

### Lab 3 – Transients and Time Constants

In this laboratory session, you will characterise the transient behaviour of circuits involving a reactive element such as capacitors and inductors. In particular, you will look at how the choice of the capacitor affects the magnitude of the current flowing through the components and observe the charging and discharging behaviour as the duty cycle of the input voltage square is changed.

Remember that by transients we mean the analysis of voltages and currents across components following an abrupt, sudden change in the source of voltage or current. The study of transients requires solving differential equations, as highlighted in lectures. Let us have a look at the RC circuit in the figure below.

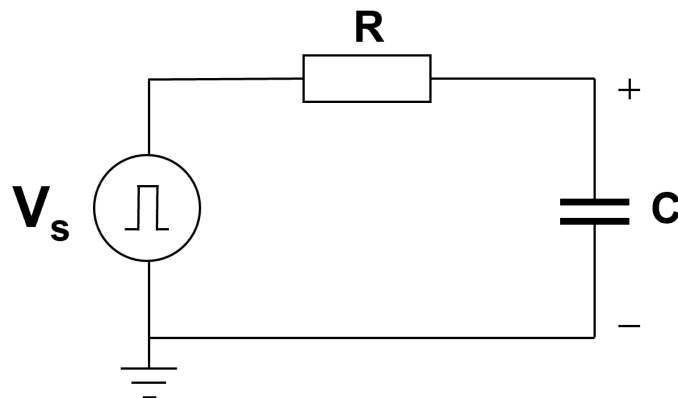


Figure 3-1 RC Circuit

The differential equation that describes this circuit is given below (it is the combination of Kirchhoff's voltage law and the equation for the current in a capacitor).

$$V_s - V_c - RC \frac{dV_c}{dt} = 0$$

We will see at the lectures that if we assume the voltage source to drop suddenly from a certain level  $K$  V to 0 V, the voltage across the capacitor will have the following expression, where  $\tau$  is the time constant of the circuit and is equal to  $RC$ .

$$V_c(t) = K e^{-t/\tau}$$

1. Recall the theory on capacitors and write the expression of the current flowing through a capacitor below:

$$I = \underline{\hspace{5cm}}$$

2. Calculate the time constant of the circuit shown in the figure above if  $R = 10 \text{ k}\Omega$  and  $C = 20 \text{ nF}$ . Write the value below:

$$\tau = \underline{\hspace{5cm}}$$

3. Set up a **PSpice simulation** for the circuit shown in the figure in the previous page. The pulsed voltage source should have an initial voltage ( $V1$ ) of 0 V, and the change should be to 4 V ( $V2$ ). The rise time (TR), fall time (TF), and delay time (TD) for the voltage source should be set to 0s. Set the period of the square wave (PER) to 10 times  $\tau$  and the pulse width (PW) to  $5\tau$ .

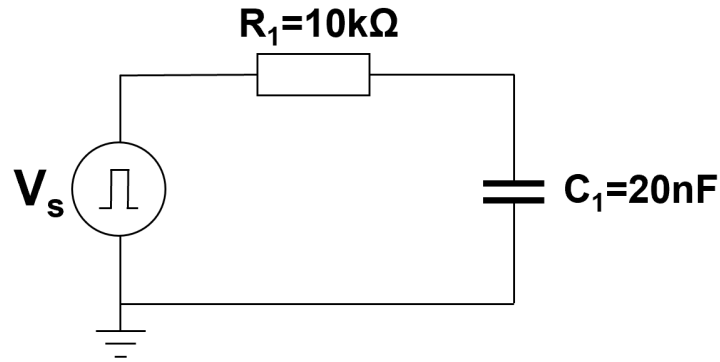


Figure 3-2 RC Circuit in PSpice

4. Run two transient simulations where you plot the voltage across the capacitor and the current through the capacitor on the same plot. Multiply the current by some factor so that the amplitude of the current can be easily read from the graph.
  - i. The first simulation should be run using the values  $R = 10 \text{ k}\Omega$  and  $C = 20 \text{ nF}$ .
  - ii. The second simulation should be run using  $R = 1 \text{ k}\Omega$  and  $C = 0.2 \text{ }\mu\text{F}$ .
  - iii. The time span for the plots should be  $20 \tau$ . The step ceiling should be  $< 0.1 \tau$  so that the rise and fall of the current is clearly shown.
5. For each of these two simulations, complete the following tables by reading values from the curves as the voltage across the capacitor falls from 4 V to 0 V during the transient. Please indicate the unit of measurement that you are using for the current.

Voltage across C [V]	Time	Current through C
3.75		
3.5		
2.5		
1.5		
0.5		
0.25		

Voltage across C [V]	Time	Current through C
3.75		
3.5		
2.5		
1.5		
0.5		
0.25		

6. Repeat the simulations, changing the duty cycle of the voltage pulse to be 0.1 and then 0.9. What do you observe? Make some notes here.

For a duty cycle of 0.1, the capacitor cannot be fully charged but discharges completely; for a duty cycle of 0.9, the capacitor cannot be fully discharged but charges completely.

Let us have a look at the RL circuit in the figure below.

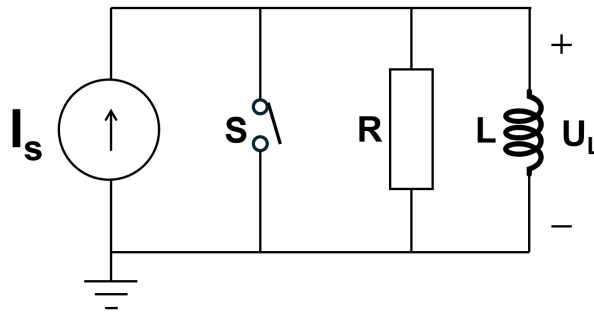


Figure 3-3 RL Circuit

The differential equation that describes this circuit is given below. Please double-check with a demonstrator if you cannot figure out where this comes from (it is the combination of Kirchhoff's current law and the equation for the voltage in an inductor).

$$I_s - \frac{L dI_L}{R dt} - I_L = 0$$

We have discussed during the lectures that if we assume the current source to drop suddenly from a certain level  $K$  A to 0 A, the voltage across the inductor will have the following expression, where  $\tau$  is the time constant of the circuit and is equal to  $L/R$ .

$$I_L(t) = K e^{-t/\tau}$$

7. Calculate the time constant of the circuit shown in the figure above if  $R = 1 \text{ k}\Omega$  and  $L = 0.2 \text{ H}$ . Write the value below.

$\tau =$  \_\_\_\_\_

8. Set up a **PSpice simulation** for the circuit shown in the figure in the previous page. The pulsed voltage source should have an initial current ( $I_1$ ) of 0 A, and the change should be to 4 mA ( $I_2$ ). The rise time (TR), fall time (TF), and delay time (TD) for the current source should be set to 0s. Set the period of the square wave (PER) to 10 times  $\tau$  and the pulse width (PW) to  $5\tau$ . And observe the simulation results.

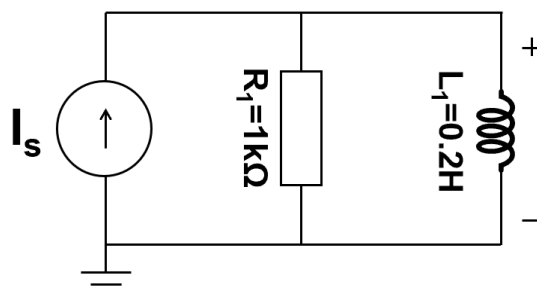


Figure 3-4 RL Circuit in PSpice

9. Now compare the similarities and differences between RC and RL transient circuits.

10. Now **build the circuit** on your breadboard using  $R = 10\text{ k}\Omega$  and  $C = 20\text{ nF}$  and an arbitrary waveform generator for your input voltage. In the generator, set the duty cycle and period of the square wave to 0.5 and  $10\tau$ , respectively. Set 4 V as peak value for our square wave.
11. Measure the actual value of the resistance of  $R$  using a multimeter and note it here

$R =$  \_\_\_\_\_

12. Measure the voltage across the capacitor using the oscilloscope. You will need to:
- Adjust and scale the waveform on the oscilloscope such that a complete exponential decay waveform starts from the upper-left corner of the screen and ends at the lower-right corner.
  - Store the image as a file to be exported from the oscilloscope.
  - Export the data as a .csv file (see lab 1 if you need to). Import the data into Excel. Plot a portion of the data that shows how the voltage across the capacitor decreases, as the voltage from the pulsed source changes from 4 V to 0 V. Using TRENDLINES, find the coefficient for the exponential fit (<https://support.office.com/en-us/article/Add-change-or-remove-a-trendline-in-a-chart-fa59f86c-5852-4b68-a6d4-901a745842ad>).
  - Calculate the value of the capacitor based upon your measured resistance of  $R$  (you can measure it using the digital multimeter), and the coefficient found from the exponential fit.

### **Hints: Fitting data by Excel**

The goal of data fitting is to find a curve that best represents the trend of data point distribution. The least squares method is a commonly used fitting technique, whose core idea is to determine a curve such that the sum of the squares of the vertical distances from all data points to the curve is minimised.

#### *Operation Instructions:*

- Input Data: Enter the data for the independent variable ( $x$ ) and dependent variable ( $y$ ) in an Excel worksheet, ensuring the data is neatly arranged in columns or rows.
- Create a Scatter Plot: Select the cell range containing the  $x$  and  $y$  values, click the **Insert** tab in the menu bar, and choose **Scatter** (under the "Charts" group).
- Add a Trendline: Right-click any data point on the generated scatter plot, then select **Add Trendline**.
- Set the Fitting Type: In the pop-up **Format Trendline** pane, under **Trendline Options**, expand the **Type** dropdown menu and select **Exponential**.
- Display the Formula and R-Squared Value: Check the boxes for **Display Equation** on chart and **Display R-squared value** on chart. This will show the equation of the fitted exponential curve and the  $R^2$  (goodness of fit) value on the chart. An  $R^2$  value closer to 1 indicates a better fitting effect.
- Adjust Chart Format (Optional): If needed, further format the chart—such as setting axis titles, scale ranges, or gridlines—to make the chart clearer and more readable.

You can make some notes in the blank space below. To have this lab session signed off by the GTAs, you need to show your plots in Excel and the calculations for the value of the capacitor  $C$ . How does your calculated value compare with the nominal value?