Chapter 29

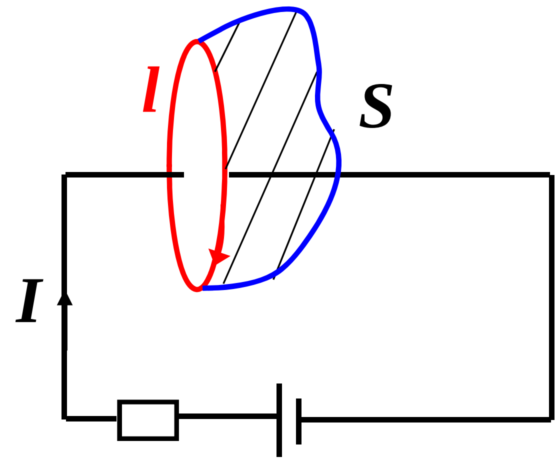
Maxwell’s Equations and Electromagnetic Waves

- Changing Electric Fields Produce Magnetic Fields
- Generalized Ampere’s Law
- Maxwell’s Equations
- Electromagnetic Waves

§29-1 Changing Electric Fields Produce Magnetic Fields

1. Problem encountered when using Ampere's law

For steady current, Ampere's law is always valid:



$$\int_l \vec{B}\cdot \mathrm{d}\vec{l} = \mu_0 I$$

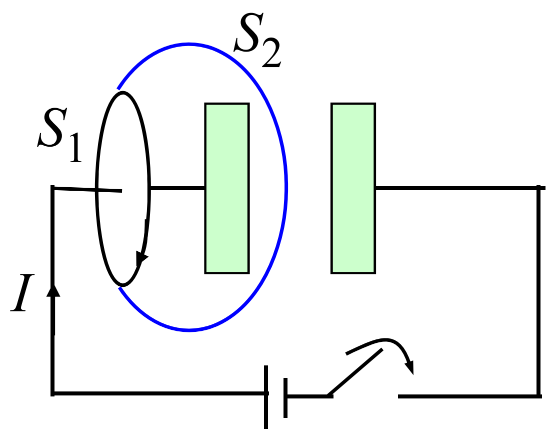
For varying current, consider S_1, Ampere's law:

$$\int_l \vec{B}\cdot \mathrm{d}\vec{l} = \mu_0 I$$

consider S_2, Ampere's law:

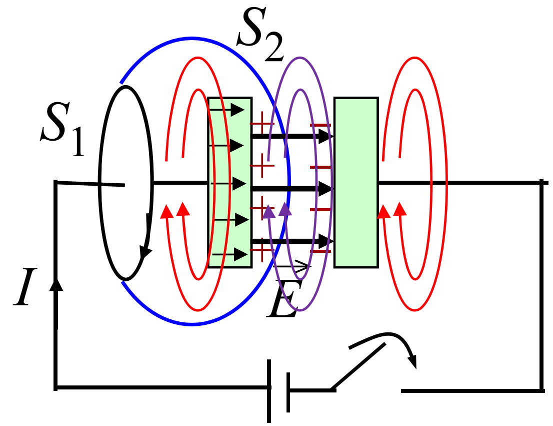
$$\int_l \vec{B}\cdot \mathrm{d}\vec{l} = 0$$

?



Maxwell:

Discontinuity of the current $\rightarrow$ contradiction



Net current out of the volume enclosed by S_1 and S_2:

$$\oint_{S_1+S_2} \vec{j}\cdot \mathrm{d}\vec{S} = -I \neq 0$$

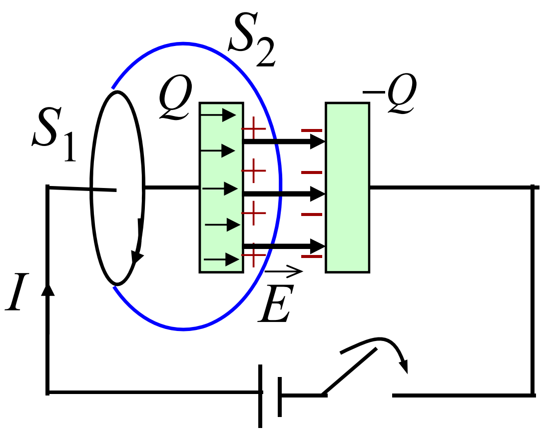
Which result is right of Ampere's law?

$$\oint \vec{B}\cdot \mathrm{d}\vec{l} = \begin{cases} \mu_0 I \rightarrow S_1 & ✓ \\ 0 \rightarrow S_2 & ✗ \end{cases}$$

In the area where the current is disconnected, varying electric field exists.

Maxwell: varying $\vec{E}$ is type of current, then the total current becomes continuous.

2. Displacement current



Electric field between plates: $E = \dfrac{\sigma}{\varepsilon_0}$

Conduction current:

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{\mathrm{d}\sigma}{\mathrm{d}t}S, \qquad j = \frac{I}{S} = \frac{\mathrm{d}\sigma}{\mathrm{d}t} = \varepsilon_0 \frac{\mathrm{d}E}{\mathrm{d}t}$$

Displacement current:

$$\vec{j}_D = \varepsilon_0 \frac{\mathrm{d}\vec{E}}{\mathrm{d}t}, \qquad I_D = \varepsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}$$

Changing electric field is a kind of current. Except for not generating resistance heat, it is equivalent to conduction current when producing magnetic fields.

单选题 10分

设置

Q: Which of the following descriptions about the displacement current is correct?

A. Displacement current refers to the changing electric field

B. Displacement current is generated by a changing magnetic field

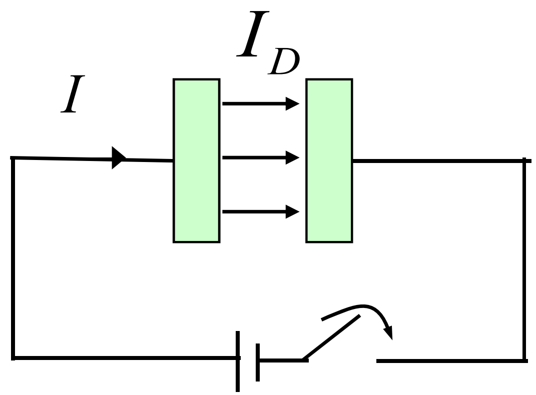
C. Displacement current has thermal effect, and follows Joule′s law

D. The magnetic effect of displacement current does not obey Ampere's law

提交

§29-2 Generalized Ampere's law

Ampere's law in a general form:



$$\oint \vec{B}\cdot \mathrm{d}\vec{l} = \mu_0 (I_c + I_D)_{encl}$$

$$= \mu_0 I_{c\_encl} + \mu_0 \varepsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}$$

(1) I_c: Conduction current, which is due to the motion of electric charges;

(2) I_D: Displacement current, which is produced by changing electric field;

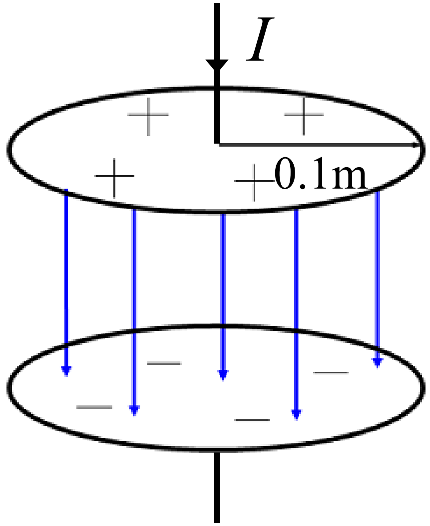
(3) Continuity of total current:

$$\oint_S (\vec{j} + \vec{j}_D)\cdot \mathrm{d}\vec{S} = 0$$

Example1: A parallel-plate capacitor is charging with $\mathrm{d}E/\mathrm{d}t = 2\times10^9 V/m\cdot s$. Determine (a) the conduction current I ; (b) magnetic field.

Solution: (a)

$$\vec{j}_D = \varepsilon_0 \frac{\mathrm{d}\vec{E}}{\mathrm{d}t}$$

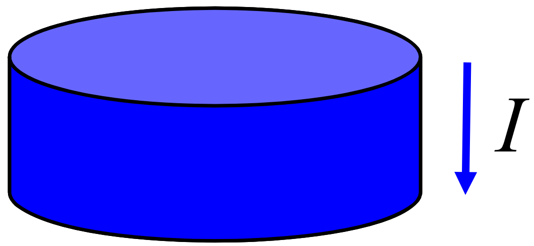


Continuity of total current:

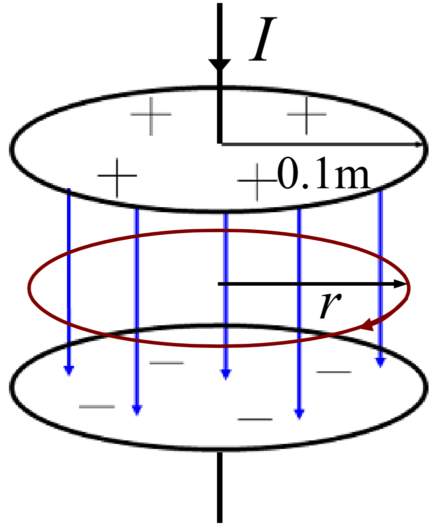
$$I = I_D = j_D S \; = \pi R^2 \varepsilon_0 \frac{\mathrm{d}E}{\mathrm{d}t}$$

$$= 5.56\times10^{-4} A$$

How the displacement current is distributed? Cylindrical symmetry



(b) magnetic field



Generalized Ampere's law:

$$B \cdot 2\pi r = \oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 (I + I_D)_{encl} = \mu_0 I_{D\_encl}$$

$$\Rightarrow \quad B = \frac{\mu_0 I_{D\_encl}}{2\pi r}$$

$$r < 0.1m: \; B = \frac{\mu_0 I_{D\_encl}}{2\pi r} = \frac{\mu_0 j_D \cdot \pi r^2}{2\pi r} = \frac{\mu_0 j_D \cdot r}{2} = \frac{\mu_0 \varepsilon_0}{2} \frac{\mathrm{d}E}{\mathrm{d}t} \cdot r$$

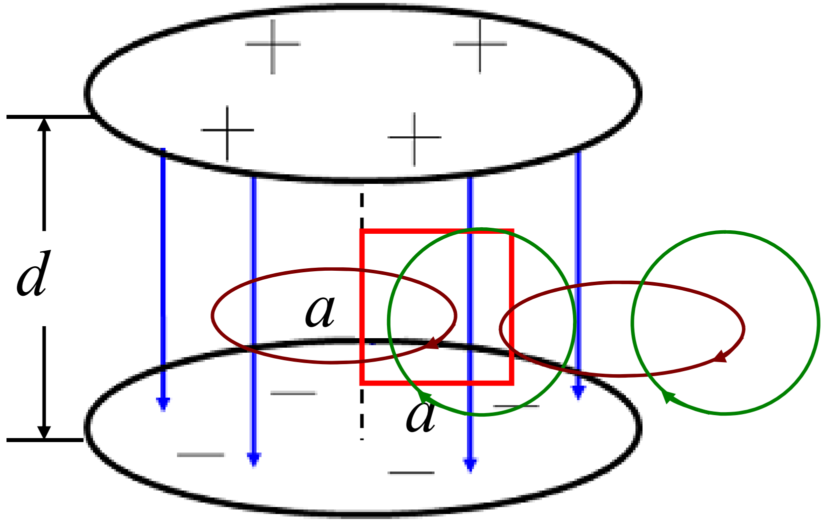
$$r > 0.1m: \; B = \frac{\mu_0 I_{D\_encl}}{2\pi r} = \frac{\mu_0 I_D}{2\pi r}$$

Question: The voltage on a capacitor is changing as $V = V_0 \cos \omega t$. What is the EMF on the square coil inside the capacitor?

$$j_D = \varepsilon_0 \frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\varepsilon_0}{d} \frac{\mathrm{d}V}{\mathrm{d}t}$$

$$B = \frac{\mu_0 j_D}{2} r \quad \Rightarrow \quad \Phi_m$$

$$\varepsilon = -\frac{\mathrm{d}\Phi_m}{\mathrm{d}t}$$



§29-3 Maxwell's equations

To generate electric field
- Electric charge, $\vec{E}_s$ (electrostatic field)
- Changing magnetic field, $\vec{E}_i$ (induced field)

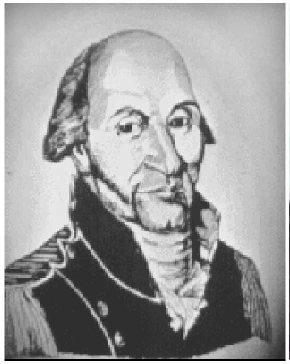
To generate magnetic field
- Conduction current, $\vec{B}_s$ (static magnetic field)
- Changing electric field, $\vec{B}_i$ (induced field)

Total field:

$$\vec{E} = \vec{E}_s + \vec{E}_i$$

$$\vec{B} = \vec{B}_s + \vec{B}_i$$

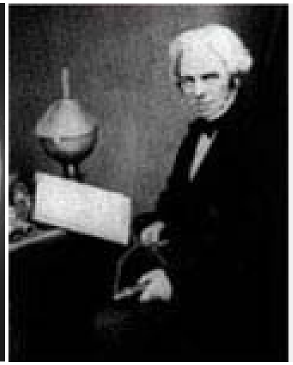
Static Field	Induced Field	Total Field
$\oint \vec{E}_s \cdot \mathrm{d}\vec{S} = \dfrac{Q}{\varepsilon_0}$	$\oint \vec{E}_i \cdot \mathrm{d}\vec{S} = 0$	$\oint \vec{E} \cdot \mathrm{d}\vec{S} = \dfrac{Q}{\varepsilon_0}$
$\oint \vec{E}_s \cdot \mathrm{d}\vec{l} = 0$	$\oint \vec{E}_i \cdot \mathrm{d}\vec{l} = -\dfrac{\mathrm{d}\Phi_m}{\mathrm{d}t}$	$\oint \vec{E} \cdot \mathrm{d}\vec{l} = -\dfrac{\mathrm{d}\Phi_m}{\mathrm{d}t}$
$\oint \vec{B}_s \cdot \mathrm{d}\vec{S} = 0$	$\oint \vec{B}_i \cdot \mathrm{d}\vec{S} = 0$	$\oint \vec{B} \cdot \mathrm{d}\vec{S} = 0$
$\oint \vec{B}_s \cdot \mathrm{d}\vec{l} = \mu_0 I_C$	$\oint \vec{B}_i \cdot \mathrm{d}\vec{l} = \mu_0 I_D$ $= \mu_0 \varepsilon_0 \dfrac{\mathrm{d}\Phi_E}{\mathrm{d}t}$	$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 (I_C + I_D)$ $= \mu_0 I_C + \mu_0 \varepsilon_0 \dfrac{\mathrm{d}\Phi_E}{\mathrm{d}t}$



Coulomb
France

Oersted
Denmark

Ampere
France



Faraday
England

Lenz
Russia

Maxwell
Scotland

All electric and magnetic phenomena could be described using only 4 equations involving electric and magnetic fields —— Maxwell's equations

→ Culmination: Electromagnetic waves

Integral Form of Maxwell's Equations:

$$\oint \vec{E}\cdot \mathrm{d}\vec{S} = \frac{Q}{\varepsilon_0}$$

$$\oint \vec{B}\cdot \mathrm{d}\vec{S} = 0$$

$$\oint \vec{E}\cdot \mathrm{d}\vec{l} = -\frac{\mathrm{d}\Phi_m}{\mathrm{d}t}$$

$$\oint \vec{B}\cdot \mathrm{d}\vec{l} = \mu_0 I_C + \mu_0\varepsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}$$

① source of E

② no magnetic charges/monopoles

③ changing $\vec{B} \rightarrow \vec{E}$

④ changing $\vec{E} \rightarrow \vec{B}$

Differential Form of Maxwell's Equations:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

(1) Basic equations for all electromagnetism;

(2) As fundamental as Newton's laws;

(3) Important outcome: electromagnetic waves.

§29-4 Electromagnetic Waves

1. Wave equations for EM wave

Free space: no charges or conduction currents.

$$\begin{cases} \nabla \cdot \vec{E} = 0 \\ \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{B} = \mu_0 \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t} \end{cases}$$

Take the curl on both sides of the third equation:

$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}(\nabla \times \vec{B}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla \times (\nabla \times \vec{E}) = \nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\nabla^2 \vec{E}$$

$$\Rightarrow \nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$ Wave equation!

You can derive the wave equation of $\vec{B}$ similarly.

2. Speed of EM wave

$$\boxed{\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \qquad \nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}}$$

3-D wave equation $\rightarrow$ 1-D (plane) wave equation (propagate along x)

$$\frac{\partial^2 \vec{E}}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}, \qquad \frac{\partial^2 \vec{B}}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

Compare with standard wave equation:

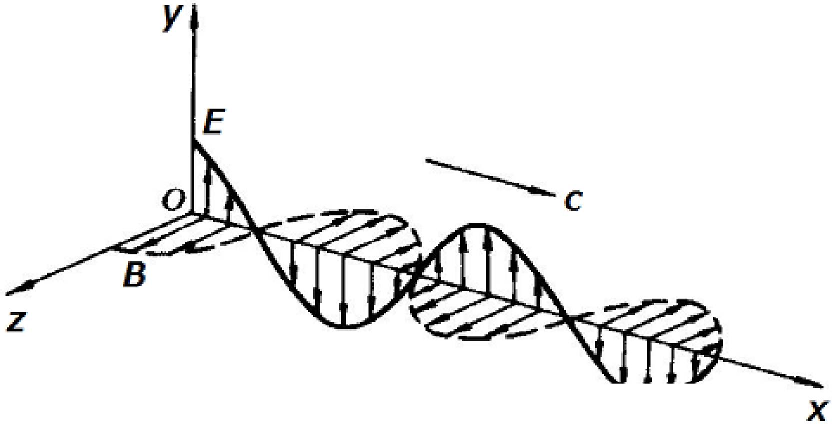
$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2} \Rightarrow v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 299792458 \pm 1 m/s \approx 3.0 \times 10^8 m/s$$

$$= c$$

3. Properties of EM wave

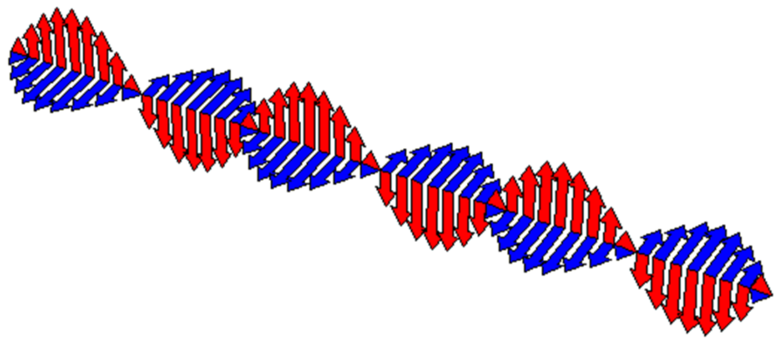
$$c^2 \frac{\partial^2 \vec{E}}{\partial x^2} = \frac{\partial^2 \vec{E}}{\partial t^2}, \qquad c^2 \frac{\partial^2 \vec{B}}{\partial x^2} = \frac{\partial^2 \vec{B}}{\partial t^2}$$

Solutions: $\begin{cases} E = E_0 \cos\omega(t - \dfrac{x}{c}) \\ B = B_0 \cos\omega(t - \dfrac{x}{c}) \end{cases}$



(1) Transverse wave

(2) In phase: $\boxed{\dfrac{E}{B} = c}$



(3) $\vec{E} \perp \vec{B}$

4. Energy in EM wave

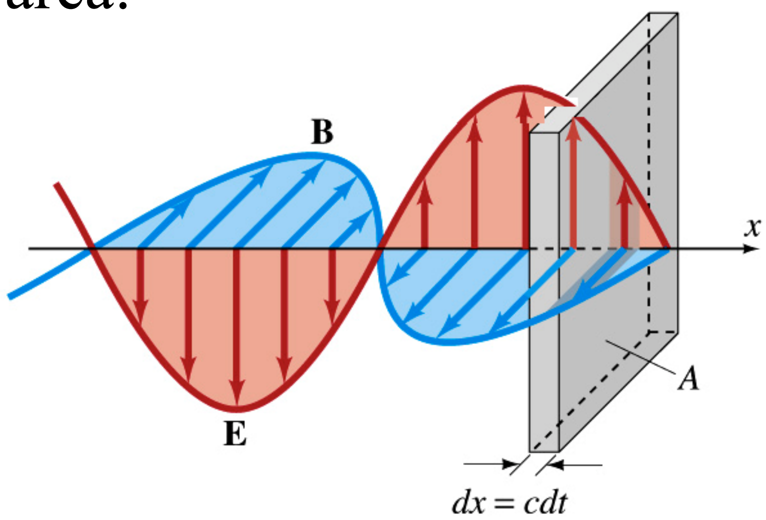
Total energy stored per unit volume in EM wave:

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0} = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}} \qquad \left(\frac{E}{B} = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right)$$

Energy transports per unit time per unit area:

$$\mathrm{d}U = u\mathrm{d}V = u \cdot A \cdot c\mathrm{d}t$$

$$\Rightarrow \quad S = \frac{\mathrm{d}U}{A \cdot \mathrm{d}t} = uc = \frac{EB}{\mu_0}$$



5. Poynting vector

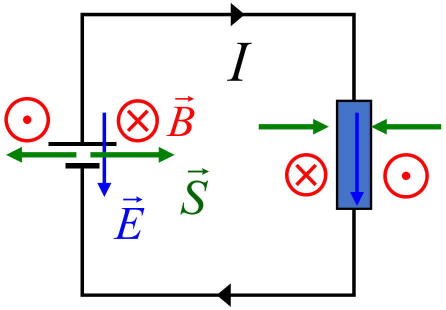
Consider the direction of energy transporting:

$$\boxed{\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})} \quad \rightarrow \text{Poynting vector}$$

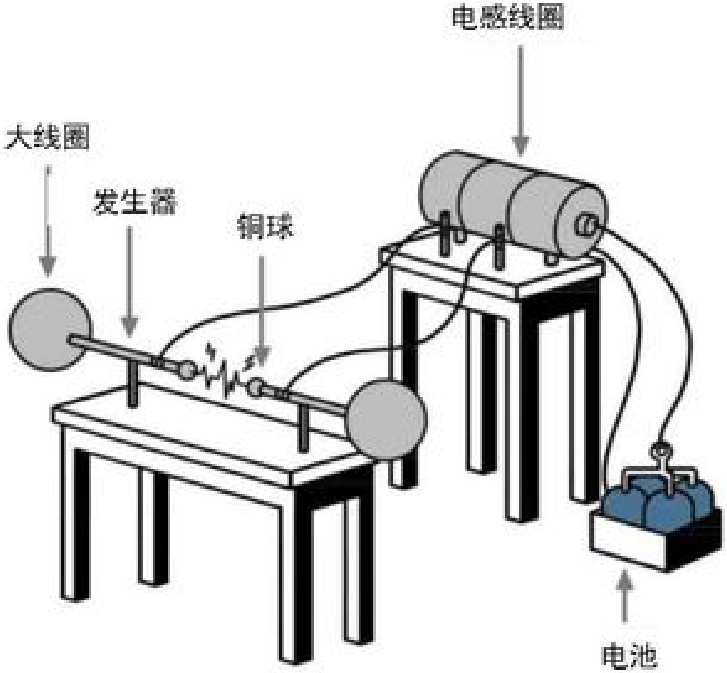
Time averaged S is intensity:

$$\boxed{\overline{S} = \frac{1}{2}\frac{E_0 B_0}{\mu_0}}$$

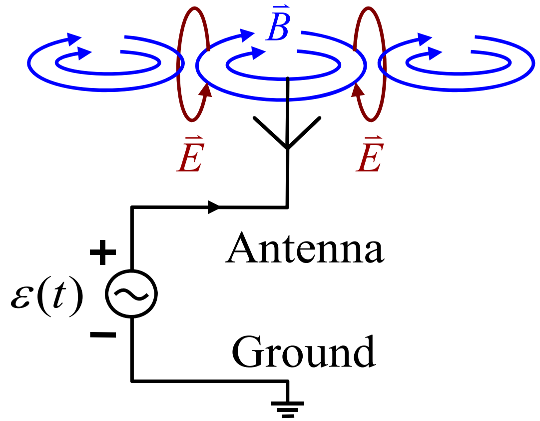
Example 2: Show the direction of energy transporting inside the battery and resistor.

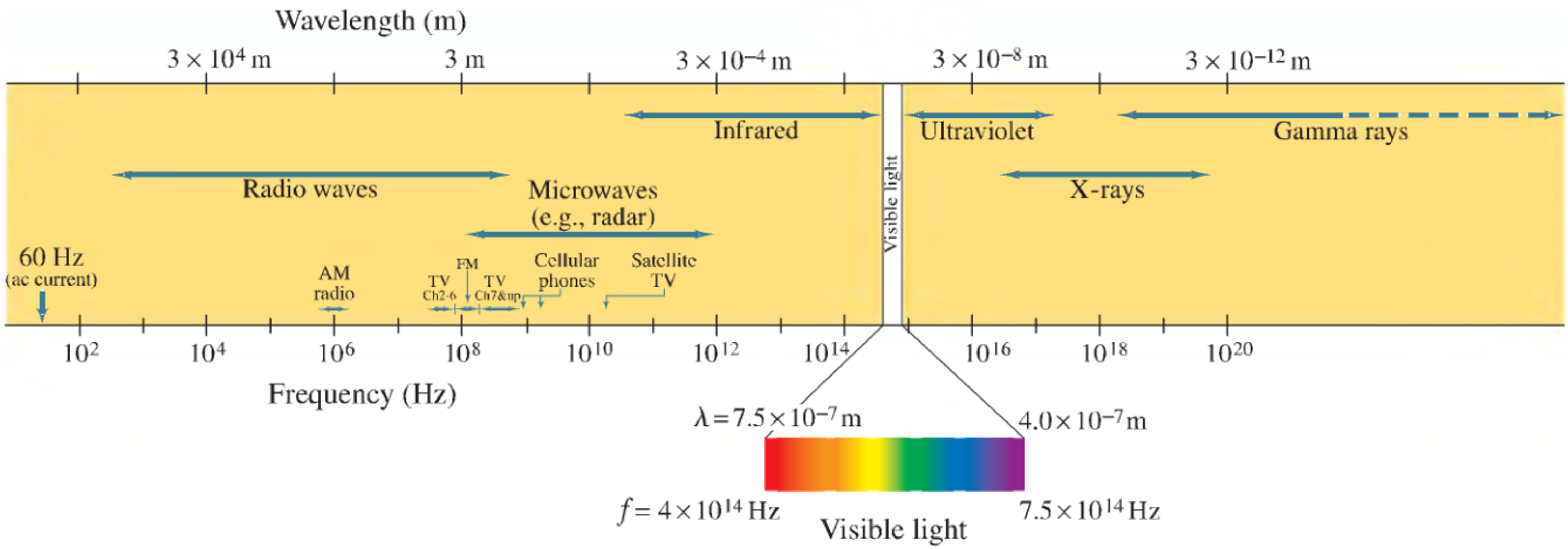


6. Hertz’s experiment



“Antenna”





Wavelength (m)
3 × 10⁴ m
3 m
3 × 10⁻⁴ m
3 × 10⁻⁸ m
3 × 10⁻¹² m
Infrared
Ultraviolet
Gamma rays
Visible light
Radio waves
Microwaves
(e.g., radar)
X-rays
60 Hz
(ac current)
AM
radio
FM
TV
Ch2-6
TV
Ch7&up
Cellular
phones
Satellite
TV
10²
10⁴
10⁶
10⁸
10¹⁰
10¹²
10¹⁴
10¹⁶
10¹⁸
10²⁰
Frequency (Hz)
λ = 7.5 × 10⁻⁷ m
4.0 × 10⁻⁷ m
f = 4 × 10¹⁴ Hz
7.5 × 10¹⁴ Hz
Visible light

Sunshine:

Example3: Radiation from the Sun reaches the Earth at a rate about 1350W/m^2. Assume it is a single EM wave, calculate E_0 and B_0.

Solution: Rate → time averaged S / intensity

$$\overline{S} = \frac{1}{2}\frac{E_0 B_0}{\mu_0} = \frac{1}{2}\varepsilon_0 c E_0^2 = \overline{u} \cdot c = \frac{1}{2} u_m c$$

$$\Rightarrow \quad E_0 = \sqrt{\frac{2\overline{S}}{\varepsilon_0 c}} = 1.01 \times 10^3 V/m$$

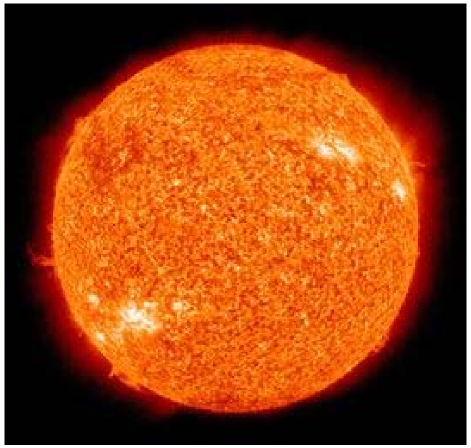
$$B_0 = \frac{E_0}{c} = 3.37 \times 10^{-6} T$$

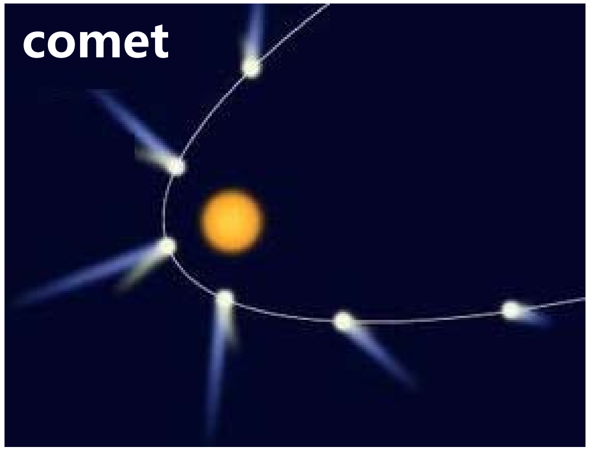
7. *Radiation pressure

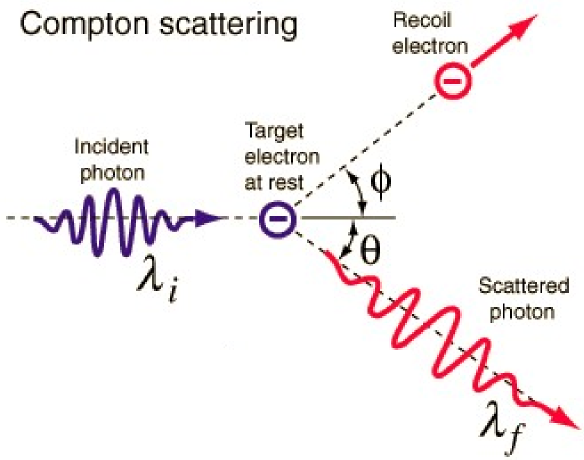
EM waves carry energy → also carry momentum

Be absorbed / reflected → radiation pressure

Absorbed: $P = \dfrac{\overline{S}}{c}$ Reflected: $P = \dfrac{2\overline{S}}{c}$







8. *Ejecting dust

Thinking: When the Sun became hot and luminous long ago, it is believed that it ejected dust particles and individual atoms out of the solar system using radiation pressure. Calculate how small the dust particle had to be in order to be ejected by comparing the radiation force with the gravitational force on the particles. Assume the particles are spherical, have a density of $2.0\times10^3 kg/m^3$, and totally absorb the Sun's radiation. The Sun has an average power output of $3.8\times10^{26} W$.

(P678, Problem 29)

Summary

1. Displacement current

$$\vec{j}_D = \varepsilon_0 \frac{\mathrm{d}\vec{E}}{\mathrm{d}t}, \qquad I_D = \varepsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}$$

2. Generalized Ampere's law

$$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 (I_c + I_D)_{encl} = \mu_0 I_c + \mu_0 \varepsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}$$

3. Continuity of total current

$$\oint_S (\vec{j} + \vec{j}_D) \cdot \mathrm{d}\vec{S} = 0$$

4. Total electromagnetic field——two way of generating field

$$\vec{E} = \vec{E}_s + \vec{E}_i \qquad \vec{B} = \vec{B}_s + \vec{B}_i$$

5. Maxwell's equations

$$\oint \vec{E}\cdot \mathrm{d}\vec{S} = \frac{Q}{\varepsilon_0}$$

$$\oint \vec{B}\cdot \mathrm{d}\vec{S} = 0$$

$$\oint \vec{E}\cdot \mathrm{d}\vec{l} = -\frac{\mathrm{d}\Phi_m}{\mathrm{d}t}$$

$$\oint \vec{B}\cdot \mathrm{d}\vec{l} = \mu_0 I_C + \mu_0\varepsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}$$

$$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla\cdot\vec{B} = 0$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times\vec{B} = \mu_0\vec{j} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

6. Electromagnetic Waves

$$c^2\frac{\partial^2\vec{E}}{\partial x^2} = \frac{\partial^2\vec{E}}{\partial t^2}, \quad c^2\frac{\partial^2\vec{B}}{\partial x^2} = \frac{\partial^2\vec{B}}{\partial t^2}$$

$$\begin{cases} E = E_0\cos\omega(t-\frac{x}{c}) \\ B = B_0\cos\omega(t-\frac{x}{c}) \end{cases}$$

$$v = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3.0\times 10^8 m/s$$

7. Properties of EM wave

(1) EM wave is transverse wave

(2) Electric field and magnetic field are in phase, and

$$\frac{E}{B} = c$$

(3) The oscillation direction of electric and magnetic fields are perpendicular to each other:

$$\vec{E} \perp \vec{B}$$

8. Energy density of EM wave

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0} = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$$

9. Poynting vector

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}), \qquad \overline{S} = \frac{1}{2}\frac{E_0 B_0}{\mu_0}$$

10. *Radiation pressure

Absorbed: $$P = \frac{\overline{S}}{c}$$

Reflected: $$P = \frac{2\overline{S}}{c}$$