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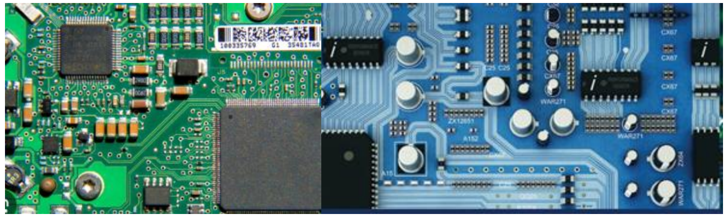


Circuit Analysis and Design

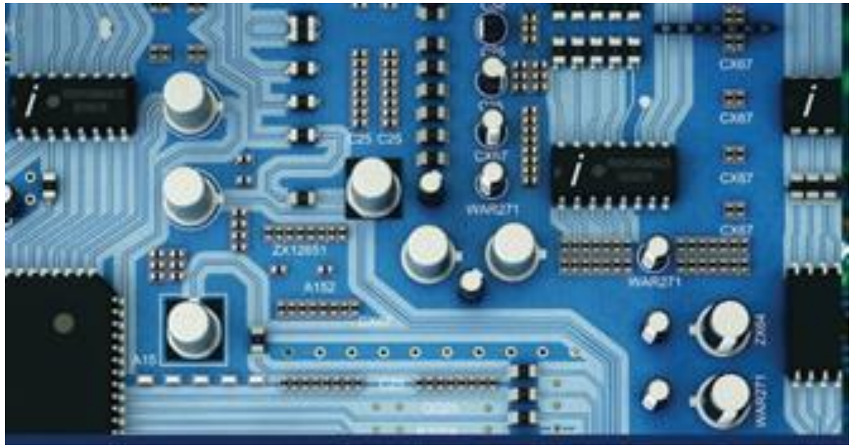
Academic year 2025/2026 – Lecture 5

Muhammad Aslam, Masood Ur Rahman, Ali Imran

{Muhammad.Aslam.2, Masood.UrRehman, Ali.Imran}@*glasgow.ac.uk*

“A good student never steal or cheat”

Agenda



- ❑Nodal analysis
- ❑Supernode
- ❑Summary

Nodal analysis

- It is defined as

“The method for calculating the voltage at the circuit nodes.”

- By knowing the voltages of the Nodes,
  - All other voltages, All currents and power can be calculated.

- Following are the laws that useful in nodal analysis:
  - **Kirchhoff’s current law**
  - Ohm’s law
  - Solve equations

Nodal Analysis – Key Features

- Nodal analysis is an application of **Kirchhoff’s current law.**

- When there are ‘**n**’ nodes in a given electrical circuit, there will be ‘**n-1**’ simultaneous **equations** to be solved.

- To obtain all the node voltages, ‘**n-1**’ equations should be solved.

- The number of non-reference nodes and the number of nodal equations obtained are equal.

Nodal Analysis

- **Nodal analysis** is a method of finding all the **unknown node voltages** of a circuit.
- The method is based on **Kirchhoff's current law (KCL):** The sum of the currents leaving a node is zero.
- Nodes can be labeled 1, 2, 3, . . . , or a, b, c, . . . (0 can be used for the reference node), and voltages on these nodes can be labeled V_1, V_2, V_3, . . . , or V_a, V_b, V_c, ...
- The node voltage of a reference node (0 V) and nodes with specified voltage sources to a reference node are known.
- For each node whose voltage is unknown, we can write a **node-voltage equation** by summing the currents leaving (entering, or some entering and the rest leaving) the node. This is tantamount to writing KCL at each node.
- The currents leaving the node through resistors can be found by applying **Ohm's law**.
- A solution to the node voltages is obtained by solving the set of node-voltage equations.
- Once all the node voltages are computed, the current in each branch can be found using Ohm's law.

Nodal Analysis : Key steps

Following main steps to be followed to solve any electrical circuit using nodal analysis:

- **Step 1: Identification of Essential Nodes:**
  - Identify principal nodes/essential nodes and select one of them as a reference node. This reference node will be treated as the ground. (0 voltage)
  - Essentials nodes/principal nodes are usually nodes with more than two branches

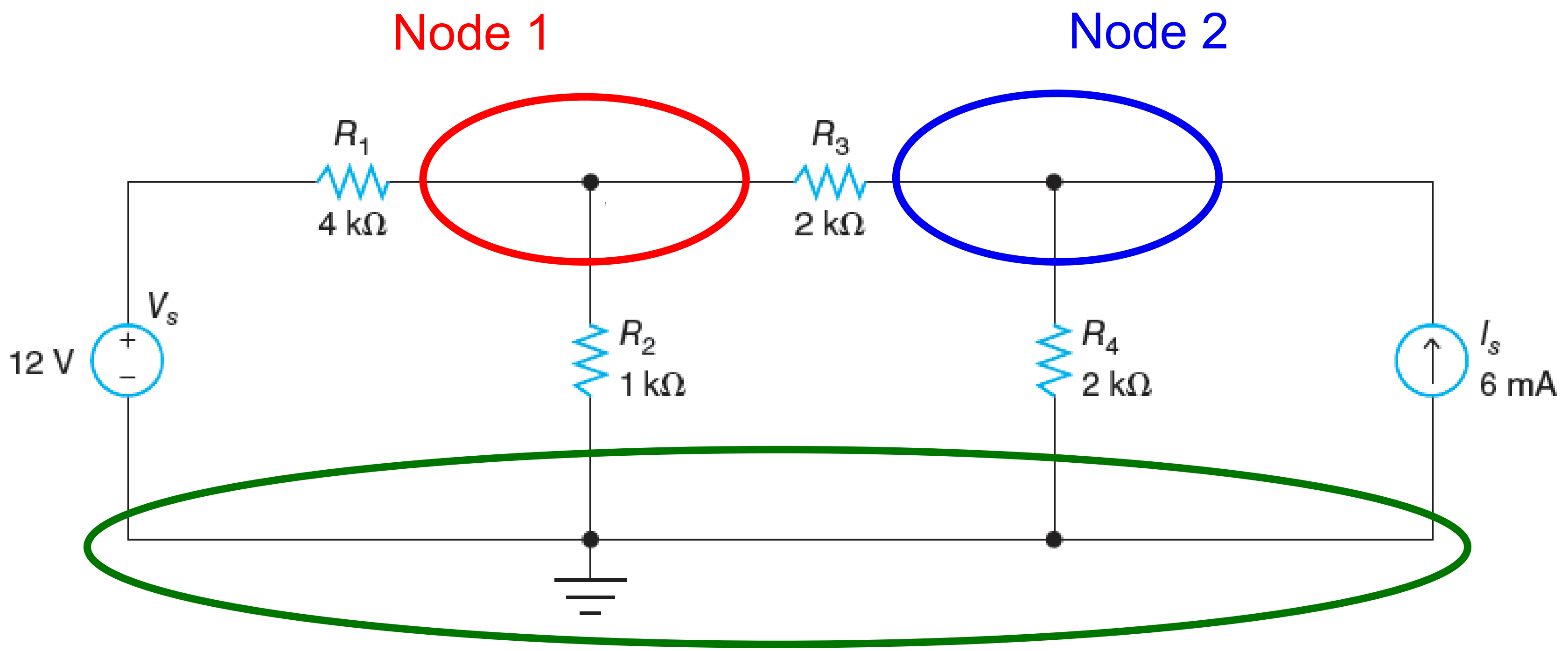
- **Step 2: Labelling the Essential Nodes with Unknown Voltages :**
  - Lebel all the unknown node voltages with respect to the ground for all the principal nodes except the reference node and the nodes whose voltage is Known .

- **Step 3: Writing Equations by applying KCL each Unknown Voltage Nodes.**
  - The nodal equations at all the nodes except the reference node should have a nodal equation. The nodal equation is obtained from **Kirchhoff's current law** and then from Ohm's law.

- **Step 4: Solve the equations**
  - Solve the equations of each node to get the required variables.
    - Substitution method
    - Cramer's Rule

A Circuit with Two Unknown Node Voltages

- **Step 1:** Identify essential nodes and Label them



Reference Node Voltage is 0

Figure 3.1

A Circuit with Two Unknown Node Voltages

- **Step 2:** Label the unknown Voltages

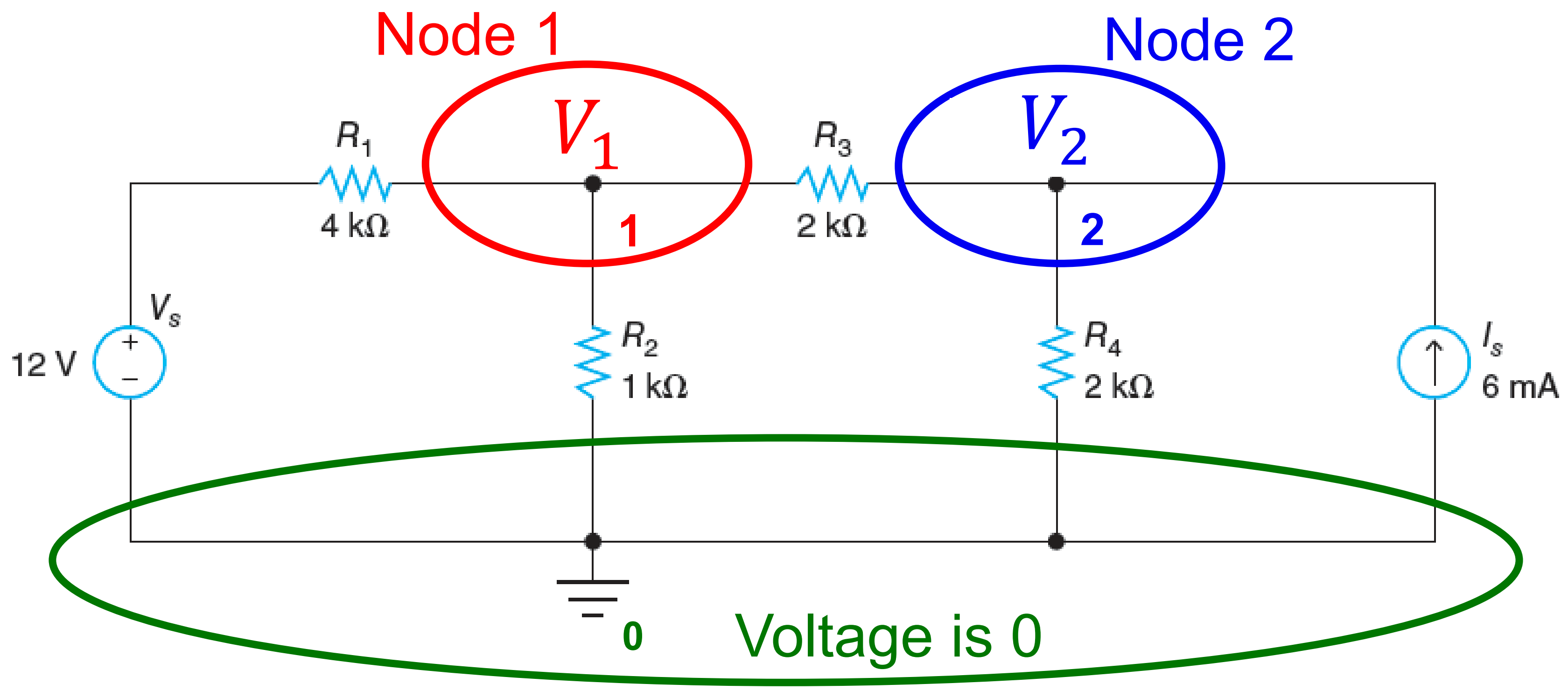


Figure 3.1

A Circuit with Two Unknown Node Voltages

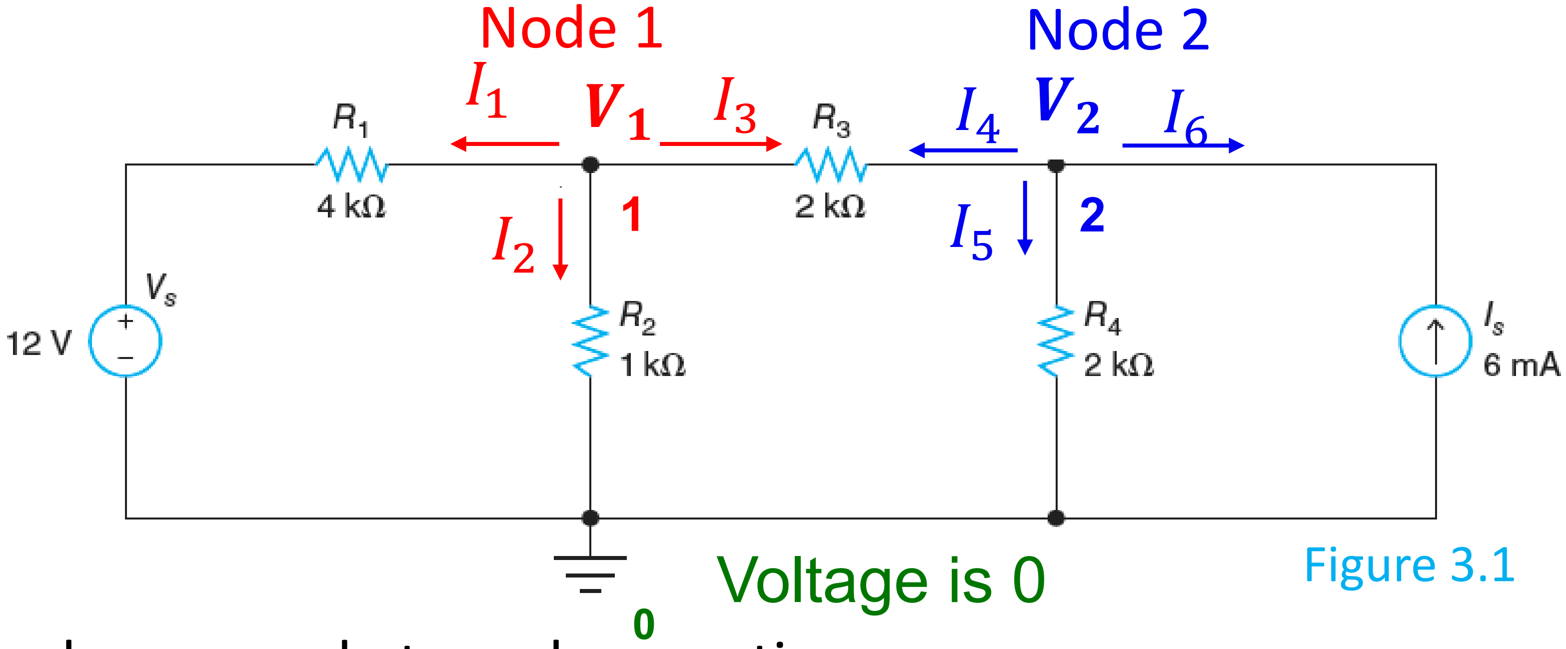


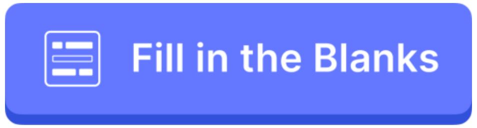
Figure 3.1

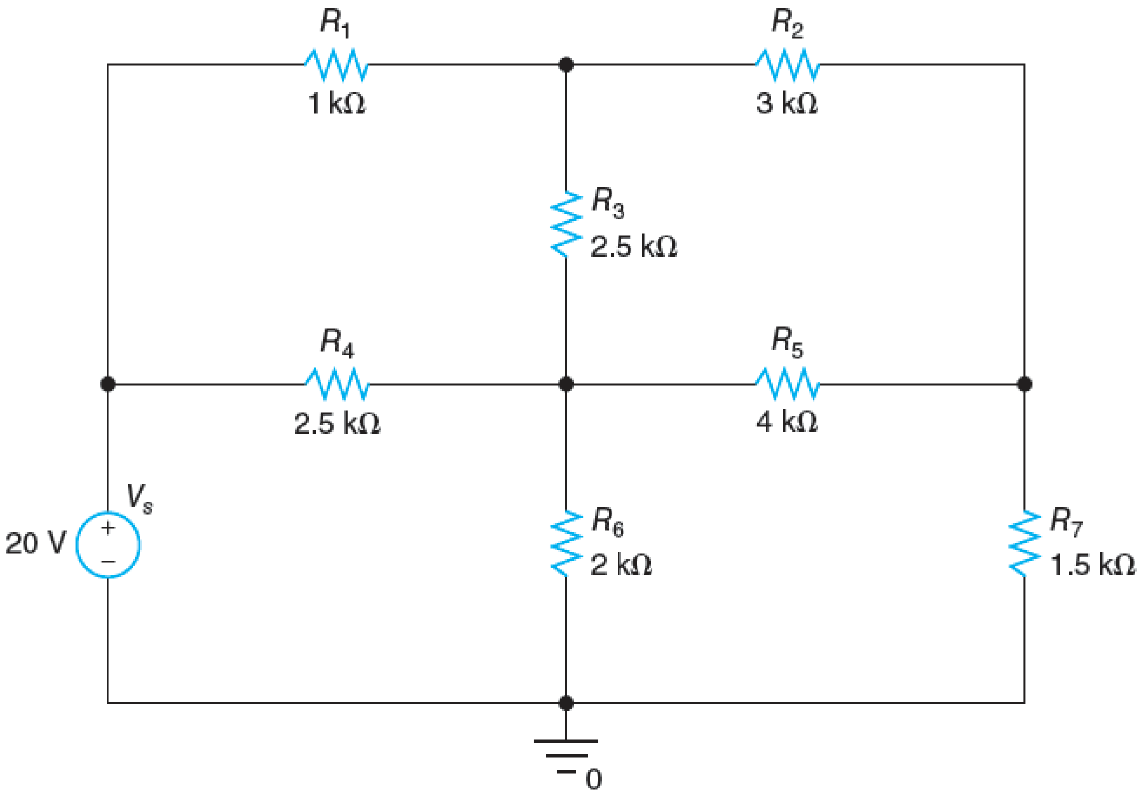
- **Step 3:** Applying KCL at each unknown node to make equations
  - Two: **Unknowns Nodal Voltages**, Apply Two **KCLs at these nodes** and get Two: **Linear Equations** (1)KCL at Node 1 → Get 1st Equation (2) KCL at Node 2 → Get 2nd Equation

- **Step 4:** Solve the two Linear Equations to get V_1 at Node1 and V_2 at Node2

- With these Nodal voltages we can find all currents and powers in the circuit

Fill in the blanks





- ❑ Fill in the blanks

a) Number of essential nodes _____

b) Number of Unknown voltage nodes _____

c) Number of Equations to be solved _____

Example: Nodal Analysis

- Applying KCL at Node 1

- **Sum of current leaving Node 1 is 0**

$$I_1 + I_2 + I_3 = 0$$

- **By Ohm's Law**

$$I_1 = \frac{V_1 - V_s}{R_1} \qquad I_2 = \frac{V_1 - 0}{R_2} \qquad I_3 = \frac{V_1 - V_2}{R_3}$$

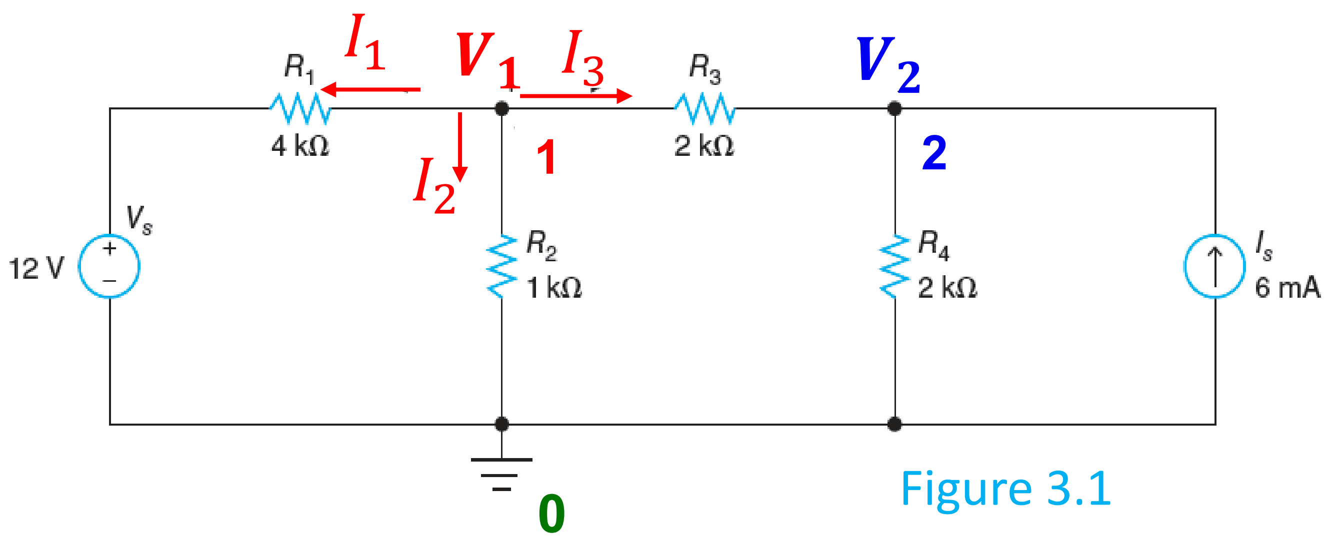


Figure 3.1

- **Substitute the currents with the values by Ohm's law**

$$\frac{V_1 - V_s}{R_1} + \frac{V_1 - 0}{R_2} + \frac{V_1 - V_2}{R_3} = 0$$

- **Substituting the values and simplifying the equation will give (1)**

$$\frac{V_1 - 12}{4000} + \frac{V_1 - 0}{1000} + \frac{V_1 - V_2}{2000} = 0$$

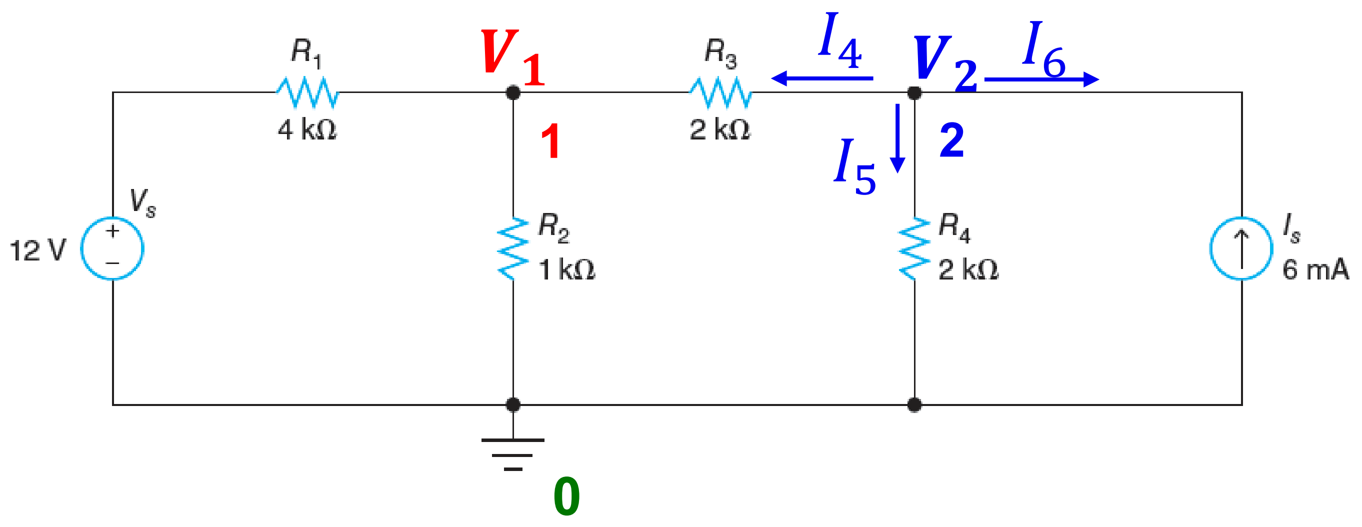
$$\boxed{\mathbf{7V_1 - 2V_2 = 12} \qquad (1)}$$

- **Multiplying both sides by 4000**

$$V_1 - 12 + 4V_1 + 2V_1 - 2V_2 = 0$$

Example: Nodal Analysis (Continue)

❑ Applying KCL at Node 2



❑ Sum of current leaving Node 2 is 0

$$I_4 + I_5 + I_6 = 0$$

❑ By Ohm's Law

$$I_4 = \frac{V_2 - V_1}{R_3} \qquad I_5 = \frac{V_2 - 0}{R_4} \qquad I_6 = -I_s$$

❑ Substitute the values by Ohm's Law

$$\frac{V_2 - V_1}{R_3} + \frac{V_2 - 0}{R_4} - I_s = 0$$

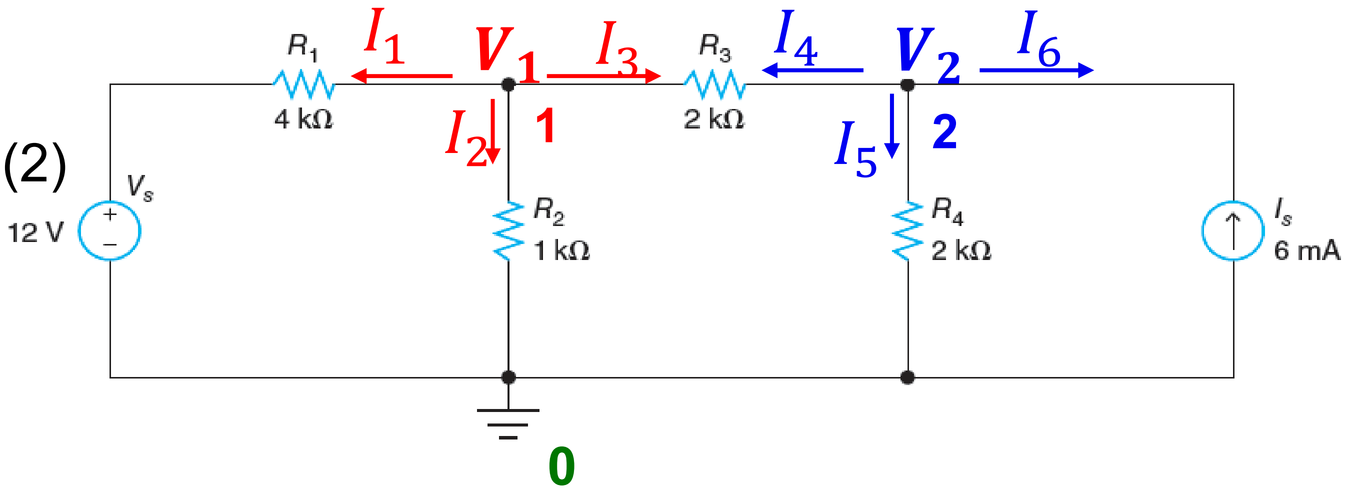
❑ Substituting the values and simplifying the equation will give (1)

$$\frac{V_2 - V_1}{2000} + \frac{V_2 - 0}{2000} - 6 \times 10^{-3} = 0$$

$$\boxed{-V_1 + 2V_2 = 12 \quad (2)}$$

❑ Multiply both sides by 2000 → $V_2 - V_1 + V_2 - 12 = 0$

Example: Nodal Analysis (Continue)



- **Step 4:** Solve the equations (1) and (2)

  - Equation (1) from Node 1

$$\boxed{7V_1 - 2V_2 = 12 \qquad (1)}$$

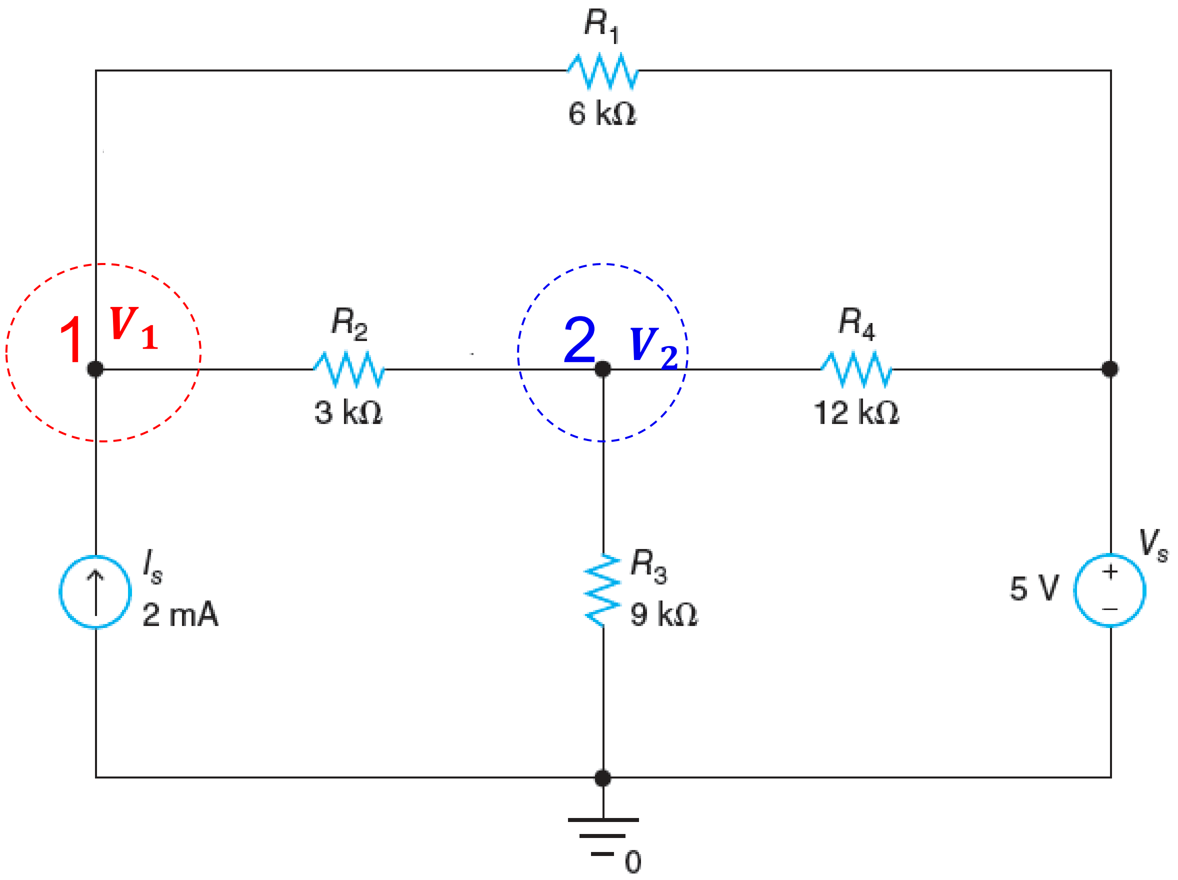
  - Equation (2) from Node 2

$$\boxed{-V_1 + 2V_2 = 12 \qquad (2)}$$

  - Adding the (1) and (2) and solving for V1 and V2

$$7V_1 - 2V_2 - V_1 + 2V_2 = 12 + 12 \rightarrow 6V_1 = 24 \rightarrow V_1 = 4 \quad \text{And} \quad V_2 = 8$$

EXAMPLE 3.1



- ❑ **Find the nodal voltages at the Unknown Nodes**
- ❑ **Step 1: Identify Unknown Nodes are ??**
- ❑ **Step 2: Label Unknown Nodes are ??**

EXAMPLE 3.1 (Continue)

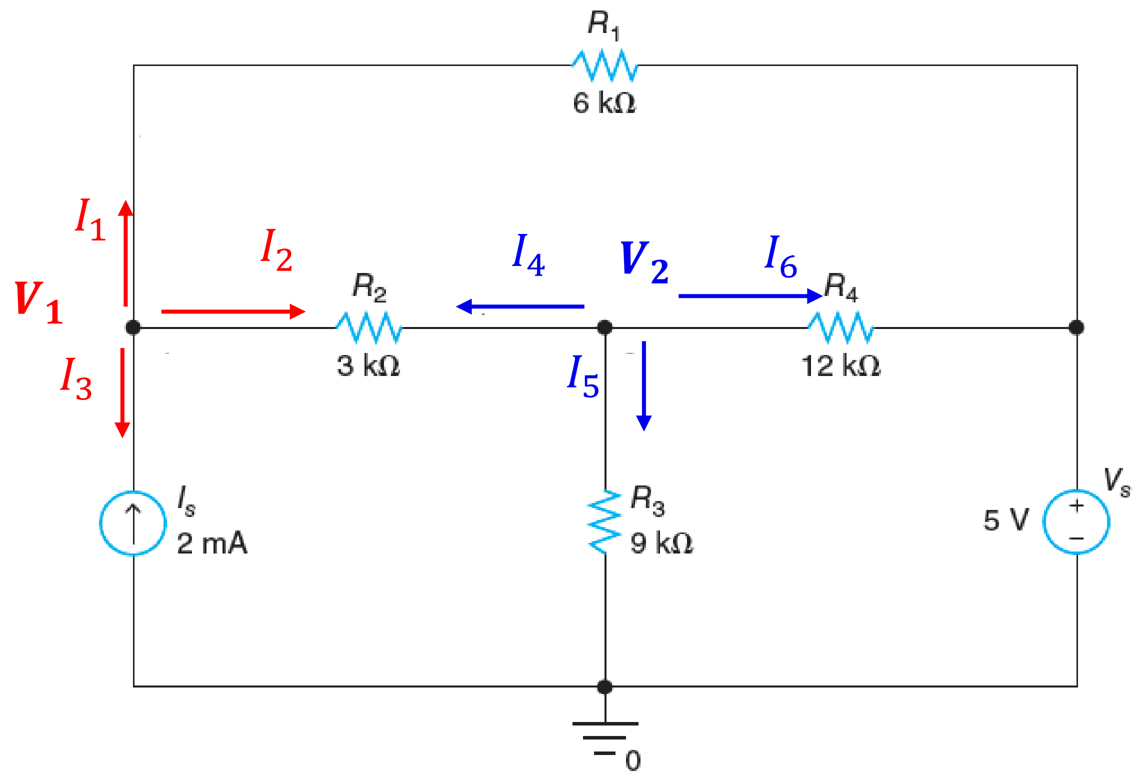
- **Step 3:** Apply KCL at Node 1

  - KCL: Sum of Current Leaving Node 1is 0

$$I_1 + I_2 + I_3 = 0$$

$$Note: I_3 = -I_S$$

  - Changing current values with those by Ohm's law



$$\frac{V_1 - V_s}{R_1} + \frac{V_1 - V_2}{R_2} - I_s = 0$$

$$\frac{V_1 - 5}{6000} + \frac{V_1 - V_2}{3000} - 2 \times 10^{-3} = 0$$

$$\mathrm{V}_1 - 5 + 2(V_1 - V_2) - 12 = 0$$

$$\boxed{3\mathrm{V}_1 - 2V_2 = 17 \quad (1)}$$

EXAMPLE 3.1 (Continue)

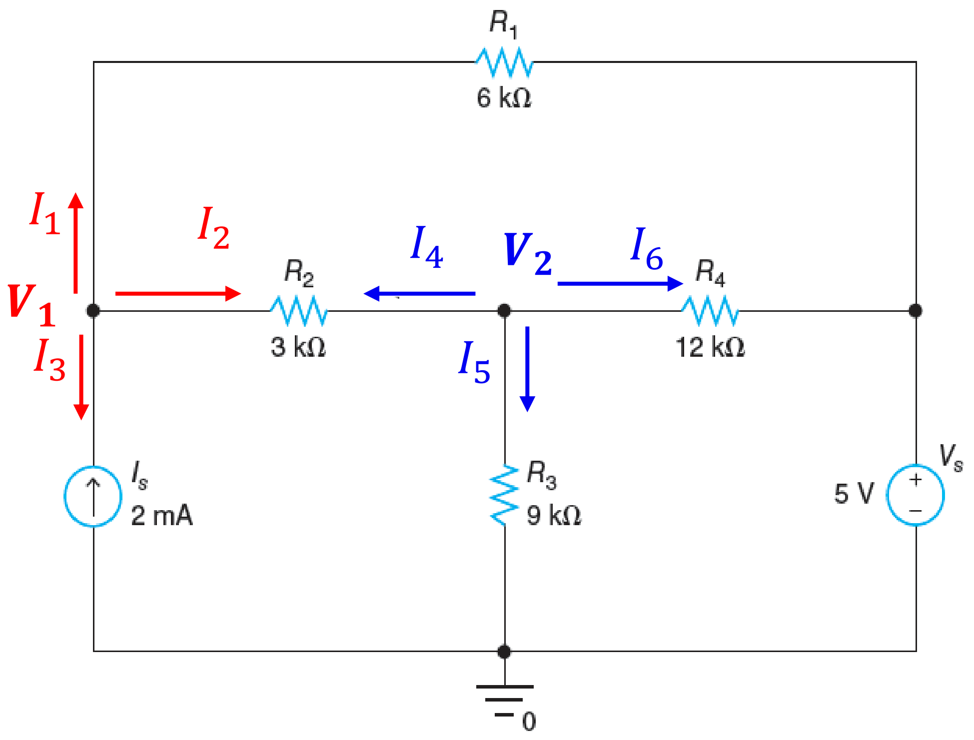
- **Step 3:** Apply KCL at Node 2
  - KCL: Sum of Current Leaving Node 2 is 0

$$I_4 + I_5 + I_6 = 0$$

  - Changing current values with those by Ohm's law

$$\frac{V_2 - V_1}{R_2} + \frac{V_2 - 0}{R_3} - \frac{V_2 - V_s}{R_4} = 0$$

$$\frac{V_2 - V_1}{3000} + \frac{V_1}{9000} + \frac{V_2 - 5}{12000} = 0$$



  - **Multiplying both sides by 36000**

$$12(\mathrm{V}_2 - V_1) + 4V_2 - 3(V_2 - 5) = 0$$

$$12\mathrm{V}_2 - 12V_1 + 4V_2 + 3V_2 - 15 = 0$$

$$\boxed{-12V_1 + 19\mathrm{V}_2 = 15 \ \ (2)}$$

EXAMPLE 3.1 (Continue)

- **Step 4:** Solve the equations (1) and (2)

  - Equation 1 from Node 1

    $$\boxed{3V_1 - 2V_2 = 17 \qquad \textbf{(1)}}$$

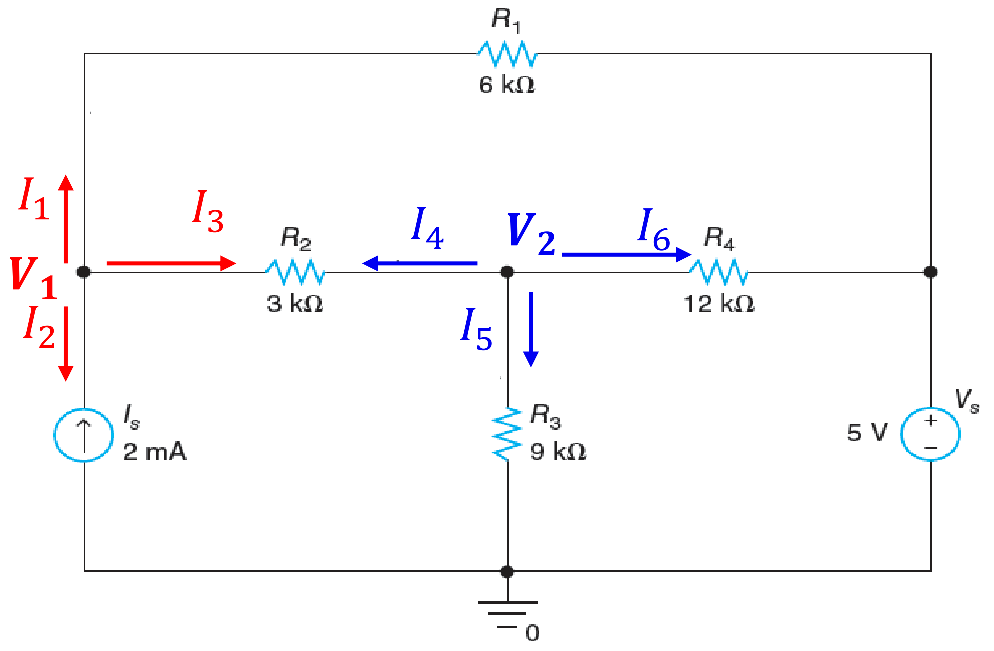
  - Equation 2 from Node 2

    $$\boxed{-12V_1 + 19V_2 = 15 \qquad (2)}$$

  - Adding the equations (1) and (2) and solving for V1 and V2

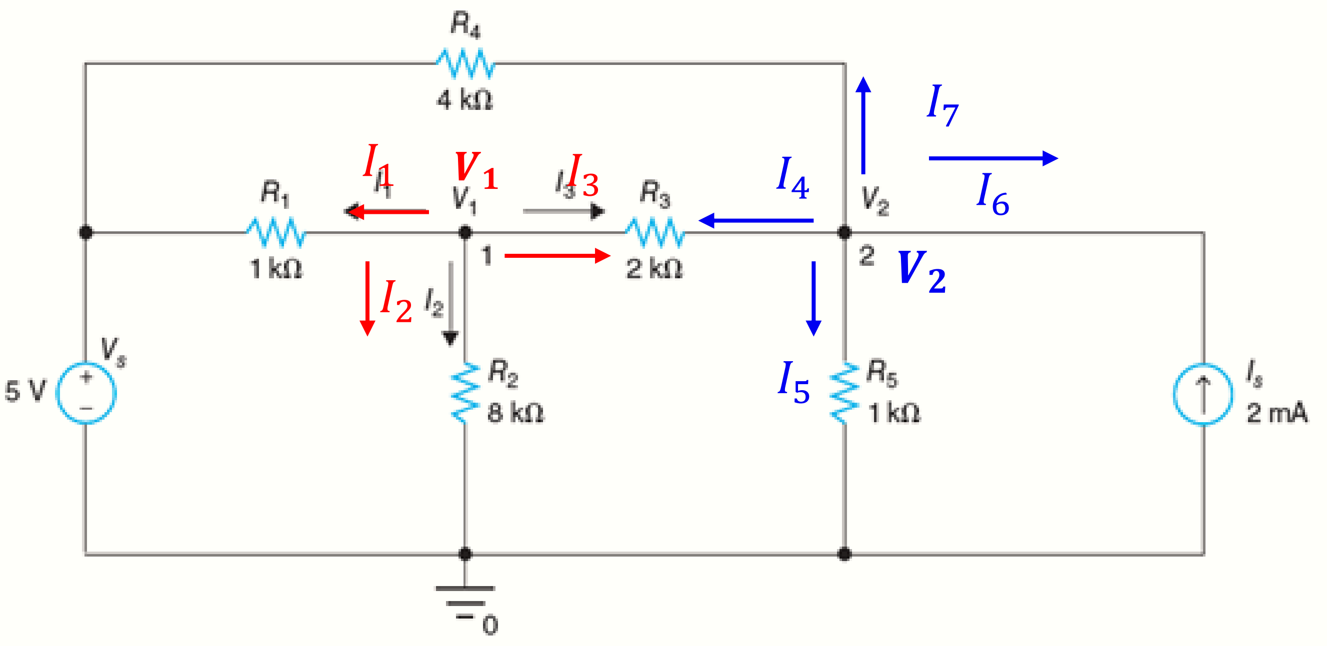
$$4(3V_1 - 2V_2) - 12V_1 + 19V_2 = 17 \times 4 + 15$$

$$11V_2 = 83 \qquad V_2 = 7.5454 \qquad V_1 = 10.6969$$



Class Task

- ❑ Find nodal voltages



Multiple Choice

Options

A. $V_1 = 3V, \quad V_2 = 4V$

B. $V_1 = 4V, \quad V_2 = 3V$

C. $V_1 = 6V, \quad V_2 = 4V$

EXAMPLE 3.3

- ❑ Step 1: Identify Essential nodes

- ❑ Step 2: Label the nodes

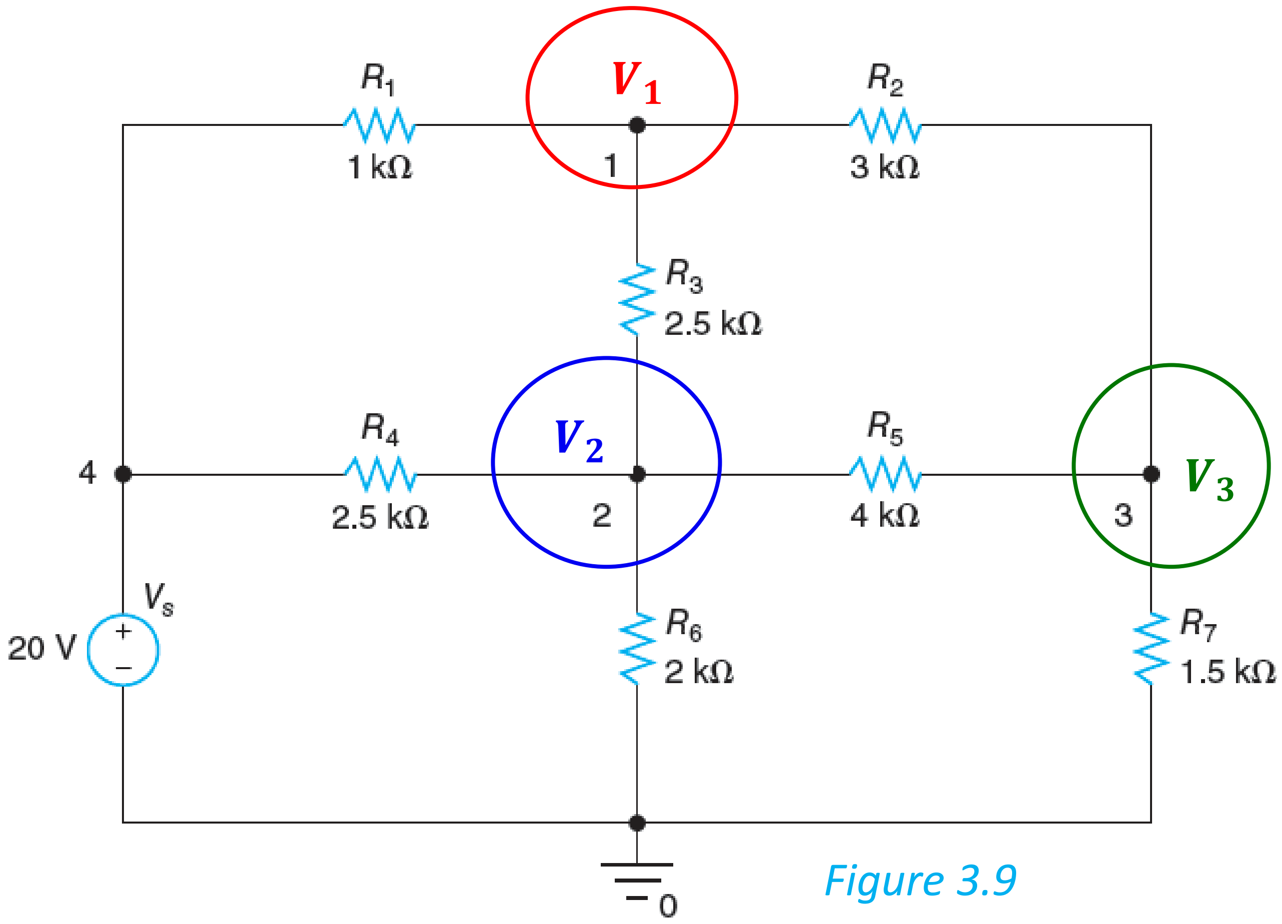
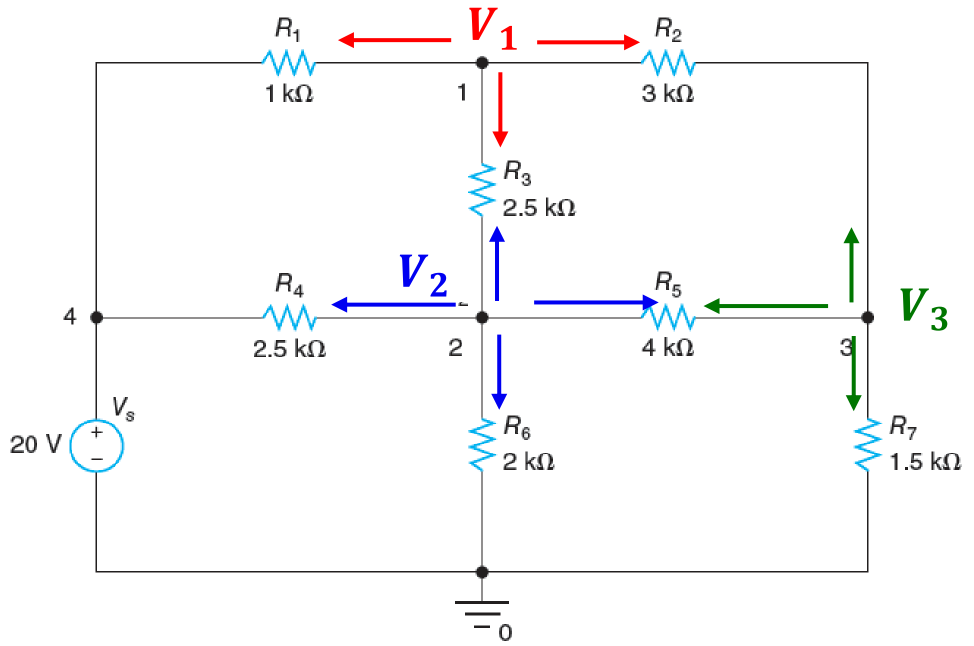


Figure 3.9

EXAMPLE 3.3 (Continue)



- Step3: Apply KCL at Node 1, Node 2 and Node 3

- Sum the currents at node 1: $\frac{V_1-20}{1000}+\frac{V_1-V_2}{2500}+\frac{V_1-V_3}{3000}=0$

- Multiply by 15000:

  $15V_1 - 300 + 6V_1 - 6V_2 + 5V_1 - 5V_3 = 0 \Rightarrow$ **$26V_1 - 6V_2 - 5V_3 = 300$ (1)**

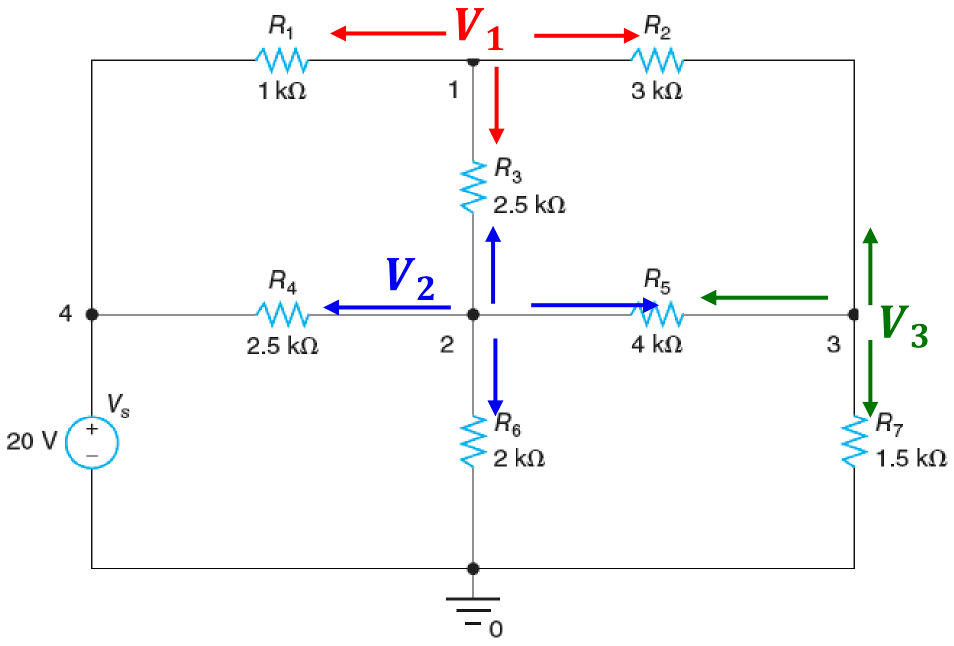
- Sum the currents leaving node 2 ➔ $\frac{V_2-20}{2500}+\frac{V_2-V_1}{2500}+\frac{V_2-V_3}{4000}+\frac{V_2}{2000}=0$

- Multiply by 20000: $8V_2 - 160 + 8V_2 - 8V_1 + 5V_2 - 5V_3 + 10V_2 = 0 \Rightarrow$ **$-8V_1 + 31V_2 - 5V_3 = 160$ (2)**

EXAMPLE 3.3 (Continue)

- **Step 3** : Apply KCL at Node 3 Sum the currents leaving node 3:

$$\frac{V_3 - V_1}{3000} + \frac{V_3 - V_2}{4000} + \frac{V_3}{1500} = 0$$



- Multiply by 12000:

$4V_3 - 4V_1 + 3V_3 - 3V_2 + 8V_3 = 0 \Rightarrow$ $\mathbf{-4V_1 - 3V_2 + 15V_3 = 0}$ **(3)**

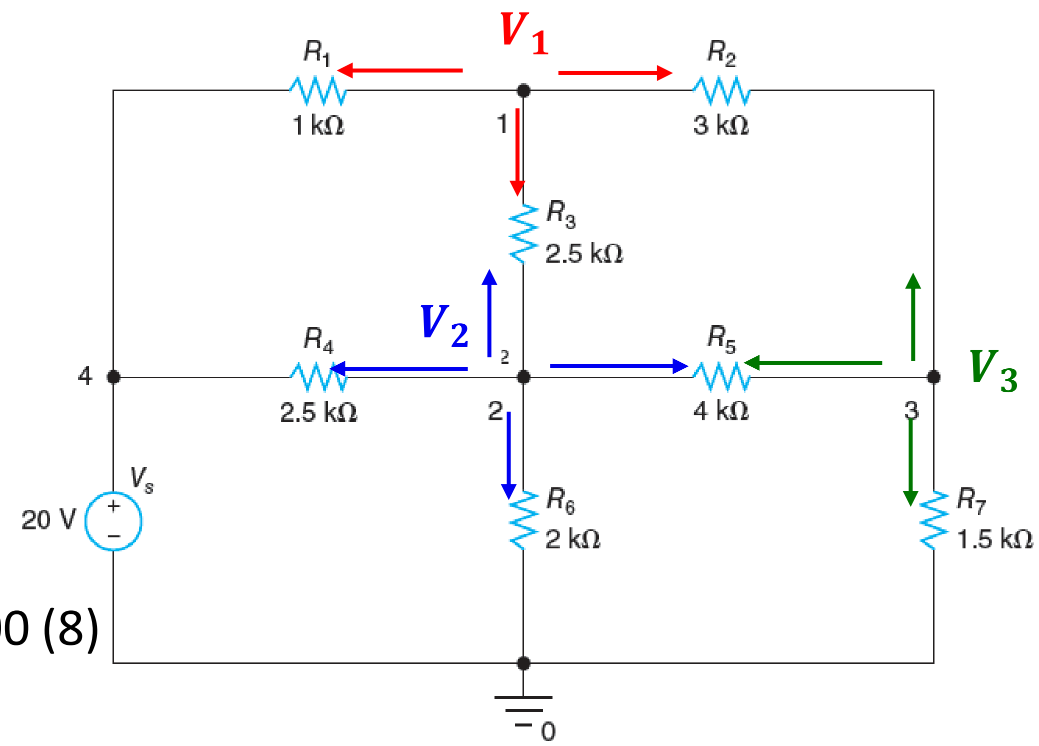
- Multiply (1) ($26V_1 - 6V_2 - 5V_3 = 300$) by 3 ➔ $78V_1 - 18V_2 - 15V_3 = 900$ (4)

- Add (3) and (4) ➔ $74V_1 - 21V_2 = 900$ (5)

- Multiply (2) ($-8V_1 + 31V_2 - 5V_3 = 160$) by 3: $-24V_1 + 93V_2 - 15V_3 = 480$ (6)

- Add (3) and (6): $-28V_1 + 90V_2 = 480$ (7)

EXAMPLE 3.3 (Continue)



- Multiply (5) ($74V_1 - 21V_2 = 900$)by 30 ➔ $2220V_1 - 630V_2 = 27000$ (8)
- Multiply (7) ($-28V_1 + 90V_2 = 480$) by 7 ➔ $-196V_1 + 630V_2 = 3360$ (9)
- Add (8) and (9) ➔ $2024V_1 = 30360$ ➔ **$V_1 = 15$ V (11)**
- Substitute (11) in (8) ➔ **V_2** $= (2220V_1 - 27000)/630$ **= 10 V** (12)
- Substitute (11) and (12) in (1) ➔ **V_3** $= (26V_1 - 6V_2 - 300)/5$ **= 6 V**

Solving Equations Methods

- If we have two or more linear equations, we can solve them in different methods. Two popular methods are
  1. Substitution Method
  2. Cramer's Rule
- **Substitution Method**
  - Solve the equations using simple substation methods as we did in the examples above
- **Cramer's Rule**
  - Read in the book to apply Cramer's Rule in solving equations OR
  - Go to this website to know about Cramer Rule https://www.purplemath.com/modules/cramers.htm

EXAMPLE 3.4

- Find V_1 and V_2

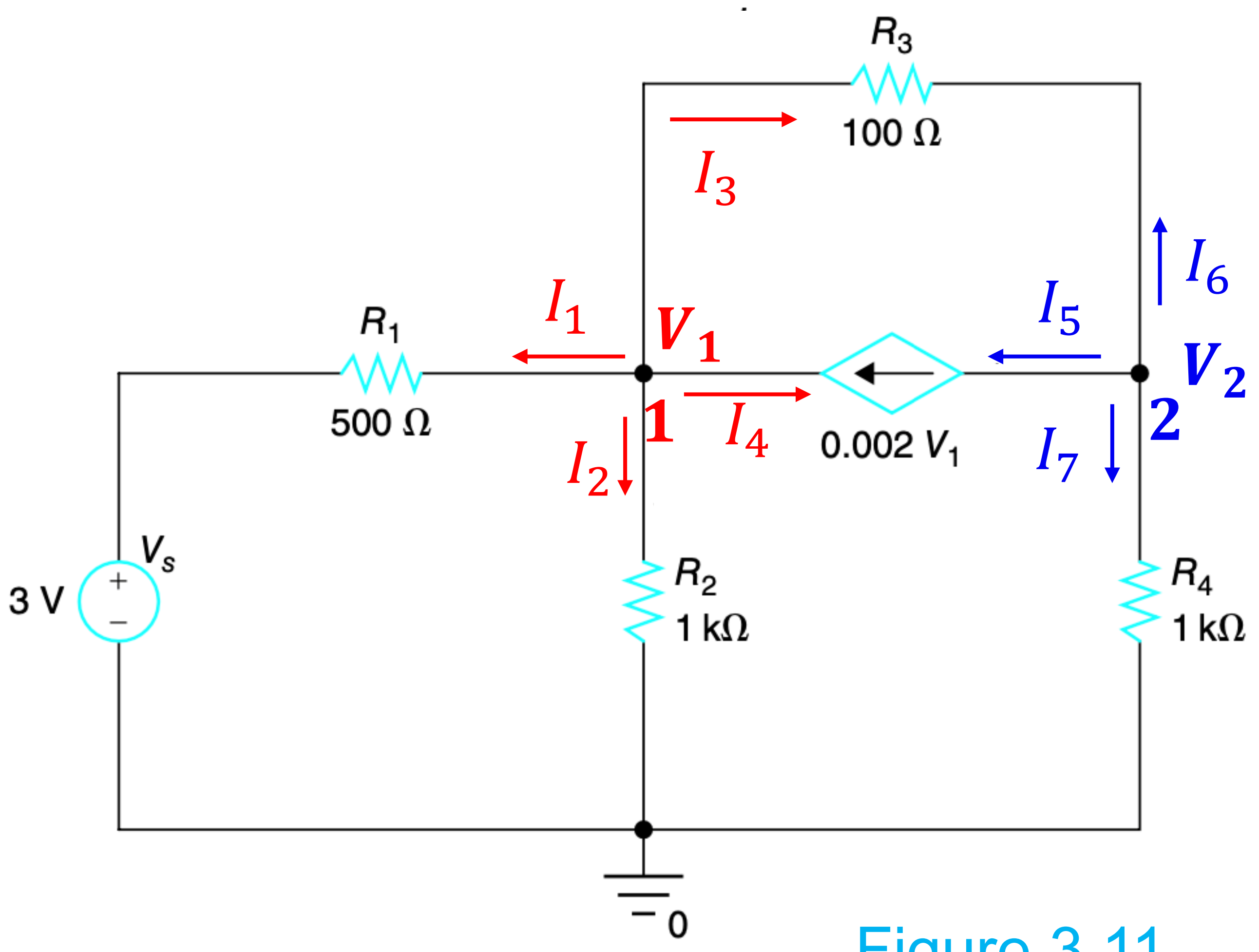


Figure 3.11

EXAMPLE 3.4 (Cont..)

- Apply KCL at Node 1

- Sum of current at Node 1 = 0

$$I_1 + I_2 + I_3 + I_4 = 0$$

$$I_4 = -I_5 = 0.002V_1$$

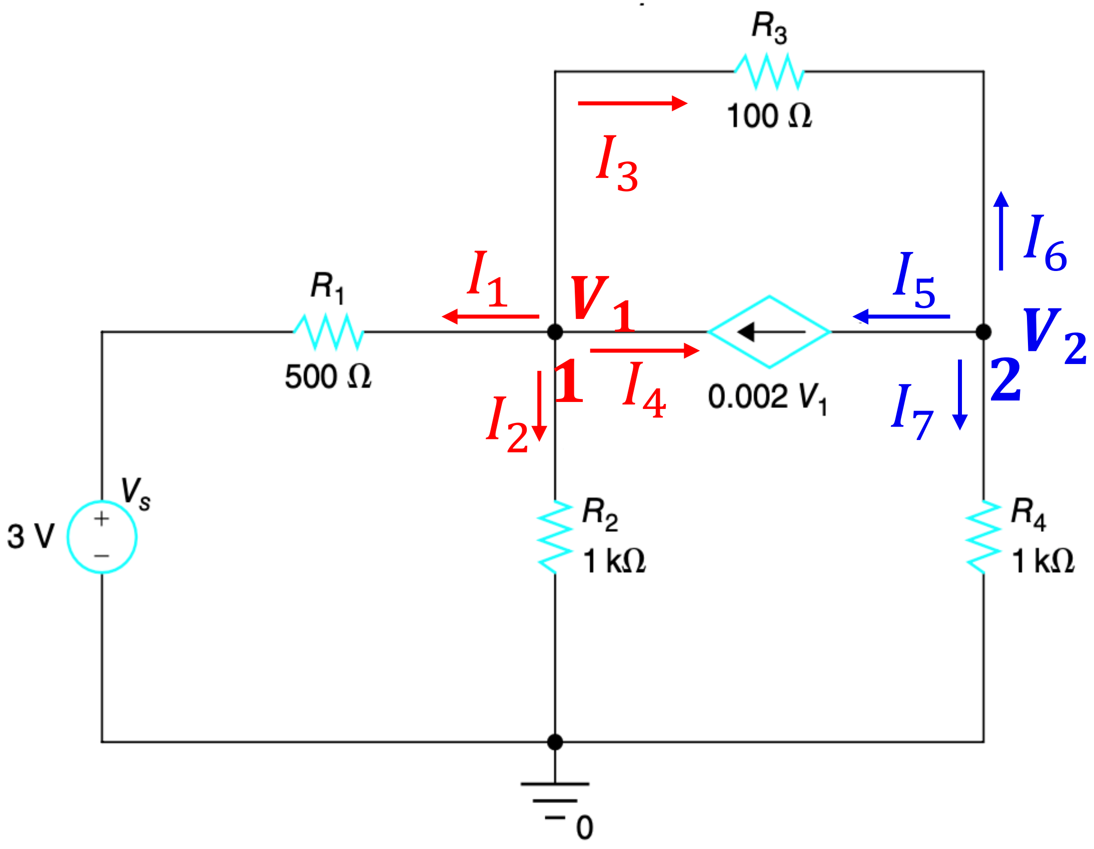
- Changing current values with those by Ohm's law

$$\frac{V_1 - V_s}{500} + \frac{V_1 - 0}{1000} + \frac{V_1 - V_2}{100} - 0.002V_1 = 0$$

$$\frac{V_1 - 3}{500} + \frac{V_1}{1000} + \frac{V_1 - V_2}{100} - 0.002V_1 = 0$$

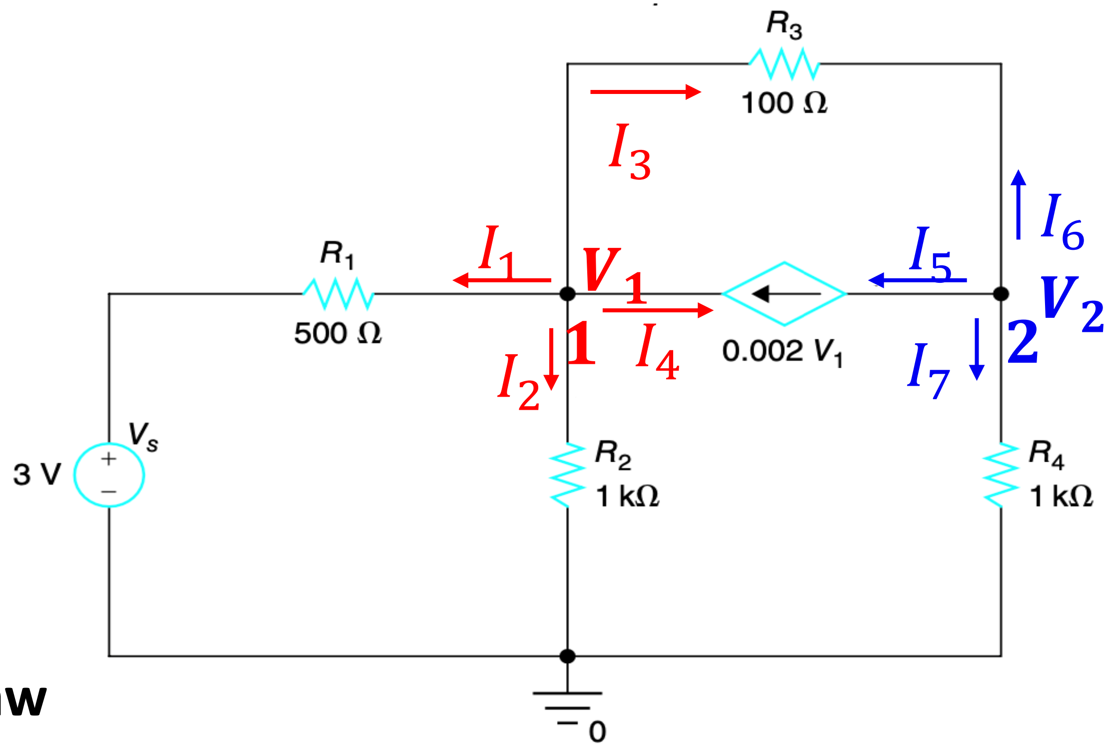
- Multiplying both sides by 1000

$$2V_1 - 6 + V_1 + 10V_1 - 10V_2 - 2V_1 = 0$$



$$11V_1 - 10V_2 = 6 \qquad (1)$$

EXAMPLE 3.4 (Cont..)



- Apply KCL at **Node 2**
  - Sum of current at Leaving **Node 2 = 0**

$$I_5 + I_6 + I_7 = 0$$

$$I_5 = 0.002V_1$$

  - Changing current values with those by Ohm's law

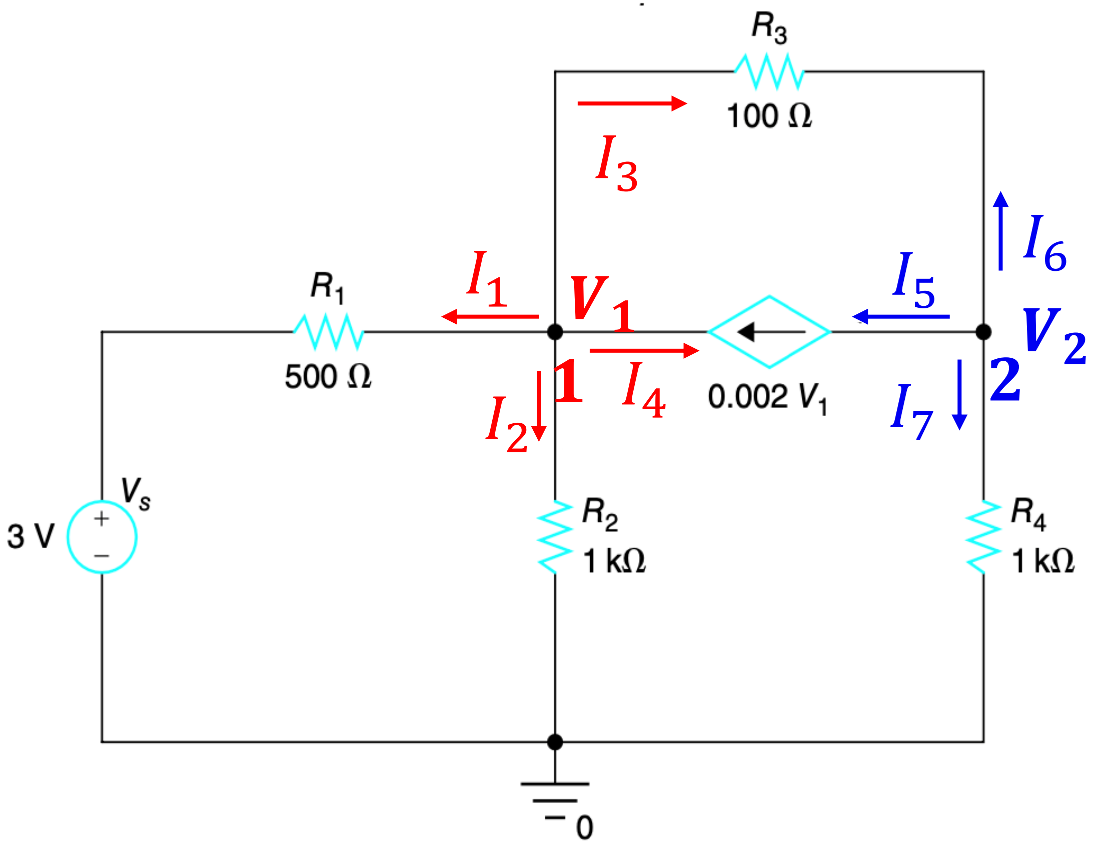
$$0.002V_1 + \frac{V_2 - V_1}{100} + \frac{V_2 - 0}{1000} = 0$$

  - Multiplying both sides by 1000

$$2V_1 + 10V_2 - 10V_1 + V_2 = 0$$

$$-8V_1 + 11V_2 = 0 \qquad (2)$$

EXAMPLE 3.4 (Cont..)



- Two equations after applying KCL at Node 1 and Node 2 are:

$$11V_1 - 10V_2 = 6 \quad (1)$$

$$-8V_1 + 11V_2 = 0 \quad (2)$$

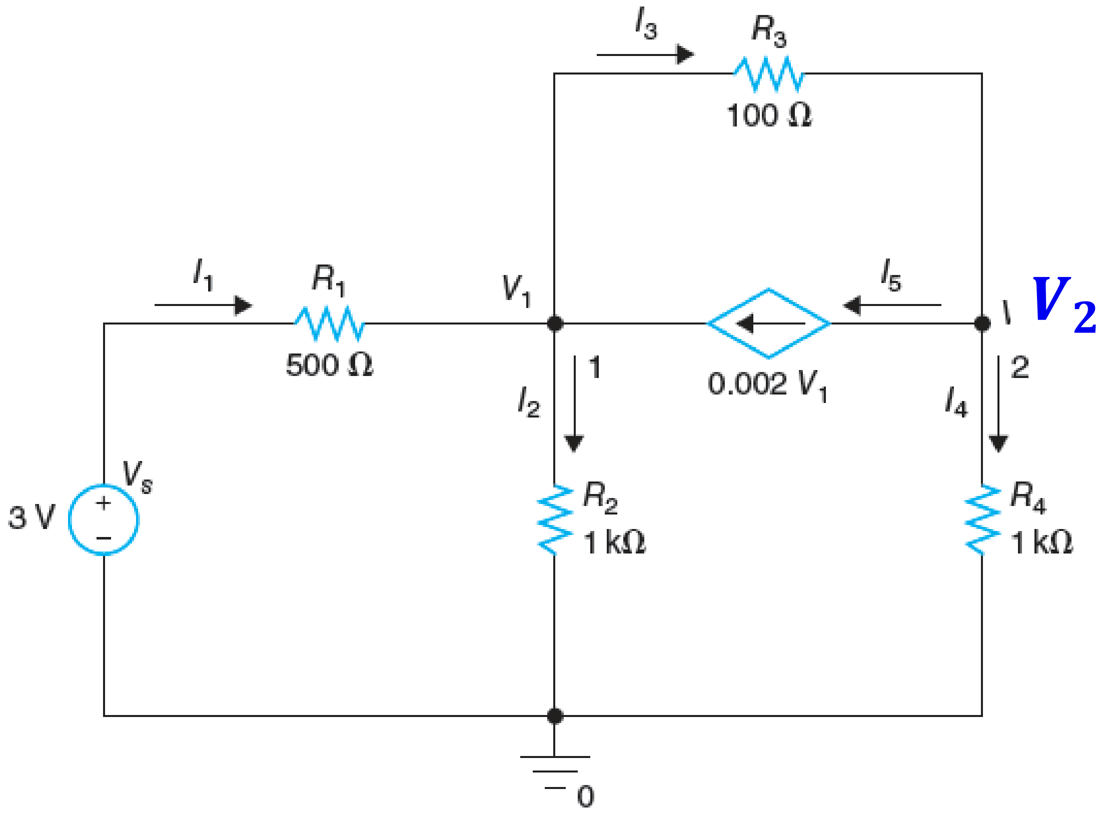
- Solving the Equation (1) and (2), we should get

$$V_1 = 1.6098\ \text{V} \qquad V_2 = 1.1707\ \text{V}$$

EXAMPLE 3.4 (Cont..)

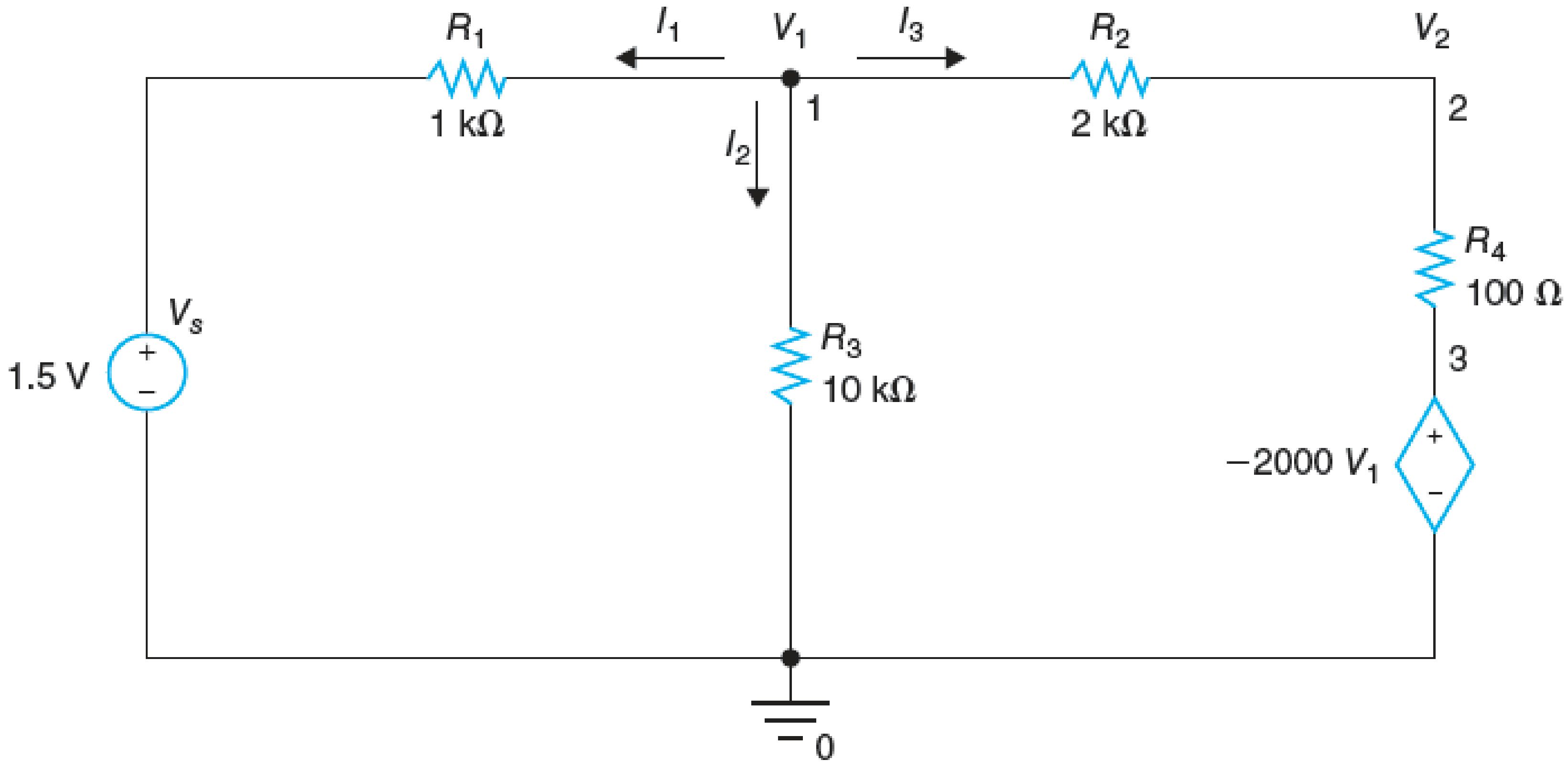
❑Currents:

- $I_1 = (V_s - V_1)/R_1 = 2.7805$ mA
- $I_2 = V_1/R_2 = 1.6098$ mA
- $I_3 = (V_1 - V_2)/R_3 = 4.3902$ mA
- $I_4 = V_2/R_4 = 1.1707$ mA
- $I_5 = 0.002V_1 = 3.2195$ mA

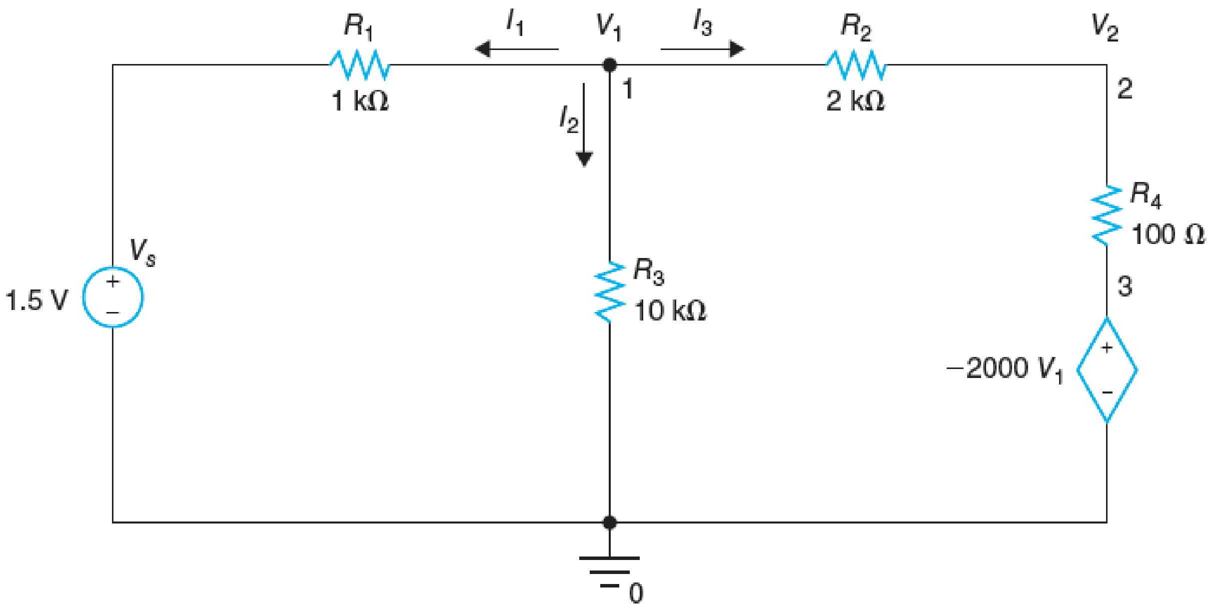


EXAMPLE 3.5

- Find V_1 and V_2



EXAMPLE 3.5 (Cont..)



KCL at Node 1

- Sum the currents leaving node 1:

$$I_1 + I_2 + I_3 = 0$$

$$I_1 = \frac{V_1 - V_s}{R_1}, \quad I_2 = \frac{V_1 - 0}{R_3} \quad I_3 = \frac{V_1 - (-2000V_1)}{R_2 + R_4}$$

$$\frac{V_1 - 1.5}{1000} + \frac{V_1}{10000} + \frac{V_1 - (-2000V_1)}{2100} = 0$$

- Multiply by 21000 ➔ $21V_1 - 31.5 + 2.1V_1 + 10V_1 + 20000V_1 = 0$ ➔ $20033.1V_1 = 31.5$ ➔ $V_1 = 1.5724$ mV

- And V_2 can be found out if we know I_3

$$I_3 = \frac{V_1 - (-2000V_1)}{2100} = 1.4982mA$$

$$V_2 = V_1 - R_2 I_3 = -2.9949\,V$$

Home Work: P3.1 to P3.32

Supernode

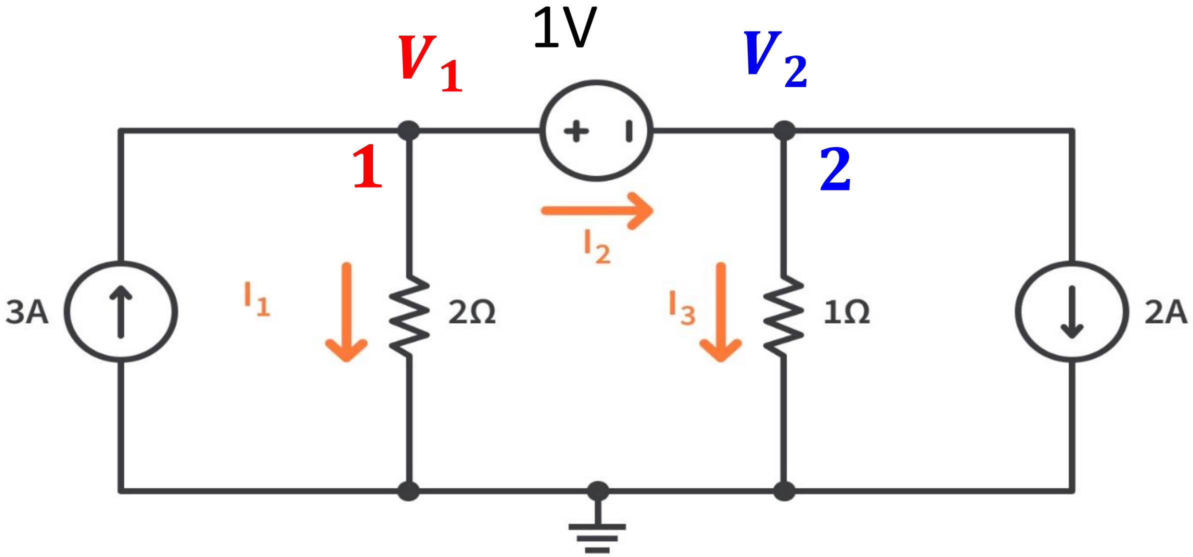
- **Super Node is a Special Situation in Nodal Analysis**, where a voltage source is between two Nodes with unknown voltages

  - Lets say we have the circuit on right
  - KCL at Node 1: Sum current leaving Node1 = 0

$$-3A + I_1 + I_2 = 0$$

  - By Ohm's Law find I_1 and I_2

$$I_1 = \frac{V_1 - 0}{2} \qquad I_2 = \frac{V_1 - V_2}{0} \text{ ???}$$



- If there is a voltage source in a circuit between two nodes whose voltages are unknown, we do not know the current through the voltage source
- What is the current source resistance ? If it is ideal source than the current source resistance is zero.
- **Solution**
  - Node 1 and Node 2 are combined to make one **Super Node**

Supernode (Cont..)

- If there is a voltage source in a circuit between two nodes whose voltages are unknown, we do not know the current through the voltage source, and it is not possible to write the node equations for the two nodes that include the voltage source. In this case, combine the two nodes to form a **supernode**.

- We can then write the node equation for this supernode.

- One additional equation, commonly referred to as a constraint equation relating the two node voltages, can be obtained by representing the voltage source as a potential drop or as a potential rise between the two nodes.

Supernode (Cont..)

- ❑ Combine the two nodes to get one Super Node
- ❑ We get two Equations for the super node

1. Constraint Equation from the source between Nodes

$$V_1 - V_2 = 1 \quad (1)$$

2. KCL for the **super node** → $-3A + I_1 + I_3 + 2A$

- By Ohm's Law

$$-3 + \frac{V_1 - 0}{2} + \frac{V_2 - 0}{1} + 2 = 0$$

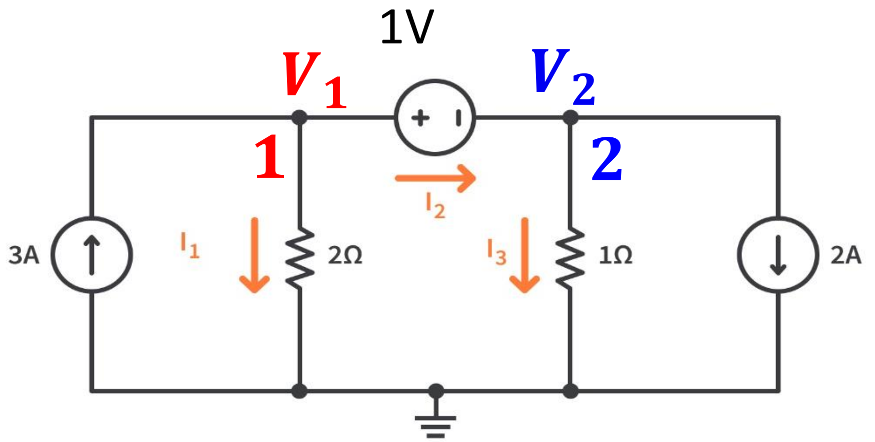
$$V_1 + 2V_2 = 2 \qquad (2)$$

- From (2) → $V_1 - V_2 = 1$ → $V_1 = V_2 + 1$
- Substitute ($V_1 = V_2 + 1$) into (2)

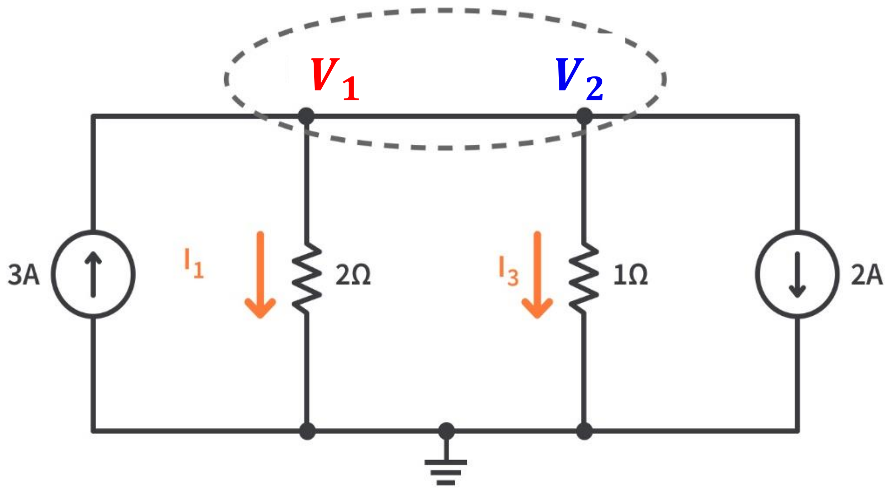
$$V_2 + 1 + 2V_2 = 2 \Rightarrow 3V_2 = 1 \Rightarrow V_2 = \frac{1}{3}V$$

- And V_1 is

$$V_1 = \frac{1}{3} + 1 \Rightarrow V_1 = \frac{4}{3}V$$



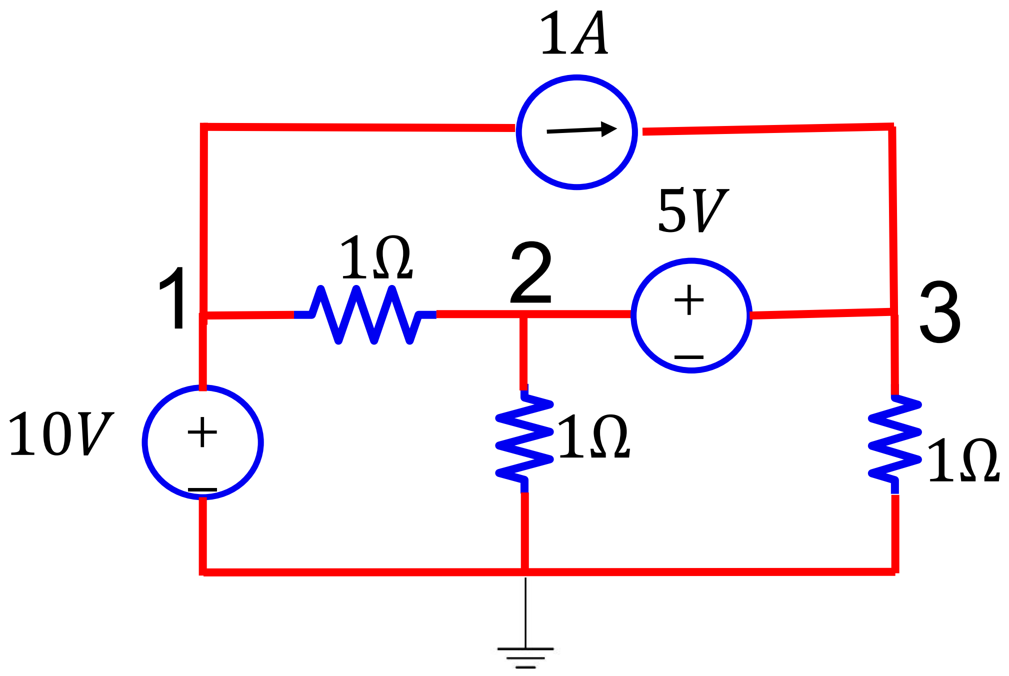
Assume no voltage source



Quiz

Multiple Choice

- ❑ Which node is super node?



Question: The super node is

A. Node 1 and Node 2

B. Node 2 and Node 3

C. Node 1 and Node 0

Supernode – Example

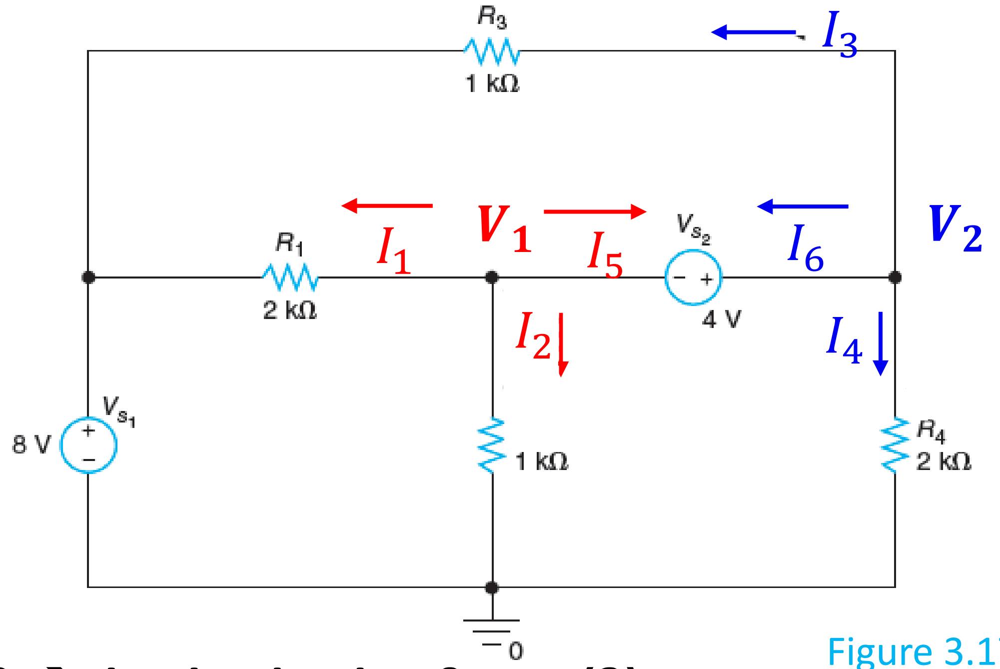


Figure 3.17

- The currents I_5, I_6 flow in opposite direction.
- Unknown currents I_5, I_6 through V_{s2} but $I_6 = -I_5$.
- KCL at node 1 ➔ $I_1 + I_2 + I_5 = 0$ (1)
- KCL at node 2 ➔ $I_3 + I_4 + I_6 = I_3 + I_4 - I_5 = 0$ (2)
- Add (1) and (2)
- $I_1 + I_2 + I_5 = 0$ and $I_3 + I_4 - I_5 = 0$, ➔ $I_1 + I_2 + I_5 + I_3 + I_4 - I_5 = 0$ ➔ $\mathbf{I_1 + I_2 + I_3 + I_4 = 0}$ **(3)**
- (3) is the sum of currents leaving nodes 1 and 2. As $I_5 + I_6 = 0$, therefore, I_5 and I_6 are not included in the sum.
- Use Ohm's Law in (3) $\dfrac{V_1 - 8}{2000} + \dfrac{V_1}{1000} + \dfrac{V_2 - 8}{1000} + \dfrac{V_2}{2000} = 0$
- Multiply by 2000 ➔ $V_1 - 8 + 2V_1 + 2V_2 - 16 + V_2 = 0 \Rightarrow 3V_1 + 3V_2 = 24$ (4)
- Since V_2 is 4V higher than V_1, the constraint equation is given by $\mathbf{V_2 = V_1 + 4}$ (5)
- Substitute (5) in (4) ➔ $3V_1 + 3V_1 + 12 = 24 \Rightarrow 6V_1 = 12 \Rightarrow V_1 = 2$ V
- Substitute (6) in (4) ➔ $V_2 = 6$ V

EXAMPLE 3.8

- Find V_1, V_2, V_3 .
- Constraint equation: $\mathbf{V_3 = V_2 + 7}$ **(1)**
- Sum the currents leaving the **supernode** consisting of node 2 and node 3:

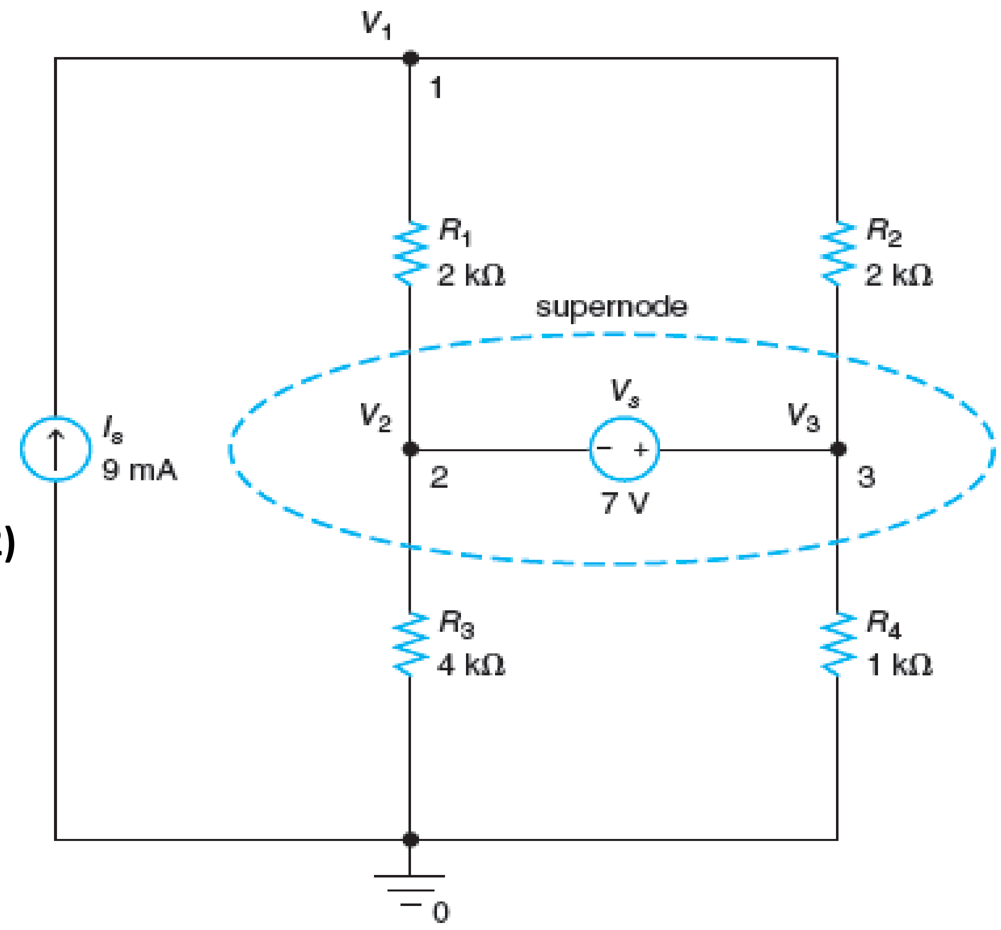
$$\frac{V_2 - V_1}{2000} + \frac{V_2}{4000} + \frac{V_3 - V_1}{2000} + \frac{V_3}{1000} = 0$$

- Multiply by 4000 ➔ $2V_2 - 2V_1 + V_2 + 2V_3 - 2V_1 + 4V_3 = 0 \Rightarrow$
  $-4V_1 + 3V_2 + 6V_3 = 0 \Rightarrow -4V_1 + 3V_2 + 6(V_2 + 7) = 0 \Rightarrow \mathbf{-4V_1 + 9V_2 = -42}$ **(2)**

- Sum the currents leaving node 1: $-0.009 + \dfrac{V_1 - V_2}{2000} + \dfrac{V_1 - V_3}{2000} = 0$

- Multiply by 2000 ➔ $\mathbf{2V_1 - V_2 - V_3 = 18}$ **(3)**

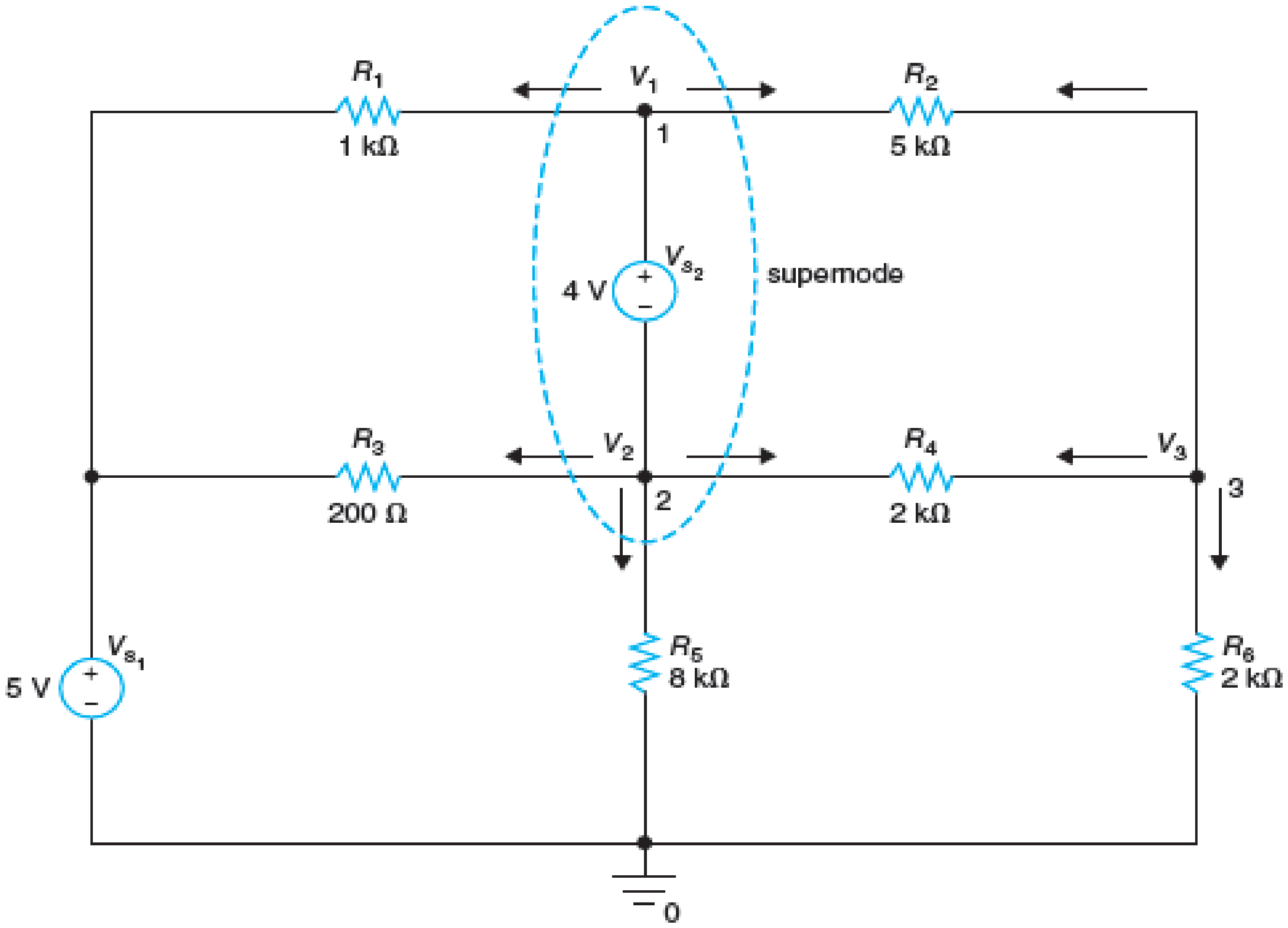
- Substitute (1) into (3)
  $2V_1 - V_2 - V_2 - 7 = 18$
  $\mathbf{2V_1 - 2V_2 = 25}$ **(4)**
- Multiply (4) by 2 ➔ $4V_1 - 4V_2 = 50$ (5)
- Add (2) and (5) ➔ $5V_2 = 8 \Rightarrow V_2 = 1.6$ V,
- From (4) ➔ $V_1 = V_2 + 12.5 = 14.1$ V
- From (1) ➔ $V_3 = V_2 + 7 = 8.6$ V



Home Work: P3.33 to P3.49

EXAMPLE 3.7

- Find V_1, V_2, and V_3.

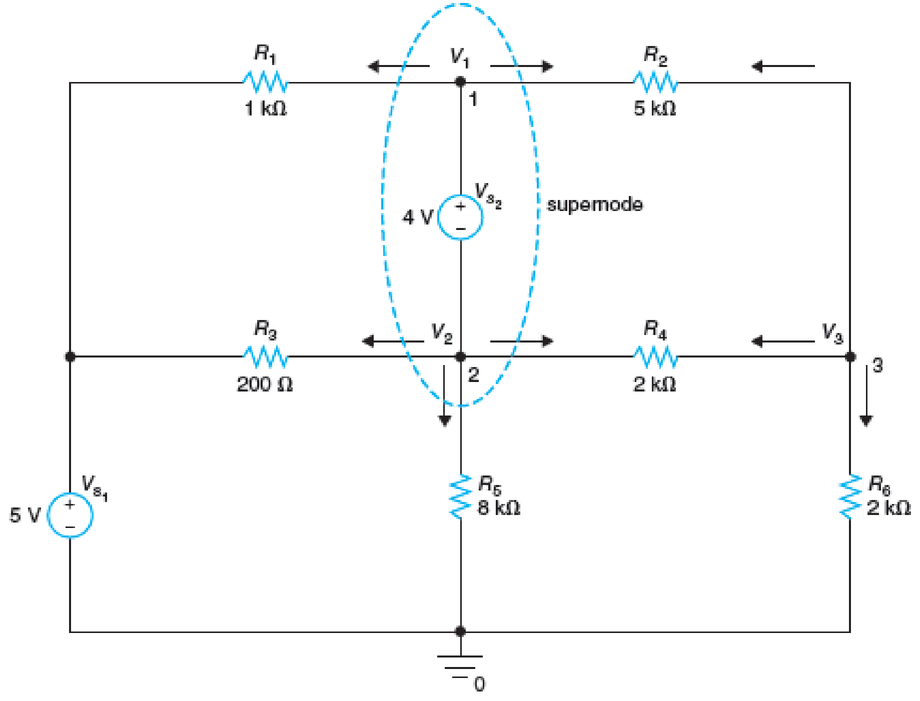


EXAMPLE 3.7 (Cont..)

- Sum the currents leaving the **supernode** consisting of node 1 and node 2:

$$\frac{V_1-5}{1000}+\frac{V_1-V_3}{5000}+\frac{V_2-5}{200}+\frac{V_2-V_3}{2000}+\frac{V_2}{8000}=0$$

- Multiply by 8000 ➔ $8V_1 - 40 + 1.6V_1 - 1.6V_3 + 40V_2 - 200 + 4V_2 - 4V_3 + V_2 \Rightarrow$ **$9.6V_1 + 45V_2 - 5.6V_3 = 240$ (1)**
- Sum the currents leaving node 3: $$\frac{V_3-V_1}{5000}+\frac{V_3-V_2}{2000}+\frac{V_3}{2000}=0$$
- Multiply by 10k ➔ $2(V_3 - V_1)+5(V_3 - V_2)+5V_3=0$ ➔ **$-2V_1 - 5V_2 + 12V_3 = 0$ (2)**
- Constraint equation: **$V_1 = V_2 + 4$ (3)**



- Substitute (3) in (1) ➔ $54.6V_2 - 5.6V_3 = 201.6$ (4)
- Substitute (3) in (2) ➔ $-7V_2 + 12V_3 = 8$ (5)
- Solve (5) for V_2 ➔ $V_2 = (12/7)V_3 - 8/7$ (6)
- Substitute (6) in (4) ➔ $54.6[(12/7)V_3 - 8/7] - 5.6V_3 = 201.6$ (7)
- Solve (7) for V_3 ➔ $V_3 = (201.6 + 54.6\times 8/7)/(54.6\times 12/7 - 5.6) = 3$ (8)
- Substitute (8) in (6) ➔ $V_2 = 4$ V (9)
- Substitute (9) in (3) ➔ $V_1 = 8$ V

Summary

- ❑ Node Analysis
- ❑ Supernode
- ❑ What will we study in next lecture.