Chapter 21

Electric Potential

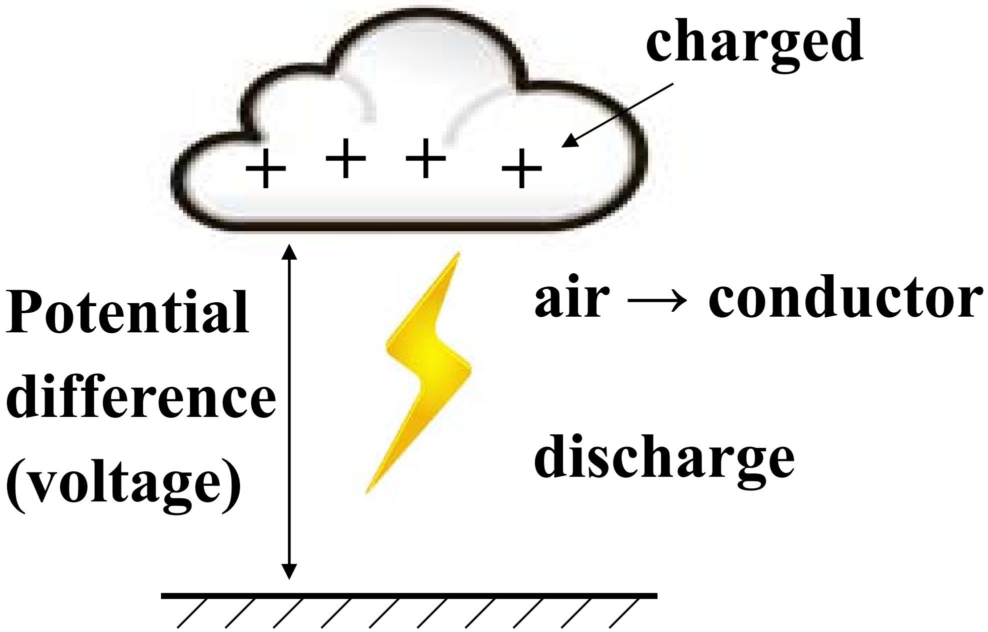
Conservative force

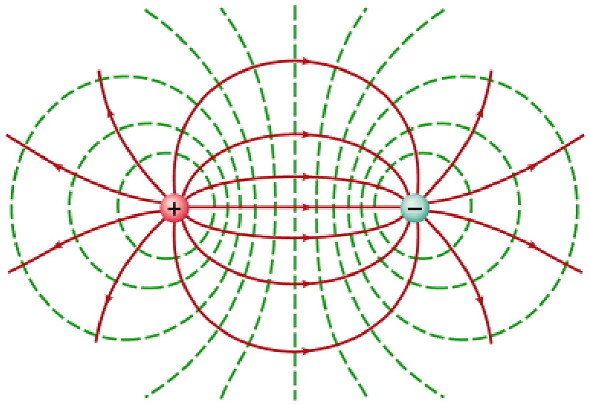
⇓

Independent of path

⇓

Potential (energy)





Conservation

Coulomb force is conservative

Comparing : $F = G\dfrac{m_1 m_2}{r^2} \sim F = k\dfrac{Q_1 Q_2}{r^2}$

Electrostatic / Coulomb force is conservative

Work done by electric field is independent on the path:

$$W = \int_{L1} q\vec{E}\cdot d\vec{l} = \int_{L2} q\vec{E}\cdot d\vec{l} \Rightarrow \boxed{\oint_C \vec{E}\cdot d\vec{l} = 0} \Rightarrow \nabla\times\vec{E} = 0$$

The Circulation Theorem of electrostatic field

Stokes Identity

$$\oint_C \vec{E}\cdot d\vec{l} = \iint_S (\nabla\times\vec{E})\cdot d\vec{S}$$

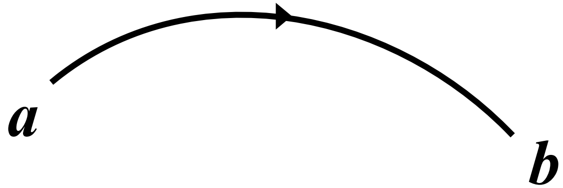
Electric potential energy $W = -\Delta U = -(U_f - U_i)$

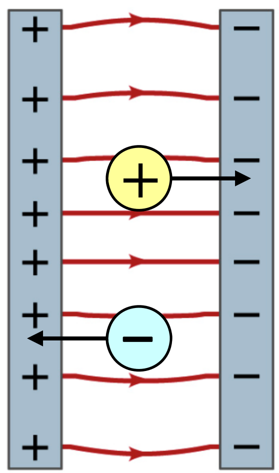
Conservative force → potential energy U

Difference in potential energy between *a* and *b*:

$$U_b - U_a = -W_{a\to b}$$

$$= -\int_a^b q\vec{E}\cdot d\vec{l}$$





***U* depends on the charge!**

Electric potential: $V = \dfrac{U}{q}$

Electric potential & potential difference

Potential difference / voltage:

$$\boxed{V_{ab} = V_a - V_b = \frac{U_a - U_b}{q} = \int_a^b \vec{E} \cdot d\vec{l}}$$

Uniform field along field lines: $V_{ab} = Ed$

If position b is the zero point: $\boxed{V_a = \int_a^{V=0} \vec{E} \cdot d\vec{l}}$

Relationship between electric field and potential

Unit of electric potential: Volt (V)

Zero point

Electric potential: $$V_a = \int_a^{V=0} \vec{E} \cdot d\vec{l}$$

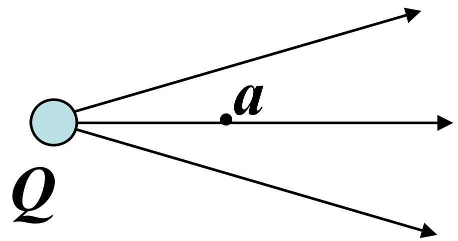
It depends on the chosen of zero point

★ **For finite size body, usually set $V = 0$ at infinity:**

$$\boxed{V_a = \int_a^{\infty} \vec{E} \cdot d\vec{l}}$$

For the field created by a point charge :

$$V_a = \int_a^{\infty} \frac{Q}{4\pi\varepsilon_0 r^2} \cdot dr = \frac{Q}{4\pi\varepsilon_0 r_a}$$



Properties of electric potential

$$\boxed{V_a = \int_a^{V=0} \vec{E} \cdot d\vec{l}} \qquad \boxed{V_{ba} = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}}$$

1) $V_a \rightarrow$ potential energy per unit (+) charge

2) V_a depends on the zero point, V_{ba} does not

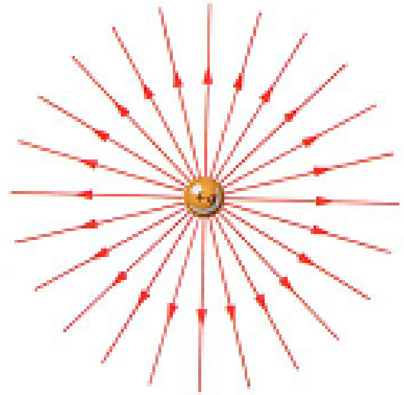
3) V_a and V_{ba} are scalars, differ from E vector

$$\boxed{U_a = qV_a = q\int_a^{V=0} \vec{E} \cdot d\vec{l}, \quad W_{ab} = qV_{ab} = q\int_a^b \vec{E} \cdot d\vec{l}}$$

Calculation of electric potential

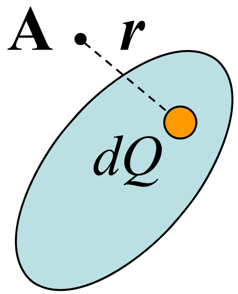
▲ **If the electric field is known:** $V_a = \int_a^{V=0} \vec{E} \cdot d\vec{l}$ ★

Potential of point charge:



$$V = \frac{Q}{4\pi\varepsilon_0 r} \quad (V = 0 \text{ at infinity})$$

▲ **System of several point charges:**



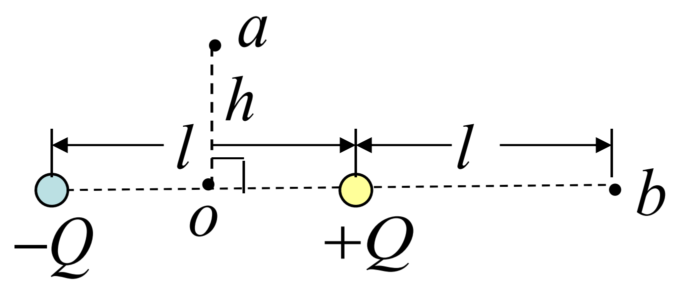
$$V = \sum \frac{Q_i}{4\pi\varepsilon_0 r_i} \quad \text{or} \quad \boxed{V = \int \frac{dQ}{4\pi\varepsilon_0 r}}$$ ★

Potential of electric dipole

Example1: **(a) Determine the potential at *o*, *a*, *b*. (b) To move charge *q* from *b* to *a*, how much work must be done by the electric field?**

Solution: (a) $V_o = \dfrac{Q}{4\pi\varepsilon_0 r} - \dfrac{Q}{4\pi\varepsilon_0 r} = 0$, $V_a = 0$

$$V_b = \frac{Q}{4\pi\varepsilon_0 l} - \frac{Q}{8\pi\varepsilon_0 l} = \frac{Q}{8\pi\varepsilon_0 l}$$

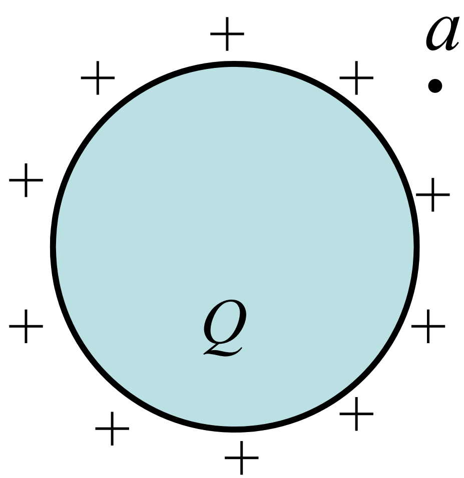


(b) $V_{ba} = V_b - V_a = \dfrac{Q}{8\pi\varepsilon_0 l}$ $\quad \therefore\ W = qV_{ba} = \dfrac{Qq}{8\pi\varepsilon_0 l}$

Charged conductor sphere

Example2: Determine the potential at a distance *r* from the center of a uniformly charged conductor sphere (Q, R) for (a) $r>R$; (b) $r=R$; (c) $r<R$.

Solution: All charges on surface:



$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \quad (r > R), \quad E = 0 \; (r < R)$$

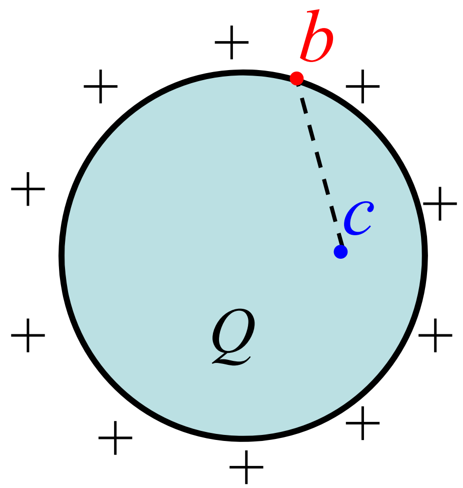
(a) V at $r>R$:

$$V_a = \int_r^\infty \frac{Q}{4\pi\varepsilon_0 r^2} dr = \frac{Q}{4\pi\varepsilon_0 r}$$

Same as point charge

(b) V at $r=R$:

$$V_b = \int_R^\infty \frac{Q}{4\pi\varepsilon_0 r^2} dr = \frac{Q}{4\pi\varepsilon_0 R}$$



(c) V at $r<R$:

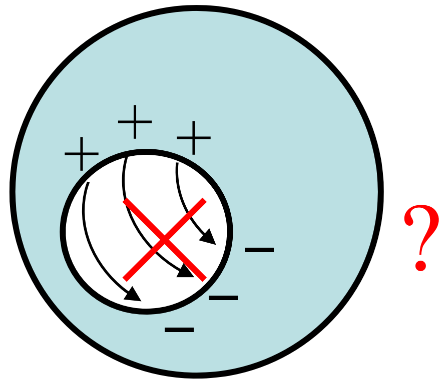
$$V_c = \int_c^b \vec{E}\cdot d\vec{l} + V_b = V_b = \frac{Q}{4\pi\varepsilon_0 R}$$

Same potential at any point!

Discussion:

(1) Conductor: equipotential body

(2) Charges on holey conductor



Uniformly charged rod

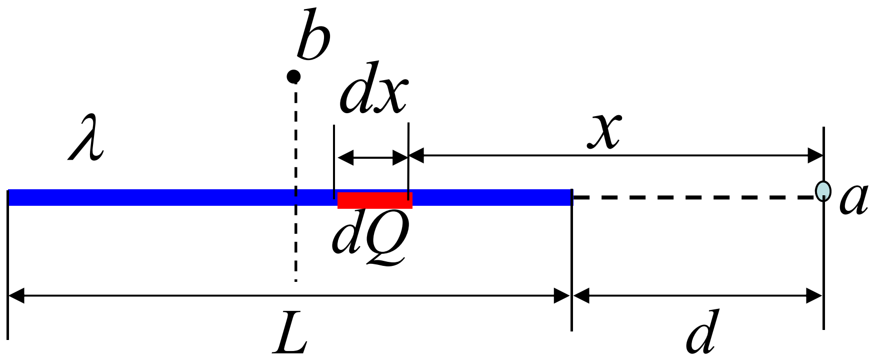
Example3: A thin rod is uniformly charged (λ, L). Determine the potential for points along the line outside of the rod.

Solution: Choose infinitesimal charge dQ

$$V_a = \int_d^{d+L} \frac{\lambda dx}{4\pi\varepsilon_0 x}$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \ln \frac{d+L}{d}$$

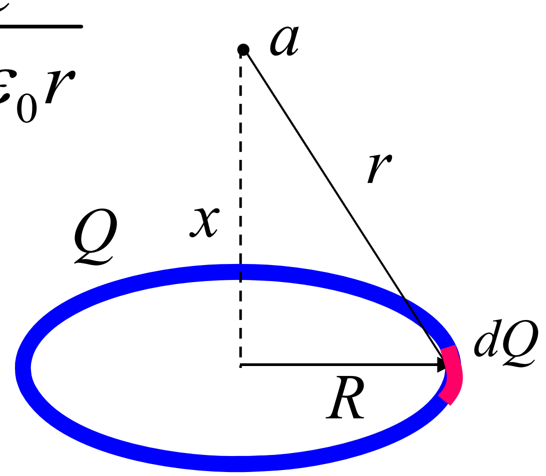
Potential at point b?



Charged ring

Example4: **A thin ring is uniformly charged (*Q*, *R*). Determine the potential on the axis.**

Solution: $$V_a = \int_{Ring} \frac{dQ}{4\pi\varepsilon_0 r} = \frac{Q}{4\pi\varepsilon_0 r}$$



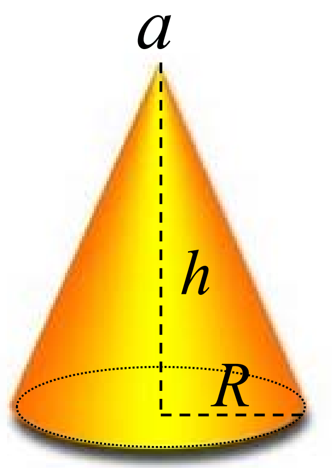
or: $$\boxed{V_a = \frac{Q}{4\pi\varepsilon_0\sqrt{x^2+R^2}}}$$

Discussion:

(1) Not uniform? (2) $x >> R$ (3) other shapes

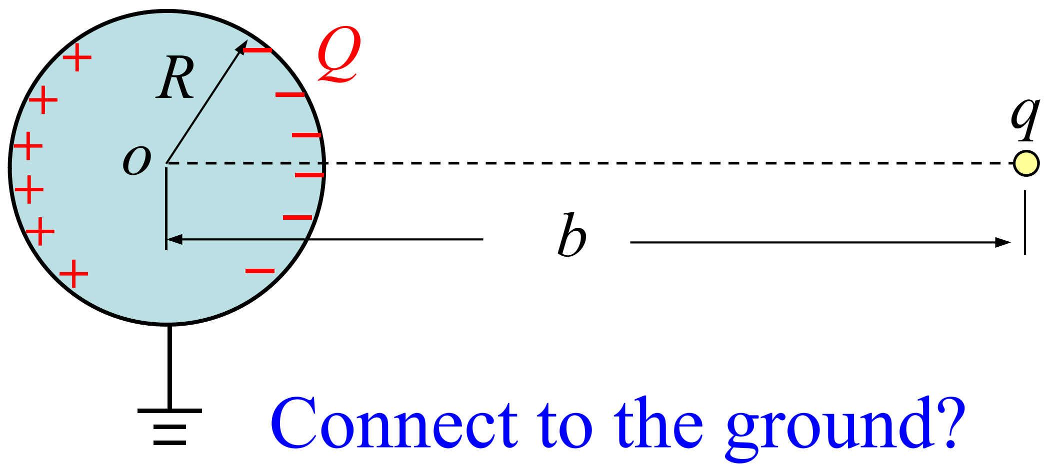
Conical surface

Question: Charges are uniformly distributed on a conical surface (σ, h, R). Determine the electric potential at top point a. ($V = 0$ at infinity)



Electrostatic induction

Question: **Put a charge q nearby a conductor sphere initially carrying no charge. Determine the electric potential on the sphere.**



*Breakdown voltage

Air can become ionized due to high electric field

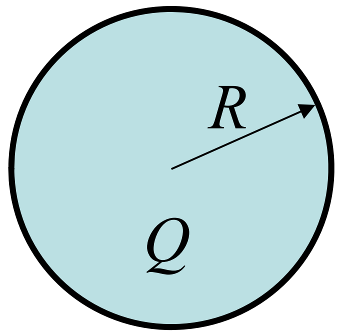
Air $\longrightarrow$ conducting $\longrightarrow$ charge flows

Breakdown of air: $\boldsymbol{E \approx 3\times10^{6} V/m}$

Breakdown voltage for a spherical conductor?

$$V = \frac{Q}{4\pi\varepsilon_0 R} = E_R \cdot R$$

$$R = 5\text{cm} \quad \Rightarrow \quad V \approx 15000\text{V}$$



Equipotential surfaces

Visualize electric potential: equipotential surfaces

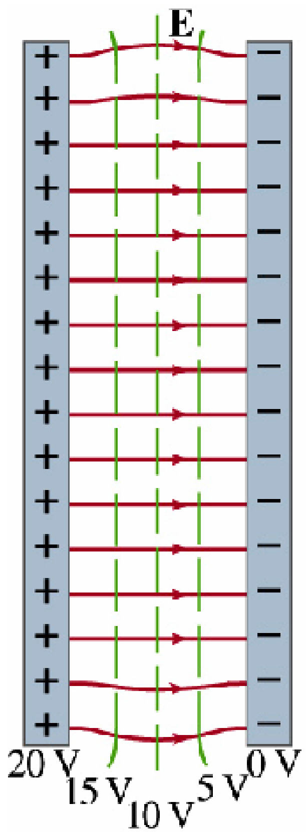
Points on surface → same potential

(1) Surfaces ⊥ field at any point.

(2) Move on surface, no work done

(3) Move along any field line, V ↘

(4) Closer surfaces → stronger field

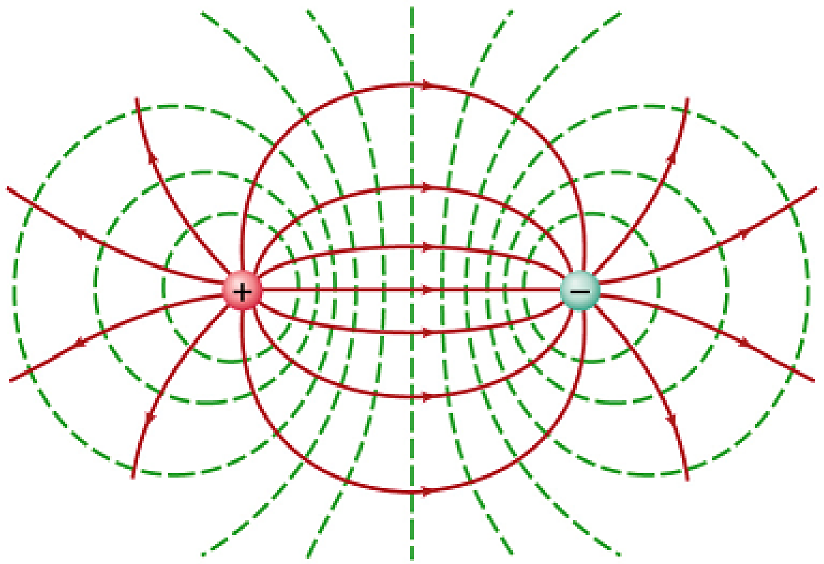


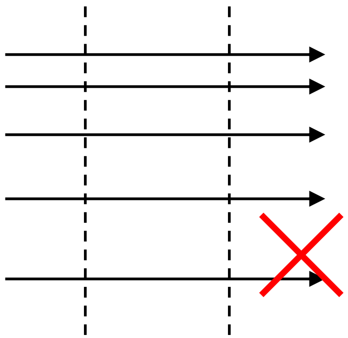
Some examples

(1) Equipotential surfaces for dipole system:

(2) Conductors

equipotential





(3) Parallel but not uniform field lines?

E determined from *V*

To determine electric field from the potential, use

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$$

In differential form: $dV = -\vec{E} \cdot d\vec{l} = -E_l dl$

Component of $\vec{E}$ in the direction of $d\vec{l}$:

$$E_l = -dV / dl \qquad \Rightarrow E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$

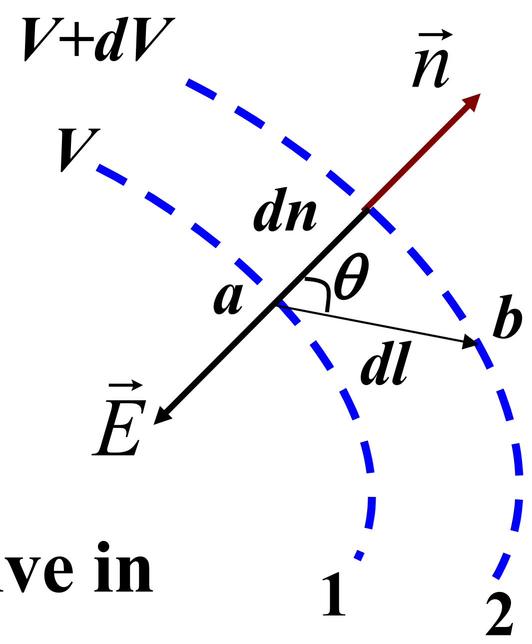
Total electric field: $$\boxed{\vec{E} = -\frac{\partial V}{\partial x}\vec{i} - \frac{\partial V}{\partial y}\vec{j} - \frac{\partial V}{\partial z}\vec{k} = -\nabla V}$$

negative gradient

Gradient of *V*

$\vec{n}$: Direction of increasing potential

$$E_l = -\frac{dV}{dl} \Rightarrow E = -\frac{dV}{dn}$$



Gradient of V : $\nabla V = \frac{dV}{dn}\vec{n}$

Gradient → partial derivative in direction that *V* changes most rapidly

$$\boxed{\vec{E} = -\nabla V = -\frac{\partial V}{\partial x}\vec{i} - \frac{\partial V}{\partial y}\vec{j} - \frac{\partial V}{\partial z}\vec{k}}$$

$$\boxed{V = \int_a^\infty \vec{E} \cdot d\vec{l}}$$

Determine *E* from *V*

Example5: **The electric potential in a region of space varies as $V = x^2yz$. Determine E.**

Solution:

$$E_x = -\frac{\partial V}{\partial x} = -2xyz,$$

$$E_y = -\frac{\partial V}{\partial y} = -x^2 z, \qquad E_z = -\frac{\partial V}{\partial z} = -x^2 y$$

$$\therefore\ \vec{E} = -2xyz\vec{i} - x^2 z\vec{j} - x^2 y\vec{k}$$

Determine *V* from *E*

Example6: The electric field in space varies as $\vec{E} = 2xy\vec{i} + (x^2 - y^2)\vec{j}$. Determine *V*.

Solution:

$$E_x = -\frac{\partial V}{\partial x} = 2xy$$

$$\Rightarrow V = \int -2xy dx = -x^2 y + C(y) \longrightarrow \frac{1}{3}y^3 + C$$

$$E_y = -\frac{\partial V}{\partial y} = x^2 - \frac{dC(y)}{dy} = x^2 - y^2$$

$$\therefore\ V = -x^2 y + \frac{1}{3}y^3 + C \qquad (\ C \text{ is constant}\ )$$