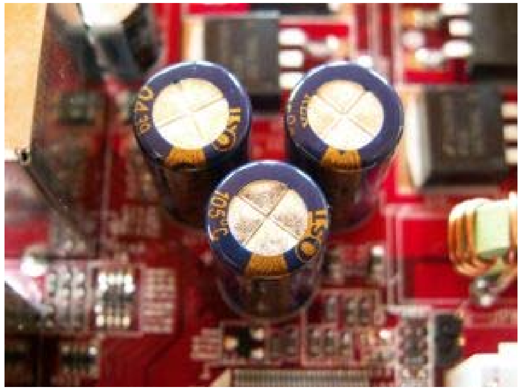
Chapter 22

Capacitance, Dielectrics, Electric Energy Storage

Capacitors

An element used to store charge and electric energy.

Commercial Capacitors

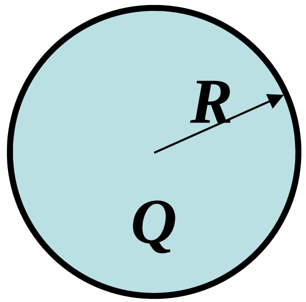


Capacitance depends on

Size, shape, relative position, insulating material

Capacitors & capacitance

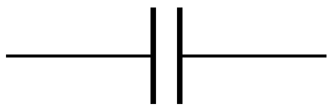
For a given capacitor: $Q \propto V$

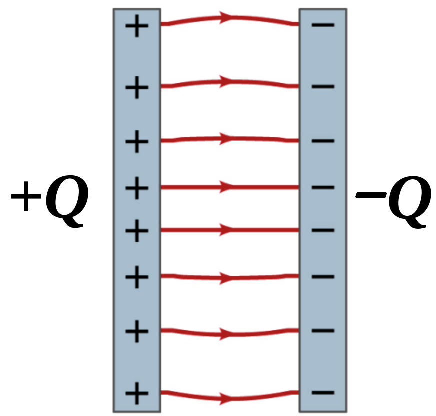


$$Q = CV \quad \text{or} \quad C = \frac{Q}{V}$$

***C*: Capacitance of capacitor**

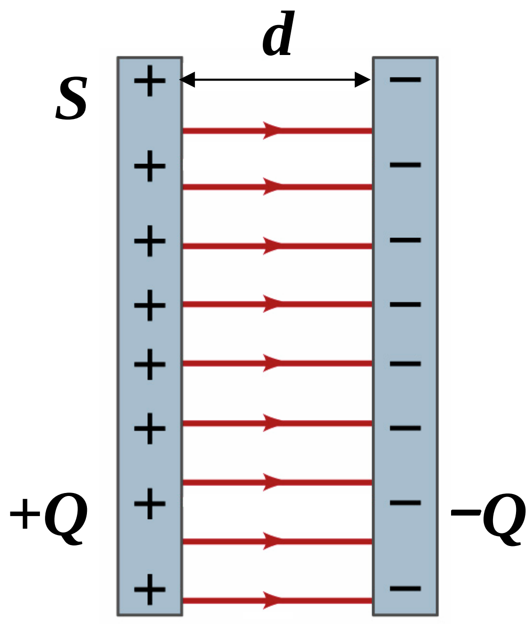
Unit: farad (F), μF & pF

Symbol in diagram: 



Determine capacitance

For a parallel-plate capacitor:



$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 S}$$

(ignoring edge effect)

$$V = Ed = \frac{\sigma d}{\varepsilon_0} = \frac{Qd}{\varepsilon_0 S}$$

$$\boxed{C = \frac{Q}{V} = \frac{\varepsilon_0 S}{d}}$$

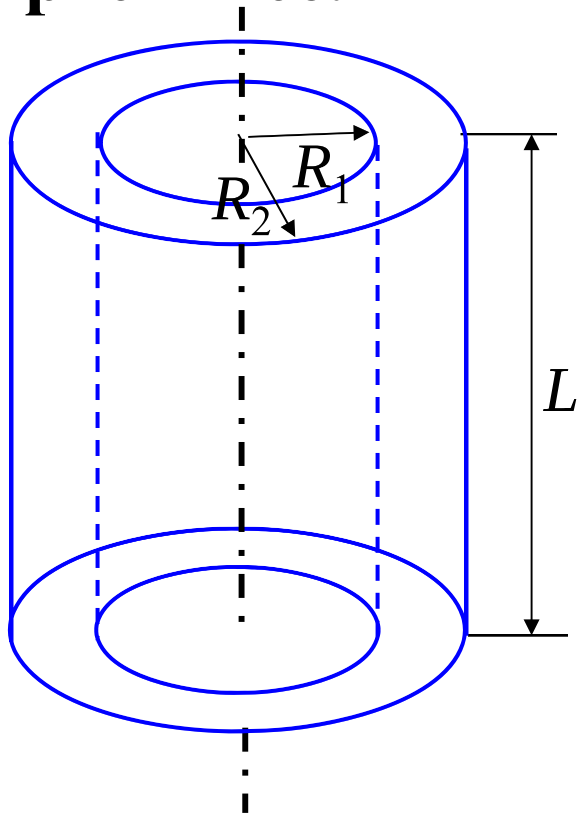
Cylindrical capacitor

Example1: A capacitor consists of two coaxial cylindrical shells (R_1, R_2), both with length $L>>R_2$. Calculate the capacitance.

$$E = \frac{\lambda}{2\pi\varepsilon_0 r},\ R_1 < r < R_2$$

$$V_{12} = \int_{R_1}^{R_2} E dr = \frac{\lambda}{2\pi\varepsilon_0} \ln\frac{R_2}{R_1}$$

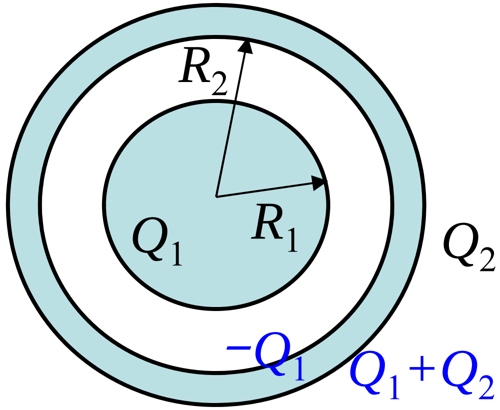
$$\boxed{C = \frac{Q}{V_{12}} = \frac{2\pi\varepsilon_0 L}{\ln(R_2/R_1)}}$$



Spherical capacitor

Question: **A capacitor consists of a sphere surrounded by a concentric spherical shell (R_1, R_2). Calculate the capacitance.**

$$E = \frac{Q_1}{4\pi\varepsilon_0 r^2}, \quad R_1 < r < R_2$$



$$V_{12} = \int_{R_1}^{R_2} E dr = \frac{Q_1}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\boxed{C = \frac{Q}{V_{12}} = \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1}}$$

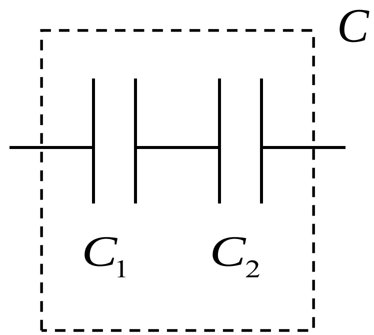
If $R_1 \gg R_2 - R_1$?

Capacitors in series and parallel

1) In series

$$Q_1 = Q_2 = Q, \qquad V = V_1 + V_2$$

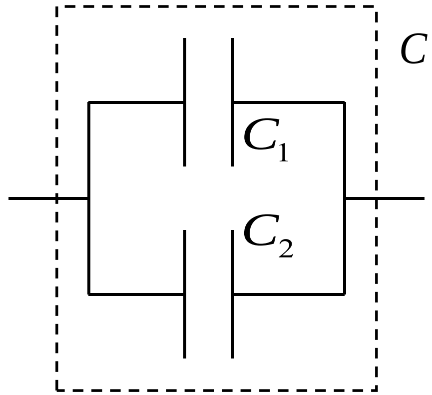
$$\boxed{\Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$$



2) In parallel

$$V_1 = V_2 = V, \qquad Q = Q_1 + Q_2$$

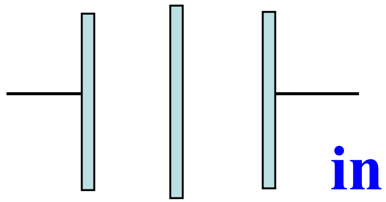
$$\boxed{\Rightarrow C = C_1 + C_2}$$



Capacitor combination

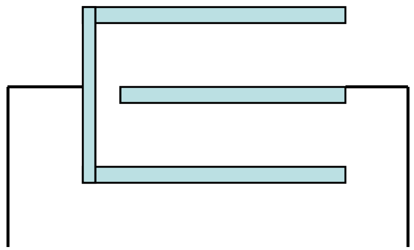
Example2: How to determine the total capacitance?

1)



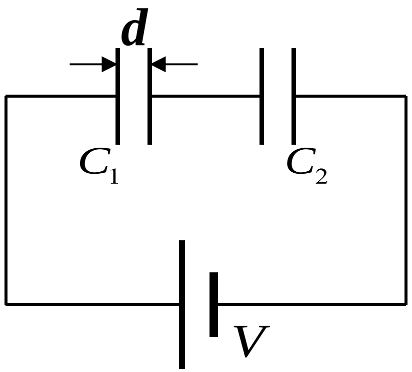
in series

2)



in parallel

Example3: How do *C*, *Q*, *V* change if $d \rightarrow 2d$?



C_1 □ $\Rightarrow C$ □ $\Rightarrow Q$ □

$\Rightarrow V_2$ □ $\Rightarrow V_1$ □

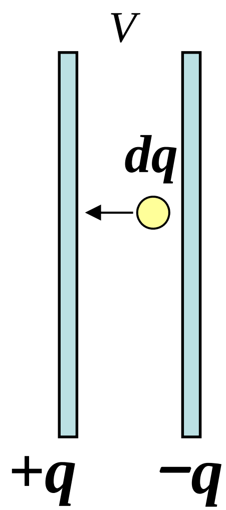
Electric energy storage

Charged capacitor stores electric energy

Work required to store charges:

$$dW = Vdq = \frac{q}{C}dq$$

$$W = \int \frac{q}{C}dq = \frac{1}{2}\frac{Q^2}{C}$$



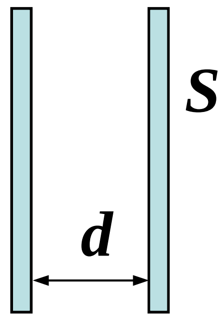
Energy stored: $$\boxed{U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV}$$

Energy in the field

How does the electric energy distribute?

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon_0 S}{d}(Ed)^2$$

$$= \frac{1}{2}\varepsilon_0 E^2 \cdot Sd$$



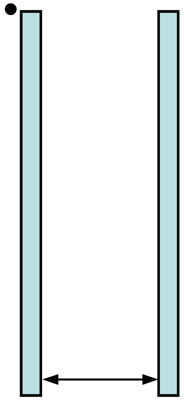
Energy per unit volume / energy density:

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$$

Electric energy is stored in the electric field!

Energy in capacitor

Example4: A parallel-plate capacitor is charged at 220V. (a) What is the capacitance; (b) how much electric energy can be stored?



Solution: (a) $$C = \frac{\varepsilon_0 S}{d}$$

$$= 8.85\times10^{-12}\,F$$

(b) Energy stored:

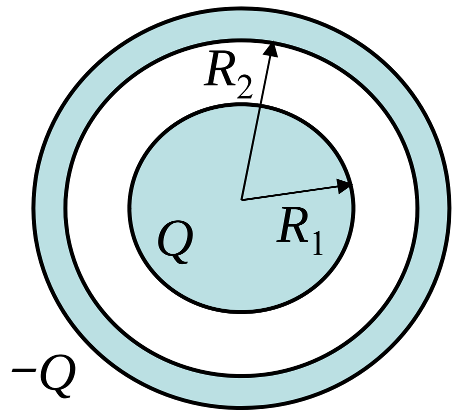
$$U = \frac{1}{2}CV^2 \qquad = 2.14\times10^{-7}\,J$$

Maximum U ?

Energy distribution

Example5: Determine the electric energy stored in a spherical capacitor.

Solution: Two common solutions



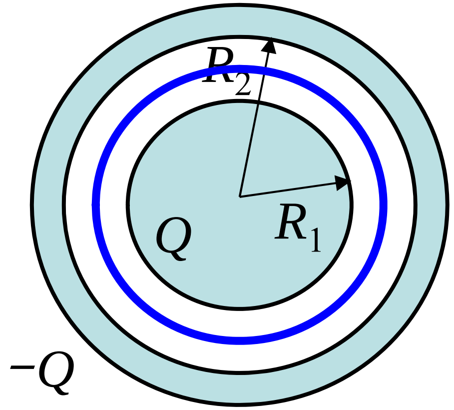
a) By using the capacitance:

$$C = \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1}$$

Energy stored: $$U = \frac{1}{2}\frac{Q^2}{C} = \frac{Q^2(R_2 - R_1)}{8\pi\varepsilon_0 R_1 R_2}$$

b) By using the energy density:

$$u = \frac{1}{2}\varepsilon_0 E^2 \qquad = \frac{1}{2}\varepsilon_0 \left(\frac{Q}{4\pi\varepsilon_0 r^2}\right)^2$$

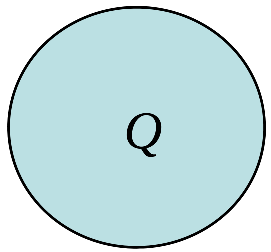


Total energy:

$$U = \int u dV \qquad = \int_{R_1}^{R_2} \frac{1}{2}\varepsilon_0 \left(\frac{Q}{4\pi\varepsilon_0 r^2}\right)^2 \cdot 4\pi r^2 dr$$

$$= \frac{Q^2}{8\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad = \frac{Q^2(R_2 - R_1)}{8\pi\varepsilon_0 R_1 R_2}$$

Conductor / nonconductor sphere?



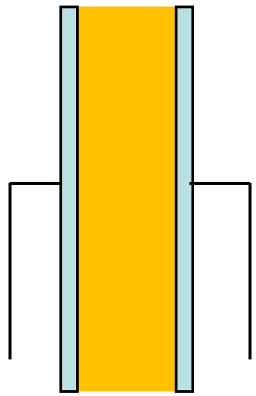
Dielectrics

Insulating material in capacitors → dielectric

① **Harder to break down → V ↗**

② **Distance between plates ↘**

③ **Increases the capacitance:**



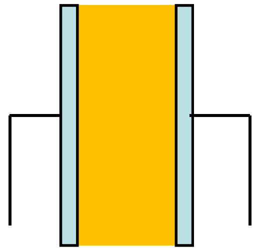
$$C = KC_0, \quad K \geq 1, \text{ called dielectric constant}$$

Also noted as ε_r : relative permittivity

Permittivity of material

Capacitor completely filled by a dielectric:

$$C = KC_0 = \frac{K\varepsilon_0 S}{d} = \frac{\varepsilon S}{d}$$



ε **: the permittivity of material**

In a specific region filled with dielectric:

ε_0 **in electric expressions** $\rightarrow$ $\varepsilon = K\varepsilon_0$

$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 S} \Rightarrow E = \frac{\sigma}{\varepsilon} = \frac{Q}{\varepsilon S}$$

Electrostatics in dielectric

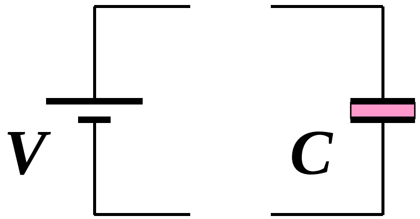
In a capacitor filled by dielectric:

$$\boxed{E = \frac{E_0}{K}}$$

field is reduced but not zero

The energy density in a dielectric: $$\boxed{u = \frac{1}{2}\varepsilon E^2}$$

Example6: How does *C*, *Q*, *U* change?



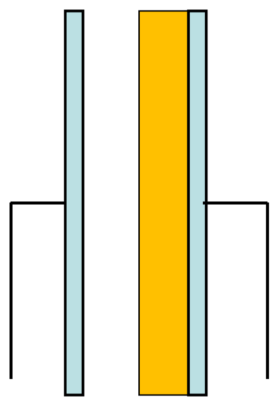
C □ $\Rightarrow Q$ □ $\Rightarrow U$ □

battery disconnected?

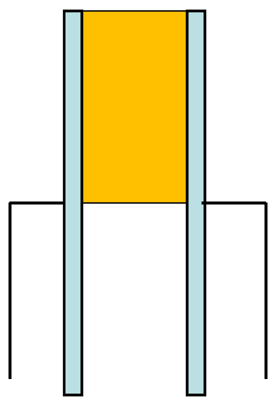
Half filled dielectric

Question: **How does the capacitance change if the capacitor is half filled with dielectric? (*S*, *d*, *K*)**

1)



2)

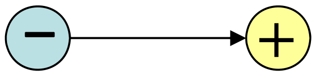


Molecular description (1)

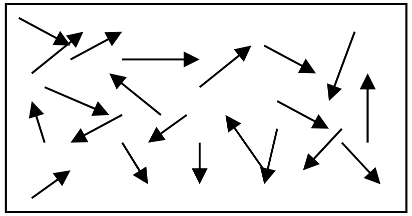
There are two different types of dielectrics:

1) Polar dielectrics, such as H_2O and CO

Have permanent electric dipole moments.



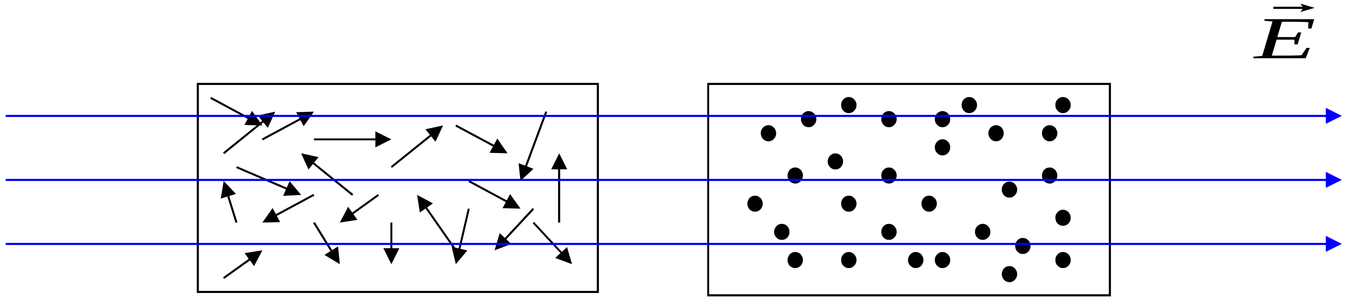
$$\vec{p} = Q\vec{l}$$



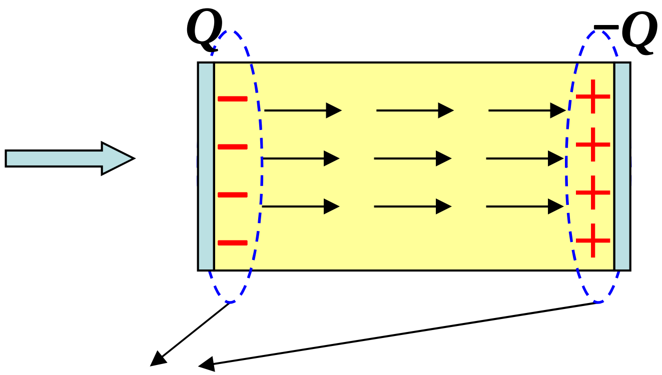
2) Nonpolar dielectrics, such as O_2 and N_2

No permanent electric dipole moments.

Molecular description (2)



Polar **Nonpolar**



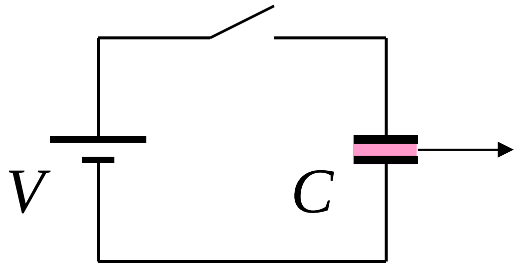
Polarization

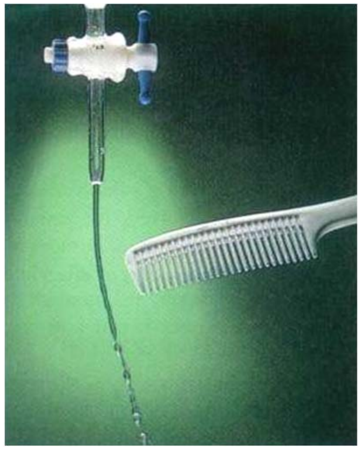
$$Q_{ind} = Q\left(1 - \frac{1}{K}\right)$$

Bound charge ⇨ **Induced field E_{ind}** ⇨ $E = \dfrac{E_0}{K}$

Thinking & question

Thinking: Why charged objects can attract small paper scraps or water flow?





Question: Move a dielectric plate out of a charged capacitor, the electric energy increases, where does the extra energy come from?

Brief review of Chapter 19-22

Coulomb's law: $$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}\hat{r}$$

Gauss's law: $$\Phi_E = \oint \vec{E}\cdot d\vec{S} = \frac{Q_{in}}{\varepsilon_0}$$

Electric potential: $$V = \int_a^\infty \vec{E}\cdot d\vec{l} \Leftrightarrow \vec{E} = -\nabla V$$

Capacitance; Electric energy; Dielectrics:

$$C = \frac{Q}{V}; \qquad U = \frac{1}{2}CV^2, \qquad u = \frac{1}{2}\varepsilon_0 E^2; \qquad \varepsilon_0 \to \varepsilon$$

Chapter 23 & 24

These chapters should be studied by yourself

Ohm's law: $V = IR$ Current & resistance

Current density: $j = \dfrac{I}{S} \quad \Leftrightarrow \quad I = \int \vec{j} \cdot d\vec{S}$

Microscopic statement of Ohm's law:

$\vec{j} = \sigma \vec{E} = \vec{E} / \rho$ conductivity & resistivity

Electromotive force (EMF) of battery