Circuit Analysis and Design

Academic Year 2025/2026 – Semester 1

Lecture 7 - Circuit Theorems

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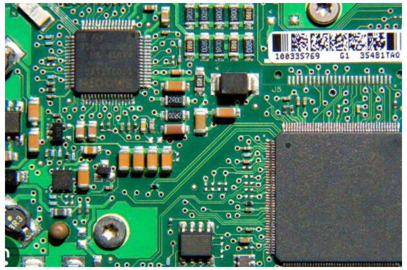
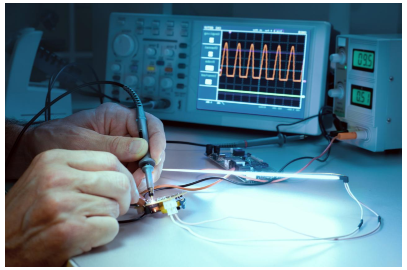




Agenda

- Introduction
- Superposition principle
- Source transformation
- Summary

Introduction

- There are a number of theorems that are useful in circuit design and analysis:
  - They enhance the understanding of circuit operation.
  - They help to simplify the circuit configuration and hence, make the choice of components easier.
- Important theorems in circuit analysis are:
  - Superposition principle
  - Source transformation
  - Thévenin’s theorem
  - Norton’s theorem
  - Maximum power transfer

Superposition Principle

- Suppose that a circuit has N independent sources with $N \geq 2$.
  - Create N circuits from the original circuit with only one independent source by deactivating the other N–1 independent sources.
    - Deactivating a current source is to open-circuit it.
    - Deactivating a a voltage source is to short-circuit it.
  - The unknown voltages and currents of the original circuit can be found by adding the voltages and currents from the N circuits with one independent source: superposition principle.
- The superposition principle reveals the contribution of each source to the voltages and currents in the circuit.
- It makes it easier to interpret the response of the circuit because we can trace the sources of the response.
- The superposition does not apply to dependent sources.

Superposition Principle

- Consider a circuit with a voltage source V_s, a current source I_s, and two resistors R_1 and R_2. We tend to find V_1 and I using superposition principle.

- Deactivate the current source by removing it from the circuit. The circuit now contains only one independent source V_s (Figure 4.2).

  – Applying the voltage divider rule:

  $V_{11} = V_s \times R_2/(R_1+R_2) = 5 \times 2/5 = 2$ V

  – The contribution of the voltage source to the voltage across R_2 is 2 V.

  – Applying Ohm's law:

  $I_a = V_s/(R_1 + R_2) = 5 / 5k = 1$ mA

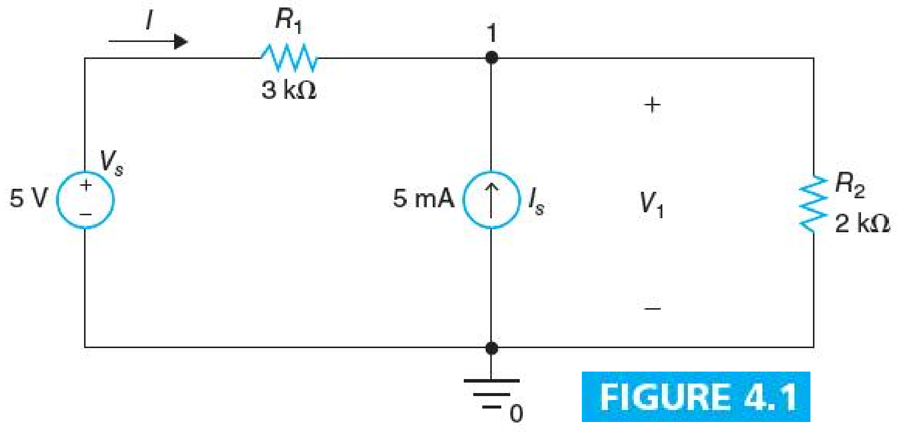


FIGURE 4.1

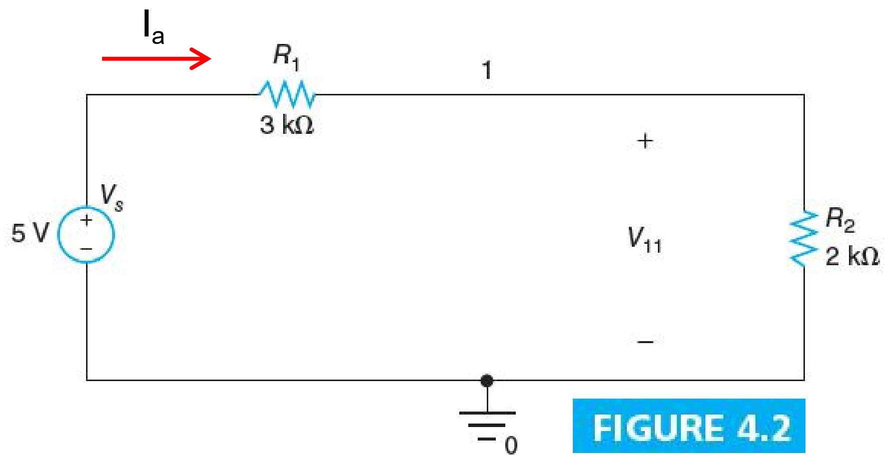


FIGURE 4.2

Superposition Principle

- Deactivate the voltage source by short-circuiting. The circuit contains only one independent source I_s (Figure 4.3).
  - Applying the current divider rule, we obtain the current through R_2:
  $I_2 = I_s \times R_1/(R_1 + R_2) = 5\text{mA} \times 3/5 = 3\text{ mA}$
  - The voltage across R_2 is given by
  $V_{12} = R_2 I_2 = 2000 \times 0.003 = 6\text{ V}$

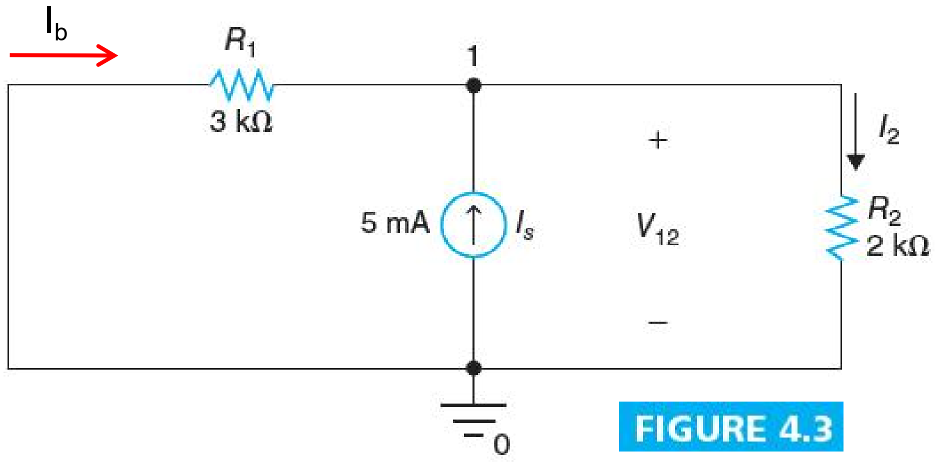


FIGURE 4.3

- The contribution of the current source to the voltage across R_2 is 6 V.
- Applying KCL gives $I_b = -(I_s - I_2) = -2\text{ mA}$
- The voltage across R_2 is given by $V = V_{11} + V_{12} = 2\text{ V} + 6\text{ V} = 8\text{ V}$
- The current I is $I = I_a + I_b = -1\text{ mA}$

EXAMPLE 4.1

- Use the superposition principle to find V_1 in the circuit.
  - When the current source is deactivated, the circuit reduces to Figure 4.5.
  - $R_a = R_1 \parallel R_3$
    $= 4 \times 16/20\ k\Omega = 3.2\ k\Omega$
  - Using voltage divider rule:
    $V_{11} = V_s \times R_a/(R_2 + R_a)$
    $= 10 \times 3.2/10 = 3.2\ V$

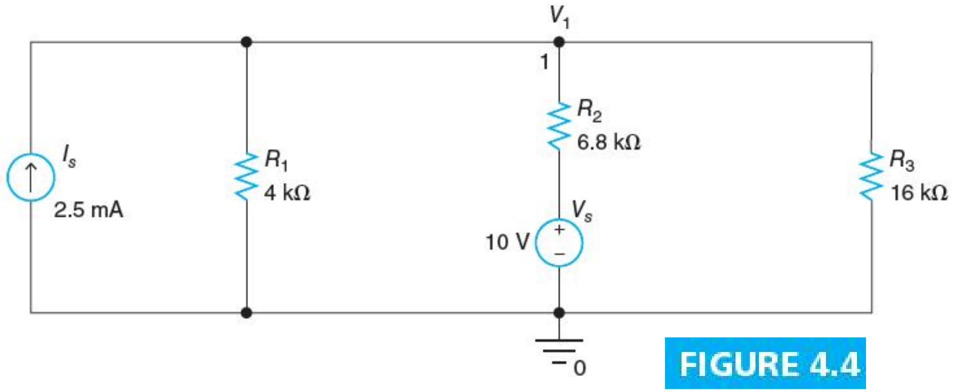


FIGURE 4.4

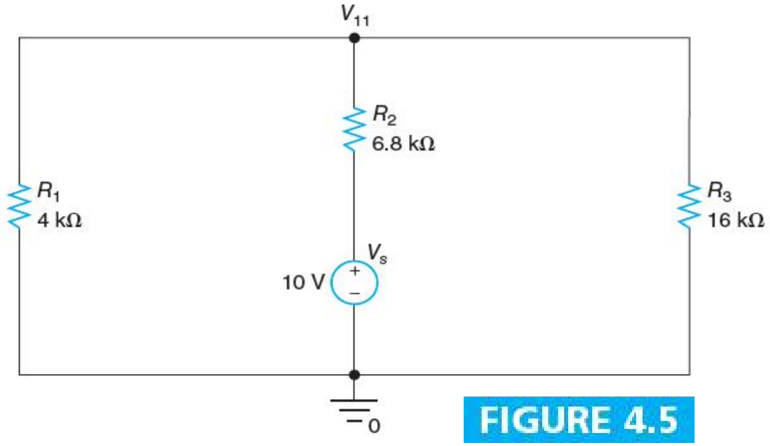


FIGURE 4.5

EXAMPLE 4.1

- When the voltage source is deactivated, the circuit reduces to Figure 4.6.

- $R_a = R_1 \parallel R_3$
  $= 4{\times}16/20\ \text{k}\Omega = 3.2\ \text{k}\Omega$

- $R_b = R_a \parallel R_2$
  $= 3.2{\times}6.8/10 = 2.176\ \text{k}\Omega$

  - Using Ohm's law
    $V_{12} = R_b \times I_s = 2.176 \times 2.5 = 5.44$ V

  - The voltage V_1 is:
    $V_1 = V_{11} + V_{12} = 3.2\ \text{V} + 5.44\ \text{V} = 8.64\ \text{V}$

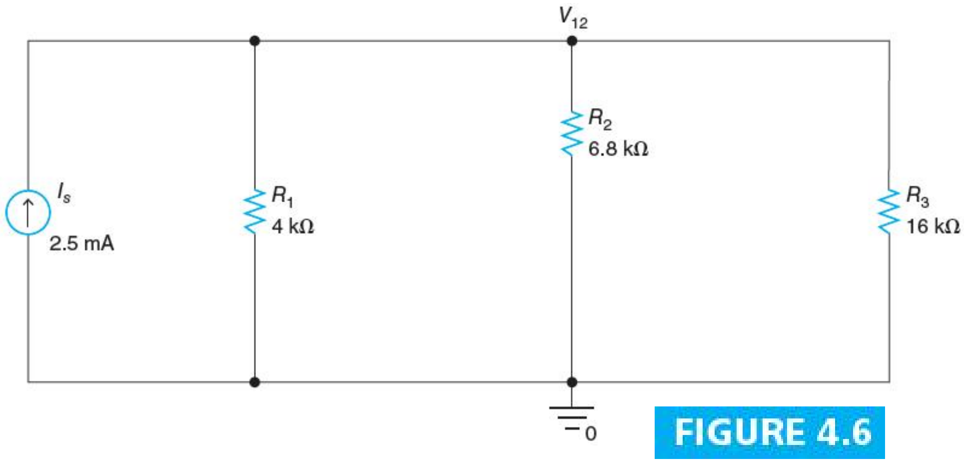


FIGURE 4.6

EXAMPLE 4.2

- Use superposition principle to find V_1 in the circuit.

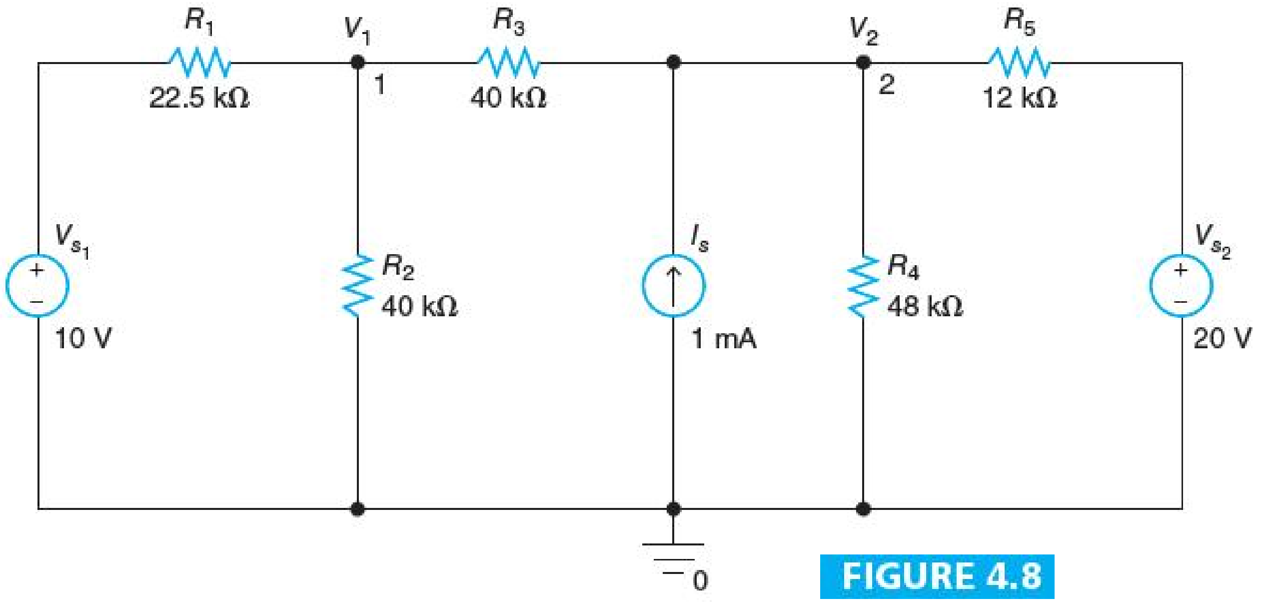


FIGURE 4.8

EXAMPLE 4.3

- Use the superposition principle to find V_o in the circuit given in Figure 4.13.
  - When I_{s2} is deactivated, the circuit reduces to Figure 4.14.
  - By voltage division rule:

  $v = (V_{11} – V_{01}) \times R_2/(R_2 + R_3) = (V_{11} – V_{01})/3$

  - At node 1:

$$-0.004+\frac{V_{11}}{4000}+\frac{V_{11}-V_{01}}{3000}=0$$

  Multiply by 12,000: $7V_{11} – 4V_{01} = 48$ (1)

  - At Node 2:

$$\frac{V_{01}-V_{11}}{3000}+\frac{V_{01}}{3000}+0.01\frac{V_{11}-V_{01}}{3}=0$$

  Multiply by 3,000: $– 8V_{o1} + 9V_{11} = 0$

  $\Rightarrow V_{11} =(8/9)V_{o1}$ (2)

  - Substituting (2) in (1):

  $(7 \times 8/9) V_{01} – 4V_{01} = 48$

  $\Rightarrow V_{o1} = 48 \times 9/20 = 21.6V$

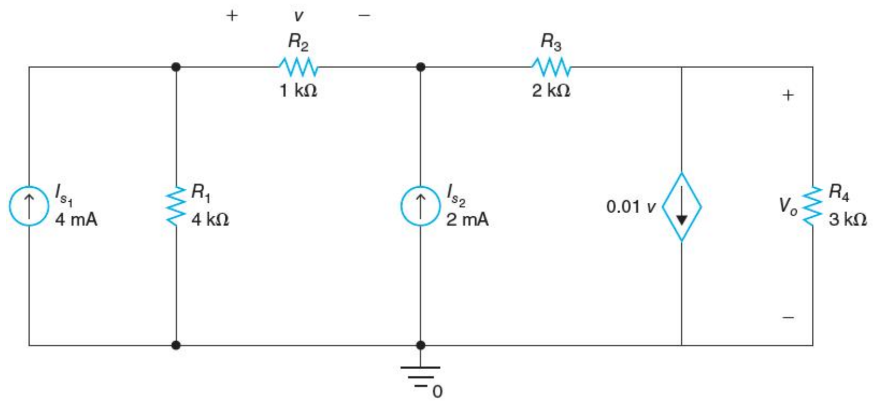


FIGURE 4.13

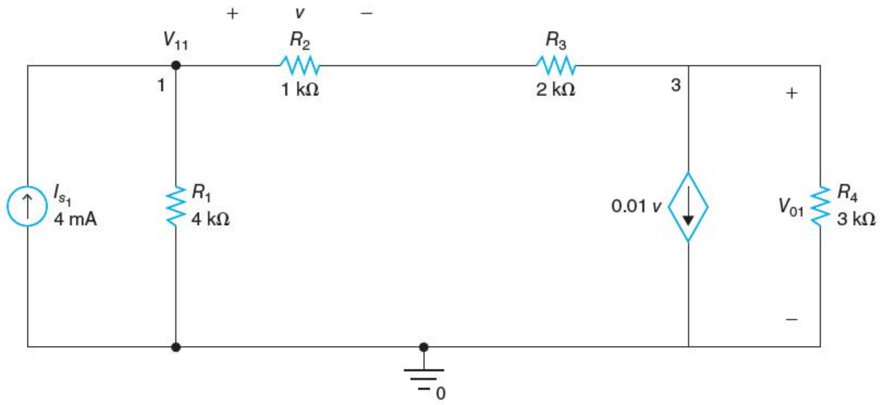


FIGURE 4.14

EXAMPLE 4.3

- When I_{s1} is deactivated, the circuit reduces to:

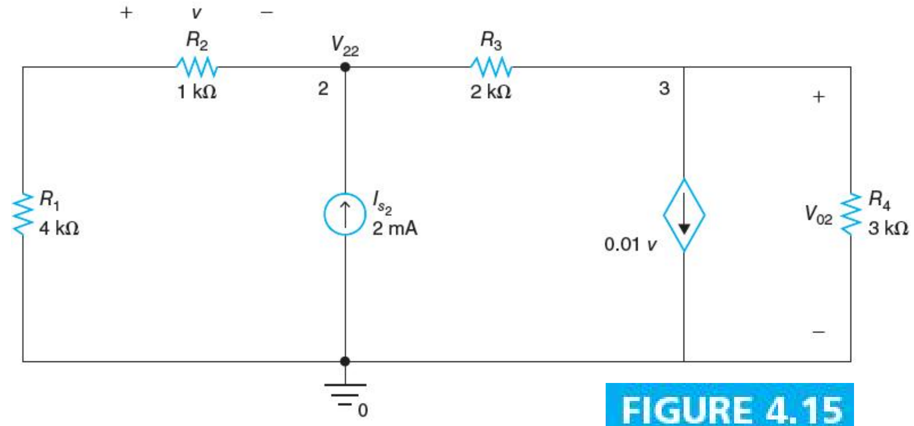


FIGURE 4.15

- Applying voltage division rule:
  $v = (-V_{22}) \times R_2/(R_1 + R_2) = (-V_{22})/5$

- At Node 2: $$\frac{V_{22}}{5000} - 0.002 + \frac{V_{22} - V_{02}}{2000} = 0$$

  Multiply by 10,000: $7V_{22} - 5V_{02} = 20$ (3)

- At Node 3: $$\frac{V_{02} - V_{22}}{2000} + \frac{V_{02}}{3000} + 0.01\frac{-V_{22}}{5} = 0$$

  Multiply 6,000: $3V_{02} - 3V_{22} + 2V_{02} - 12V_{22} = 0, \Rightarrow V_{22} = (1/3)V_{02}$ (4)

- Substitute (4) into (3): $(7/3)V_{02} - 5V_{02} = 20$
  $\Rightarrow (-8/3)\,V_{02} = 20 \Rightarrow V_{02} = -60/8 = -7.5$ V

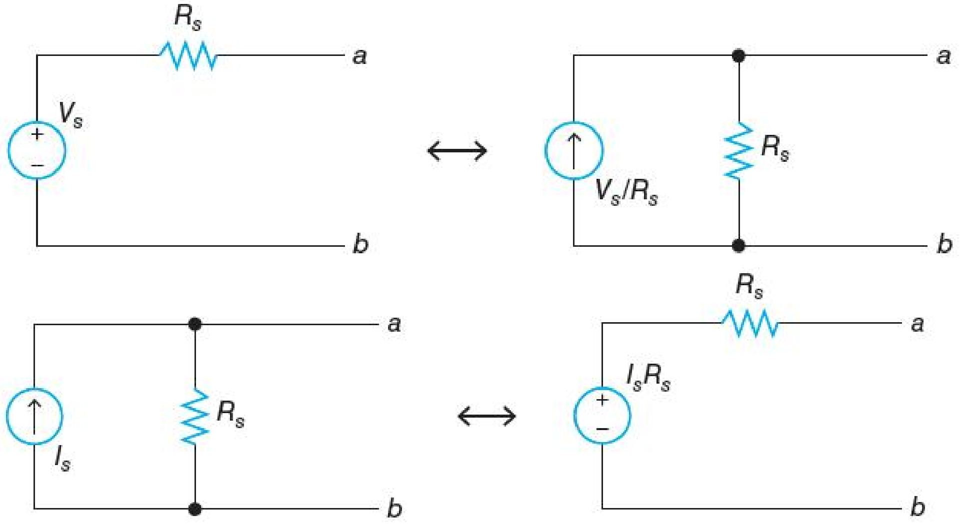
- $V_0 = V_{01} + V_{02} = 21.6\text{ V} - 7.5\text{ V} = 14.1\text{ V}$

Source Transformation

- A circuit consisting of a voltage source with voltage V_s and a series resistor with resistance R_s, is equivalent to a circuit consisting of a current source with current V_s/R_s and a parallel resistor with resistance R_s.

- Similarly, a circuit consisting of a current source with current I_s and a parallel resistor with resistance R_s is equivalent to a circuit consisting of a voltage source with voltage I_sR_s and a series resistor with resistance R_s.

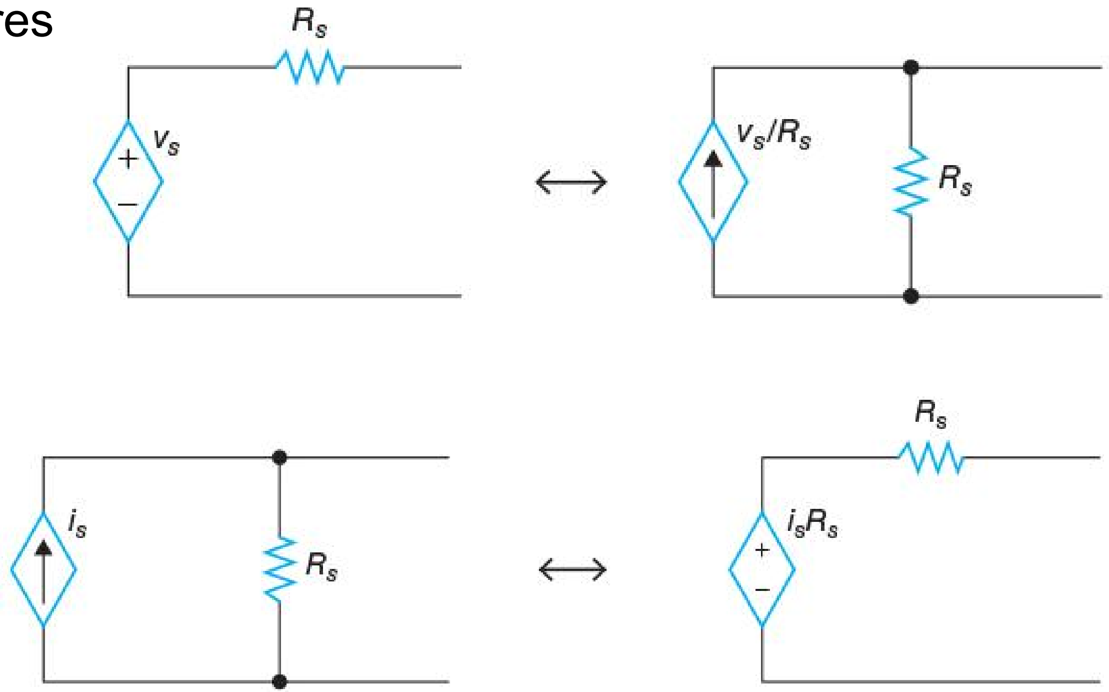
- Equivalence means that the circuits have the same open-circuit voltage across *a* and *b*, the same short-circuit current through *a* and *b*, and the same resistance looking into the circuit from *a* and *b* after deactivating the source.



Source Transformation

- The source transformations apply to dependent sources as well.

- The equivalence of a dependent voltage source and a series resistor, and a dependent current source and a parallel res



Source Transformation

- We are interested in finding V_o using source transformation.
  - I_s and R_1 can be transformed to a voltage source V_{s1} and a series resistor R_1 (Figure 4.23).
  - $R_a = R_1 + R_2 = 2k\Omega$
  - V_{s1} and R_a can be transformed to a current source I_{s1} and a parallel resistor R_a (Figure 4.24).

    $I_{s1} = V_{s1}/R_a = 1.5$ mA
  - V_s and R_3 can be transformed to a current source I_{s2} and a parallel resistor R_3 (Figure 4.24).

    $I_{s2} = V_s/R_3 = 2$ mA

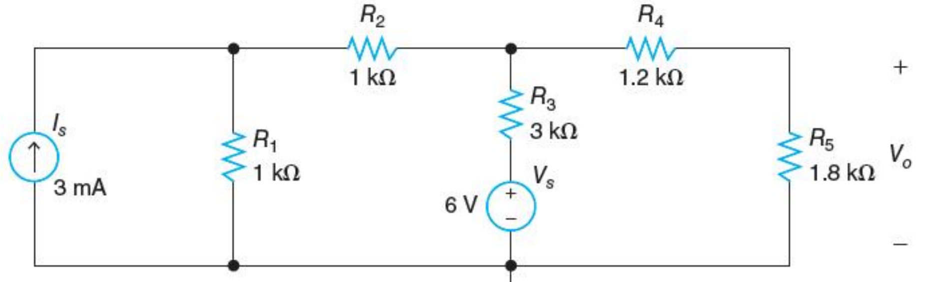


FIGURE 4.22

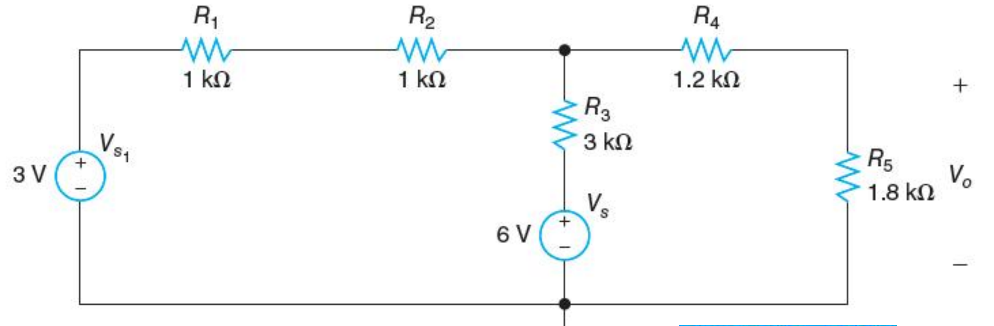


FIGURE 4.23

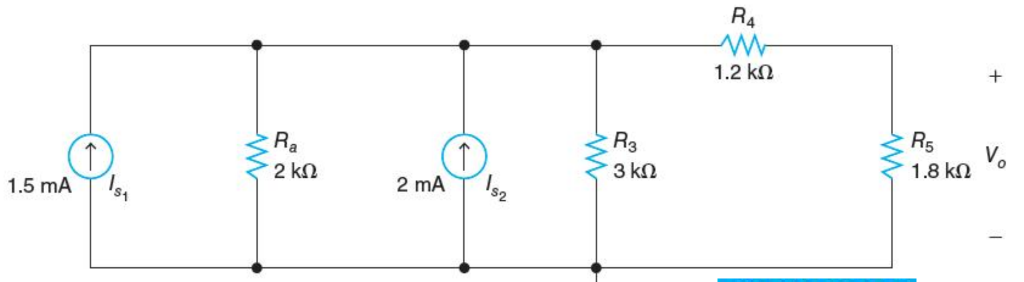
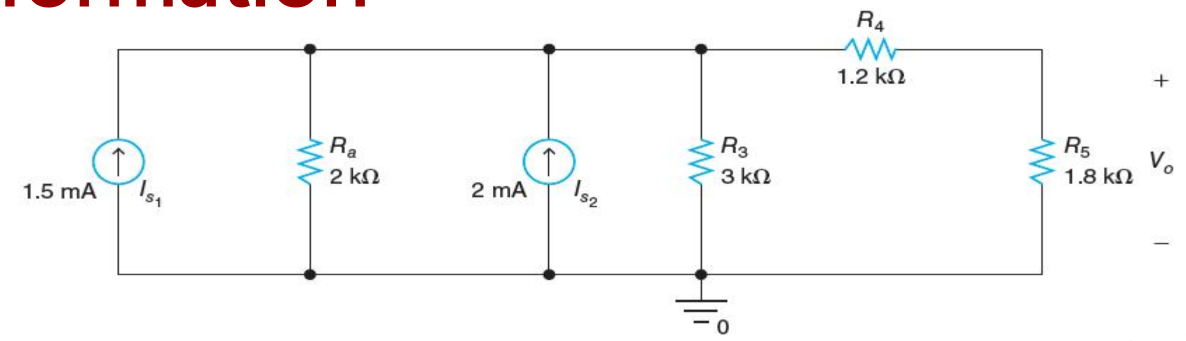


FIGURE 4.24

Source Transformation



- The two current sources can be combined into one current source with current

  $I_{s3} = I_{s1} + I_{s2} = 3.5$ mA,

  and parallel resistors R_a and R_3 can be combined into an equivalent resistor with resistance

  $R_b = R_a \,||\, R_3 = 1.2\ k\Omega$.

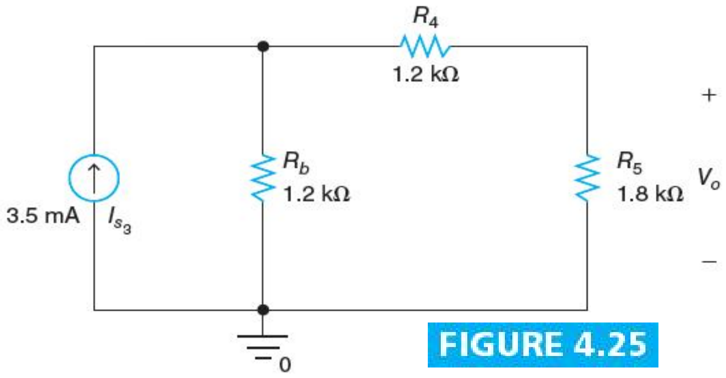


FIGURE 4.25

- I_{s3} and R_b can be transformed to a voltage source with voltage $V_{s2} = R_b I_{s3} = 4.2$ V and a series resistor R_b.

- According to the voltage divider rule, we obtain

  $V_o = V_{s2} \times R_5/(R_b + R_4 + R_5)$
  $= 4.2\ V \times 1.8/(1.2 + 1.2 + 1.8) = 1.8$ V

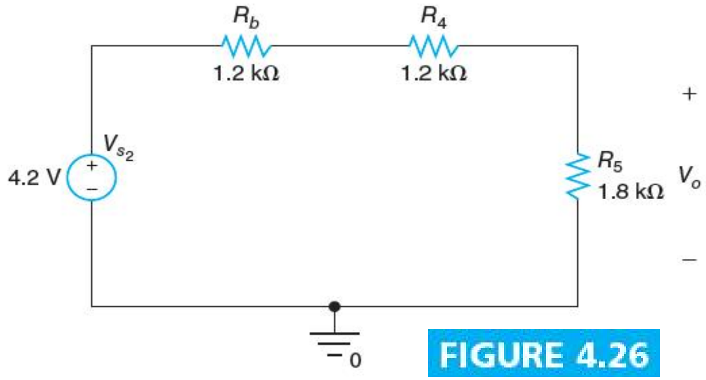


FIGURE 4.26

Source Transformation

- We are interested in finding V_o in the circuit.

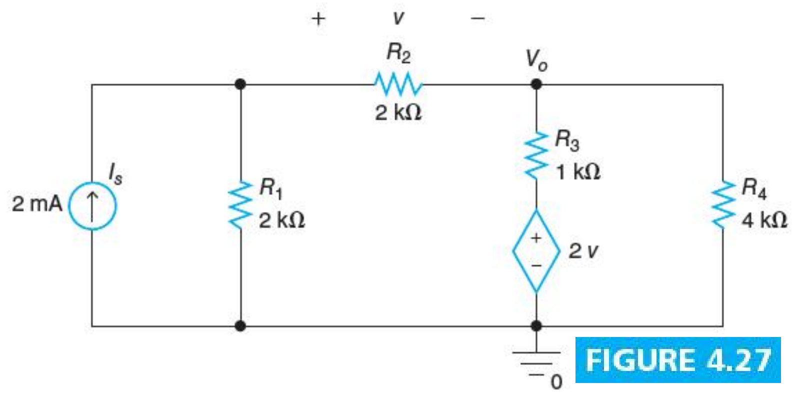


FIGURE 4.27

  - I_s and R_1 are transformed to V_{s1} with voltage 4 V and a series resistor R_1 (Figure 4.29).
  - The voltage source 2v and series resistor R_3 can be transformed to a current source with current $2v/R_3$ and a parallel resistor R_3.
  - The equivalent resistance of the parallel connection of R_3 and R_4,

$$R_a = R_3 \parallel R_4 = 0.8\ \text{k}\Omega.$$

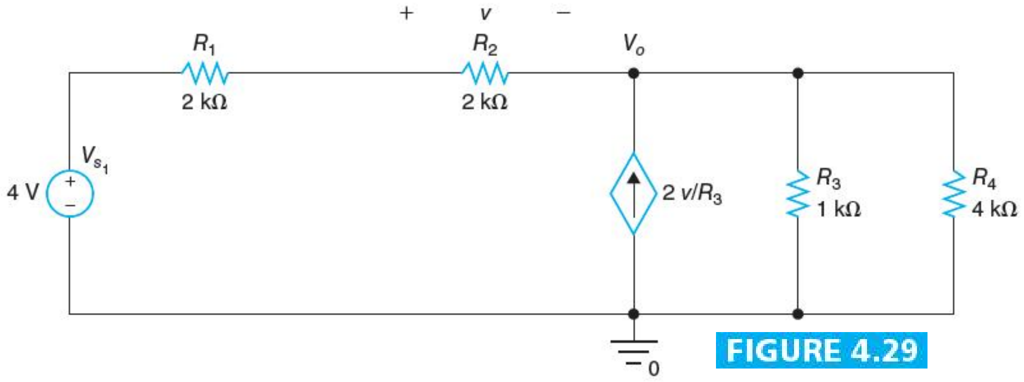
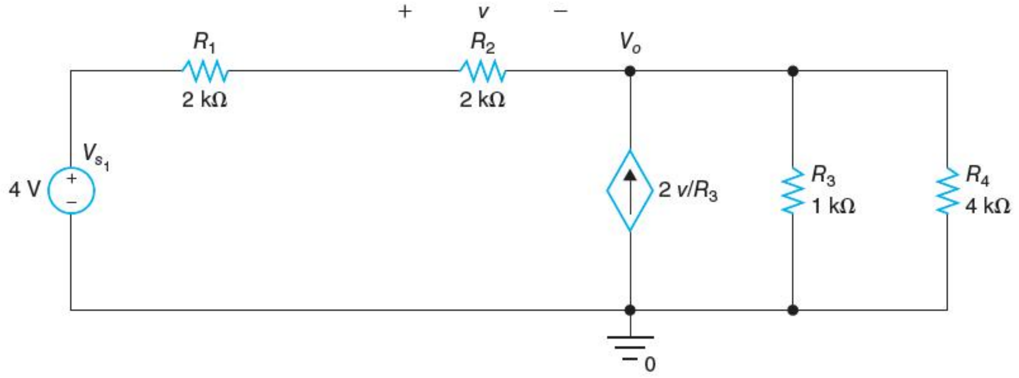


FIGURE 4.29

Source Transformation



- The current source with current $2v/R_3$ and a parallel resistor R_a can be transformed to a voltage source with voltage $2vR_a/R_3$ and a series resistor R_a (Figure 4.30).

- Collecting the voltage drops around the mesh, we obtain:

  $– 4 + 2000i + 2000i + 800i + 2(2000i)800/1000 = 0$

  $\Rightarrow i = 4/8000 = 0.5$ mA

- The voltage V_o is given by

  $V_o = V_s – 2000i – 2000i$

  $V_o = 4$ V – 1 V – 1 V = 2 V

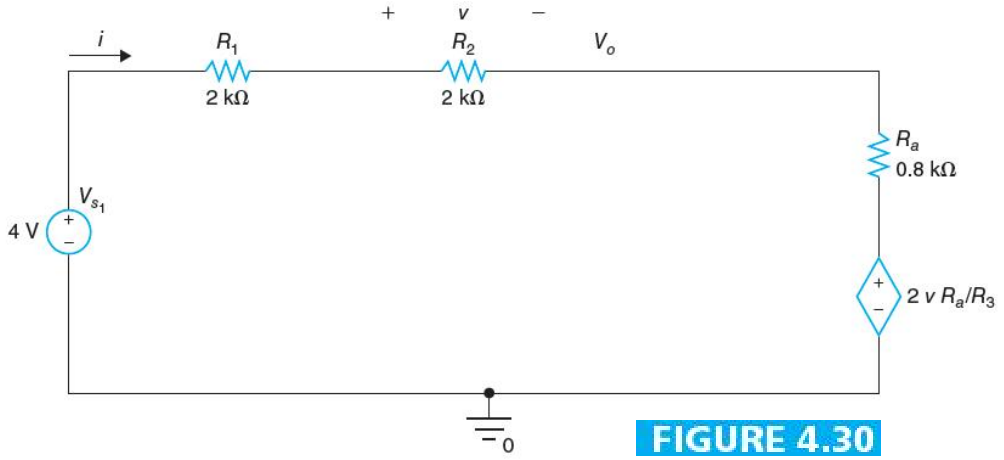


FIGURE 4.30

EXAMPLE 4.4

- Use source transformation to find V_1 in the circuit.

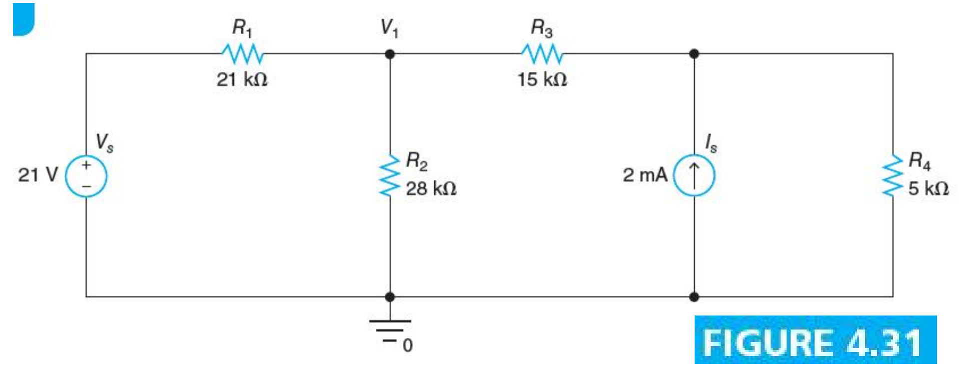


FIGURE 4.31

- V_s and R_1 are transformed to $I_{s1} = V_s/R_1 = 1$ mA and R_1 (Figure 4.32).

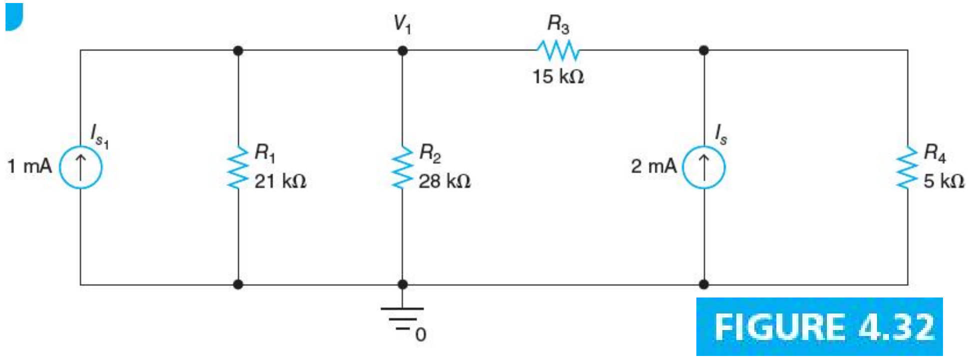


FIGURE 4.32

- Let $R_a = R_1 \parallel R_2 = 12\ \text{k}\Omega$. Then, the parallel connection of R_1 and R_2 can be replaced by R_a.

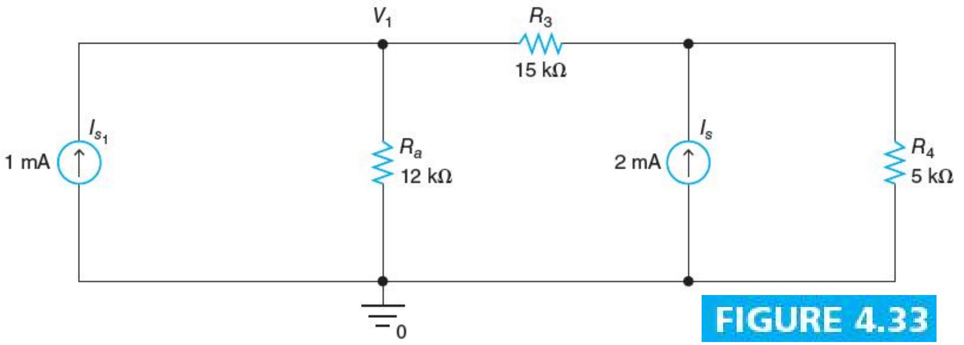


FIGURE 4.33

EXAMPLE 4.4

- I_s and R_4 are transformed to $V_{s1} = R_4 I_s = 10$ V and R_4 (Figure 4.34).
- Let $R_b = R_3 + R_4 = 20\ \text{k}\Omega$.
- V_{s1} and R_b are transformed to $I_{s2} = V_{s1}/R_b = 0.5$ mA and R_b in parallel (Figure 4.35).
- $R_c = R_a \,||\, R_b = 7.5\ \text{k}\Omega$
- $I_{s3} = I_{s1} + I_{s2} = 1.5$ mA
- $V_1 = R_c I_{s3} = 11.25$ V

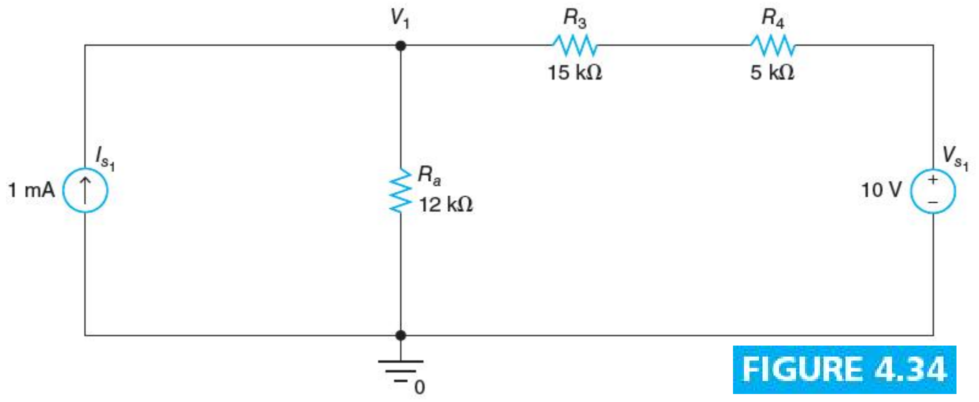


FIGURE 4.34

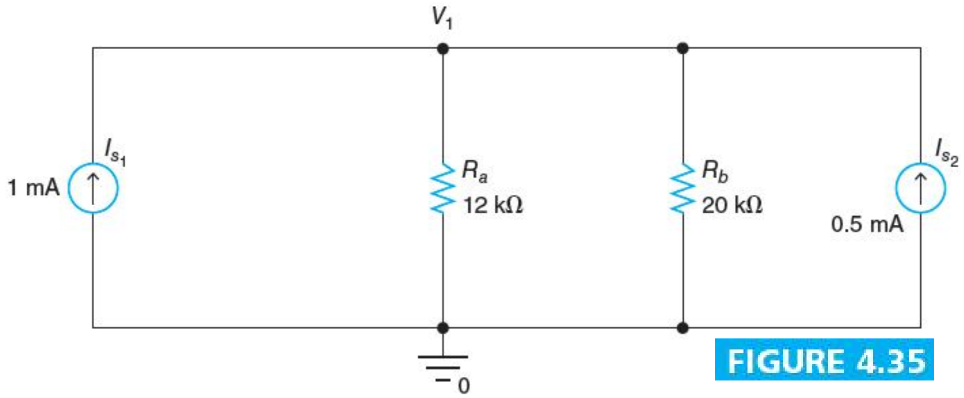


FIGURE 4.35

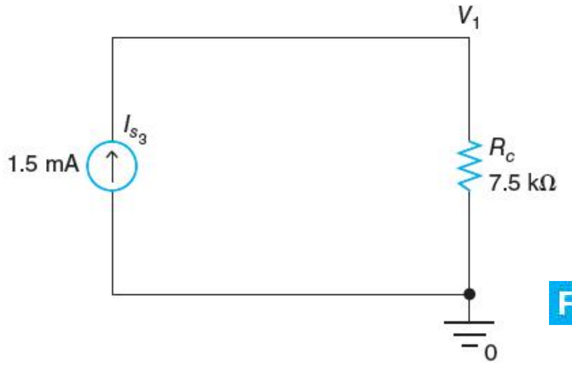


FIGURE 4.36

EXAMPLE 4.5

- Use source transformation to find V and I for the circuit.

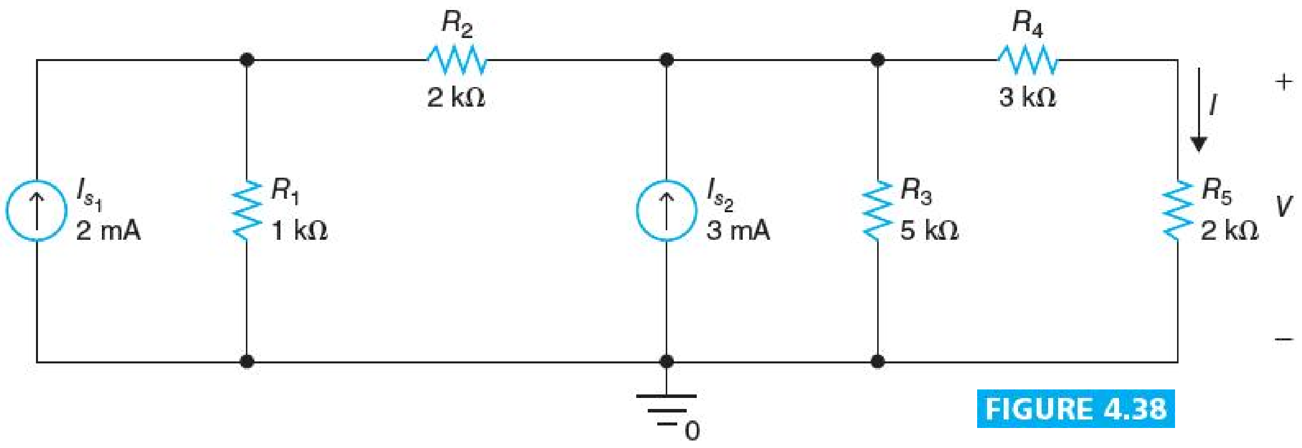


FIGURE 4.38

EXAMPLE 4.6

- Use source transformation to find V_1.
  - Transform the VCCS and parallel resistor R_3 to a VCVS with voltage
    $$0.005v \times 1000 = 5v$$
    and a series resistor R_3 (Figure 4.46).
  - $R_4 = R_2 + R_3 = 4\ k\Omega$
  - Transform VCVS and series resistor R_4 to VCCS with current
    $$5v/R_4 = 0.00125v$$
    and parallel resistor R_4 (Figure 4.47).

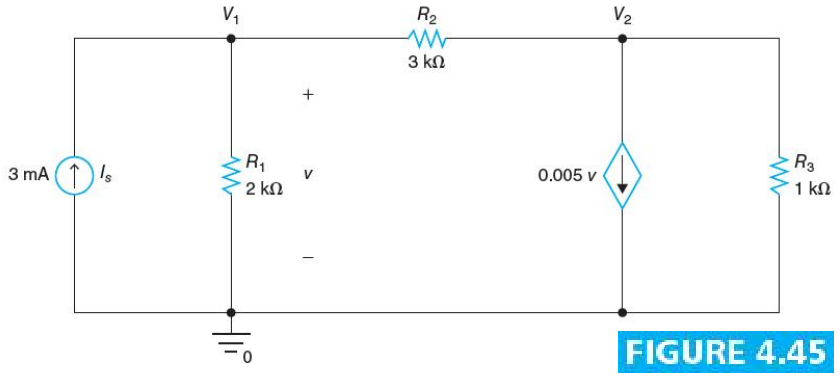


FIGURE 4.45

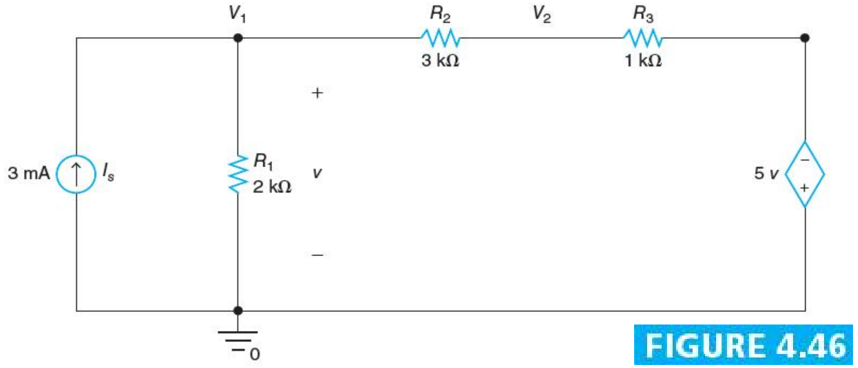


FIGURE 4.46

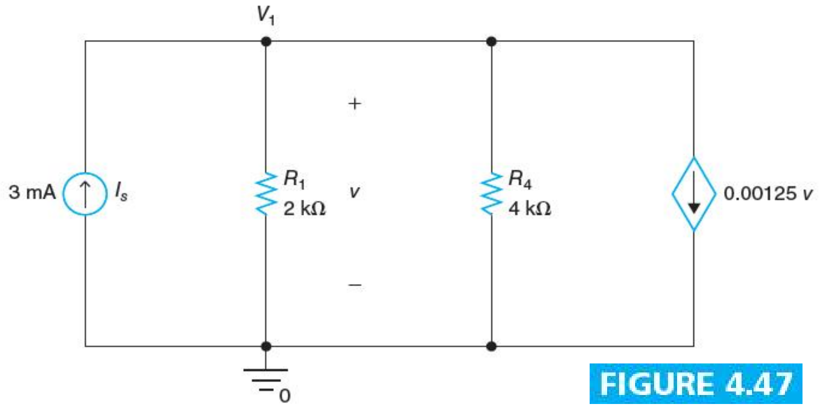


FIGURE 4.47

EXAMPLE 4.6

- $R_{eq} = R_1 \parallel R_4 = 1.3333\ \text{k}\Omega$

- Adding the two currents:

  $0.003 - 0.00125v$

- $v = (0.003 - 0.00125v)\times 1333.3333 = 4 - 1.6667v$

- $v = V_1 = 4/2.66667\ \text{V} = 1.5\ \text{V}$

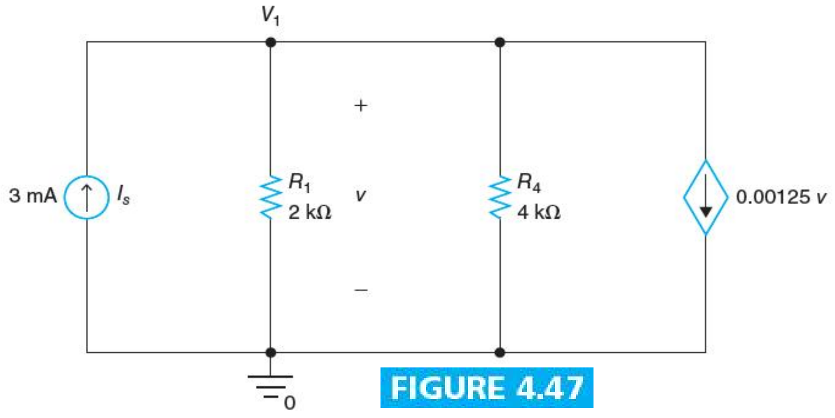


FIGURE 4.47

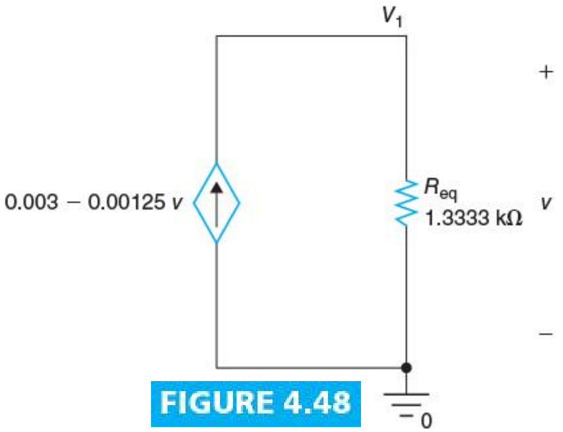


FIGURE 4.48

Summary

- Superposition principle is an effective way to analyse the circuits and tracing the impact of individual sources.

- Source transformations make design easier by providing a way to effectively simplify the circuit.

- What will we study in next lecture.