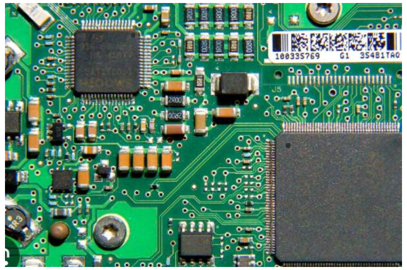
Circuit Analysis and Design

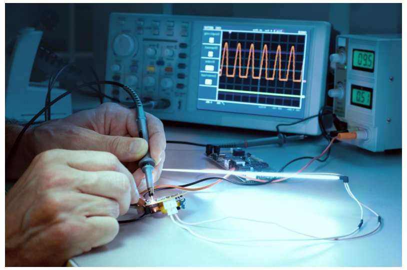
Academic Year 2025/2026 – Semester 1

Lecture 11 - Inductors

Masood Ur-Rehman, Muhammad Aslam and Ali Imran

{masood.urrehman, muhammad.aslam.2, ali.imran}@glasgow.ac.uk





Agenda

01

Inductors

Fundamental principles, physical structure, and energy storage mechanisms

02

Series Connection

Combining inductors in series configurations and calculating equivalent inductance

03

Parallel Connection

Understanding parallel inductor networks and their equivalent values

04

Practical Applications

Real-world examples and problem-solving techniques

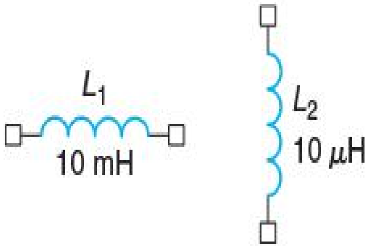
Learning Objectives: By the end of this lecture, you will understand how inductors store energy in magnetic fields, analyze inductor circuits using series and parallel combination rules, and apply these concepts to practical circuit design problems.

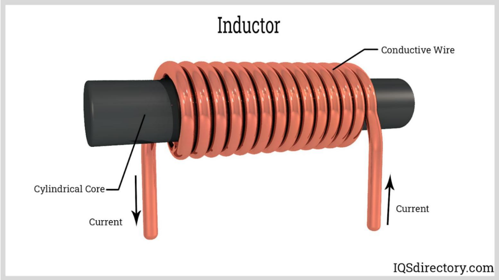
Inductors

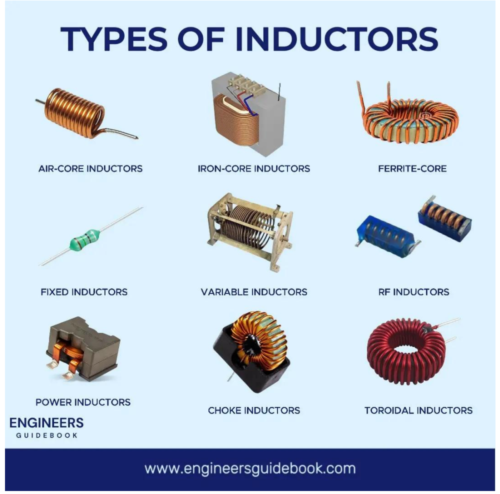
- An inductor is a passive circuit element that can store energy in the form of a magnetic field. The circuit symbol for inductors is shown in Figure 6.26.
- A simple inductor consists of a solenoid, which is a wire wound in helix.
- The core can be filled with air or a ferromagnetic material.
- When current flows through an inductor, it introduces a magnetic field. According to Faraday's law, a changing magnetic field induces an electromotive force.
- The inductance of an inductor is the measure of the inductor to generate magnetic flux for the given change of current.
- *Inductance is measured in henrys (H), named after American scientist Joseph Henry who discovered self-inductance independently of Faraday in 1832.*
- The amount of energy stored in the inductor depends on the inductance and the current through the inductor.

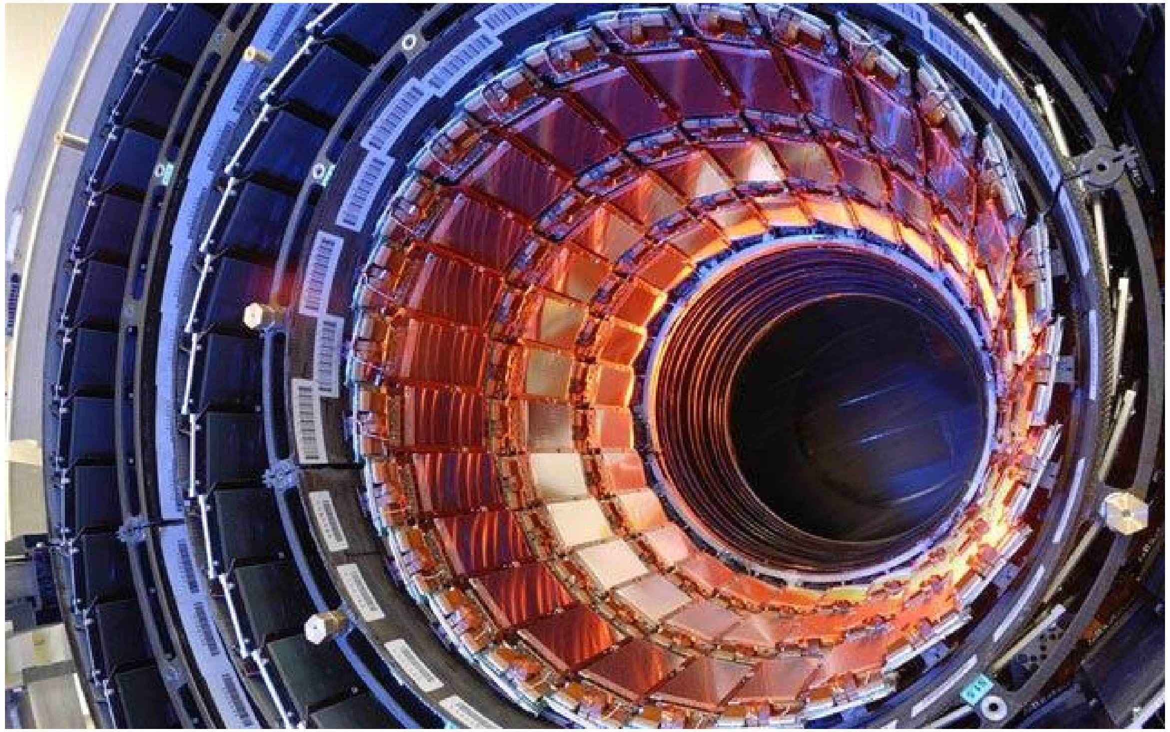
FIGURE 6.26

Circuit symbol for inductors.









Fun Fact: *The world's largest inductor is the Large Hadron Collider's toroidal magnet system, which uses superconducting coils storing 10 gigajoules of energy—enough to melt 18 tons of gold!*

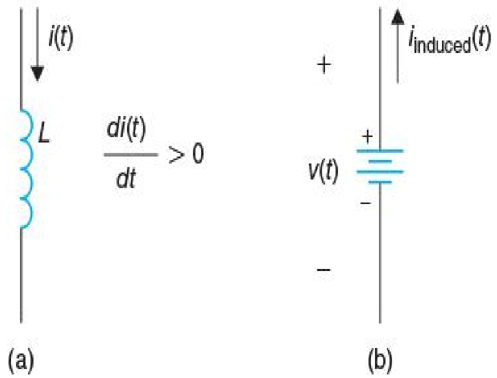
Current-Voltage Relation of an Inductor

- If an inductor is traversed in the direction of current, and the time rate of change of the current is positive, di(t)/dt > 0, as shown in Figure 6.32(a), according to Lenz's law, the current $i_{induced}(t)$ generated from the induced emf will flow in the opposite direction of i(t) to oppose the increase in i(t), as shown in Figure 6.32(b).

- To generate the induced current in the direction shown in Figure 6.32(b), the polarity of the induced voltage should be the one shown in Figure 6.32(b). Thus, the voltage v(t) is given by

FIGURE 6.32

Polarity of the voltage $v(t)$ for $\frac{di(t)}{dt} > 0$.



$$v(t) = L\frac{di(t)}{dt} \quad (1)$$

Fun Fact: This opposition to current change makes inductors invaluable in electrical engineering. In automotive ignition systems, when you suddenly interrupt current through an ignition coil (an inductor), the rapid di/dt creates a voltage spike of 20,000-50,000 volts—enough to jump the spark plug gap and ignite the fuel-air mixture. The same principle applies to why turning off inductive loads (motors, transformers) can create destructive voltage spikes that require protection circuits.

Current-Voltage Relation of an Inductor (Continued)

- If Equation (1) is integrated, we obtain

$$i(t)=\frac{1}{L}\int_{-\infty}^{t} v(\lambda)d\lambda = i(0)+\frac{1}{L}\int_{0}^{t} v(\lambda)d\lambda$$

where i(0) is the current through the inductor at t = 0.

- The instantaneous power on the inductor is given by

$$p(t)=i(t)v(t)=i(t)\left[L\frac{di(t)}{dt}\right]=Li(t)\frac{di(t)}{dt}$$

- The energy stored in the inductor is given by

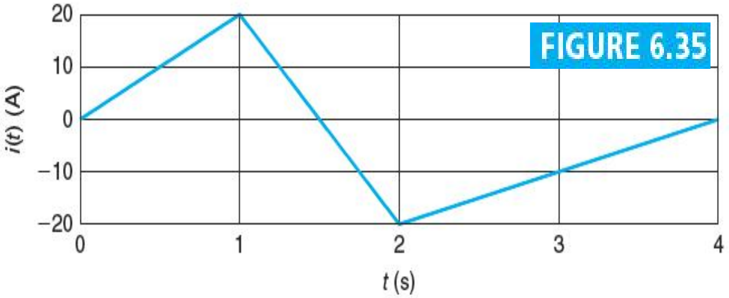
$$w(t)=\int_{-\infty}^{t} p(\lambda)d\lambda = L\int_{-\infty}^{t} i(\lambda)\frac{di(\lambda)}{d\lambda}d\lambda = L\int_{-\infty}^{t} i(\lambda)di(\lambda)=\frac{1}{2}Li^2(t)$$

- If the current through the inductor is constant, i(t) = I, the energy stored is $W = 0.5LI^2$.

EXAMPLE 6.8

- The current through an inductor with inductance of 100 mH is given by

$$i(t)=\begin{cases} 20t, & 0\le t<1 \\ -40t+60, & 1\le t<2 \\ 10t-40, & 2\le t<4 \\ 0, & \text{otherwise} \end{cases} \text{ A}$$



- The voltage across the inductor is given by?

EXAMPLE 6.8

- The current through an inductor with inductance of 100 mH is given by

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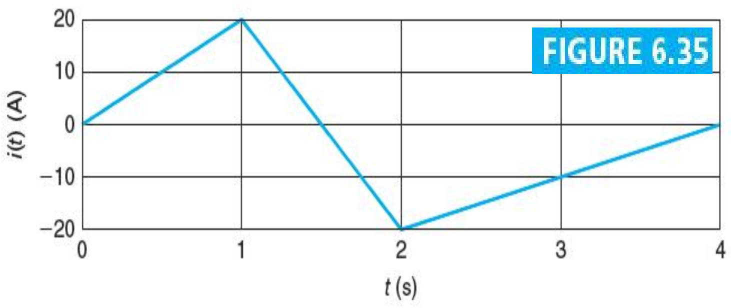
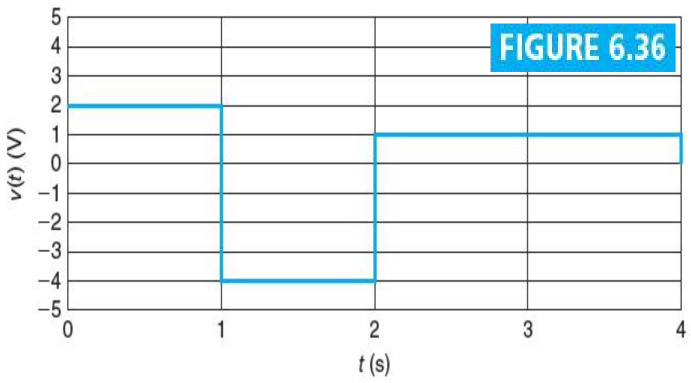


- The voltage across the inductor is given by

$$v(t)=L\frac{di(t)}{dt}=\begin{cases}0.1\times 20, & 0\le t<1\\ 0.1\times(-40), & 1\le t<2\\ 0.1\times 10, & 2\le t<4\\ 0, & \text{otherwise}\end{cases}\ \text{V}=\begin{cases}2, & 0\le t<1\\ -4, & 1\le t<2\\ 1, & 2\le t<4\\ 0, & \text{otherwise}\end{cases}\ \text{V}$$

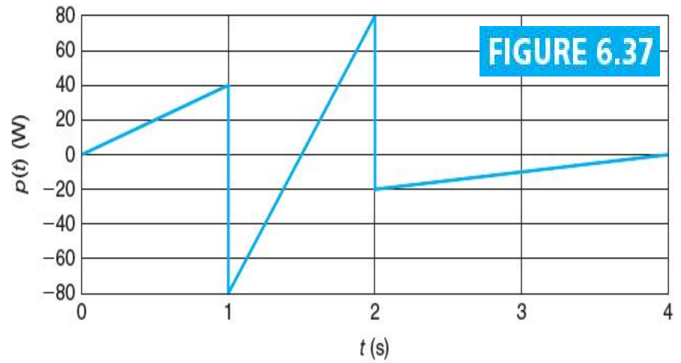


- The current i(t) is shown in Figure 6.35 and voltage v(t) is shown in Figure 6.36.

EXAMPLE 6.8 (Continued)

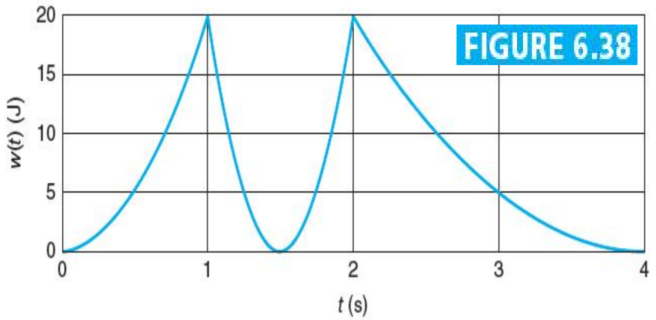
- The instantaneous power absorbed (p(t) > 0) or released (p(t) < 0) is given by

$$p(t)=v(t)i(t)=\begin{cases} 40t, & 0\le t<1 \\ 160t-240, & 1\le t<2 \\ 10t-40, & 2\le t<4 \\ 0, & \text{otherwise} \end{cases} \text{ W}$$



- The energy stored in the inductor is given by

$$w(t)=\frac{1}{2}Li^2(t)=\begin{cases} 20t^2, & 0\le t<1 \\ 80(t-1.5)^2, & 1\le t<2 \\ 5(t-4)^2, & 2\le t<4 \\ 0, & \text{otherwise} \end{cases} \text{ J}$$



- The power p(t) is shown in Figure 6.37 and energy w(t) is shown in Figure 6.38.

- The energy stored in the inductor increases during intervals 0 < t < 1 s and 1.5 s ≤ t < 2 s and the energy stored in the inductor decreases during intervals 1 s ≤ t < 1.5 s and 2 s ≤ t < 4 s.

EXAMPLE 6.9

- A signal, $v(t) = \cos(2\pi 100t)$, $t \geq 0$, V is applied to an inductor with inductance of 100 mH.
- The current through the inductor is given by

$$i(t) = \frac{1}{L}\int_0^t v(\lambda)d\lambda = \frac{1}{0.1}\int_0^t \cos(2\pi 100\lambda)d\lambda$$

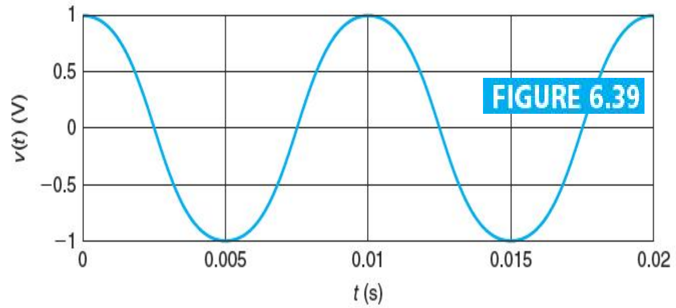
$$= \frac{10}{2\pi 100}\sin(2\pi 100t) = 0.01592\sin(2\pi 100t)\,\mathrm{A}$$

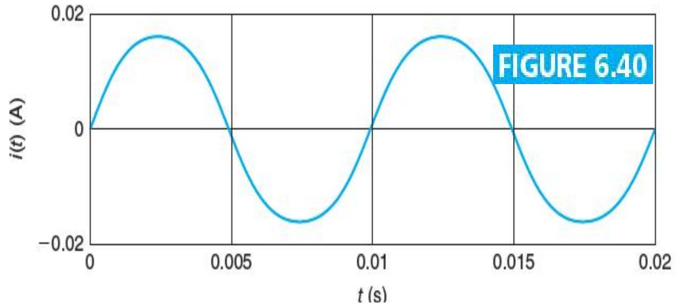
- The instantaneous power of the inductor is given by

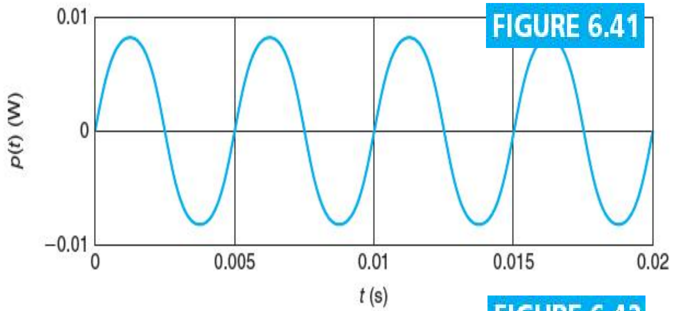
$$p(t) = v(t)i(t) = \frac{\sin(2\pi 200t)}{400\pi} = 0.007958\sin(2\pi 200t)\,\mathrm{W}$$

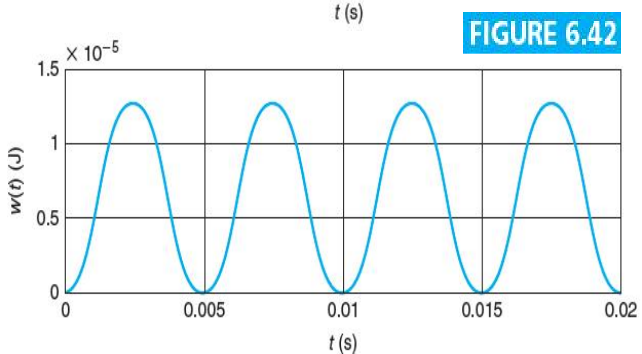
- The energy stored in the inductor is given by

$$w(t) = \frac{1}{2}Li^2(t) = 1.2665\times 10^{-5}\left[\sin(2\pi 100t)\right]^2\,\mathrm{J}$$







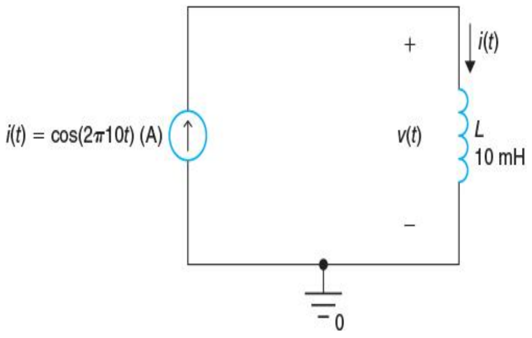


Sinusoidal Current to an Inductor

- A sinusoidal current, $i(t) = \cos(2\pi 10t)$ A is applied to an inductor with inductance 0.01 H as shown in Figure 6.43.

FIGURE 6.43

An inductor with sinusoidal input.



- The voltage across the inductor is given by

$$v(t) = L\frac{di(t)}{dt} = 0.01 \times (-1) \times (2\pi 10) \times \sin(2\pi 10t)$$

$$= -0.6283\sin(2\pi 10t) = 0.6283\cos(2\pi 10t + 90^{o})\,\mathrm{V}$$

- The phase of voltage is 90^{o}, compared to 0^{o} for the current. The current lags the voltage by 90^{o}.

- The current crosses zero T/4 s later than the voltage as shown in Figure 6.44. T is a period given by 0.1 s. T/4 s is equivalent to $360^{o}/4 = 90^{o}$

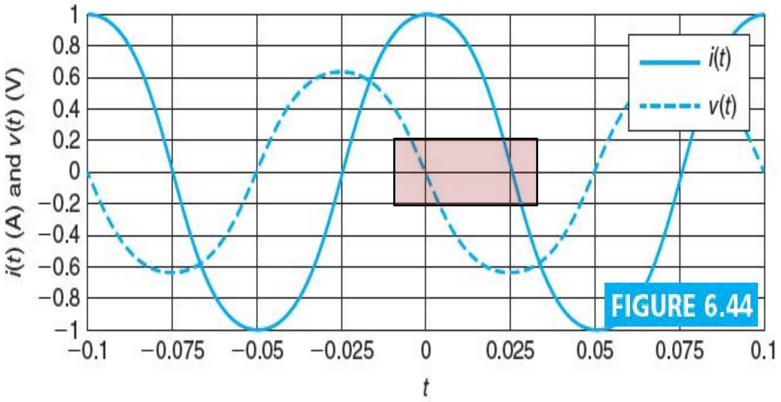


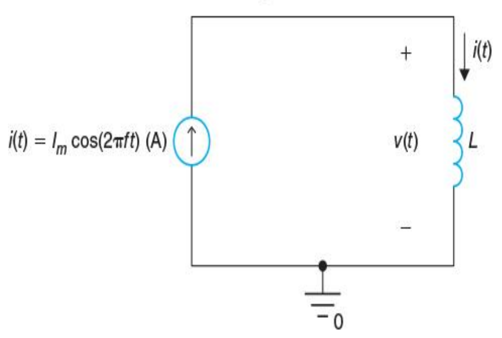
FIGURE 6.44

Sinusoidal Current to an Inductor (Continued)

- A sinusoidal current, $i(t) = I_m \cos(2\pi ft)$ is applied to an inductor with inductance L as shown in Figure 6.45.

FIGURE 6.45

An inductor with sinusoidal input.



- The voltage across the inductor is given by

$$v(t) = L\frac{di(t)}{dt} = LI_m \times (-1) \times (2\pi f) \times \sin(2\pi ft) = -LI_m 2\pi f \times \sin(2\pi ft)$$

- The amplitude of the voltage is proportional to the frequency of the current applied. As the frequency decreases, the amplitude decreases as shown in Figure 6.46.

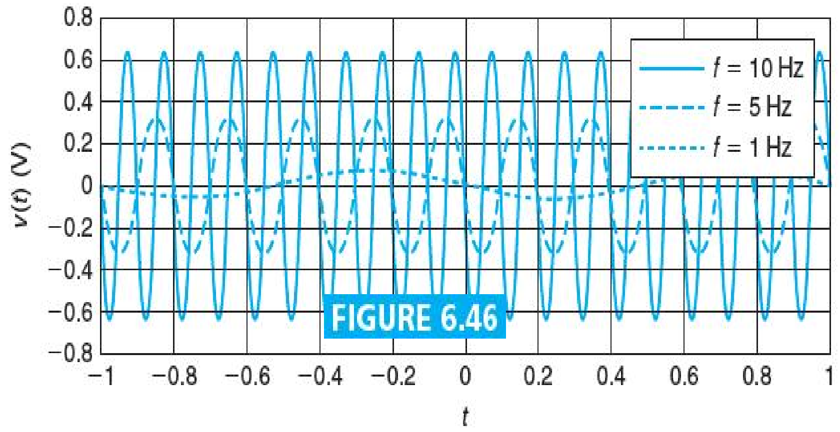


FIGURE 6.46

- For a dc current (f = 0), the voltage across the inductor is zero in the steady state.

- The inductor acts as a short circuit for a dc voltage, and acts as an open circuit for high frequency voltage.

Series Connection of Inductors

- Figure 6.47(a) shows *n* inductors connected in series. The current through the *n* inductors is i(t).

- Let $v_1(t)$ be the voltage across L_1, $v_2(t)$ be the voltage across L_2, . . ., $v_n(t)$ be the voltage across L_n, then the total voltage v(t) across all *n* inductors is

  $$v(t) = v_1(t) + v_2(t) + \ldots + v_n(t) \quad (1)$$

- Substitution of the i-v relation $$v_k(t) = L_k \frac{di(t)}{dt}, \quad 1 \le k \le n$$ to Equation (1) yields

$$v(t) = v_1(t) + v_2(t) + \ldots + v_n(t) = L_1 \frac{di(t)}{dt} + L_2 \frac{di(t)}{dt} + \ldots + L_n \frac{di(t)}{dt}$$

$$= \left(L_1 + L_2 + \ldots + L_n\right) \frac{di(t)}{dt} = L_{eq} \frac{di(t)}{dt}$$

where the equivalent inductance, L_{eq}, is given by

$$L_{eq} = L_1 + L_2 + \ldots + L_n$$

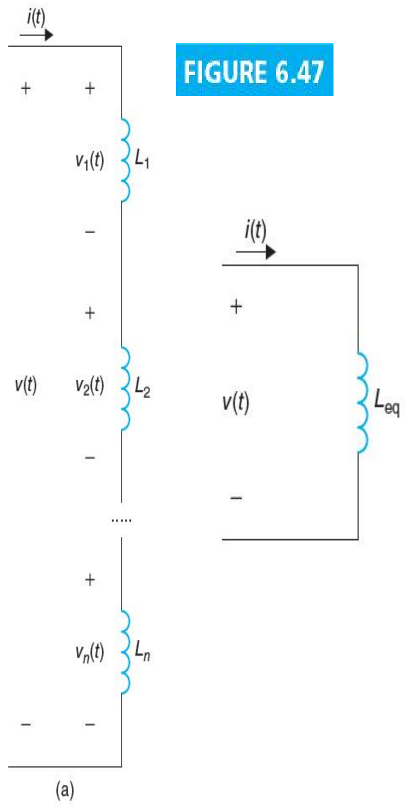


FIGURE 6.47

Parallel Connection of Inductors

- Figure 6.48(a) shows n inductors connected in parallel. The voltage across the n inductors is v(t).

- Let $i_1(t)$ be the current through L_1, $i_2(t)$ be the current through L_2, . . ., $i_n(t)$ be the current through L_n, then the total current i(t) through all n inductors is

$$i(t) = i_1(t) + i_2(t) + \ldots + i_n(t) \quad (1)$$

- Substitution of the i-v relation to Equation (1) yields

$$i_k(t) = \frac{1}{L_k}\int_{-\infty}^{t} v(\lambda)d\lambda,\ 1 \le k \le n$$

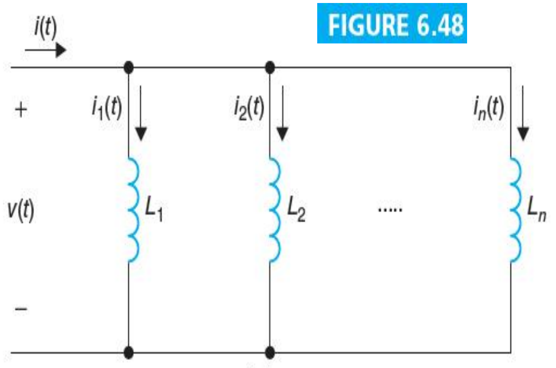
$$i(t) = \frac{1}{L_1}\int_{-\infty}^{t} v(\lambda)d\lambda + \frac{1}{L_2}\int_{-\infty}^{t} v(\lambda)d\lambda + \ldots + \frac{1}{L_n}\int_{-\infty}^{t} v(\lambda)d\lambda$$

$$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_n}\right)\int_{-\infty}^{t} v(\lambda)d\lambda = \frac{1}{L_{eq}}\int_{-\infty}^{t} v(\lambda)d\lambda$$

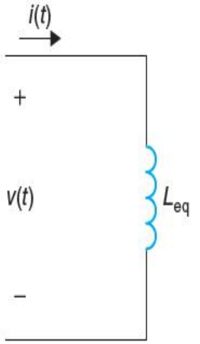
where equivalent inductance, L_{eq} is

$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_n}}$$

FIGURE 6.48



(a)



(b)

Parallel Connection of Inductors (Continued)

- Notice that the equation for the equivalent inductance of the parallel connected inductors is similar to the equivalent resistance of parallel connected resistors.

- The equivalent inductance of two inductors with inductances L_1 and L_2 connected in parallel is given by

$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}} = \frac{L_1 L_2}{L_1 + L_2}$$

- The equivalent inductance of three inductors with inductances L_1, L_2, and L_3 connected in parallel is given by

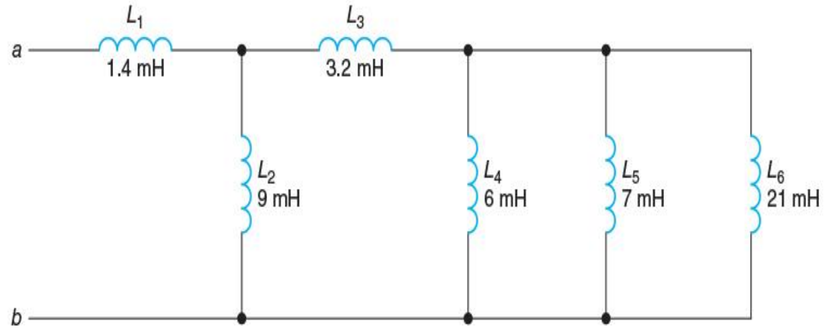
$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}} = \frac{L_1 L_2 L_3}{L_1 L_2 + L_1 L_3 + L_2 L_3}$$

EXAMPLE 6.10

Find the equivalent inductance L_{eq} between *a* and *b* for the circuit shown in Figure 6.49.

FIGURE 6.49

Circuit for EXAMPLE 6.10.



EXAMPLE 6.10

Find the equivalent inductance L_{eq} between *a* and *b* for the circuit shown in Figure 6.49.

- Let $L_a = L_4 \parallel L_5 \parallel L_6$. Then,

$$L_a = \frac{1}{\frac{1}{L_4}+\frac{1}{L_5}+\frac{1}{L_6}} = \frac{1}{\frac{1}{6}+\frac{1}{7}+\frac{1}{21}} \text{mH} = \frac{42}{\frac{42}{6}+\frac{42}{7}+\frac{42}{21}} \text{mH} = \frac{42}{7+6+2} \text{mH} = \frac{42}{15} \text{mH} = 2.8 \text{mH}$$

- Let $L_b = L_3 + L_a$. Then,

  $L_b = 3.2$ mH + 2.8 mH = 6 mH

- Let $L_c = L_2 \parallel L_b$. Then,

  $L_c = L_2L_b/(L_2 + L_b)$

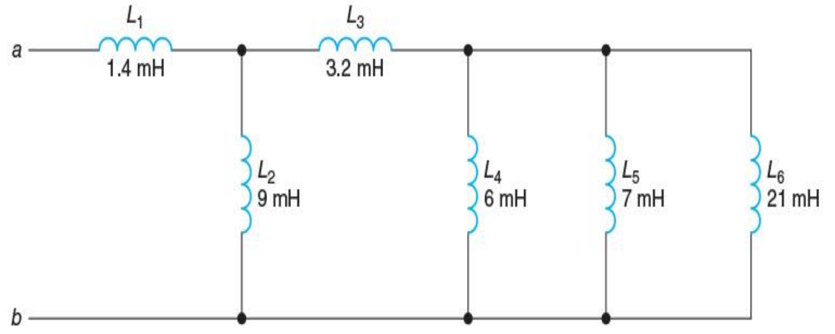
  = 54/15 mH = 3.6 mH

- $L_{eq} = L_1 + L_c$ = 1.4 mH + 3.6 mH

  = 5 mH

FIGURE 6.49

Circuit for EXAMPLE 6.10.



EXAMPLE 6.11

For the circuit shown in Figure 6.51, find the equivalent circuit consisting of a single inductor and a single capacitor connected in parallel between *a* and *b.*

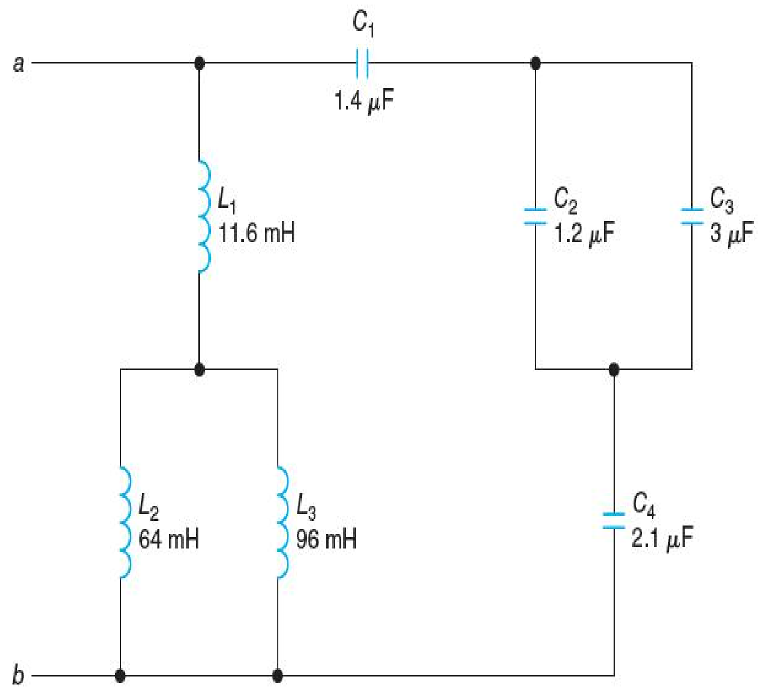


FIGURE 6.51

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FIGURE 6.51

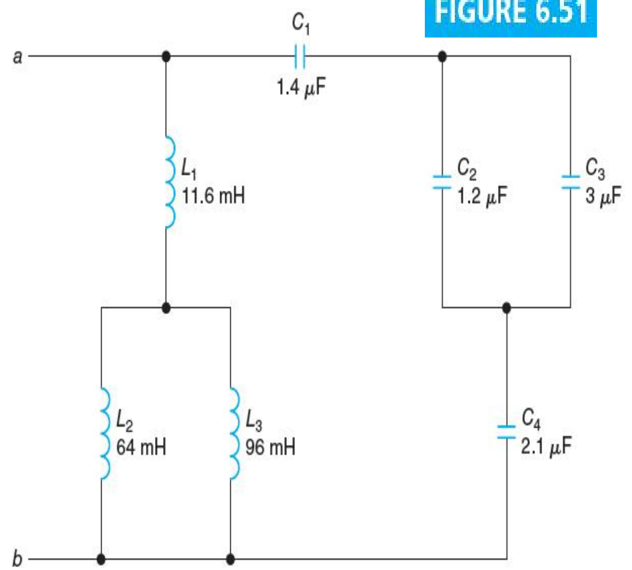


FIGURE 6.52

EXAMPLE 6.11

For the circuit shown in Figure 6.51, find the equivalent circuit consisting of a single inductor and a single capacitor connected in parallel between *a and b.*

- Let $C_5 = C_2 + C_3 = 4.2\ \mu F$. The equivalent capacitance is

$$C_{eq} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_5}+\frac{1}{C_4}} = \frac{1}{\frac{1}{1.4}+\frac{1}{4.2}+\frac{1}{2.1}}\mu F = \frac{4.2}{6}\mu F = 0.7\mu F$$

- .

FIGURE 6.51

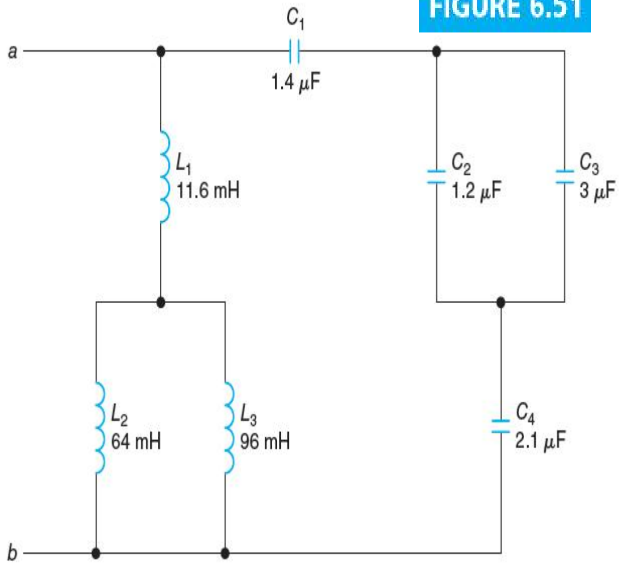
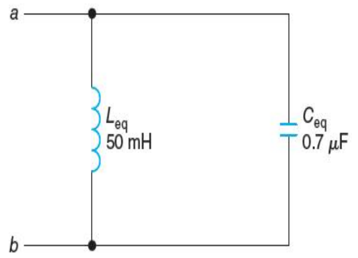


FIGURE 6.52



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For the circuit shown in Figure 6.51, find the equivalent circuit consisting of a single inductor and a single capacitor connected in parallel between *a and b.*

- Let $C_5 = C_2 + C_3 = 4.2\ \mu F$. The equivalent capacitance

$$C_{eq} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_5}+\frac{1}{C_4}} = \frac{1}{\frac{1}{1.4}+\frac{1}{4.2}+\frac{1}{2.1}}\mu F = \frac{4.2}{6}\mu F = 0.7\mu F$$

- Let $L_a = L_2 \,||\, L_3$. Then,

$$L_a = \frac{L_2 \times L_3}{L_2 + L_3} = \frac{64 \times 96}{64+96} mH = \frac{6144}{160} mH = 38.4 mH$$

- $L_{eq} = L_1 + L_a = 11.6\ mH + 38.4\ mH = 50\ mH$

- The circuit with L_{eq} and C_{eq} is shown in Figure 6.52.

FIGURE 6.51

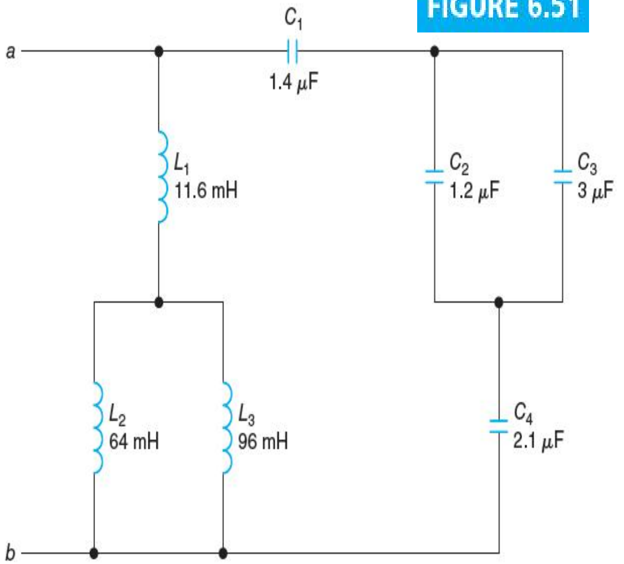
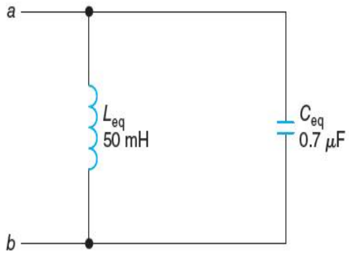


FIGURE 6.52



Summary (Continued)

- An inductor is a passive circuit element that can store energy in the form of a magnetic field.

- The voltage – current relation of an inductor is given by

$$v(t) = L\frac{di(t)}{dt}$$

- The current can be represented as

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\lambda)\,d\lambda$$

- The energy stored in the inductor is given by

$$w(t) = \frac{1}{2}Li^2(t)$$

Summary (Continued)

- If *n* inductors are connected in series, the equivalent inductance, L_{eq}, is given by

$$L_{eq} = L_1 + L_2 + \ldots + L_n$$

- If *n* inductors are connected in parallel, the equivalent inductance, L_{eq}, is given by

$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_n}}$$

- What we will study next