Circuit Analysis and Design

Academic year 2024/2025 – Semester 1 – Lecture 2

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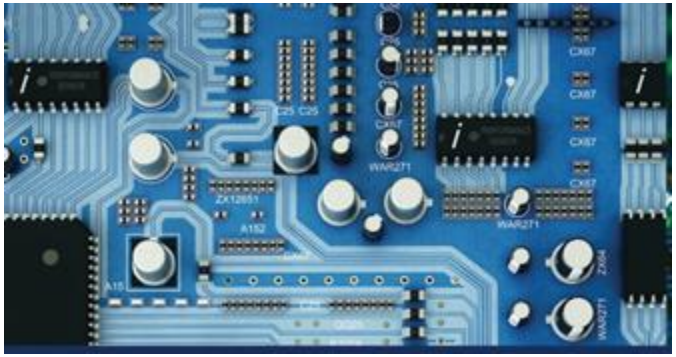
"A good student never steal or cheat"

Agenda

- Review of previous lecture
- Independent sources
- Dependent sources
- Elementary signals
- Summary



Independent Sources

- These source **directly convert energy** from another form to electrical energy
- A **voltage source** with voltage V_s **provides** a **constant potential difference** to the circuit connected between the positive terminal and the negative terminal. The circuit notations for voltage source are shown in Figure 1.6.
- If a positive charge Δq is moved from the negative terminal to the positive terminal through the voltage source, the potential energy of the charge is increased by $\Delta q V_s$.
- If a negative charge with magnitude Δq is moved from the positive terminal to the negative terminal through the voltage source, the potential energy of the charge is increased by $\Delta q V_s$.
- A **current source** with **current I_s** provides a **constant current of I_s** amperes to the circuit connected to the two terminals. The circuit notation for current source is shown in Figure 1.7.

FIGURE 1.6

Circuit symbols for voltage sources.

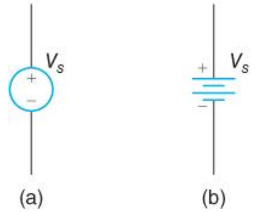
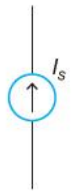


FIGURE 1.7

A circuit symbol for the current source.



DC Sources and AC Sources

- If the **voltage** from the voltage **source is constant with time**, the voltage source is called the direct **current (dc) voltage source**. Likewise, if the **current from** the current source is **constant with time,** the current source is called the **direct current (dc) current source**.
- If the **voltage** from the voltage source is a **sinusoid** as shown in Figure 1.8, the voltage source is called **alternating current (ac)** voltage source. Likewise, if the **current** from the current source is a sinusoid, the **current source** is called **alternating current (ac) current source**.
- A circuit notation for ac voltage source and ac current source are shown in Figure 1.9.

FIGURE 1.8

Plot of a cosine wave with period T, amplitude V_m, and phase zero.

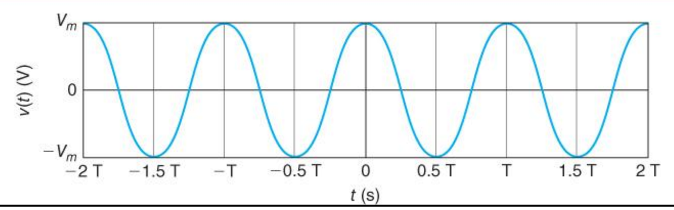
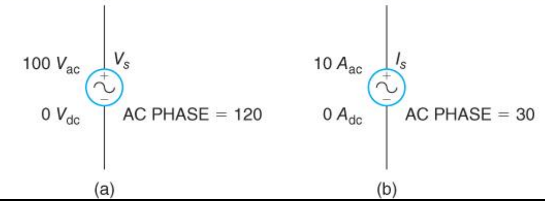


FIGURE 1.9

Circuit symbols for (a) ac voltage source; (b) ac current source.

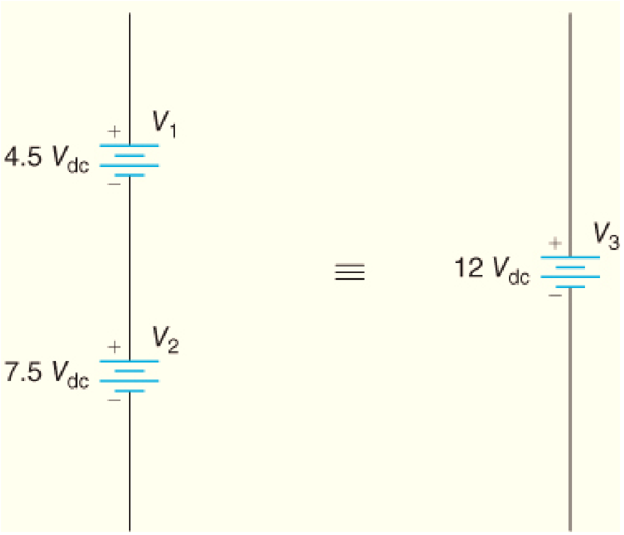


From Electric Circuits by James S. Kang (Cenange Learning)

Equivalent Voltage Source and Equivalent Current Source

- When **dc voltage sources** are connected in **series**, they can be **combined into single** equivalent dc voltage source as shown in Figure

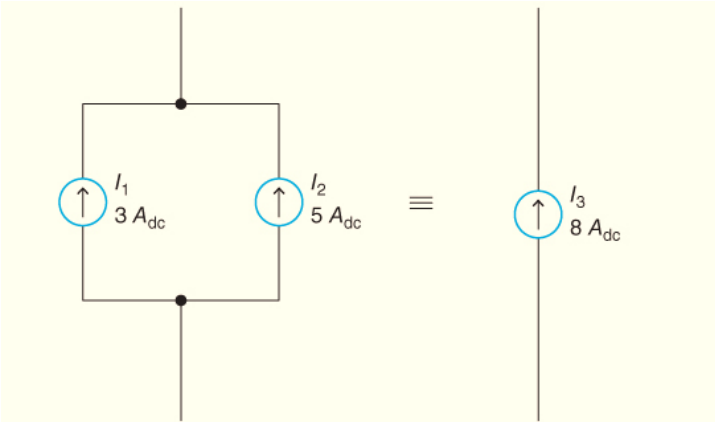
$$V_3 = V_1 + V_2 = 4.5\,V + 7.5\,V = 12\,V$$



Equivalent voltage source

- When **dc current sources** are connected in **parallel**, they can be combined **into single equivalent dc current source** as shown in Figure

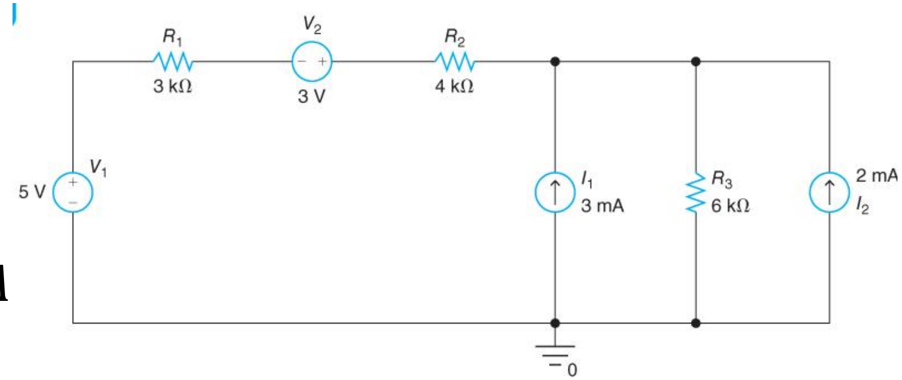
$$I_3 = I_1 + I_2 = 3\,A + 5\,A = 8\,A$$



Equivalent voltage source

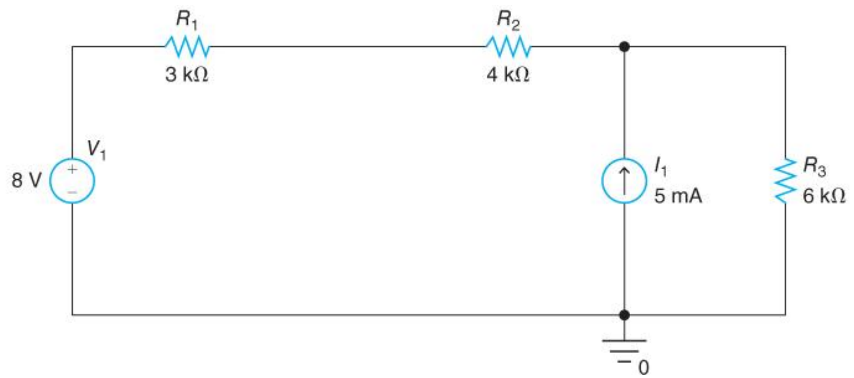
EXAMPLE 1.4

- Redraw the circuit shown in Figure 1.12 with one voltage source and one current source without affecting the voltages across and currents through the resistors in the circuit.



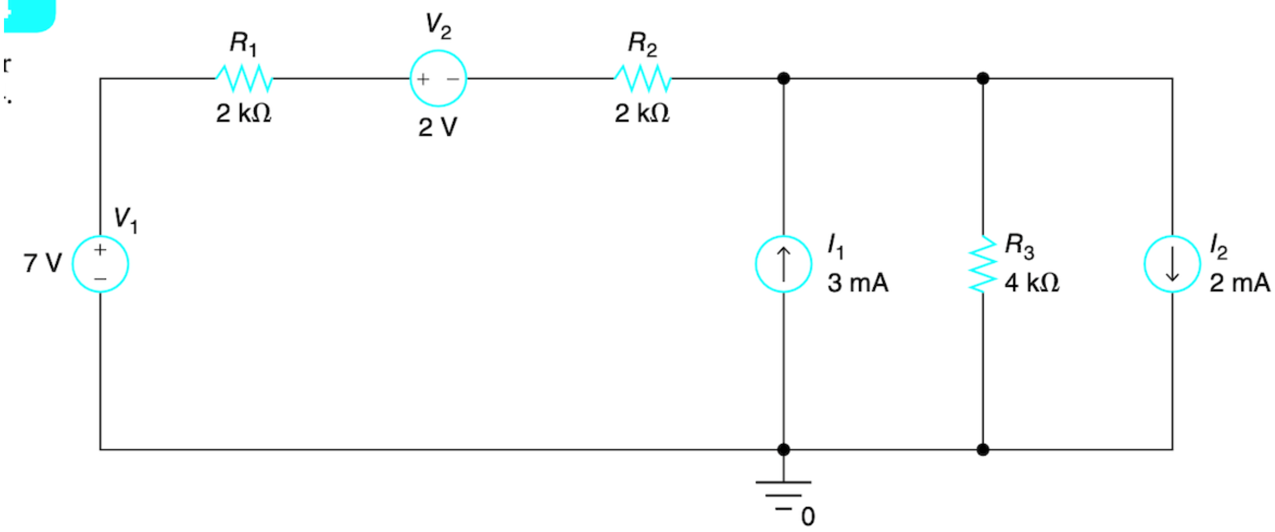
- $V_3 = V_1 + V_2 = 5\,V + 3\,V = 8\,V$
- $I_3 = I_1 + I_2 = 3\,mA + 2\,mA = 5\,mA$

- Equivalent circuit with one voltage source and one current source is.



Class Task

- What is the equivalent voltage and current source in the following circuit.



Options

A. V=7V, I = 5 mA

B. V = 5V, I=1mA

C. V = 7V, I = 3mA

Can you redraw the new circuit with one voltage source and one current source

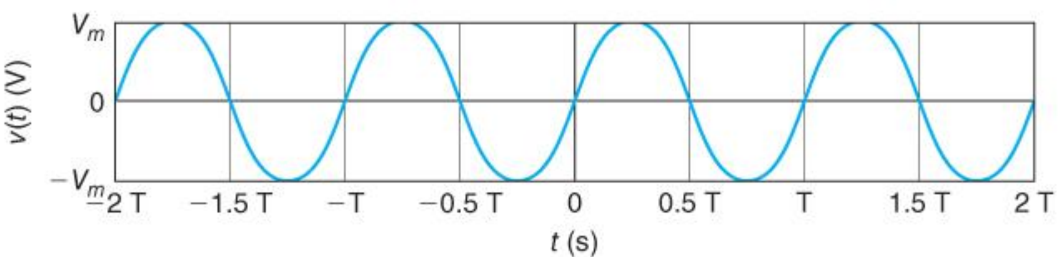
Multiple Choice

Sinusoidal Signal

- An ac voltage waveform can be represented as:
  - $\boldsymbol{v(t) = V_m cos(2\pi t/T + \phi)\ V}$

  $\boldsymbol{V_m}$ = peak amplitude (V), **T** = period (s), **ϕ** = phase (rad or deg).
- **If f** = 1/T = frequency (Hz), then
  - $\boldsymbol{v(t) = V_m\, cos(2\pi f t + \phi)\ V}$
- **ω = 2πf = 2π/T** = angular velocity (rad/s).
  - $\boldsymbol{v(t) = V_m\, cos(\omega t + \phi)\ V}$

$$v(t) = V_m \cos\left[2\pi\left(t - T/4\right)/T\right] = V_m \cos\left(\frac{2\pi}{T}t - \frac{\pi}{2}\right) = V_m \sin\left(\frac{2\pi}{T}t\right) = V_m \sin\left(2\pi f t\right) = V_m \sin\left(\omega t\right)$$

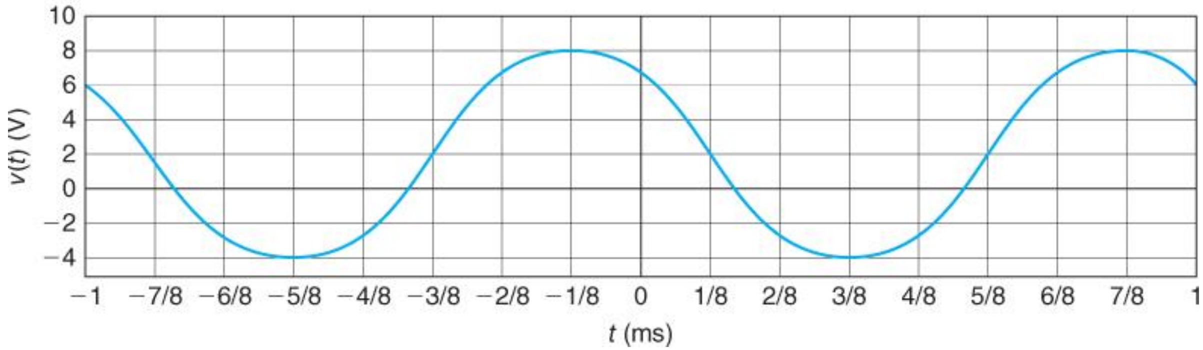


- An ac current waveform can be written as

$$i(t) = I_m \cos\left(\frac{2\pi t}{T} + \emptyset\right) = \ I_m \cos(2\pi f + \emptyset) = I_m \cos(\omega t + \emptyset)\ A$$

EXAMPLE 1.5

- Find the equation of the sinusoidal signal shown in Figure 1.17.

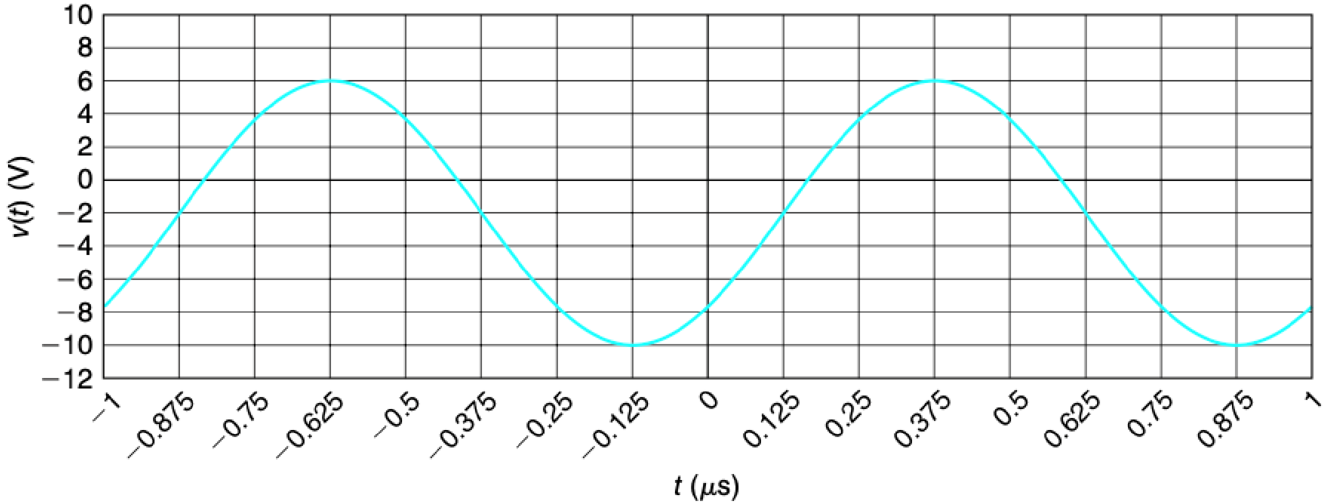


- T = 1 ms, f = 1/T = 1000 Hz = 1 kHz, $\omega = 2\pi f = 6283.1853$ rad/s
- Peak-to-peak amplitude = $V_{p\text{-}p} = 8 – (– 4) = 12$ V,
- peak amplitude = $V_m = V_{p\text{-}p}/2 = 6$ V
- DC offset = average amplitude = $V_{dc} = [8 + (– 4)]/2 = 2$ V
- Cosine wave is shifted to the left by T/8 ms, which is π/4 rad = 45°.
- The equation is given by:

$$\boldsymbol{v(t) = 2 + 6\cos(2\pi 1000t + 45^o)\, V}$$

Class Task

- **Find the equation of the sinusoid**



- **Options**

A. **v(t)** = $-2 + 8\cos(2\pi 10^6 t - 135^o)\ V$

B. **v(t)** = $\mathbf{6}\cos(2\pi 10^6 t - 135^o)\ V$

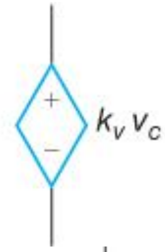
C. **v(t)** = $\mathbf{2 - 8}\cos(2\pi 10^6 t - \mathbf{90}^o)\ V$

Multiple Choice

Dependent Sources

- Voltage or current on **dependent sources depend** solely on **controlling voltage or controlling current**. The dependent sources are used to model integrated circuit (IC) devices.
- Depending on whether dependent source is a voltage source or a current source and whether dependent source is controlled by a voltage or a current, there are **four different dependent sources**:

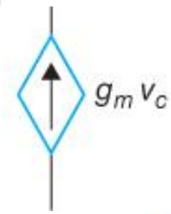
1. Voltage-Controlled Voltage Source (VCVS):



$$\boldsymbol{v_d = k_v\, v_c}$$

k_v is constant

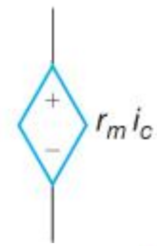
2. Voltage-Controlled Current Source (VCCS):



$$\boldsymbol{i_d = gm\, v_c}$$

g_m is conductance

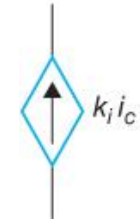
3. Current-Controlled Voltage Source (CCVS):



$$\boldsymbol{v_d = rm\, i_c}$$

r_m resistance

4. Current-Controlled Current Source (CCCS):

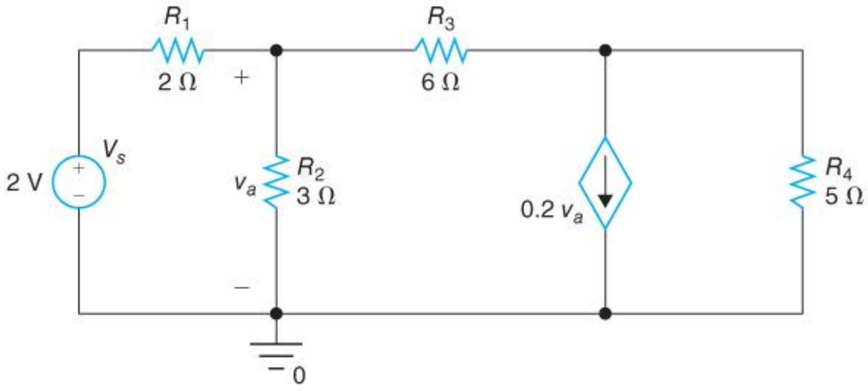


$$\boldsymbol{i_d = k_i\, i_c},$$

$\boldsymbol{k_i}$ is ?

EXAMPLE 1.6

- In the circuit shown controlling voltage, across R_2, is $v_a = 0.9851$ V.
- Find controlled current through the VCCS.



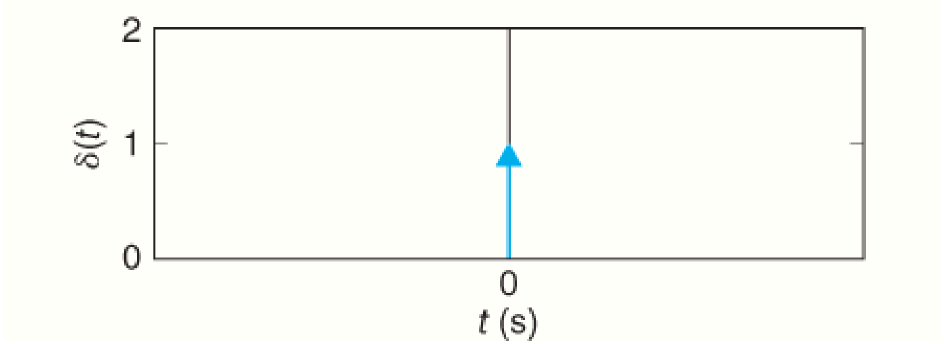
- Current through the VCCS in the direction indicated in Figure 1.23 (↓) is

$0.2\ v_a = 0.2\ (A/V) \times 0.9851\ V = 0.1970\ A$

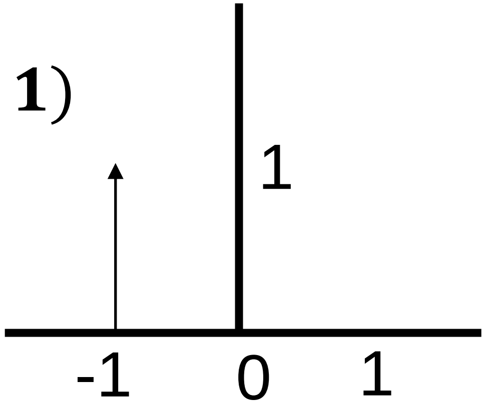
Dirac Delta Function

- *Represented as*

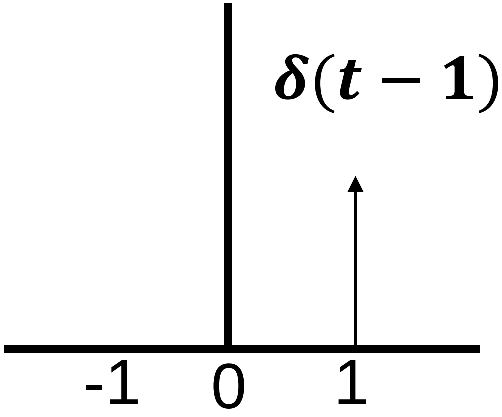
$$\boldsymbol{\delta(t)}$$



$$\boldsymbol{\delta(t+1)}$$



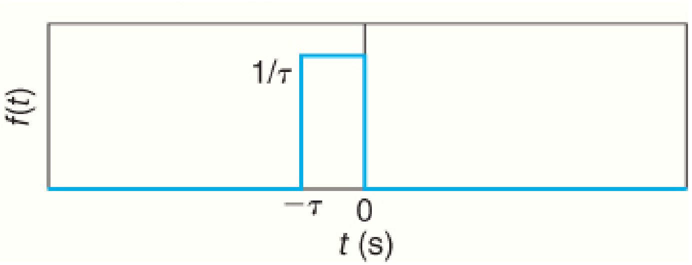
$$\boldsymbol{\delta(t-1)}$$



Dirac Delta Function

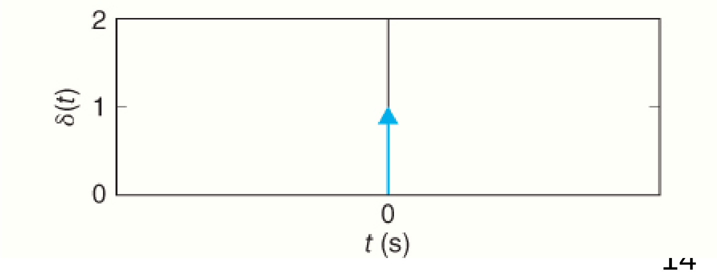
- A rectangular pulse with height 1/τ and width τ is shown in Figure 1.25. The pulse is centered at –τ/2 and the area of the pulse is one. The rectangular pulse can be written as

$$f(t) = \frac{1}{\tau} rect\left(\frac{t + \frac{\tau}{2}}{\tau} \right)$$



- If pulse width τ is decreased to zero, height of the pulse is increased to infinity while maintaining the area at one. The limiting form of a rectangular pulse shown in Figure 1.25 as τ→0 is defined as Dirac delta function (or delta function) and is denoted by δ(t):

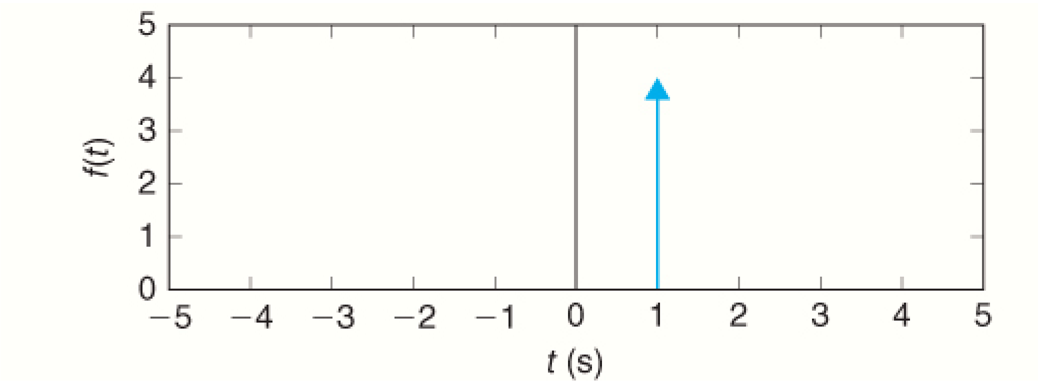
$$\delta(t) = \lim_{\tau \to 0} \frac{1}{\tau} rect\left(\frac{t + \frac{\tau}{2}}{\tau} \right)$$



EXAMPLE 1.7

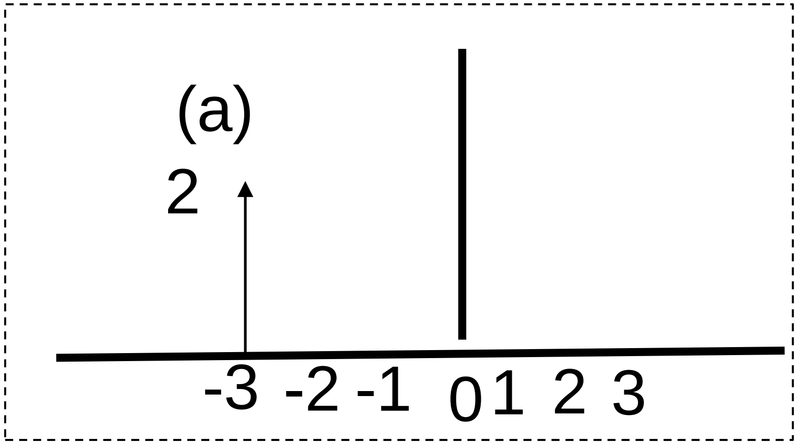
- Plot $f(t) = 4\,\delta(t - 1)$.

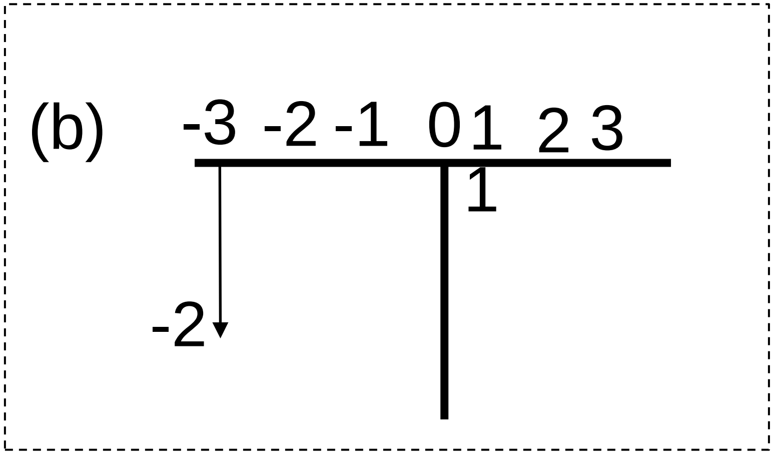
- The Dirac delta function is located at $t = 1$ and has area of 4. The signal $f(t)$ is shown in Figure 1.27.

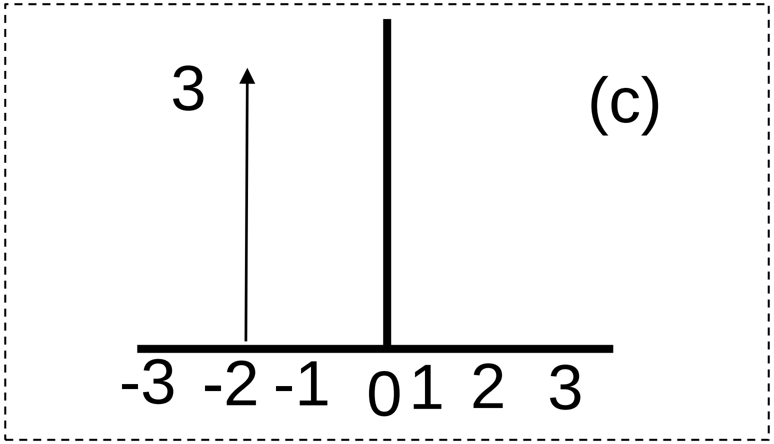


Class Task

- **Plot** $f(t) = -2\boldsymbol{\delta}(\boldsymbol{t}+\mathbf{3})$







Multiple Choice

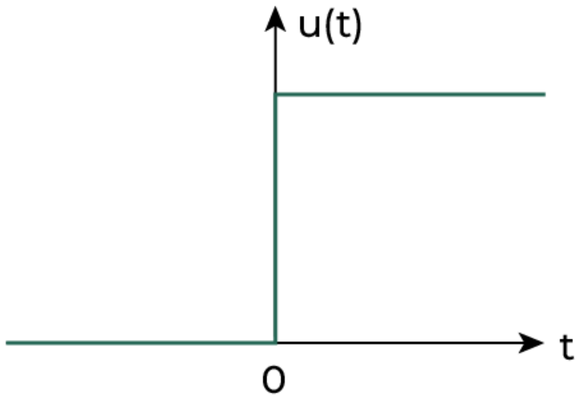
Sifting Property

- When a continuous signal f(t) is multiplied by $\delta(t - a)$ and integrated from $-\infty$ to ∞, we obtain f(a), that is,

$$\int_{-\infty}^{\infty} f(t)\delta(t-a)dt = f(a)$$

- This is called **sifting property of the delta function**. It sifts out a single value of f(t), at the location of the delta function (t = a).

Step Function



$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t > 0 \end{cases}$$

Integrate $\delta(t)$

$$u(t) = \int \delta(t)dt \; = \int 1\,\mathrm{dt} = 1 \text{ for } \mathrm{t} \geq 0$$

Step Function

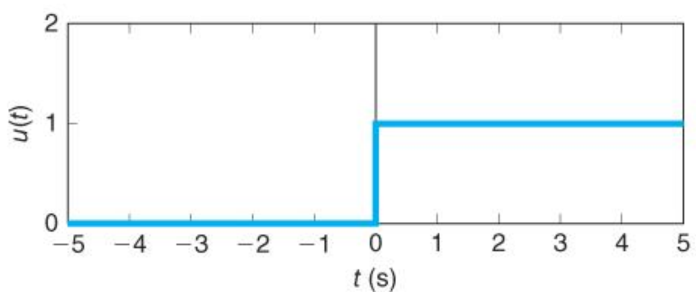
- Unit step function u(t) is **integral** of the **Dirac delta function δ(t).** If a rectangular pulse $f(t) = \left(\frac{1}{\tau}\right) rect\left[\frac{\left(t+\frac{\tau}{2}\right)}{\tau}\right]$ is integrated, we obtain

$$\int_{-\infty}^{t} f(\lambda)d\lambda = \frac{1}{\tau}\int_{-\infty}^{t} rect\left(\frac{\lambda+\frac{\tau}{2}}{\tau}\right)d\lambda = \begin{cases} 0, & t<-\tau \\ \frac{t}{\tau}+1, & -\tau\le t<0 \\ 1, & 0\le t \end{cases}$$

- Unit step function is defined as limiting form of this equation. In the limit as $\tau \to 0$, this equation becomes

$$u(t) = \begin{cases} 0, & t<0 \\ 1, & 0\le t \end{cases}$$

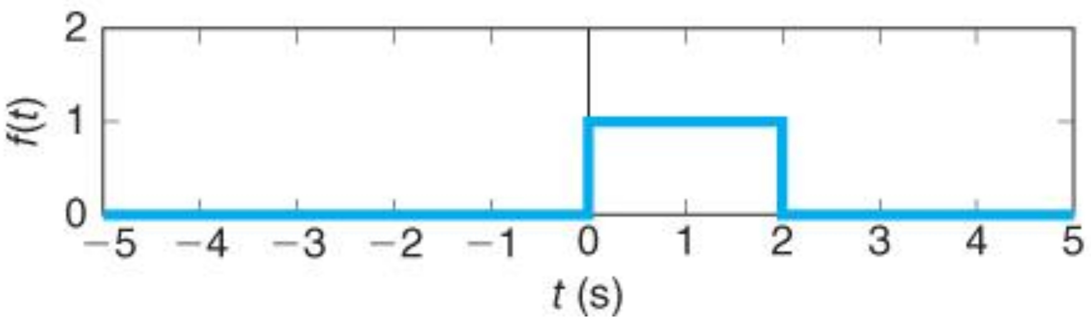
- Notice that at t = 0, u(t) = 1. The unit step function is shown in Figure 1.29.



EXAMPLE 1.8

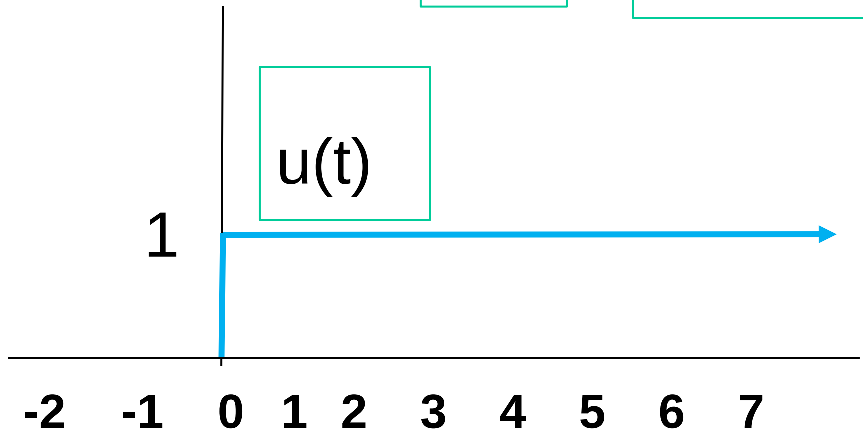
- Plot $f(t) = u(t) - u(t - 2)$.

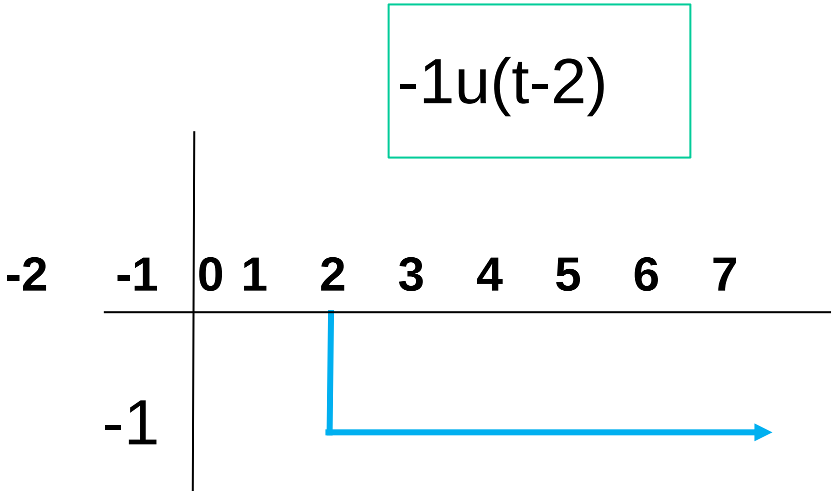
- Notice that $u(t) = 1$ for $t \geq 0$ and zero for $t < 0$, and $u(t - 2) = 1$ for $t \geq 2$ and zero for $t < 2$.
- Thus, $u(t) - u(t - 2) = 0$ for $t \geq 2$, and $u(t) - u(t - 2) = 1$ for $0 \leq t < 2$, and zero for $t < 0$. The signal $f(t)$ is shown in Figure 1.30.

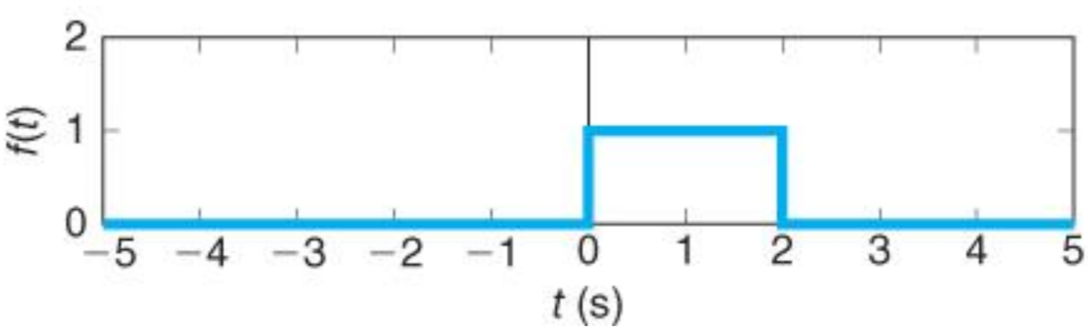


EXAMPLE 1.8 (continue)

- Plot f(t) = $\boxed{u(t)} - \boxed{u(t-2)}$.





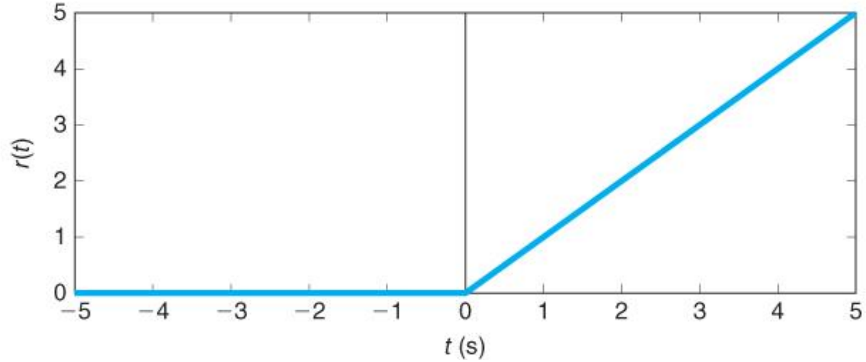


Ramp Function

- A unit ramp function is defined by

$$r(t) = t\,u(t)$$

- Unit ramp function is shown here



- Unit ramp function is the integral of the unit step function:

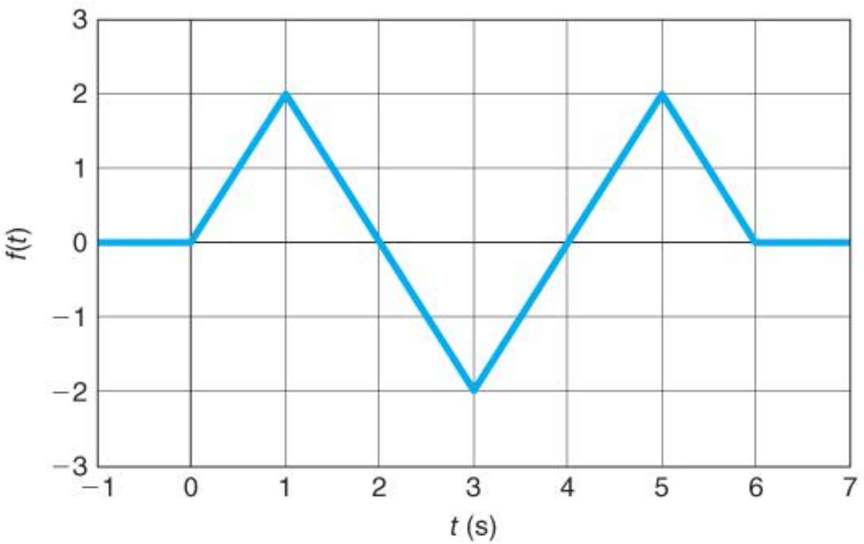
$$r(t) = \int_{-\infty}^{t} u(\lambda)\,d\lambda$$

- Derivative of unit ramp function is unit step function:

$$u(t) = \frac{dr(t)}{dt}$$

EXAMPLE 1.9

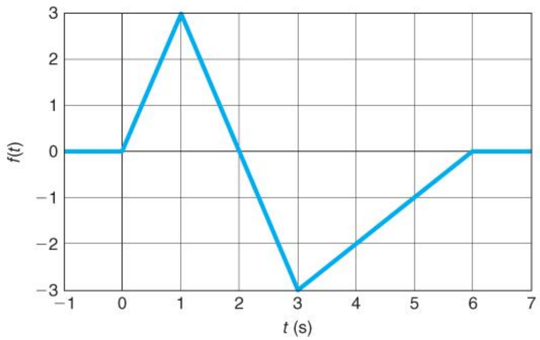
- Plot $f(t) = 2tu(t) – 4(t – 1)u(t – 1) + 4(t – 3)u(t – 3) – 4(t – 5)u(t – 5) + 2(t – 6)u(t – 6)$.
- For $t < 0$, $f(t) = 0$
- For $0 \le t < 1$, f(t) is a linear line with slope of 2.
- For $1 \le t < 3$, f(t) is a linear line with slope of –2. (Add 2 and -4)
- For $3 \le t < 5$, f(t) is a linear line with slope of 2. (-2 + 4)
- For $5 \le t < 6$, f(t) is a linear line with slope of –2. (2-4)
- For $6 \le t$, $f(t) = 0$. (-2 + 2)



EXAMPLE 1.10

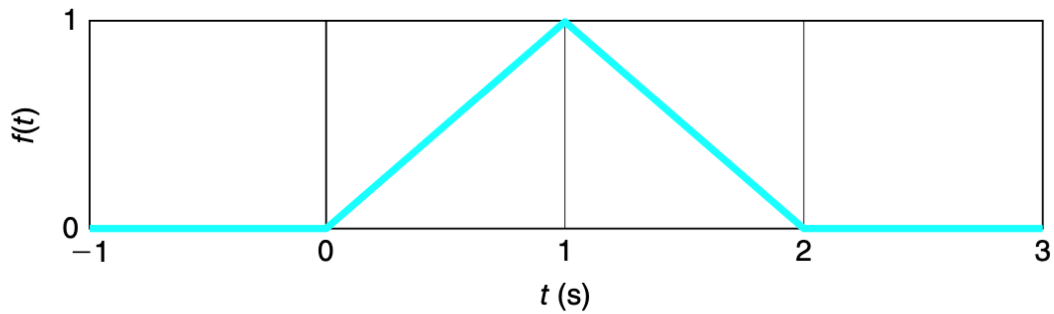
- Find equation of the waveform shown in Figure 1.35.
- For $t < 0$, $f(t) = 0$.
- For $0 \le t < 1$, $f(t)$ is a linear line with slope of 3. Thus, $f(t) = 3tu(t)$.
- For $1 \le t < 3$, $f(t)$ is a linear line with slope of –3. To change the slope from 3 to –3, we need to add $-6(t - 1)u(t - 1)$. At this point, we have $f(t) = 3tu(t) - 6(t - 1)u(t - 1)$.
- For $3 \le t < 6$, $f(t)$ is a linear line with slope of 1. To change the slope from –3 to 1, we need to add $4(t - 3)u(t - 3)$. At this point, we have $f(t) = 3tu(t) - 6(t - 1)u(t - 1) + 4(t - 3)u(t - 3)$.
- For $6 \le t$, $f(t) = 0$. To change the slope from 1 to 0, we need to add $-(t - 6)u(t - 6)$.
- The final equation is given by

$$f(t) = 3tu(t) - 6(t - 1)u(t - 1) + 4(t - 3)u(t - 3) - (t - 6)u(t - 6).$$



Class work

- **Find the equation**



Options

A. $tu(t) + 2(t-1)u(t-1) - (t-2)u(t-2)$

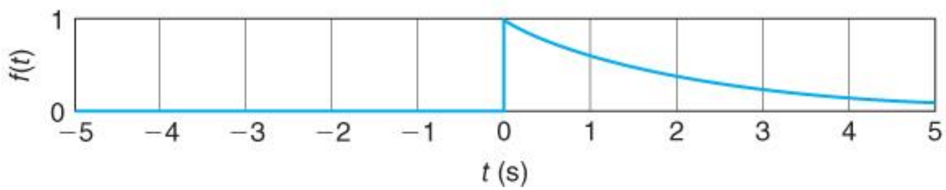
B. $tu(t) - 2(t-1)u(t-1) + (t-2)u(t-2)$

C. $tu(t) + (t-2)u(t-2)$

Multiple Choice

Exponential Decay

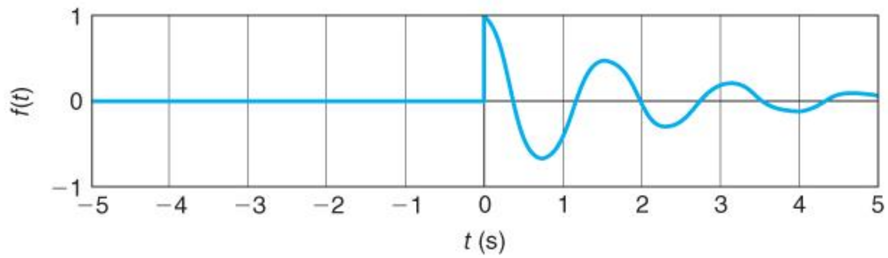
- A signal that decays exponentially can be written as $f(t) = e^{-at}u(t)$, $a > 0$.
- The signal $f(t)$ for $a = 0.5$ is shown in Figure 1.37.



- Damped cosine and damped sine can be written respectively as

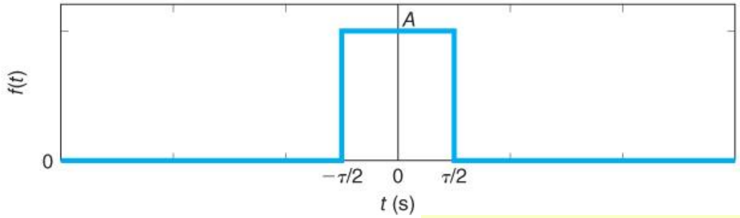
  $f(t) = e^{-at}\cos(bt)u(t)$, $a > 0$

  $f(t) = e^{-at}\sin(bt)u(t)$, $a > 0$
- Damped cosine signal is shown 1.38 for $a = 0.5$ and $b = 4$.



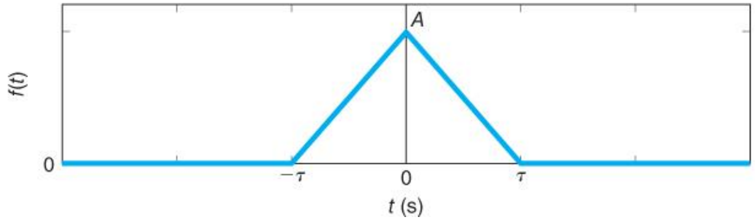
Rectangular Pulse and Triangular Pulse

- **A rectangular** pulse with amplitude A pulse width τ is shown in Figure. Center of the pulse is at t = 0.



- Rectangular pulse shown in is denoted by: $$f(t) = A\,rect\left(\frac{t}{\tau}\right)$$

- **A triangular** pulse with amplitude **A** and base 2τ is shown in Figur. The center of the pulse is at t = 0.
- Triangular pulse shown in Figure 1.42 is denoted by $$f(t) = A\,tri\left(\frac{t}{\tau}\right)$$



EXAMPLE 1.11

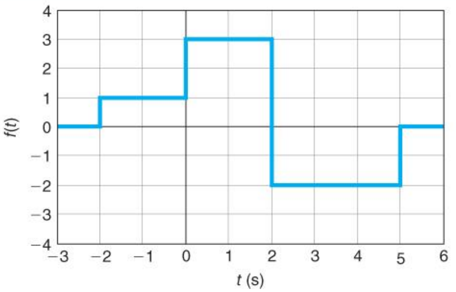
- Plot

$$f(t) = rect\left(\frac{t+1}{2}\right) + 3rect\left(\frac{t-1}{2}\right) - 2rect\left(\frac{t-3.5}{3}\right)$$

- The first rectangle is centered at t = –1 and has a height of 1 and width of 2. The second rectangle is centered at t = 1 and has a height of 3 and width of 2. The third rectangle is centered at t = 3.5 and has a height of –2 and width of 3. The waveform f(t) is shown in Figure 1.40.

FIGURE 1.40

Waveform $f(t)$.

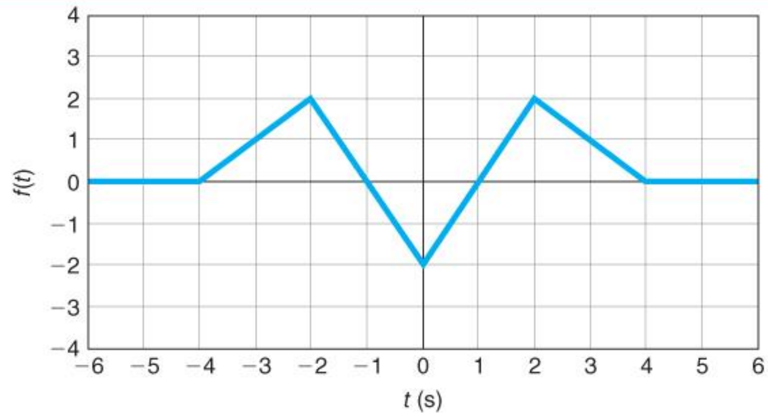


EXAMPLE 1.12

- Plot $$f(t) = 2tri\left(\frac{t+2}{2}\right) - 2tri\left(\frac{t}{2}\right) + 2tri\left(\frac{t-2}{2}\right)$$
- The first triangle is centered at t = –2 and has a height of 2 and base of 4. The second triangle is centered at t = 0 and has a height of -2 and base of 4. The third triangle is centered at t = 2 and has a height of 2 and base of 4. The waveform f(t) is shown in Figure 1.43.

FIGURE 1.43

Waveform $f(t)$.



Summary

- The seven base units of the International System of Units (SI) along with derived units relevant to electrical and computer engineering are presented.
- The electric field E is a force per unit charge.
- When electric field E is integrated along the path against the field, we get work (force $\times$ displacement) done per unit charge.
- The potential difference between points A (final) and B (initial) is defined as the work done per unit charge against the force: $v_{AB} = v_A - v_B = dw_{AB}/d_q$ (J/C)
- The current is defined as the rate of change of charge: i(t) = dq(t)/dt
- The power is the product of current and voltage: p(t) = i(t)v(t)
- The energy is the integral of power: $w(t) = \int_{-\infty}^{t} p(\lambda)d\lambda$
- Power is the derivative of energy: p(t) = dw(t)/dt
- Four types of dependent sources: VCVS, VCCS, CCVS, CCCS
- Elementary signals: Dirac delta, step, ramp, exponential, rectangular pulse, triangular pulse.
- What will we study in next lecture