# Circuit Analysis and Design

## Academic Year 2025/2026 – Semester 1

## Lecture 15 – Operational Amplifier Circuits

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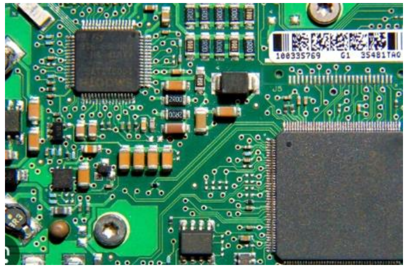
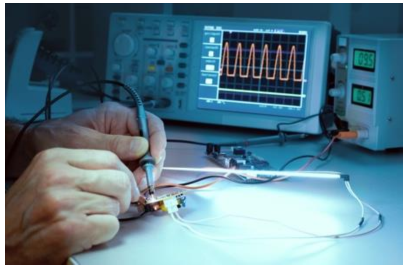
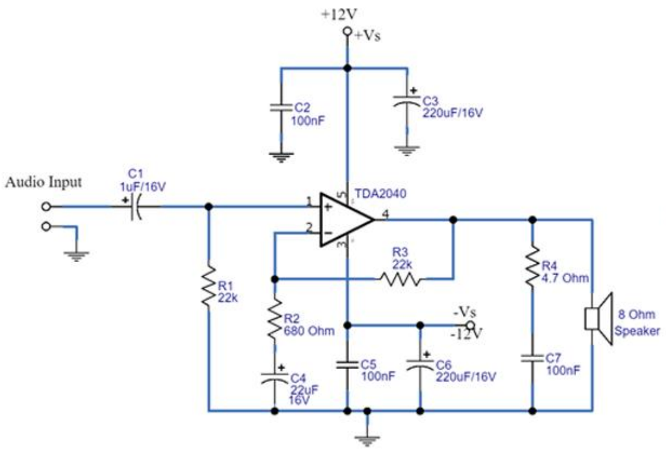




# Agenda

- ❑ Ideal op amp
- ❑ Sum and difference
- ❑ Analysis of inverting configuration
- ❑ Analysis of non-inverting configuration




# Introduction

- An operational amplifier (commonly called op amp or op amp*)* is a device that can be used to perform mathematical operations such as
  - Addition, Subtraction, amplification, attenuation, integration, and differentiation.
  - It is a versatile integrated circuit (IC) chip that is widely used in amplifiers, filters, signal conditioning, and instrumentation circuits.

- Circuit symbol for an op amp is shown in Figure 5.1 and Figure 5.2 shows pin configuration for a typical 8-pin package.

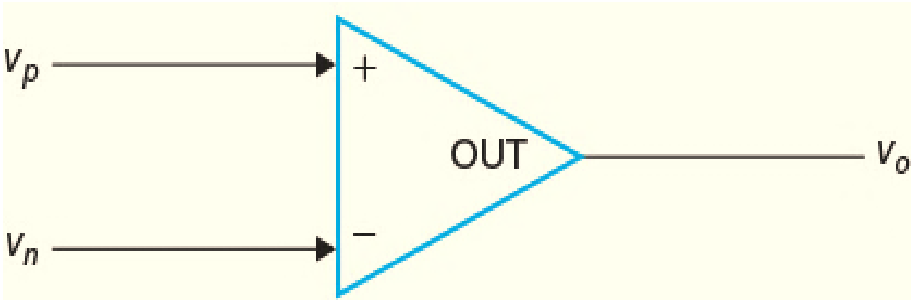



FIGURE 5.1
Circuit symbol for an op amp.

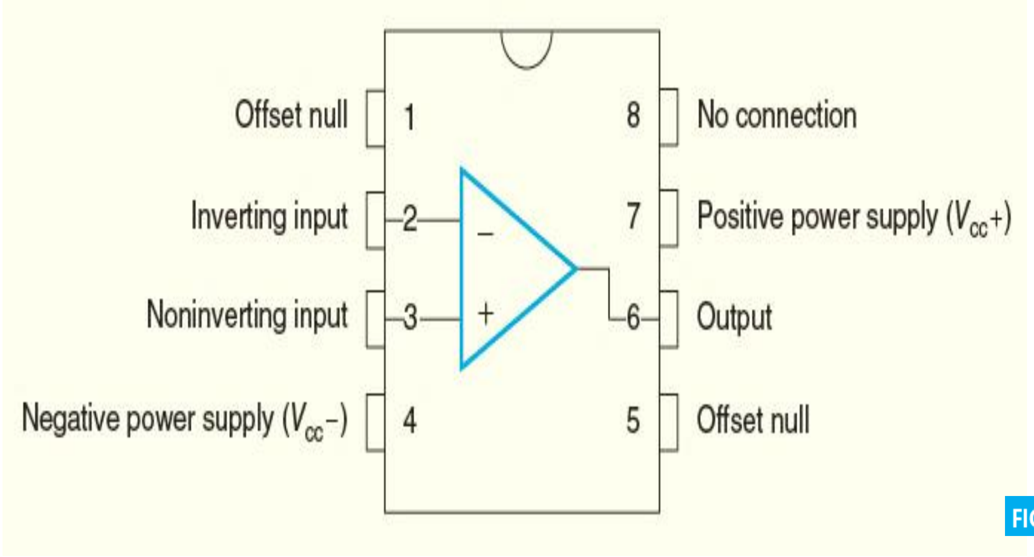


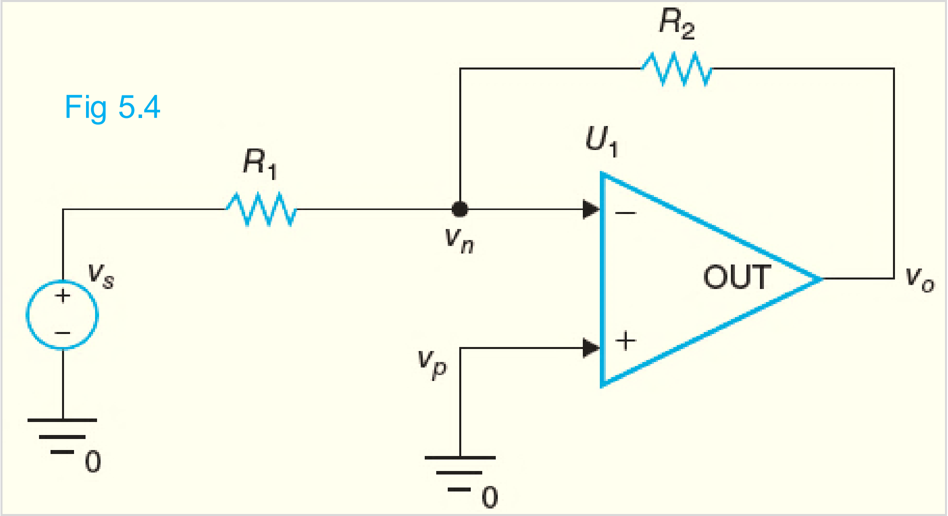

FIGURE 5.2

# Configurations OP-AMP

## ❑ There are two configurations to operate an OP-AMP

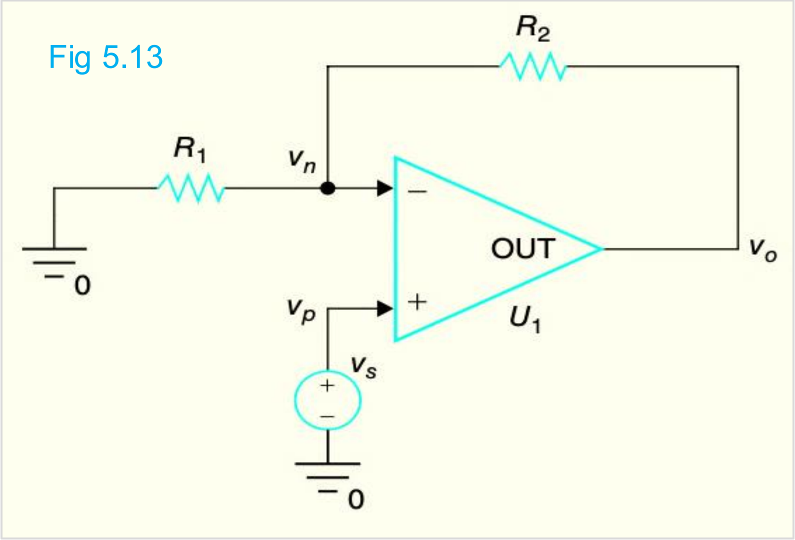
### Inverting Configuration

- Figure 5.4 shows inverting configuration of an op amp.
- Input voltage $v_s$ is applied to inverting input through a resistor $R_1$.
- Resistor $R_2$ provides a feedback path between the output terminal and the inverting input terminal.
- Output voltage $v_o$ is inverted compared to input $v_n$




### Non-Inverting Configuration

- Figure 5.13 shows non-inverting configuration of an op amp.
- Input voltage $v_s$ is applied to non-inverting input.
- Inverting terminal is connect to ground through a resister resistor $R_1$.
- Resistor $R_2$ provides a feedback path between the output terminal and the inverting input terminal.
- Output voltage $v_o$ is in phase with input voltage $v_p$



# Op Amp Equivalent Circuit (VCVS Model)

- Op amps can be modeled as a voltage-controlled voltage source (VCVS), as shown in Figure 5.3. In the model,

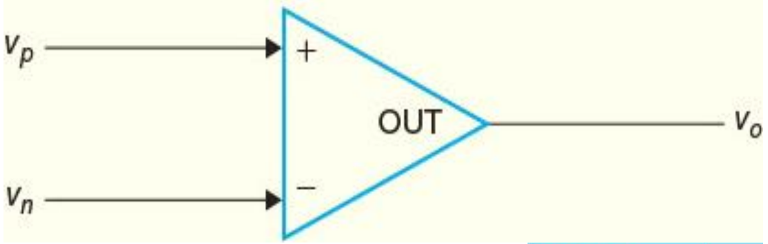
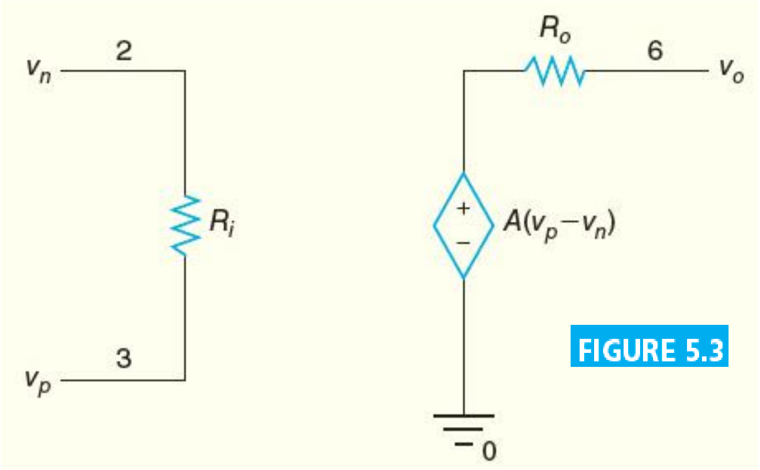



FIGURE 5.1
Circuit symbol for an op amp.



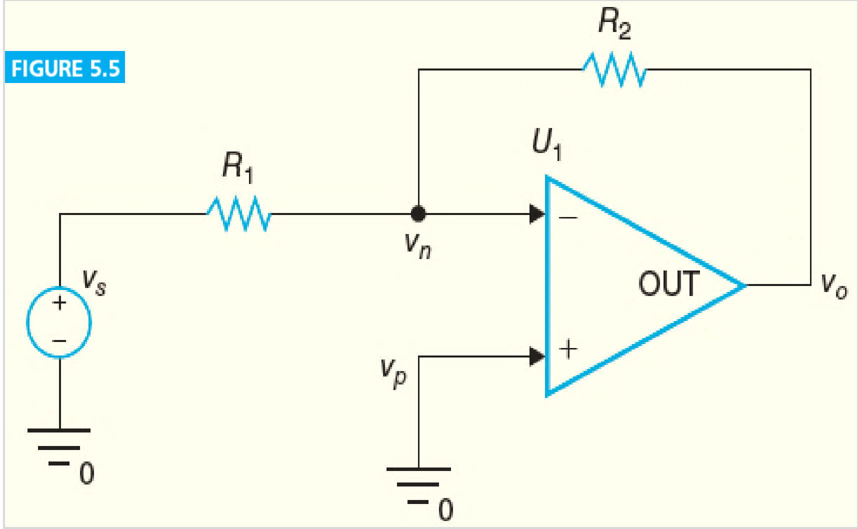

  - $v_n$ : is the voltage on the Inverting input (pin 2),
  - $v_p$ : is the voltage on the Noninverting input (pin 3),
  - $v_o$ : is the voltage on the Output (pin 6),
  - $R_i$ : is the Input Resistance,
  - $R_o$ : is the Output Resistance,
  - A : is the unloaded voltage Gain.

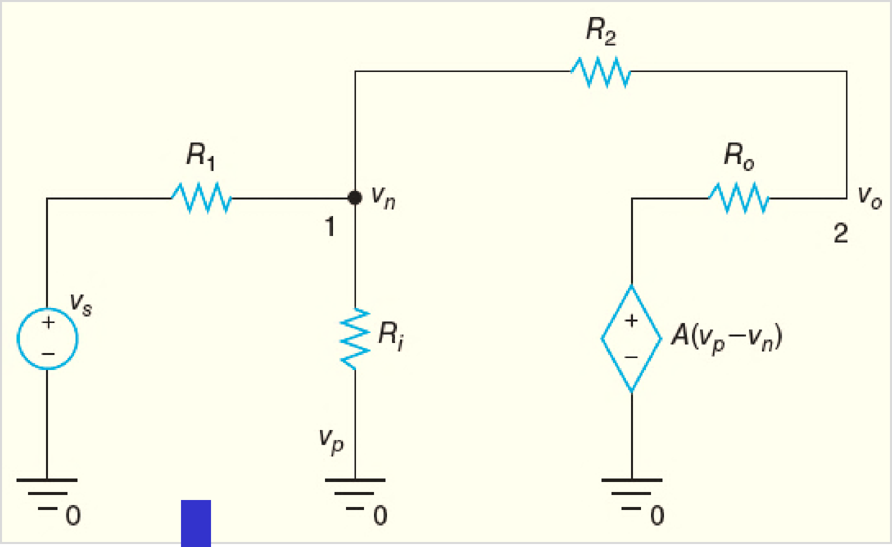
- In general, the input resistance $R_i$ is large, the output resistance $R_o$ is small, and the gain A is large.

# Ideal OP-AMP

- Let's, consider the inverting configuration. When the op amp is replaced by the model(VCVS) shown in Figure 5.3, we obtain the circuit shown in Figure 5.5.

FIGURE 5.5



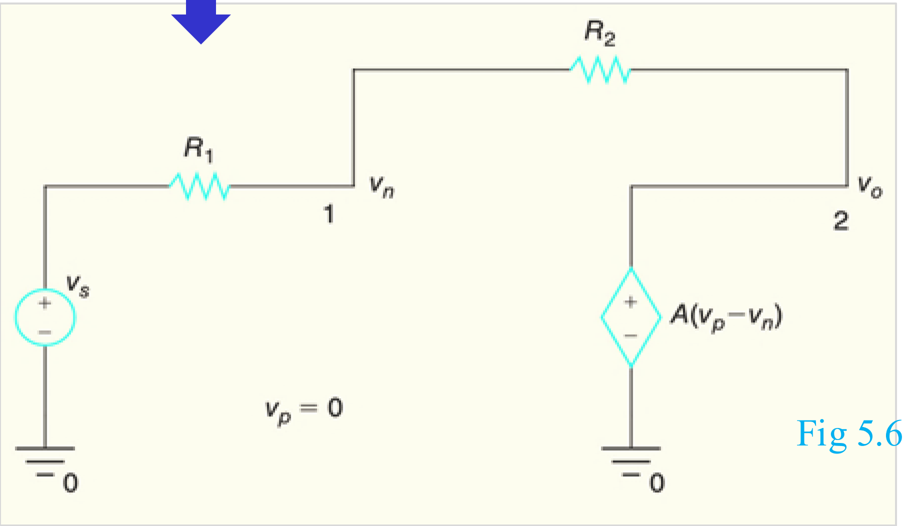




Fig 5.6

- **An Ideal OP-AMP Assumes**
  - $R_i = \propto, R_o = 0, A$ **is large** in Fig. 5.5

we can get Fig 5.6, then we can prove that

$$\boxed{v_d = v_p - v_n = 0 \;\; OR \; v_p = v_n \; and \; i_{R_i} = 0}$$

*$v_p$ and $v_n$ are virtually same OR $i_R$ = 0 i.e. virtually short*

# Ideal OP-AMP (Cont..)

- **For an Idea OP-AMP after assuming $R_i = \propto, R_o = 0$, in a circuit we can prove the following**

1. Current flowing into op-amp from positive terminal is zero $(i_p = 0)$
2. Current flowing into op-amp from negative terminal is zero $(i_n = 0)$
3. Voltage difference $v_d$ between $v_p$ and $v_n$ is $0$, that means $v_p = v_n$

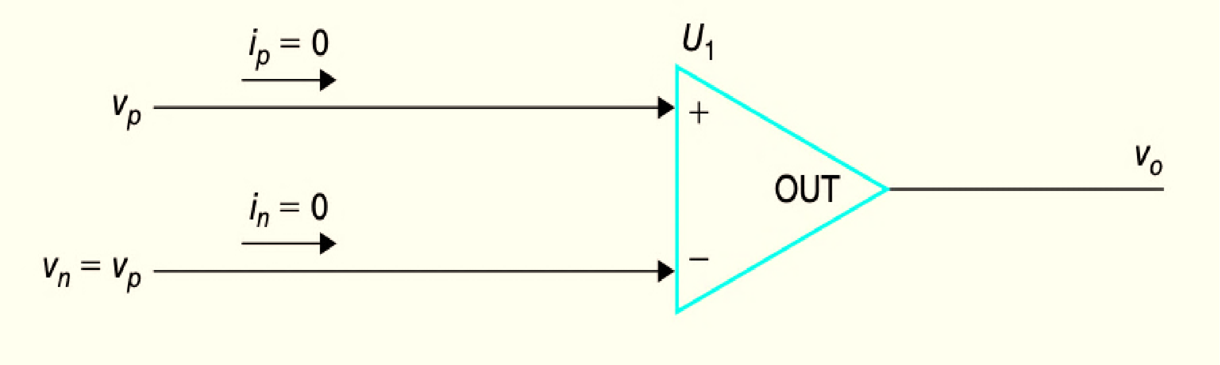



**FIGURE 5.10**
An op amp showing zero input currents and a virtual short.

- **These assumptions make analysis very simple, and the results of practical and ideal op-amp are not that much difference**
- **In the analysis, generally, we try to compute output voltage $v_o$**

# Quiz

Will be shown and solved in the Class

# Summing Amplifier (Inverting Config.)

- An inverting amplifier with two inputs is shown in Figure 5.30.

- **KCL: Sum the currents leaving at node 1 = 0 : Note for ideal OP-AMP voltage at Node 1 is $v_n = v_p = 0$ or ($V = U_1 = 0$)**

$$\frac{0-v_1}{R_1} + \frac{0-v_2}{R_2} + \frac{0-v_o}{R_f} = 0$$

- **Solve for $V_o$:**

$$\boxed{v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2\right)}$$

- **If** $\boxed{R_1 = R_f,\ R_2 = R_f,\quad V_o = -(V_1 + V_2)}$

- If $R_1 = R_f/k_1$, $R_2 = R_f/k_2$, we obtain

$$v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2\right) = -\left(k_1 v_1 + k_2 v_2\right)$$

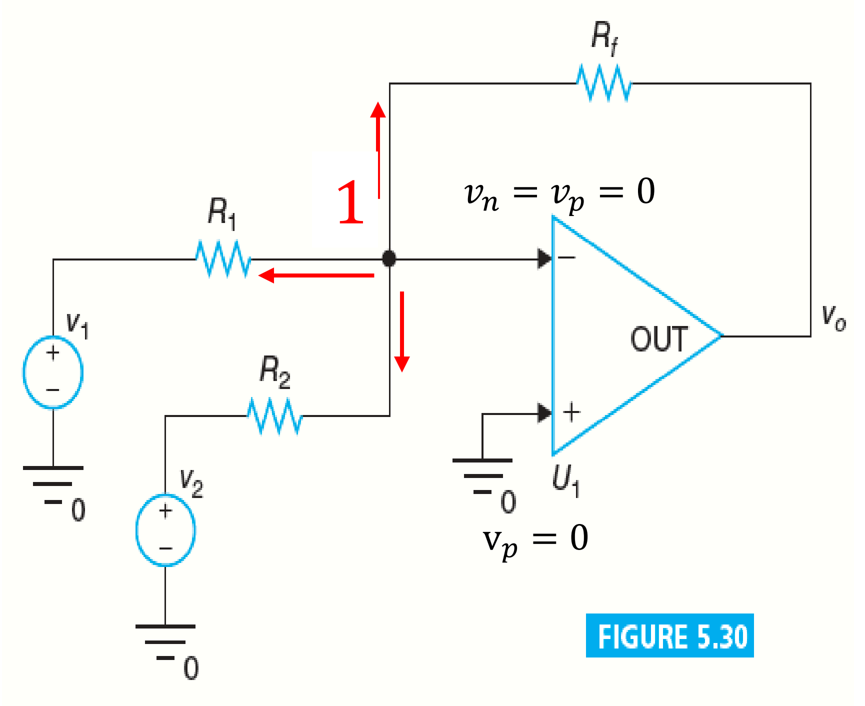


FIGURE 5.30

# Summing Amplifier (Inverting Config.)

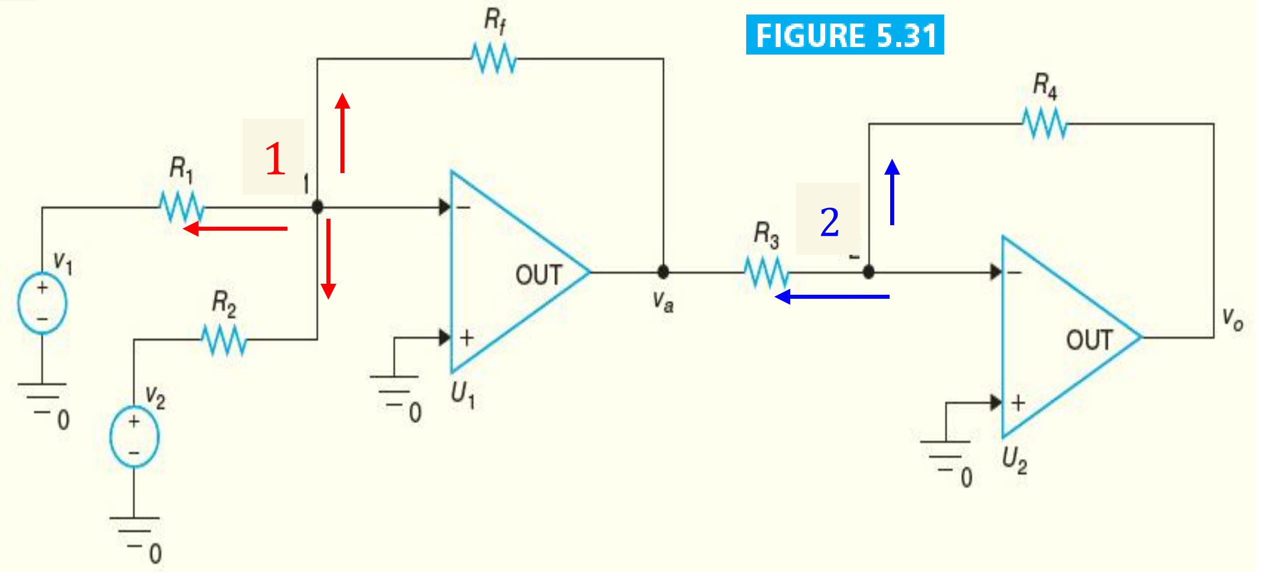
- To get positive sum instead of negative, connect another OP-AMP as shown in Figure 5.31
- Let $v_a$ be voltage at the output of first op amp. KCL at Node 2 :Sum of currents leaving = 0

$$\frac{0-v_a}{R_3}+\frac{0-v_0}{R_4}=0 \Rightarrow v_a=-\frac{R_3}{R_4}v_0 \qquad (1)$$

- Sum currents leaving Node 1 = 0:

$$\frac{0-v_1}{R_1}+\frac{0-v_2}{R_2}+\frac{0-v_a}{R_f}=0$$

$$v_a=-\left(\frac{R_f}{R_1}v_1+\frac{R_f}{R_2}v_2\right) \qquad (2)$$




- Substituting $v_a$ from 1 in 2, and solve for $v_o$ is

$$v_o=\frac{R_4}{R_3}\left(\frac{R_f}{R_1}v_1+\frac{R_f}{R_2}v_2\right)$$

If $R_3=R_4, R_f=R_1=R_2=R$ $\Rightarrow v_0=v_1+v_2$

# Summing Amplifier (Inverting Config.)

- An inverting amplifier with N inputs is shown in Figure 5.32.
- Sum the currents leaving node 1: $\frac{0-v_1}{R_1}+\frac{0-v_2}{R_2}+\ldots+\frac{0-v_N}{R_N}+\frac{0-v_o}{R_f}=0$
- Solve for $V_o$: $v_o=-\left(\frac{R_f}{R_1}v_1+\frac{R_f}{R_2}v_2+\ldots+\frac{R_f}{R_N}v_N\right)$
- If $R_1 = R_2 = \ldots = R_N = R_f$,

$$v_o = -(v_1 + v_2 + \ldots + v_N)$$

- If $R_1 = R_f/k_1$, $R_2 = R_f/k_2$, …, $R_N = R_f/k_N$, we obtain

$$v_o=-\left(k_1v_1+k_2v_2+\ldots+k_Nv_N\right)$$

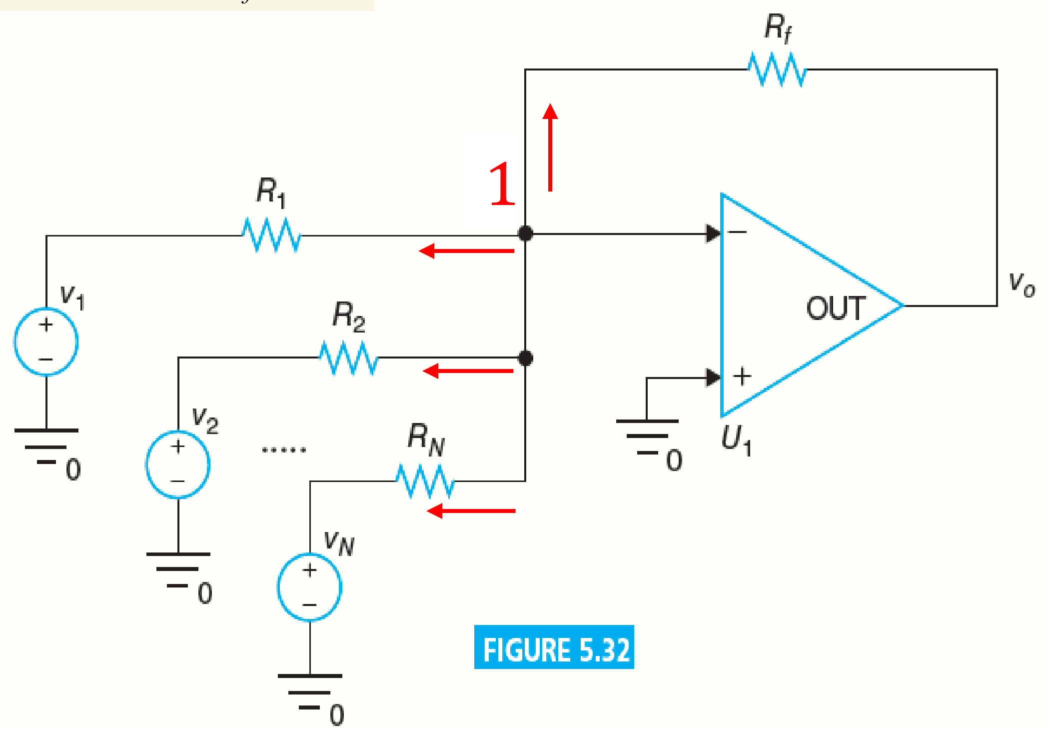


FIGURE 5.32

# Summing Amplifier (Noninverting Configuration)

- A non-inverting amplifier with two inputs is shown in Figure 5.33.
- Voltage divider rule on $R_5$-$R_4$:

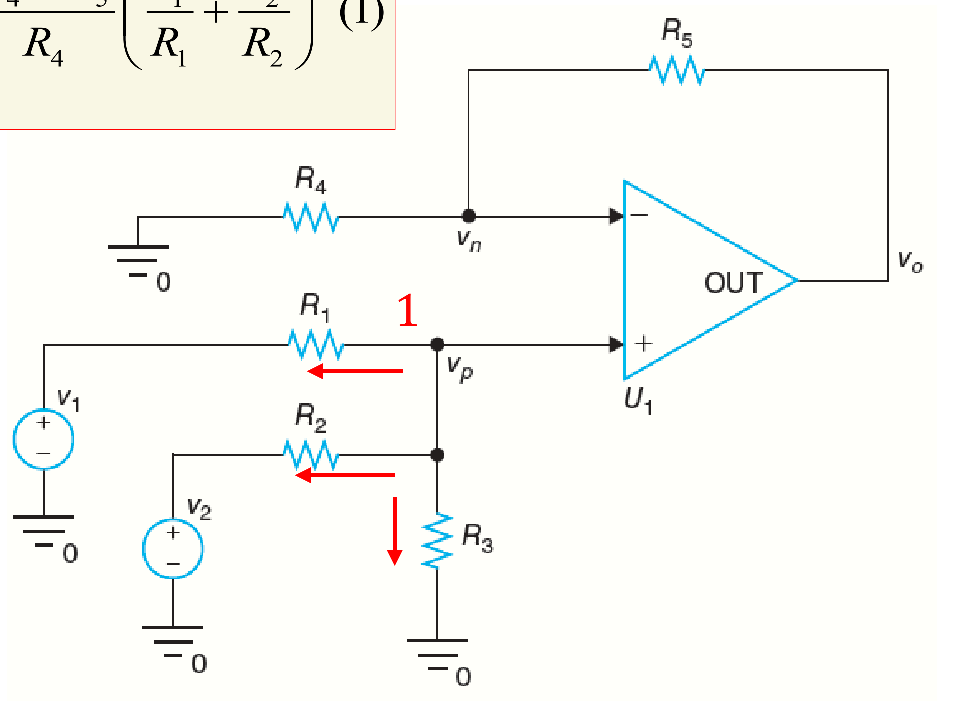
$$v_n = \frac{R_4}{R_4 + R_5} v_o \Rightarrow v_o = \frac{R_4 + R_5}{R_4} v_n$$

- Sum the currents leaving node 1: (note $v_p = v_n$)

$$\frac{v_p - v_1}{R_1} + \frac{v_p - v_2}{R_2} + \frac{v_p}{R_3} = 0 \Rightarrow \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) v_p = \frac{v_1}{R_1} + \frac{v_2}{R_2},$$

$$v_o = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \frac{R_4 + R_5}{R_4} \left(\frac{v_1}{R_1} + \frac{v_2}{R_2}\right) \quad (1)$$

FIGURE 5.33




- If $R_1 = R_2 = R_3 = R_4 = R$ and $R_5 = 2R$, then, $V_o$ becomes $V_o = V_1 + V_2$

- If $R_3 = R_4 = R$, $R_1 = R/k_1$, $R_2 = R/k_2$, and $R_5 = R(k_1 + k_2)$, $v_o$ becomes $v_o = k_1 v_1 + k_2 v_2$ (2)

# EXAMPLE 5.7

- **Design an op amp circuit for** $V_o = 0.5V_1 + 3V_2$.
  - $k_1 = 0.5$, $k_2 = 3$
  - **Let** $R = 3\ k\Omega$. **Then,**
  - $R_3 = R_4 = R = 3\ k\Omega$
  - $R_1 = R/k_1 = 6\ k\Omega$
  - $R_2 = R/k_2 = 1\ k\Omega$
  - $R_5 = R(k_1 + k_2) = 10.5\ k\Omega$
  - **The circuit is shown in Figure 5.34.**

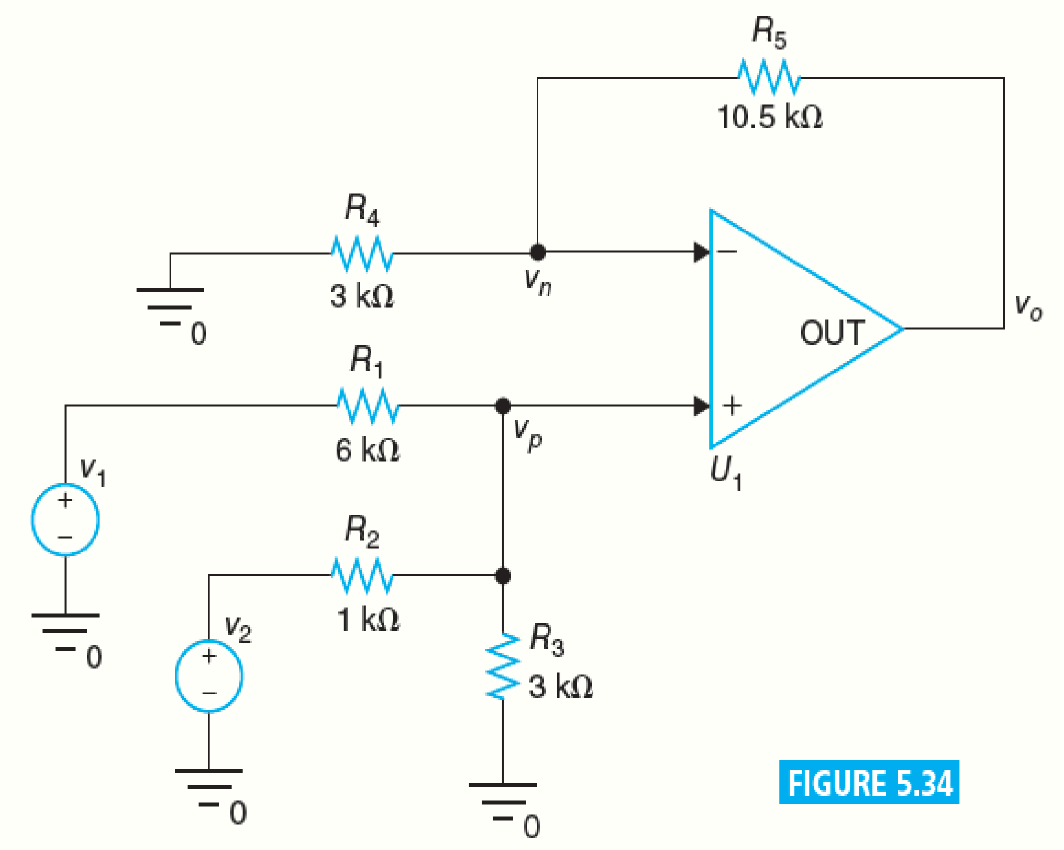



FIGURE 5.34

If $R_3 = R_4 = R$, $R_1 = R/k_1$, $R_2 = R/k_2$, and
$R_5 = R(k_1 + k_2)$,
$\mathbf{v_o = k_1 v_1 + k_2 v_2}$
Summing Amplifier (Noninverting Configuration)

# Quiz

Will be shown and solved in the Class

# EXAMPLE 5.8

- Design a circuit for **converting a polar binary signal $V_1$** (1 is represented by a pulse with 5 V, and 0 is represented by a pulse with – 5 V) **to a unipolar binary signal $v_o$** (1 is represented by a pulse with 5 V, and 0 is represented by a pulse with 0 V).

- **If** $R_1 = R_2 = R_3 = R_5 = R$ **and** $R_4 = 2R$, $v_o$ **becomes**

$$v_o = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} \frac{R_4+R_5}{R_4}\left(\frac{v_1}{R_1}+\frac{v_2}{R_2}\right) = \frac{1}{2}\left(v_1+v_2\right)$$

- **The circuit is shown in Figure 5.36 ($v_2$ = 5 V).**
- **Alternate choice:**

  $k_1 = k_2 = 0.5$

  $R = 10\ k\Omega$, $R_3 = R_4 = R = 10\ k\Omega$, $R_1 = R/k_1 = 20\ k\Omega$,

  $R_2 = R/k_2 = 20\ k\Omega$, $R_5 = R(k_1 + k_2) = 10\ k\Omega$

- **Figure 5.37 shows sample waveforms.**

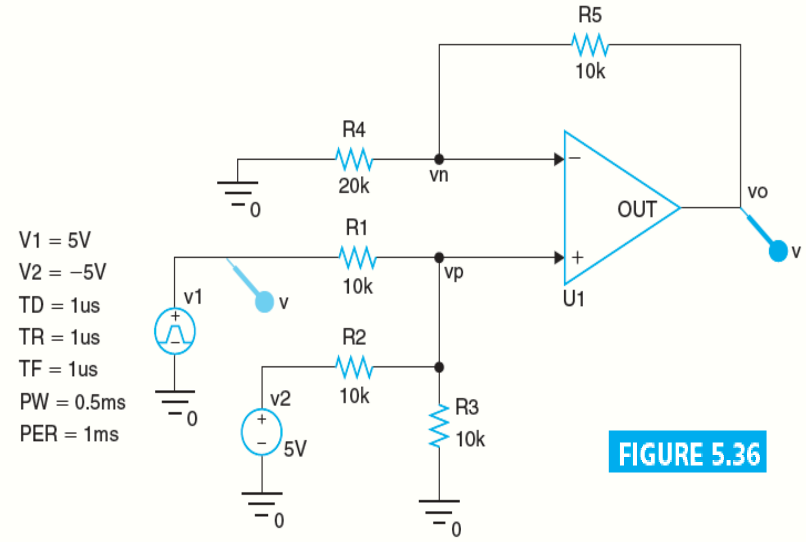



FIGURE 5.36

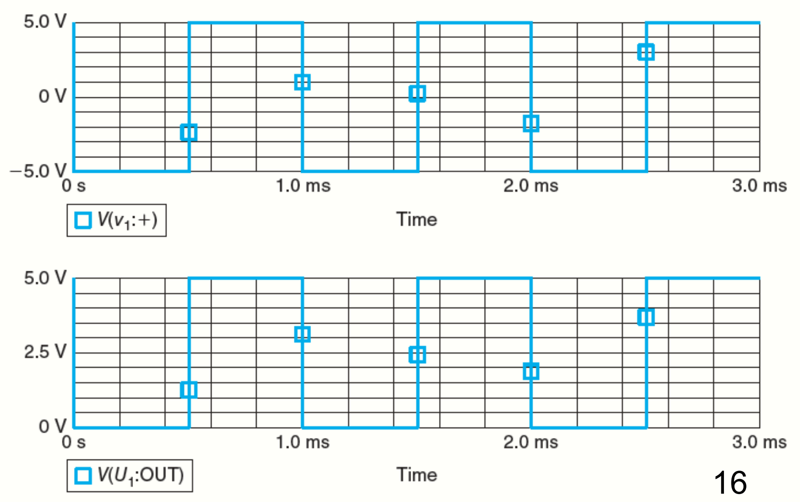



FIGURE 5.37

# Summing Amplifier (Noninverting Configuration, N Inputs)

- A non-inverting summing amplifier with N inputs is shown in Figure 5.38.
- Voltage divider rule on $R_{N+3}$ - $R_{N+2}$: $v_n = \dfrac{R_{N+2}}{R_{N+2}+R_{N+3}} v_o \Rightarrow v_o = \dfrac{R_{N+2}+R_{N+3}}{R_{N+2}} v_n$
- Sum the currents leaving node 1: Note Vn = Vp

$$\frac{v_p - v_1}{R_1} + \ldots + \frac{v_p - v_N}{R_N} + \frac{v_p}{R_{N+1}} = 0 \Rightarrow \left(\frac{1}{R_1} + \ldots + \frac{1}{R_{N+1}}\right) v_p = \frac{v_1}{R_1} + \ldots + \frac{v_N}{R_N}$$

$$v_o = \frac{1}{\dfrac{1}{R_1} + \ldots + \dfrac{1}{R_{N+1}}} \frac{R_{N+2}+R_{N+3}}{R_{N+2}} \left(\frac{v_1}{R_1} + \ldots + \frac{v_N}{R_N}\right) \quad (1)$$

- If $R_1 = R_2 = \ldots = R_{N+2} = R$ and $R_{N+3} = NR$, $v_o$ becomes

$$v_o = v_1 + v_2 + \ldots + v_N$$

- If $R_{N+1} = R_{N+2} = R$, $R_1 = R/k_1, \ldots, R_N = R/k_N$, and $R_{N+3} = R(k_1 + k_2 + \ldots + k_N)$,

$$v_o = k_1 v_1 + k_2 v_2 + \ldots + k_N v_N \quad (2)$$

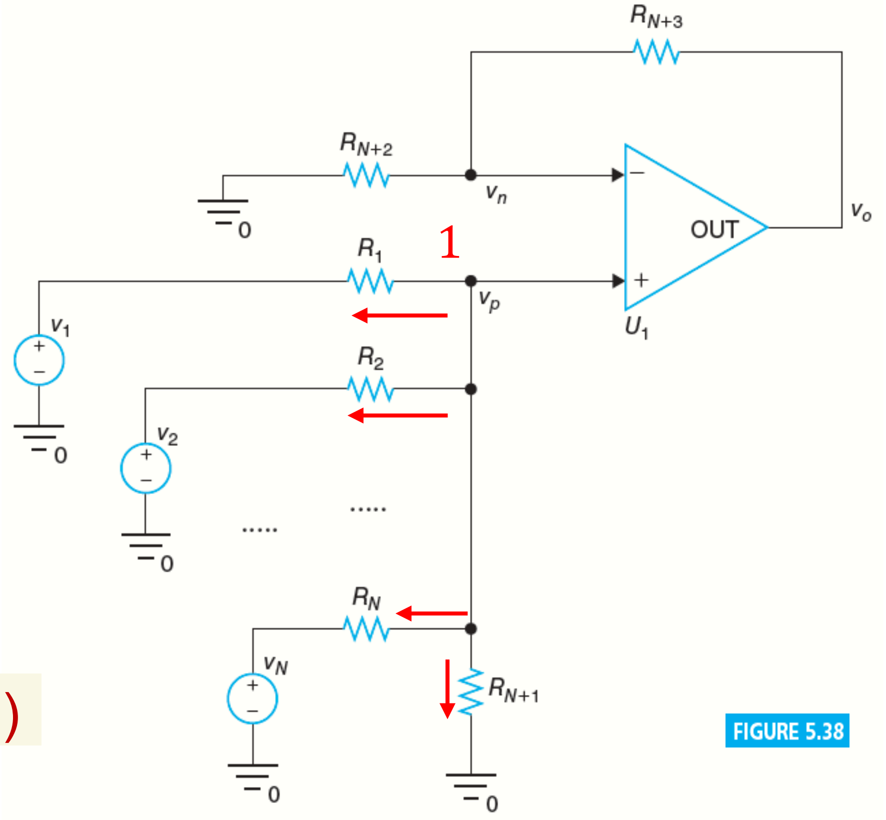


FIGURE 5.38

# Difference Amplifier

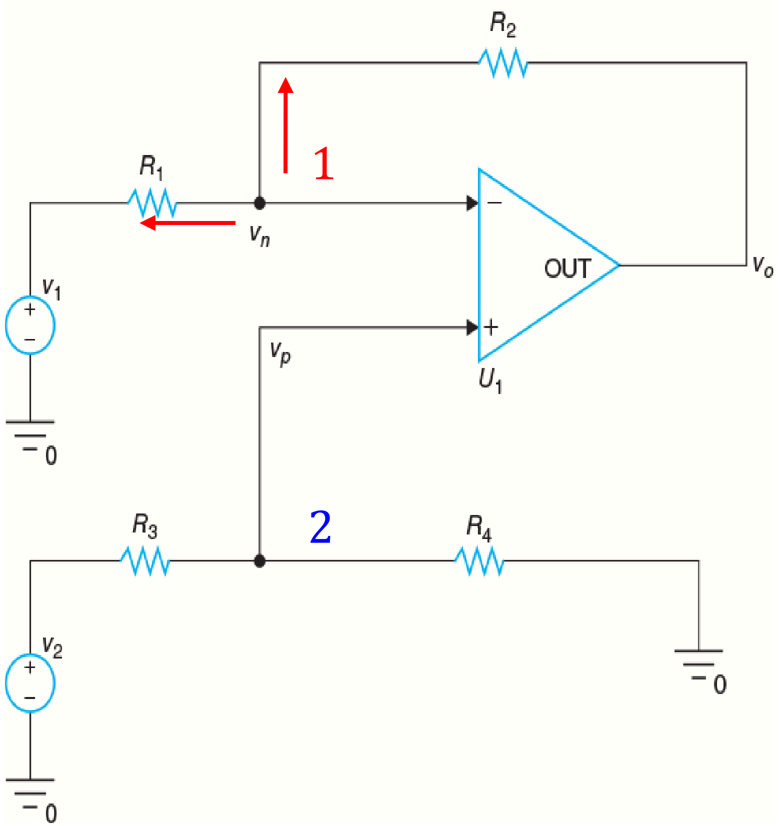
- A difference amplifier is shown in Figure 5.41.
- Voltage divider rule on $R_3$-$R_4$: $$v_p = \frac{R_4}{R_3 + R_4} v_2 \quad (1)$$
- Sum the currents leaving node 1: $$\frac{v_n - v_1}{R_1} + \frac{v_n - v_o}{R_2} = 0 \Rightarrow v_o = \frac{R_1 + R_2}{R_1} v_n - \frac{R_2}{R_1} v_1 \quad (2)$$
- Substitute Equation (1) into Equation (2): $$v_o = \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) v_2 - \frac{R_2}{R_1} v_1 \quad (3)$$
- If $R = R_1 = R_2 = R_3 = R_4$, Equation (3) becomes

  $v_o = v_2 - v_1$

  $v_o$ is the difference of $v_2$ and $v_1$.

FIGURE 5.41

Difference amplifier.



# EXAMPLE 5.9

- Design a circuit for converting a **unipolar binary signal $v_2$** (1 is represented by a pulse with 5 V, and 0 is represented by a pulse with 0 V) **to a polar binary signal $v_o$** (1 is represented by a pulse with 5 V, and 0 is represented by a pulse with – 5V).

$V_o = 2(v_2 - 2.5)$

- **If** $R_3 = R_1$, $R_4 = R_2$, $R_2 = 2R_1$, $v_1 = 2.5$ V, **we get**

$$v_o = \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)v_2 - \frac{R_2}{R_1}v_1 = 2(v_2 - 2.5)$$

- **We choose** $R_1 = R_3 = 10\ \text{k}\Omega$, $R_2 = R_4 = 20\ \text{k}\Omega$ **in the circuit shown in Figure 5.42.**

- **Figure 5.43 shows sample waveforms.**

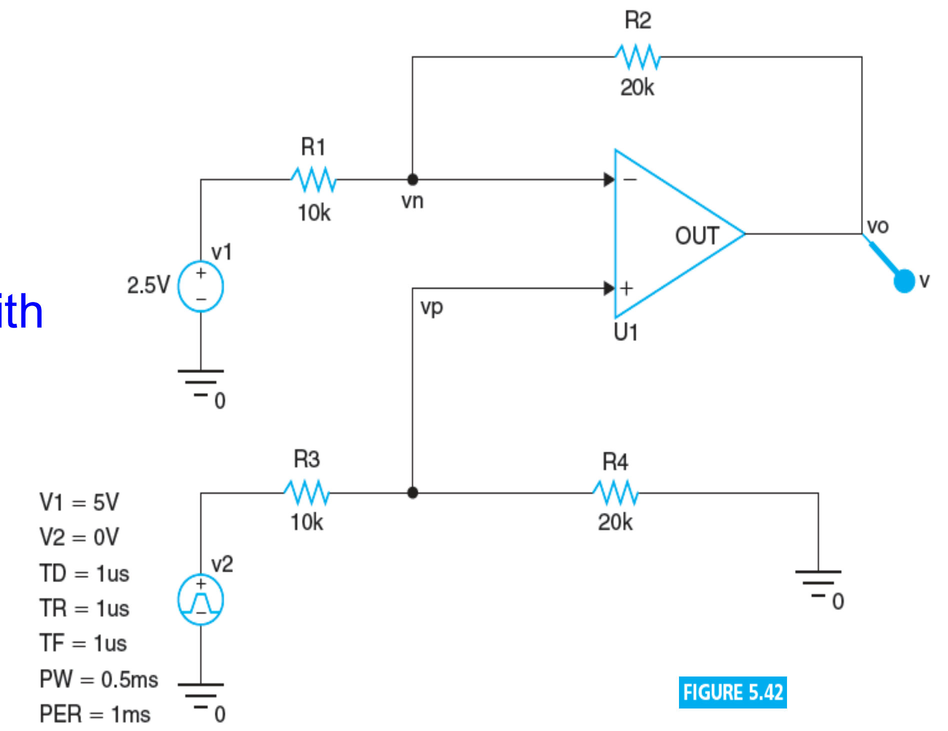



FIGURE 5.42

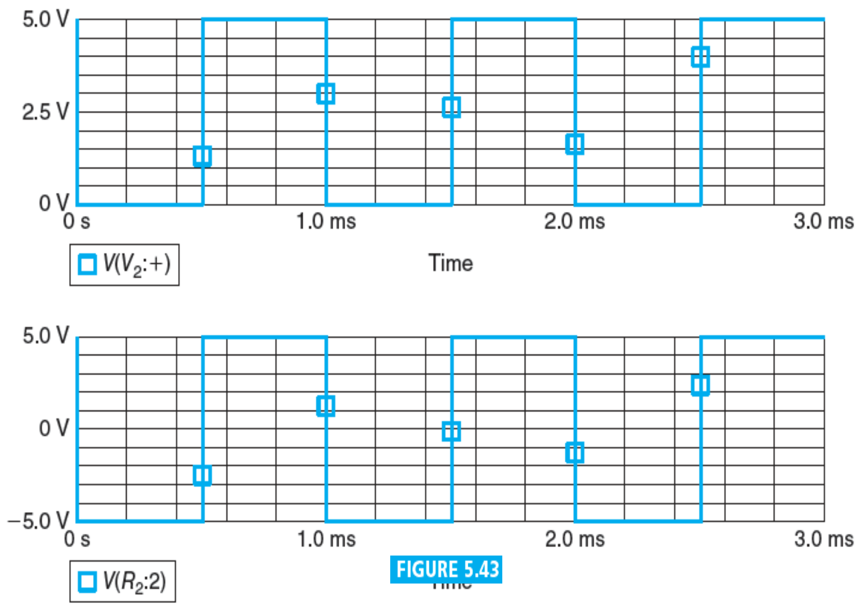



FIGURE 5.43

# Op Amp Circuit to Implement $v_o = 7v_1 - 2v_2$

- We can use the superposition principle to show that the circuit shown in Figure 5.44 provides an output voltage given by $v_o = 7v_1 - 2v_2$.

- Deactivate $v_2$ by short-circuiting it. $v_a = 0$. $U_2$ is configured as a noninverting amplifier. Thus,

$$v_o = (1 + R_4/R_3)v_1 = (1 + 6)v_1 = 7v_1$$

- Deactivate $v_1$ by short-circuiting it. From the voltage divider rule, the voltage at node 1 is given by $v_2/3$. Thus, $v_a = v_2/3$. $U_2$ is configured as an inverting amplifier. Thus,

$$v_o = (-R_4/R_3)v_a = (-6)v_2/3 = -2v_2$$

- Adding the two outputs, we obtain

$$v_o = 7v_1 - 2v_2$$

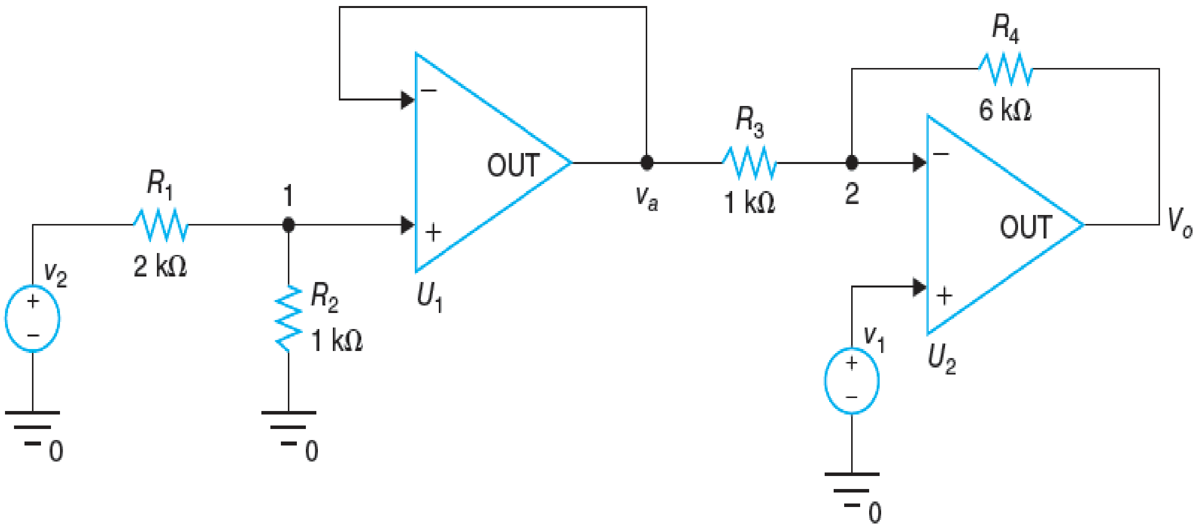


FIGURE 5.44

# Analysis of OP-AMP

- **Before in these sections, We assumed that $R_i = \infty$ and $R_o = 0$.**

- **In the coming sections, we will analyze circuits, including $R_i$ and $R_o$ and see how these considerations impact, differently from our assumptions**

- **We will analyze both configurations of OP-AMP, i.e.**
  - **Analysis of Inverting OP-AMP and**
  - **Analysis of Non-inverting OP-AMP**

# Analysis of Inverting Configuration

- A model for an inverting configuration is shown in Figure 5.52. Notice that $V_p = 0$.
- KCL: Summing the currents leaving node 2 = 0, we obtain

$$\frac{v_o - v_n}{R_2} + \frac{v_o + Av_n}{R_o} = 0 \Rightarrow v_n = \frac{R_o + R_2}{R_o - R_2 A} v_o, \quad v_o = \frac{R_o - R_2 A}{R_o + R_2} v_n \qquad (1)$$

- KCL: Summing the currents leaving node 1 = 0, we obtain

$$\frac{v_n - v_s}{R_1} + \frac{v_n}{R_i} + \frac{v_n - v_o}{R_2} = 0 \Rightarrow \left(\frac{1}{R_1} + \frac{1}{R_i} + \frac{1}{R_2}\right) v_n - \frac{v_o}{R_2} = \frac{v_s}{R_1} \qquad (2)$$

- Solving Equations (1) and (2) for $V_n$ and $V_o$, we get

$$v_n = \frac{R_i (R_o + R_2) v_s}{R_o R_i + R_o R_1 + R_i R_2 + R_1 R_2 + R_1 R_i + A R_1 R_i} \qquad (3)$$

$$v_o = \frac{-R_i(-R_o + R_2 A) v_s}{R_o R_i + R_o R_1 + R_i R_2 + R_1 R_2 + R_1 R_i + A R_1 R_i} \qquad (4)$$

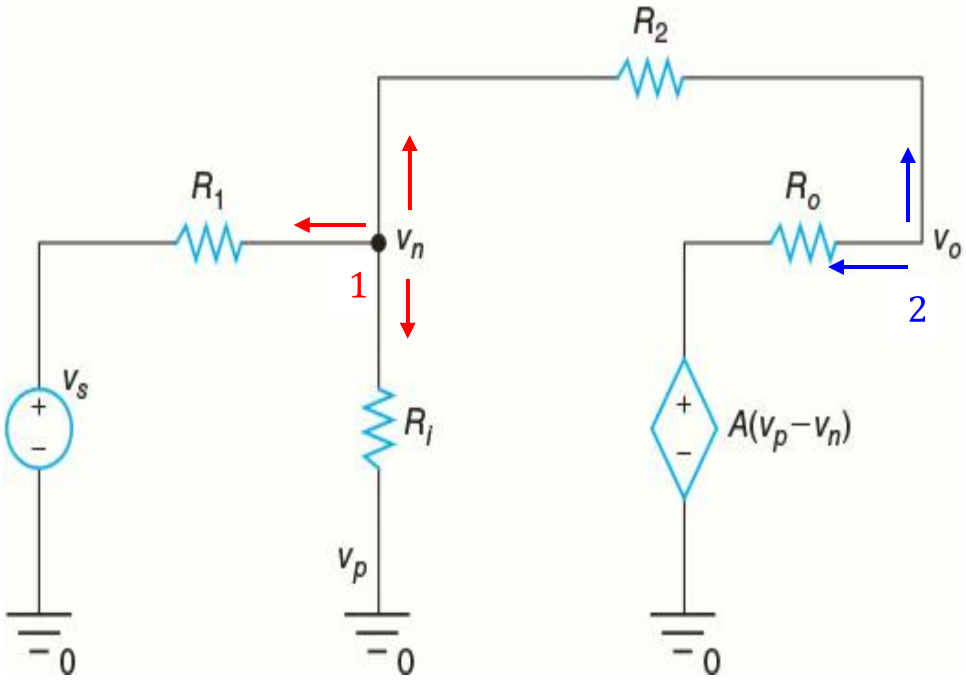



FIGURE 5.52
A model for an inverting configuration.

# Analysis of Inverting Configuration (Cont.)

- If $R_o = 0$ and $R_i = \infty$, and $A >> (R_1 + R_2)$ Equations (3) and (4) become, respectively

$$v_n = \frac{R_2 v_s}{R_1 + R_2 + AR_1} \cong \frac{R_2}{AR_1} v_s \approx 0 \quad (5)$$

$$v_o = \frac{-R_2 A v_s}{R_1 + R_2 + AR_1} \cong -\frac{R_2}{R_1} v_s \quad (6)$$

- Current flowing into the OP-AMP from negative terminal is : $I_n$

$$i_n = \frac{v_n}{R_i} = \frac{(R_0 + R_2)v_s}{R_o R_i + R_o R_1 + R_i R_2 + R_1 R_2 + R_1 R_i + AR_1 R_i} \approx \frac{R_2 v_s}{R_1 A R_i} \approx 0$$

- Current through $R_1$ is given by

$$i_1 = \frac{v_s - v_n}{R_1} = \frac{R_o + R_2 + R_i + AR_i}{R_o R_i + R_o R_1 + R_i R_2 + R_1 R_2 + R_1 R_i + AR_1 R_i} v_s$$

- **Input Resistance** is given by

$$R_{in} = \frac{v_s}{i_1} = \frac{R_o R_i + R_o R_1 + R_i R_2 + R_1 R_2 + R_1 R_i + AR_1 R_i}{R_o + R_2 + R_i + AR_i} \approx R_1$$

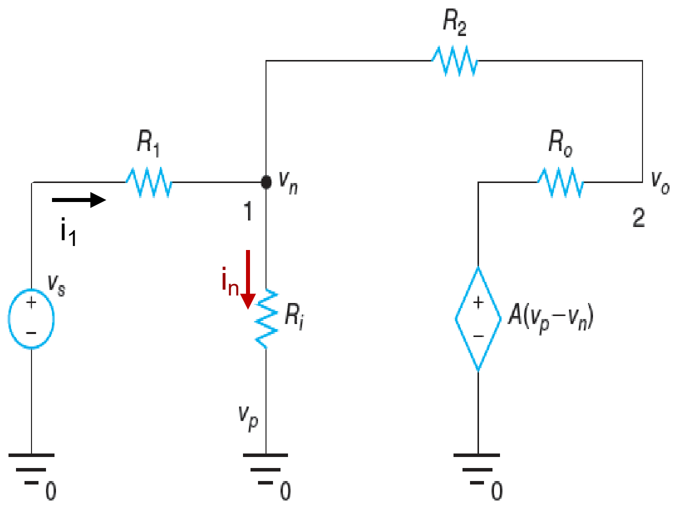



FIGURE 5.52

# Analysis of Inverting Configuration (Cont.)

- **Output Resistance:** Apply test voltage $V_t$ between node 2 and ground as shown in Figure 5.53. From the voltage divider rule, $V_n$ is given by

$$v_n = \frac{R_1 \| R_i}{R_2 + (R_1 \| R_i)} v_t = \frac{\dfrac{R_1 R_i}{R_1 + R_i}}{R_2 + \dfrac{R_1 R_i}{R_1 + R_i}} v_t = \frac{R_1 R_i}{R_1 R_2 + R_2 R_i + R_1 R_i} v_t$$

- The current out of the test voltage source is

$$i_t = \frac{v_t - v_n}{R_2} + \frac{v_t + A v_n}{R_o} = \frac{v_t - \dfrac{R_1 R_i}{R_1 R_2 + R_2 R_i + R_1 R_i} v_t}{R_2} + \frac{v_t + A \dfrac{R_1 R_i}{R_1 R_2 + R_2 R_i + R_1 R_i} v_t}{R_o}$$

- Output Resistance is the ratio of $V_t$ to $i_t$: $R_{out} = v_t / i_t$

$$\boxed{R_{out} = \frac{R_o (R_1 R_2 + R_2 R_i + R_1 R_i)}{R_o R_1 + R_o R_i + A R_1 R_i + R_1 R_2 + R_2 R_i + R_1 R_i} \cong \frac{R_o (R_2 + R_1)}{A R_1} \approx 0}$$

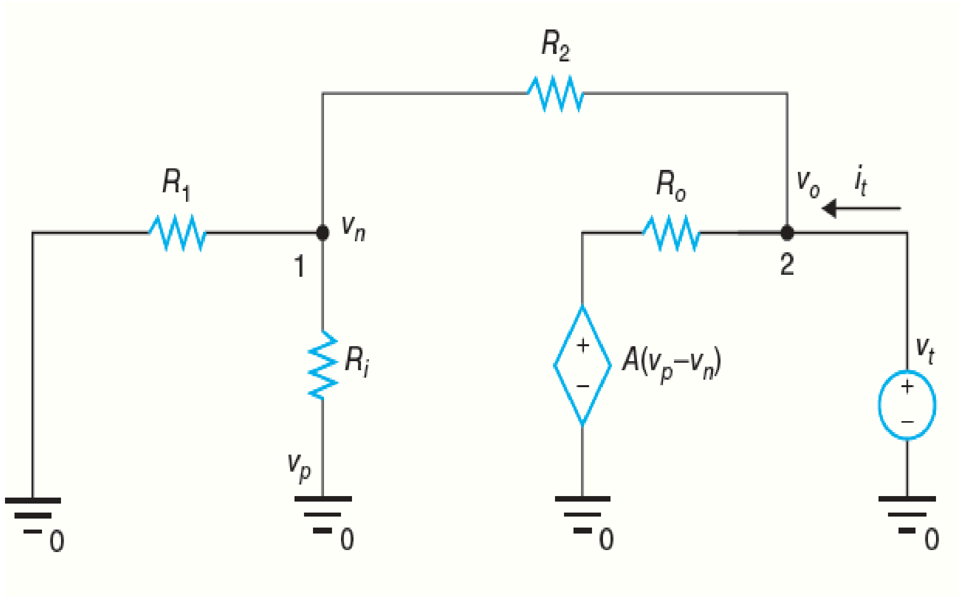



FIGURE 5.53

- The output resistance is close to zero for the inverting configuration.

# EXAMPLE 5.10

- **Find $V_o$ in the circuit shown in Figure 5.54.**
- **Summing the currents leaving Node 1 = 0, we obtain**

$$\frac{V_n - 1}{1000} + \frac{V_n}{1000} + \frac{V_n - V_o}{5000} = 0$$

- **Multiplication by 5000 yields**

  $5V_n - 5 + 5V_n + V_n - V_o = 0 \Rightarrow 11V_n - V_o = 5$ (1)

- **Summing the currents leaving Node 2 = 0, we obtain**

$$\frac{V_o - V_n}{5000} + \frac{V_o - 1000(0 - V_n)}{1000} = 0$$

- **Multiplication by 5000 yields**

  $V_o - V_n + 5V_o + 5000V_n = 0 \Rightarrow 4999V_n + 6V_o = 0$ (2)

- **Multiply Equation (1) by 6:** $66V_n - 6V_o = 30$ (3)
- **Add Equations (2) and (3):** $5065V_n = 30$
- $V_n = 30/5065 = 0.005923$ V (4)
- **Substitute Equation (4) into Equation (1):**

  $V_o = 11V_n - 5 = -4.934847$ V

- **Due to small value of A, $V_o$ is off from – 5 V (ideal model).**

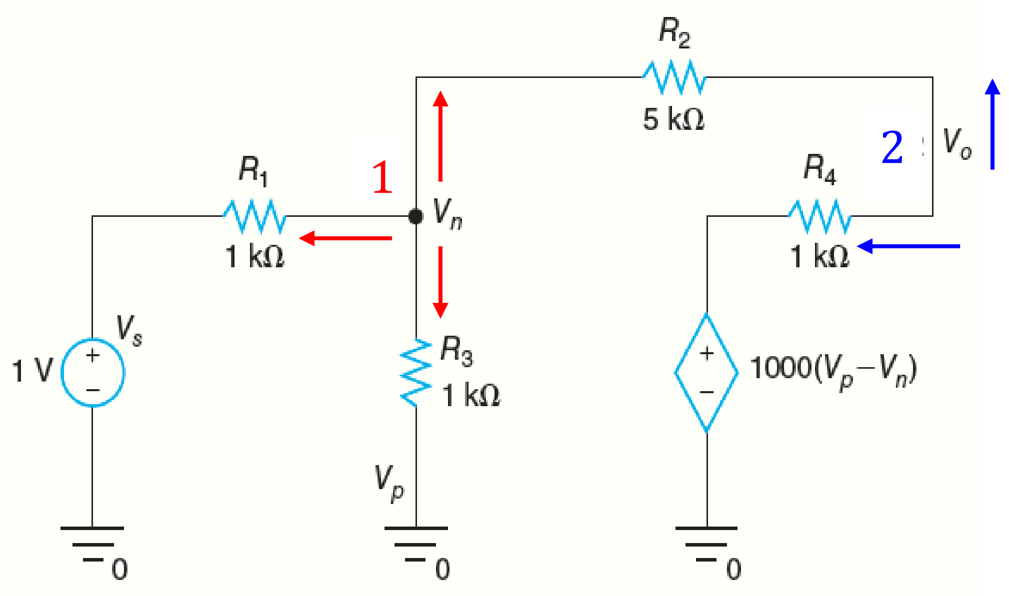



FIGURE 5.54

# Analysis of Noninverting Conf.

- A model for a noninverting configuration is shown in Figure 5.56.
- KCL: Summing the currents leaving node 2 = 0, we obtain

$$\frac{v_o - v_n}{R_2} + \frac{v_o - A(v_s - v_n)}{R_o} = 0 \quad (1)$$

- KCL: Summing the currents leaving node 1 = 0

$$\frac{v_n}{R_1} + \frac{v_n - v_s}{R_i} + \frac{v_n - v_o}{R_2} = 0 \Rightarrow \left(\frac{1}{R_1} + \frac{1}{R_i} + \frac{1}{R_2}\right) v_n = \frac{v_s}{R_i} + \frac{v_o}{R_2} \quad (2)$$

- Solving Equations (1) and (2) for $V_n$ and $V_o$, we get

$$v_n = \frac{R_1(R_o + R_2 + AR_i)v_s}{R_o R_i + R_o R_1 + R_i R_2 + R_1 R_2 + R_1 R_i + AR_1 R_i} \quad (3)$$

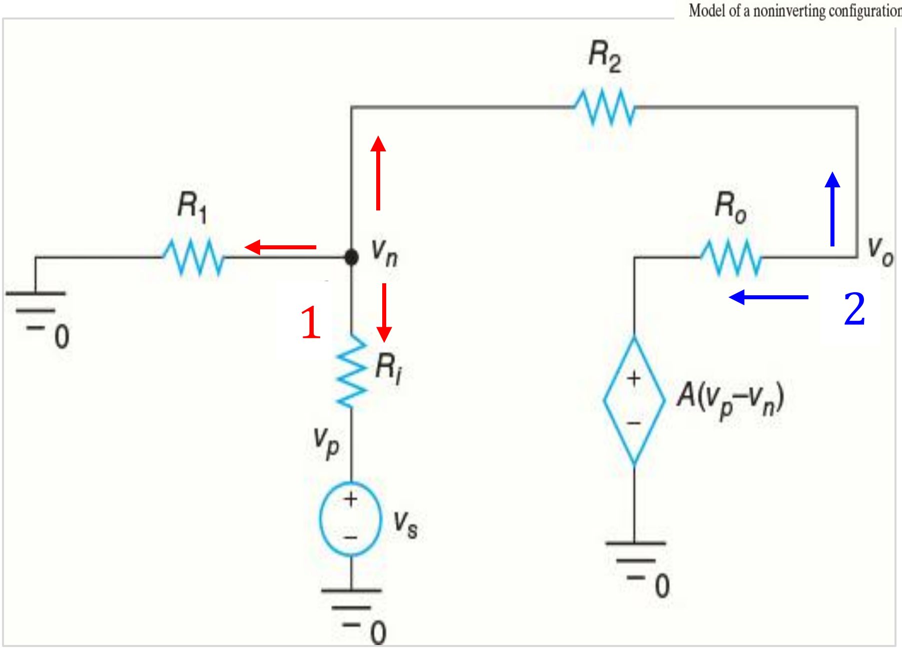
$$v_o = \frac{(R_o R_1 + R_1 R_i A + R_2 R_i A)v_s}{R_o R_i + R_o R_1 + R_i R_2 + R_1 R_2 + R_1 R_i + AR_1 R_i} \quad (4)$$

- Current in OP-AMP from Negative Terminal is

$$i_n = \frac{v_n - v_s}{R_i} = \frac{((R_0 R_1 + R_1 R_2 + AR_i R_1)v_s)}{R_i(R_0 R_i + R_0 R_1 + R_i R_2 + R_1 R_2 + R_1 R_i + AR_1 R_i)} - \frac{v_s}{R_i}$$

FIGURE 5.56

Model of a noninverting configuration.



# Analysis of Noninverting Configuration (Cont.)

- **The current through $R_i$ is given by**

$$i_i = \frac{v_s - v_n}{R_i} = \frac{v_s}{R_i} - \frac{(R_oR_1 + R_1R_2 + AR_1R_i)v_s}{R_i(R_oR_i + R_oR_1 + R_iR_2 + R_1R_2 + R_1R_i + AR_1R_i)} = \frac{(R_o + R_2 + R_1)v_s}{R_oR_i + R_oR_1 + R_iR_2 + R_1R_2 + R_1R_i + AR_1R_i} \approx 0$$

- **The input resistance is given by**

$$R_{in} = \frac{v_s}{i_i} = \frac{R_oR_i + R_oR_1 + R_iR_2 + R_1R_2 + R_1R_i + AR_1R_i}{R_o + R_2 + R_1} \cong \frac{AR_1R_i}{R_o + R_2 + R_1} \approx \frac{AR_1R_i}{R_2 + R_1}$$

- **If $R_o = 0$ and $R_i = \infty$, equations (3) and (4) become:**

$$v_n = \frac{AR_1v_s}{R_1 + R_2 + AR_1} \approx v_s \quad (5)$$

$$v_o = \frac{A(R_1 + R_2)v_s}{R_1 + R_2 + AR_1} \cong \left(1 + \frac{R_2}{R_1}\right)v_s \quad (6)$$

$$i_n = \frac{v_s}{R_i} - \frac{v_s}{R_i} = 0 \quad (7)$$

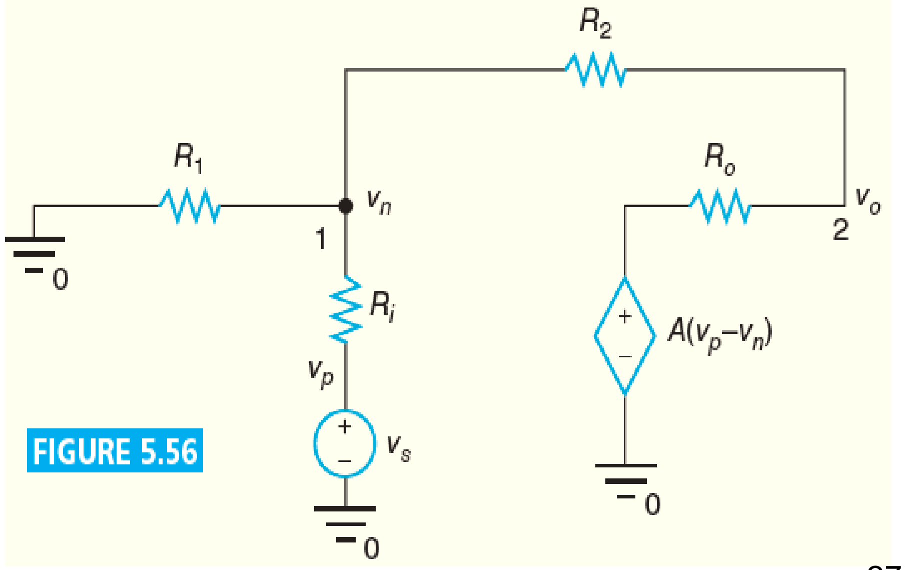


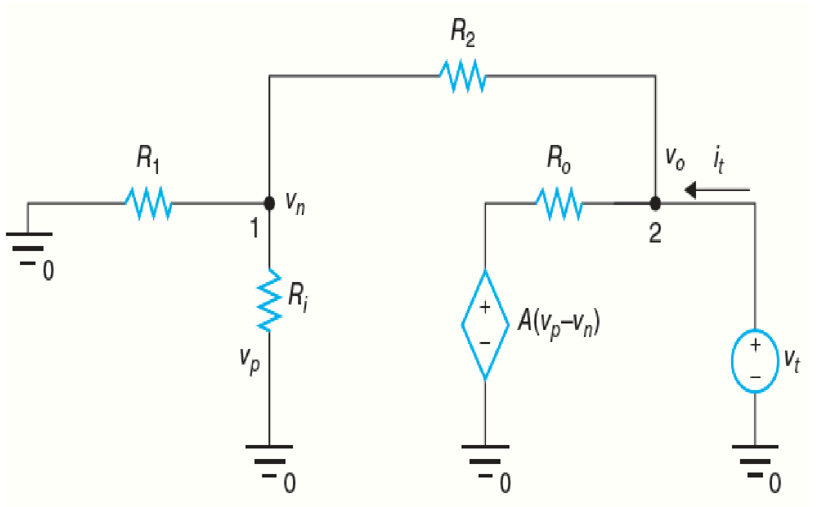
FIGURE 5.56

# Analysis of Noninverting Configuration (Cont.)

- **Output resistance**, Apply **test voltage between node 2 and ground** as shown in Figure 5.57. From the voltage divider rule, $v_n$ is given by

$$v_n = \frac{R_1 \| R_i}{R_2 + (R_1 \| R_i)} v_t = \frac{\dfrac{R_1 R_i}{R_1 + R_i}}{R_2 + \dfrac{R_1 R_i}{R_1 + R_i}} v_t = \frac{R_1 R_i}{R_1 R_2 + R_2 R_i + R_1 R_i} v_t$$

FIGURE 5.57




- The current out of the test voltage source is

$$i_t = \frac{v_t - v_n}{R_2} + \frac{v_t + A v_n}{R_o} = \frac{v_t - \dfrac{R_1 R_i}{R_1 R_2 + R_2 R_i + R_1 R_i} v_t}{R_2} + \frac{v_t + A \dfrac{R_1 R_i}{R_1 R_2 + R_2 R_i + R_1 R_i} v_t}{R_o}$$

- The output resistance is the ratio of $v_t$ to $i_t$:

$$\boxed{R_{out} = \frac{R_o (R_1 R_2 + R_2 R_i + R_1 R_i)}{R_o R_1 + R_o R_i + A R_1 R_i + R_1 R_2 + R_2 R_i + R_1 R_i} \cong \frac{R_o (R_2 + R_1)}{A R_1} \approx 0}$$

- The output resistance is close to zero for noninverting configuration.

# EXAMPLE 5.11

- Find $V_o$ in the circuit shown in Figure 5.58.
- Summing the currents leaving node 1, we obtain $$\frac{V_n - 1}{2000} + \frac{V_n}{1000} + \frac{V_n - V_o}{9000} = 0$$
- Multiplication by 18000 yields

  $9V_n - 9 + 18V_n + 2V_n - 2V_o = 0 \Rightarrow 29V_n - 2V_o = 9 \quad (1)$

- Summing the currents leaving node 2, we obtain $$\frac{V_o - V_n}{9000} + \frac{V_o - 2000(1 - V_n)}{3000} = 0$$
- Multiplication by 9000 yields

  $V_o - V_n + 3V_o - 6000 + 6000V_n = 0 \Rightarrow 5999V_n + 4V_o = 6000 \quad (2)$

- Multiply Equation (1) by 2: $58V_n - 4V_o = 1 \quad (3)$
- Add Equations (2) and (3): $6057V_n = 6018$
- $V_n = 6018/6057 = 0.99356117$ V (4)
- Substitute Equation (4) into Equation (1):

  $V_o = (29/2)V_n - (9/2) = 9.90664$ V

- Due to small value of A, $V_o$ is off from 10 V (ideal model).

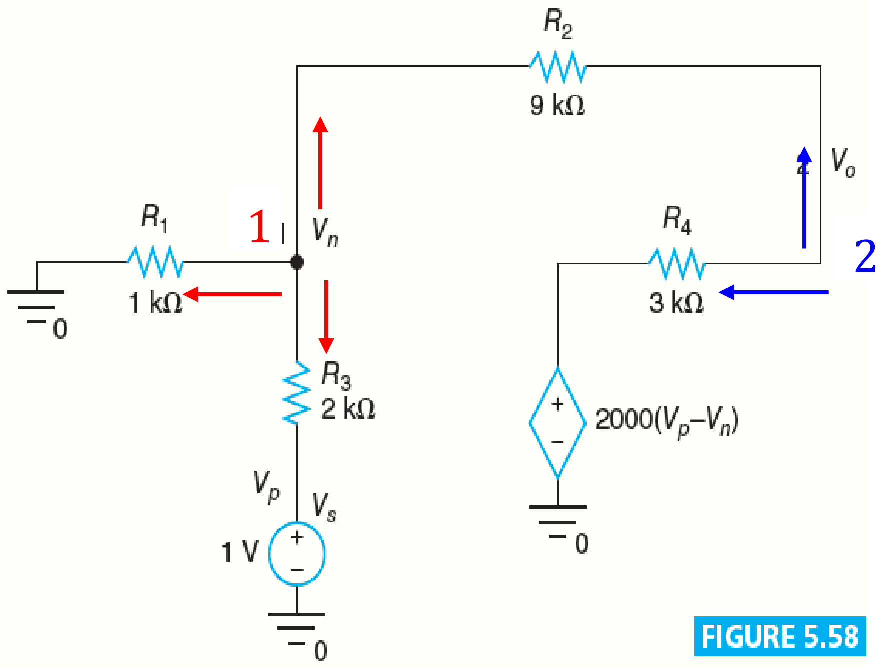



FIGURE 5.58

# Summary

- Without the negative feedback component from the output, the output $v_o$ will be large due to large gain A. The negative feedback component provides comparable gain in the denominator to offset the effects of large gain in the numerator.
- In the ideal op amp model, assume large input resistance ($R_i = \infty$), small output resistance ($R_o = 0$), and large gain A. Then, the current flowing into (or out of) the two input terminals is zero, that is, $i_p = 0$, and $i_n = 0$. Also, the voltage at the negative input terminal is equal to the voltage at the positive input terminal ($v_n = v_p$). This is called virtual short.
- In the inverting configuration of op amp, $v_o$, $v_n$, $i_{Ri}$, $R_{in}$, $R_{out}$ are given by

$$v_o = -\frac{R_2}{R_1} v_s \qquad v_n \cong \frac{R_2}{AR_1} v_s \approx 0 \qquad i_{R_i} = \frac{v_d}{R_i} = \frac{-v_n}{R_i} \cong -\frac{R_2}{R_1 A R_i} v_s \approx 0 \qquad R_{in} = \frac{v_s}{i_1} \approx R_1 \qquad R_{out} \cong \frac{R_o(R_2 + R_1)}{AR_1} \approx 0$$

- In the noninverting configuration of op amp, $v_o$, $v_n$, $i_{Ri}$, $R_{in}$, $R_{out}$ are given by

$$v_o \cong \left(1 + \frac{R_2}{R_1}\right) v_s \qquad v_n = \frac{AR_1 v_s}{R_1 + R_2 + AR_1} \approx v_s \qquad i_{R_i} = \frac{v_s - v_n}{R_i} \approx 0 \qquad R_{in} \approx \frac{AR_1 R_i}{R_2 + R_1} \qquad R_{out} \cong \frac{R_o(R_2 + R_1)}{AR_1} \approx 0$$

# Summary (Continued)

- **A summing amplifier can be designed in inverting configuration or in noninverting configuration.**

- **In the inverting configuration, for N inputs, the output can be**
  $v_o = -(k_1v_1 + k_2v_2 + \ldots + k_Nv_N)$
- **In the noninverting configuration, for N inputs, the output can be**
  $v_o = k_1v_1 + k_2v_2 + \ldots + k_Nv_N$

- **The output of a difference amplifier is given by**

$$v_o = \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)v_2 - \frac{R_2}{R_1}v_1$$

**If** $R = R_1 = R_2 = R_3 = R_4$**, the output is given by** $v_o = v_2 - v_1$