Chapter 25

Magnetism

Magnets

Buckyball

clip

Compass

Magnetic Spoon
Compass in ancient China

Magnets: interact on iron objects / other magnets

1) North pole (N-pole) & south pole (S-pole)

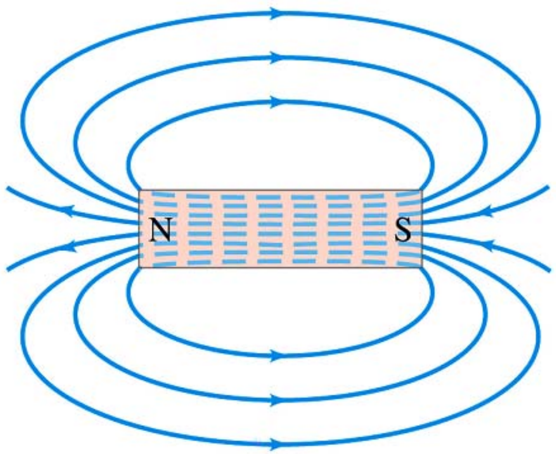
2) Opposite poles attract, like poles repel

3) Magnetic monopoles have not been found

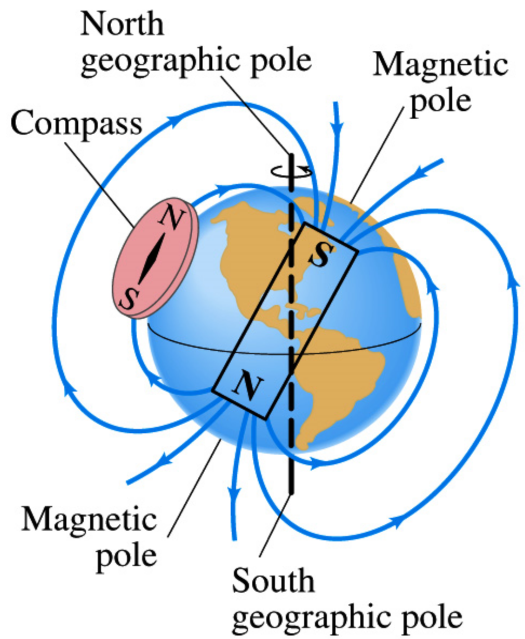
Magnetic field

Magnets interact each other by magnetic field

Magnets $\underset{\text{interact on}}{\overset{\text{create}}{\Longleftrightarrow}}$ **magnetic field**



magnetic field lines



Inner: S ⇒ N;   Outer: N ⇒ S

Currents produce magnetism

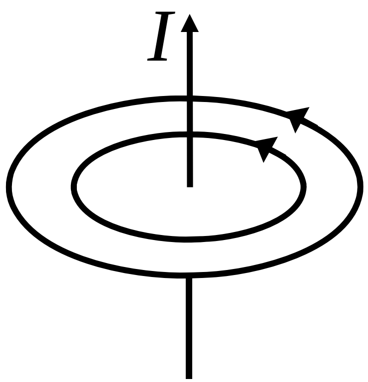
H. C. Oersted found in 1820:

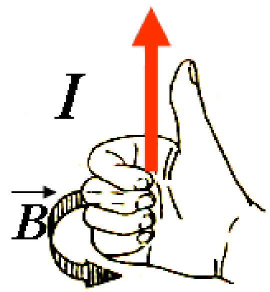
An electric current produces a magnetic field.

magnet ⟷ magnet

magnet ⟷ current

current ⟷ current





A. M. Ampère: **(molecular currents)**

Magnetism is caused by electric currents.

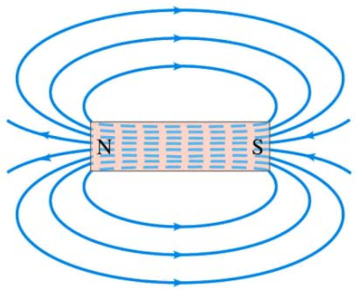
Nature of magnetism

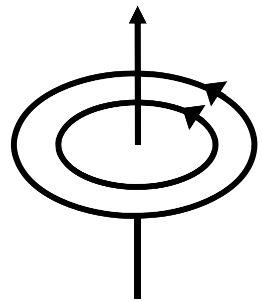
Magnetism is the interaction of electric currents or moving charges.

A. Einstein: electromagnetic field in relativity

Magnetic field:

1) Created by / interacts on currents or moving charges



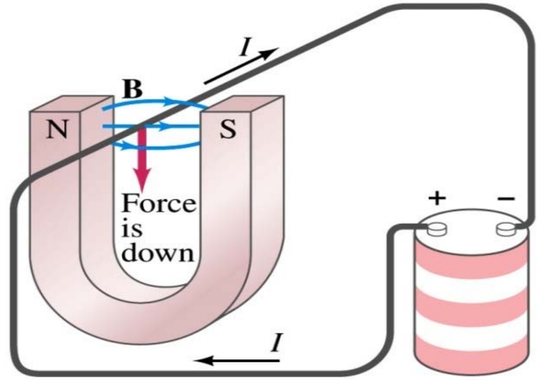


2) Closed field lines without beginning or end

Force on a current

Magnetic field exerts a force on a current (wire)

It's called Ampère force

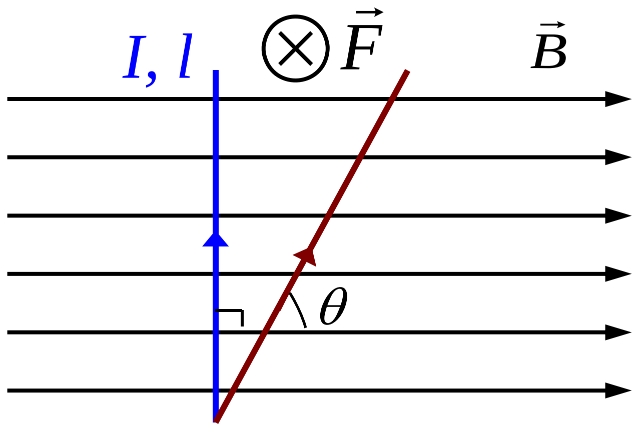


1) Uniform field:

$$F = IlB \quad \left(\vec{l} \perp \vec{B}\right)$$

or $F = IlB\sin\theta$

$$\boxed{\vec{F} = I\vec{l} \times \vec{B}}$$



You can also decide the direction of the Ampere force using your left hand.

Definition of B

Define magnetic field $\vec{B}$ **by using:** $\vec{F} = I\vec{l} \times \vec{B}$

SI unit for *B* : Tesla (T), 1 T = 1 N/A·m = 10^4 G

2) General case: nonuniform *B* & curved wire

$$\boxed{d\vec{F} = Id\vec{l} \times \vec{B}} \quad \Rightarrow \quad \boxed{\vec{F} = \int Id\vec{l} \times \vec{B}}$$

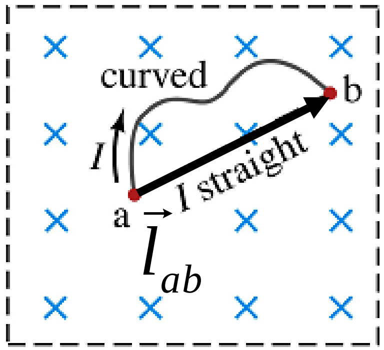
where $d\vec{l}$ **is a infinitesimal length of the wire**

integral over the current-carrying wire

F on curved wire

Example1: Uniform magnetic field $\vec{B}$. Show that any curved wire connecting points A and B is exerted by the same magnetic force.

Solution: Total magnetic force:

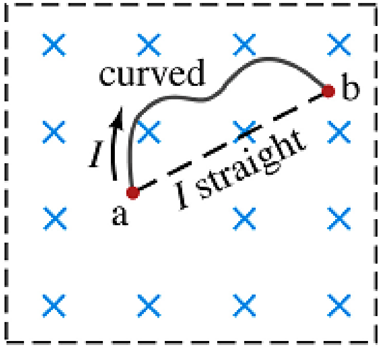


$$\vec{F} = \int I d\vec{l} \times \vec{B} = I\left(\int d\vec{l}\right) \times \vec{B}$$

$$= I\vec{l}_{ab} \times \vec{B}$$

It is equivalent to a straight wire from A to B

$$\vec{F} = \int I d\vec{l} \times \vec{B} = I\vec{l}_{ab} \times \vec{B}$$

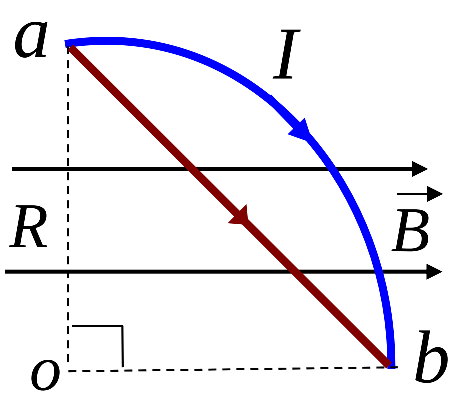


Discussion:

1) It is valid only when B is uniform

2) Total force exerted on a current loop (coil)?

3)



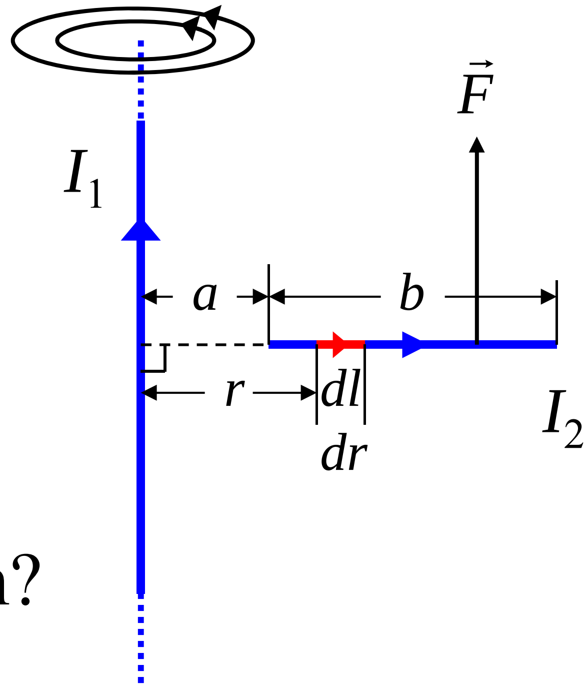
$$F_{\overset{\frown}{ab}} = F_{ab} = I \cdot \sqrt{2}R \cdot B \cdot \sin 45^\circ$$

$$F_{\overset{\frown}{ab}} = F_{ao} = IRB$$

$\odot$

Nonuniform field

Example2: **I_1 and I_2 are on the same plane. What is the force on I_2, if the magnetic field created by I_1 is** $B = k\frac{I_1}{r}$?



Solution: Total force on I_2

$$F = \int I_2 B dl = \int_a^{a+b} k\frac{I_1 I_2}{r} dr$$

$$= kI_1 I_2 \ln\frac{a+b}{a}$$

direction?

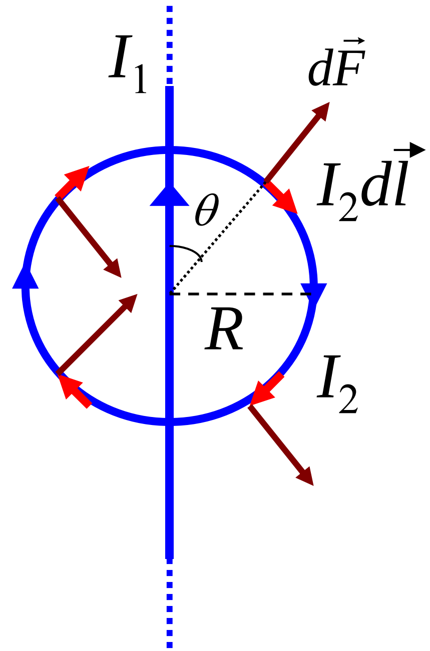
Interaction of currents

Question: Infinitely long current I_1 and circular current I_2 are insulated and on the same plane. What is the force between them?

$$dF = k\frac{I_1 I_2}{R\sin\theta}\cdot R d\theta$$

$$F = \int \sin\theta dF \qquad = kI_1I_2\int d\theta$$

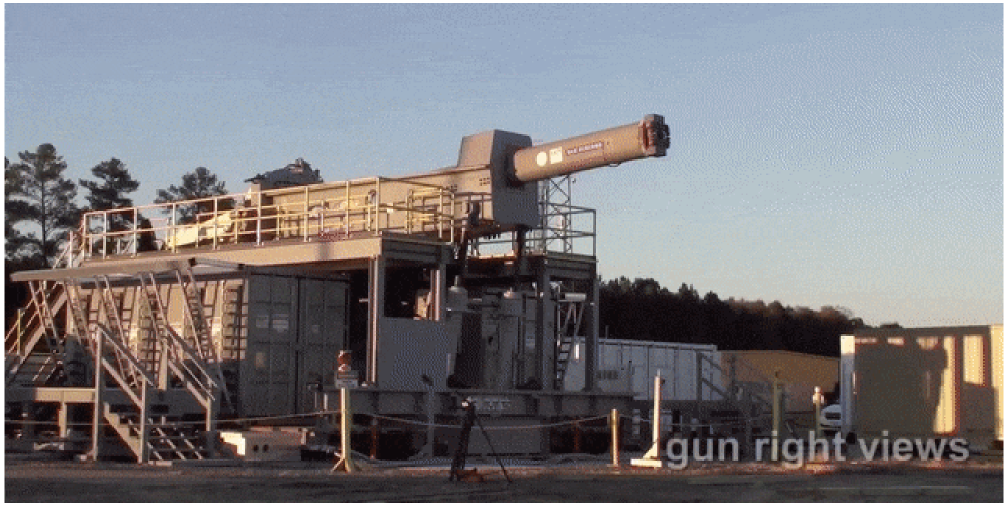
$$= 2\pi k I_1 I_2$$

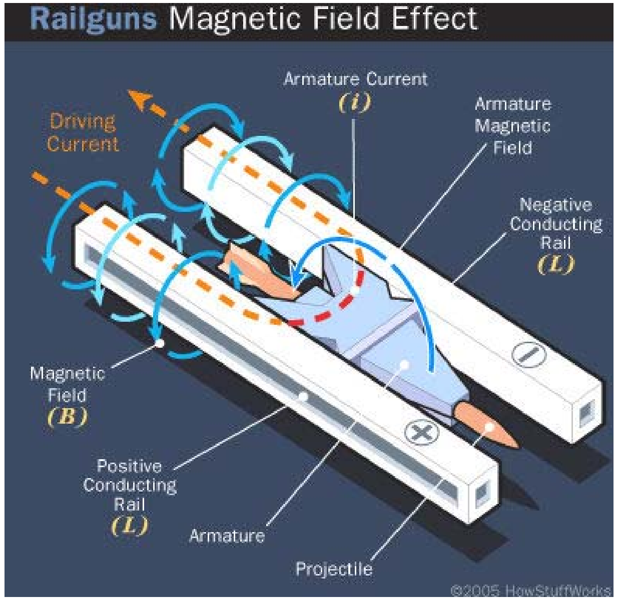


*Railgun

Armature: the part of a generator , motor etc that turns around to produce electricity, movement etc

Weapon for space war in future → railgun





High power ~ 10^7 J

High velocity ~ 10 Mach

about 3 km/s

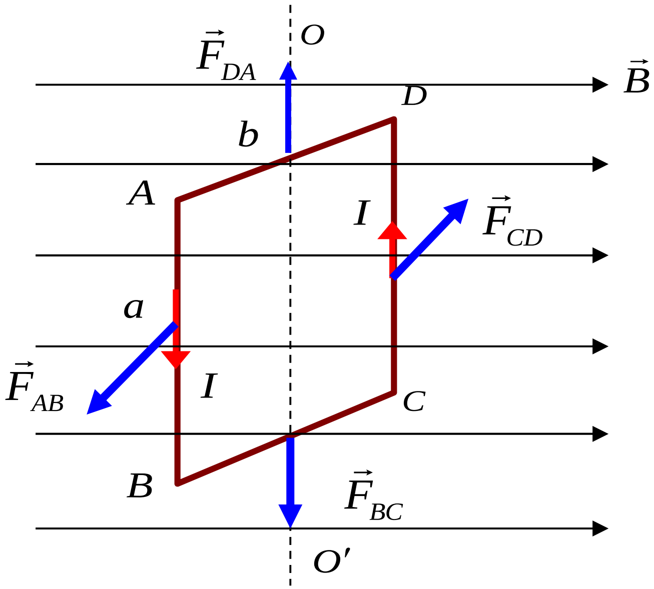
Battery & rail

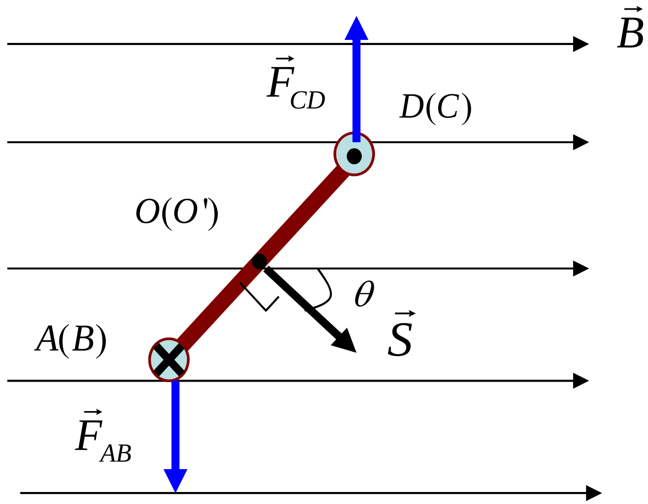
Torque on a current loop (1)

Rectangular current loop in a uniform field

$$\vec{F}_{total} = \vec{F}_{AB} + \vec{F}_{BC} + \vec{F}_{CD} + \vec{F}_{DA} = 0$$

$$\tau = 2 \times F_{AB} \frac{b}{2} \sin\theta = BIab \sin\theta = BIS \sin\theta$$



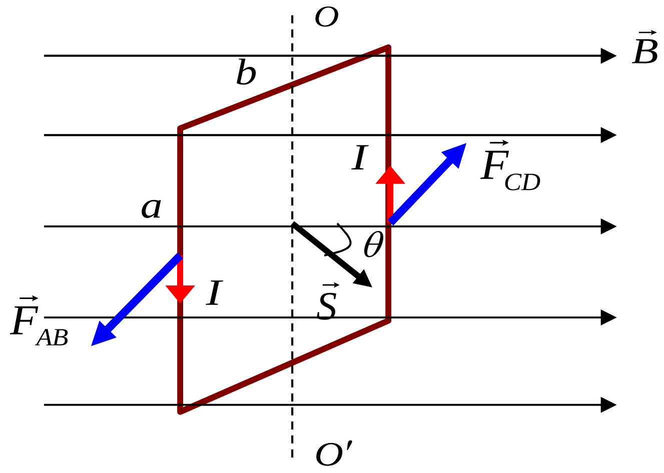


Torque on a current loop (2)

Torque on the loop:

$$\tau = BIS\sin\theta$$

Magnetic dipole moment:



$$\vec{\mu} = I\vec{S}$$

where the direction is defined by right-hand rule

$$\boxed{\vec{\tau} = \vec{\mu}\times\vec{B}}$$

compare with $\begin{cases} \vec{p} = Q\vec{l} \\ \vec{\tau} = \vec{p}\times\vec{E} \end{cases}$

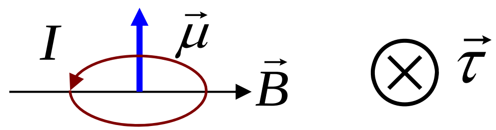
Magnetic dipoles

$$\vec{\mu} = I\vec{S} \qquad\qquad \boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$

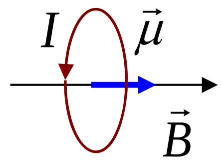
These are valid for any plane current loop

A small circular current → a magnetic dipole

1) θ=π/2: maximum torque



2) θ=0 or π: stable / unstable equilibrium

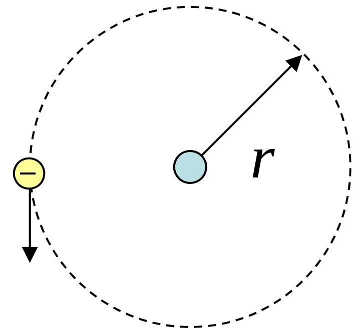


3) N loops coil / solenoid: $\vec{\mu} = NI\vec{S}$

Magnetic moment of an atom

Example3: **Show that μ of an electron inside a hydrogen atom is related to angular momentum L of the electron by $\mu=eL/(2m)$.**

Solution: $\mu = IS \quad = \frac{1}{2}evr$



$$I = \frac{Q}{T} \quad = e\frac{v}{2\pi r} \qquad S = \pi r^2$$

Orbital angular momentum: $L = mvr$

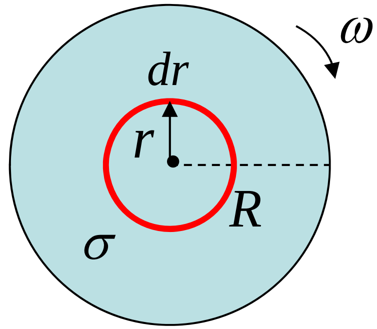
$$\therefore \ \mu = \frac{eL}{2m}$$

Classic / quantum model

μ of rotating charged body

Example4: A uniformly charged disk is rotating about the center axis (σ, R, ω). Determine the magnetic moment.

Solution: Magnetic moment $\mu = IS$



Rotating charge: $I = \dfrac{Q}{T} = \dfrac{\omega}{2\pi} Q$

Total magnetic moment:

direction?

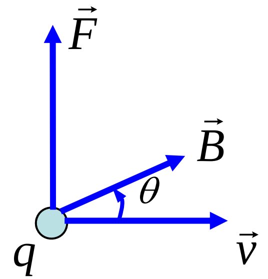
$$\mu = \int_0^R \frac{\omega}{2\pi} \sigma \cdot 2\pi r dr \cdot \pi r^2 = \frac{1}{4} \pi \omega \sigma R^4$$

$\otimes \vec{\mu}$

Force on moving charges

Magnetic field exerts a force on a moving charge:

$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$ $\rightarrow$ **Lorentz force**



1) Magnitude: $F = qvB\sin\theta$

2) Direction: right-hand rule, sign of *q*

If *q* < 0, $\vec{F}$ **has an opposite direction to** $\vec{v} \times \vec{B}$

3) Lorentz force doesn't do work on the charge!

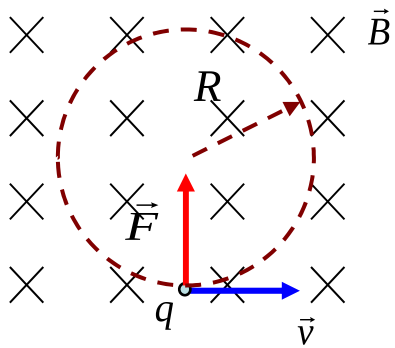
Motion in a uniform field (1)

Point charge moves in a uniform magnetic field

1) $\vec{v} \parallel \vec{B}$ **:** $\vec{F} = q\vec{v} \times \vec{B} = 0$ **Free motion**

2) $\vec{v} \perp \vec{B}$ **:** **Uniform circular motion**

$$F = qvB = \frac{mv^2}{R}$$

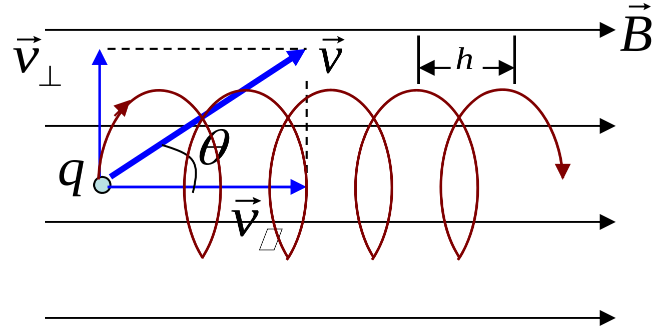


$$R = \frac{mv}{qB}, \qquad T = \frac{2\pi R}{v} = \frac{2\pi m}{qB}$$

Motion in a uniform field (2)

3) General case:

$$\vec{v} = \vec{v}_\perp + \vec{v}_\parallel$$



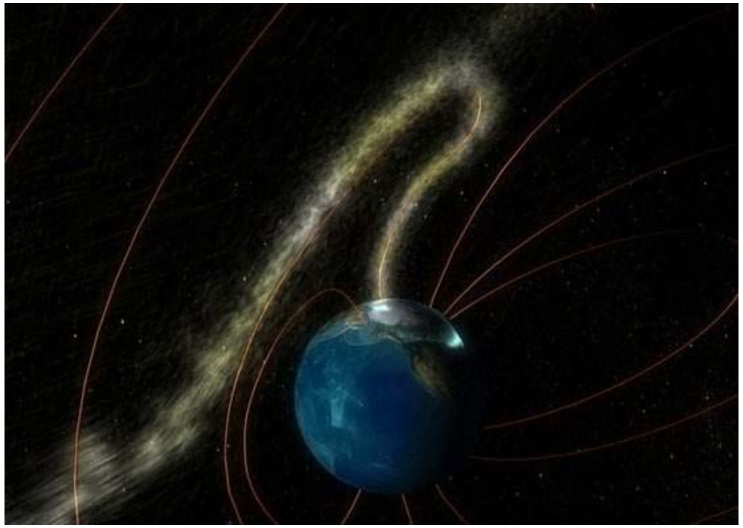
Free motion + uniform circular motion

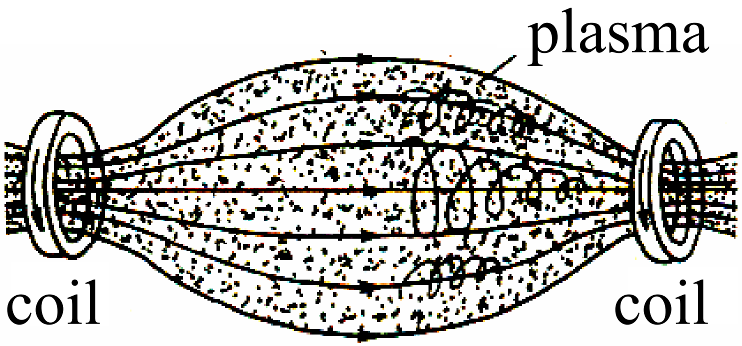
Combination: the charge moves in a helix

$$R = \frac{mv\sin\theta}{qB}, \qquad T = \frac{2\pi m}{qB}, \qquad h = \frac{2\pi mv\cos\theta}{qB}$$

*Aurora & magnetic confinement

Aurora: Caused by high-energy charges from the Solar wind





Magnetic confinement: "magnetic mirror"

Lorentz equation

Example5: A proton moves under both magnetic and electric field. Determine the components of the total force on the proton. (All in SI units)

$$\vec{B} = 0.4\vec{i} + 0.2\vec{j},\ \vec{E} = (3\vec{i} - 4\vec{j}) \times 10^3,\ \vec{v} = (6\vec{i} + 3\vec{j} - 5\vec{k}) \times 10^3$$

Solution: Total force $\boxed{\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})}$

$$= e\left(3\vec{i} - 4\vec{j}\right) \times 10^3 + e(6\vec{i} + 3\vec{j} - 5\vec{k}) \times 10^3 \times (0.4\vec{i} + 0.2\vec{j})$$

$$= (6.4\vec{i} - 9.6\vec{j}) \times 10^{-16}\, N$$

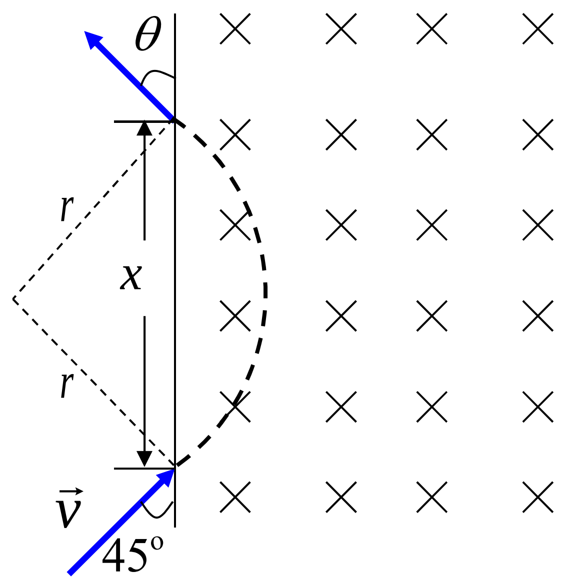
Different regions

Example6: **A proton moving in a field-free region abruptly enters a uniform magnetic field as the figure. (a) At what angle does it leave? (b) At what distance *x* does it exit from the field?**

Solution: Circular motion

(a) $\theta = 45^{\circ}$

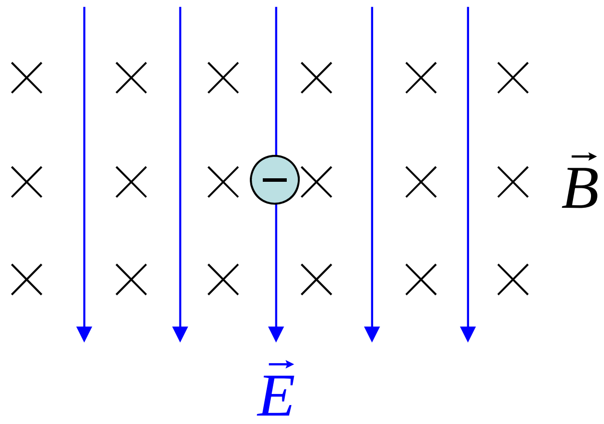
(b) $r = \dfrac{mv}{eB} \quad \Rightarrow x = \dfrac{\sqrt{2}mv}{eB}$



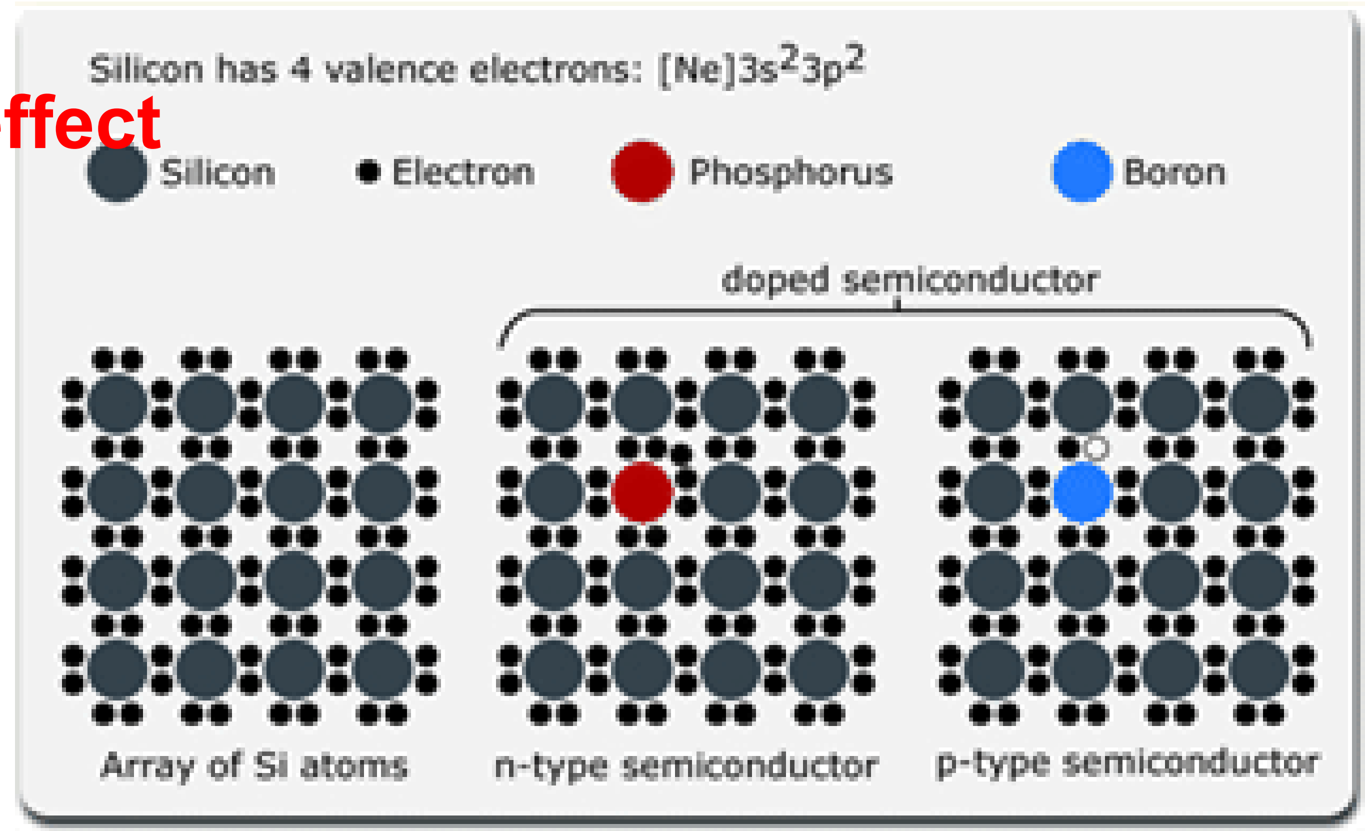
Challenging question

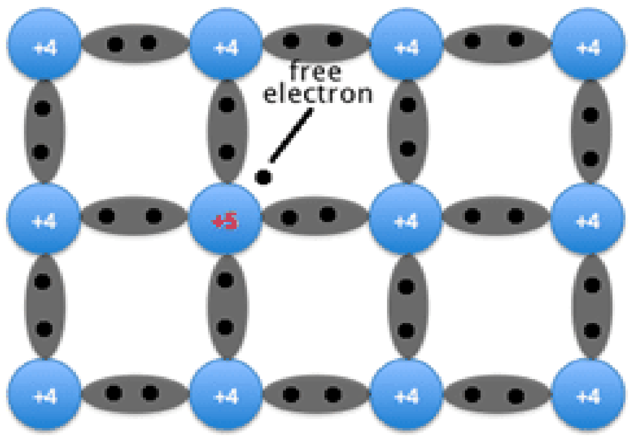
Question: **A electron is released from rest. If the field is shown as the figure, how does the electron move under the field? What is the path?**

The path is a cycloid

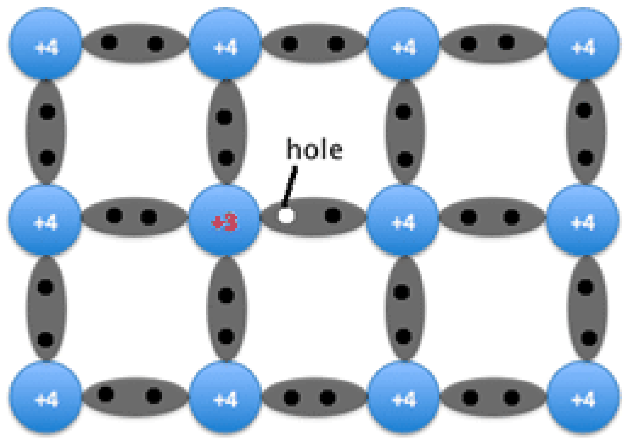


The Hall effect





N-type Semiconductor



P-type Semiconductor

The Hall effect

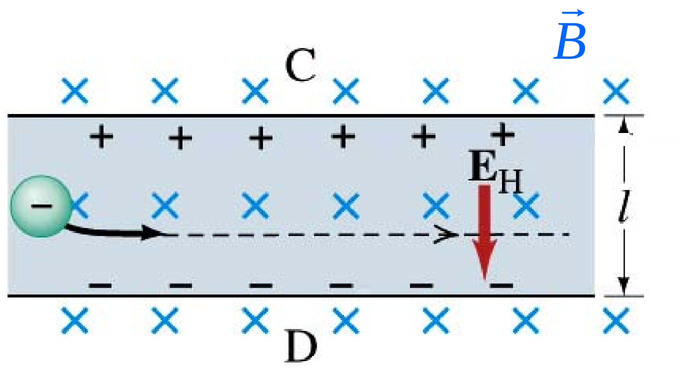
Current-carrying conductor / semiconductor

placed in a magnetic field → Hall voltage

Lorentz force → Hall field → equilibrium

$$eE_H = evB \quad \Rightarrow E_H = vB \quad \Rightarrow V_H = E_H l = vBl$$

$$\boxed{V_H = KI_H B}$$



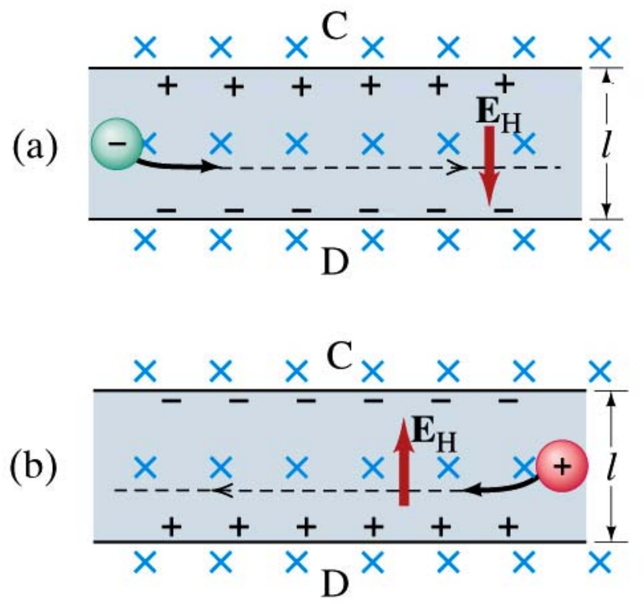
I_H → Hall current

Applications of Hall effect

1) Distinguish the types of semiconductors

$$V_C > V_D \rightarrow N-type$$

$$V_C < V_D \rightarrow P-type$$



2) Measure magnetic field

Hall sensor / switch

3) Measure the carrier concentration