mean lifetime of a muon at rest is 2.2×10^{-6}s, and it is traveling at v=0.6c relative to the lab. (a) What is the mean lifetime measured in the lab? (b) How far does it travel before decaying?

Solution: (a) Mean lifetime at rest: proper time

$$\Delta t=\frac{\Delta t_0}{\sqrt{1-v^2/c^2}}=\frac{5}{4}\Delta t_0=2.75\times10^{-6}\,\mathrm{s}$$

(b) Distance travel before decaying:

$$d=v\Delta t=\left(1.8\times10^{8}\right)\left(2.75\times10^{-6}\right)=495\mathrm{m}$$

Time dilation of space station:

Example 2: A space station moves around the Earth with v=7700m/s. A spaceman stays in the station for 100 days, how much younger will he become?

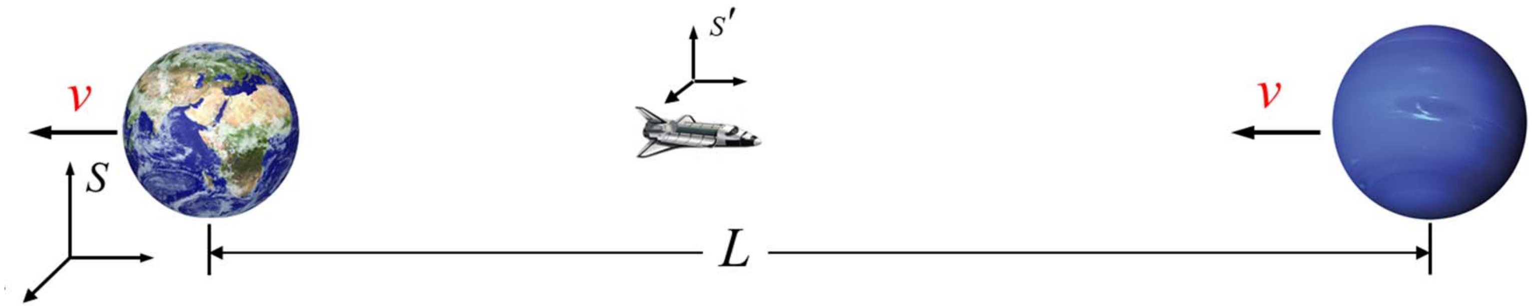
Solution: Time dilation:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2 / c^2}} = \Delta t_0 \cdot \frac{1}{\sqrt{1 - (2.57 \times 10^{-5})^2}} = \Delta t_0 \times \left(1 + 3.3 \times 10^{-9}\right)$$

$$\Rightarrow \quad \Delta t - \Delta t_0 = 3.3 \times 10^{-9} \times \Delta t_0 = 0.029\text{s}$$

Ignorable for human Correction of GPS time

§ 31-6 Length Contraction



1. Observer in S

$$\Delta t = L_0 / v$$

2. Observer in S′

Event 1 and event 2 occurred at the same place (spacecraft), the time interval is proper time:

$$\Delta t_0 = \Delta t\sqrt{1-v^2/c^2} = \frac{L}{v} \qquad \Rightarrow \qquad L = L_0\sqrt{1-v^2/c^2}$$

3. Length contraction

$$\boxed{L = L_0\sqrt{1 - v^2 / c^2} = L_0 / \gamma}$$

The length of an object is measured to be shorter when it is moving relative to the observer than when it is at rest.

1) L_0 is called Proper length (measured at rest with respect to the object);

2) Not noticeable in everyday life;

3) Relativistic effect & universality;

4) Occurs only along the direction of motion.

Painting's contraction:

Example 3: A painting (1.5m×1.0m) is hanging on a spaceship with v=0.9c relative to Earth. What are the dimensions as seen (a) in spaceship; (b) on Earth?

1.0m

1.5m

$0.9c$

Solution: (a) In spaceship

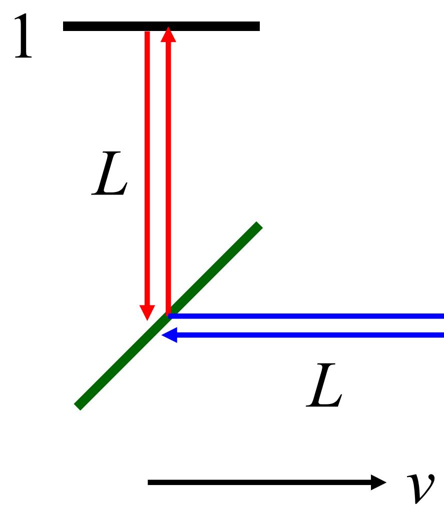
Looks perfectly normal!

(b) On Earth:

$$L = L_0 \sqrt{1 - v^2 / c^2} = 1.5 \times \sqrt{1 - 0.9^2} = 0.65\text{m}$$

So it has dimensions 0.65m×1.0m.

*Lorentz contraction:



$$t_1 = \frac{2L/c}{\sqrt{1-v^2/c^2}}$$

$$t_2 = \frac{L\sqrt{1-v^2/c^2}}{c-v} + \frac{L\sqrt{1-v^2/c^2}}{c+v}$$

$$= \frac{2cL\sqrt{1-v^2/c^2}}{c^2-v^2} = \frac{2L/c}{\sqrt{1-v^2/c^2}}$$

No time difference for any observer!

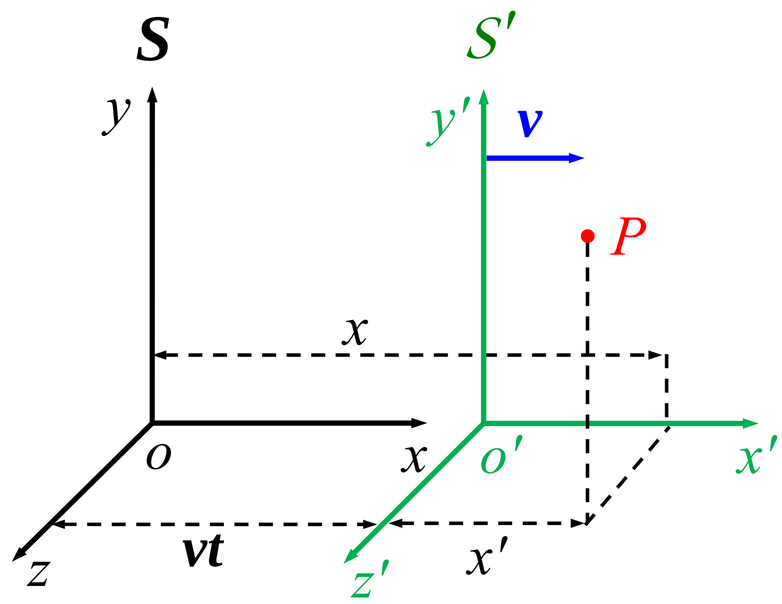
Lorentz contraction $\Rightarrow$ Lorentz transformation

§ 31-7 Lorentz Transformation

See textbook for the derivation of Lorentz transformation.

$S' \Leftarrow S$ $\qquad\qquad$ $S \Leftarrow S' \ (v \to -v)$

$$\begin{cases} x' = \dfrac{x - vt}{\sqrt{1 - \dfrac{v^2}{c^2}}} \\ y' = y \\ z' = z \\ t' = \dfrac{t - \dfrac{v}{c^2}x}{\sqrt{1 - \dfrac{v^2}{c^2}}} \end{cases} \qquad \begin{cases} x = \dfrac{x' + vt'}{\sqrt{1 - \dfrac{v^2}{c^2}}} \\ y = y' \\ z = z' \\ t = \dfrac{t' + \dfrac{v}{c^2}x'}{\sqrt{1 - \dfrac{v^2}{c^2}}} \end{cases}$$



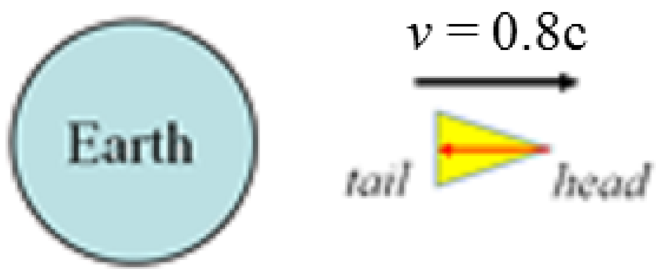
1) Space-time relationship in relativity;

2) $v << c \to$ Galilean transformation;

3) Space and time are relative, 4-D vectors.

Thinking Question:

A spaceship (proper length L_0=90m) is moving relative to the Earth with v = 0.8c. A light signal travels from the head of the ship to the tail of the ship. What is the distance traveled of the light signal measured by an observer on the Earth?



Lorentz velocity transformation:

$$\begin{cases} \mathrm{d}x' = \gamma(\mathrm{d}x' - v\mathrm{d}t) \\ \mathrm{d}y' = \mathrm{d}y \\ \mathrm{d}z' = \mathrm{d}z \\ \mathrm{d}t' = \gamma(\mathrm{d}t - v\mathrm{d}x / c^2) \end{cases}$$

In S: $u_x = \dfrac{\mathrm{d}x}{\mathrm{d}t}, \quad u_y = \dfrac{\mathrm{d}y}{\mathrm{d}t}, \quad u_z = \dfrac{\mathrm{d}z}{\mathrm{d}t}$

In S': $u'_x = \dfrac{\mathrm{d}x'}{\mathrm{d}t'}, \quad u'_y = \dfrac{\mathrm{d}y'}{\mathrm{d}t'}, \quad u'_z = \dfrac{\mathrm{d}z'}{\mathrm{d}t'}$

$$\frac{\mathrm{d}x'}{\mathrm{d}t'} = \frac{\mathrm{d}x - v\mathrm{d}t}{\mathrm{d}t - v\mathrm{d}x / c^2} = \frac{\dfrac{\mathrm{d}x}{\mathrm{d}t} - v}{1 - \dfrac{v}{c^2}\dfrac{\mathrm{d}x}{\mathrm{d}t}} \quad \Rightarrow \quad u'_x = \frac{u_x - v}{1 - u_x v / c^2}$$

$$\frac{\mathrm{d}y'}{\mathrm{d}t'} = \frac{\mathrm{d}y}{\gamma(\mathrm{d}t - v\mathrm{d}x / c^2)} = \frac{\mathrm{d}y/\mathrm{d}t}{\gamma\left(1 - \dfrac{v}{c^2}\dfrac{\mathrm{d}x}{\mathrm{d}t}\right)} \quad \Rightarrow \quad u'_y = \frac{u_y/\gamma}{1 - u_x v / c^2} \qquad u'_z = \frac{u_z/\gamma}{1 - u_x v / c^2}$$

$S' \Leftarrow S$

$$\begin{cases} u'_x = \dfrac{u_x - v}{1 - \dfrac{u_x v}{c^2}} \\ u'_y = \dfrac{u_y/\gamma}{1 - \dfrac{u_x v}{c^2}} \\ u'_z = \dfrac{u_z/\gamma}{1 - \dfrac{u_x v}{c^2}} \end{cases}$$

$S \Leftarrow S' \; (v \to -v)$

$$\begin{cases} u_x = \dfrac{u'_x + v}{1 + \dfrac{u_x v}{c^2}} \\ u_y = \dfrac{u'_y/\gamma}{1 + \dfrac{u_x v}{c^2}} \\ u_z = \dfrac{u'_z/\gamma}{1 + \dfrac{u_x v}{c^2}} \end{cases}$$

(1) y, z components are also affected by v;

(2) Light speed is independent of observer;

$$u'_x = c \quad \Rightarrow \quad u_x = c$$

(3) c is the ultimate speed;

(4) $u,\ v << c \to$ Classical

$$\Rightarrow \begin{cases} u'_x = u_x - v \\ u'_y = u_y \\ u'_z = u_z \end{cases}$$

The ultimate speed:

Example 4: Two spaceships move with same speed 0.9c relative to Earth, but in opposite direction. What is the speed of one ship relative to the other?

Solution:

$$u_x = \frac{u'_x + v}{1 + u'_x v / c^2} = \frac{0.9c + 0.9c}{1 + \frac{0.9c \times 0.9c}{c^2}} = 0.994c$$

Example 5: Show that the speed of light is always c in two different frames with relative speed v.

Solution:

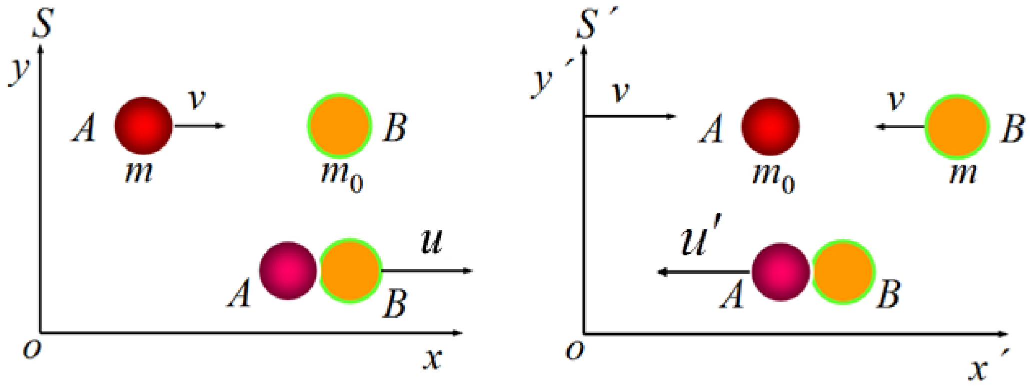
$$u_x = \frac{u'_x + v}{1 + u'_x v / c^2} = \frac{c + v}{1 + v / c} = c$$

How about light traveling in other direction?

§ 31-8 Relativistic Momentum

1. Relativistic mass

Two identical balls A and B made inelastic collision with relative speed v.



S (B before collision):

Ball A: v, m; **ball B**: 0, m_0; **after collision**: u, $m+m_0$. **m_0: rest mass**

Conservation of momentum: $mv = (m+m_0)u$ (1)

S' (A before collision):

Ball A: 0, m_0; **ball B**: $-v$, m; **after collision**: u', $m+m_0$.

Conservation of momentum: $-mv = (m+m_0)u'$ (2)

Lorentz velocity transformation:

$$\left.\begin{aligned} u' &= \frac{u-v}{1-uv/c^2} && (3) \\ mv &= (m+m_0)u && (1) \\ -mv &= (m+m_0)u' && (2) \end{aligned}\right\} \Rightarrow \boxed{m = \frac{m_0}{\sqrt{1-v^2/c^2}} = \gamma m_0}$$

***m*: relativistic mass**

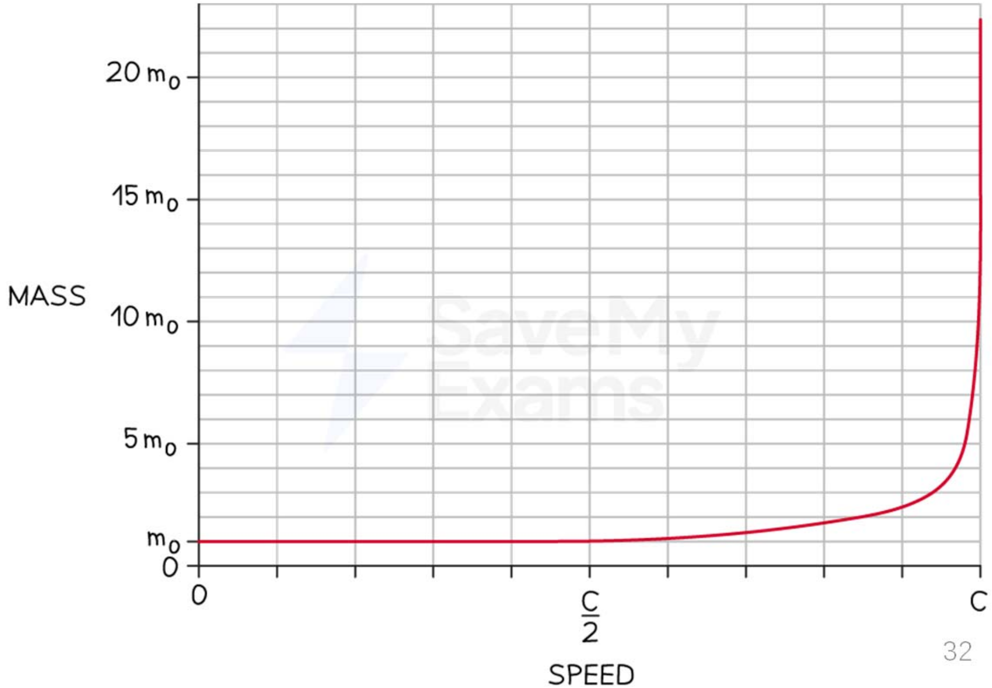
(1) $v \ll c$, $m = m_0$;

(2) $v = c$ (photon:光子), $m_0 = 0$;

(3) For a mass object,

$v \to c,\ m \to \infty,\ a = F/m \to 0,$

No further acceleration, c is the ultimate speed of mass objects.



2. Relativistic dynamics

$$\boxed{m = \frac{m_0}{\sqrt{1 - v^2 / c^2}} = \gamma m_0} \qquad \boxed{\vec{p} = m\vec{v} = \frac{m_0 \vec{v}}{\sqrt{1 - v^2 / c^2}}}$$

Newton's second law in relativity:

$$\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(m\vec{v}) = m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} + \vec{v}\frac{\mathrm{d}m}{\mathrm{d}t}$$

Equation $\vec{F} = m\vec{a}$ is not valid in relativity.

Ultimate speed *c*:

Mass object: (1) $m \to \infty$; (2) $p \to \infty$; (3) $F = \mathrm{d}p/\mathrm{d}t \to 0$.

§ 31-9 Kinetic energy, Mass-Energy Equation $E=mc^2$

1. Kinetic energy

Assume work-energy principle is still valid. Work done to accelerate a particle from rest:

$$E_k = \int_0^{\upsilon} \vec{F}\cdot \mathrm{d}\vec{r} = \int_0^{\upsilon} \frac{\mathrm{d}(m\vec{\upsilon})}{\mathrm{d}t}\cdot \mathrm{d}\vec{r} = \int_0^{\upsilon} \vec{\upsilon}\cdot \mathrm{d}(m\vec{\upsilon})$$

$$\because \quad \vec{\upsilon}\cdot \mathrm{d}(m\vec{\upsilon}) = \upsilon^2 \mathrm{d}m + m\vec{\upsilon}\cdot \mathrm{d}\vec{\upsilon} = \upsilon^2 \mathrm{d}m + m\upsilon \mathrm{d}\upsilon$$

$$m = \frac{m_o}{\sqrt{1-\upsilon^2/c^2}} \quad \Rightarrow \quad \mathrm{d}m = \frac{m_o \upsilon \mathrm{d}\upsilon}{c^2(1-\upsilon^2/c^2)^{3/2}} = \frac{m\upsilon \mathrm{d}\upsilon}{c^2-\upsilon^2}$$

$$\boxed{E_k = \int_{m_o}^{m} c^2 \mathrm{d}m = mc^2 - m_0 c^2}$$

Relativistic kinetic energy

2. Mass-energy equation

Kinetic energy: $E_k = mc^2 - m_0c^2$

Rest Energy: $E_0 = m_0c^2$

Total Energy: $\boxed{E = E_0 + E_k = mc^2}$ —— Mass-energy equation

In a system, when the mass changes by an amount Δm, the energy of the system changes by an amount:

$$\Delta E = c^2 \Delta m$$

- It is one of the most famous equations in physics;
- It relates the concepts of energy and mass;
- It is confirmed in nuclear/particle experiments.

Nuclear reactions:

Example: In a nuclear fission reaction:

$$n + U^{235} \rightarrow B_a^{141} + K_r^{92} + 3n$$

1mol: 236.133g 235.918g

Mass decrease: Δm=236.133g-235.918g=0.215g

Energy released: $\Delta E = c^2 \Delta m = 1.93 \times 10^{13}$J

$= 5.37 \times 10^6$ KWH=4600T TNT-equivalent

Comparing with chemical reactions

3. Relationship of quantities

$$E_k = mc^2 - m_0c^2 = m_0c^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right) = \frac{1}{2}m_0v^2 + \frac{3}{8}m_0\frac{v^4}{c^2} + \ldots$$

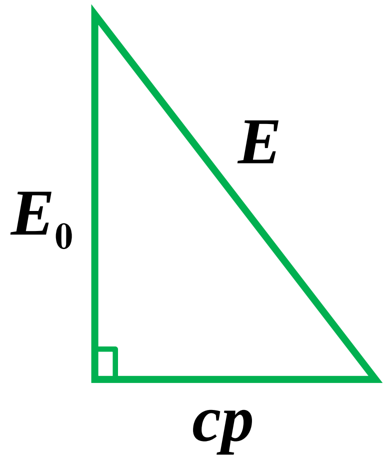
Only when $v \ll c$:

$$\left(\frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{1\cdot 3}{2\cdot 4}x^2 - \ldots\right)$$

$$E_k = \frac{1}{2}m_0v^2$$

$$\left.\begin{aligned} E &= mc^2 = \frac{m_0c^2}{\sqrt{1-v^2/c^2}} \\ p &= \frac{m_0v}{\sqrt{1-v^2/c^2}} \end{aligned}\right\} \Rightarrow \boxed{E^2 - p^2c^2 = E_0^2}$$

Lorentz invariant



Photon: $m_0 = 0 \rightarrow E = pc$

High speed pion:

Example 6: A pion (m_0 =2.4×10^{-28} kg) travels at v=0.8c. What is its momentum and kinetic energy?

Solution: Relativity factor

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{5}{3}$$

Momentum: $p = \gamma m_0 v = 9.6\times 10^{-20}\,\mathrm{kg}\cdot\mathrm{m/s}$

Kinetic energy: $E_k = m_0 c^2(\gamma - 1) = 1.4\times 10^{-11}\,\mathrm{J}$

Comparing with classical kinetic energy:

$$E_k = \frac{1}{2} m_0 v^2 = 6.9\times 10^{-12}\,\mathrm{J}$$

High energy electron:

Example 7: Determine: (a) rest energy of an electron (m=9.11×10^{-31}kg, q= – e= –1.60×10^{-19}C); (b) speed of electron accelerated from rest by electric potential 20kV (teletube) or 5.0 MV (X-ray machine).

Solution: (a) $E_0 = m_0c^2 = 8.2\times10^{-14}J = 0.51MeV$

(b) Total energy: $E = \gamma m_0 c^2 = m_0 c^2 + qV$

$$V = 20\text{kV} \Rightarrow \gamma = 1.04 \Rightarrow v = \frac{c}{\gamma}\sqrt{\gamma^2 - 1} = 0.27c$$

$$V = 5\text{MV} \Rightarrow \gamma = 10.8 \Rightarrow v = \frac{c}{\gamma}\sqrt{\gamma^2 - 1} = 0.996c$$

Energy in collision:

Example 8: Two identical particles of rest mass m_0 move oppositely at equal v, then a completely inelastic collision occurs and results a single particle. What is the rest mass of the new particle?

Solution: Conservation of momentum

$$0=mv-mv=MV \rightarrow V=0$$

Conservation of energy:

$$2mc^2=\frac{2m_0c^2}{\sqrt{1-v^2/c^2}}=M_0c^2$$

$$\therefore \quad M_0=\frac{2m_0}{\sqrt{1-v^2/c^2}}$$

No energy loss!

Summary

1. Relativity principle

The basic laws of mechanics are the same in all inertia reference frames. or: **All inertial reference frames are equivalent for the description of mechanical phenomena.**

2. Galilean transformation

$$\begin{cases} x' = x - vt \\ y' = y \\ z' = z \\ t' = t \end{cases} \qquad \begin{cases} \upsilon'_x = \upsilon_x - v \\ \upsilon'_y = \upsilon_y \\ \upsilon'_z = \upsilon_z \end{cases} \qquad \begin{cases} a'_x = a_x \\ a'_y = a_y \\ a'_z = a_z \end{cases}$$

3. Michelson-Morley Experiment

Found no significant fringe shift, which approves that the absolute reference frame ‘ether’ does not exist.

4. Postulates of the Special Theory of Relativity

(1) The laws of physics have the same form in all inertial reference frames.

(2) Light propagates through empty space with a definite speed *c* independent of the speed of the source or observer.

5. Simultaneity

- Simultaneity of events depends on the observer;
- Simultaneity is not an absolute concept, but is relative;
- Time is no longer an absolute quantity, but rather related to motion.

6. Time dilation

$$\boxed{\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2 / c^2}} = \frac{\Delta t_0}{\sqrt{1 - \beta^2}} = \gamma \Delta t_0}$$

Δt_0 : Proper time (the two events occur at the same point in place)

7. Length contraction

$$\boxed{L = L_0\sqrt{1 - v^2/c^2} = L_0/\gamma}$$

L_0 : Proper length (measured at rest with respect to the object)

8. Lorentz transformation

$S' \Leftarrow S$

$$\begin{cases} x' = \dfrac{x - vt}{\sqrt{1-\dfrac{v^2}{c^2}}} \\ y' = y \\ z' = z \\ t' = \dfrac{t - \dfrac{v}{c^2}x}{\sqrt{1-\dfrac{v^2}{c^2}}} \end{cases}$$

$S \Leftarrow S' \ (v \to -v)$

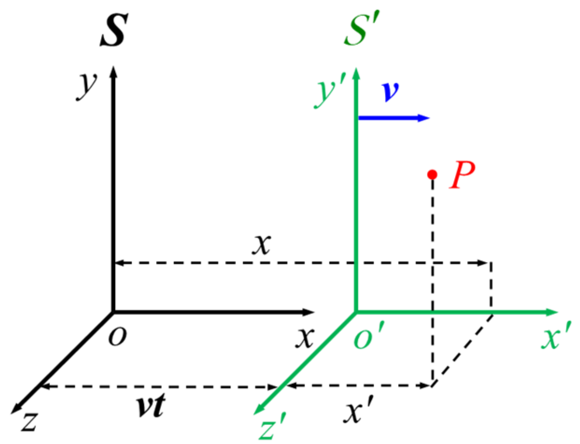
$$\begin{cases} x = \dfrac{x' + vt'}{\sqrt{1-\dfrac{v^2}{c^2}}} \\ y = y' \\ z = z' \\ t = \dfrac{t' + \dfrac{v}{c^2}x'}{\sqrt{1-\dfrac{v^2}{c^2}}} \end{cases}$$

$S' \Leftarrow S$

$$\begin{cases} u'_x = \dfrac{u_x - v}{1 - \dfrac{u_x v}{c^2}} \\ u'_y = \dfrac{u_y/\gamma}{1 - \dfrac{u_x v}{c^2}} \\ u'_z = \dfrac{u_z/\gamma}{1 - \dfrac{u_x v}{c^2}} \end{cases}$$

$S \Leftarrow S' \ (v \to -v)$

$$\begin{cases} u_x = \dfrac{u'_x + v}{1 + \dfrac{u_x v}{c^2}} \\ u_y = \dfrac{u'_y/\gamma}{1 + \dfrac{u_x v}{c^2}} \\ u_z = \dfrac{u'_z/\gamma}{1 + \dfrac{u_x v}{c^2}} \end{cases}$$



(1) $v << c \to$ Galilean transformation; (2) Light speed is independent of observer:

$$u'_x = c \quad \Rightarrow \quad u_x = c$$

9. Relativistic mass and momentum

$$\boxed{m = \frac{m_0}{\sqrt{1-v^2/c^2}} = \gamma m_0} \quad \boxed{\vec{p} = m\vec{v} = \frac{m_0\vec{v}}{\sqrt{1-v^2/c^2}}} \quad \boxed{\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}}$$

(1) v<<c, m=m_0; (2) v=c (photon:光子), m_0=0;

(3) For a mass object, $v\rightarrow c$, $m\rightarrow\infty$, $a=F/m\rightarrow 0$, no further acceleration, c is the ultimate speed of mass objects.

10. Relativistic energy

Kinetic energy: $E_k = mc^2 - m_0c^2$

Rest Energy: $E_0 = m_0c^2$

Total Energy: $\boxed{E = E_0 + E_k = mc^2}$

11. Relationship of quantities

$$E_k = mc^2 - m_0c^2 = m_0c^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right) = \frac{1}{2}m_0v^2 + \frac{3}{8}m_0\frac{v^4}{c^2} + \ldots$$

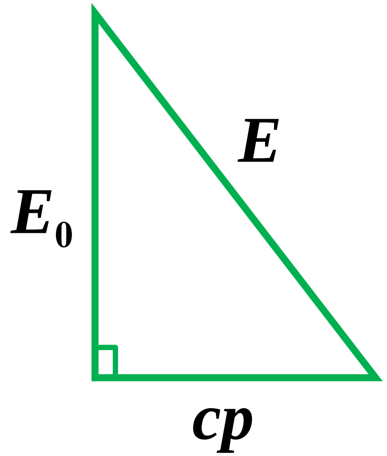
$$\left(\frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{1\cdot 3}{2\cdot 4}x^2 - \ldots\right)$$

Only when $v \ll c$:

$$E_k = \frac{1}{2}m_0v^2$$

Energy triangle:

$$\boxed{E^2 - p^2c^2 = E_0^2}$$



Photon: $m_0 = 0 \rightarrow E = pc$