电子科技大学
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Circuit Analysis and Design

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Lecture 6

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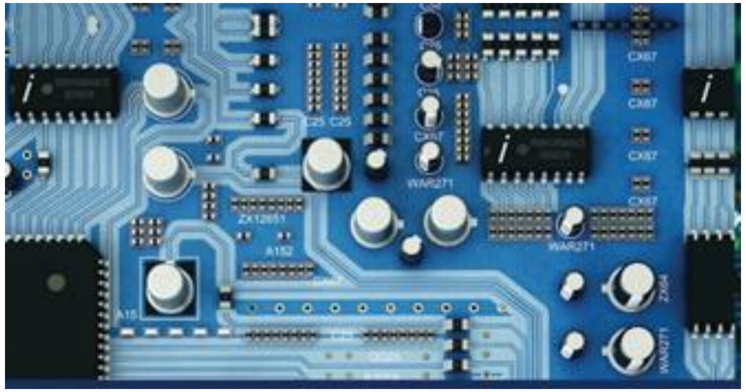
"A good student never steal or cheat"

AGENDA



- ❑MESH ANALYSIS
- ❑SUPERMESH
- ❑SUMMARY

MESH ANALYSIS

- Another useful method to calculate circuit parameters, Defined as:

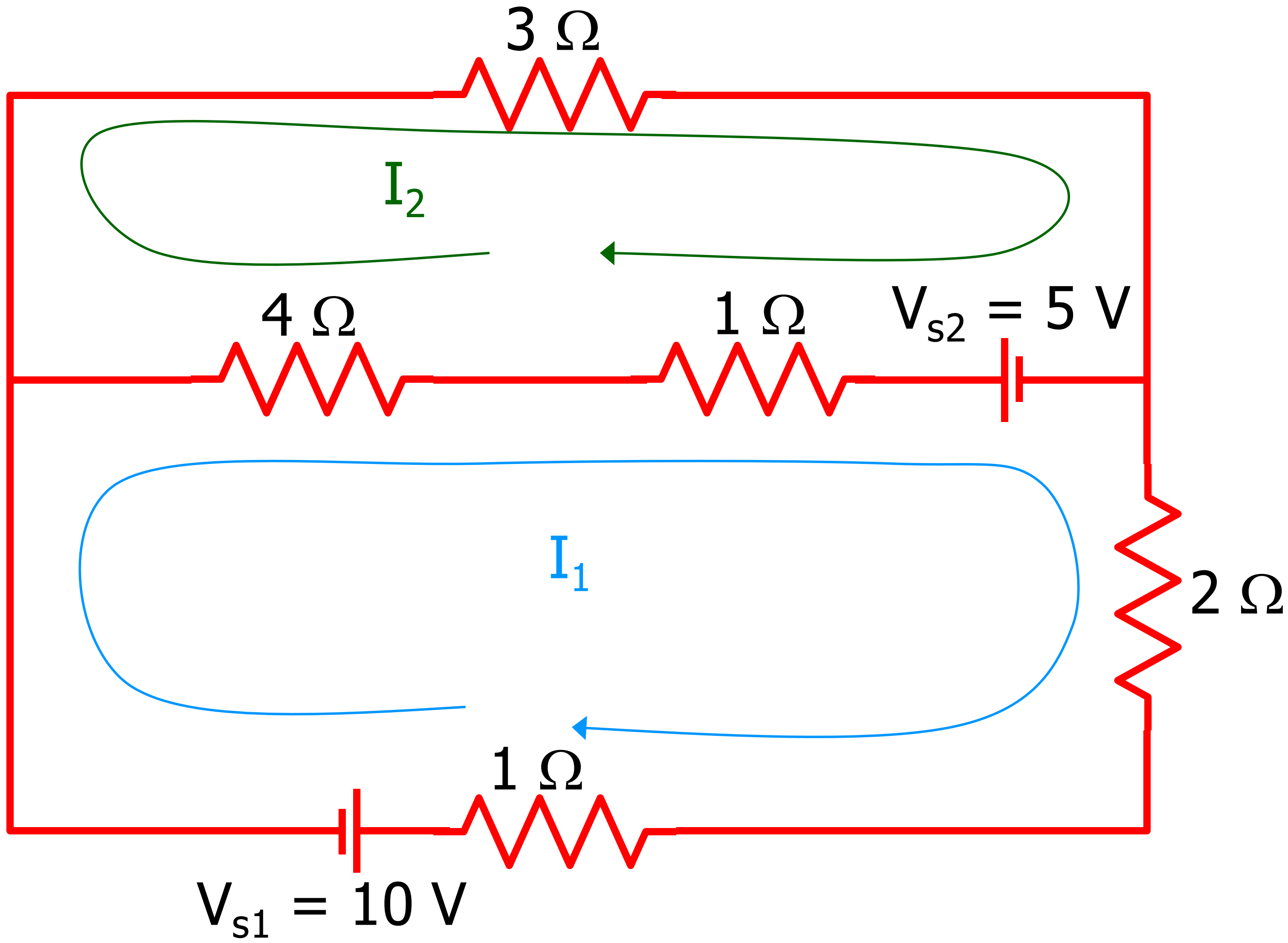
  The method in which the current flowing through a mesh in a circuit is calculated.

- It is based on
  - Kirchhoff’s Voltage Law (KVL)
  - Ohm’s law to calculate the voltage drops

- Knowing mesh currents, all other currents, all voltages and power can be computed

MESH

- ❑ *A mesh is a closed loop, which does not contain further loop inside it*



There are two Meshes.

Mesh 1.

Mesh 2.

COMPARE: MESH ANALYSIS VS NODAL ANALYSIS

Nodal Analysis	Mesh Analysis
Find: **Unknown Nodal Voltages**	Find: **Unknown Mesh Currents**
Based on **Kirchhoff's Current Law (KCL)** → **Sum of currents leaving a node = 0**	Based on **Kirchhoff's Voltage Law (KVL)** → **Sum of voltage drops in a Mesh = 0**
Use **Ohm's Law** to find **Branch Current**	Use **Ohm's Law** to find **Branch Voltages**
No. of Nodal Equations = No. of Unknown Nodal Voltages	No. of Mesh Equations = No. of Unknown Mesh Currents
Known Voltage Node: Voltage Source Connected to the Node and ground	**Known Mesh Current:** Current Source connect to a branch of that Mesh only
Supernode: Voltage source connect between to Unknown Voltage Nodes	**Supermesh:** Current Source connected between two Unknown Current Meshes

MESH ANALYSIS

- A mesh is a loop that does not contain any other loops.

- For each mesh, define a mesh current as the current flowing around the mesh in the clockwise direction.

- If a mesh contains a current source, the mesh current is the same as the current from the current source if they point in the same direction. If the mesh current is negative, the physical current flows in the opposite direction to the mesh current.

- If there is a branch in a mesh that is shared by another mesh, the physical current flowing through the branch is the difference in mesh currents sharing the branch.

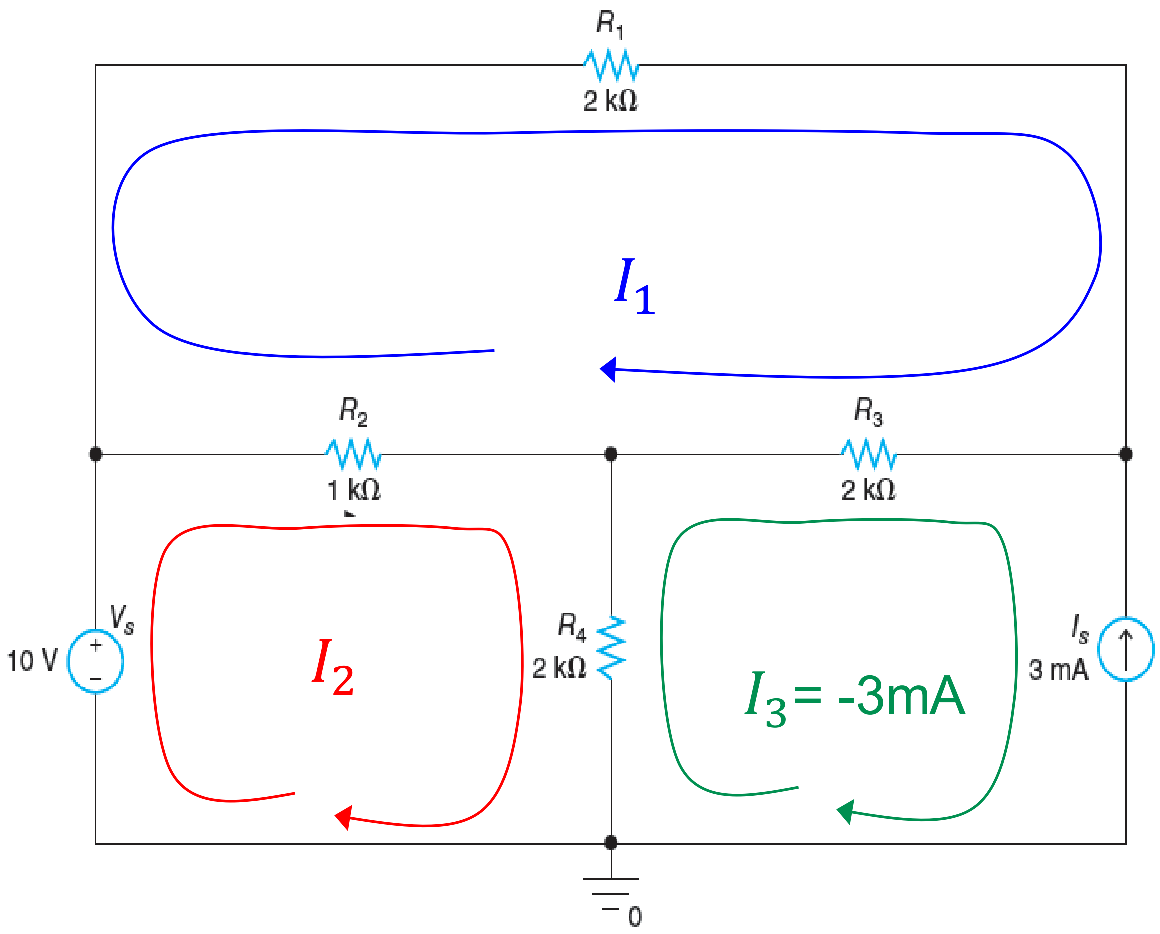
MESH ANALYSIS (cont..)

- The mesh analysis is based on KVL:
  - **The sum of voltage drops around a mesh is zero.**
  - For each mesh, sum the voltage drops around the mesh in the clockwise direction and set that equal to zero.
  - The voltage drop across a resistor is the product of the resistance and the net current (physical current) through the resistor. i.e. Ohm's Law
  - If the resistor is not shared by another mesh, the voltage drop is the product of the resistance and the mesh current.
  - If the resistor is shared by another mesh, the voltage drop is the product of the resistance and the difference in mesh currents.
  - The voltage drop across a voltage source V_s from positive terminal to negative terminal is V_s, and $-V_s$ from negative terminal to positive terminal.

STEPS TO APPLY MESH ANALYSIS

- **Step 1: Identification of the Meshes in the Circuit**
- **Step 2: Label the Meshes with respective Currents**
  - Assign a current to each mesh flowing in clockwise direction
  - If a mesh contains a current source connected to its branch only, the mesh current = the current source, but direction could be opposite.
  - Label voltage drops across all the resistors in a mesh, voltage across a source is known and follow the sign convention.
- **Step 3: Applying KVL on Meshes with Unknown Current**
  - KVL: Sum of voltage drop = 0
  - Apply the Ohm's Law to find voltage drop across resistors i.e. : $V_R = IR$
  - Get Equations in terms of unknown Mesh Currents as variable (No. of equation = No. of Unknown mesh currents)
- **Step 4: Solve the Equations to find Unknown Mesh currents**

EXAMPLE: MESH ANALYSIS



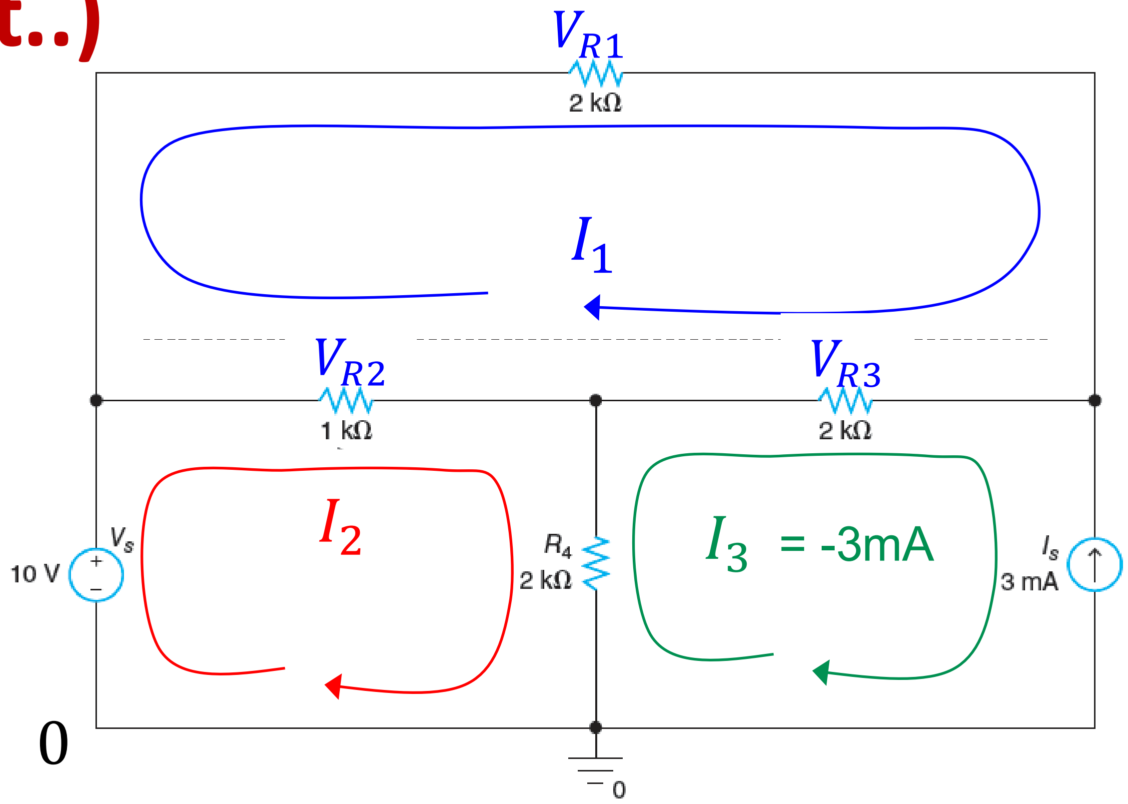
- **Step1:** Identify Meshes and assign currents to each
  - There are **three meshes**

- **Step 2:** Assign currents to each mesh are $\boldsymbol{I_1, I_2, and\ I_3}$ flowing in **clockwise direction**

- Notice that mesh current I_3 flows in the opposite direction to I_s. Thus, $\mathbf{I_3 = -I_s = -3\ mA}$

- Therefore, only two unknowns $\boldsymbol{I_1}$ and $\boldsymbol{I_2}$:
  - Two KVLs around Mesh 1 and Mesh 2 respectively to find $\boldsymbol{I_1\ and\ I_2}$

EXAMPLE: MESH ANALYSIS (Cont..)



- **Step 3: KVL around Mesh 1:**
  - Sum of voltage drops around mesh 1 in the clockwise is 0

$$V_{R1} + V_{R3} + V_{R2} = 0$$

  - Substitute values ➔ Ohm's law

$$2000\mathrm{I}_1 + 2000(I_1 - I_3) + 1000(I_1 - I_2) = 0$$

  - Divide by 1000

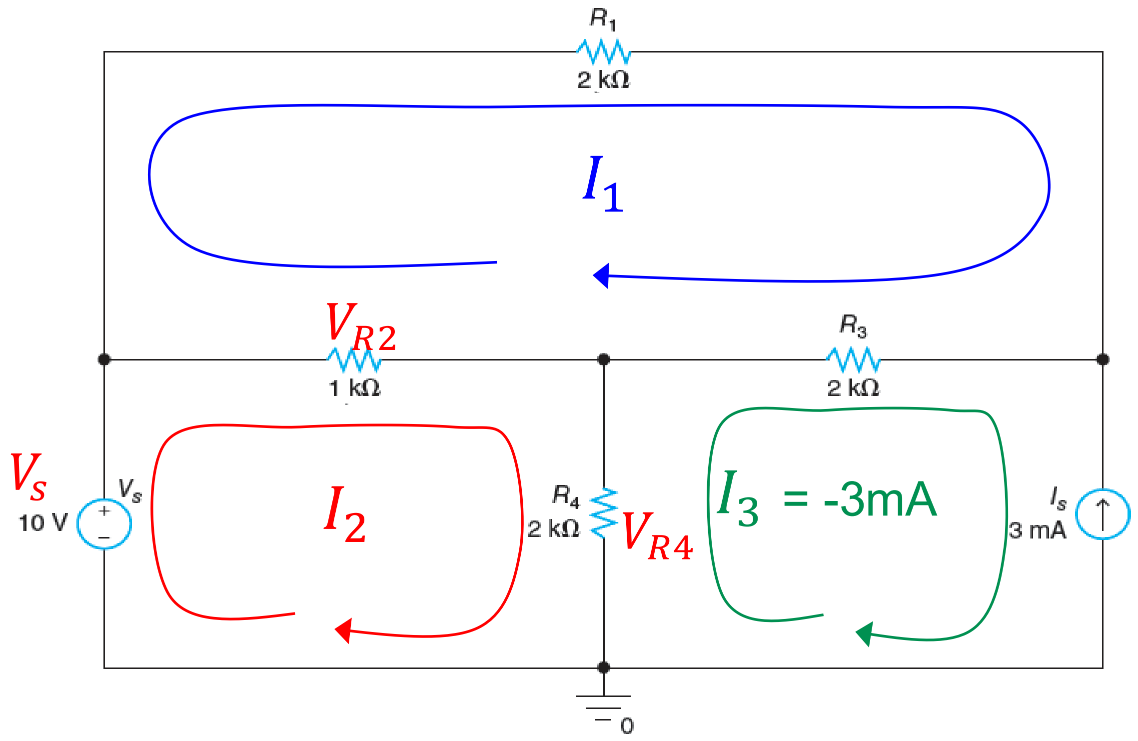
$$\mathbf{2}\mathrm{I}_1 + \mathbf{2}(I_1 - I_3) + (I_1 - I_2) = 0$$

$$\mathbf{2}\mathrm{I}_1 + \mathbf{2}(I_1 - (-3 \times 10^{-3})) + 1(I_1 - I_2)) = 0$$

$$\boxed{5\mathrm{I}_1 - \mathrm{I}_2 = -0.006 \quad (1)}$$

EXAMPLE: MESH ANALYSIS (Cont..)

- **Step 3: Applying KVL around Mesh 2**



- Sum of voltage drops around Mesh 2 in clockwise direction = 0

$$-V_S + V_{R2} + V_{R4} = 0$$

- Substitute values ➔ Ohm's law

$$-10 + 1000(I_2 - I_1) + 2000(I_2 - I_3) = 0$$

- Divide by 1000

$$-0.01 + (I_2 - I_1) + 2(I_2 - (-3 \times 10^{-3})) = 0$$

$$-I_1 + 3I_2 - 0.01 + 0.006 = 0$$

$$\boxed{-I_1 + 3I_2 = 0.004 \qquad (2)}$$

EXAMPLE: MESH ANALYSIS (Cont..)

- **Step 4:** Solve the two equations to get the unknown current I_1 and I_2

$$\boxed{5I_1 - I_2 = -0.006 \qquad (1)}$$

$$\boxed{-I_1 + 3I_2 = 0.004 \qquad (2)}$$

- Multiply (2) by 5

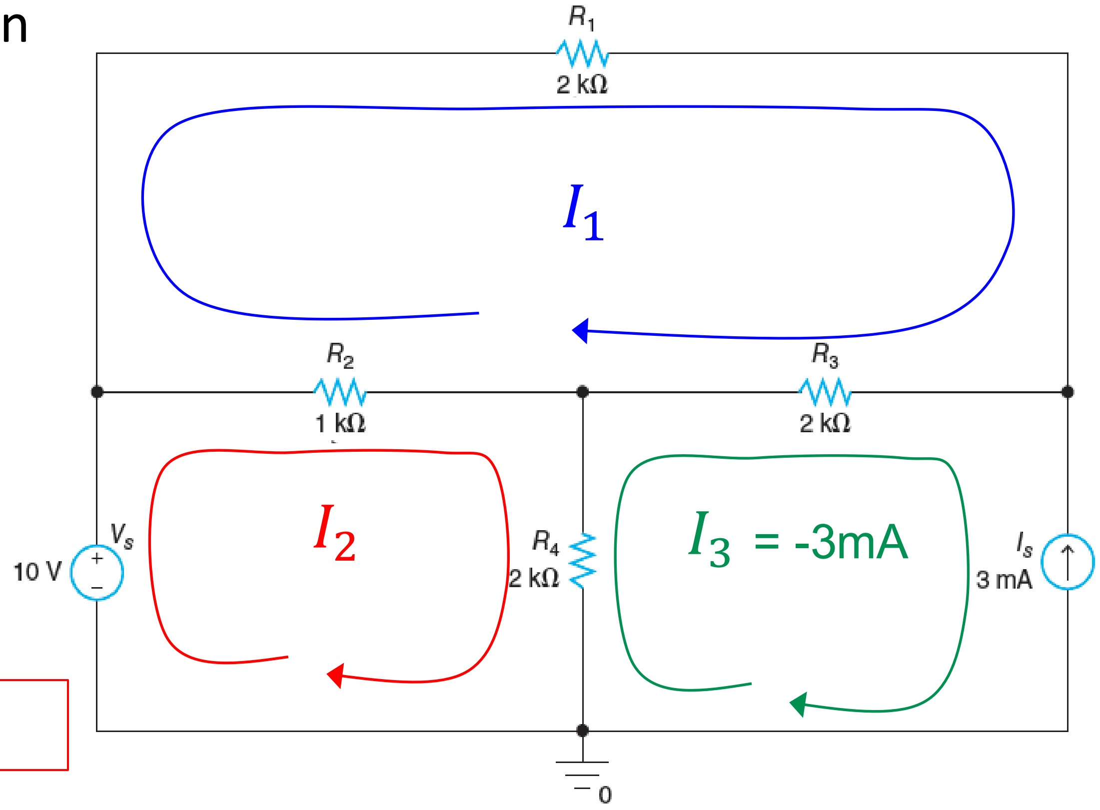
$$-5I_1 + 15I_2 = 0.02 \quad (3)$$

- Add (1) and (3)

$$\boxed{14I_2 = 0.014 \Rightarrow I_2 = 0.001\,A = 1\,mA}$$

- From Equation (2):

$$\boxed{I_1 = 3I_2 - 0.004 \Rightarrow I_1 = -\,0.001\,A = -\,1mA}$$



- Note: I_1 is negative → Actually it is flowing in the opposite direction to our initial assumption

EXAMPLE: MESH ANALYSIS (Cont..)

❑ Actual Branch Currents

- Since $\mathbf{I_1 = -1\ mA}$, the physical current of 1 mA flows through $\mathbf{R_1}$ from right to left : (←)

- The physical current through $\mathbf{R_2}$ from left to right (→) is given by

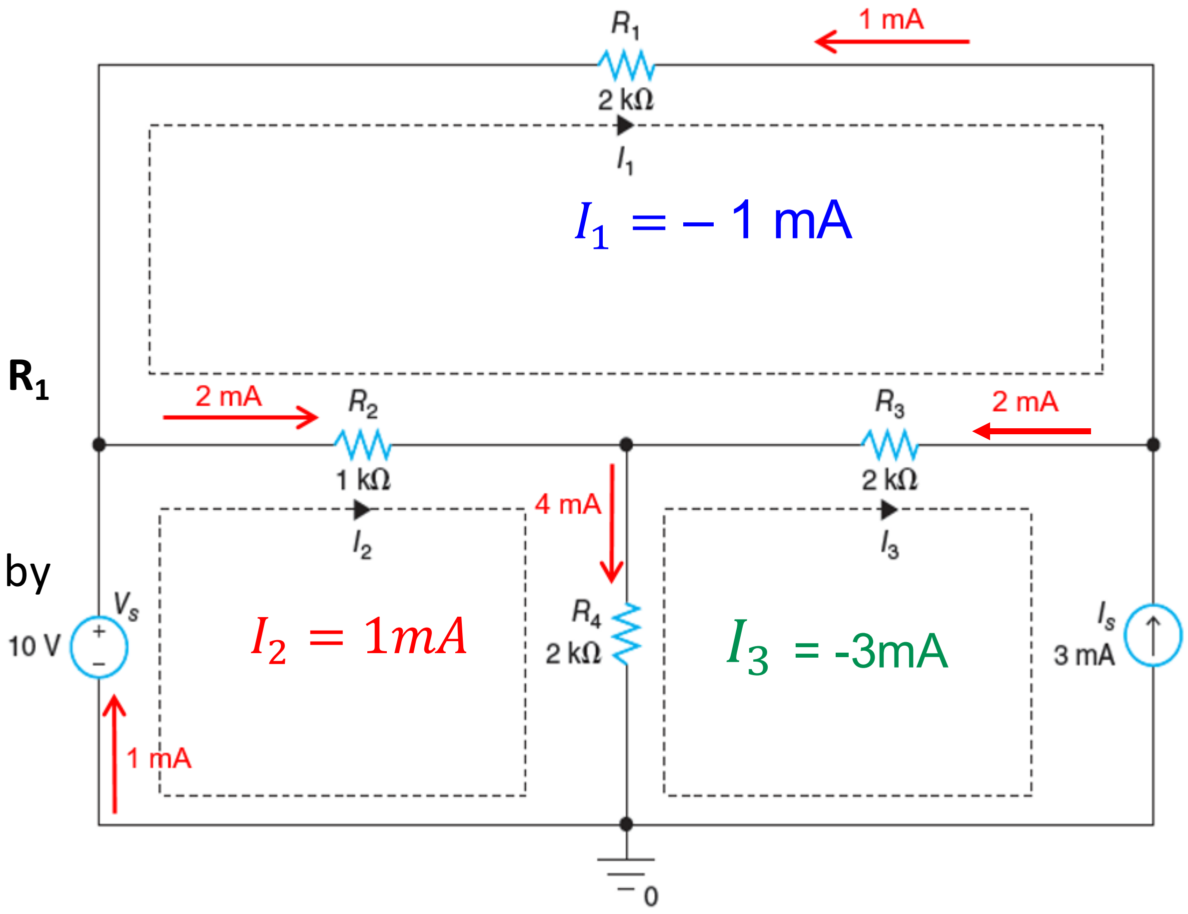
  $$I_2 - I_1 = 1 - (-1) = 2\ mA$$

- The physical current through $\mathbf{R_3}$ from right to left (←) is

  $$I_1 - I_3 = -1 - (-3) = 2\ mA$$

- The physical current through $\mathbf{R_4}$ flowing top to bottom (↓)is

  $$I_2 - I_3 = 1 - (-3) = 4\ mA\ )$$



EXAMPLE: MESH ANALYSIS(Cont..)

- Actual Voltages across elements

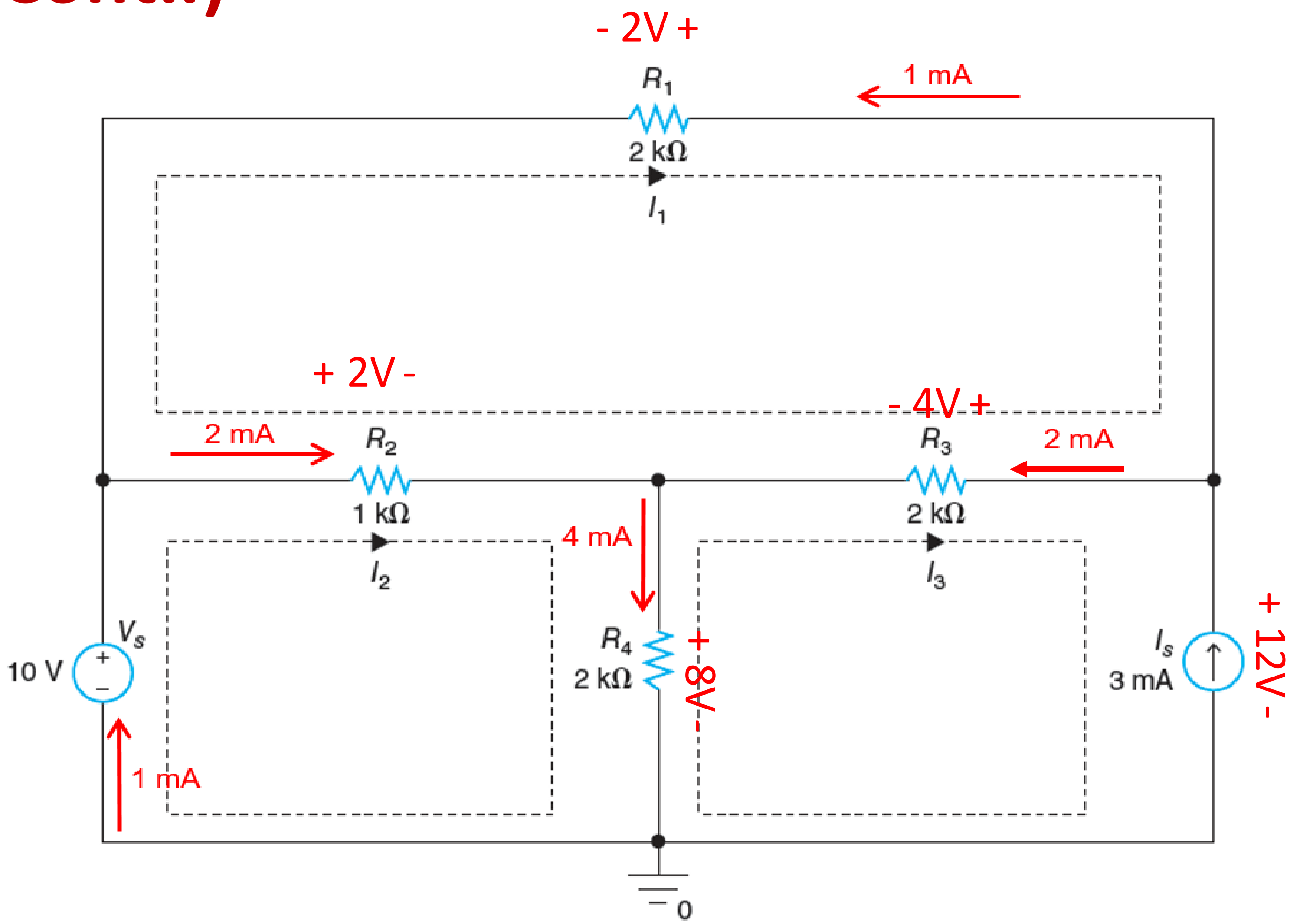
  - The voltage across R_1 from right to left is
    $V_{R1} = R_1(-I_1) = 2000 \times 0.001 = 2$ V.

  - The voltage across R_2 from left to right is
    $V_{R2} = R_2(I_2 - I_1) = 1000 \times 0.002 = 2$ V

  - The voltage across R_3 from right to left is
    $V_{R3} = R_3(I_1 - I_3) = 2000 \times 0.002 = 4$ V

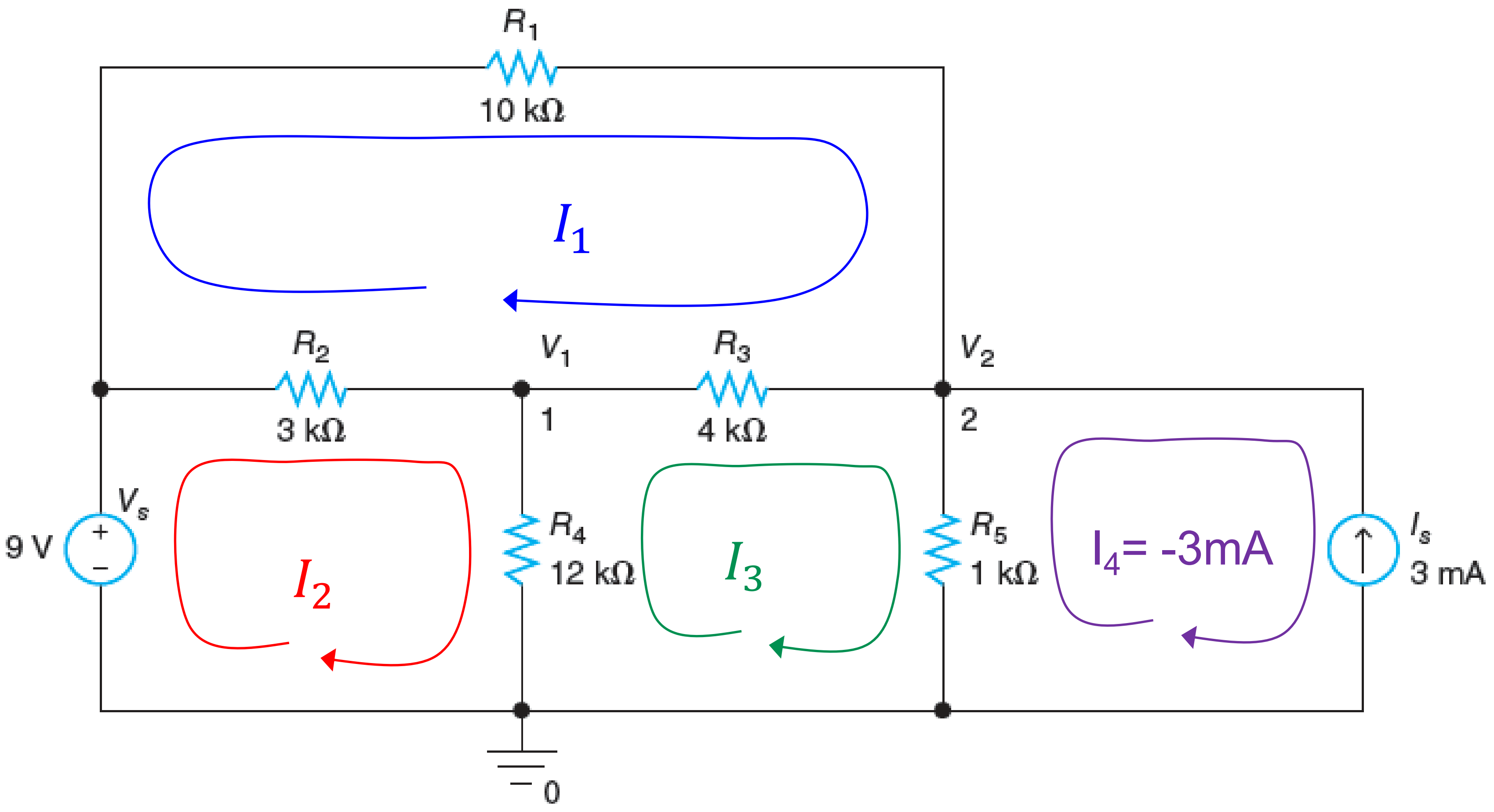
  - The voltage across R_4 from top to bottom is
    $V_{R4} = R_4(I_2 - I_3) = 2000 \times 0.004 = 8$ V

  - The voltage across the current source is
    $V_{R3} + V_{R4} = 4\text{ V} + 8\text{ V} = 12$ V



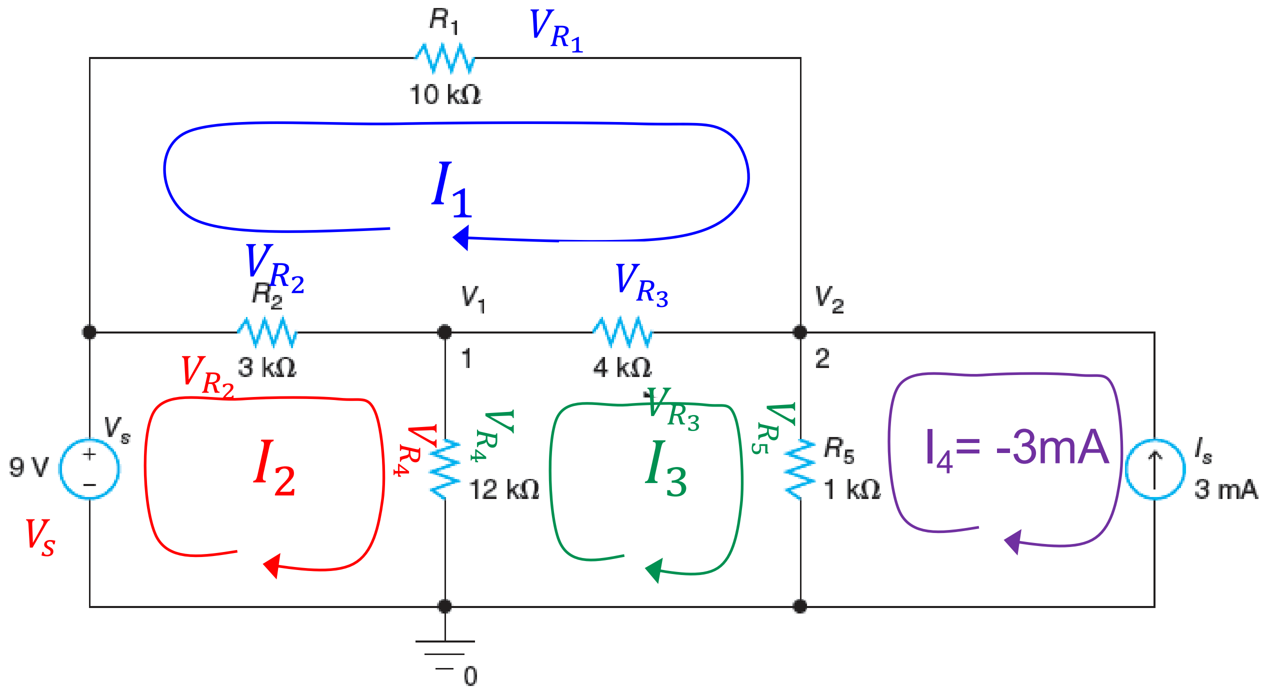
EXAMPLE 3.12

- Find I_1, I_2, I_3, I_4, V_1, V_2, and currents through branches



EXAMPLE 3.12 (cont..)

- I_4 = -3mA, Therefore for apply 3 KVLs on Meshes 1, 2 and 3 to find 3 Unknown Mesh Currents I_1, I_2, and I_3



- Sum of voltage drop around **Mesh 1** ➔ $V_{R_1} + V_{R_3} + V_{R_2} = 0$ ➔ $10000I_1 + 4000(I_1 - I_3) + 3000(I_1 - I_2) = 0$ ➔ $17000I_1 - 3000I_2 - 4000I_3 = 0$ ➔ $\boxed{\mathbf{17I_1 - 3I_2 - 4I_3 = 0 \quad (1)}}$

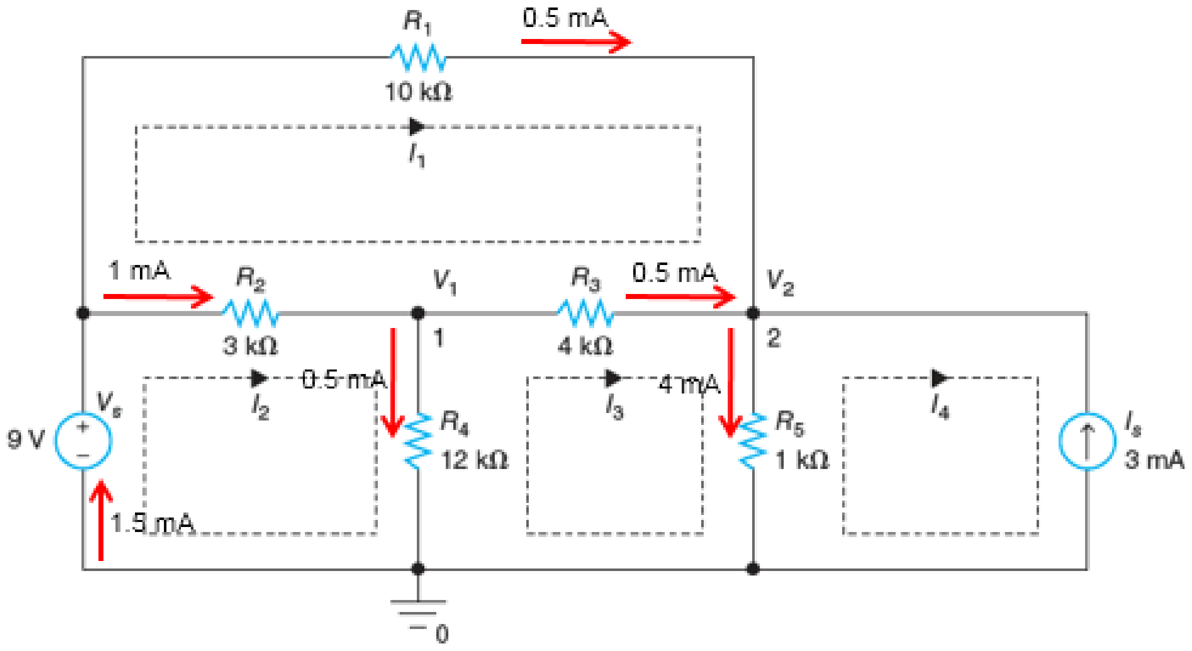
- Sum the voltage drops around **Mesh 2** ➔ $V_s + V_{R_2} + V_{R_4} \Rightarrow -9 + 3000(I_2 - I_1) + 12000(I_2 - I_3) = 0$ ➔ $-3000I_1 + 15000I_2 - 12000I_3 = 9$ ➔ $\boxed{\mathbf{-3I_1 + 15I_2 - 12I_3 = 0.009 \ (2)}}$

- Sum the voltage drops around **Mesh 3** ➔ $V_{R_4} + V_{R_3} + V_{R_5}$ ➔ $12000(I_3-I_2)+4000(I_3-I_1)+1000(I_3 - I_4) = 0$ ➔ $-4000I_1 - 12000I_2 + 17000I_3 = -3$ ➔ $\boxed{\mathbf{-4I_1 - 12I_2 + 17I_3 = -0.003 \ (3)}}$

- Multiply (1) by 5 ➔ $85I_1 - 15I_2 - 20I_3 = 0$ (4)
- Add (2) and (4) ➔ $82I_1 - 32I_3 = 0.009$ (5)

EXAMPLE 3.12 (cont..)

- $17I_1 - 3I_2 - 4I_3 = 0$ **(1)**
- $-3I_1 + 15I_2 - 12I_3 = 0.009$ **(2)**
- $-4I_1 - 12I_2 + 17I_3 = -0.003$ **(3)**
- $82I_1 - 32I_3 = 0.009$ **(5)**



- Multiply (1) by 4 ➔ $68I_1 - 12I_2 - 16I_3 = 0$ **(6)**
- (6) – (3) ➔ $72I_1 - 33I_3 = 0.003$ **(7)**
- Multiply (5) by 33 ➔ $2706I_1 - 1056I_3 = 0.297$ **(8)**
- Multiply (7) by 32 ➔ $2304I_1 - 1056I_3 = 0.096$ **(9)**

- (8) – (7): $402I_1 = 0.201 \Rightarrow I_1 = 0.0005\text{ A} = 0.5\text{ mA}$
- From (5)➔ $I_3 = (82I_1 - 0.009)/32 = 1\text{ mA}$
- From (1)➔ $I_2 = (17I_1 - 4I_3)/3 = 1.5\text{ mA}$
- **V1 and V2 can be computed as**

$$\boldsymbol{V_1 = R_4(I_2 - I_3) = 12000\times 0.0005 = 6\ V}$$
$$\boldsymbol{V_2 = R_5(I_3 - I_4) = 1000\times 0.004 = 4\ V}$$

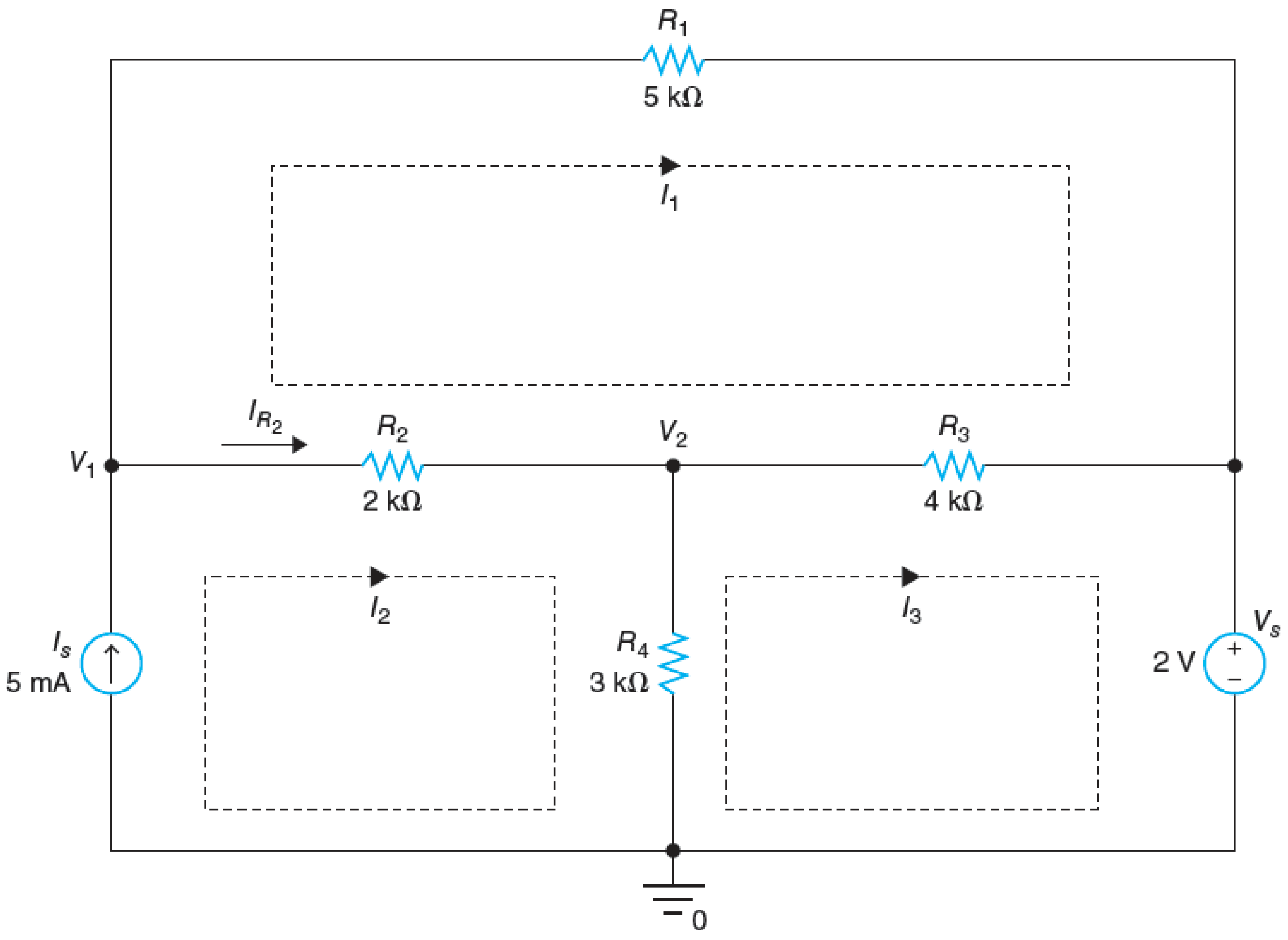
- And the currents through R_1, R_2, R_3, R_4, R_5 can be calculated as:

  I_{R1} ($\rightarrow$) = I_1 = 0.5 mA, I_{R2} ($\rightarrow$) = $I_2 - I_1$ = 1 mA
  I_{R3} ($\rightarrow$) = $I_3 - I_1$ = 0.5 mA
  I_{R4} ($\downarrow$) = $I_2 - I_3$ = 0.5 mA, I_{R5} ($\downarrow$) = $I_3 - I_4$ = 4 mA

CLASS TASK

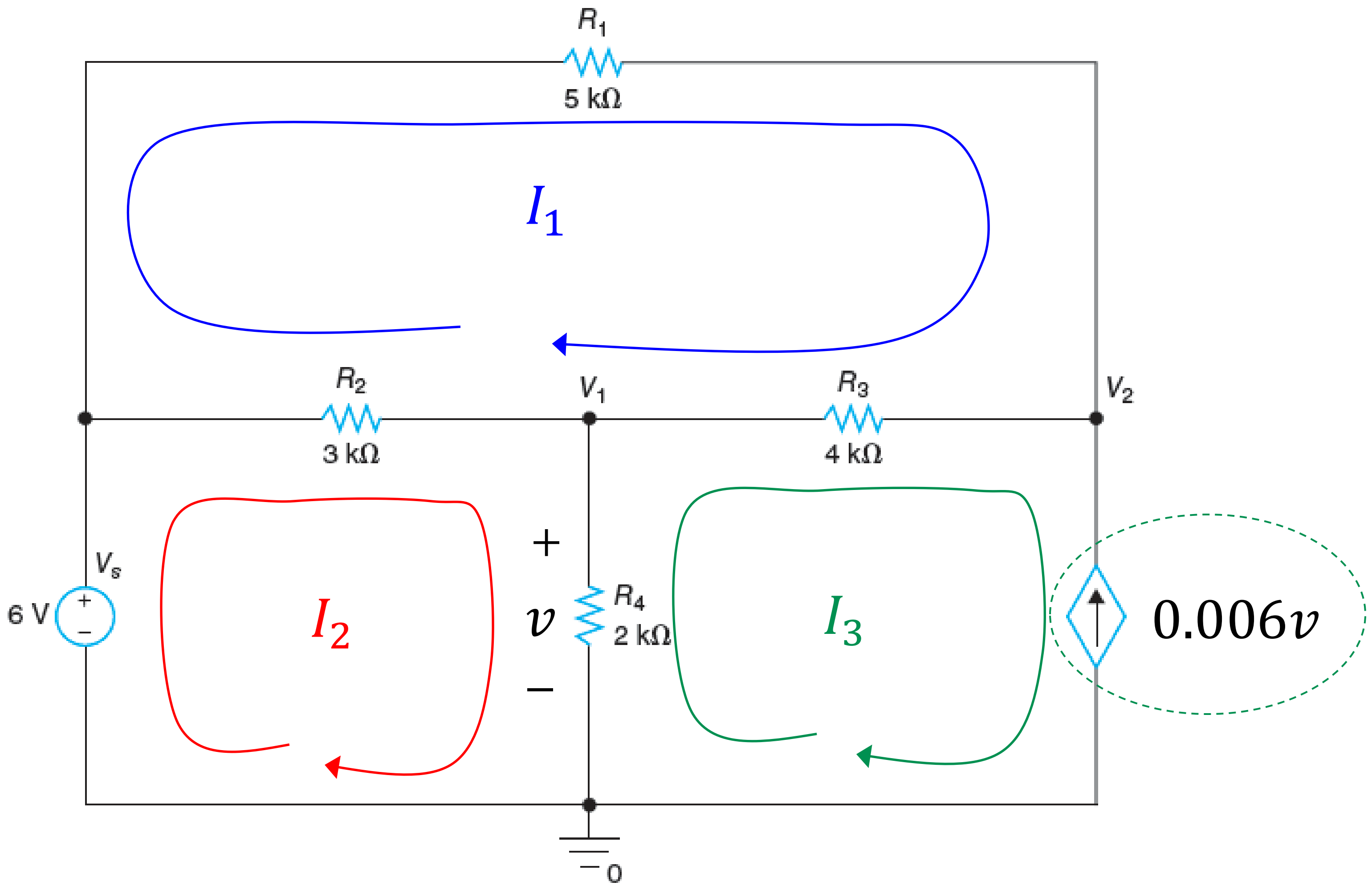
- Find I_1, I_2, I_3, V_1, V_2 Use Mesh Analysis

Solutions will be Provided in class

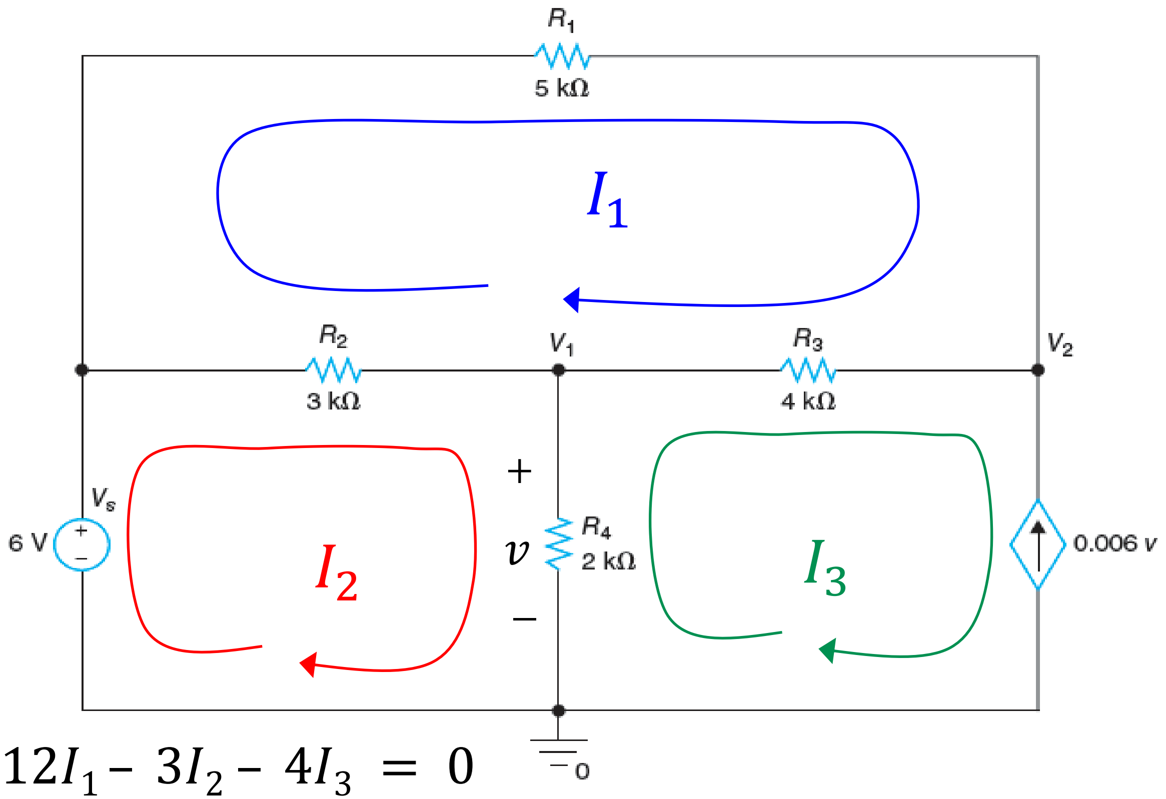


EXAMPLE 3.13 : VOLTAGE CONTROL CURRENT SOURCE EXAMPLE

- Find I_1, I_2, I_3, V_1, V_2,and powers



EXAMPLE 3.13 (cont..)



- Get equation from Dependent source →
  $V_1 = v = 2000(I_2 - I_3)$
- And we know, $I_3 = -0.006V_1$, Substitute I_3 in V_1
  $V_1 = v = -12(I_2 - I_3) \rightarrow 12I_2 = 11I_3$
  → $\boxed{\mathbf{I_3 = (12/11)I_2 = 1.09091I_2 \quad (1)}}$
- KVL around Mesh 1: Sum the voltage drops around mesh 1
  ➔ $5000I_1 + 4000(I_1 - I_3) + 3000(I_1 - I_2) = 0$
- Substitute values → $12000I_1 - 3000I_2 - 4000I_3 = 0 \Rightarrow 12I_1 - 3I_2 - 4I_3 = 0$
- From (1) : $I_3 = (12/11)I_2 = 1.09091I_2 \rightarrow 12I_1 - 3I_2 - 4(12/11)I_2 = 0 \rightarrow 132I_1 - 81I_2 = 0$

  $\boxed{\rightarrow \boldsymbol{I_2} = \left(\frac{\mathbf{132}}{\mathbf{81}}\right)\boldsymbol{I_1} \quad (2)}$
- KVL around Mesh 2: Sum the voltage drops around **mesh 2**

  $-6 + 3000(I_2 - I_1) + 2000(I_2 - I_3) = 0 \rightarrow 3I_1 + 5I_2 - 2I_3 = 0.006$ → $\boxed{\mathbf{3I_1 + (31/11)I_2 = 0.006 \quad (3)}}$
- Substitute (2) → (3) ➔ $-33I_1 + 31(132/81)\ I_1 = 0.006$
- $I_1 = 3.767442$ mA → $I_2 = 6.139535$ mA and $I_3 = 6.69767$mA
- $V_1 = R_4(I_2 - I_3) = -1.11628$ V $V_2 = V_1 + R_3(I_1 - I_3) = -12.83721$ V

EXAMPLE 3.13 (cont..)

- The three mesh currents are

  I_1 = 3.767442 mA I_2 = 6.139535 mA

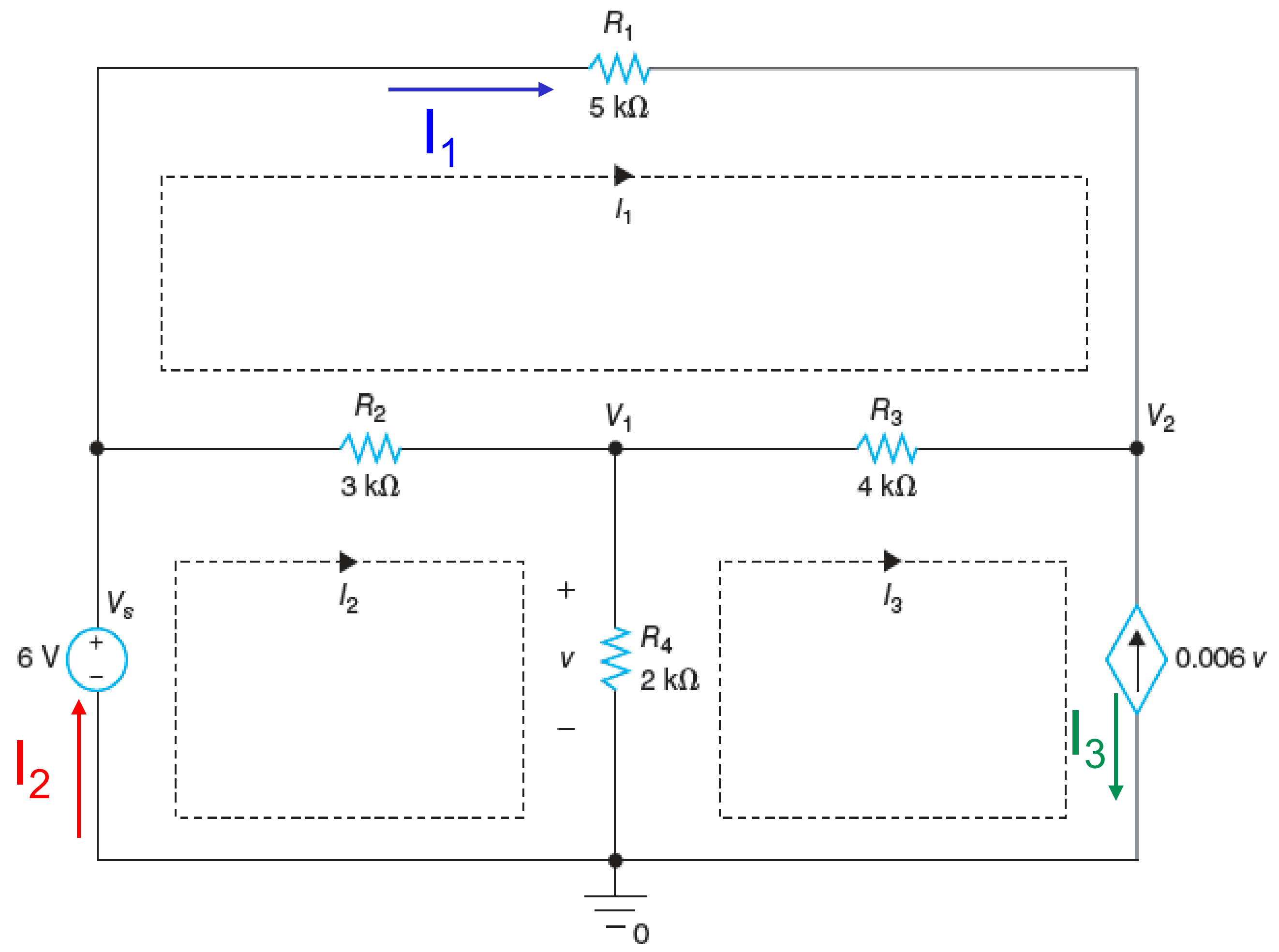
  I_3 = 6.69767mA

- Voltages V_1 and V_2 are

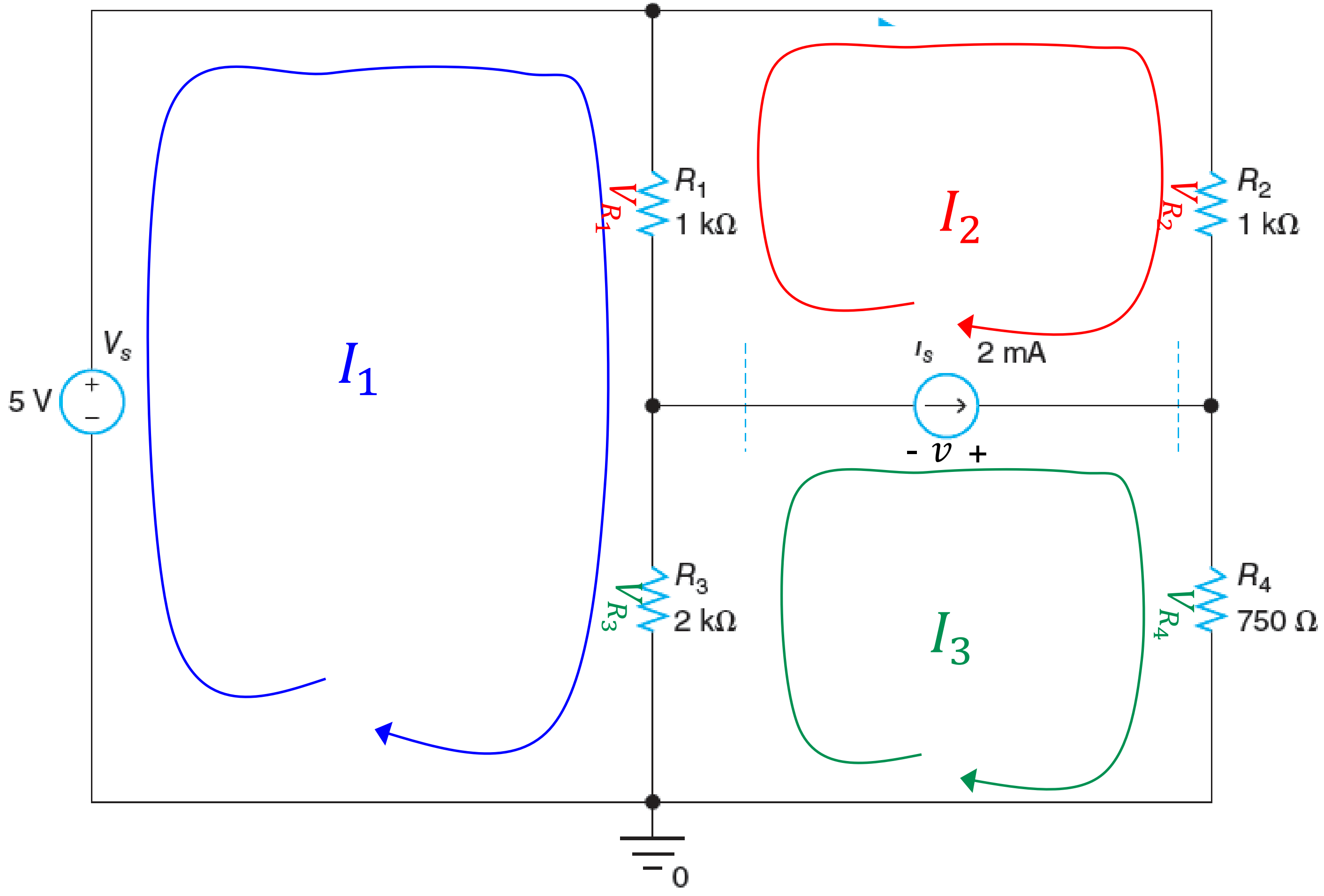
  V_1 = –1.11628 V and

  V_2 = –12.83721 V

- $P_{R1} = (I_1)^2R_1$ = 70.9681 mW
- $P_{R2} = (I_1 - I_2)^2R_2$ = 16.8805 mW
- $P_{R3} = (I_1 - I_3)^2R_3$ = 34.3451 mW
- $P_{R4} = (I_2 - I_3)^2R_4$ = 0.6230 mW
- $P_{Vs} = - I_2V_s$ = – 36.83721 mW
- $P_{VCCS} = I_3V_2$ = – 85.97945 mW
- $P_{R1} + P_{R2} + P_{R3} + P_{R4} + P_{Vs} + P_{VCCS} = 0$

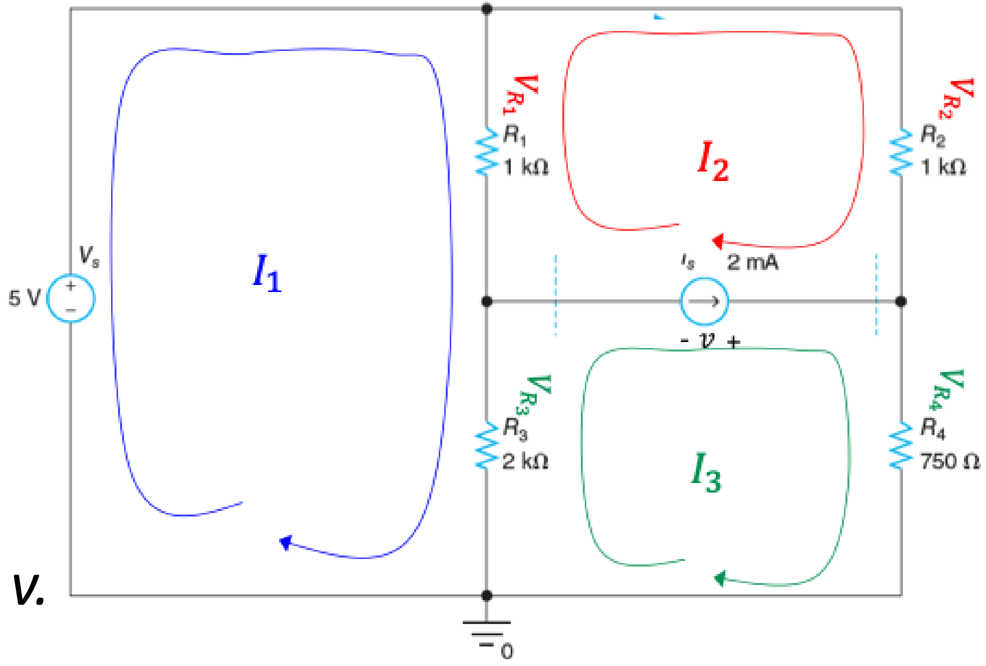


SUPERMESH



R_1
1 kΩ
V_{R_1}
R_2
1 kΩ
V_{R_2}
I_2
V_s
5 V
I_1
I_s
2 mA
- v +
R_3
2 kΩ
V_{R_3}
I_3
R_4
750 Ω
V_{R_4}
0

SUPERMESH (cont..)



- If there is a current source that is a common branch between two different meshes, we do not know the voltage drop across the current source.

  - Let the unknown voltage across the current source be *v*.

  - Write the mesh equation for each mesh: *Mesh 2:* $v + V_{R_1} + V_{R_2} = 0$, *Mesh 2:* $-v + V_{R_3} + V_{R_4} = 0$ and add the two equations to remove the unknown voltage *v*.

  - *Add Mesh1 and Mesh2:* $V_{R_1} + V_{R_2} + V_{R_3} + V_{R_4} = 0$ → this is called SuperMesh Equation

- The voltage *v* is removed when added because in one equation, the voltage drop across the current source is *v* and in the other equation, the voltage drop across the current source is *-v*.

SUPERMESH (cont..)

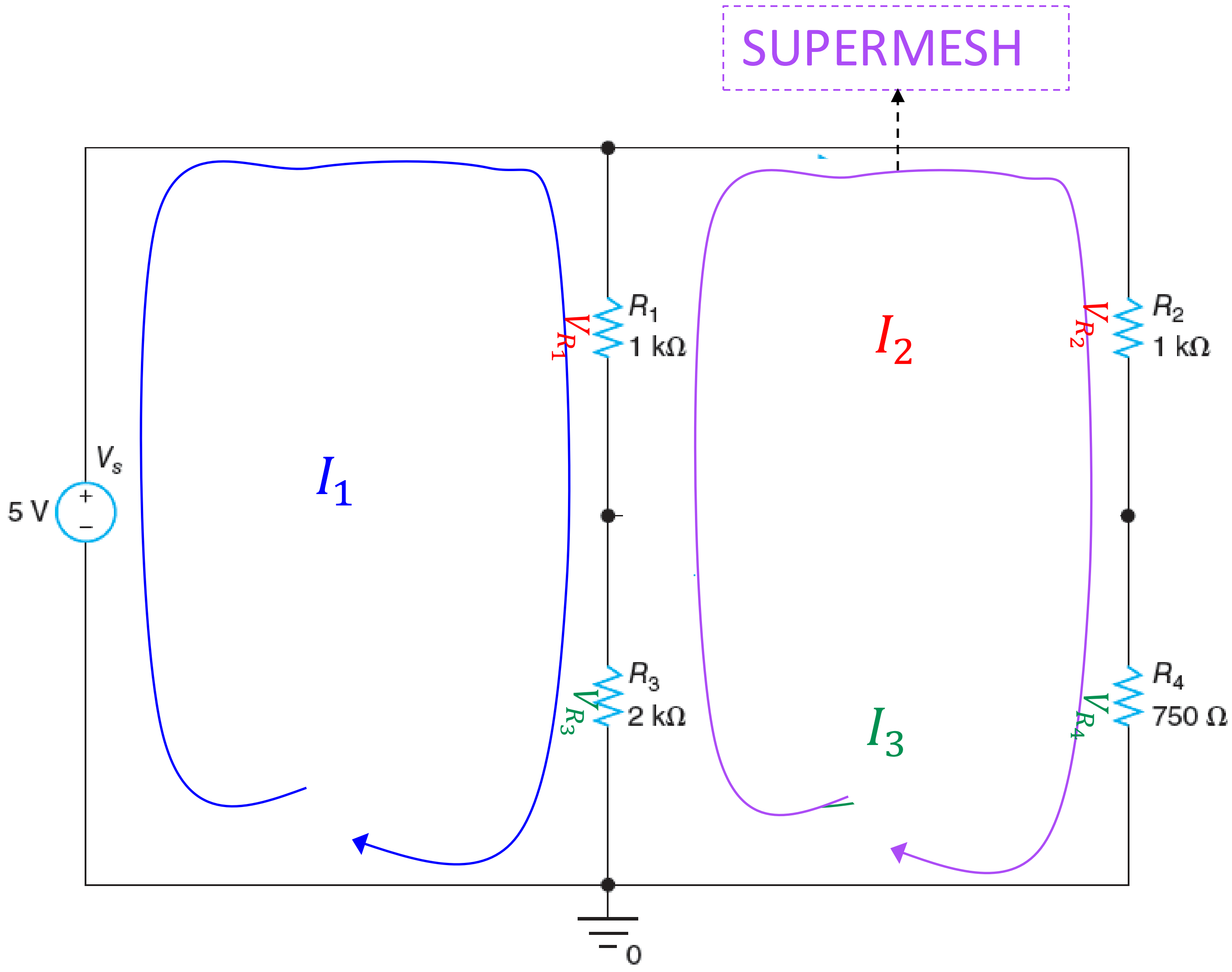
- We get TWO equations from the Supermesh:

1. One by applying KVL on the SuperMesh

$$V_{R_1} + V_{R_2} + V_{R_3} + V_{R_4} = 0$$

2. One from the current source

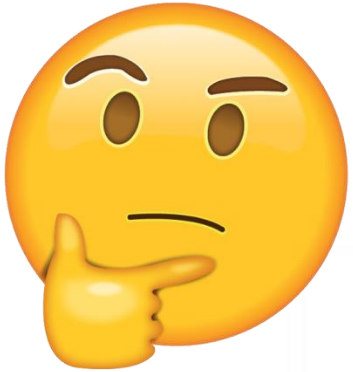
$$I_3 - I_2 = 2mA$$

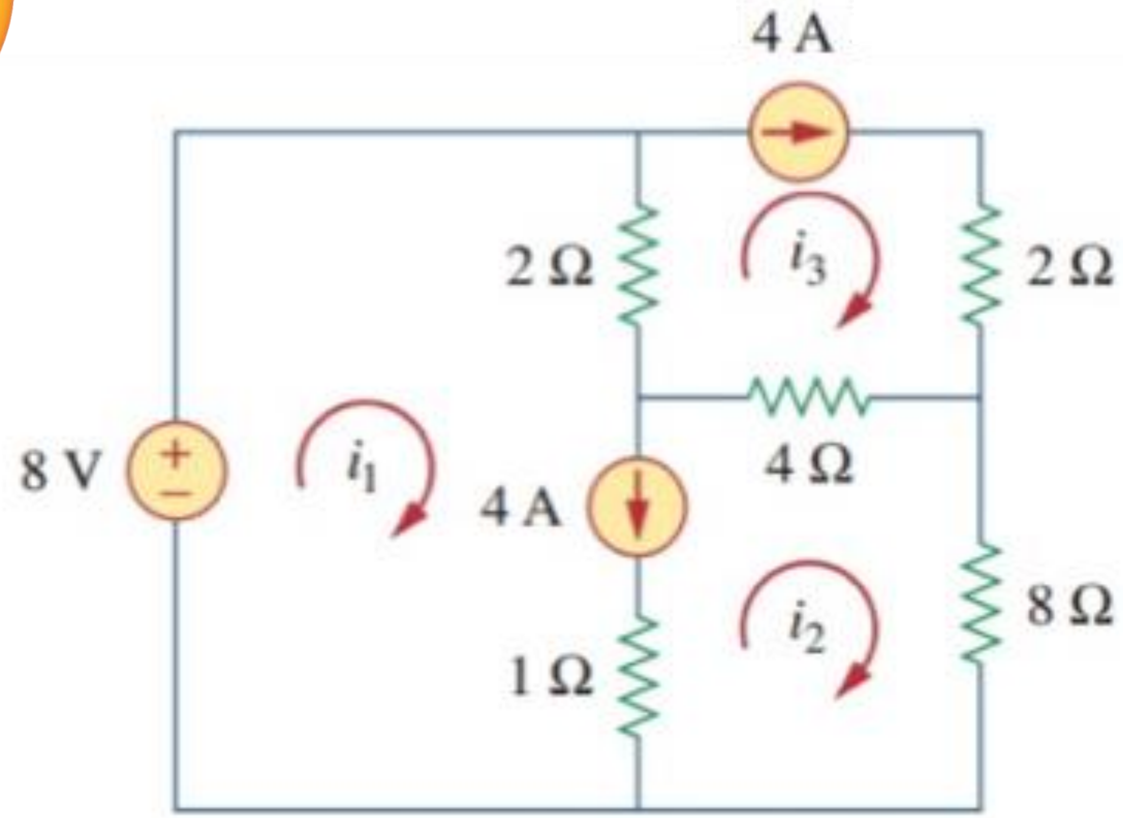


SUPERMESH (cont..)

- Alternatively, the sum of the two equations can be obtained directly by defining a supermesh consisting of the two meshes, excluding the current sourc i.e. $V_{R_1} + V_{R_2} + V_{R_3} + V_{R_4} = 0$. One Equation
- When the voltage drops around the supermesh are added, we get the same equation that we obtain by adding the two equations.
- The extra equation needed to find the mesh currents is obtained by representing the current of the current source by the difference of the two mesh currents. $I_3 - I_2 = 2mA$
- The current of the current source is obtained by subtracting the mesh current pointing in the opposite direction from the mesh current pointing in the same direction as the current source.

QUIZ

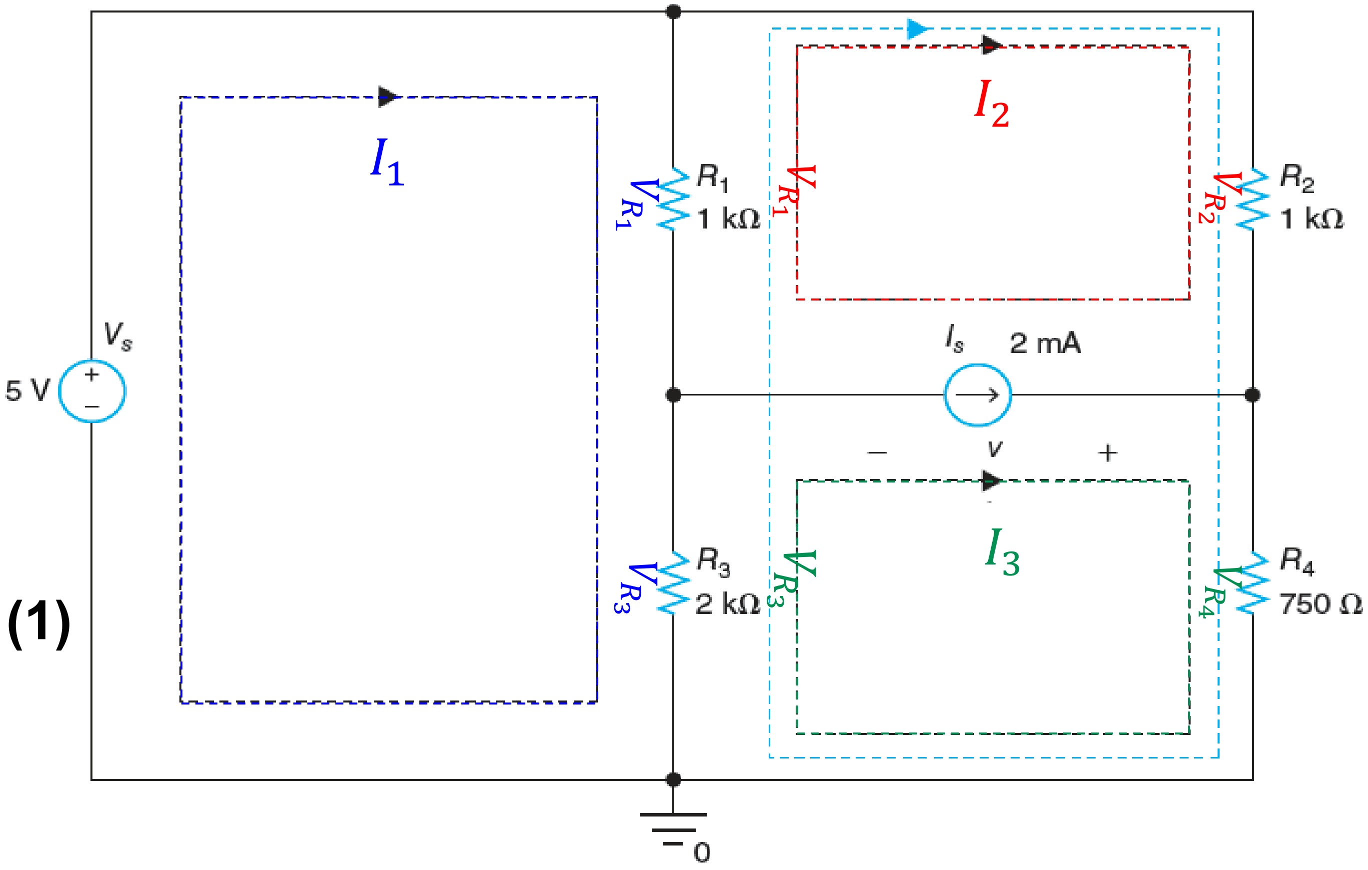




- ❑ Identify the Supermesh in this circuit. Which two meshes form a Supermesh?

A. Mesh 1 & 2
B. Mesh 2 and 3
C. Mesh 1 and 3

EXAMPLE 3.15



- ❑ Find I_1, I_2, I_3, v

- ○ One Equation from Current Source of Supermesh

  $I_3 - I_2 = Is = 2\text{ mA} = 0.002\text{ A} \Rightarrow \mathbf{I_3 = I_2 + 0.002}$ **(1)**

- ○ Applying KVL around Super Mesh: Sum the voltage drops around supermesh ➔

  $V_{R_3} + V_{R_1} + V_{R_2} + V_{R_4} = 0 \rightarrow 2000(I_3 - I_1) + 1000(I_2 - I_1) + 1000I_2 + 750I_3 = 0 \rightarrow$

  $-3000I_1 + 2000I_2 + 2750I_3 = 0 \Rightarrow \mathbf{-3I_1 + 2I_2 + 2.75I_3 = 0}$ **(2)**

- ○ Substitute (1) to (2) ➔ $\mathbf{-3I_1 + 4.75I_2 = -0.0055}$ **(3)**

EXAMPLE 3.15 (Cont..)

- **$3I_1 + 4.75I_2 = -0.0055$ (3)**

- Sum voltage drops around mesh 1➔

$$-V_s + V_{R_1} + V_{R_3} = 0$$

$\Rightarrow -5 + 1000(I_1 - I_2) + 2000(I_1 - I_3) = 0$

$\Rightarrow 3000I_1 - 1000I_2 - 2000I_3 = 5$

$\Rightarrow 3I_1 - I_2 - 2I_3 = 0.005$

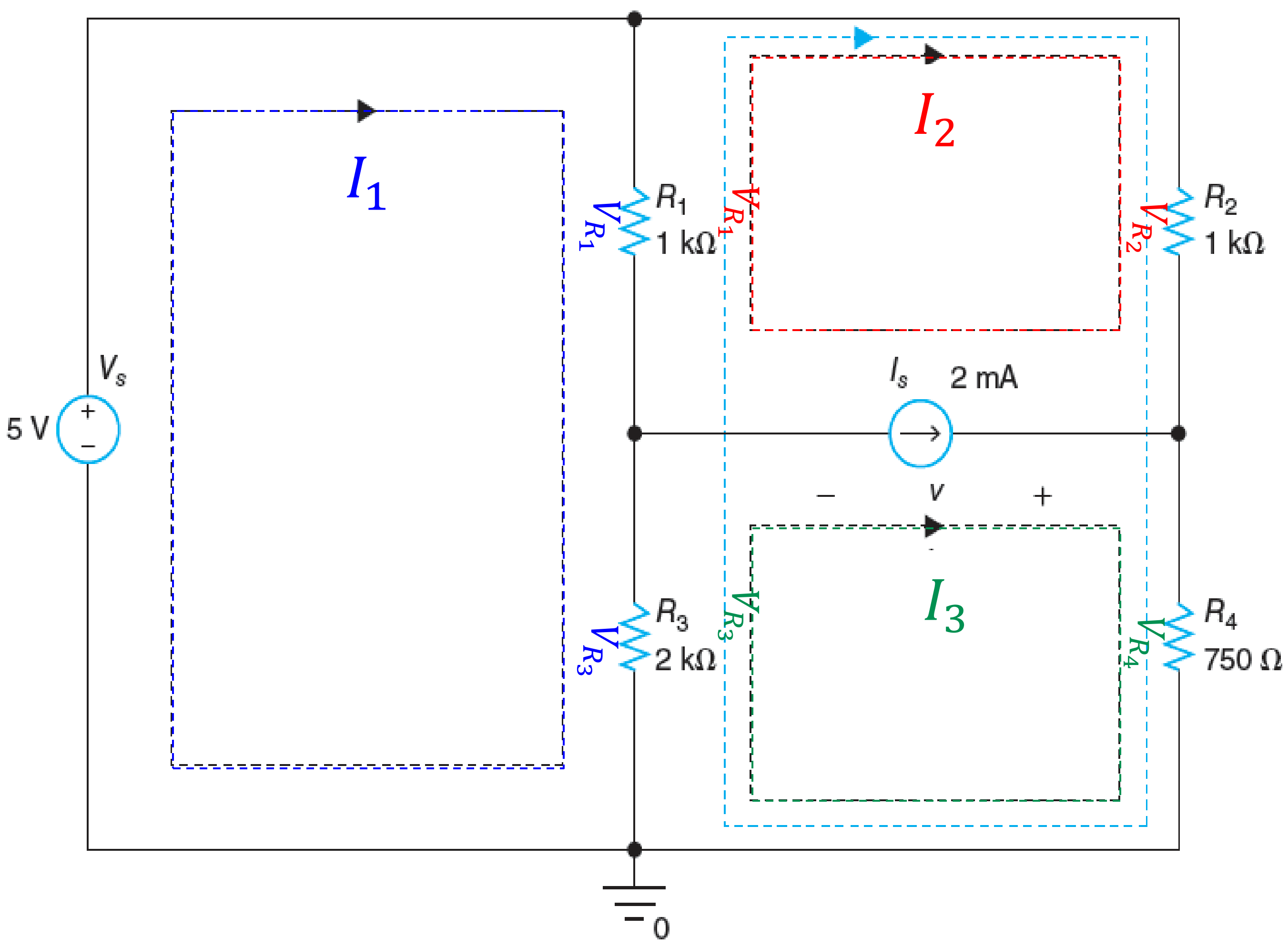
$\Rightarrow 3I_1 - I_2 - 2(I_2 + 0.002) = 0.005$

$\Rightarrow$ **$3I_1 - 3I_2 = 0.009$ (4)**

(3) + (4) ➔ $1.75I_2 = 0.0035 \Rightarrow I_2 = 2$ mA

$I_1 = (3I_2 + 0.009)/3 = 5$ mA, $I_3 = 4$ mA

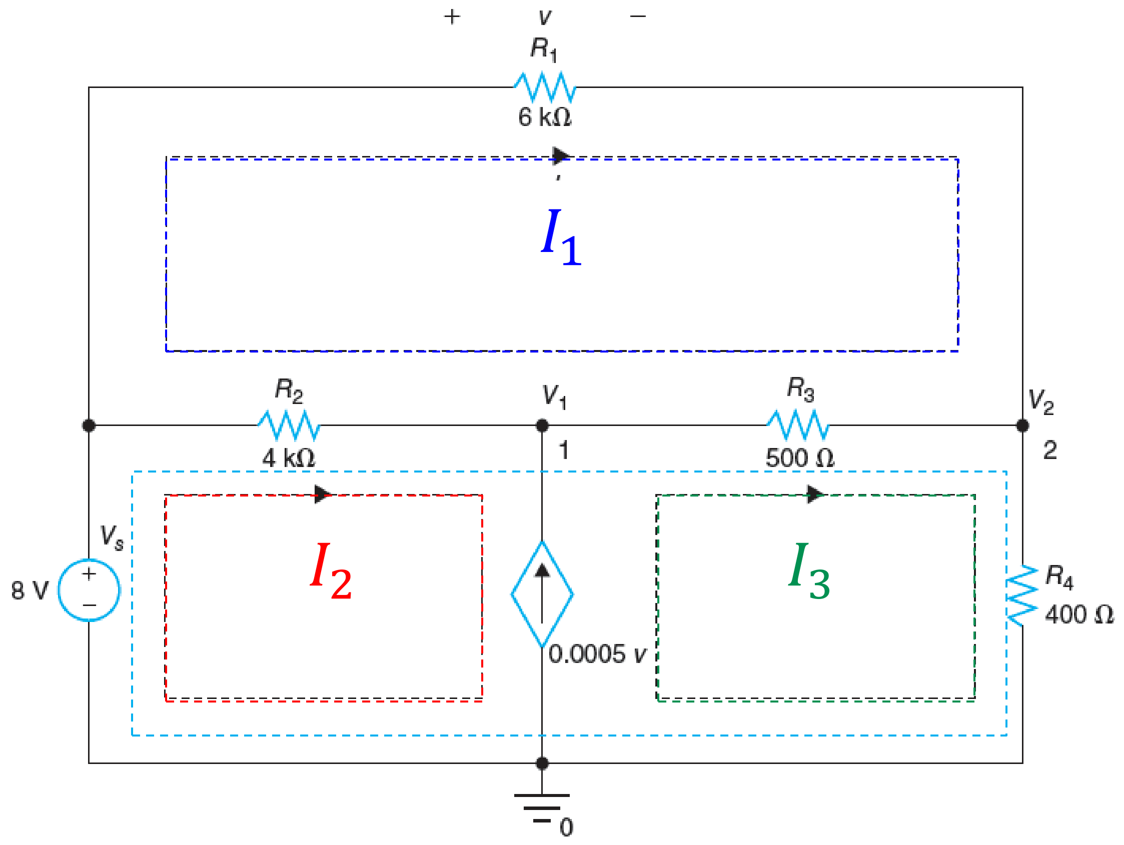
$v = R_4I_3 - R_3(I_1 - I_3) = 1$ V



EXAMPLE 3.17

- Find I_1, I_2, I_3, V_1, V_2.

- **Supermesh** is Mesh 2 & Mesh 3
  - Equation from Supermesh
  - $I_3 - I_2 = 0.0005v = 0.0005 \times 6000 I_1 \Rightarrow \mathbf{I_3 = 3I_1 + I_2}$ **(1)**



- Lets apply KVL on Supermesh and the Mesh 1

- **Supermesh** → $-V_s + V_{R_2} + V_{R_3} + V_{R_4}$

→ $-8 + 4000(I_2 - I_1) + 500(I_3 - I_1) + 400I_3 = 0$

→ $\mathbf{-4500I_1 + 4000I_2 + 900I_3 = 8}$ **(2a)**

→ **(1) int (2a)** → $-4500I_1 + 4000I_2 + 900(3I_1 + I_2) = 8$

→ $-4500I_1 + 4000I_2 + 900(3I_1 + I_2) = 8$

→ $-1800I_1 + 4900I_2 = 8$

→ $\mathbf{-1.8I_1 + 4.9I_2 = 0.008}$ **(2)**

- **KVL on Mesh 1** → $V_{R_1} + V_{R_3} + V_{R_2}$

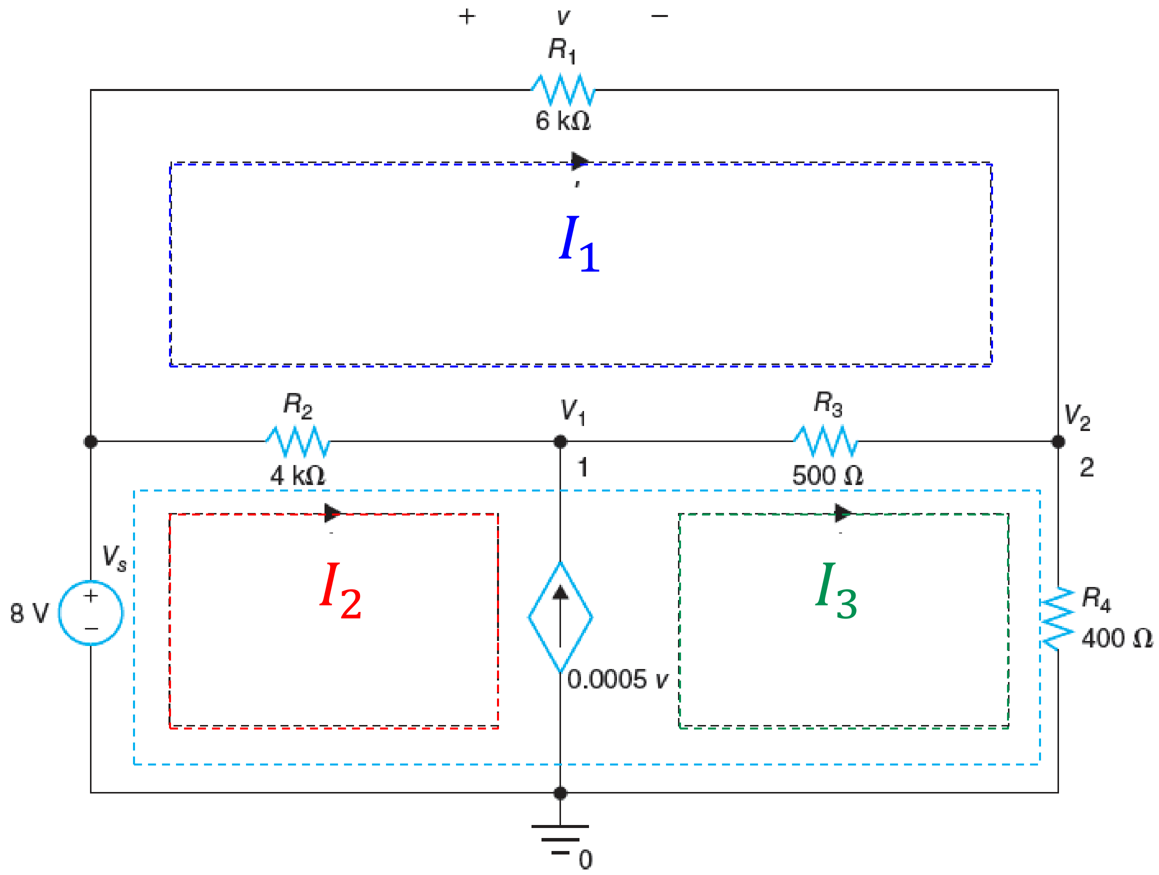
→ $6000I_1 + 500(I_1 - I_3) + 4000(I_1 - I_2) = 0$

→ $10500I_1 - 4000I_2 - 500I_3 = 0$

→ $10500I_1 - 4000I_2 - 500(3I_1 + I_2) = 0$

→ $\mathbf{9000I_1 - 4500I_2 = 0 \Rightarrow I_2 = 2I_1}$ **(3)**

EXAMPLE 3.17 (Cont..)



$I_3 = 3I_1 + I_2$ (1)

$-1.8I_1 + 4.9I_2 = 0.008$ (2)

$I_2 = 2I_1$ (3)

- Substitute (3) into (2) ➔ $-1.8I_1 + 4.9(2I_1) = 0.008 \Rightarrow 8I_1 = 0.008 \Rightarrow I_1 = 1$ mA
- Substitute I_1 into (3) → $I_2 = 2$ mA,
- Substitute I_1 and I_2 into (1) → $I_3 = 5$ mA,
- Then we can compute: $V_2 = R_4 I_3 = 2$ V
- $V_1 = V_s - R_2(I_2 - I_1) = 4$ V

Summary

- A mesh is a loop that does not contain any other loops. For each mesh, define a mesh current as the current flowing around the mesh in the clockwise direction.
- The mesh analysis is based on KVL: The sum of voltage drops around a mesh is zero. For each mesh, sum the voltage drops around the mesh in the clockwise direction and set that equal to zero. The voltage drop across a resistor is the product of the resistance and the net current (physical current) through the resistor. If the resistor is not shared by another mesh, the voltage drop is the product of the resistance and the mesh current. If the resistor is shared by another mesh, the voltage drop is the product of the resistance and the difference in mesh currents.
- If there is a current source that is a common branch between two different meshes, we do not know the voltage drop across the current source. Define a supermesh consisting of the two meshes, excluding the current source. When the voltage drops around the supermesh are added, we get the same equation that we obtain by adding the two equations for two meshes.
- The extra equation needed to find the mesh currents is obtained by representing the current of the current source by the difference of the two mesh currents. The current of the current source is obtained by subtracting the mesh current pointing in the opposite direction from the mesh current pointing in the same direction as the current source.
- What we will study next