3.11 The Millikan Oil Drop Experiment

Prelab Assignment 3.11

(1)What forces act on the oil droplet when it is stationary in an electrostatic field? If the field is removed and it reaches its terminal speed, what forces would be acting on it?

(2)An oil droplet in a Millikan apparatus is determined to have a mass of 3.3×10^{-15} kg. It is observed to float between two parallel plates separated by a distance of 0.95 cm with 340 V of potential difference between them. Determine how many excess (extra) electrons are on the droplet.

(3)What is Stokes' law?

3.11.1 Introduction and Objectives

The oil drop experiment was performed by Robert A. Millikan and his Ph. D. student, Harvey Fletcher, from 1909 to 1913 to measure the elementary electric charge. The elementary charge is one of the fundamental physical constants and its accurate value is of great importance. The Nobel Prize in Physics 1923 was awarded to Robert A. Millikan "for his work on the elementary charge of electricity and on the photoelectric effect".

After performing this experiment and analyzing the data, you should be able to:

(1)Analyze the forces on the oil drop in the stationary and motion states.

(2)Measure the elementary charge.

3.11.2 Required Equipment

(1)Millikan oil drop apparatus, MOD-5C(Fig. 3.11-1)

(2)A CCD camera

(3)A monitor

Fig. 3.11-1 Monitor and Millikan oil drop apparatus

3.11.3 Theory

In 1907, Robert Millikan's young collaborator, Harvey Fletcher, got the idea of watching the motion of a single, charged, microscopic oil droplet under the influence of gravity and a uniform electric field between two parallel metal plates. An analysis of the forces acting on a charged droplet will allow determination of its charges.

Figure 3.11-2 shows the forces acting on the drop when it is stationary in the electric field. In Fig. 3.11-2, F_E is the electric field force, F_g is the gravity force, and F_b is the buoyant force exerted by the air. Since the density of air is only about one-thousandth that of oil, the buoyant force may be neglected and we have

$$q\frac{V}{d} = mg \tag{3.11-1}$$

where q is the electronic charge carried by the oil drop, V is voltage applied on the parallel metal plates, d is the distance between the plates, m is the mass of the oil drop, and g is the local acceleration of gravity.

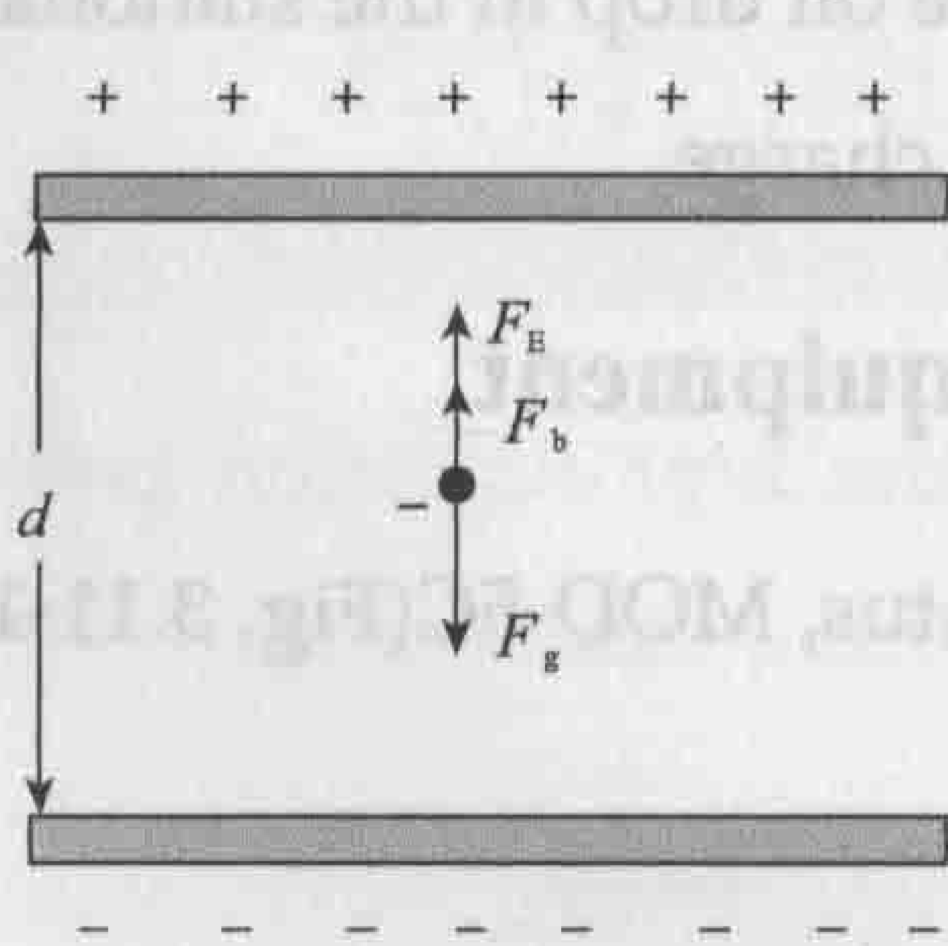

Fig. 3.11-2 The forces acting on the drop when it is stationary in the electric field

To eliminate m from Eq. (3.11-1), one uses the expression for the volume of a sphere:

$$m = \frac{4}{3}\pi r^3 \rho \tag{3.11-2}$$

where r is the radius of the oil drop, ρ is the density of the oil.

Thus, we have

$$q\frac{v}{d} = \frac{4}{3}\pi r^3 \rho g \tag{3.11-3}$$

When the voltage is removed the oil drop will fall and reach its terminal velocity in a few milliseconds. Fig. 3.11-3 shows the forces acting on the drop when it is falling in air and has reached its terminal velocity. In Fig. 3.11-3, F_v is the viscous resistance exerted by air, F_g is the gravitational force, and F_b is the buoyant force.

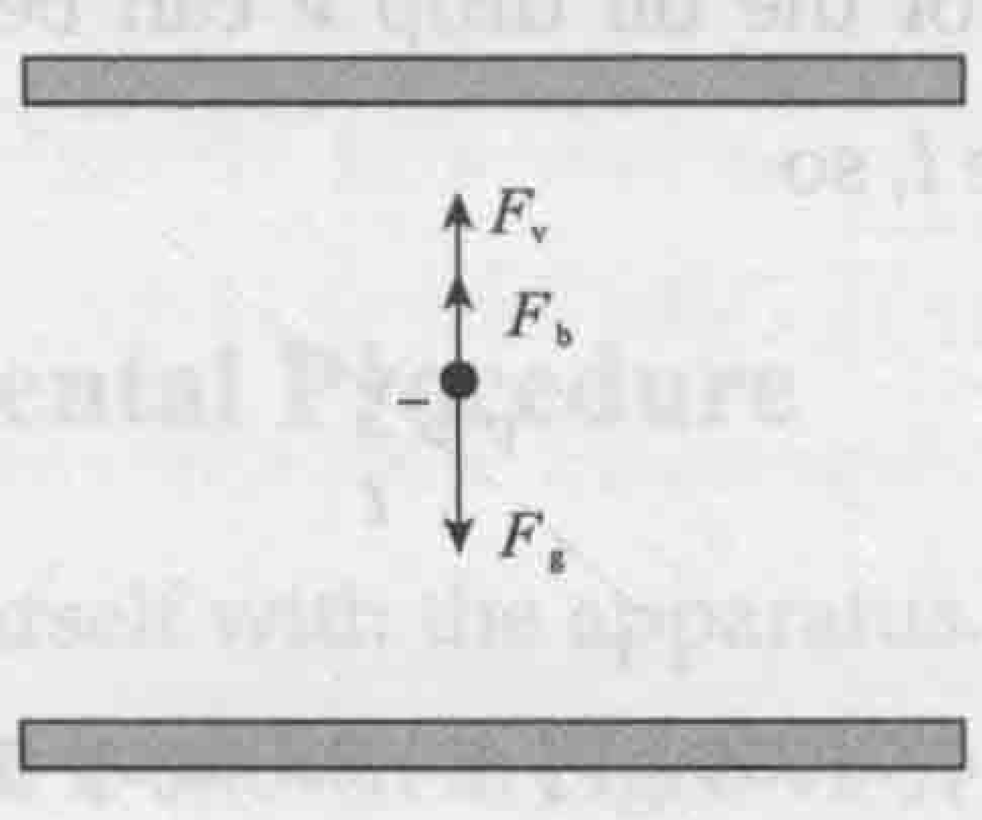

Fig. 3.11-3 Forces acting on the drop when reaching its terminal velocity

According to the Stokes' law, the viscous resistance is

$$F_v = 6\pi r \eta v \tag{3.11-4}$$

where r is the radius of the oil drop, η is the coefficient of viscosity of the air, v is the terminal velocity of the oil drop.

Neglecting the buoyant force, we have

$$\frac{4}{3}\pi r^3 \rho g = 6\pi r \eta v$$

and

$$r^2 = \frac{9\eta v}{2\rho g} \tag{3.11-5}$$

If the radius of the oil drop is as small as about 10^{-7} m, which is the order of magnitude of the mean free path of air molecules under normal laboratory conditions, a correction should be applied to the viscosity.

$$\eta' = \frac{\eta}{1+\dfrac{b}{pr}} \tag{3.11-6}$$

where η' is the corrected value of the viscosity, p is local atmospheric pressure, and b is a constant, 6.17×10^{-6} m·cm(Hg).

The terminal velocity of the oil drop v can be determined by measuring the falling distance l in time t, so

$$v = \frac{l}{t} \tag{3.11-7}$$

Thus, the electronic charge q carried by an oil drop can be derived from the Eqs. (3.11-3) and (3.11-5)~(3.11-7)

$$q = \frac{18\pi}{\sqrt{2\rho g}}\left[\frac{\eta l}{t\left(1+\dfrac{b}{pr}\right)}\right]^{\frac{3}{2}} \frac{d}{V} \tag{3.11-8}$$

where $r = \sqrt{\dfrac{9\eta l}{2\rho g t}}$.

The parameters used in our apparatus in Eq. (3.11-8) are depicted in Table 3.11-1.

Table 3.11-1 The parameters of density of the oil ρ, acceleration of gravity g, viscosity of air η, the distance of fall at constant speed l, correction constant b, atmospheric pressure p, the separation distance of the plates d

parameters	magnitude	unit
ρ	981	kg/m^3
g	9.79	m/s^2
η	1.83×10^{-5}	kg (m · s)
l	2.00×10^{-3}	m
b	6.17×10^{-6}	m·cm(Hg)
p	76.0	cm(Hg)
d	5.00×10^{-3}	m

Finally, we have the equation for calculating the charges of the oil drop

$$q = ne = \frac{1.43\times10^{-14}}{[t(1+0.02\sqrt{t})]^{\frac{3}{2}}} \cdot \frac{1}{V} \quad (3.11\text{-}9)$$

where n is an integer.

3.11.4 Experimental Procedure

(1)First, familiarize yourself with the apparatus. A picture of the apparatus as it should be set up for you is shown in Fig. 3.11-1.

(2)Turn on the power of the monitor and the oil drop apparatus.

(3)Introducing the droplets into the chamber. Spray droplets of oil from the atomizer through the port into the oil drop chamber. You can see hundreds of droplets on the monitor. Adjust the focusing hand wheel of the CCD so the images of the droplets are sharp.

(4)Selection of the droplet. Put the plate charging switch in the "balance" position. Tune the potentiometer to apply a voltage of about 200 volts across the parallel plates to arrest one droplet's fall. Fine tune the potentiometer to make the droplet stationary in the field. Record the balance voltage, V, registered on the voltmeter.

NOTE: The balance voltage should be in the range from 100 to 500 volts. The optimal voltage is about 200 volts.

(5)When you find an appropriately sized and charged oil droplet, fine tune the focus of the viewing scope to make it appear as a pinpoint of bright light.

(6)Put the plate charging switch in the "up" position to pull the droplet up. When it reaches the top line, put the switch in the "balance" position as soon as possible to make it stationary. In the "up" position, the voltage is greater than the balance voltage. So, the droplet goes up.

(7)Put the switch in the "down" position to measure the fall time. Keep your right hand on the timer controls and your left hand on the plate charging switch. When the droplet reaches the second horizontal line, start timing; when it reaches the sixth horizontal line, stop timing and put the switch in the "balance" position. Record the time registered on the timer.

NOTE: The fall time should be in the range from 10 to 50 seconds. The optimal time is 20 s.

(8)Pull the droplet up and repeat Steps (5) and (6) two more times to reduce the random errors of the time and voltage measurements. Record the balance voltage and fall time in Data Table 3.11-1.

(9)Select nine more independent droplets and repeat the experiment to complete Data Table 3.11-1.

3.11.5 Experimental Data

Data Table 3.11-1 ***Purpose*: To measure the electric charges carried by an oil droplet**

Oil droplets	Balance voltage V/V		Fall time t/s	
	Measurement	Average	Measurement	Average
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Student's name and number:______________ Instructor's initial:______________

3.11.6 Calculations

(1)Compute the mean values of the balance voltage and the fall time. For the first oil droplet

$$\overline{V_1} = \frac{V_{11} + V_{12} + V_{13}}{3}$$

$$\overline{t_1} = \frac{t_{11} + t_{12} + t_{13}}{3}$$

Compute the averaged values of the balance voltages and the fall time for the ten oil droplets and enter them in Data Table 3.11-1.

(2)Use the following equation to calculate the charges carried by an oil droplet. Show the sample calculation for the first droplet and list all the charges in Table 3.11-2.

$$\overline{q} = ne = \frac{1.43\times10^{-14}}{[\overline{t_1}(1+0.02\sqrt{\overline{t}})]^{\frac{3}{2}}} \cdot \frac{1}{\overline{V}}$$

Data Table 3.11-2 Charges and the number of excess electrons on selected droplets

Oil droplets	Charges $\overline{q}$ /$\times10^{-19}$C	the number of excess electrons, n	Elementary charge e/$\times10^{-19}$C
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
		Average	

(3)Compute the number of excess electrons for every droplet. Suppose that the elementary charge $e = 1.602\times10^{-19}$ C and the number of excess electrons is

$$n = \left[\frac{q}{e}\right]$$

"[ ]" means rounding the quotient to an integer.

Show the sample calculation for the first oil droplet and enter all the results in Data Table 3.11-2.

(4)Compute the elementary charge.

$$e = \frac{\overline{q}}{n}$$

Show the sample calculation for the first oil droplet and enter all the results in Data Table 3.11-2.

(5)Compute the averaged elementary charge. Enter the final result in Data Table 3.11-2.

$$\overline{e} = \frac{1}{10}\sum_{i=1}^{10} e_i$$

(6)Compute the relative error of the elementary charge.

$$E = \frac{\overline{e} - e}{e}\times100\% \quad (e = 1.602\times10^{-19}\ \text{C})$$

3.11.7 Post Lab Questions

(1)You can observe that some oil droplets go down very fast after introducing the droplets into the chamber. Why?

(2)Some oil droplets go down very slowly after removing the potential difference applied on the two parallel plates. Are these droplets good for measurement? Why?

(3)Give some examples to show the importance of the accurate value of elementary charge.