3.9 Measurement of the Apex Angle of a Prism and the Wavelengths of Mercury Lights Using a Spectrometer

Prelab Assignment 3.9

(1)What is autocollimation? Describe the concept briefly in words with the aid of a diagram. (Hint: Autocollimation is one of methods to measure the focal length of a convex lens.)

(2)Some of the gratings used in the lab have 600 lines/mm etched on them.

Ⅰ. Find the distance, *d*, between the lines for such a grating.

Ⅱ. Suppose the wavelength of the incident light is 435.83 nm, compute the first-order diffraction angle.

3.9.1 Introduction and Objectives

A spectrometer is an instrument used to measure properties of light over a specific portion of the electromagnetic spectrum. In its simplest form, a spectrometer is nothing more than a prism or a diffraction grating and a protractor. However, because of the need for very sensitive detection and precise measurement, a real spectrometer is a bit more complicated.

After performing this experiment and analyzing the data, you should be able to:

(1)Explain the functions of various components of a spectrometer.

(2)Determine the apex angle of a glass prism.

(3)Observe the spectrum of a mercury vapor lamp and record the angle of deviation for the spectral lines.

3.9.2 Required Equipment

- A spectrometer(Fig. 3.9-1)
- A glass prism
- A diffraction grating
- A mercury vapor lamp

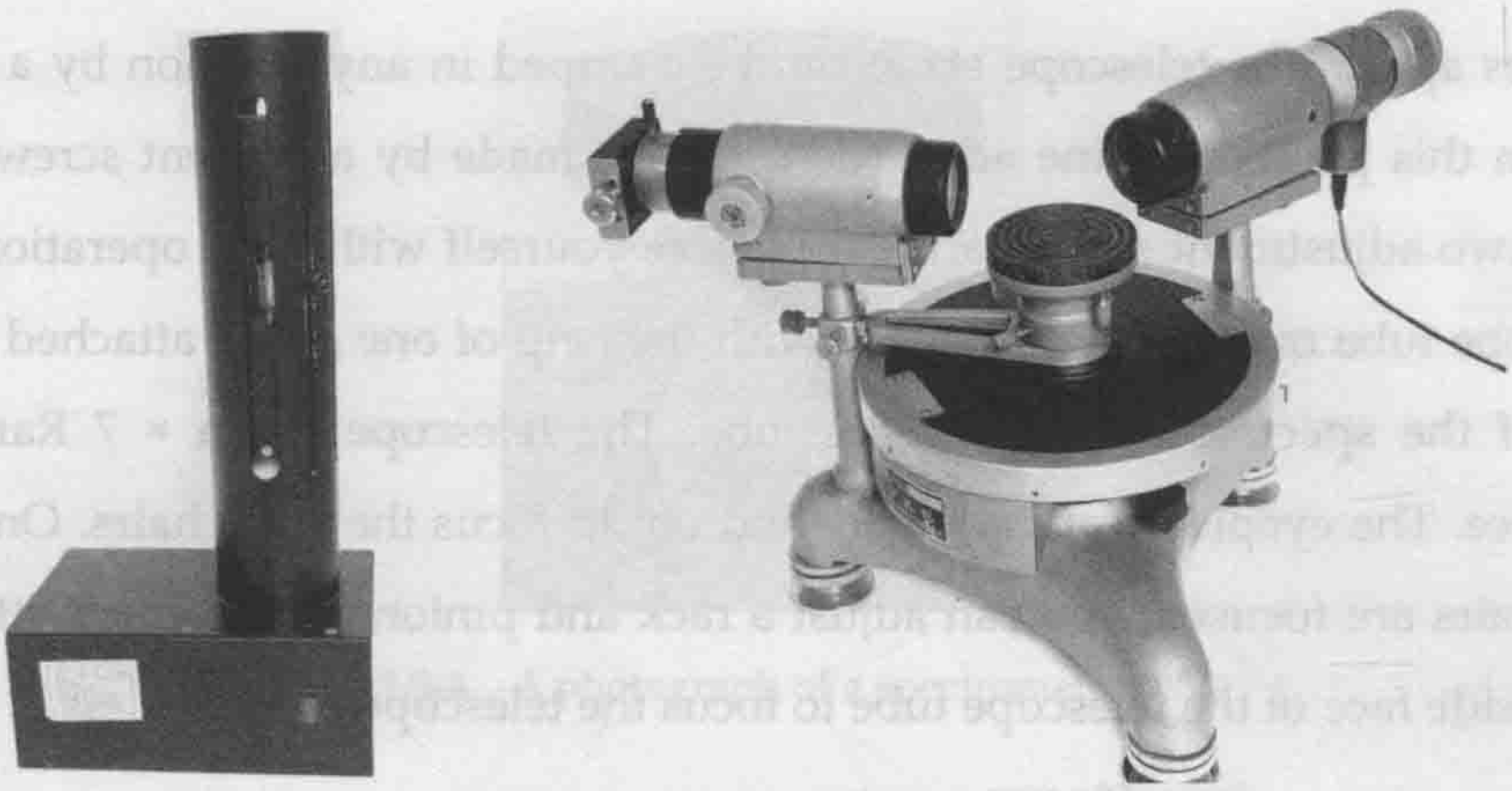


Fig. 3.9-1 A mercury vapor lamp and a spectrometer

3.9.3 Theory

3.9.3.1 The Structure of a Spectrometer

As shown in Fig. 3.9-2, a spectrometer consists of three basic components: a collimator, a diffracting element (a prism or a grating), and a telescope.

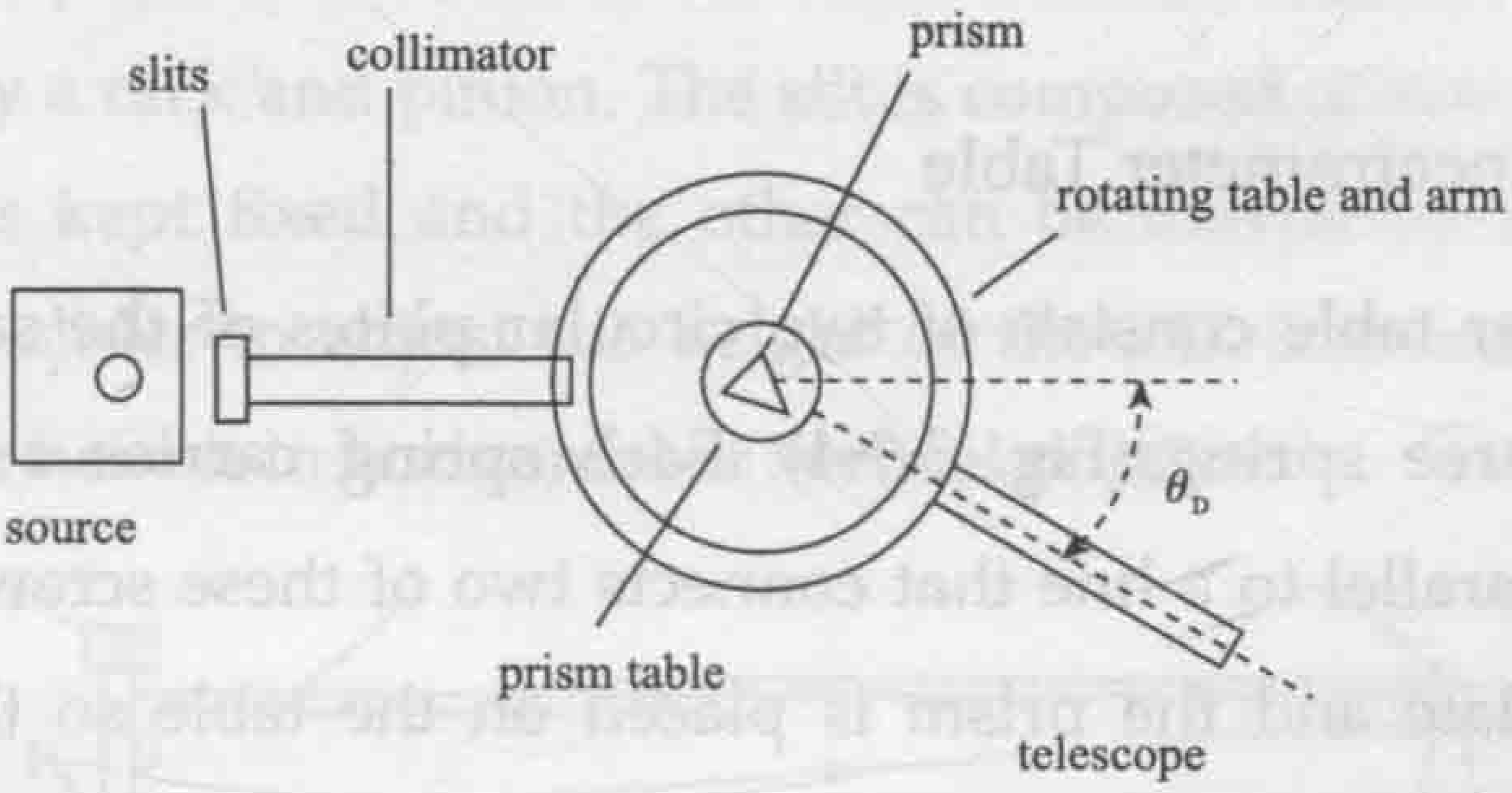


Fig. 3.9-2 Schematic diagram of an optical spectrometer

3.9.3.2 A Telescope

The schematic of a telescope is shown in Fig. 3.9-3. An astronomical (i. e. inverting) telescope with an achromatic objective and eyepiece is mounted on one arm of the spectrometer. The arm can be turned around a vertical axis passing through the center of the spectrometer. A graduated circular scale is attached to the telescope arm. This scale can be read by two verniers 180 degrees apart. The telescope stage can be clamped in any position by a screw and, in this position a fine adjustment can be made by a tangent screw. Find these two adjustment screws and familiarize yourself with their operation. The telescope tube may be slightly tilted with the help of one screw attached to the arm of the spectrometer under the tube. The telescope has a × 7 Ramsden eyepiece. The eyepiece can be slid in and out to focus the cross hairs. Once the crosshairs are focused, you can adjust a rack and pinion arrangement attached to the side face of the telescope tube to focus the telescope.

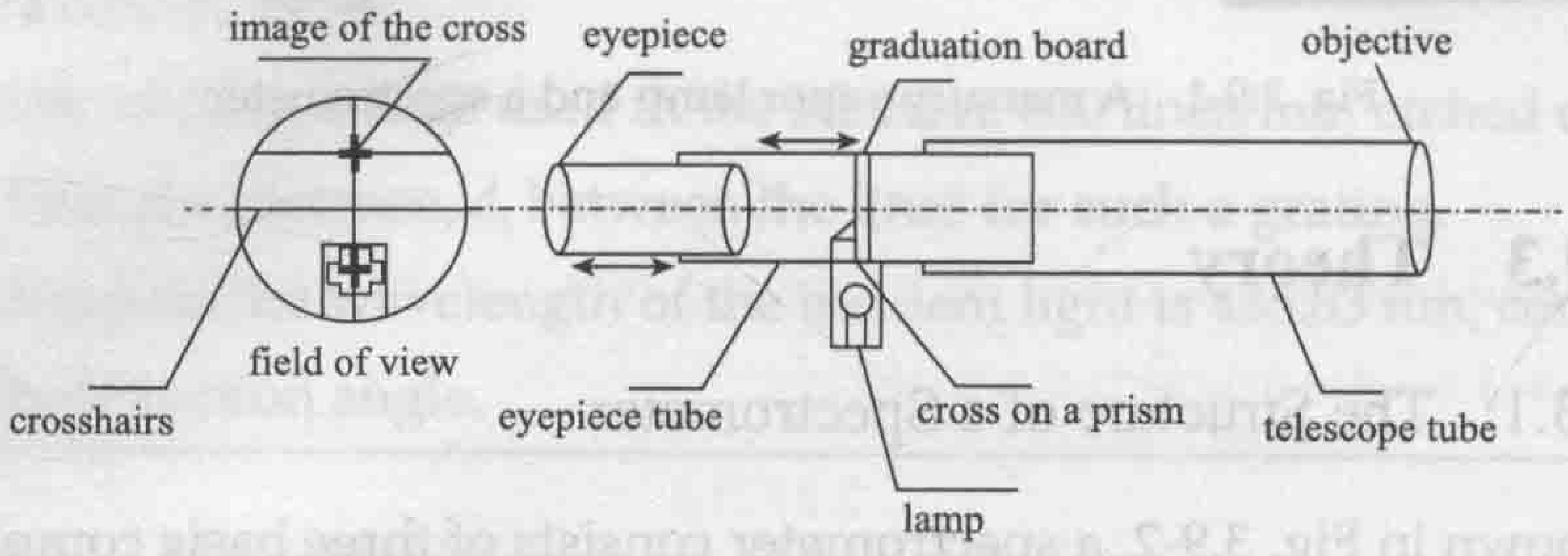


Fig. 3.9-3 Schematic diagram of a telescope and its field of view

3.9.3.3 Spectrometer Table

The spectrometer table consists of two circular plates of the same radius and separated by three springs(Fig. 3.9-4). Each spring carries a leveling screw. Straight lines, parallel to a line that connects two of these screws are engraved on the upper plate and the prism is placed on the table so that a reflecting surface is perpendicular or parallel to these lines. The vertical axis of rotation of the spectrometer passes through the center of the spectrometer table. The height of the spectrometer table can be adjusted with a clamping screw that fixes the table to the circular verniers. Thus the table can be moved around the vertical axis and its position (relative to the telescope arm scale) can be read by the verniers. Like the telescope arm it can also be fixed at any desired angle by a clamping screw and rotated very slowly using a tangent screw. Familiarize yourself with these adjustments.

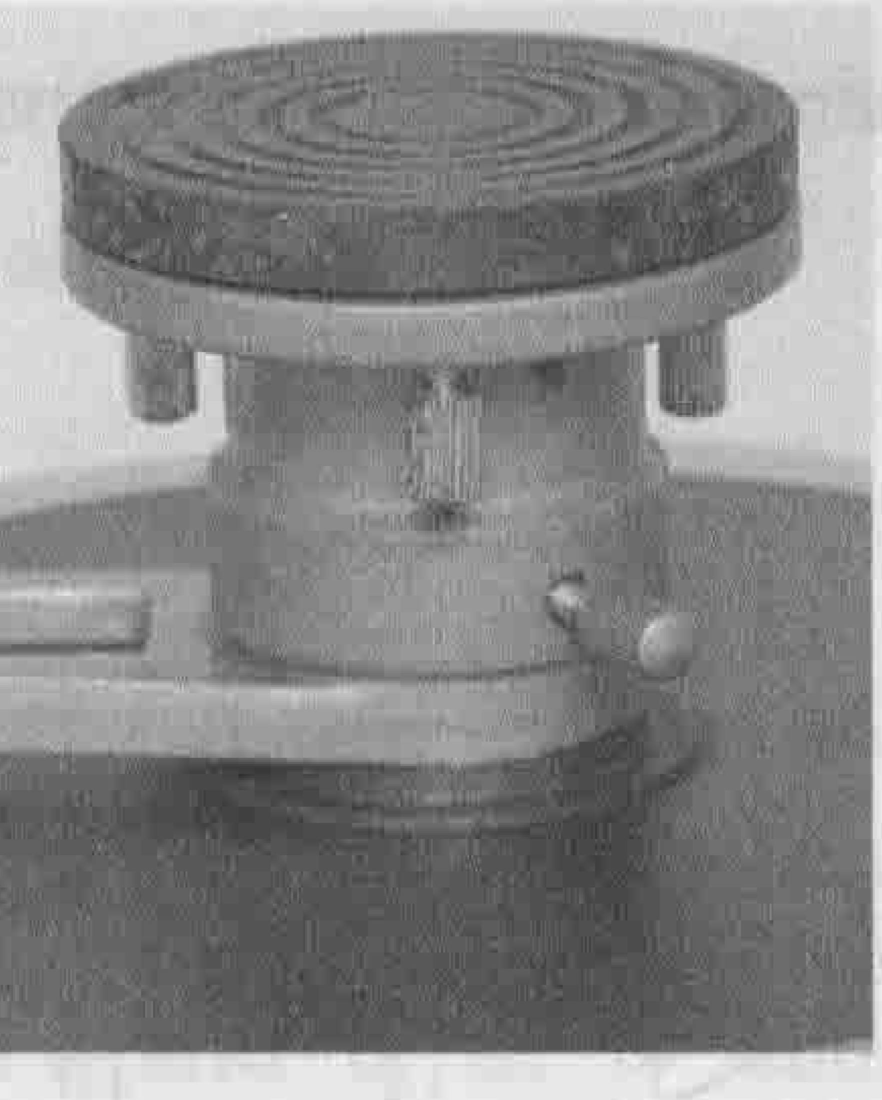


Fig. 3.9-4 A photograph of a spectrometer table

3.9.3.4 Collimator

The schematic of a collimator is shown in Fig. 3.9-5. The purpose of the collimator is to produce a parallel beam of light i. e. an object at infinity. It consists of a tube mounted horizontally on another arm of the spectrometer. This arm is fixed to the spectrometer base. The end of the tube facing the prism table has an achromatic converging lens and the other end carries a sliding tube attached to an adjustable vertical slit. The distance between the slit and the lens can be varied by a rack and pinion. The slit is composed of two sharp edges out of which one is kept fixed and the other can be moved by using the screw. There is also a slider to adjust the height of the slit. The entire "slit" can be rotated to either have it in horizontal or vertical position.

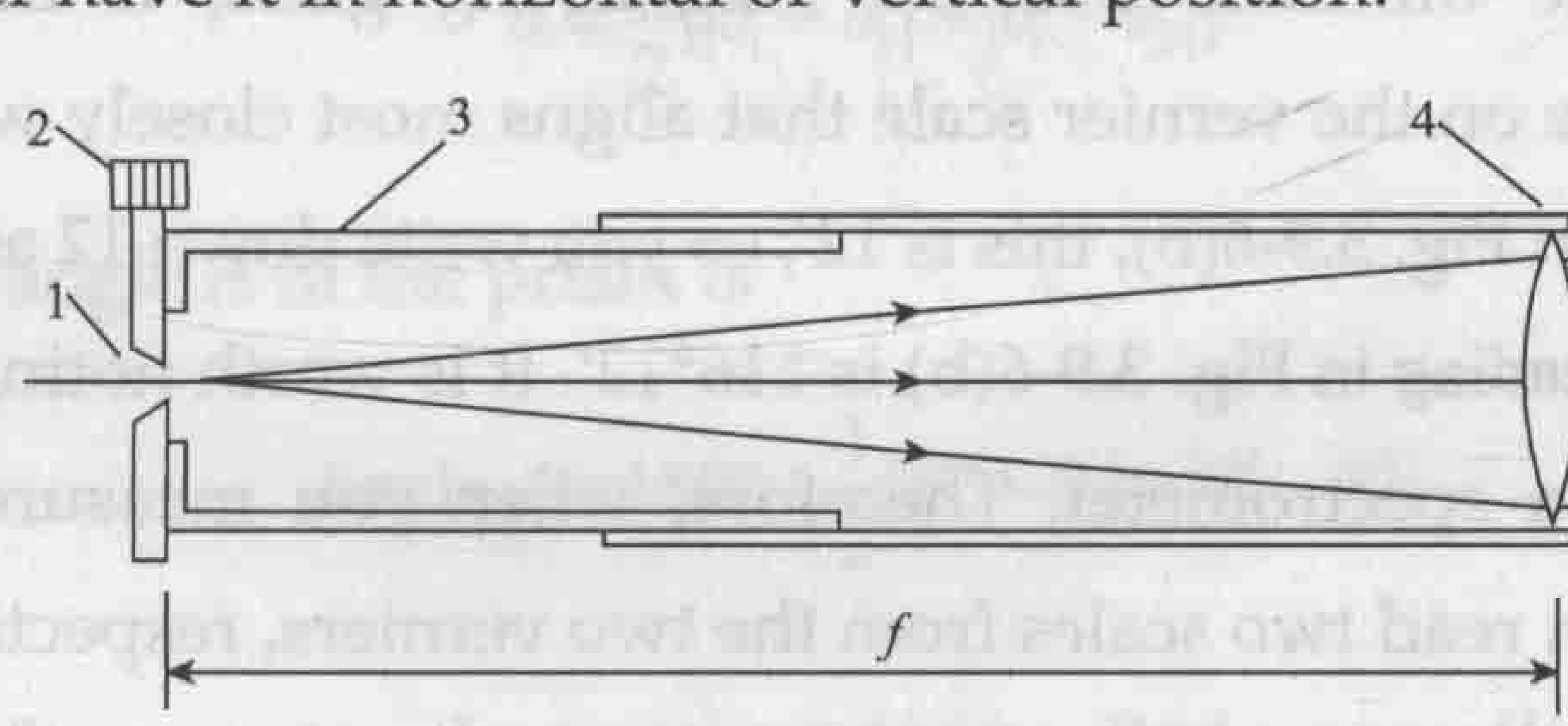
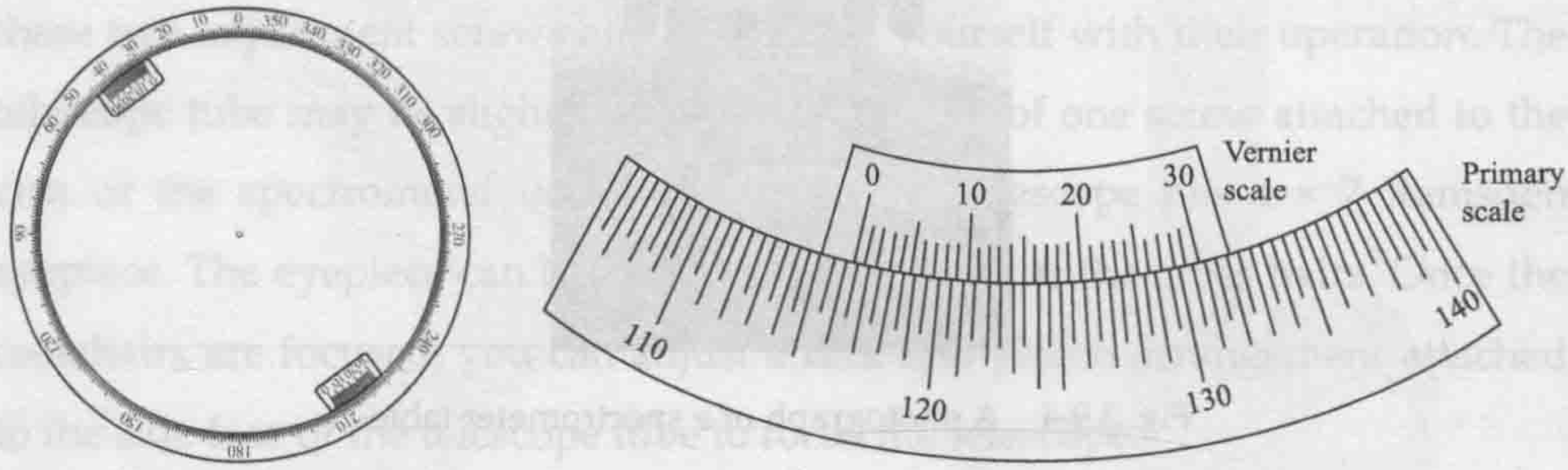


Fig. 3.9-5 A collimator: 1. slit; 2. screw; 3. tube; 4. convex lens

3.9.3.5 Scales of a Spectrometer

A spectrometer consist of a graduated circular scale attached to the telescope arm and two verniers 180 degrees apart (see Fig. 3.9-6). The primary circular scale is in 0.5° or 30′ increment. There are thirty equal parts on the vernier. So, the smallest division is 1′.



(a) (b)

Fig. 3.9-6 Scales of a spectrometer

(a) schematic diagram of the scales; (b) vernier scales

Follow the procedure below to read the scales shown in Fig. 3.9-6 (b).

(1)Find the Zero. Locate where the zero mark of the vernier scale aligns with the degree plate.

(2)Record the degrees to the nearest, lowest 0.5°. When the zero of the top scale is between two lines on the degree plate, use the smaller value. In Fig. 3.9-6(b), the zero is between the lines 116°0′ and 116° 30′.116°0′ is the smaller of the two, so you record the degrees as 116°0′.

(3)Record the minutes. Use the magnifying glass, if there is one, to determine the line on the vernier scale that aligns most closely with any line on the degree plate. In Fig. 3.9-6(b), this is 12′, so you write down 12 as the minutes.

So, the final reading in Fig. 3.9-6(b) is 116°12′. It is worth noting that there are two verniers for a spectrometer. Therefore, when you measure the telescope arm angle you can read two scales from the two verniers, respectively.

3.9.3.6 Determination of the Apex Angle of a Prism by Autocollimation Method

Figure 3.9-7 shows a diagram to measure the apex angle of a standard triangular prism. At position 1, adjust the telescope to make its optical axis perpendicular to the side surface *AB*. Then rotate the telescope to position 2 and make the telescope's optical axis again perpendicular to the side surface AC. In the quadrilateral in Fig. 3.9-7, we have

$$\text{Angle } A = 180° - \alpha \tag{3.9-1}$$

At position 1, we can read two angles, θ_1 and θ_1', from the two verniers 180 degrees apart (see Fig. 3.9-6). At position 2, we can also read two angles, θ_2 and θ_2'. The difference between θ_1 and θ_2, or θ_1' and θ_2' is the angle α. So,

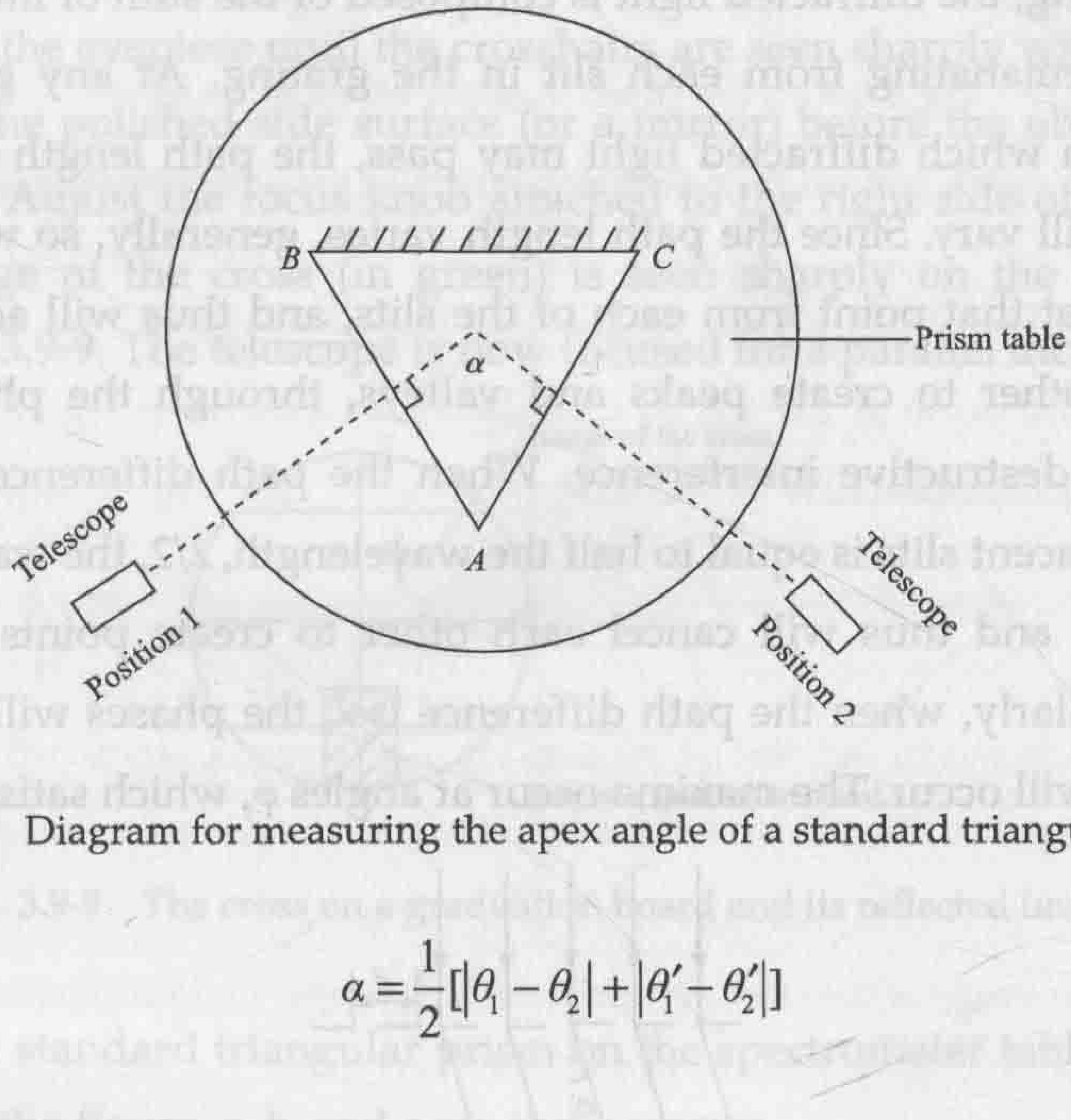


Fig. 3.9-7 Diagram for measuring the apex angle of a standard triangular prism

$$\alpha = \frac{1}{2}[|\theta_1 - \theta_2| + |\theta_1' - \theta_2'|] \tag{3.9-2}$$

Thus, the apex angle A of the prism is

$$\text{Angle } A = 180° - \frac{1}{2}[|\theta_1 - \theta_2| + |\theta_1' - \theta_2'|] \tag{3.9-3}$$

This method of measuring the apex angle is called *autocollimation*.

3.9.3.7 Measuring Wavelengths by a Diffraction Grating

Gratings may be of the *reflective* or *transmissive* type and they are called reflection grating or diffraction grating, respectively. We use a diffraction grating in this experiment. The term *diffraction* is a bit misleading, because the structure in the pattern observed is dominated by interference effects. However, the interference effect is the consequence of diffraction.

In Fig. 3.9-8, there is an idealized grating made up of a set of slits of spacing d. Normally, d is wider than the wavelength of interest to cause diffraction. Assume a plane wave of wavelength λ at normal incidence (perpendicular to the grating). Each slit in the grating acts as a quasi point-source from which light propagates in all directions. After light interacts with the grating, the diffracted light is composed of the sum of interfering wave components emanating from each slit in the grating. At any given point in space through which diffracted light may pass, the path length to each slit in the grating will vary. Since the path length varies, generally, so will the phases of the waves at that point from each of the slits, and thus will add or subtract from one another to create peaks and valleys, through the phenomenon of additive and destructive interference. When the path difference between the light from adjacent slits is equal to half the wavelength, $\lambda/2$, the waves will all be out of phase, and thus will cancel each other to create points of minimum intensity. Similarly, when the path difference is λ, the phases will add together and maxima will occur. The maxima occur at angles φ, which satisfy

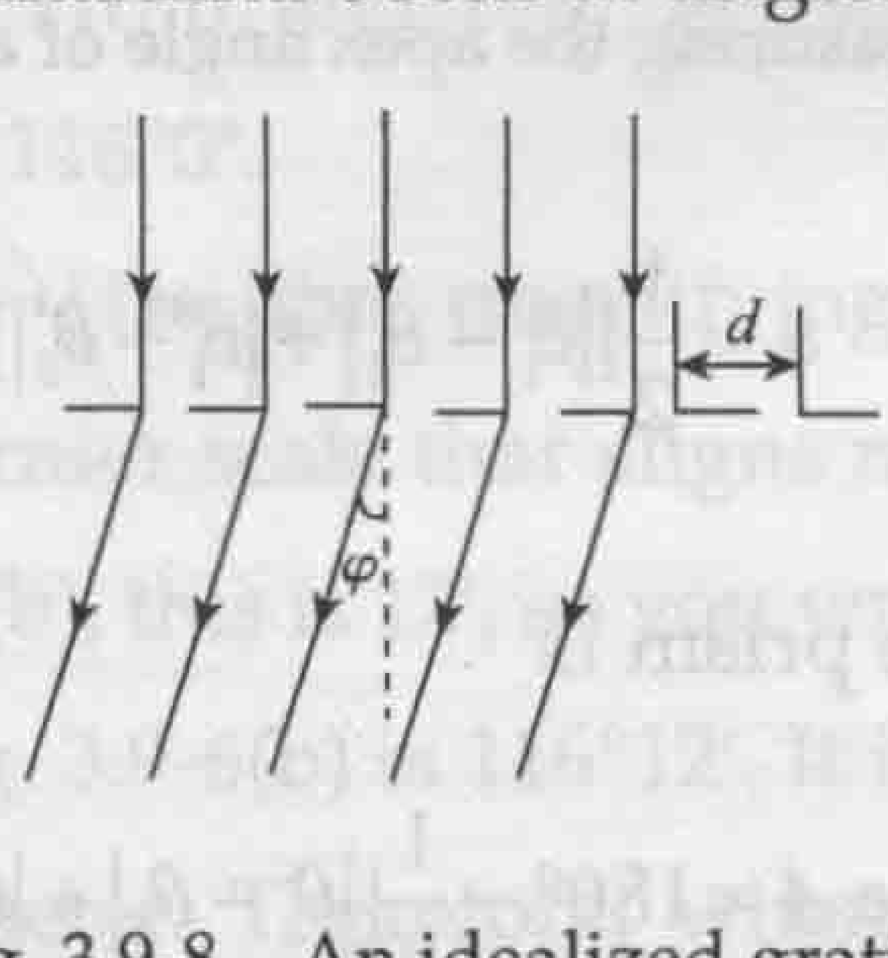


Fig. 3.9-8 An idealized grating

$$d\sin\varphi = k\lambda \quad (k=0, \pm1, \pm2, \cdots) \tag{3.9-4}$$

The above relationship between the grating spacing d and the angles of the incident and diffracted beams of light is known as the grating equation.

In this experiment a spectrometer is first carefully set up for a parallel light. It is then used to observe the emissions from a mercury vapor lamp. A diffraction grating is placed on the spectrometer table, which diffracts the green spectral line from the lamp to a certain angle. This angle is measured and the result used to determine the spacing *d* of the grooves on the diffraction grating. This same grating is then used to measure the wavelengths of a number of other lines in the spectra of mercury vapor discharge lamps.

3.9.4 Experimental Procedure

3.9.4.1 Determination of the Apex Angle

(1)Turn on the power of the lamp of the telescope.

(2)Rotate the eyepiece until the crosshairs are seen sharply with your eye.

(3)Place the polished side surface (or a mirror) before the objective lens of the telescope. Adjust the focus knob attached to the right side of the telescope until the image of the cross (in green) is seen sharply on the crosshairs, as shown in Fig. 3.9-9. The telescope is now focused for a parallel incident beam.

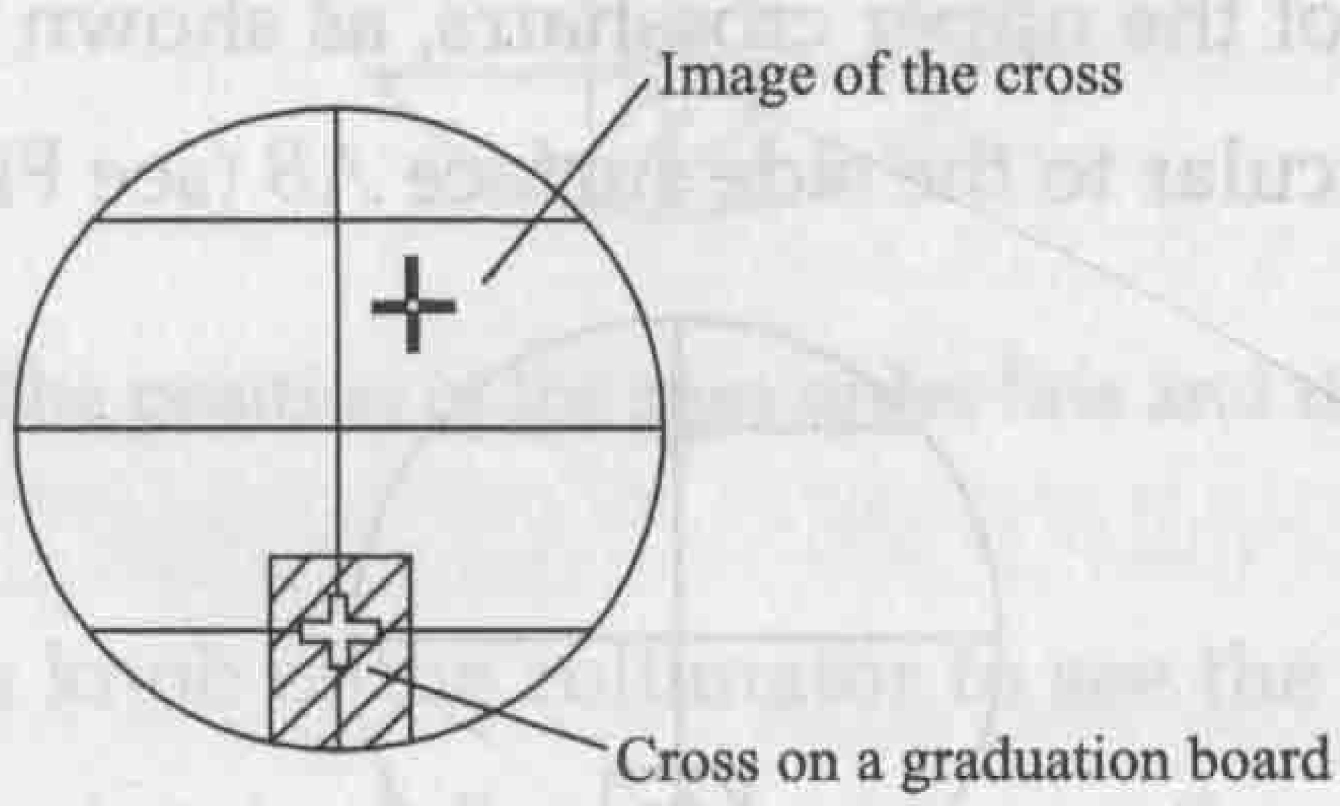


Fig. 3.9-9 The cross on a graduation board and its reflected image

(4)Place a standard triangular prism on the spectrometer table as shown in Fig. 3.9-10. In the figure, *a*, *b*, and *c* are three screws.

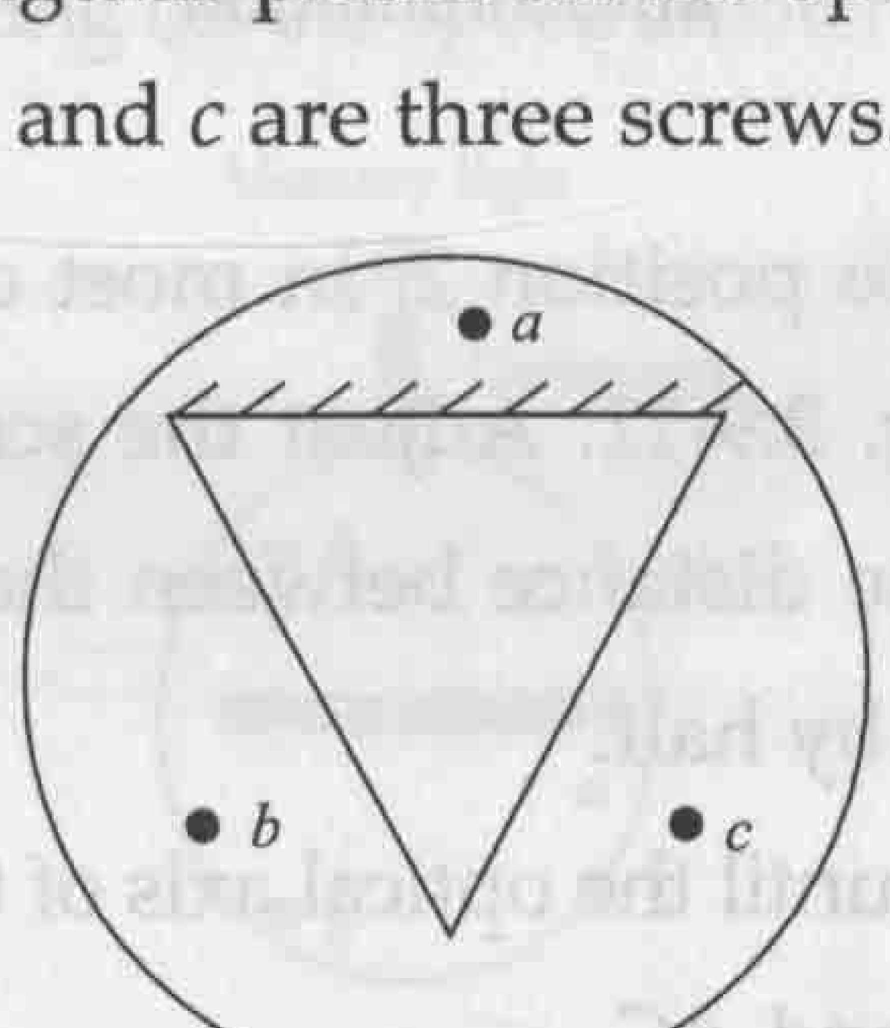


Fig. 3.9-10 The position of the prism on the table

(5)Refer to Fig. 3.9-7. Rotate the telescope to position 1 until you can see the image of the cross as shown in Fig. 3.9-11 from the eyepiece. The image of the cross could be under or above the upper node of the crosshairs.

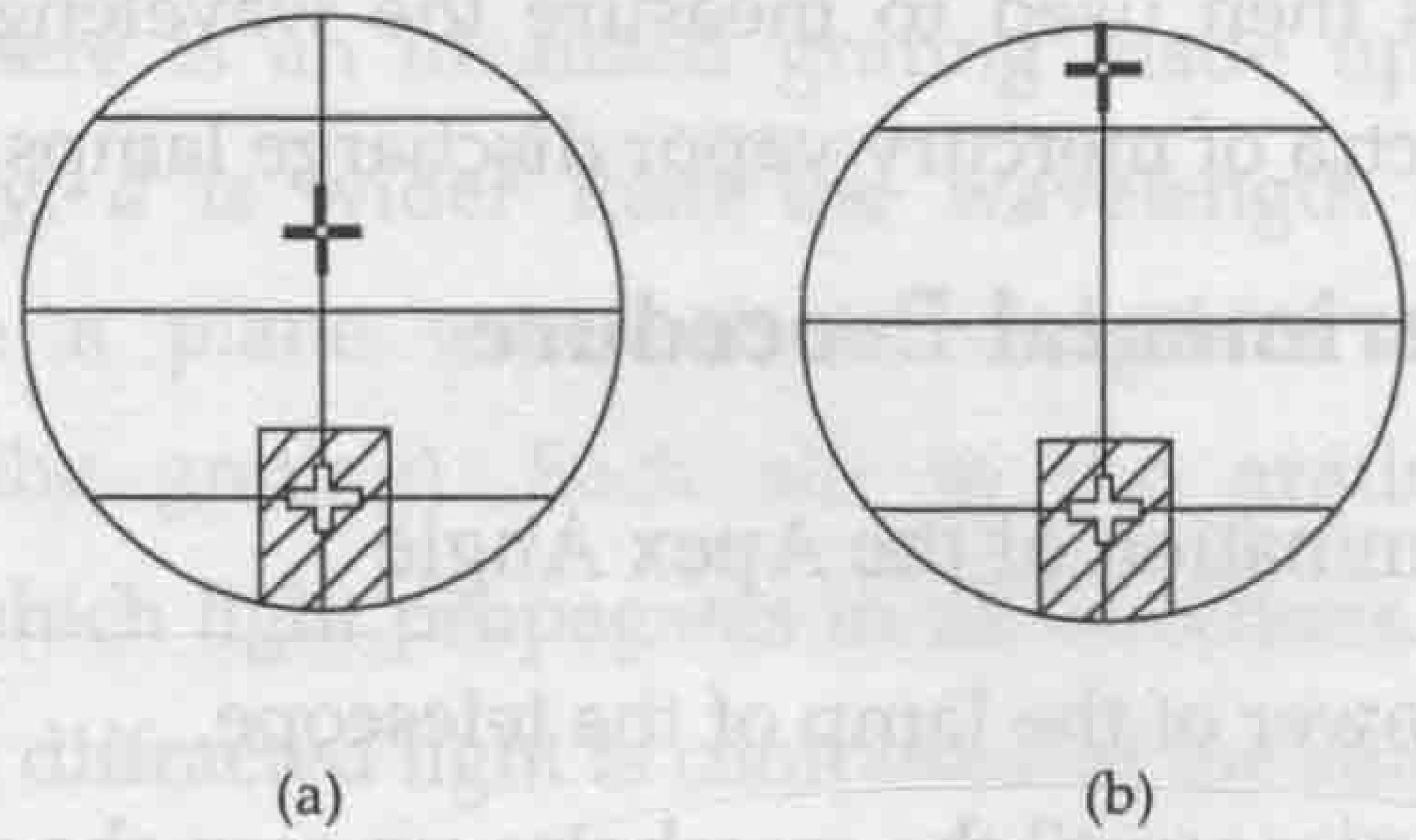


Fig. 3.9-11 The position of the image of the cross

(a) under the upper node of the crosshairs; (b) above the upper node of the crosshairs

(6)Adjust the tilting screw under the telescope to let the image of cross overlap with the node of the upper crosshairs, as shown in Fig. 3.9-12. Now the optical axis is perpendicular to the side surface *AB* (see Fig. 3.9-7).

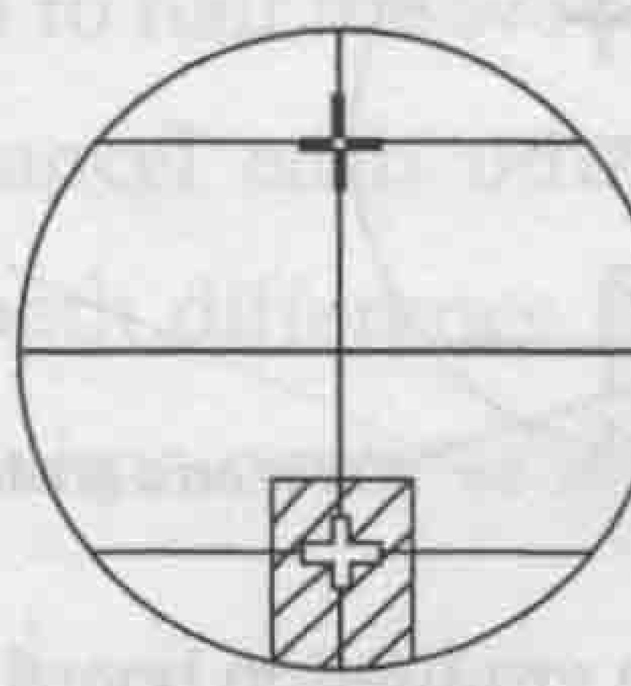


Fig. 3.9-12 The image of the cross coincides with the upper node of the crosshairs. This is autocollimation

(7)Rotate the telescope to position 2. In most cases, you can see the image of the cross as shown in Fig. 3.9-11. Adjust the screw *c* under the prism table (see Fig. 3.9-10) to reduce the distance between the image of the cross and the node of the upper crosshairs by half.

(8)Repeat Steps (5) to (7) until the optical axis of the telescope is perpendicular to both the side surfaces *AB* and *AC*.

(9)Rotate the telescope to position 1 and read two angles, θ_1 and θ_1', from the two verniers 180 degrees apart. Rotate the telescope to position 2 and read two angles, θ_2 and θ_2'. Enter these angles in Data Table 3.9-1.

(10)Repeat Step (9) to complete Data Table 3.9-1

3.9.4.2 Measuring wavelengths of the Mercury Lights by Diffraction Grating

(1)Take the prism off the spectrometer.

(2)Illuminate the slit and move the telescope so it is in-line with collimator. Loosen the screw that clamps the slit and rotate the slit to a vertical orientation so that its image will be parallel with the vertical crosshair (see Fig. 3.9-13). Retighten the screw.

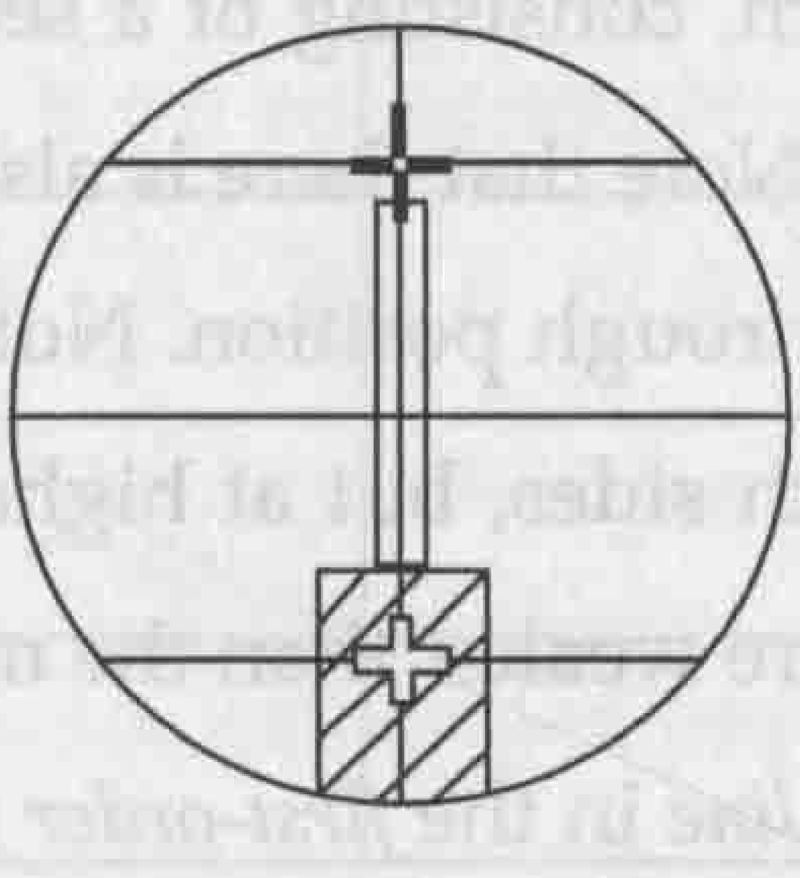


Fig. 3.9-13 The position of the zero order line and the green cross

(3)Rotate the focus knob of the collimator to see the image of the slit, i. e. the zero-order diffraction line sharply. Adjust the screw on the slit to change the width of zero-order line and set it about 0.5 mm. The collimator is now producing a parallel beam of light from the slit.

(4)Place a diffraction grating on the spectrometer table as shown in Fig. 3.9-14.

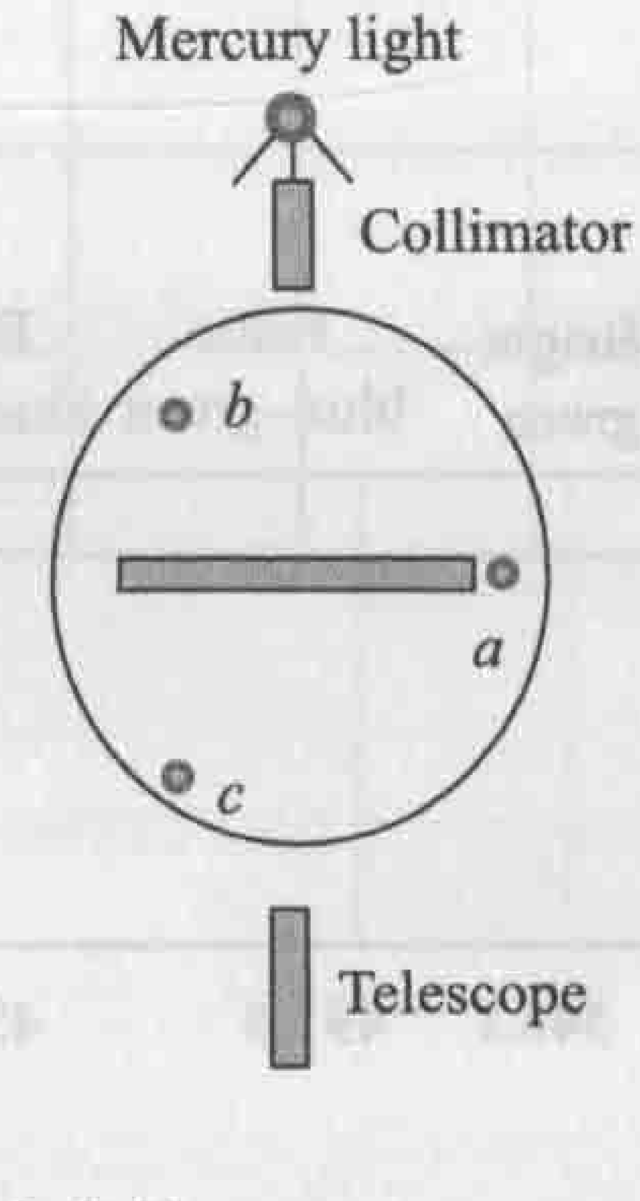


Fig. 3.9-14 Positioned diffraction grating on spectrometer table

(5)At the straight-through position, where the telescope is in line with the collimator, you can see the image of the cross from the field of view of the telescope. The image of the cross may be below or above the upper node of the crosshairs, as shown in Fig. 3.9-11. Adjust the tilting screw of the telescope or the screw under the spectrometer table to align it with the upper horizontal cross wire, as shown in Fig. 3.9-13. Now, you are ready for measurements.

(6)Move the telescope off to an angle of about 20 degrees to, say, the left side (k = −1), while looking through it. You should then be able to find the "first-order" diffraction pattern, consisting of a set of bright differently-colored lines, as shown in Fig. 3.9-15. Note that there is also an equivalent set of lines on the right side of the straight-through position. Note also that each set of colored lines is repeated again on both sides, but at higher angles due to higher-order diffractions. However, these are weaker than the main first-order sets.

(7)Locate the *bright green line* in the *first-order* mercury spectrum and, using the telescope's slow motion drive, carefully align the cross-wires with this line. Record the telescope's angular positions, φ_{G-L}, in Data Table 3.9-2.

(8)Rotate the telescope and align the cross-wires with the yellow doublet. Record the telescope's angular positions, φ_{Y-L1} and φ_{Y-L2}, in Data Table 3.9-2.

(9)Swing the telescope to the right-side of the straight-through position and again align the cross-wires with the first-order green and yellow doublet lines and record the telescope's angular positions, φ_{G-R}, φ_{Y-R1} and φ_{Y-R2}, in Data Table 3.9-2.

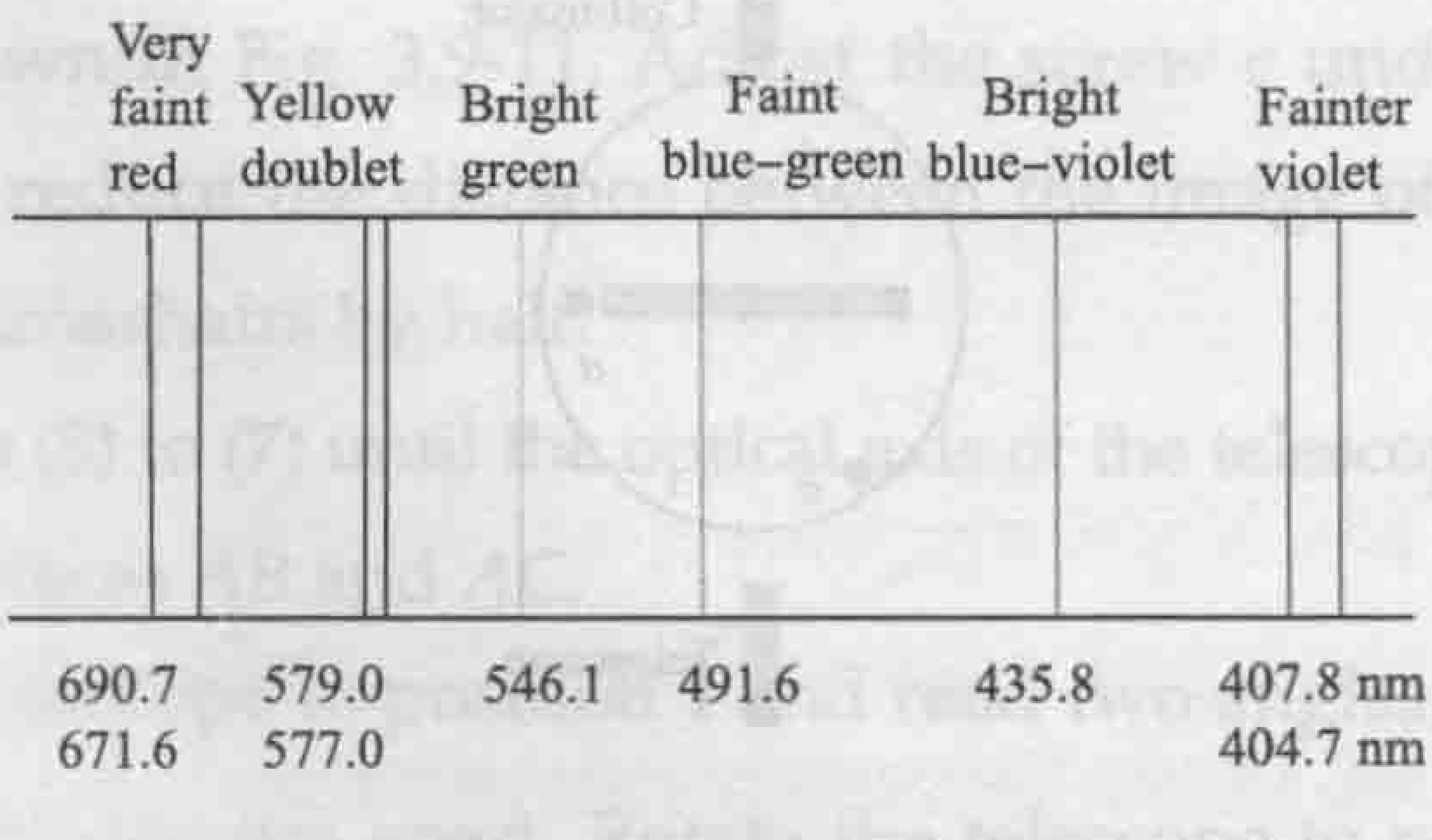


Fig. 3.9-15 Visible lines in the emission spectrum of mercury

3.9.5 Experimental Data

Data Table 3.9-1 ***Purpose*: To measure the apex angle of a prism**

Instrument error:__________

Position of telescope	Left side (position 1)		Right side (position 2)	
Trial	Vernier 1	Vernier 2	Vernier 1	Vernier 2
	θ_1 (°, ′)	θ_1' (°, ′)	θ_2 (°, ′)	θ_2' (°, ′)
1				
2				
3				
Average				

Data Table 3.9-2 **(*Purpose*: To measure the wavelengths of lines in the spectra of mercury**

Diffraction order	$K = -1$ (left side)						$k = +1$ (right side)					
Lines	Yellow 2		Yellow 1		Green		Green		Yellow 1		Yellow 2	
Trial	$\varphi_{Y\text{-}L21}$	$\varphi_{Y\text{-}L22}$	$\varphi_{Y\text{-}L11}$	$\varphi_{Y\text{-}L12}$	$\varphi_{G\text{-}L1}$	$\varphi_{G\text{-}L2}$	$\varphi_{G\text{-}R1}$	$\varphi_{G\text{-}R2}$	$\varphi_{Y\text{-}R11}$	$\varphi_{Y\text{-}R12}$	$\varphi_{Y\text{-}R21}$	$\varphi_{Y\text{-}R22}$
1												
2												
Average												

Student's name and number:______________Instructor's initial:______________

3.9.6 Calculations

3.9.6.1 Compute the Apex Angle of a Prism and its Uncertainty

(1)The apex angle: Angle $\bar{A}=180°-\frac{1}{2}[|\bar{\theta}_1-\bar{\theta}_2|+|\bar{\theta}_1'-\bar{\theta}_2'|]$ [Note: If the difference between the two angles is about 240 degrees, the difference should be subtracted from 360°. For example, $\bar{\theta}_1=355°45'$ and $\bar{\theta}_2=115°43'$, their difference is not $355°45'-115°43'=240°2'$ but $360°-(355°45'-115°43')=119°58'$].

(2)The Type A evaluation of uncertainty in θ_1, θ_1', θ_2, θ_2'.

(3)The Type B evaluation of uncertainty in θ_1, θ_1', θ_2, θ_2'.

(4)The combined uncertainty in θ_1, θ_1', θ_2, θ_2'.

(5)The uncertainty in apex angle *A*: $\sigma_A=\frac{1}{2}(\sqrt{\sigma^2_{\theta 1}+\sigma^2_{\theta 1'}+\sigma^2_{\theta 2}+\sigma^2_{\theta 2'}})$.

(6)The final result of the apex angle: $A=\bar{A}\pm\sigma_A$.

3.9.6.2 Compute the Wavelengths of Yellow Doublet Lines

(1)The diffraction angles of spectral lines of green and yellow doublets tint. For example, for green line,

$$\bar{\varphi}=\frac{1}{4}(|\varphi_{G-L1}-\varphi_{G-R1}|+|\varphi_{G-L2}-\varphi_{G-R2}|)$$

(2)The diffraction space *d*, $d=\frac{\lambda_{green}}{\sin\varphi_{green}}$ (λ_{green}= 546.07 nm).

(3)The wavelengths of yellow doublet lines, $\lambda_{Y1}=d\times\sin\varphi_{Y1}$; $\lambda_{Y2}=d\times\sin\varphi_{Y2}$.

(4)The relative error of λ_{Y1} and λ_{Y2}. Find the accepted values in Fig. 3.9-15.

3.9.7 Post Lab Questions

(1)In Fig. 3.9-12, when the image of the cross coincides with the upper node of the crosshairs we say that the prism, the telescope, and the cross are autocollimation. Why?

(2)How can you adjust the collimator to produce a parallel beam of light?

(3)For a spectrometer, the eccentric deviation is the inconsistency between the center of the prism table and that of the dial disk. In Fig. 3.9-16, O is the center of the dial disk and O' is the center of the prism table.

i. Prove that $\varphi = \frac{1}{2}(\varphi_1 + \varphi_2)$.

ii. Use this conclusion to explain why there are two verniers on the dial disk.

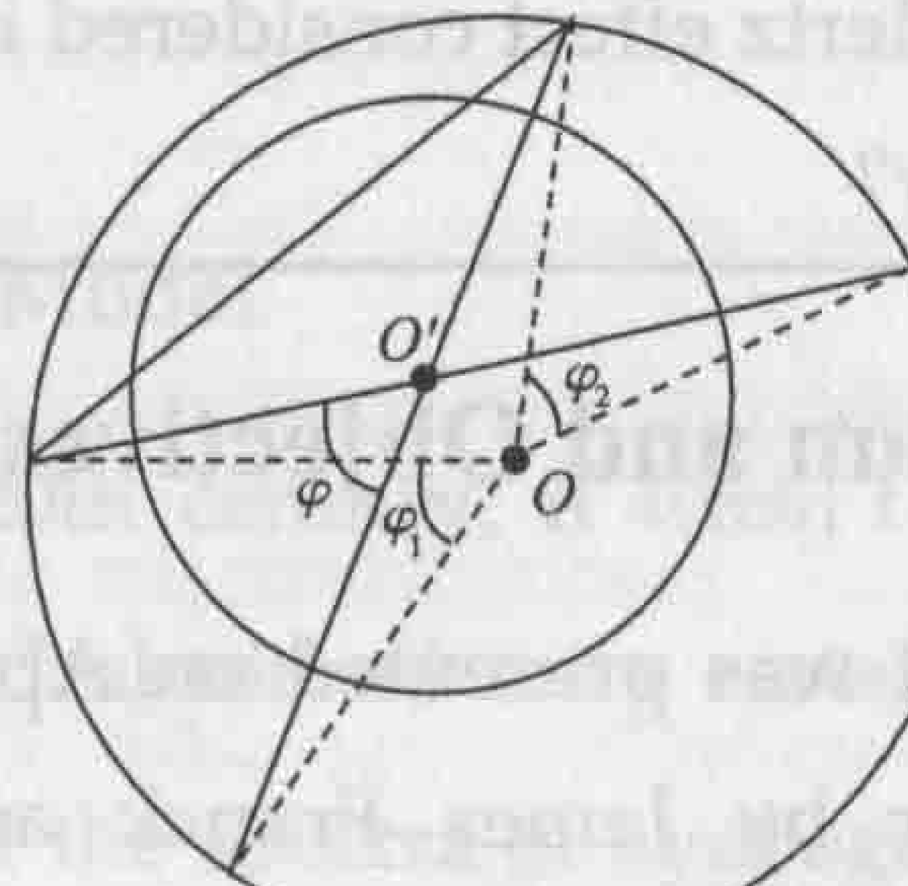


Fig. 3.9-16 The diagram of eccentric deviation