Chapter 27
Electromagnetic Induction and Faraday's Law

- Electromagnetic Induction
- Faraday's Law of Induction; Lenz's Law
- EMF Induced in a Moving Conductor
- A Changing Magnetic Flux Produces an Electric Field
- Applications of induction

- An electric current produces a magnetic field;
- A magnetic field exerts a force on an electric current or moving electric charge.

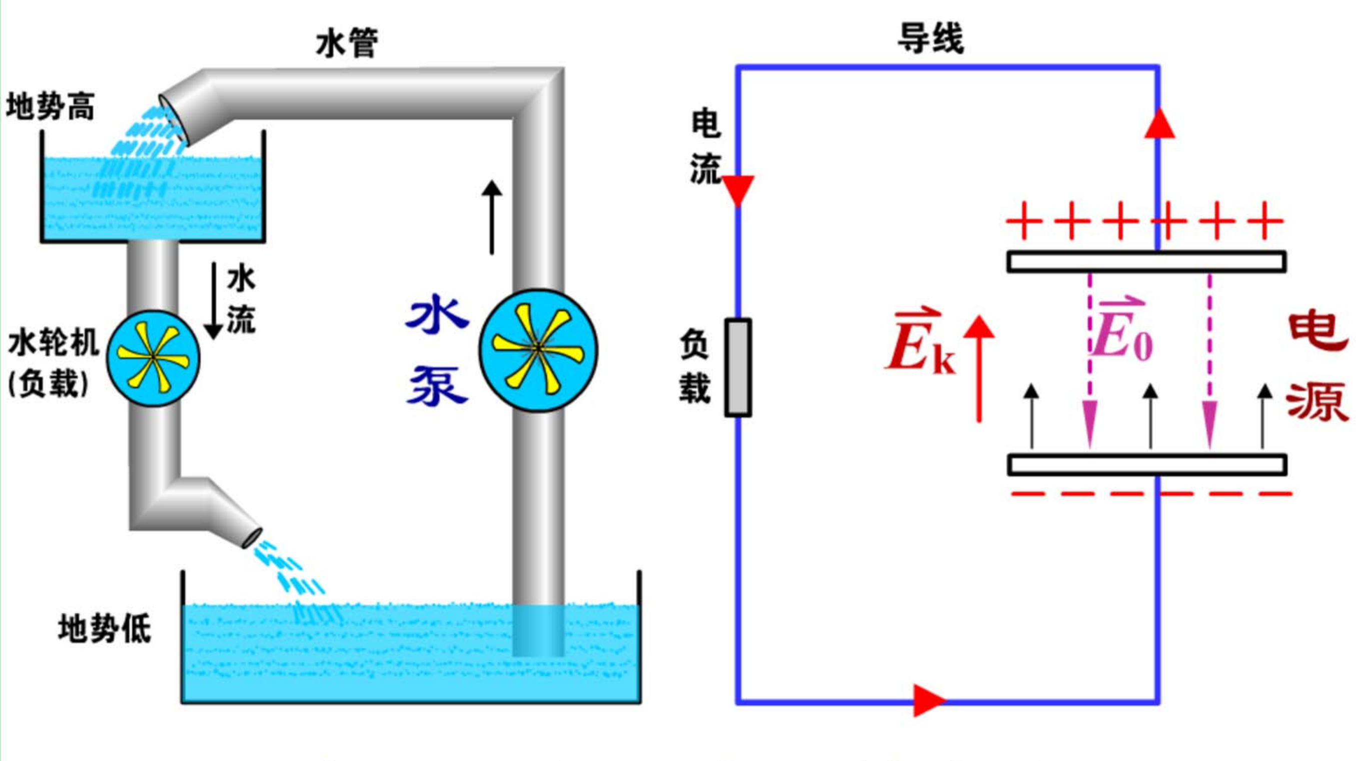
Scientists then began to wonder: **if electric currents produce a magnetic field, is it possible that a magnetic field can produce an electric current?**

Ten years later, the American Joseph Henry (1797-1878) and the Englishman Michael Faraday (1791-1867) independently found that it was possible.

EMF (Electromotive force) of a battery (power supply)

EMF: moving a positive unit charge from the negative electrode to the positive one of the battery, the work done by the non-electrostatic force $\vec{E}_k$:

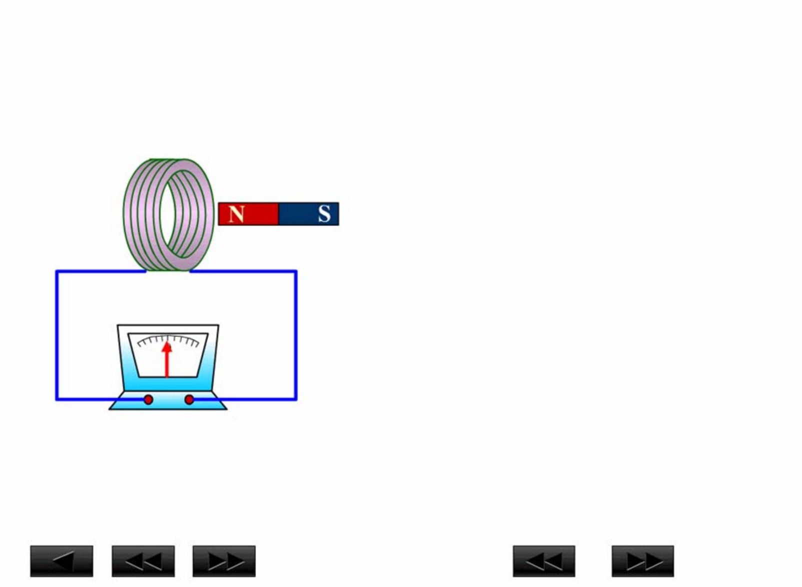
$$\varepsilon = \int_{-}^{+} \vec{E}_k \cdot \mathrm{d}\vec{l}$$

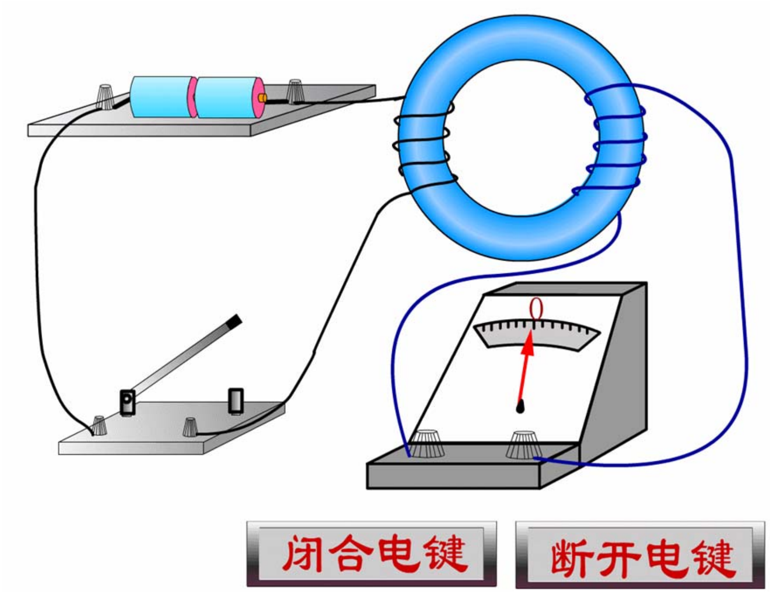


电源和水泵作用的类比

§27-1 Electromagnetic Induction

Faraday's experiments:





As long as the magnetic flux passing through a closed circuit changes over time, a current will appear in the circuit (Electromagnetic induction).

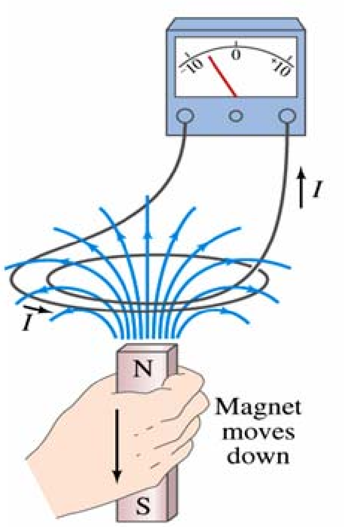
The condition for electromagnetic induction:

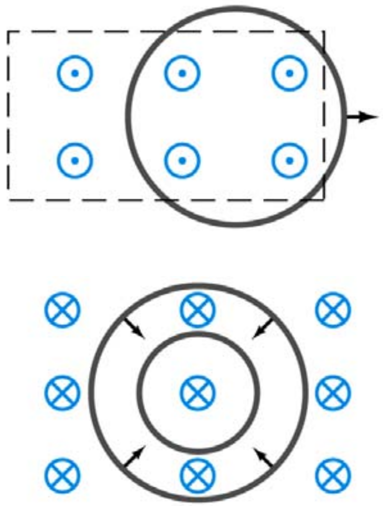
$$\frac{\mathrm{d}\Phi_m}{\mathrm{d}t} \neq 0$$

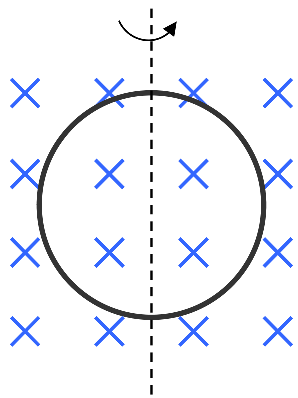
To achieve this condition, there are two means:

$$\Phi_m = \int \vec{B} \cdot \mathrm{d}\vec{S} = \int B \cos\theta \mathrm{d}S$$

(1) By changing *B*　　(2) By changing S of the loop　　(3) By changing θ







§27-2 Faraday's Law of Induction; Lenz's Law

1. Faraday's law of induction

The EMF induced in a circuit is equal to the rate of change of magnetic flux through the circuit.

$$\varepsilon = -\frac{\mathrm{d}\Phi_m}{\mathrm{d}t}$$

(1) EMF is produced even if the circuit is not closed;

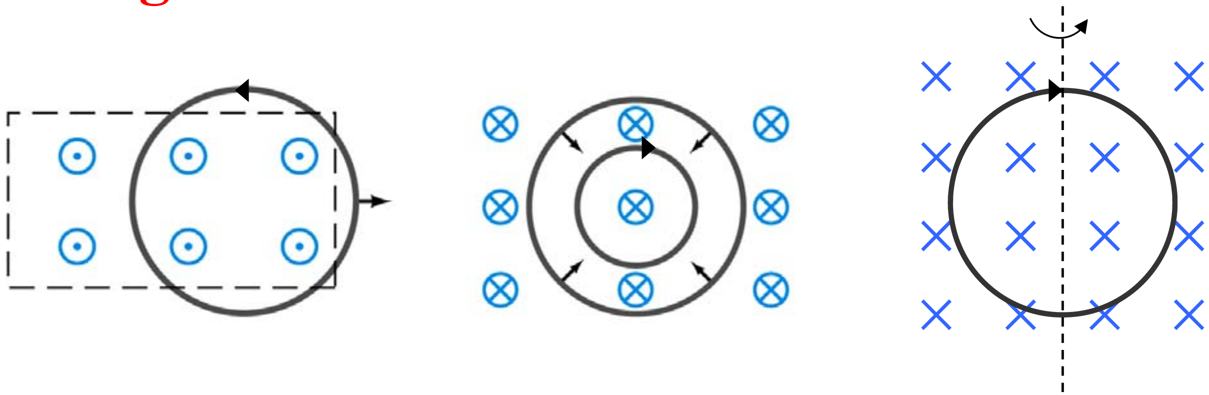
(2) If the circuit contains N closely wrapped loops :

$$\varepsilon = -N\frac{\mathrm{d}\Phi_m}{\mathrm{d}t}$$

(3) Induction current in a closed circuit: $I = \frac{\varepsilon}{R}$

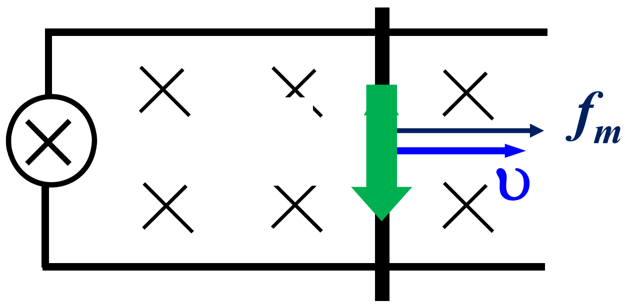
2. Lenz's law

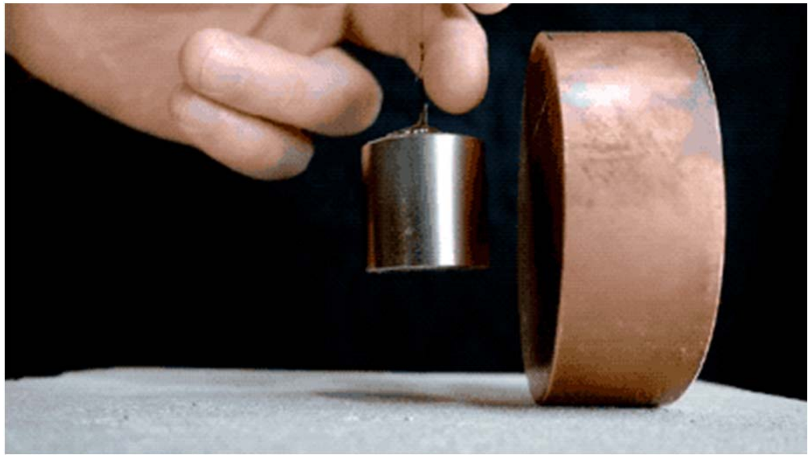
An induced EMF is always in a direction that opposes the original change in flux that caused it.

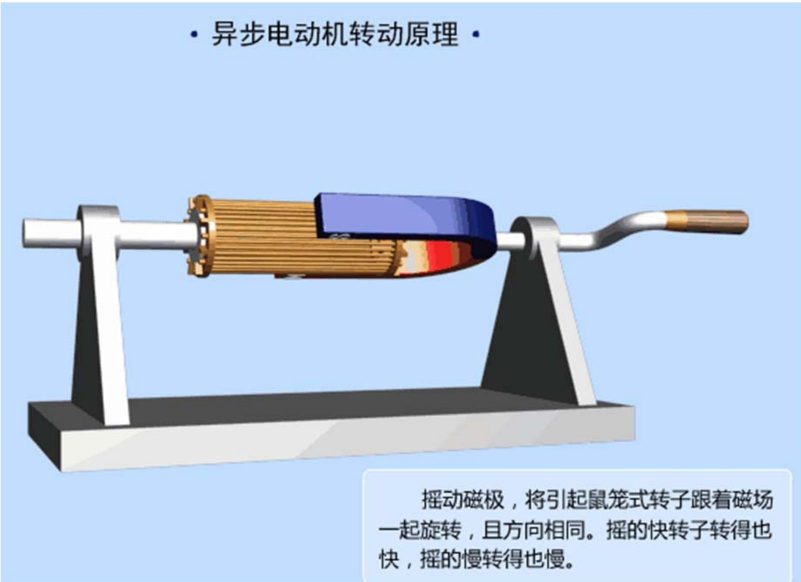


Complies with the conservation of energy:

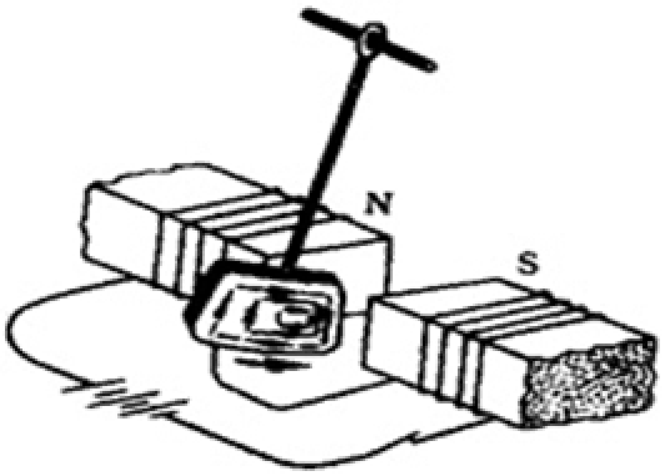
If contrary to Lenz's law → Ampère force does positive work → no external energy required for the circuit to work → contradicts the conservation of energy.







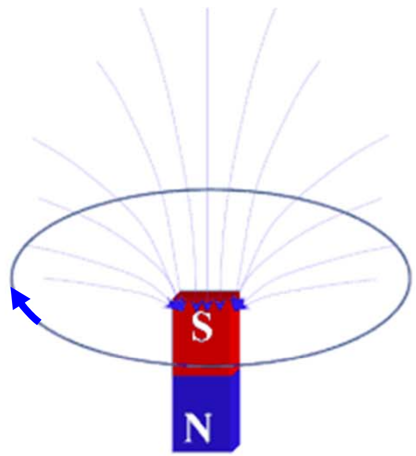
• 异步电动机转动原理 •
摇动磁极，将引起鼠笼式转子跟着磁场一起旋转，且方向相同。摇的快转子转得也快，摇的慢转得也慢。

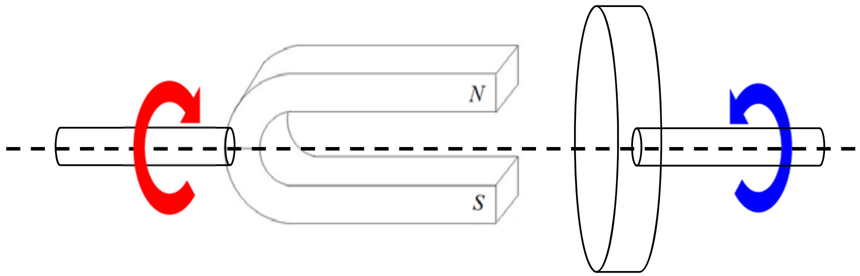


N
S

Usually, it's difficult to find a clear circuit loop, so, Lenz’s law is often expressed as:

The effect of induced current always resists the cause of induced current. (effect of the result is always against the reason)





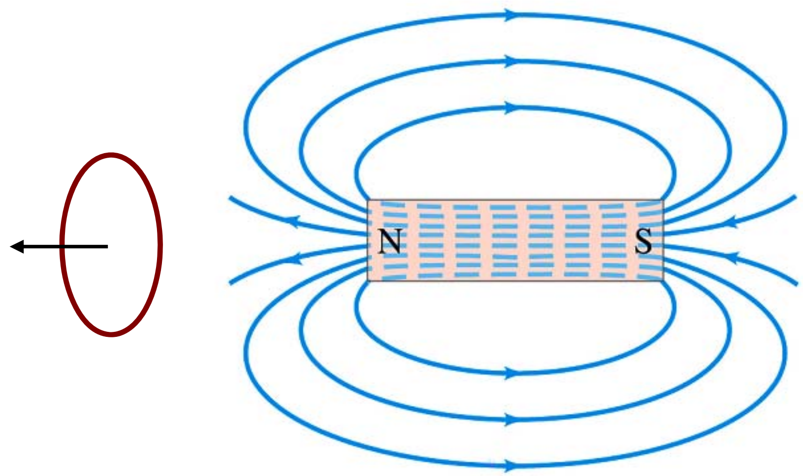
Induced charges

Example: At time t_1 and t_2, the magnetic flux passing through the area enclosed by a circuit is Φ_1 and Φ_2, determine the induced charge through any cross-section of the circuit at time interval $t_1 \rightarrow t_2$?

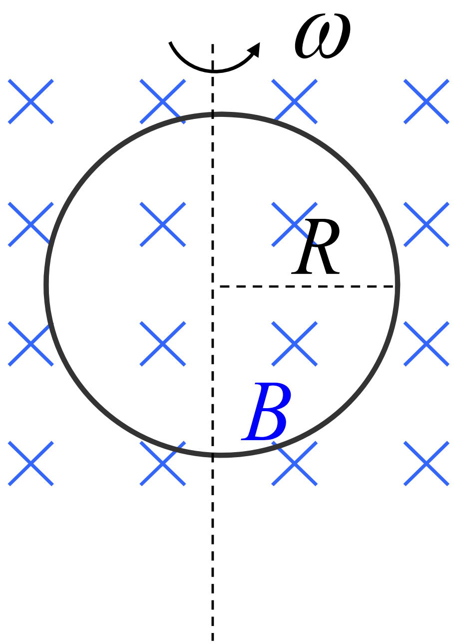
$$\boldsymbol{q_i} = \int_{t_1}^{t_2} I_i \mathrm{d}t = \int_{\Phi_1}^{\Phi_2} -\frac{1}{R} \mathrm{d}\Phi_m$$

$$\Rightarrow \quad q_i = \frac{\Phi_1 - \Phi_2}{R}$$

Thinking: A small coil moves far away from the position in the figure shown below. How to determine the total induced charge going through the coil?



Example1 (Rotating coil): A circular coil is rotating in uniform magnetic field. Determine the EMF.



Solution: The magnetic flux:

$$\Phi_B = BS\cos\theta = \pi R^2 B\cos(\omega t + \theta_0)$$

$$\therefore \ \varepsilon = -\frac{\mathrm{d}\Phi_m}{\mathrm{d}t} = \pi R^2 \omega B\sin(\omega t + \theta_0)$$

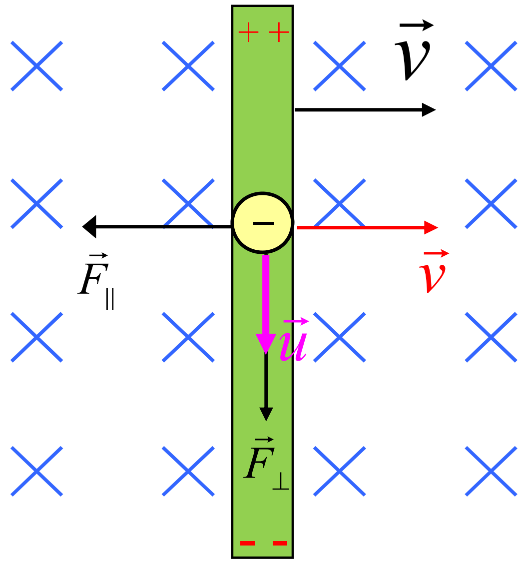
If the magnetic field is changing as:

$$B = B_0 e^{-\beta t}$$

$$\varepsilon = \pi R^2 \omega B_0 e^{-\beta t}\sin(\omega t + \theta_0) + \pi R^2 \beta B_0 e^{-\beta t}\cos(\omega t + \theta_0)$$

§27-3 EMF Induced in a Moving Conductor

Motional EMF: Caused by a component of Lorentz force.



$\vec{F}_{\perp} = -e\vec{v}\times\vec{B}$ does positive work on e

$\vec{F}_{\parallel} = -e\vec{u}\times\vec{B}$ does negative work on e

Total Lorentz force $\vec{F} = \vec{F}_{\parallel} + \vec{F}_{\perp}$ does no work on e.

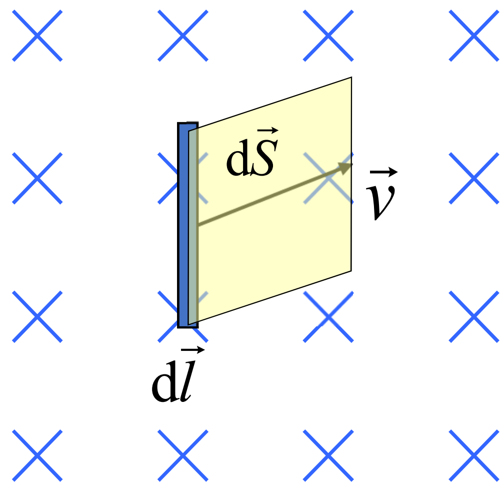
Component $\vec{F}_{\perp}/(-e)$ acts as the non-electrostatic force $\vec{E}_k$ to produce the motional EMF:

$$\varepsilon = \int_{-}^{+} \vec{E}_k \cdot \mathrm{d}\vec{l} = \int_{-}^{+} (\vec{v}\times\vec{B})\cdot \mathrm{d}\vec{l}$$

The result can also be derived by Faraday' law

$\mathbf{d}\vec{l}$ moves with velocity $\vec{v}$

It sweeps out an area dS:



$$\mathrm{d}\vec{S} = \left(\vec{v}\,\mathrm{d}t\right)\times \mathrm{d}\vec{l}$$

Magnetic flux:

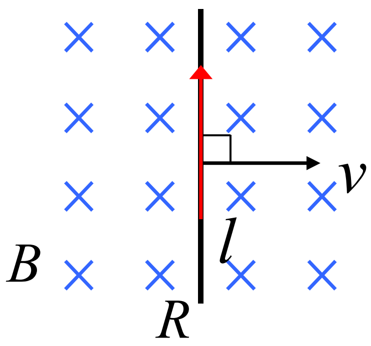
$$\mathrm{d}\Phi_m = \vec{B}\cdot \mathrm{d}\vec{S} = \vec{B}\cdot\left(\vec{v}\times \mathrm{d}\vec{l}\right)\mathrm{d}t$$

EMF:

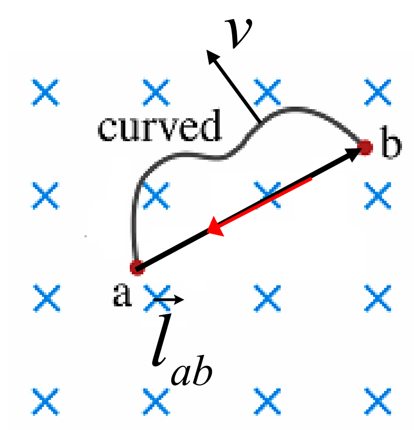
$$\varepsilon = -\frac{\mathrm{d}\Phi_m}{\mathrm{d}t} = -\vec{B}\cdot\left(\vec{v}\times \mathrm{d}\vec{l}\right) = \left(\vec{v}\times\vec{B}\right)\cdot \mathrm{d}\vec{l}$$

Example2 (Motion in uniform field): Determine the EMF induced in the conductor in a uniform magnetic field.

(a)



(b)



EMF: $\varepsilon = Blv$

Force: $F = BIl = B^2 l^2 v / R$

Power: $P = Fv = I^2 R$

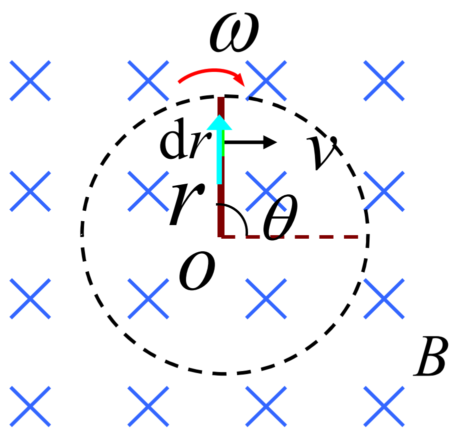
$\rightarrow$ straight wire:

$$\varepsilon = (\vec{v} \times \vec{B}) \cdot \vec{l}_{ab}$$

Example3 (Rotates in uniform field): A conductor rod rotates about axis $\boldsymbol{O}$. Determine the induced EMF. (B, L, ω)

Solution: For an infinitesimal:

$$d\varepsilon = B \cdot \omega r \cdot dr$$



$$S = \frac{1}{2} L^2 \theta$$

Total EMF in the conductor:

$$\varepsilon = \int_0^L B\omega r dr = \frac{1}{2} B\omega L^2$$

Faraday's law:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d\theta}{dt} \cdot \frac{1}{2} BL^2 = \frac{1}{2} B\omega L^2$$

单选题 10分

设置

Question: As the figure shows, a copper disk is placed in a uniform magnetic field, and rotates clockwise. The field $\vec{B}$ is perpendicular to the face of the disk. Which of the following is right ?

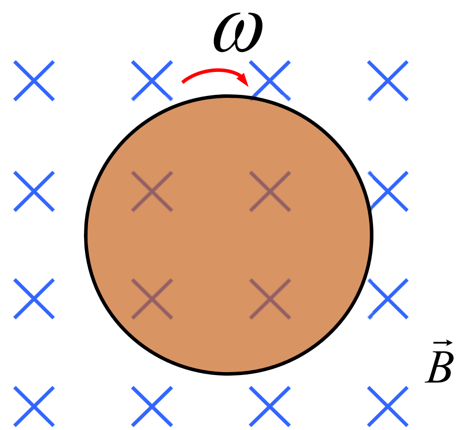
A Current with clockwise direction is induced inside the disk;

B Current with anticlockwise direction is induced inside the disk;

C Eddy current is induced inside the disk;

D EMF is induced on the disk, and the edge has the highest potential;

E EMF is induced on the disk, and the center has the highest potential;

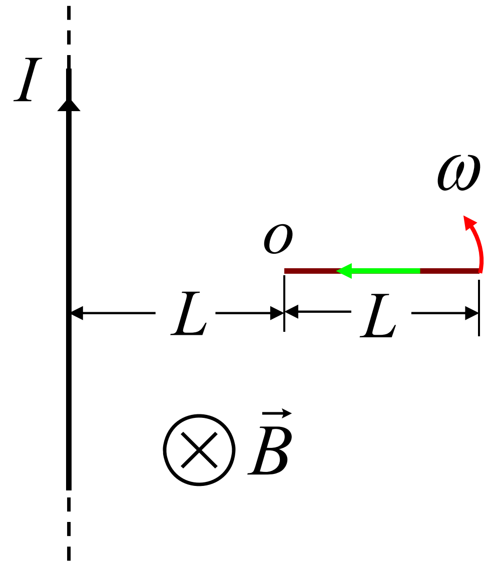


提交

Rotates in nonuniform field

Question: A conductor rod rotates about axis ***o***. Determine the induced EMF when the two wires are perpendicular to each other.

$$\varepsilon = \frac{\mu_0 \omega I L}{2\pi}(1 - \ln 2)$$

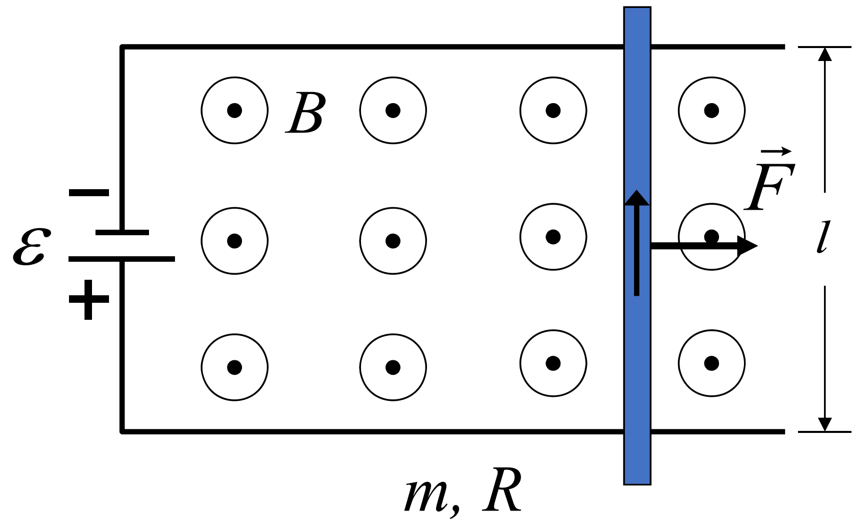


Example4 (Motion on rails): Conducting rod rests on frictionless parallel rails with an EMF source. Determine the speed of rod if the source outputs: (a) constant I; (b) constant EMF. (c) What is the terminal speed?

Solution: (a)

$$F = ma = BIl$$

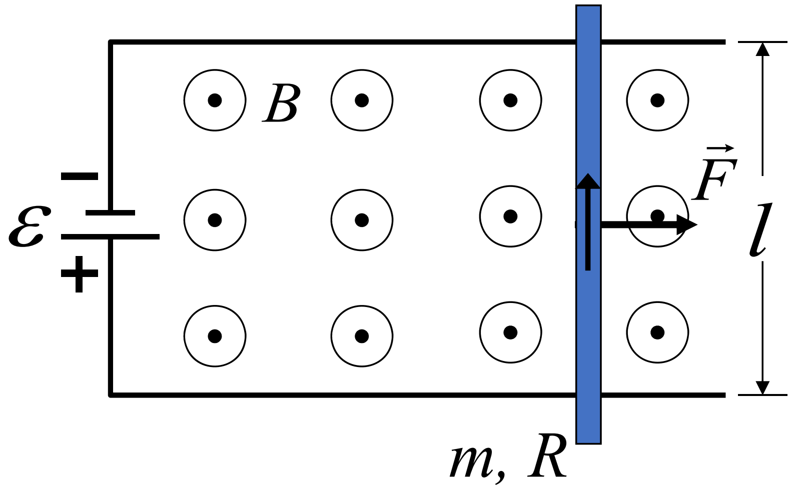
$$\Rightarrow \quad v = at = \frac{BIl}{m} t$$



(b) source outputs constant EMF: $$I = \frac{\varepsilon - Blv}{R}$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{BIl}{m} = \frac{(\varepsilon - Blv)Bl}{mR} \quad \Rightarrow \quad \int_0^t \frac{Bl}{mR}\mathrm{d}t = \int_0^v \frac{\mathrm{d}v}{\varepsilon - Blv}$$

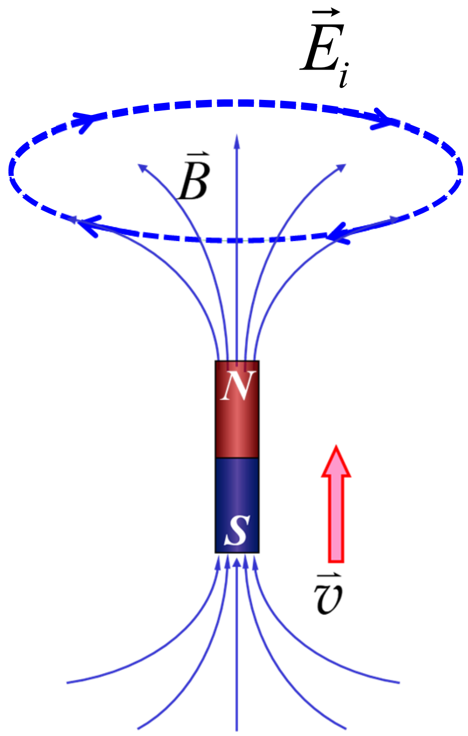
$$\Rightarrow \qquad v = \frac{\varepsilon}{Bl}\left(1 - e^{-\frac{B^2 l^2 t}{mR}}\right)$$



(c) Terminal speed

$$v = \frac{\varepsilon}{Bl}$$

§27-4 A Changing Magnetic Flux Produces an Electric Field



When the magnetic field changes:

$\rightarrow$ Induced EMF $\rightarrow$ induced current (closed coil)

In the space where the magnetic field changes:

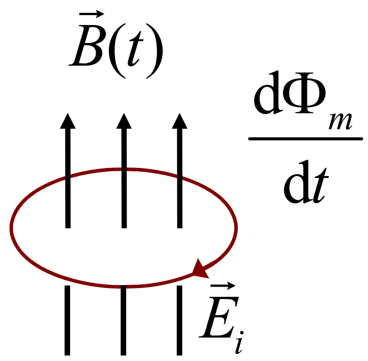
electric field is produced (Maxwell)

Induced (vortex) electric field $\vec{E}_i$:

- Produced by changing magnetic field;
- Field lines form closed loops, so it is nonconservative;
- Acts on electric charges as the electrostatic field.

General form of Faraday's law:

$$\boldsymbol{\varepsilon} = \oint \vec{\boldsymbol{E}}_i \cdot \mathbf{d}\vec{\boldsymbol{l}} = -\frac{\mathbf{d}\Phi_m}{\mathbf{d}\boldsymbol{t}} = -\frac{\mathbf{d}}{\mathbf{d}\boldsymbol{t}}\int \vec{\boldsymbol{B}}\cdot \mathbf{d}\vec{\boldsymbol{S}}$$



$$\boxed{\Rightarrow \quad \oint \vec{\boldsymbol{E}}_i \cdot \mathbf{d}\vec{\boldsymbol{l}} = -\int \frac{\partial \vec{\boldsymbol{B}}}{\partial \boldsymbol{t}} \cdot \mathbf{d}\vec{\boldsymbol{S}}} \quad \Leftrightarrow \quad \nabla \times \vec{\boldsymbol{E}}_i = -\frac{\partial \vec{\boldsymbol{B}}}{\partial \boldsymbol{t}}$$

Comparison between induced electric field and electrostatic field:

Induced electric field	**Electrostatic field**
nonconservative	**conservative**
$\oint_L \vec{\boldsymbol{E}}_i \cdot \mathrm{d}\vec{\boldsymbol{l}} = -\frac{\mathrm{d}\boldsymbol{\Phi}_m}{\mathrm{d}\boldsymbol{t}} \neq 0$	$\oint_L \vec{\boldsymbol{E}}_{静} \cdot \mathrm{d}\vec{\boldsymbol{l}} = 0$
Produced by changing magnetic field	**Produced by charges**

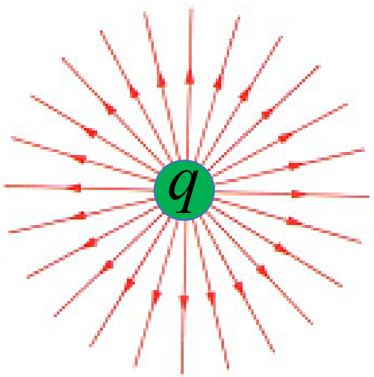
Summary of fields:

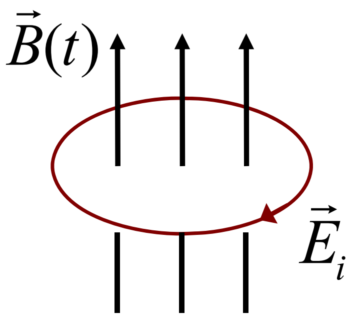
Electrostatic / **induced electric /** **magnetic field**

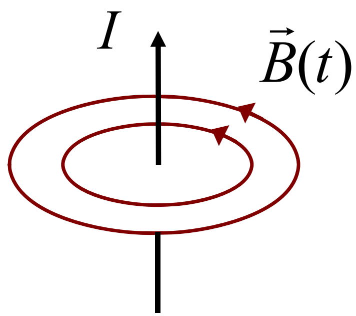
$$\oint \vec{E}_s \cdot d\vec{l} = 0 \qquad \oint \vec{E}_i \cdot d\vec{l} = -\frac{d\Phi_m}{dt} \qquad \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$\oint \vec{E}_s \cdot d\vec{S} = \frac{Q_{encl}}{\varepsilon_0} \qquad \oint \vec{E}_i \cdot d\vec{S} = 0 \qquad \oint \vec{B} \cdot d\vec{S} = 0$$

Field lines: vortex Closed loops

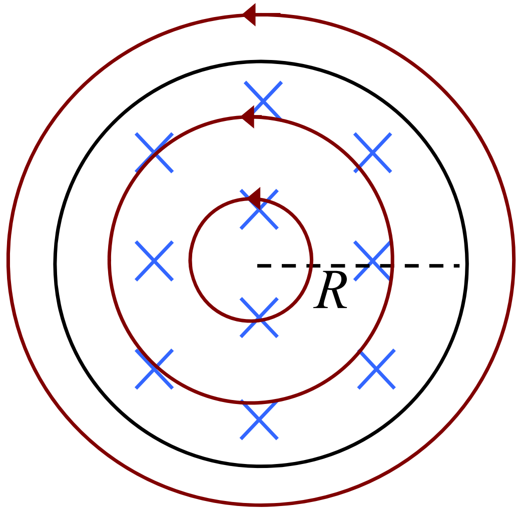






Example5 (Vortex electric field): Uniform magnetic field in cylindrical space changes as dB/dt=C>0. Determine the induced electric field.

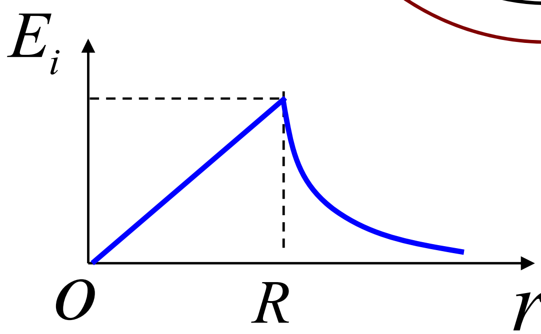
Solution: Analyze the symmetry



$$\oint \vec{E}_i \cdot \mathrm{d}\vec{l} = E_i \cdot 2\pi r = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = C \cdot S$$

$$r < R:\ E_i = \frac{C \cdot \pi r^2}{2\pi r} = \frac{C}{2} r$$

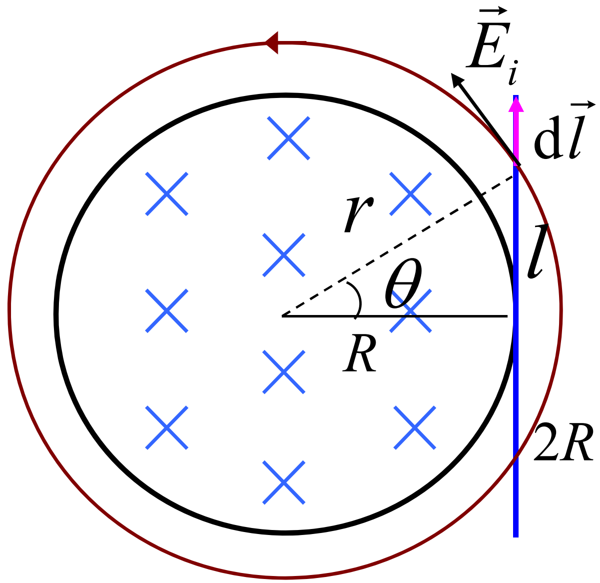
$$r > R:\ E_i = \frac{C \cdot \pi R^2}{2\pi r} = \frac{CR^2}{2r}$$



Example6 (EMF in a wire): Magnetic field in a solenoid changes as $dB/dt = C > 0$. A straight wire lies tangent to the solenoid at its center. What is the EMF in wire?

Solution: Induced electric field:

$$r > R: \; E_i = \frac{CR^2}{2r}$$



$$\varepsilon = \int_L \vec{E}_i \cdot d\vec{l} = \int \frac{CR^2}{2r} \cos\theta \, dl$$

$$R = r\cos\theta$$

$$l = R\tan\theta$$

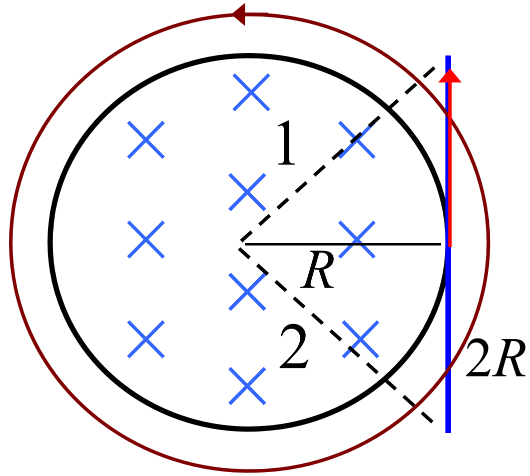
$$= \int_{-\pi/4}^{\pi/4} \frac{CR^2}{2} d\theta = \frac{\pi R^2 C}{4}$$

Another Solution:

Imagine a closed circuit

$$\varepsilon_1 = \int \vec{E}_i \cdot \mathrm{d}\vec{l} = 0, \qquad \varepsilon_2 = 0$$

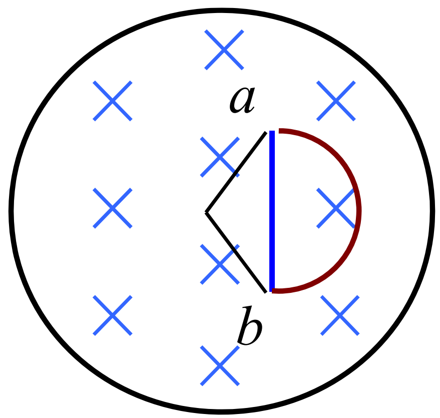
$$\therefore \quad \varepsilon = \varepsilon_\Delta = -\frac{\mathrm{d}\Phi_m}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\pi R^2 B}{4}\right) = \frac{\pi R^2 C}{4}$$



Discussion:

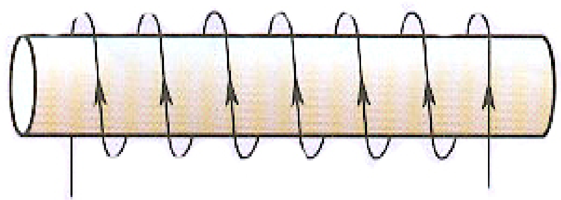
$$\oint \vec{E}_i \cdot \mathrm{d}\vec{l} = -\frac{\mathrm{d}\Phi_m}{\mathrm{d}t} \neq 0\,!$$

$$\varepsilon_{\overline{ab}} = \varepsilon_{\overset{\frown}{ab}} \ ? \qquad \varepsilon_{\overline{ab}} < \varepsilon_{\overset{\frown}{ab}}$$



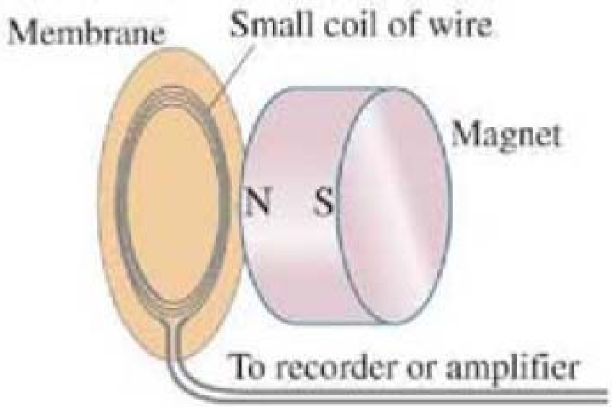
Self induction

Thinking: If a solenoid with current I is cut off from the battery, will I drop abruptly to 0? How does I change over time?

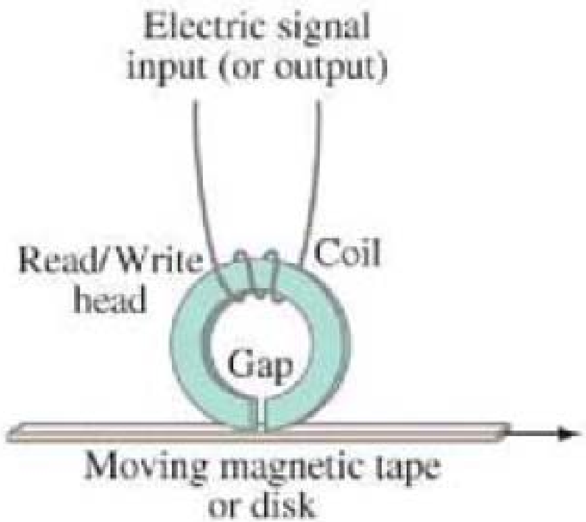


§27-5 Applications of induction

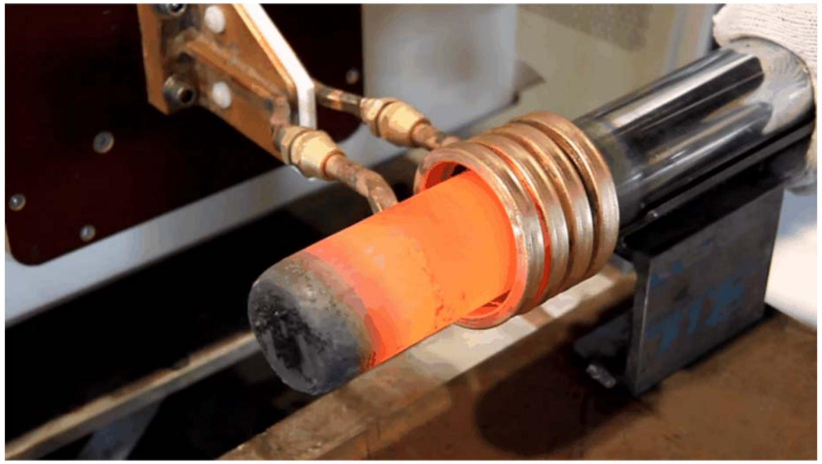
Sound systems / microphones:



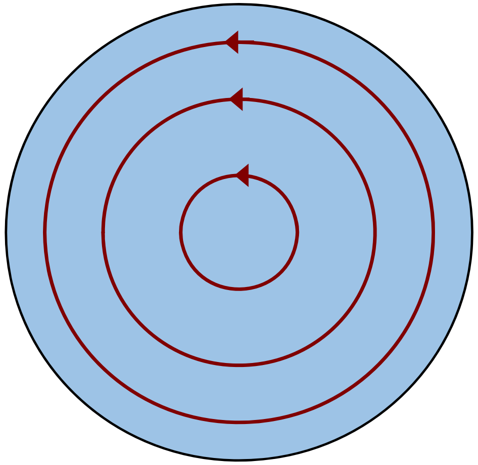
Recording tape/computer memory/swipe card



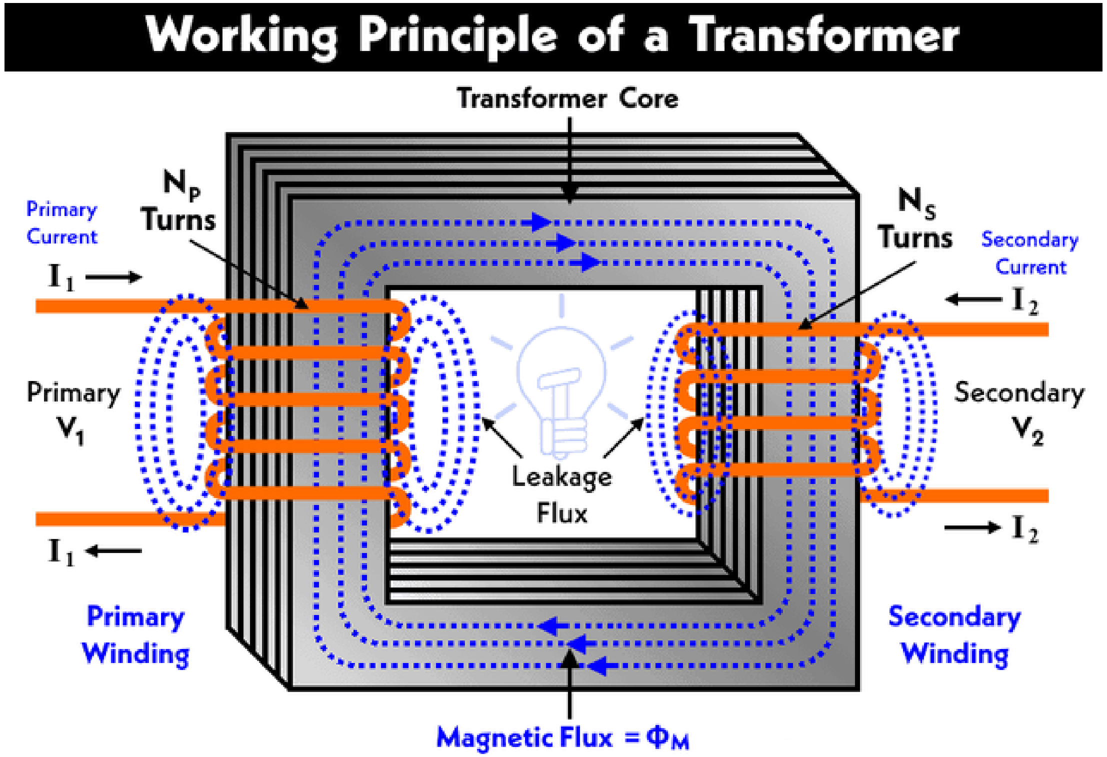
Eddy currents



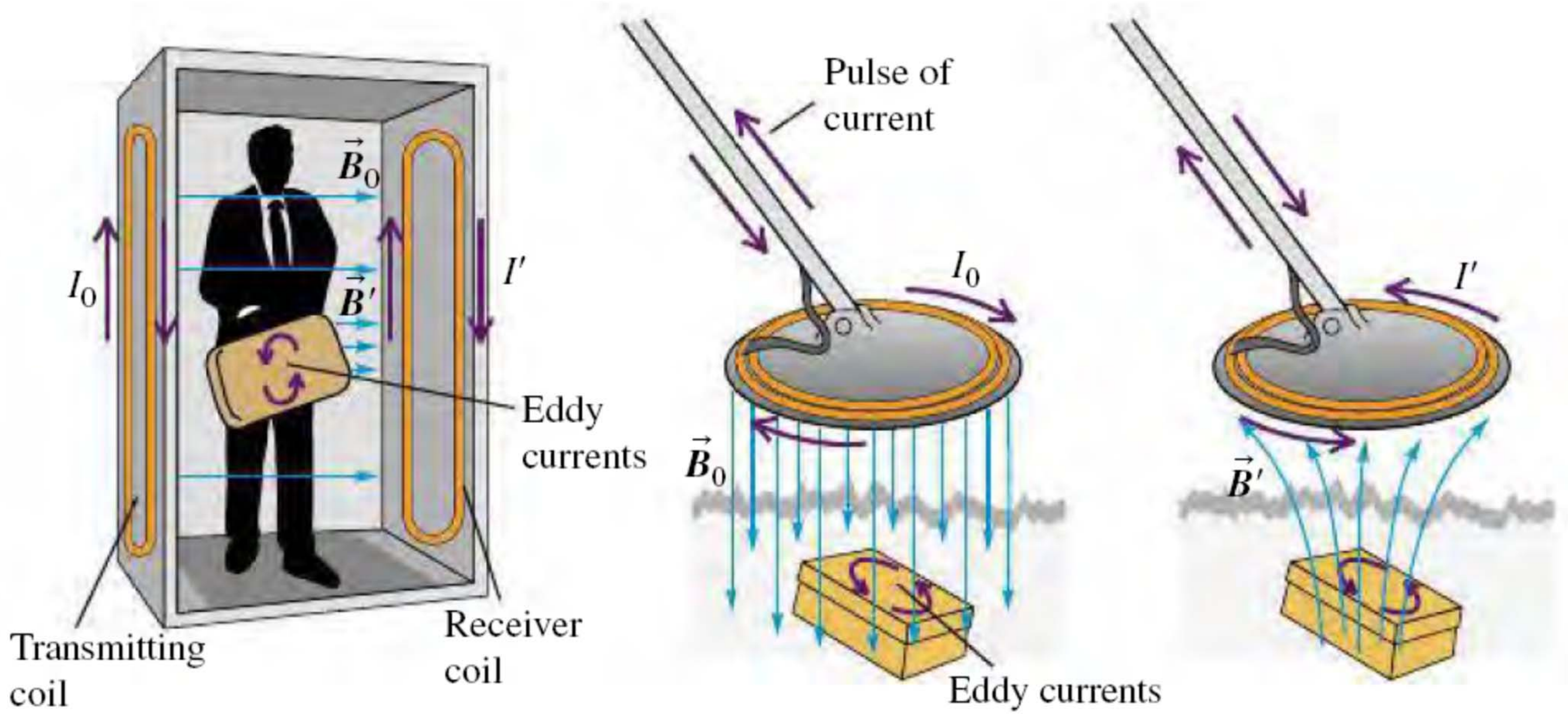
Induction stove



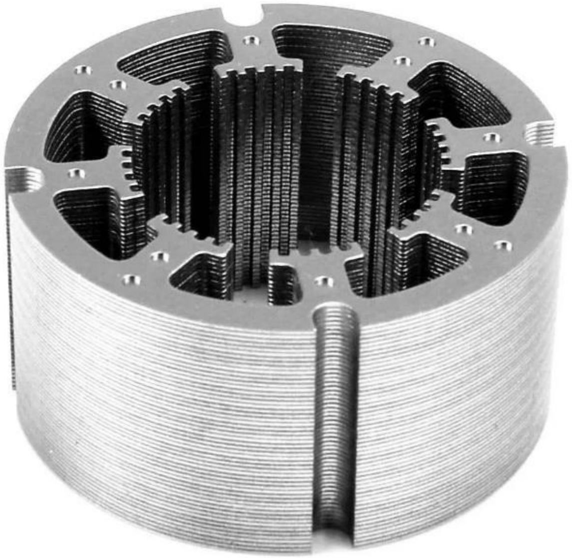
Transformers:



Metal detector:



Reducing the induced currents:



Summary

1. Faraday's law of induction

$$\varepsilon = -\frac{\mathrm{d}\Phi_m}{\mathrm{d}t}$$

2. Lenz's law

An induced EMF is always in a direction that opposes the original change in flux that caused it. (effect of the result is always against the reason)

3. Motional EMF

$$\varepsilon = \int_{-}^{+} \vec{E}_k \cdot \mathrm{d}\vec{l} = \int_{-}^{+} (\vec{v} \times \vec{B}) \cdot \mathrm{d}\vec{l}$$

4. Induced (vortex) electric field

Induced electric field	Electrostatic field
nonconservative	conservative
$\oint_L \vec{E}_i \cdot \mathrm{d}\vec{l} = -\frac{\mathrm{d}\Phi_m}{\mathrm{d}t} \neq 0$	$\oint_L \vec{E}_{静} \cdot \mathrm{d}\vec{l} = 0$
Produced by changing magnetic field	Produced by charges

5. Summary of fields

Electrostatic /	induced electric /	magnetic field
$\oint \vec{E}_s \cdot \mathrm{d}\vec{l} = 0$	$\oint \vec{E}_i \cdot \mathrm{d}\vec{l} = -\frac{\mathrm{d}\Phi_m}{\mathrm{d}t}$	$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I_{encl}$
$\oint \vec{E}_s \cdot \mathrm{d}\vec{S} = \frac{Q_{encl}}{\varepsilon_0}$	$\oint \vec{E}_i \cdot \mathrm{d}\vec{S} = 0$	$\oint \vec{B} \cdot \mathrm{d}\vec{S} = 0$