Electric field involving conductors

Q1: What is conductor? How does it affect the electric field?

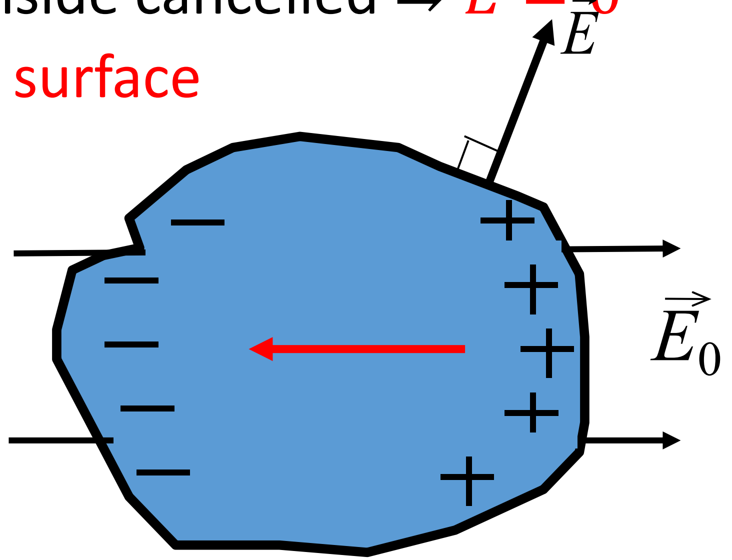
- Conductor is a kind substance which can supply flowing positive, negative charge or both.
- Perfect conductor is an ideal model, which supply sufficient charge and zero resistance.
- Usual conductor includes: metal (electron), electrolyte solution (both positive and negative charges), plasma (both positive and negative charge), …

Q2: What happens when a conductor is in electric field generated by charge on itself or elsewhere?

- Electrostatic equilibrium: no charge inside or on the surface moves!
- Charge redistribution ⇒ built-in electric field ⇒ electric field inside cancelled ⇒ $\vec{E} = 0$ inside everywhere ⇒ no charge inside ⇒ charge is only on the surface

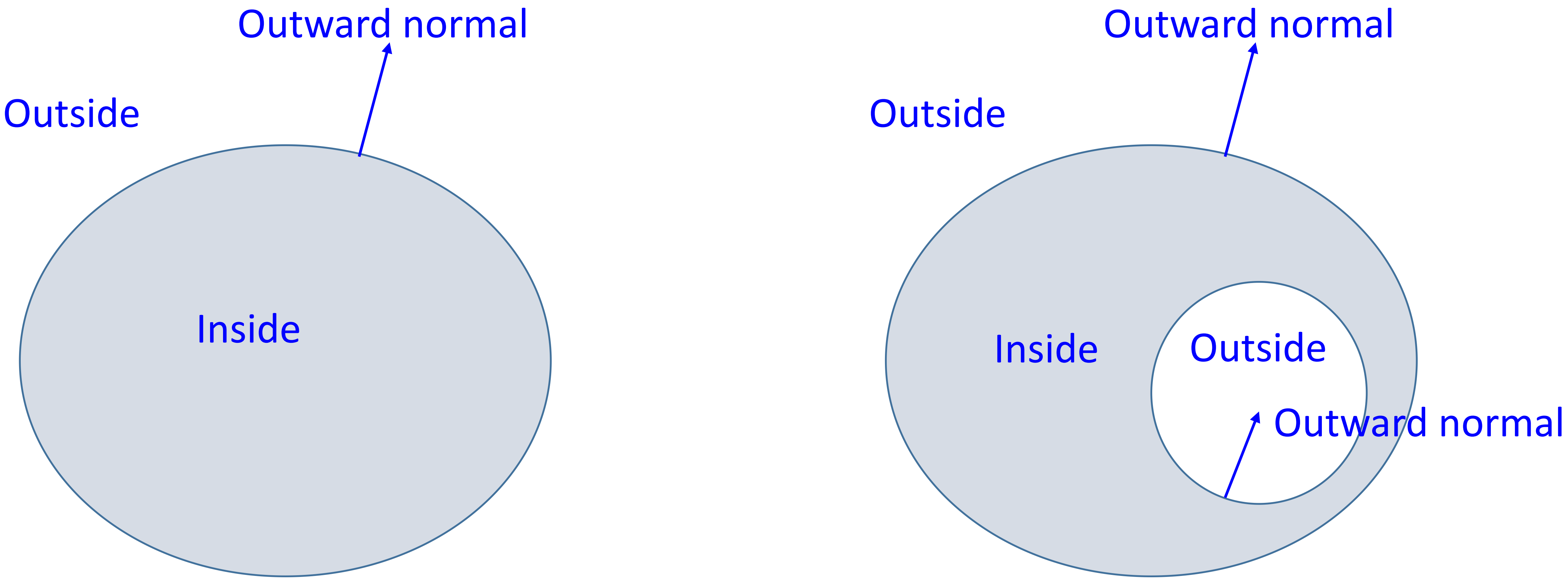
$$\oint_{any\,S} \vec{E} \cdot \mathrm{d}\vec{S} \equiv 0 \Rightarrow any\ Q_{in} = 0 \Rightarrow no\ charge\ inside$$

- Charge on the surface does not move ⇒ $\vec{E} \perp surface$



Key feature: no charge moves!

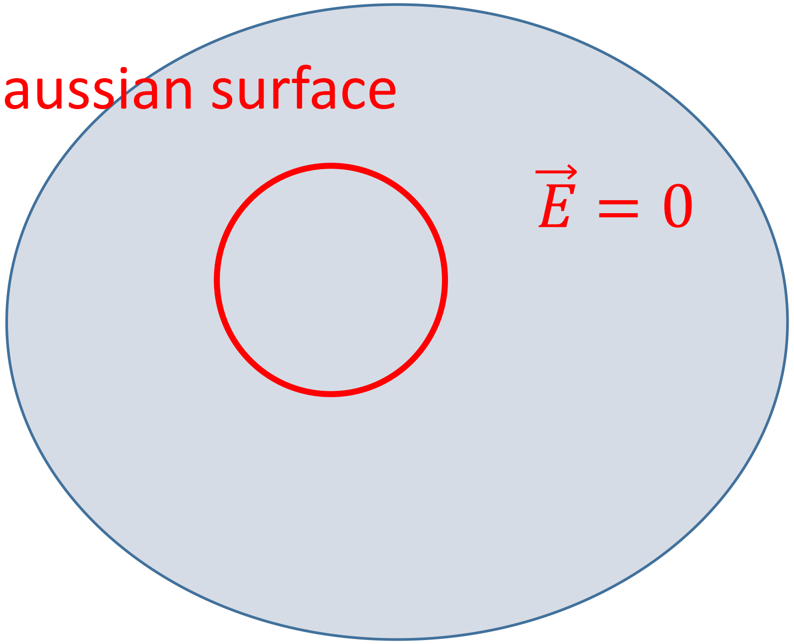
Q3: To a conductor, what is inside?



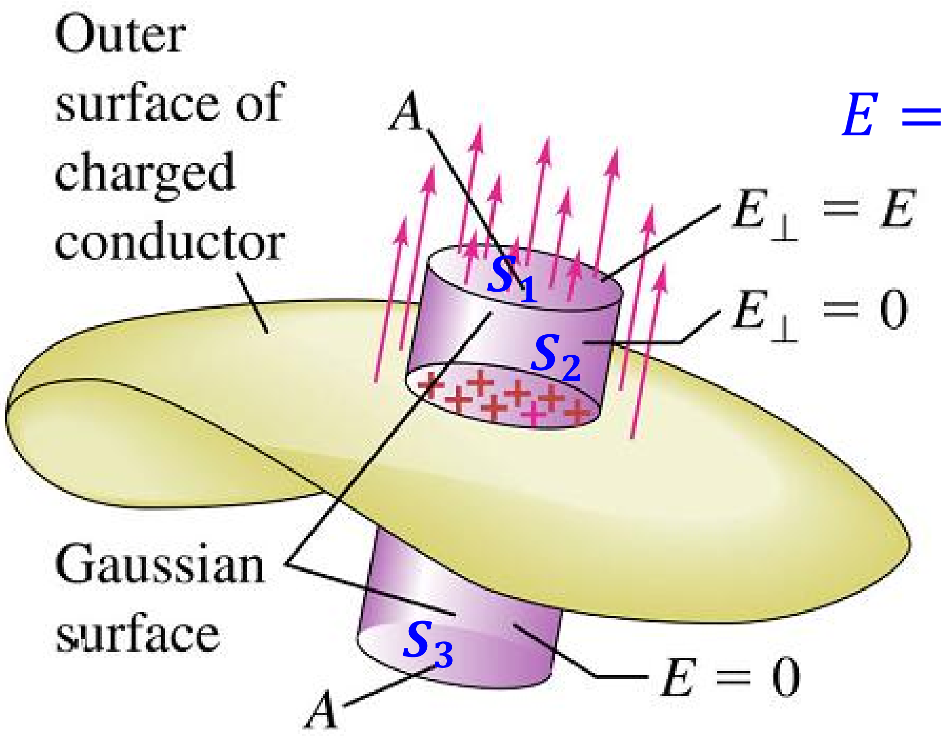
Outward normal: pointing from conductor to other

Q4: Why do we say conductor as a situation of using Gauss's Law to calculate $\vec{E}$?

Inside the conductor



Outside and near the conductor



$$E = \frac{\sigma}{\varepsilon_0}$$

$$\oint_{any\ S} \vec{E} \cdot d\vec{S} \equiv 0$$

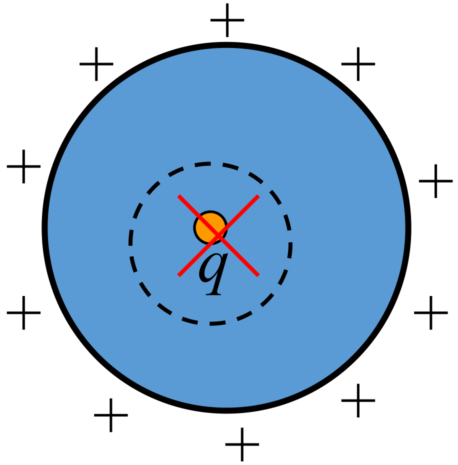
$$\oint_S \vec{E} \cdot d\vec{S} = \int_{in} \vec{E} \cdot d\vec{S} + \int_{out,side} \vec{E} \cdot d\vec{S} + \int_{out,flat} \vec{E} \cdot d\vec{S}$$

It's very easy to calculate electric flux! $= \int_{out,flat} \vec{E} \cdot d\vec{S} = E \cdot A = \frac{\sigma A}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{\varepsilon_0}$

It's very easy to calculate electric flux!

Q5: What is the charge distribution on a conductor?

Case 1: solid charged conductor without hole



All the charges are on the surface

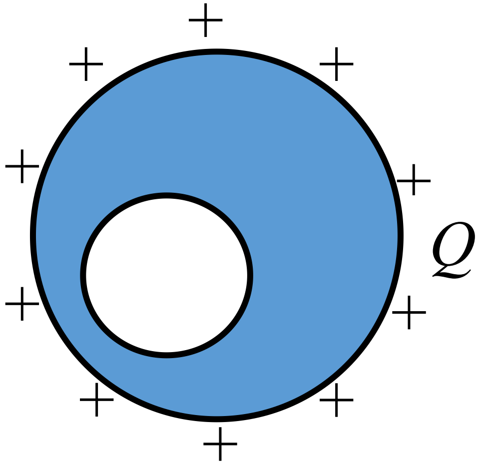
Because $\vec{\boldsymbol{E}} = \mathbf{0}$ everywhere inside conductor,

for any closed surface $\boldsymbol{S}$,

$$\oint_{S} \vec{\boldsymbol{E}} \cdot \mathbf{d}\vec{\boldsymbol{S}} = \mathbf{0} = \frac{\boldsymbol{Q_{in}}}{\boldsymbol{\varepsilon_0}}$$

Therefore any closed surface $\boldsymbol{S}$ there is no charge!

Case 2: holey conductor charged by $\boldsymbol{Q}$ without charge in the hole



All the charges are on outer surface

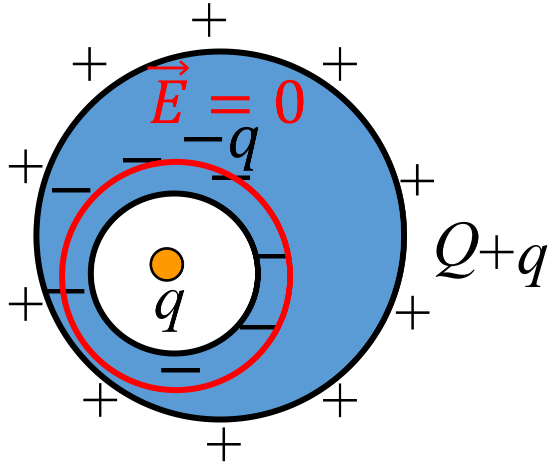
The reason is similar to case 1

Q5: What is the charge distribution on a conductor?

Case 3: holey conductor charged by $\boldsymbol{Q}$ with a charge q in the hole

Charge $-q$ on inner surface

Charge $Q+q$ on outer surface



Gaussian surface

Because

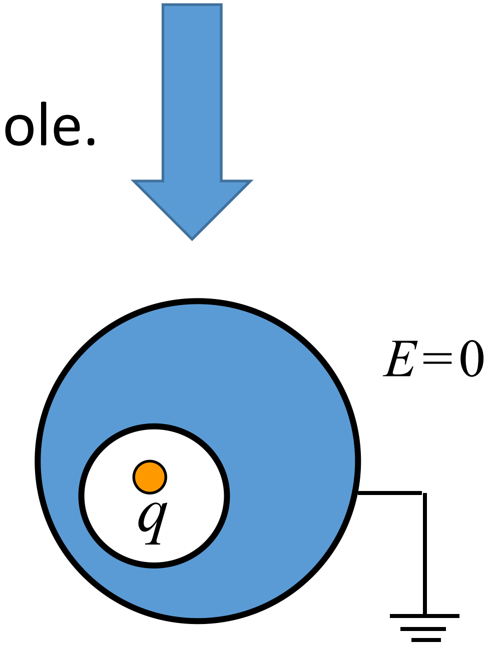
$$\oint_S \vec{E}\cdot \mathrm{d}\vec{S} = 0 \Rightarrow Q_{in} = 0 \Rightarrow Q_{inner\ surface} = 0$$

Here S is a closed surface inside the conductor and enclosing the whole hole.

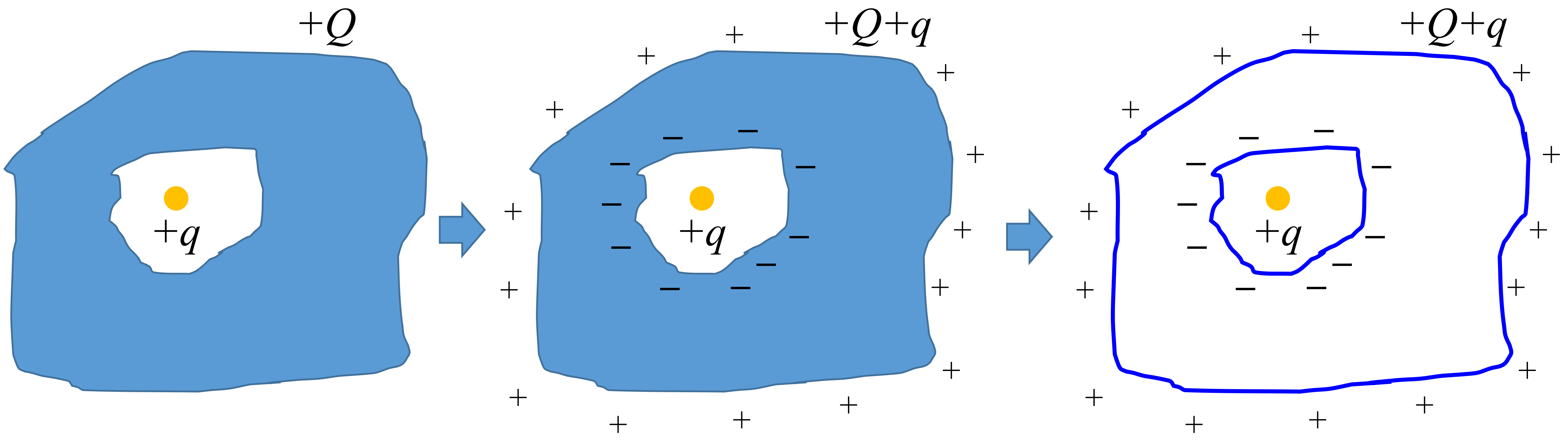
Faraday cage

Electrostatic shielding

A body placed inside the cavity of conductor will not be affected by the electric field outside.



Q6: How to deal with problems involving conductor?



Initial state

Electrostatic equilibrium

$\Downarrow$

Charge redistribution

Charges and charged surface

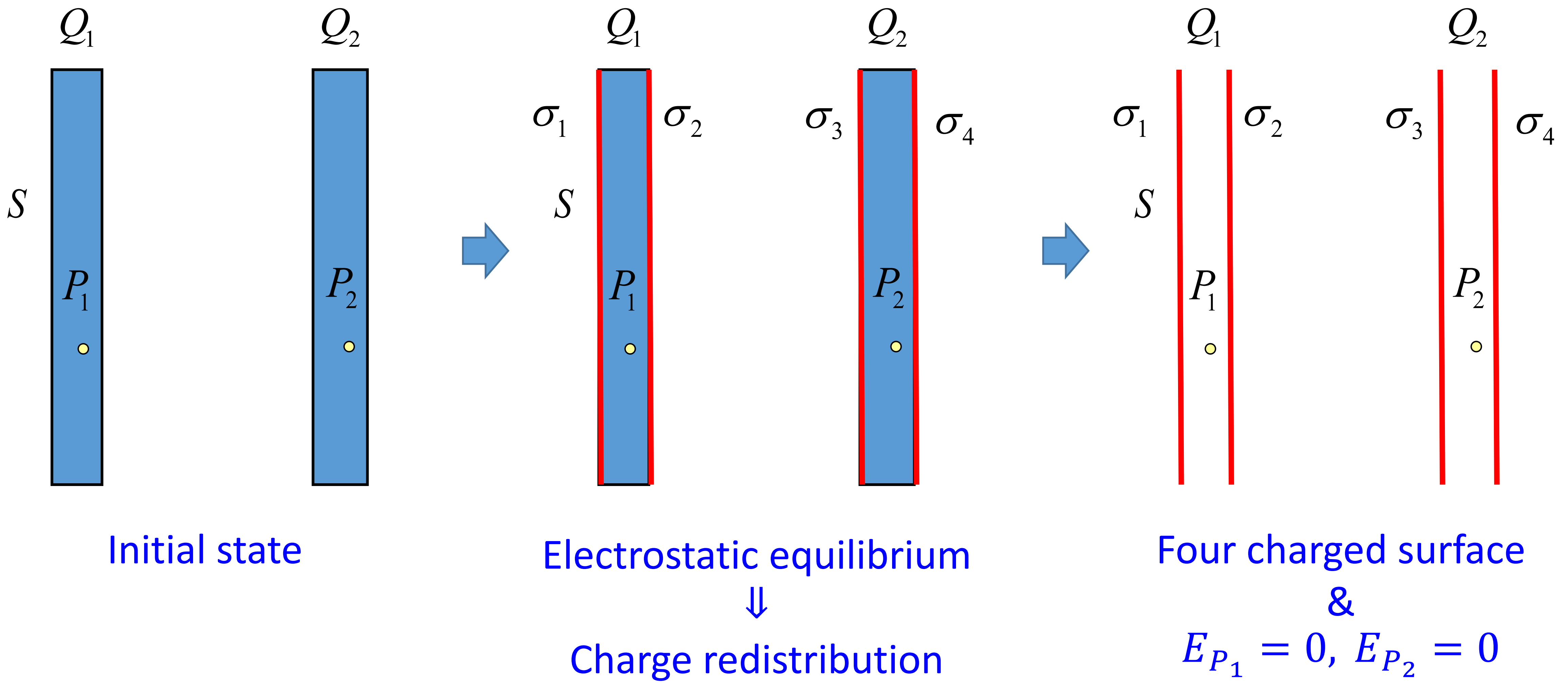
Principle of superposition of $\vec{E}$

Example4: Flat metal plates

Two large flat metal plates with charges Q_1 and Q_2. Determine:

(a) charges on each surface;

(b) electric field between the plates.



Example4: Flat metal plates

Two large flat metal plates with charges Q_1 and Q_2. Determine:

(a) charges on each surface;

(b) electric field between the plates.

Each charged plane generates two uniform field!

Solution:

$$(\sigma_1+\sigma_2)S=Q_1$$

$$(\sigma_3+\sigma_4)S=Q_2$$

$$\frac{\sigma_1}{2\varepsilon_0}-\frac{\sigma_2}{2\varepsilon_0}-\frac{\sigma_3}{2\varepsilon_0}-\frac{\sigma_4}{2\varepsilon_0}=0$$

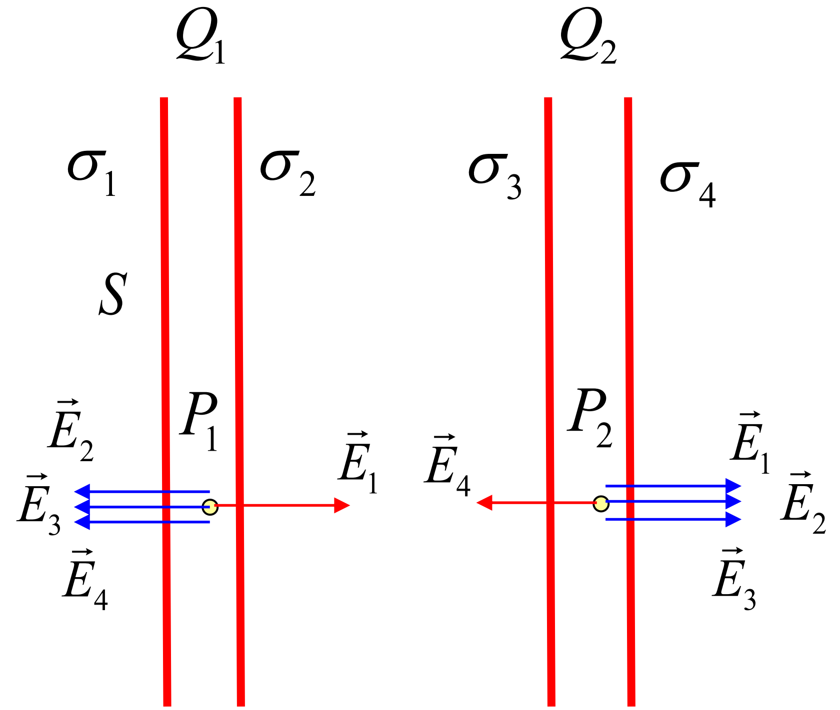
$$\frac{\sigma_1}{2\varepsilon_0}+\frac{\sigma_2}{2\varepsilon_0}+\frac{\sigma_3}{2\varepsilon_0}-\frac{\sigma_4}{2\varepsilon_0}=0$$

(a)

$$\begin{cases}\sigma_1=\sigma_4=\dfrac{Q_1+Q_2}{2S}\\[2ex]\sigma_2=\dfrac{Q_1-Q_2}{2S}=-\sigma_3\end{cases}$$

(b) E between plates:

$$E=\frac{\sigma_2}{\varepsilon_0}=\frac{Q_1-Q_2}{2\varepsilon_0 S}$$



Four charged surface

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$E_{P_1}=0,\ E_{P_2}=0$