Circuit Analysis and Design

Academic Year 2025/2026 – Semester 1

Lecture 17 – RC and RL Filter Circuits

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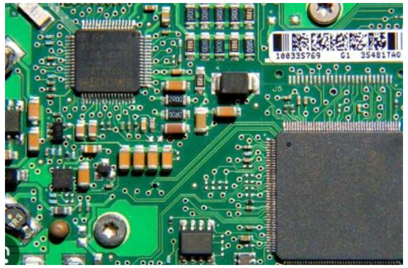
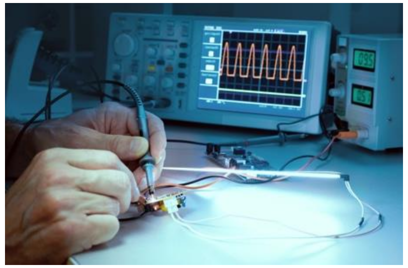




Agenda

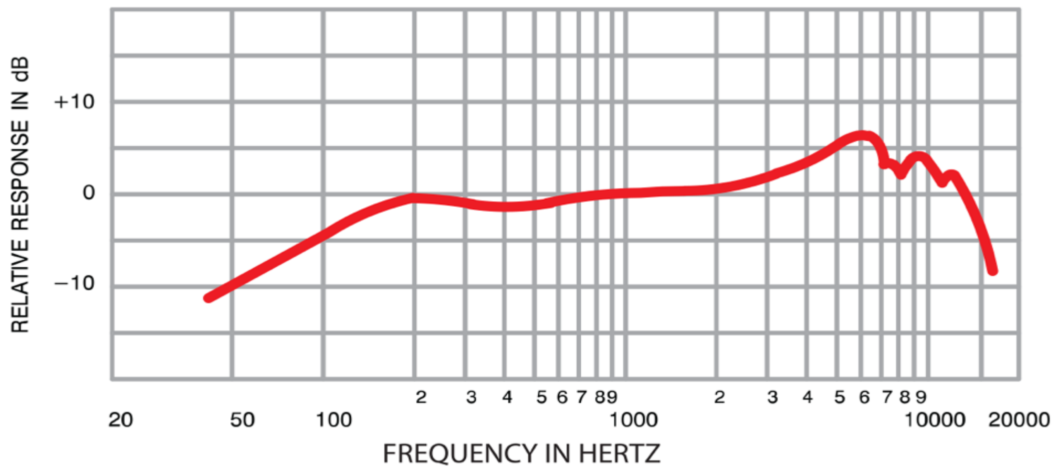
- ❑Frequency response/Transfer function
- ❑Ideal and practical filters
- ❑RC, RL filter circuits (First order filters)

Frequency Response

- If no fixed frequency is defined for the phasor-transformed circuits, it will be interesting to see how they respond to sinusoids as the frequency ω is scanned from zero to infinity.
- The **frequency response** of a circuit is the variation in its behavior with change in signal frequency (but constant amplitude).

Microphone

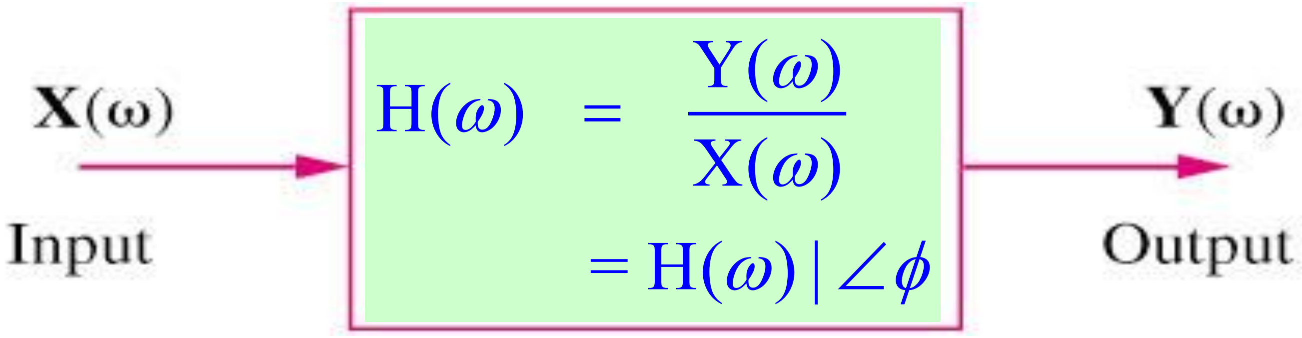
Frequency response of a Microphone



- Microphone frequency response is the frequency-specific output sensitivity of a mic. It details the relative output levels of the sound/audio frequencies a mic is able to reproduce.

Transfer Function

- The **transfer function *H*(ω)** of a circuit is the frequency dependent ratio of the phasor output *Y*(ω) to a phasor input *X*(ω).
- By plotting the magnitude of the transfer function |*H(ω)*|, called the magnitude response, and the phase of the transfer function ∠*H(ω)*, called the phase response, we can gain more insight into the behavior of the circuit as the frequency is varied.



$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = H(\omega)|\angle\phi$$

$$H(\omega) = \text{Voltage gain} = \frac{V_o(\omega)}{V_i(\omega)}$$

$$H(\omega) = \text{Transfer Impedance} = \frac{V_o(\omega)}{I_i(\omega)}$$

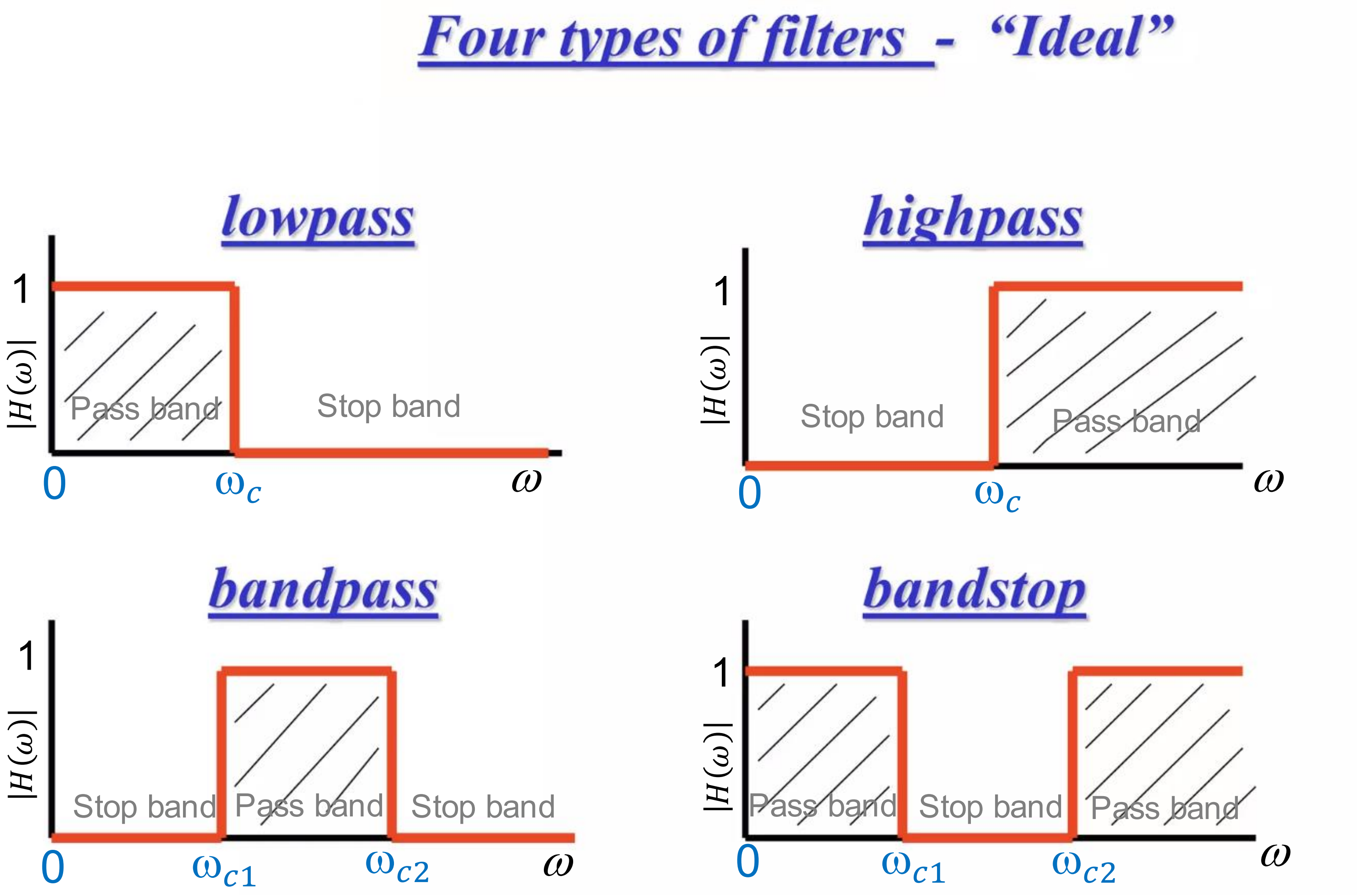
$$H(\omega) = \text{Current gain} = \frac{I_o(\omega)}{I_i(\omega)}$$

$$H(\omega) = \text{Transfer Admittance} = \frac{I_o(\omega)}{V_i(\omega)}$$

Filters

- As a frequency-selective device, a filter is designed to pass/allow/permit certain frequencies and block/reject/attenuate other frequencies.
- Filters are circuits which are characterized according to their transfer function characteristics.
- Common types of filters are :
  - Lowpass filter (LPF)
  - Highpass filter (HPF)
  - Bandpass filter (BPF), and
  - Bandstop filter (BSF)
- The filters that cannot be physically realizable are called ideal filters.
  - The ideal filters have a gain transition from 0 in the stopband to 1 in the passband.
- Typical used in:
  - Audio crossovers, Speaker systems, Noisy power supplies, Radio and TV tuners etc.

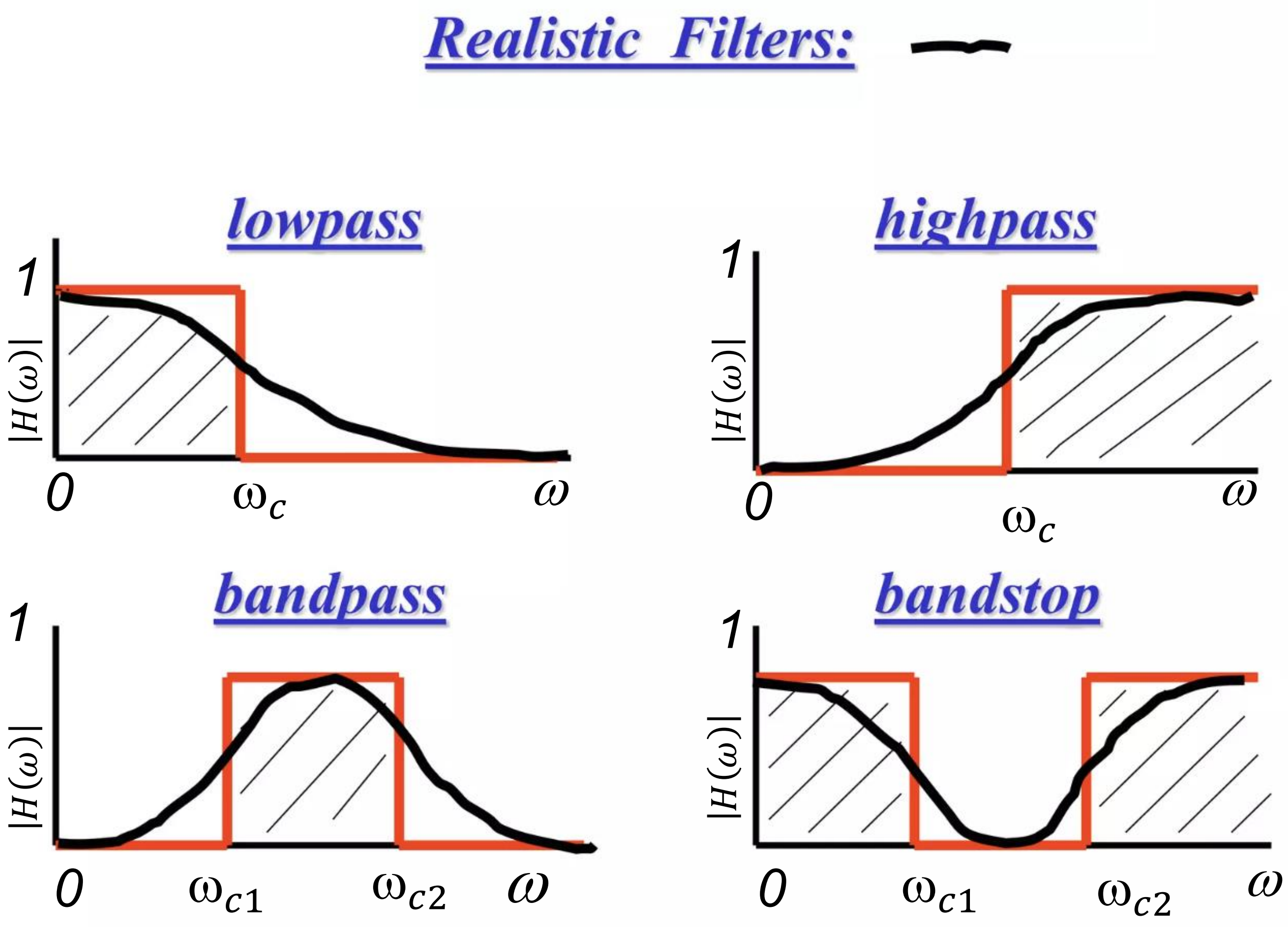
Ideal Filters



ω_c *is* cut-off frequency, ω_{c1} the lower cut-off and ω_{c2} the upper cut-off

Practical Filters

- In practical filters, the gain in the passband cannot be 1 for all frequencies, and the gain in the stopband cannot be zero for all frequencies.
- The transitions from passband to stopband and stopband to passband are gradual.
- The magnitude response changes continuously as a function of radian frequency ω, and the transition from passband to stopband and stopband to passband happens over a finite width of ω.



ω_c *is* cut-off frequency, ω_{c1} the lower cut-off and ω_{c2} the upper cut-off

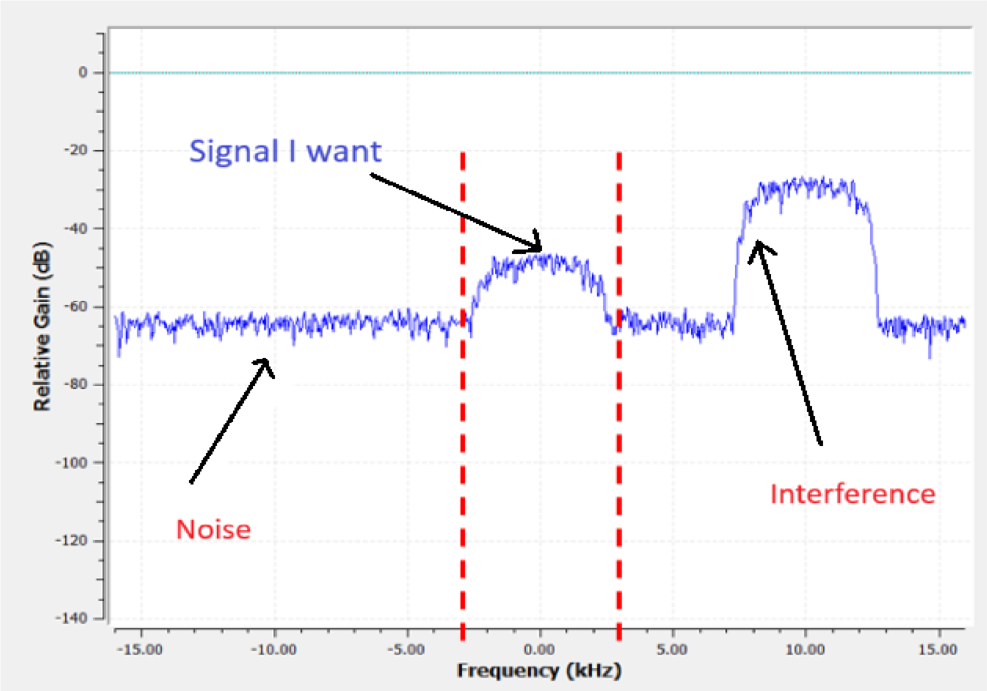
Practical Filters (Cont..)

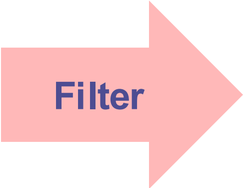
- Filters may be classified as either digital or analog.
  - Digital filters are implemented using a digital computer or special purpose digital hardware.
  - Analog filters are usually implemented with different combinations of R, L, and C components and operational amplifiers. They may be classified as either passive or active.

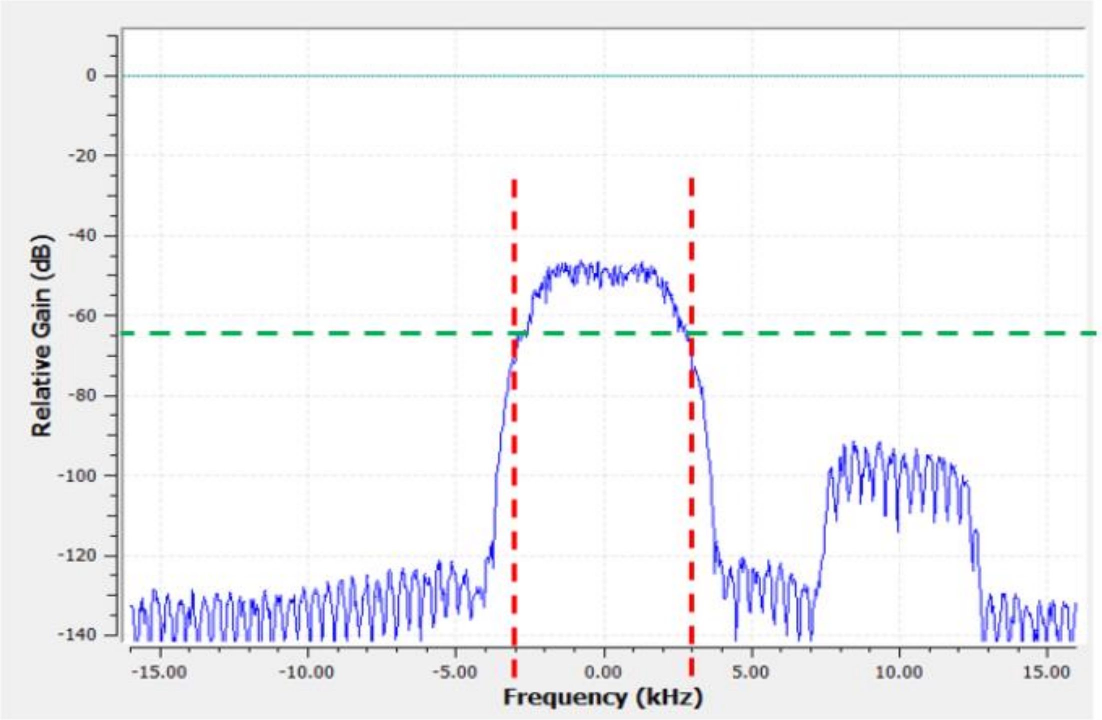
- A passive filter is one that contains only passive components R, L, and C.

- An active filter is one that, along with R, L, and C components, also contains active elements such as transistors and operational amplifier.

Example: Typical use of Filter







Original Signal

Signal after applying the bandpass filter

Key Filter Parameters

- **Roll-off:** The slope of the filter's response in the transition region between the passband and stopband.

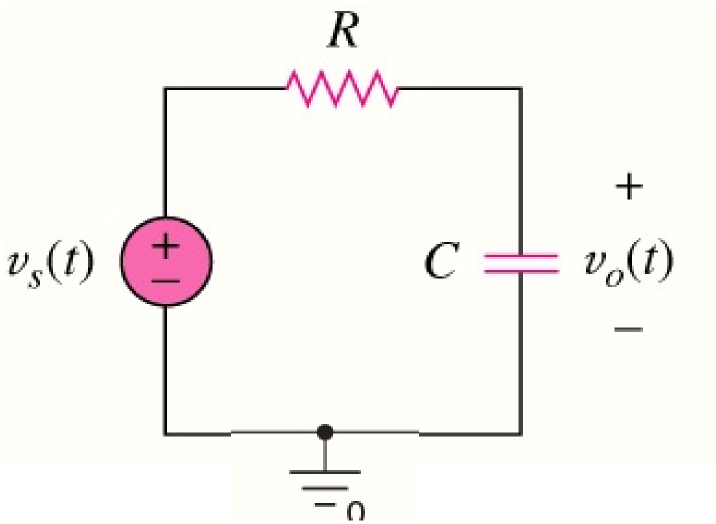
- **Resonance frequency:** A frequency at which the capacitive reactance and inductive reactance are equal in magnitude, thereby resulting in a purely resistive impedance.

- **Cutoff frequency/half-power/critical/corner frequencies:** A frequency that defines the regions of passbands and stopbands. It is sometimes taken as the half-power point (a frequency for which the output of the circuit is −3 dB of the nominal passband value).

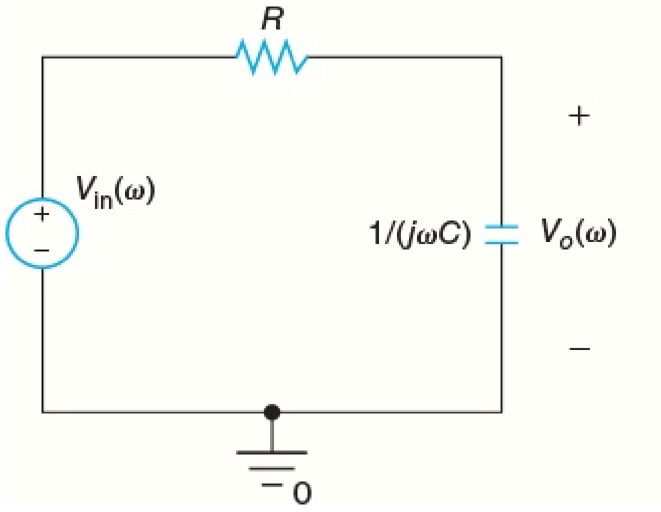
- **Bandwidth (for BPF and BSF):** Width of the passband/stopband defined by the difference between lower and upper cutoff frequencies.

- **Quality factor:** Defines the selectivity of the filter (a quantitative measure of the “sharpness” of the filter response). Higher Q results in higher selectivity and lower bandwidth.

Transfer Function of the RC Circuit



Time Domain RC Circuit



Frequency Domain RC Circuit

- Applying VDR

$$V_o(\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} V_{\text{in}}(\omega) = \frac{1}{RCj\omega + 1} V_{\text{in}}(\omega) = \frac{\frac{1}{RC}}{j\omega + \frac{1}{RC}} V_{\text{in}}(\omega)$$

- Dividing by $V_{in}(\omega)$ on both sides, we obtain the transfer function $H(\omega)$

$$H(\omega) = \frac{V_o(\omega)}{V_{\text{in}}(\omega)} = \frac{\frac{1}{RC}}{j\omega + \frac{1}{RC}}$$

Transfer Function of the RC Circuit (Cont..)

$$H(\omega) = \frac{V_o(\omega)}{V_{\text{in}}(\omega)} = \frac{\frac{1}{RC}}{j\omega + \frac{1}{RC}}$$

- Changing the numerator and denominator to polar coordinates, we obtain

$$H(\omega) = \frac{V_o(\omega)}{V_{\text{in}}(\omega)} = \frac{\frac{1}{RC}e^{j0}}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}\, e^{j\tan^{-1}\left(\frac{\omega}{\frac{1}{RC}}\right)}}$$

$$= \frac{\frac{1}{RC}}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}} e^{-j\tan^{-1}(\omega RC)}$$

$$|H(\omega)| = \frac{\frac{1}{RC}}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}}$$

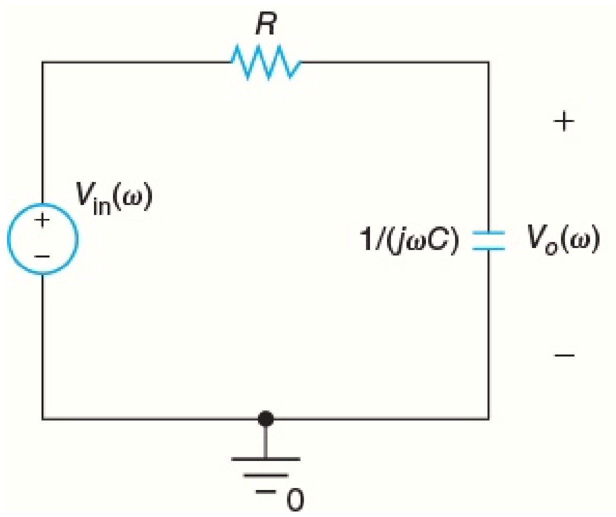
Magnitude response

$$\angle H(\omega) = -\tan^{-1}\left(\frac{\omega}{\frac{1}{RC}}\right) = -\tan^{-1}(\omega RC)$$

Phase response

Transfer Function of the RC Circuit (Cont..)

If $\omega = 0$ (dc input signal), $|H(\omega)| = 1$ and $\angle H(\omega) = 0$. The output signal is identical to the input signal. When $\omega = 0$, the impedance of the capacitor is $1/(j0C) = \infty$, and the capacitor can be treated as an open circuit. There is no current through the resistor, and the voltage drop across the resistor is zero. Thus, the output voltage is the same as the input voltage. If $\omega = \infty$, $|H(\omega)| = 0$ and $\angle H(\omega) = -90°$. When $\omega = \infty$, the impedance of the capacitor is $1/(j\infty C) = 0$, and the capacitor can be treated as a short circuit. The voltage drop across the capacitor is zero. Thus, the output voltage is zero.



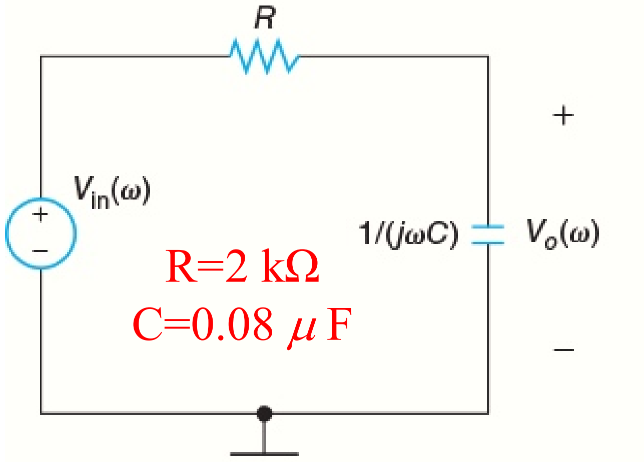
At $\omega = 1/(RC)$,
$|H(\omega)| = 1/\sqrt{2} = 0.7071$,
$20 \log_{10}(1/\sqrt{2}) = -3.01$ dB
$\angle H(\omega) = -\tan^{-1}(1) = -45^{\circ}$

$\omega_0 = 1/(RC)$ is called 3 dB cutoff frequency.
The gain decreased by 3 dB.

$$\omega_0 = \frac{1}{RC} \text{ rad/s}$$

$$H(\omega) = \frac{\omega_0}{j\omega + \omega_0}$$

Transfer Function of the RC Circuit (Cont..)



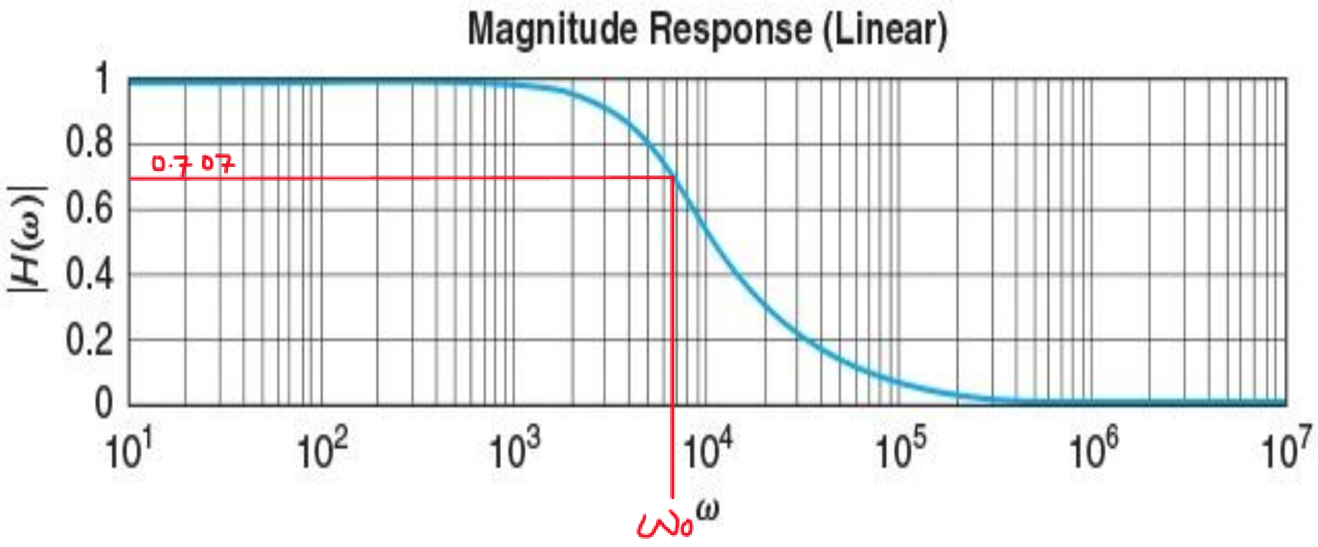
$$|H(\omega)| = \frac{1/_{RC}}{\sqrt{\omega^2 + (1/_{RC})^2}}$$

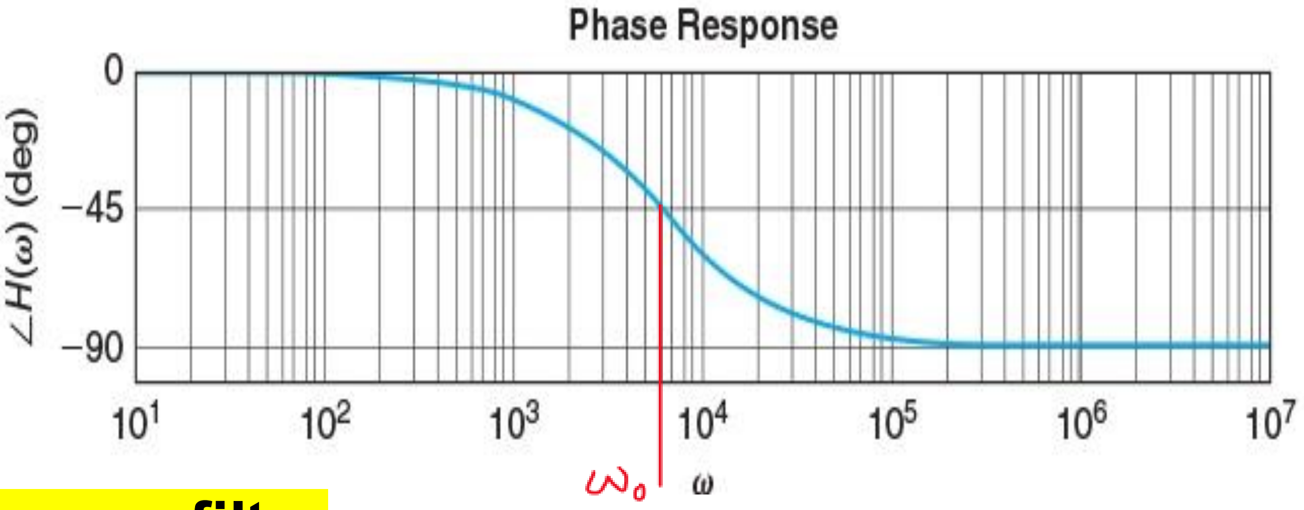
$$\angle H(\omega) = -tan^{-1}(\omega RC)$$

If R = 2 $k\Omega$, C = 0.08 μF,

ω_0 = 1/(RC) = 6250 rad/s

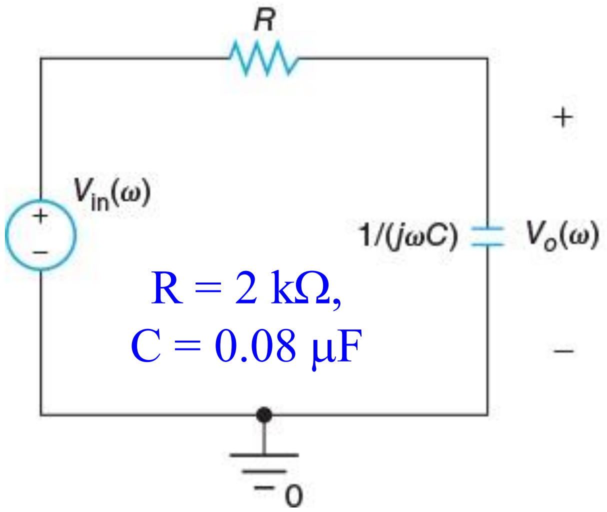
$f_0 = \omega_0/(2\pi)$ = 994.7184 Hz



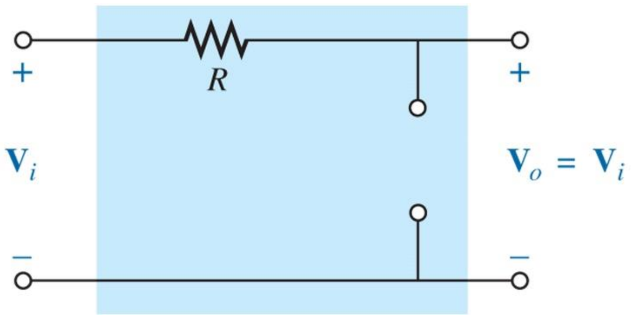


This is lowpass filter

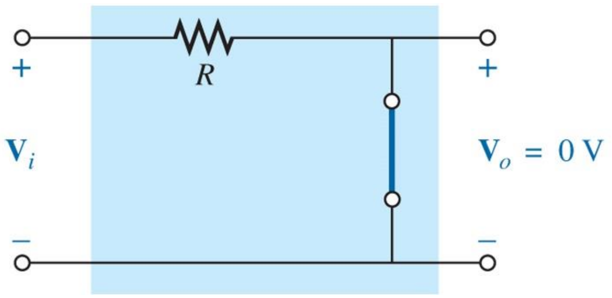
Transfer Function of the RC Circuit (Cont..)



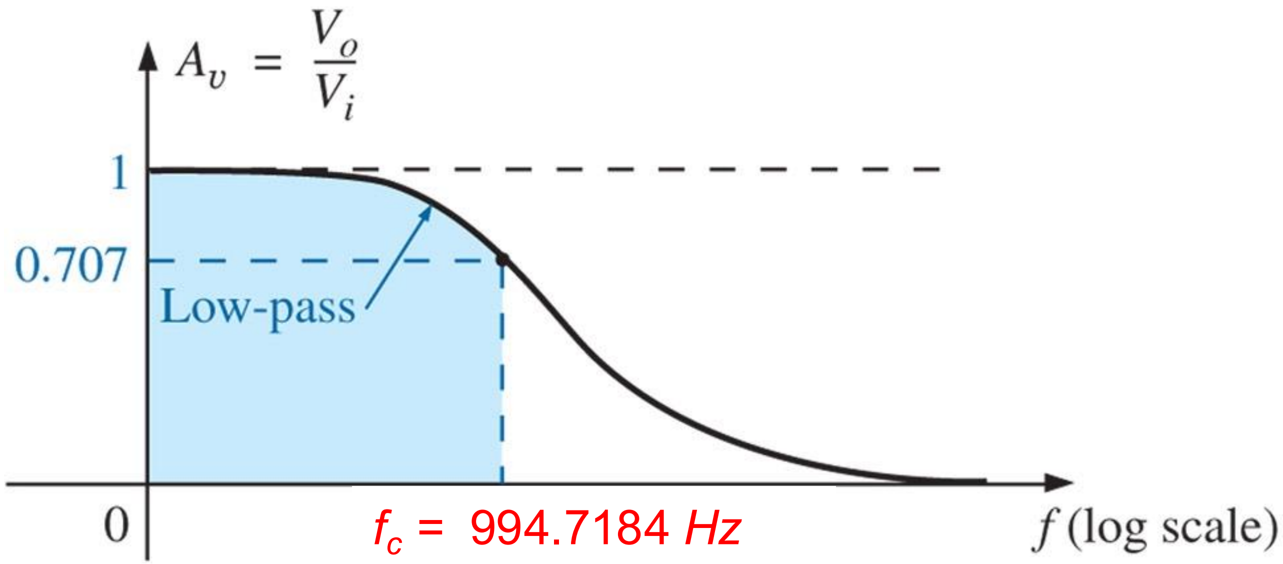
$$z_c = \frac{1}{j\omega C}$$



At low frequencies



At high frequencies



Magnitude plot for a low-pass filter

$\omega_0 = 1/(RC) = 6250\ rad/s$

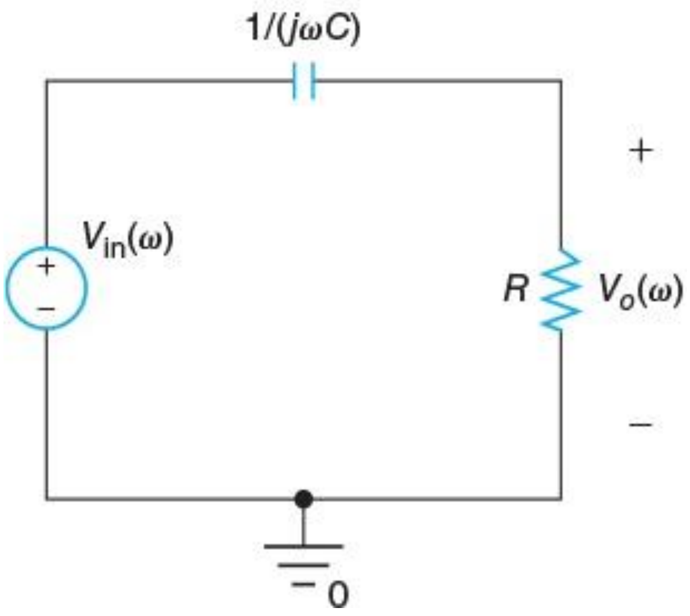
$f_0 = \omega_0/(2\pi) = 994.7184\ Hz$

Transfer Function of the CR Circuit

- Application of the voltage divider rule yields

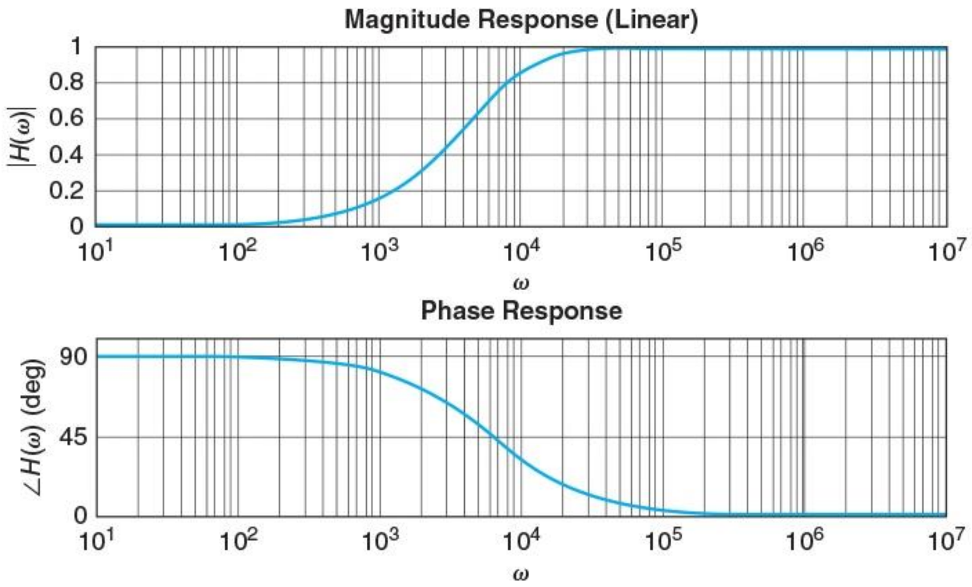
$$H(\omega) = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega}{j\omega + \frac{1}{RC}} = \frac{\omega}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}} e^{j\left[\frac{\pi}{2} - \tan^{-1}(\omega RC)\right]}$$

$$|H(\omega)| = \frac{\omega}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}}, \qquad \angle H(\omega) = \frac{\pi}{2} - \tan^{-1}(\omega RC)$$

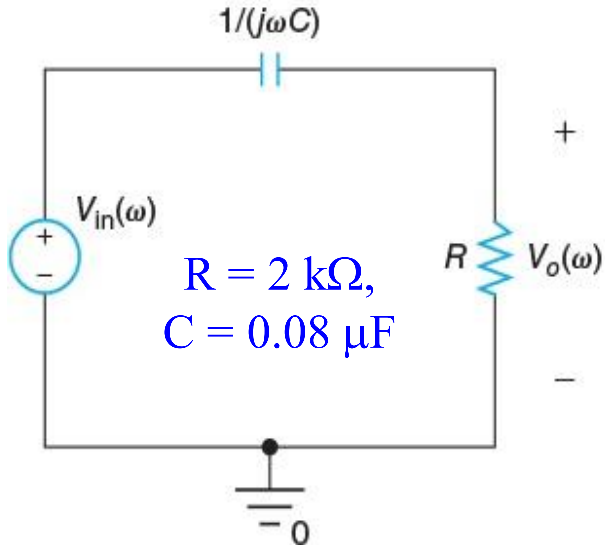


- At $\omega = 1/(RC)$, $|H(\omega)| = 1/\sqrt{2} = 0.7071$, $20 \log_{10}(1/\sqrt{2}) = -3.01$ dB
  $\angle H(\omega) = 90^o - \tan^{-1}(1) = 45^o$
- $\omega_0 = 1/(RC)$ is called 3 dB cutoff frequency.
- If R = 2 kΩ, C = 0.08 μF,
  $\omega_0 = 1/(RC) = 6250$ rad/s
  $f_0 = \omega_0/(2\pi) = 994.7184$ Hz

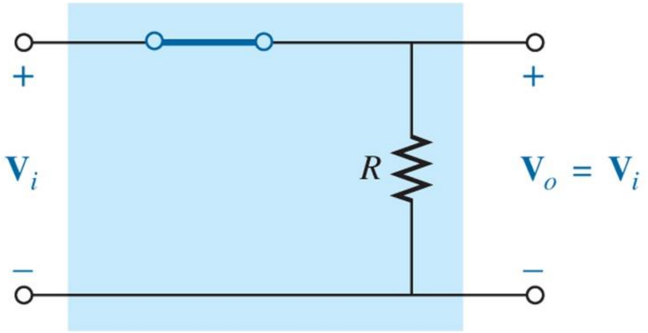
This is an HPF.



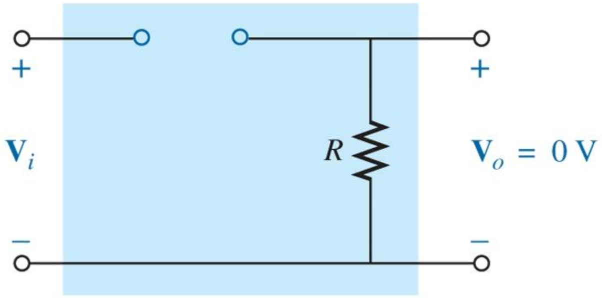
Transfer Function of the CR Circuit (cont.)



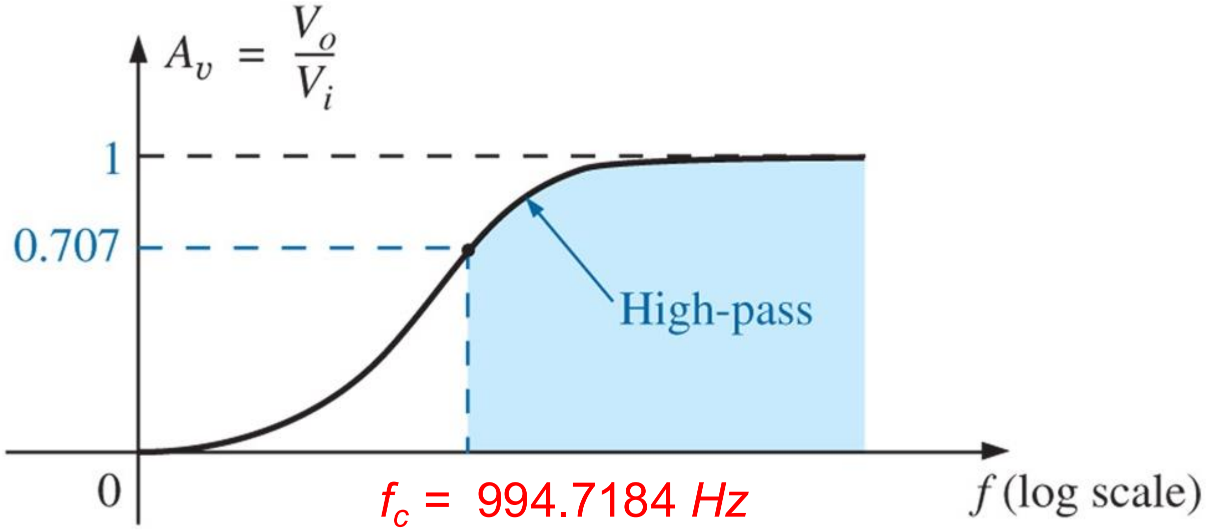
$$z_c = \frac{1}{j\omega C}$$



At high frequencies



At low frequencies

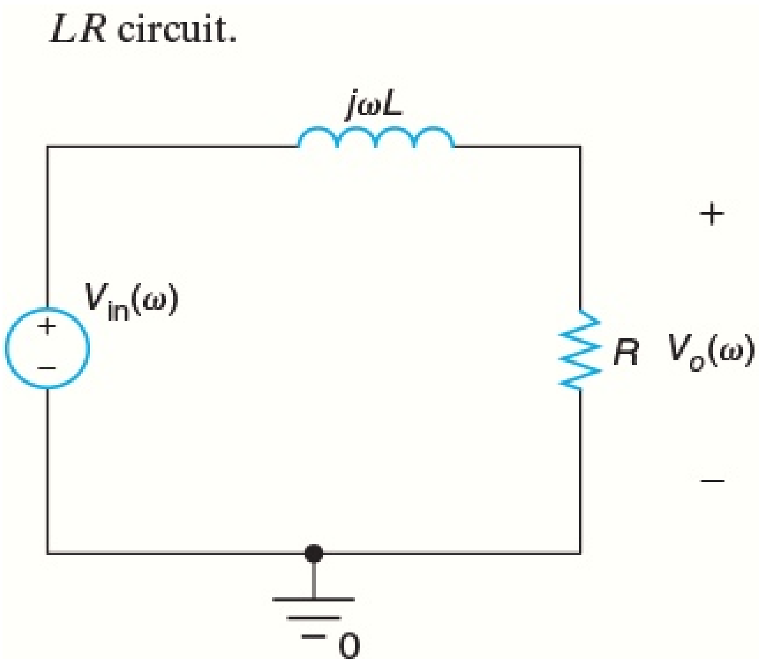


Magnitude plot for a high-pass filter

$\omega_0 = 1/(RC) = 6250\ rad/s$

$f_0 = \omega_0/(2\pi) = 994.7184\ Hz$

Transfer Function of the LR Circuit



- Application of the voltage divider rule yields

$$H(\omega) = \frac{R}{j\omega L + R} = \frac{\frac{R}{L}}{j\omega + \frac{R}{L}} = \frac{\frac{R}{L}}{\sqrt{\omega^2 + \left(\frac{R}{L}\right)^2}} e^{-j\tan^{-1}\left(\frac{\omega L}{R}\right)},$$

$$|H(\omega)| = \frac{\frac{R}{L}}{\sqrt{\omega^2 + \left(\frac{R}{L}\right)^2}}, \angle H(\omega) = -\tan^{-1}\left(\frac{\omega}{\frac{R}{L}}\right)$$

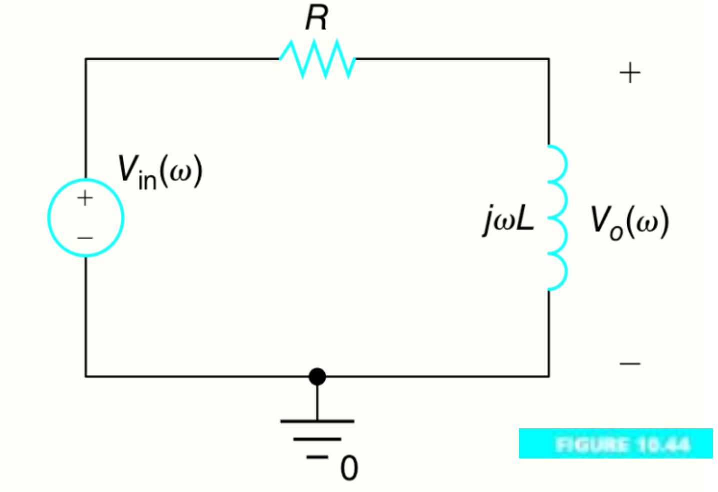
- At $\omega = R/L$, $|H(\omega)| = 1/\sqrt{2} = 0.7071$, $20\log_{10}(1/\sqrt{2}) = -3.01\ dB$
  $\angle H(\omega) = -\tan^{-1}(1) = -45^o$
- $\omega_0 = R/L$ is the $3\ dB$ cutoff frequency.

This is an LPF.

Transfer Function of the RL Circuit

❑ Find $H(\omega) = V_o(\omega)/V_{in}(\omega)$ and state filter type for the circuit shown in Figure 10.44.

$$V_o(\omega) = \frac{j\omega L}{R+j\omega L}V_{in}(\omega) = \frac{j\omega}{R/_L+j\omega}V_{in}(\omega) \qquad H(\omega) = \frac{V_o(\omega)}{V_{in}(\omega)} = \frac{j\omega}{R/_L+j\omega}$$



- Changing to polar coordinates

$$H(\omega) = \frac{\omega e^{j\frac{\pi}{2}}}{\sqrt{\omega^2+(R/_L)^2}e^{j\tan^{-1}(\frac{\omega L}{R})}} = \frac{\omega}{\sqrt{\omega^2+(R/_L)^2}}e^{j[(\frac{\pi}{2}-\tan^{-1}(\frac{\omega L}{R})]}$$

$$|H(\omega)| = \frac{\omega}{\sqrt{\omega^2+(R/_L)^2}} \qquad \angle H(\omega) = \frac{\pi}{2} - \tan^{-1}(\frac{\omega L}{R})$$

At $\omega = R/L$, $|H(\omega)| = 1/\sqrt{2} = 0.7071$, $20\log_{10}(1/\sqrt{2}) = -3.01$ dB

$\angle H(\omega) = \frac{\pi}{2} - \tan^{-1}(1) = 45^o$

$\omega_0 = R/L$ is the 3 dB cutoff frequency.

This is an HPF.

Summary

- The transfer function $H(\omega)$ is the ratio of the output to input and defines the behavior of the circuit in response to frequency change.
- A filter is a device that passes certain frequencies and blocks other frequencies. Common types of filters are lowpass filter (LPF), highpass filter (HPF), bandpass filter (BPF), and bandstop filter (BSF).
- The filters that cannot be realized are called ideal filters.
- In practical filters, the gain in the passband cannot be 1 for all frequencies, and the gain in the stopband cannot be zero for all frequencies. Also, the transitions from passband to stopband and stopband to passband are gradual.
- A simple first order LPF can be implemented in RC circuit or LR circuit. A simple first order HPF can be implemented in CR circuit or RL circuit.
- What will we study next?