3.4 Measurement of the Young's Modulus of Wire by Elongating

Prelab Assignment 3.4

(1)What's the instrument error for a vernier caliper with the smallest division of 0.02 mm? What's its reading error?

(2)What's the definition of Young's modulus? In this experiment, which quantities will be measured in order to determine the Young's modulus? In Eq. (3.4-5), which quantity is the most critical? Why?

(3)Describe the main functions of an optical lever in this experiment. What's the magnification of the optical lever in Fig. 3.4-3?

3.4.1 Introduction and Objectives

Young's modulus, also known as the tensile modulus or elastic modulus, is a quantity used to characterize materials. It is named after Thomas Young, the 19th century British scientist. It is an inherent material property (the term modulus refers to an inherent material property). Prior to Young's contribution, engineers were required to apply Hooke's $F = kx$ relationship to identify the deformation (x) of a body subject to a known load (F), where the constant (k) is a function of both the geometry and material under consideration. Thomas Young pointed out that Young's modulus depends only on the material, not its geometry, thus allowing a revolution in engineering strategies.

After performing this experiment and analyzing the data, you should be able to:

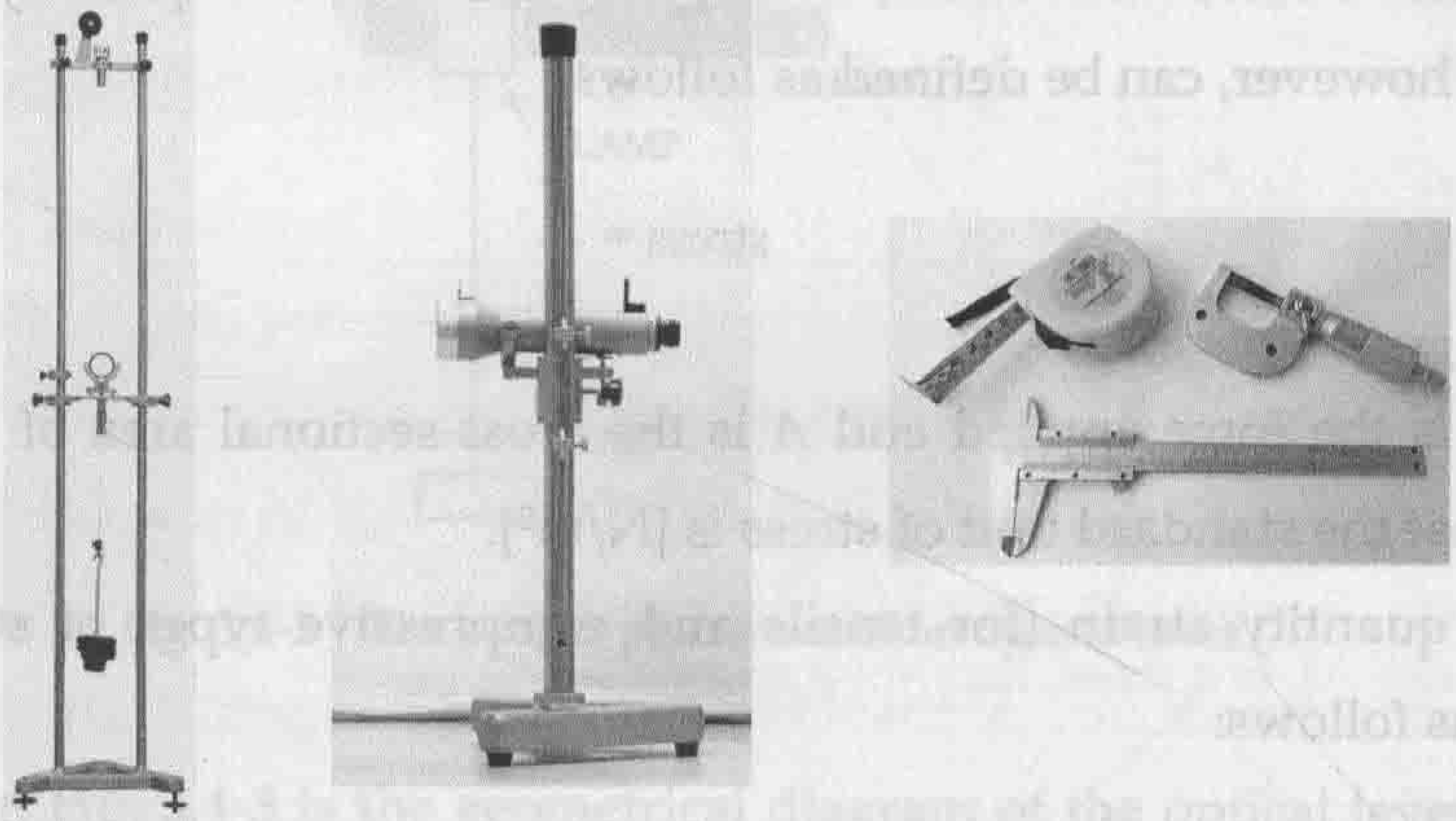
(1)Study elasticity of solids and to determine their Young's modulus.

(2)Use accurate measurement instruments correctly to measure the small lengths of objects.

(3)Understand the methods employed to eliminate the systematic error in some specific situations.

3.4.2 Required Equipment

- Vertical stand to support the wire(Fig. 3.4-1)
- Optical lever
- Set of kilogram weights
- Vernier caliper
- Telescope
- Micrometer screw gauge
- Steel tape



(a) (b) (c)

Fig. 3.4-1 Experimental equipment: (a) vertical stand; (b) telescope; and (c) instruments for length measurement

3.4.3 Theory

In most applications and example problems used in introductory physics courses, objects are assumed to be rigid for the purpose of simplification. Applied forces and torques, therefore, cause equilibrium, translational motion, and/or rotational motion to occur without deformation. Personal experience, however, tells us otherwise. When supposedly rigid materials are subject to great forces, permanent deformation is a definite possibility. Car crashes are an unfortunately common example. Stories of buildings and bridges collapsing under duress are less common, but they do occur.

Engineers need to be extremely cognizant of the properties of the materials that they use in their designs. When subject to a particular stress, or force per unit area, materials will respond with a particular strain, or deformation. If the stress is small enough, the material will return to its original shape after the stress is removed, exhibiting its elasticity. If the stress is greater, the material may be incapable of returning to its original shape, causing it to be permanently deformed. At some even greater value of stress, the material will break or fracture.

There are different types of stress: tension or tensile stress, compression or compressive stress, shear stress, and hydraulic stress. The quantity for all types of stress, however, can be defined as follows:

$$\text{stress} = \frac{F}{A} \qquad (3.4\text{-}1)$$

where F is the force applied and A is the cross-sectional area of the material. Notice that the standard unit of stress is [N/m^2].

The quantity strain (for tensile and compressive types of stress) can be defined as follows:

$$\text{strain} = \frac{\Delta L}{L} \qquad (3.4\text{-}2)$$

where L is the original length of the material, and ΔL is the change in length that results after the stress is applied. Notice that strain is a dimensionless quantity.

Young's modulus, E, is a constant that describes the ratio of stress to strain for a material experiencing either tensile or compressive stress in elastic deformation.

$$E = \frac{\text{Stess}}{\text{Strain}} = \frac{F \times L}{A \times \Delta L} \qquad (3.4\text{-}3)$$

Given the fact that ΔL is generally a very small length for most materials, measuring Young's modulus accurately can be a difficult task. In this experiment we use an optical lever (see Fig. 3.4-2) to measure the small elongations. Optical lever is made up of one small circular plane mirror and three metal toes which form one isosceles triangle. The two front metal toes must be placed in one of the grooves in the flat and the back toe must be on the surface of the clamp, as shown in Fig. 3.4-2. The length of the back metal toe can be adjusted if necessary. The distance *b* from the back metal toe to the line between two front metal toes is called the length of optical lever.

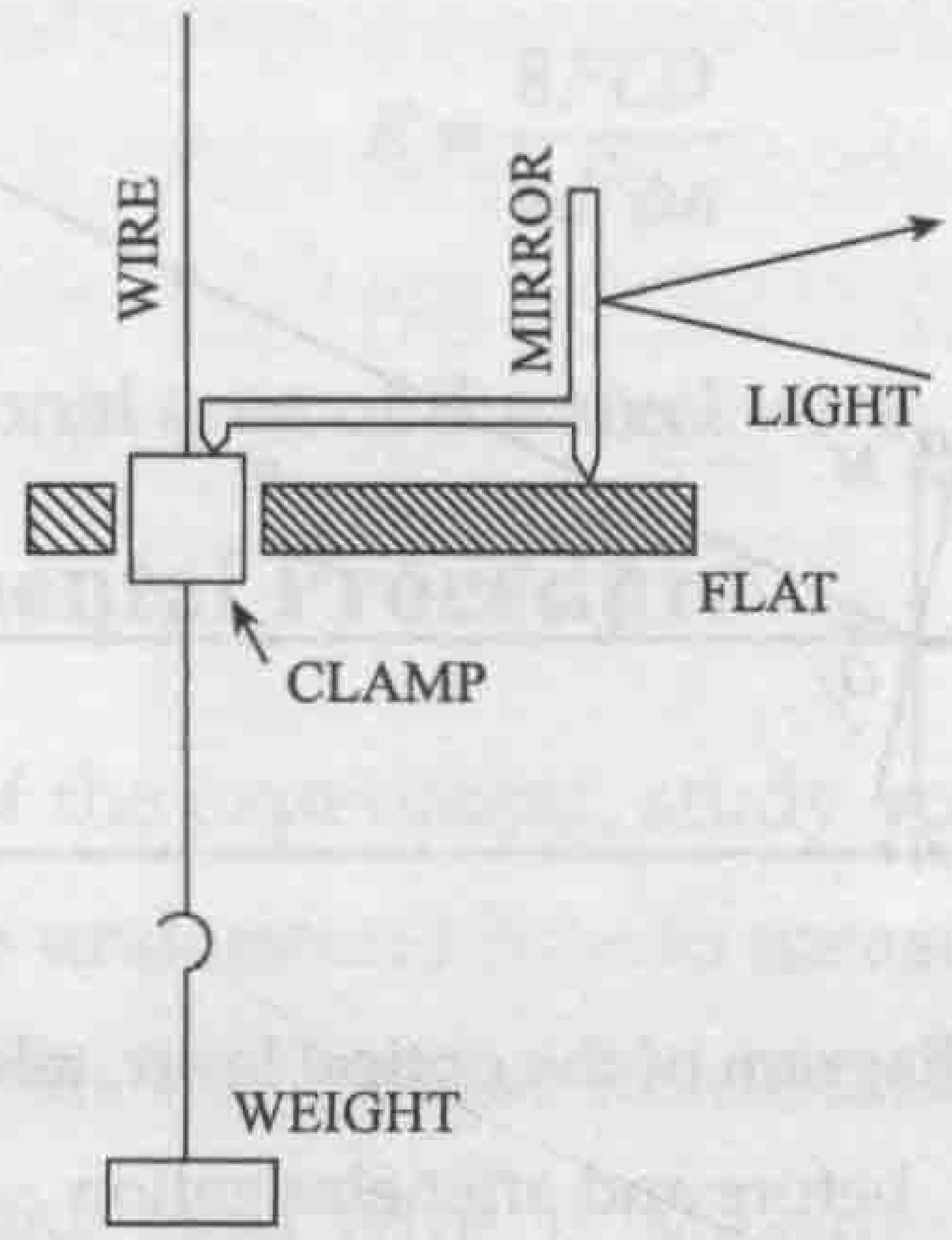


Fig. 3.4-2 The proper placement of an optical lever

Shown in Fig. 3.4-3 is the geometrical diagram of the optical lever and the steel rule before and after elongation. The normal of the plane mirror M is accordant with the axes (BO) of the telescope when there is no force on the wire. At this point, the reading on the measuring scale from the telescope is n_0. When weight is added, the wire is stretched by ΔL. The back metal toe of the optical lever is dropped so that the angle α of the normal of the plane mirror is formed. At this time, the reading on the measuring scale from the telescope is n_1. According to the reference law of light, $\angle n_0 OB' = \angle B'O n_1 = \alpha$ and $\angle n_0 OB'' = 2\alpha$, where OB' is the normal of the plane mirror after the tilt. The difference in readings of measuring scale between the initial and final situations is,

$$\Delta n = |n_1 - n_0|$$

From the geometry, we know

$$\tan \alpha = \frac{\Delta L}{b}$$

$$\tan 2\alpha = \frac{\Delta n}{D}$$

where, D is the distance between the plane mirror and the measuring scale (meter stick near the telescope).

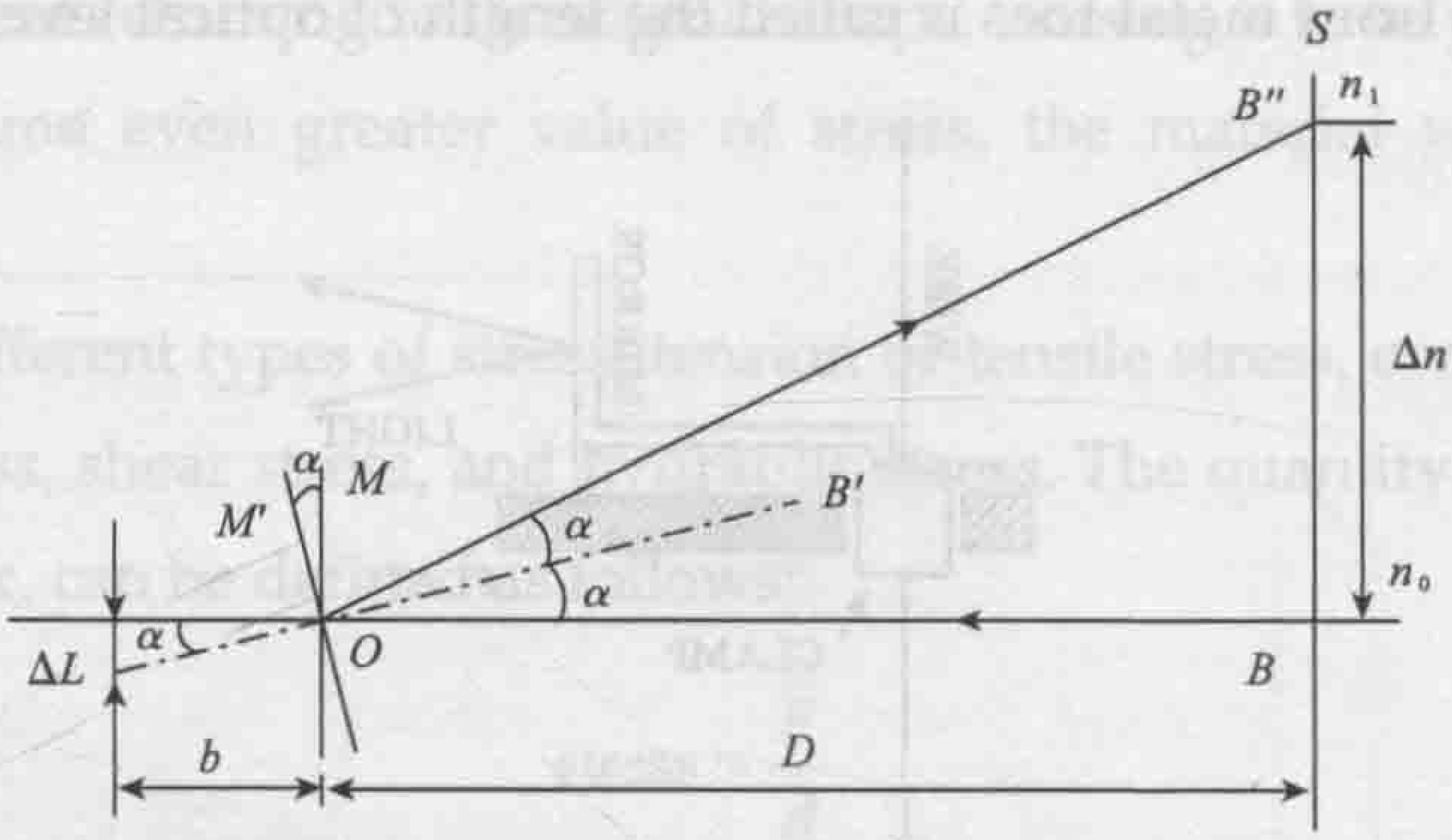


Fig. 3.4-3 The geometrical diagram of the optical lever, telescope, and the steel rule (S) before and after elongation

Because ΔL is a small quantity and $\Delta L \ll b$,

$$\tan\alpha \cong \alpha \cong \frac{\Delta L}{b}$$

Similarly, $\Delta n \ll D$,

$$\tan 2\alpha \cong 2\alpha = \frac{\Delta n}{D}$$

Then,

$$\frac{\Delta L}{b} = \frac{\Delta n}{2D}$$

Thus,

$$\Delta n = \frac{2D}{b}\cdot\Delta L = k\cdot\Delta L \tag{3.4-4}$$

where, k is the magnification for the optical lever, $k = 2D/b$.

In this experiment, b is 4～8 cm, D is 1～2 m, so the magnification for the optical lever could be 25~100. The elongation of the wire ΔL is a small length quantity, which is difficult to measure directly. However, using the optical lever, the small quantity is magnified into a quantity that can be read easily.

Finally, using Eq. (3.4-4) ΔL may be substituted into Eq. (3.4-3) considering that $A = \pi d^2/4$, the Young's modulus can then be written as

$$E = \frac{8FLD}{\pi d^2 \Delta n} \tag{3.4-5}$$

where, A is the cross sectional area of the steel wire.

3.4.4 Experimental Procedure

(1)At the beginning of the experiment, study your apparatus very carefully and work with it until you understand how to measure elongation.

(2)Add two weights on the weight holder. Use a micrometer caliper to measure the diameter of the steel wire, d. Measurement is performed in three directions at each of three points (upper end, middle, and lower end) along its length. It is worth noting that before the first reading, you should note the "zero reading" of the micrometer. Enter your measurements in Data Table 3.4-1. (Hint: Read the next section to learn to use a micrometer.)

(3)Use the steel tape to measure the original length of the steel wire, L_0. Enter your measurements in Data Table 3.4-1.

(4)Place the optical lever such that the two front metal toes are put in one of the transverse grooves on the flat while the back metal toe is placed on the surface of the clamp (see Fig. 3.4-2). Adjust the height of the telescope to the same height as the optical lever. Attention: do not touch the wire and do not put the back toe into the cracks between the clamp and the flat (see Fig. 3.4-2).

(5)Adjust the telescope and the optical lever to see the image of the meter stick by following the steps below.

i. Move the telescope to point to the plane mirror by aligning the gap (or circle) and post on the telescope and the plane mirror in one line.

ii. Put the eye outside the telescope in the direction of gap (or circle) -post; looking for the image of the steel rule from the outside of the telescope. If you cannot see the meter stick in the mirror, adjust the height of the telescope or adjust the screw bolt in front of the plane mirror to alter the tilting of the plane mirror (see Fig. 3.4-3).

iii. Put your eye on the telescope and adjust the eye piece to make the cross clear. Rotate the bolt on the right side of the telescope (to adjust the image distance) to see the image of the measuring scale clearly (see Fig. 3.4-4) so that the horizontal line is near the 0 cm mark while the two weights are on the weight holder. Attention: if you can not see the image of the meter stick or part of the vision is blurry, you can adjust the screw under the telescope.

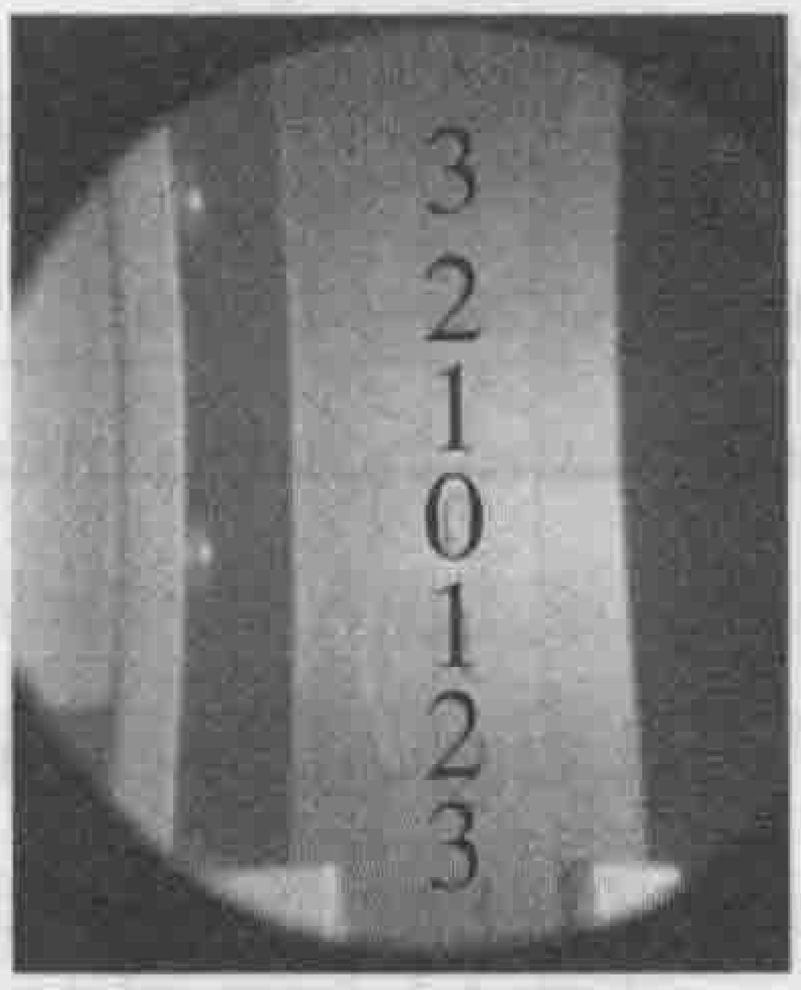


Fig. 3.4-4 The image of the scale marks of the meter stick

iv. Add weights to the weight holder. The telescope will give readings of the measuring scale. Load the measured wire successively in steps of one kilogram weight up to nine kilograms, taking the telescope readings after each addition. Attention: Only read the scale when the system (the loaded wire) is stationary. Enter the readings in Data Table 3.4-2.

v. Add one more weight up to ten kilograms without reading the scale. Unload one weight per time and read the telescope at each step. Enter the readings in Data Table 3.4-2.

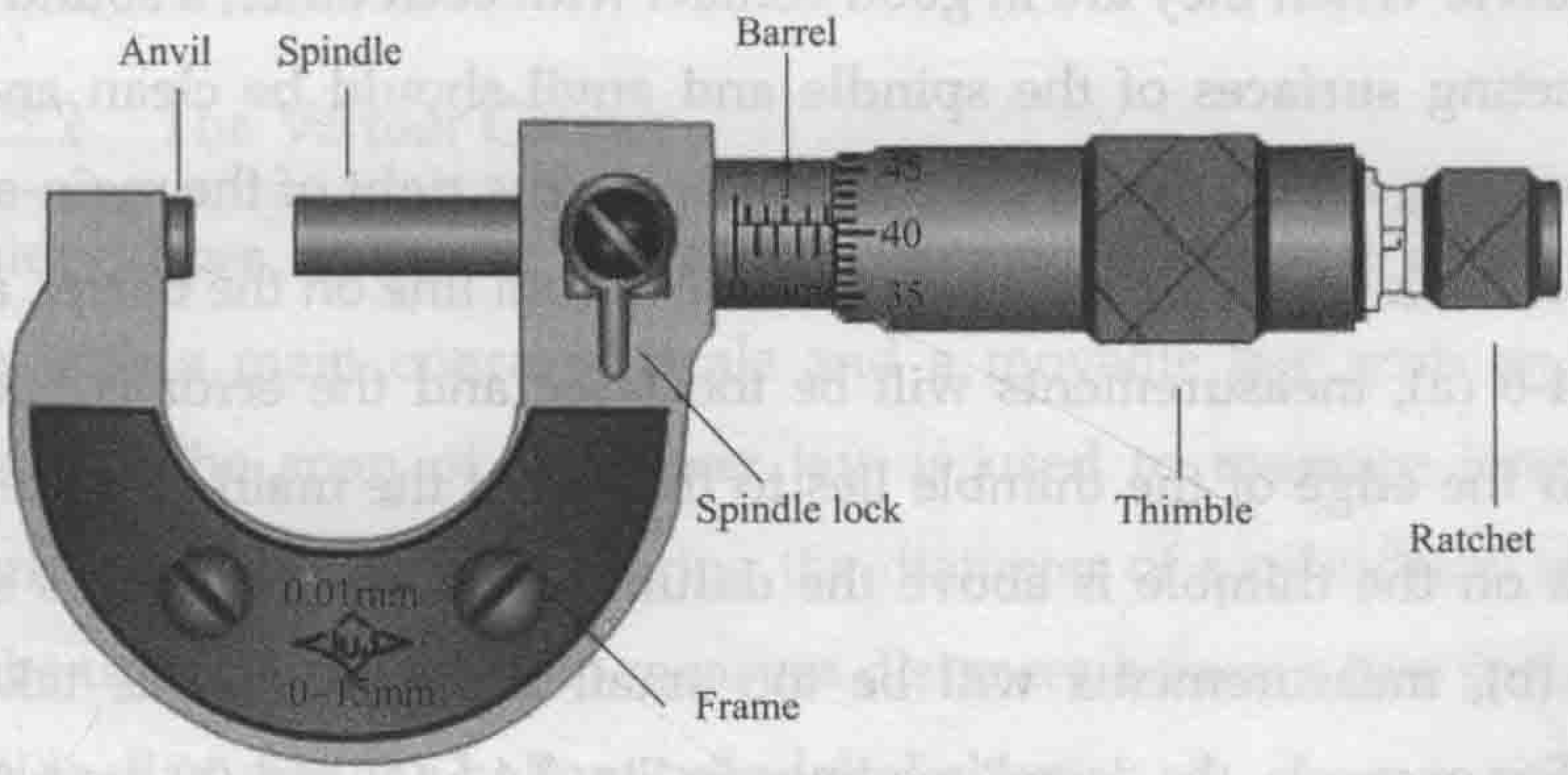
(6)Use the steel tape to measure the distance between the optical lever and the steel rule, D_0. Enter it in Data Table 3.4-1.

(7)Measure the length of the optical lever, b_0, by using the vernier caliper. Put the optical lever on a piece of white paper and press. You can see three dots corresponding to the three toes. The distance b is from the back metal toe to the line across the two front metal toes.

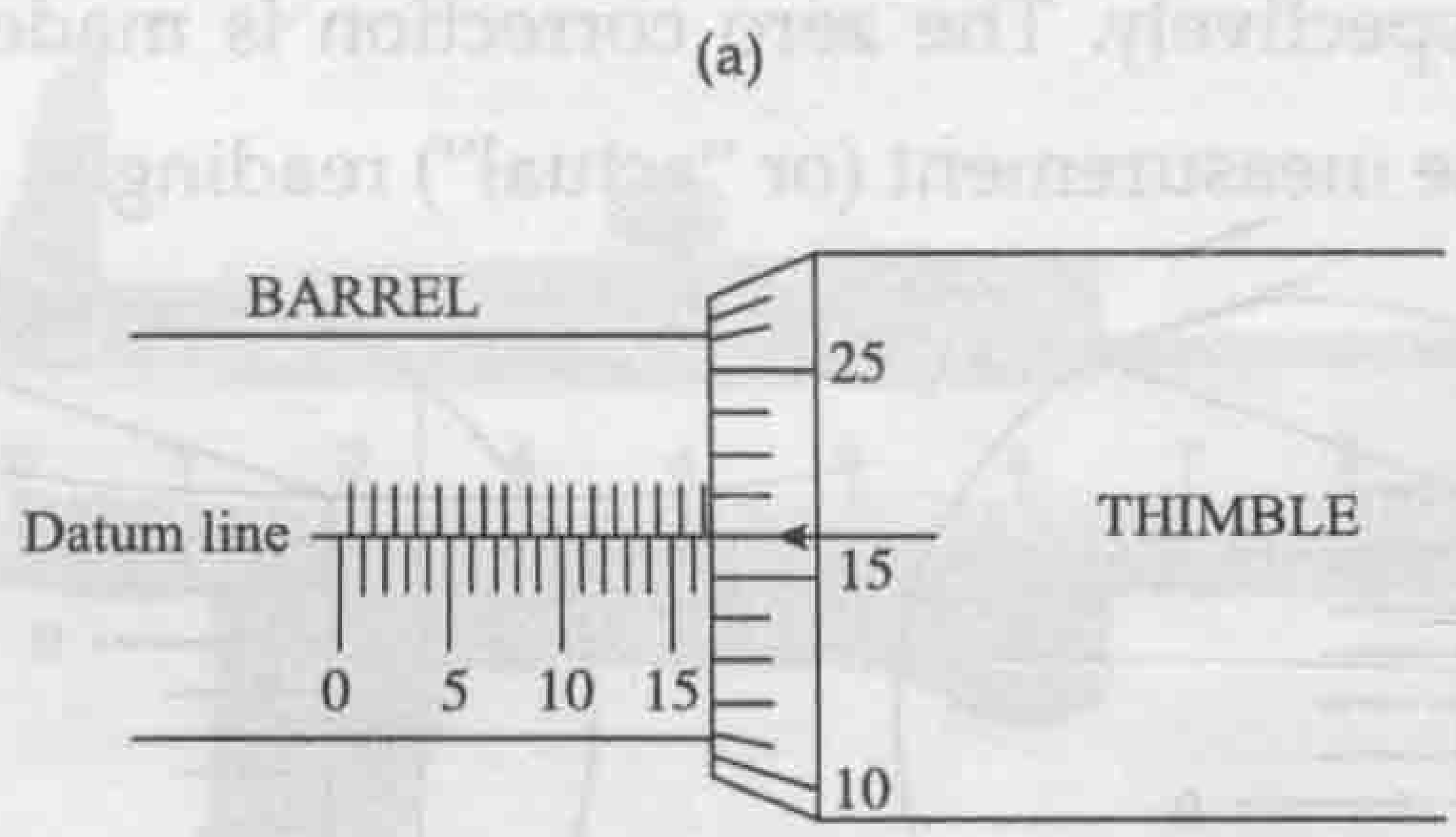
3.4.5 Accurate Instruments for Measuring Lengths

3.4.5.1 The Micrometer Caliper (Micrometer screw Gauge)

The micrometer caliper [see Fig. 3.4-5(a)] provides for accurate measurements of small lengths. A micrometer is particularly convenient in measuring the diameters of thin wires and the thicknesses of thin sheets. The axial main scale on the barrel (or "sleeve") is calibrated in the 1.0 mm and 0.5 mm scale divisions below and above the datum line. The thimble scale is calibrated in 0.01 mm. One complete rotation of the thimble (50 divisions) moves it through 0.5 mm, which is the minor division calibrated on the barrel.



(a)

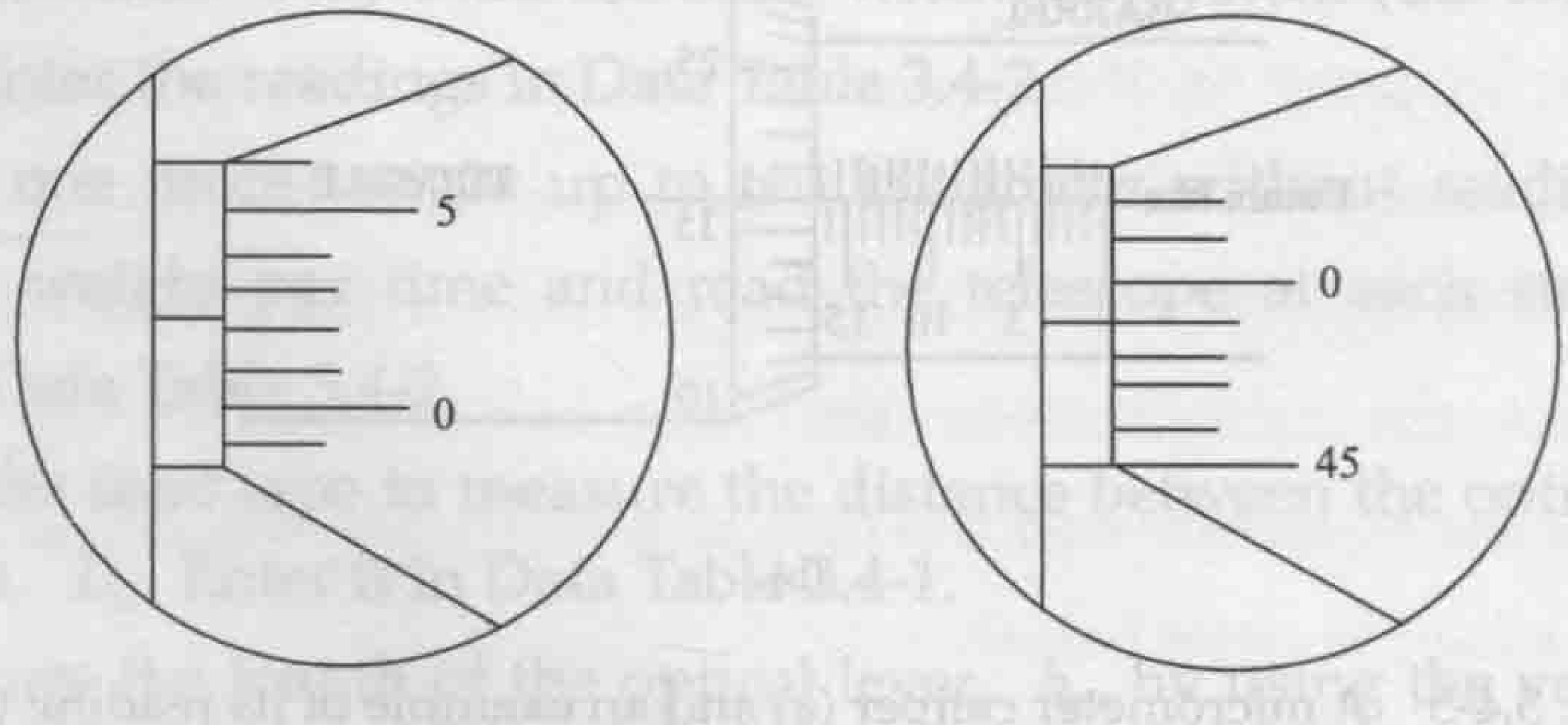


(b)

Fig. 3.4-5 A micrometer caliper (a) and an example of its reading (b)

Measurements are taken by noting the position of the edge of the thimble on the main scale and the position of the datum (or "reading") line on the thimble scale. For example, for the drawing in Fig. 3.4-5(b), the micrometer has a reading of 16.661 mm. The edge of the thimble is past the half millimeter mark after the 16 mm major division. Therefore, on the main scale is a reading of 16.000 mm plus one 0.500 mm division (scale above datum line), giving 16.500 mm. In addition, the datum line is past the 0.16 mm scale mark on the thimble. So, the reading on the thimble scale is 0.161 mm, where the last digit 1 is the estimated or doubtful figure. Thus, the final measurement is 16.500 + 0.161 = 16.661 (mm).

As with most instruments, a zero check should be made and a zero correction applied to each reading if necessary. As for a micrometer caliper, a zero reading is made by rotating the ratchet until the spindle comes into contact with the anvil. When they are in good contact with each other, a sound is heard. The contacting surfaces of the spindle and anvil should be clean and free of dust. In zeroing, if the edge of the thimble lies to the right of the main-scale zero or the zero mark on the thimble is below the datum line on the barrel, as shown in Fig. 3.4-6 (a), measurements will be too large and the error is taken to be positive. If the edge of the thimble lies to the left of the main-scale zero or the zero mark on the thimble is above the datum line on the barrel, as shown in Fig. 3.4-6 (b), measurements will be too small and the error is taken to be negative. For example, the "zero" readings in Fig. 3.4-6 (a) and (b) are + 0.023 mm and – 0.010 mm, respectively. The zero correction is made by subtracting the zero reading from the measurement (or "actual") reading.



(a) (b)

Fig. 3.4-6 Zero reading of a micrometer
(a) positive error; (b) negative error

Summarizing the corrections in equation form,

Corrected reading = actual reading – zero reading

For example, for a positive zero reading of +0.023 mm as in Fig. 3.4-6 (a),

Corrected reading = actual reading – 0.023 mm

For a negative zero reading of – 0.010 mm as in Fig. 3.4-6 (b), then

Corrected reading = actual reading – (– 0.010) mm

= actual reading + 0.010 mm

3.4.5.2 The Vernier Caliper

The vernier caliper, as shown in Fig. 3.4-7, commonly called a vernier, consists of a rule with a main engraved scale and a movable jaw with an engraved vernier scale. The span of the lower jaw is used to measure length and is particularly convenient for measuring the diameter of a cylindrical object. The span of the upper jaw is used to measure distances between two surfaces, such as the inside diameter of a hollow cylindrical object.

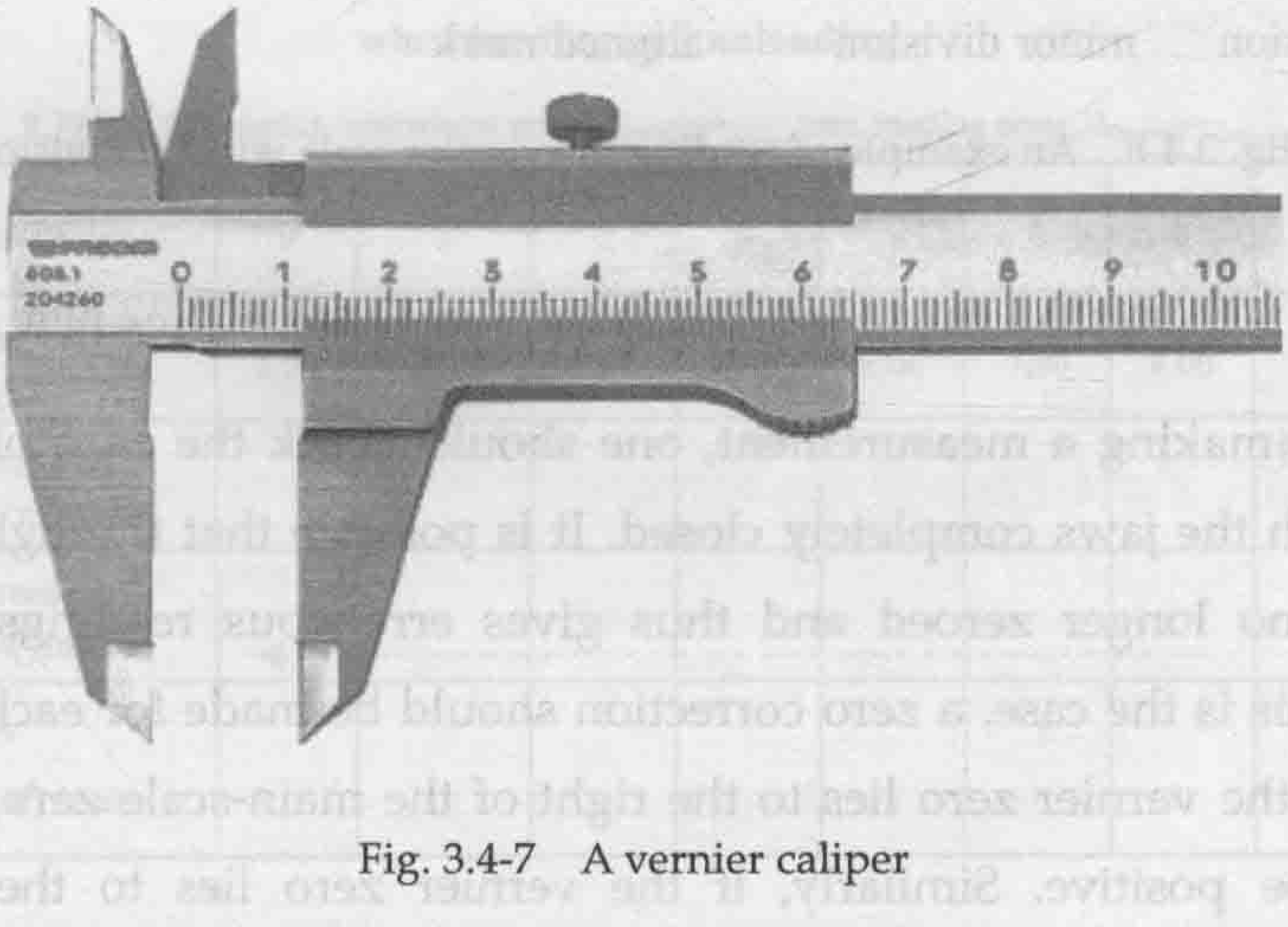
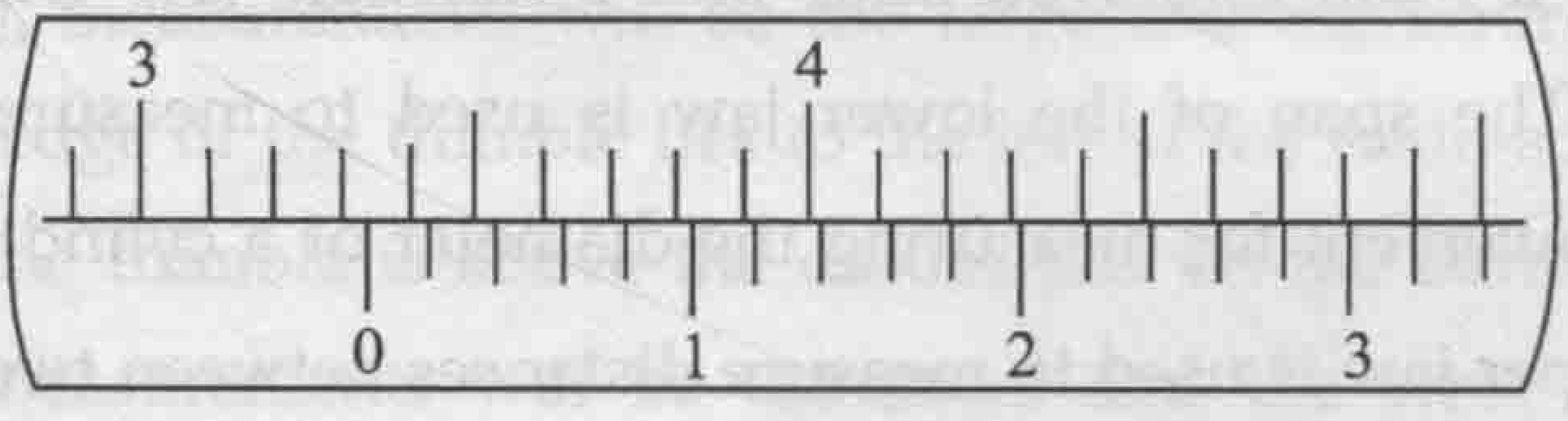


Fig. 3.4-7 A vernier caliper

The main scale is calibrated in centimeters with a millimeter least count.

The movable vernier scale has 50 divisions and one division is 1/50 of 1 mm, or 0.02 mm. If a vernier has 20 divisions on the vernier scale, one division is 0.05 mm. A measurement is made by closing the jaws on the object to be measured and reading where the zero mark on the vernier scale falls on the main scale.

In Fig. 3.4-8, the first two significant figures are read directly from the main scale. The vernier zero mark is past the 3 mm line after the 3 cm major division mark, so there is a reading of 3.3 cm. The next significant figures are obtained by referring to the vernier scale markings below the main scale. There is always a vernier mark that coincides with a mark on the main scale and this vernier mark number is the last two digits. In the figure, the sixteenth mark (or the second mark after 3 on the vernier scale) to the right of the vernier zero coincides with a mark on the main scale, so the last two significant figures are 32 (0.032 cm or 0.32 mm). Therefore, the measurement is



3 cm	+	0.3 cm	+	(0.002×16) cm = 3.332 cm = 33.32 mm
major division		minor division		aligned mark

Fig. 3.4-8 An example of reading the vernier scale with 50 divisions

$$3 \text{ cm} + 0.3 \text{ cm} + (0.002 \times 16) \text{ cm} = 3.332 \text{ cm} = 33.32 \text{ mm}$$

Before making a measurement, one should check the zero of the vernier caliper with the jaws completely closed. It is possible that through misuse the caliper is no longer zeroed and thus gives erroneous readings (systematic error). If this is the case, a zero correction should be made for each reading. In zeroing, if the vernier zero lies to the right of the main-scale zero, the error is taken to be positive. Similarly, if the vernier zero lies to the left of the main-scale zero the error is negative. In both cases, the zero correction is made by subtracting the zero reading from the measurement reading.

3.4.6 Experimental Data

Data Table 3.4-1 *Purpose*: To record the instrument parameters

(Unit: mm)

Diameter, d_i	Zero reading, d_0		Instrument error, $\Delta_{instr.}$			0.005	Reading error, Δ_{Read}			
	Trials	Point 1			Point 2			Point 3		
		1	2	3	4	5	6	7	8	9
	Actual reading, d_i'									
	Corrected reading $d_i = d_i' - d_0$									
Instrument error of tape, Δ_{tape}=(0.2×reading/1000+0.3)							Length of the optical lever			
L_0 (original reading)			D_0 (original reading)				b_0(original reading)			
Instrument error, $\Delta_{L\text{-}tape}$			Instrument error, $\Delta_{D\text{-}tape}$				Instrument error, $\Delta_{b\text{-}vernier}$			
Estimated error, $\Delta_{L\text{-}eval.}$		2	Estimated error, $\Delta_{D\text{-}eval.}$			5	Estimated error, $\Delta_{b\text{-}eval.}$			0.2
L (after rounding to an integer)			D (after rounding to an integer)				b (after rounding to one decimal place)			

Data Table 3.4-2 *Purpose*: To record the meter stick readings observed from the telescope when loading and unloading

g_{local}= 9.79m/s^2 (Chengdu), Instrument error, $\Delta_{\Delta n\text{-}stick}$=____mm, reading error, $\Delta_{\Delta n\text{-}read}$=____mm

Trial	0	1	2	3	4	5	6	7	8
Total mass of the weights, m/kg	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
Meter stick reading, loading n_i'/mm									
Meter stick reading, unloading n_i''/mm									
Mean, $n_i = \frac{1}{2}(n_i' + n_i'')$ /mm									

Your name and student number:__________ Instructor's initial:__________

3.4.7 Calculations

(1)Compute the diameter of the wire.

$$\overline{d} = \frac{1}{9}(d_1 + d_2 + \cdots + d_9)$$

(2)Use the following equation to compute the change in length after loading 4 kg weights.

$$\overline{\Delta n} = \frac{1}{4}\left[|n_7 - n_3| + |n_6 - n_2| + |n_5 - n_1| + |n_4 - n_0|\right]$$

(3)Use the following equation to compute the Young's modulus of the measured wire.

$$\overline{E} = \frac{8FLD}{\pi \overline{d}^2 b \overline{\Delta n}}$$

where $F = mg = 4 \times g$ (N).

(4)Use the propagation of uncertainties to compute the uncertainty of the measured Young's modulus.

$$\sigma_E = \overline{E} \times \sqrt{\left(\frac{\sigma_L}{L}\right)^2 + \left(\frac{\sigma_D}{D}\right)^2 + \left(2\frac{\sigma_d}{d}\right)^2 + \left(\frac{\sigma_b}{b}\right)^2 + \left(\frac{\sigma_{\overline{\Delta n}}}{\overline{\Delta n}}\right)^2}$$

Firstly, you must determine the uncertainties, σ_L, σ_D, σ_d, σ_b, and $\sigma_{\overline{\Delta n}}$ of the direct measurements.

(5)Express the final measurement result in the following form.

$$E = \overline{E} \pm \sigma_E (\mathrm{N/\,m^2})$$

3.4.8 Post Lab Questions

(1)Which measurement is more critical, the length of the wire or its elongation? Why? Explain why the length of the wire is measured with a meter stick while a very sensitive length measuring device is required for the elongation?

(2)The initial load on the hanger was not included in our calculation. Why not? Could we have used a different value as our initial load? Explain.