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# Circuit Analysis and Design

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*“A good student never steal or cheat”*

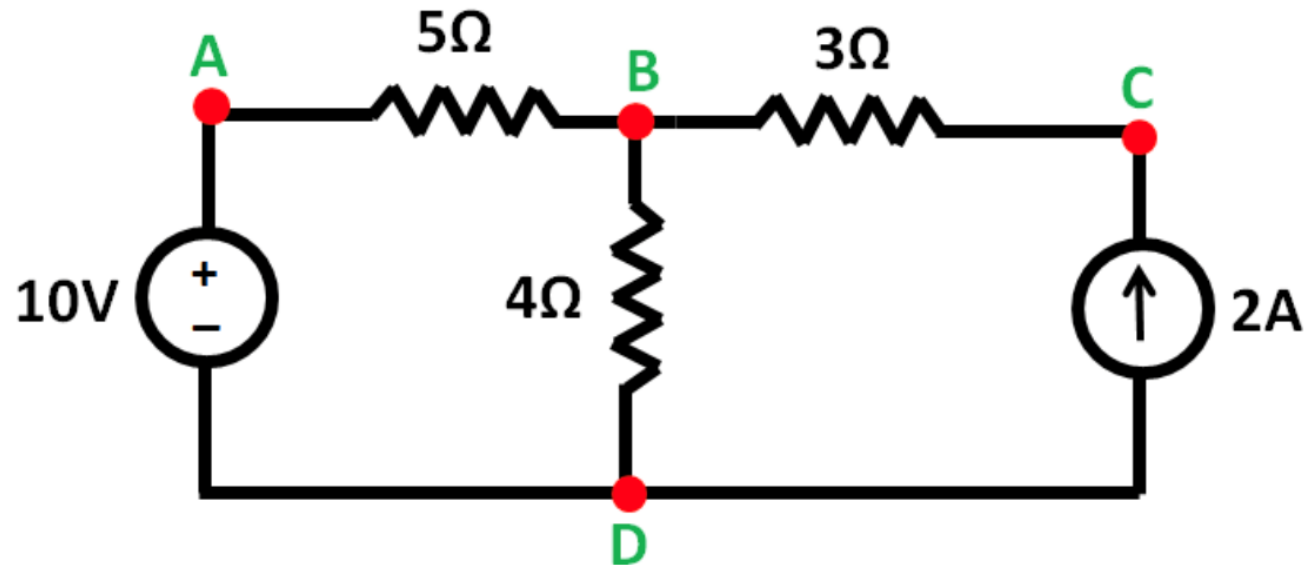


# Agenda

- ☐ Definition of node, branch, path, loop, and mesh
- ☐ Resistor
- ☐ Ohm's law
- ☐ Kirchhoff's current law (KCL)
- ☐ Kirchhoff's voltage law (KVL)
- ☐ Equivalent resistance of series connection of resistors
- ☐ Equivalent resistance of parallel connection of resistors
- ☐ Summary

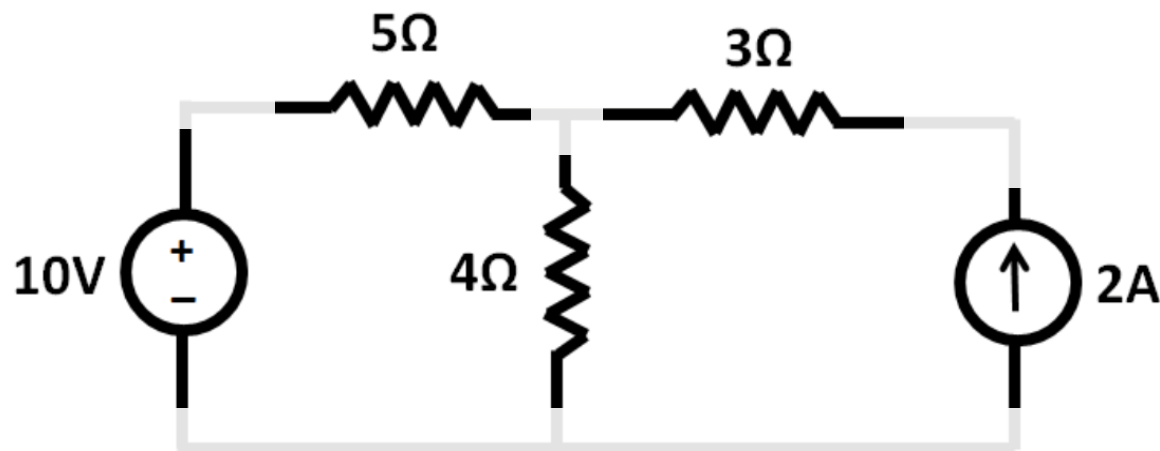
# Circuit

□ A circuit is an interconnection of elements, which can be voltage sources, current sources, resistors, capacitors, inductors, coupled coils, transformers, op amps, etc.



# What is Branch

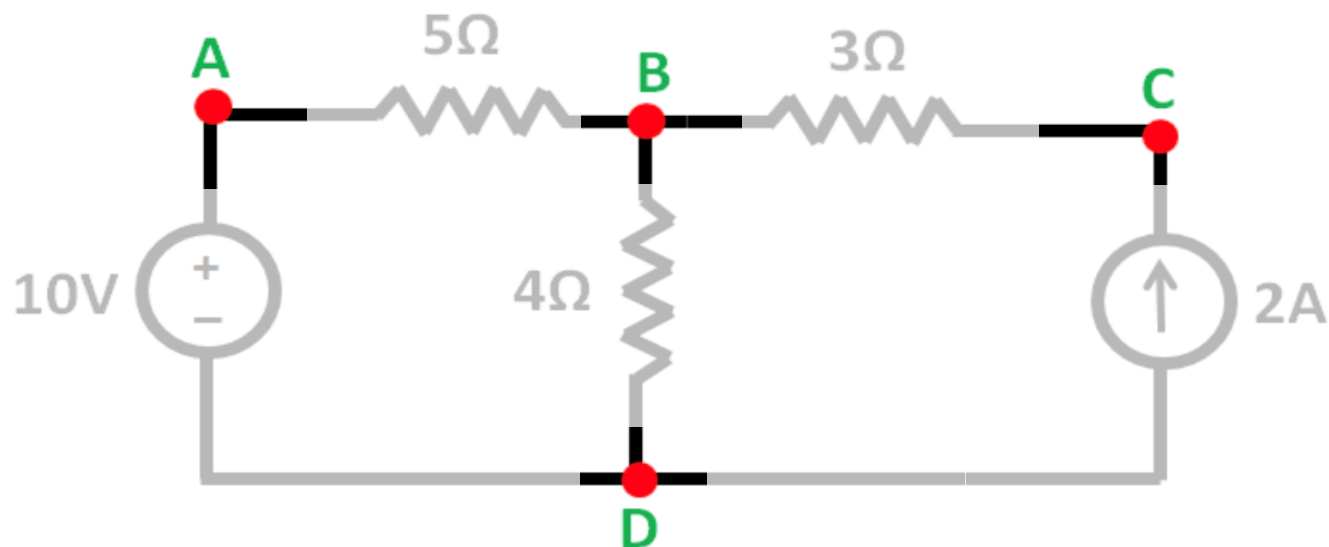
- ❑ A branch represents the single circuit element like resistor, capacitor, inductor, voltage, or current source



- ❑ For example, for the circuit shown in figure 1, there are five branches. A 10 V voltage source, 2A current source, 4  $\Omega$ , 5  $\Omega$ , and 3  $\Omega$  resistors.
- ❑ Current in a branch is always the same.

# What is Node?

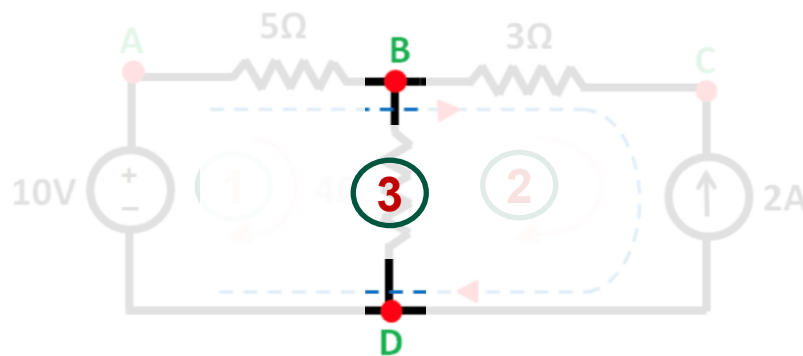
- A node is a point in the circuit where two or more circuit elements (or branch) are connected)



- For example, the above circuit contains Four Nodes. The node A, B, C, and D

# What is Loop

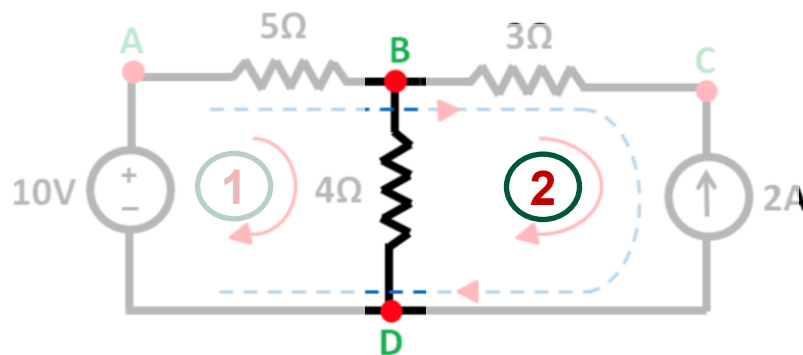
- Any closed path in circuit is called a loop
  - A closed path formed by starting at a node, passing through a set of nodes, and returning to starting node without passing through any node more than once.



- For example, the circuit contains **three loops**,
  - 1<sup>st</sup> loop **A-B-D-A**, 2<sup>nd</sup> loop **B-C-D-B**, 3<sup>rd</sup> loop is **A-B-C-D-A**

# What is Mesh

- A mesh is a closed path in the circuit, which does not contain any other close path inside it.
- 1<sup>st</sup> loop **A-B-D-A**, 2<sup>nd</sup> loop **B-C-D-B** does not contain any loop inside, Therefore there are two meshes in this circuit



- All meshes are loops but not oops are meshes

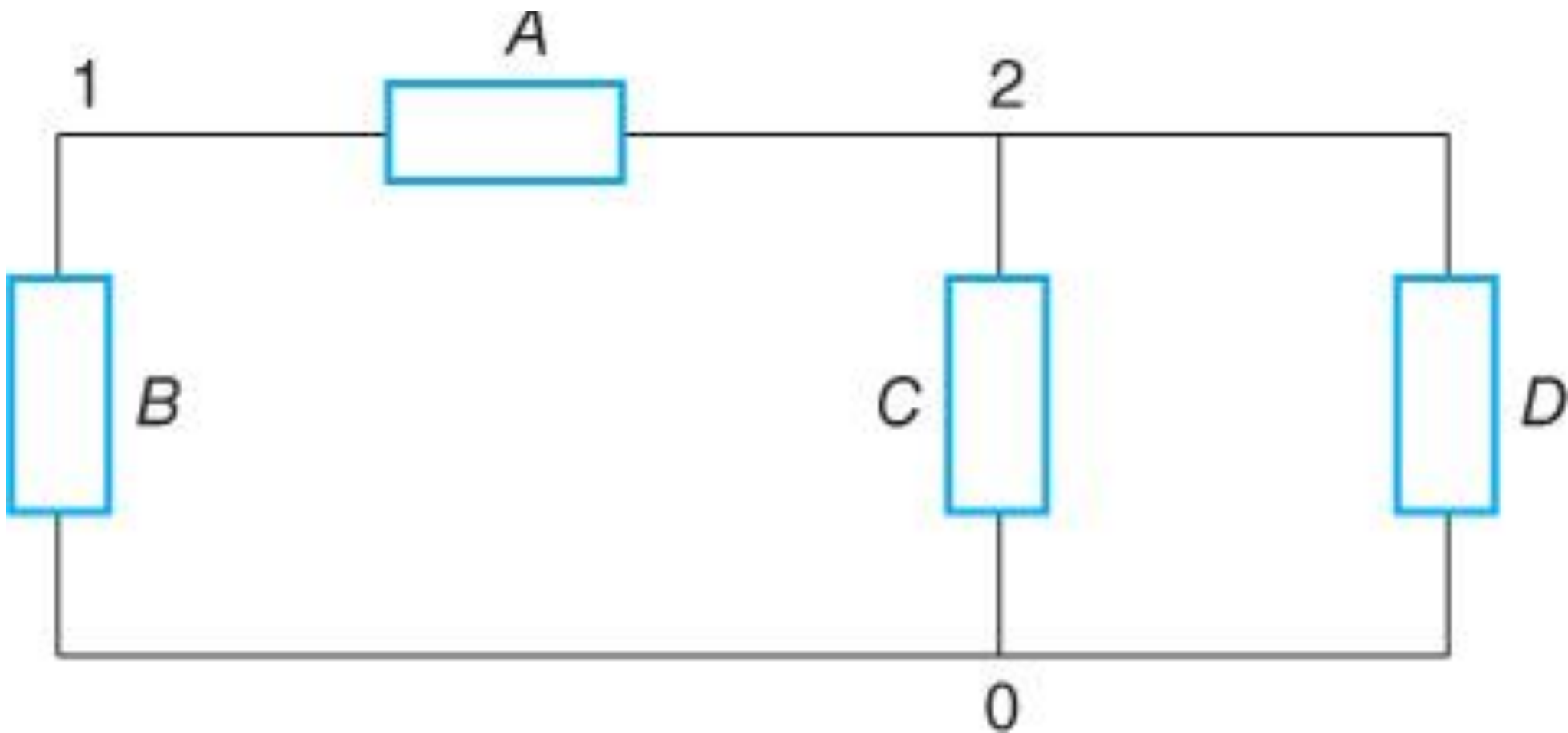
## Further Definitions, Node, Loop, and Mesh

- ❑ A **simple node** is a node that connects two elements.
- ❑ An **essential node** is a node that connects three or more elements.
- ❑ A **path** in a circuit is a series of connected elements from a node to another node that does not go to the same node more than once.
- ❑ The **ground node** where the voltage is at ground level is usually taken to be the reference node.
- ❑ The voltage of a node measured with respect to a reference node is called **node voltage**.
- ❑ The current through a mesh is called **mesh current**.



## EXAMPLE 2.1

□ Find all the nodes, loops, and meshes for the following circuit.



## EXAMPLE 2.1

□ Find all the nodes, loops, and meshes for the circuit shown in Figure 2.1.

There are three nodes (labeled as 0, 1, 2).

Node 1 is a simple node

nodes 2 and 0 are essential nodes.

There are three loops in the circuit

0-B-1-A-2-D-0

0-B-1-A-2-C-0

0-C-2-D-0

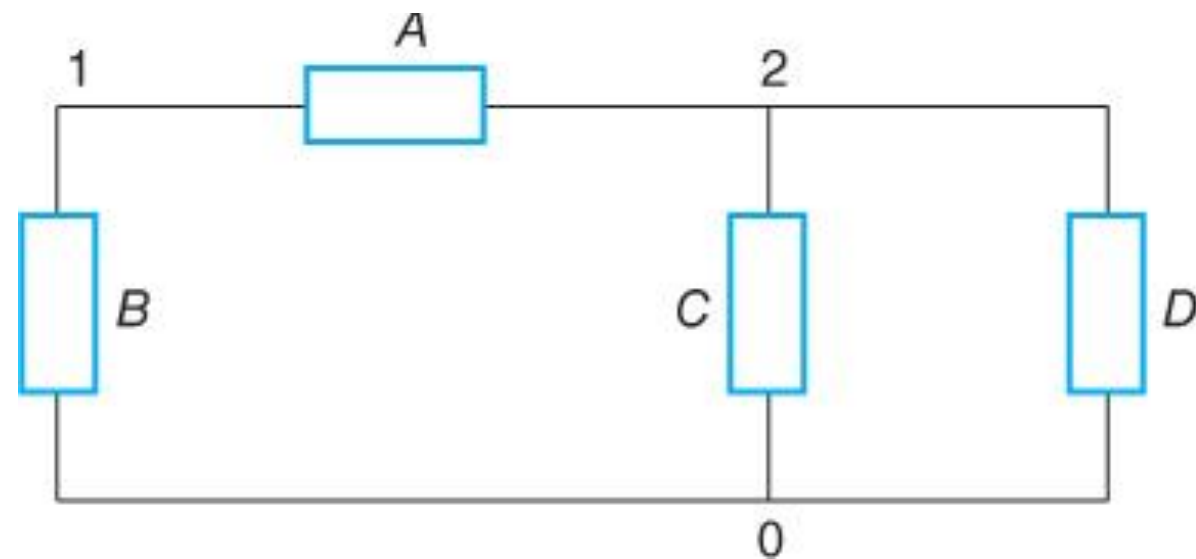
There are two meshes in the circuit

0-B-1-A-2-C-0

0-C-2-D-0

The loop 0-B-1-A-2-D-0 contains two meshes

0-B-1-A-2-C-0 and 0-C-2-D-0.

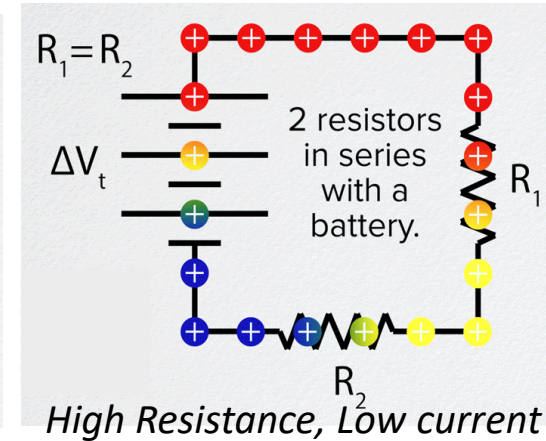
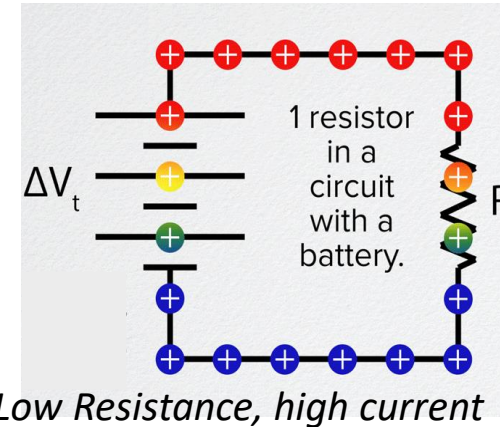


# Resistors

❑ Resistor is a circuit component that regulates flow of current.

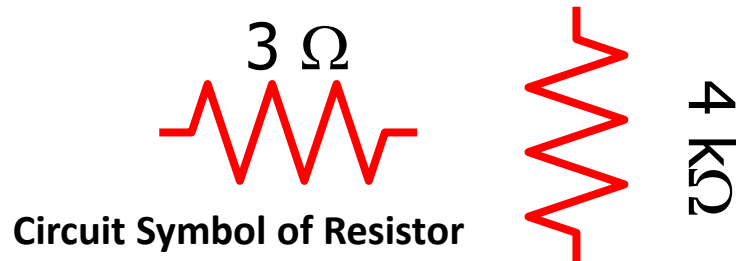
❑ **Resistance** of resistor is measures of **its ability to limit current**.

- When resistance value is large, the amount of current flow through the resistor is small.
- If resistance value is small, the amount of current flow through the resistor is large



❑ Resistance value of a resistor is determined by conductivity (or resistivity) of material used to make it, as well as its dimensions.

❑ **Low-power** resistors can be made from carbon composition material made of **fine granulated graphite mixed with** clay. For **high power**, **wire-wound resistors** can be used. The wire-wound resistors are constructed by twisting a wire made of **nichrome** or similar material around a ceramic core.



# Resistance

- ❑ The current density is defined as the amount of current through the unit area. If  $A$  is the cross-sectional area of a wire that carries a constant current  $I$ , the current density is given by:  $J = \frac{I}{A}$
- ❑ It can be shown that the current density is proportional to the electric field intensity:  $J = \sigma E$ ,  $\sigma$  is the conductivity of the material.
- ❑ Let  $\ell$  be the length of the wire and  $V$  be the potential difference (voltage) between the ends of the wire. The potential difference generates a constant electric field  $E$  inside the conductor. The potential difference  $V$  is related to the electric field through  $V = E\ell$
- ❑ Substituting  $E = V/\ell$  and  $J = I/A$  into  $J = \sigma E$ , we obtain  $I/A = \sigma V/\ell$ .  
Thus,  $V = [\ell/(\sigma A)]I$ .
- ❑ The resistance is defined as

$$R = \frac{\ell}{\sigma A}$$

## Resistance (Continued)

❑ The resistance is proportional to the length of the wire and inversely proportional to the cross-sectional area of the wire and conductivity of the material.

❑ The resistivity  $\rho$  of the material is the inverse of the conductivity:

$$\rho = 1/\sigma$$

❑ In terms of the resistivity, the resistance is given by:  $R = \frac{\rho \ell}{A}$

❑ The resistance is proportional to the length of the wire and resistivity, and inversely proportional to the cross-sectional area of the wire.

❑ In terms of resistance  $R$ , Equation,  $V = \frac{\ell}{\sigma A} I = RI$  (Ohms Law)  
The unit for resistance is ohm ( $\Omega$ ).

# Resistance (Continued)

Example: What is the resistance of a wire with radius 1 mm, length 10 m, conductivity  $5 \times 10^4$  S/m?.

- (A)  $R=63.662 \Omega$
- (B)  $R=636.62 \Omega$
- (C)  $R=6.3662 \Omega$
- (D)  $R=63 \text{ k}\Omega$

Solution:

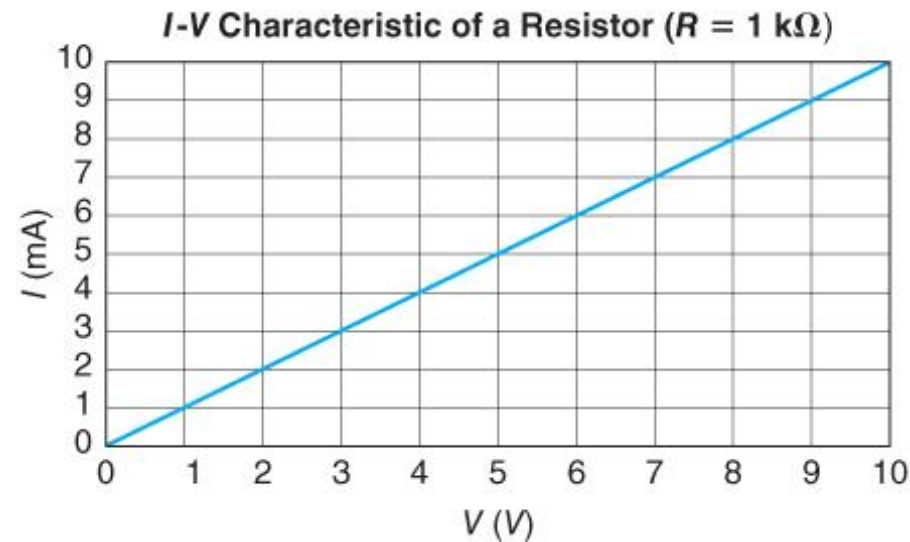
$$R = \frac{\ell}{\sigma A}$$
$$= \frac{10}{\pi \times 0.001^2 \times 5 \times 10^4} = 63.662 \Omega$$

# Ohm's Law ( $V = RI$ , $I = V/R$ , $R = V/I$ )

- ❑ The voltage-current relation of a resistor is given by:  $V = RI$
- ❑ The **voltage across a resistor is proportional to the current through the resistor**. The proportionality constant in this linear relation is the resistance  $R$ . For the given current  $I$ , the voltage across the resistor increases as  $R$  increases.
- ❑ The current through the resistor is proportional to the voltage across the resistor.
- ❑ Conductance defined by  $G = 1/R$ . For the given voltage  $V$ , the current through the resistor decreases as  $R$  increases.
- ❑ The unit for conductance is siemens (S).

FIGURE 2.5

I-V characteristic of a resistor.



## Ohm's Law and Power absorbed by Resistor

- ❑ The resistance  $R$  of a resistor is given by  $R = \frac{V}{I}$
- ❑ The resistance of a resistor is the ratio of voltage to current.
- ❑ Power absorbed by a resistor is given by  $P = IV = VI$  (Watt)
- ❑ The power absorbed by a resistor is the product of the current through the resistor and the voltage across the resistor. Substituting  $V = IR$  into  $P = IV$ , we get

$$P = I^2R \text{ (Watt)}$$

- ❑ The power absorbed by a resistor is the product of the square of the current through the resistor and the resistance value. Substituting  $I = \frac{V}{R}$  into  $P = IV$ , we get  $P = \frac{V^2}{R}$  (Watt)
- Using different units for current, voltage, and resistance in power calculations will lead to power being expressed in different units.

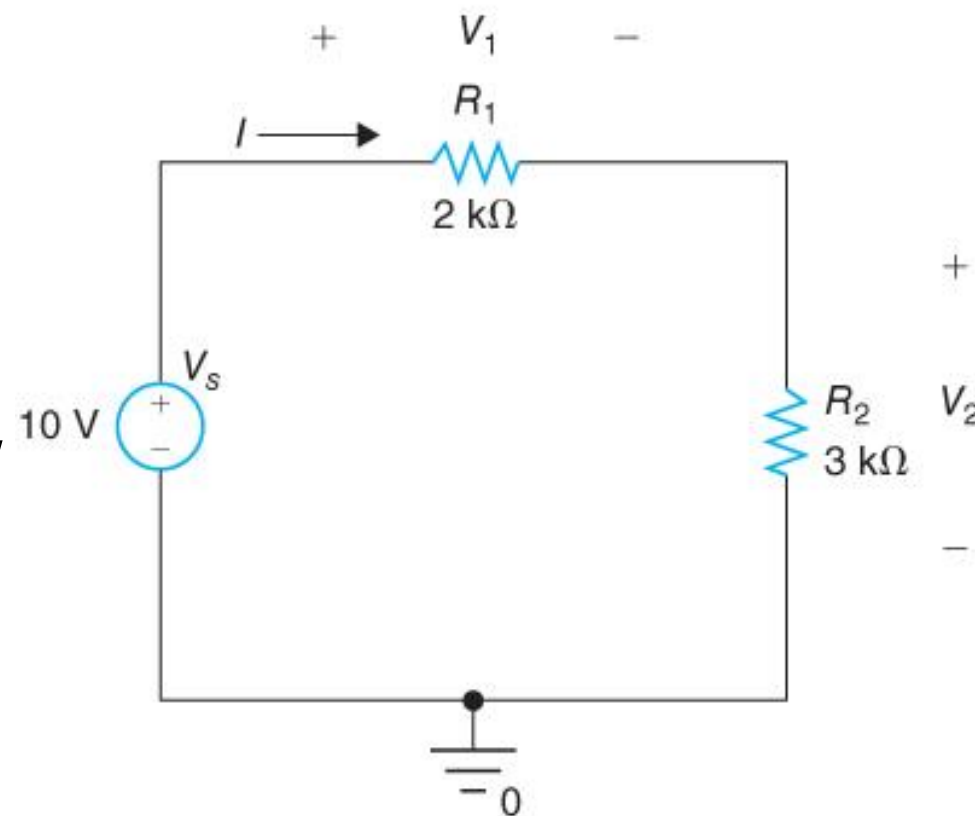


# Ohm's Law and Power absorbed by Resistor

□ Example: Given  $I = 2 \text{ mA}$ , find  $V_1$ ,  $V_2$ , and powers.

- $V_1 = R_1 \times I = 2000 \times 0.002 = 4 \text{ V}$  (Ohm's law)
- $V_2 = R_2 \times I = 3000 \times 0.002 = 6 \text{ V}$  (Ohm's law)
- $P_{R1} = I \times V_1 = 0.002 \times 4 = 0.008 \text{ W} = 8 \text{ mW}$
- $P_{R2} = I \times V_2 = 0.002 \times 6 = 0.012 \text{ W} = 12 \text{ mW}$
- $P_{V_s} = -I \times V_s = -0.002 \times 10 = -0.02 \text{ W} = -20 \text{ mW}$
- Power absorbed by  $R_1$  and  $R_2 = 20 \text{ mW}$
- Power generated by  $V_s = -20 \text{ mW}$
- Power absorbed = Power released

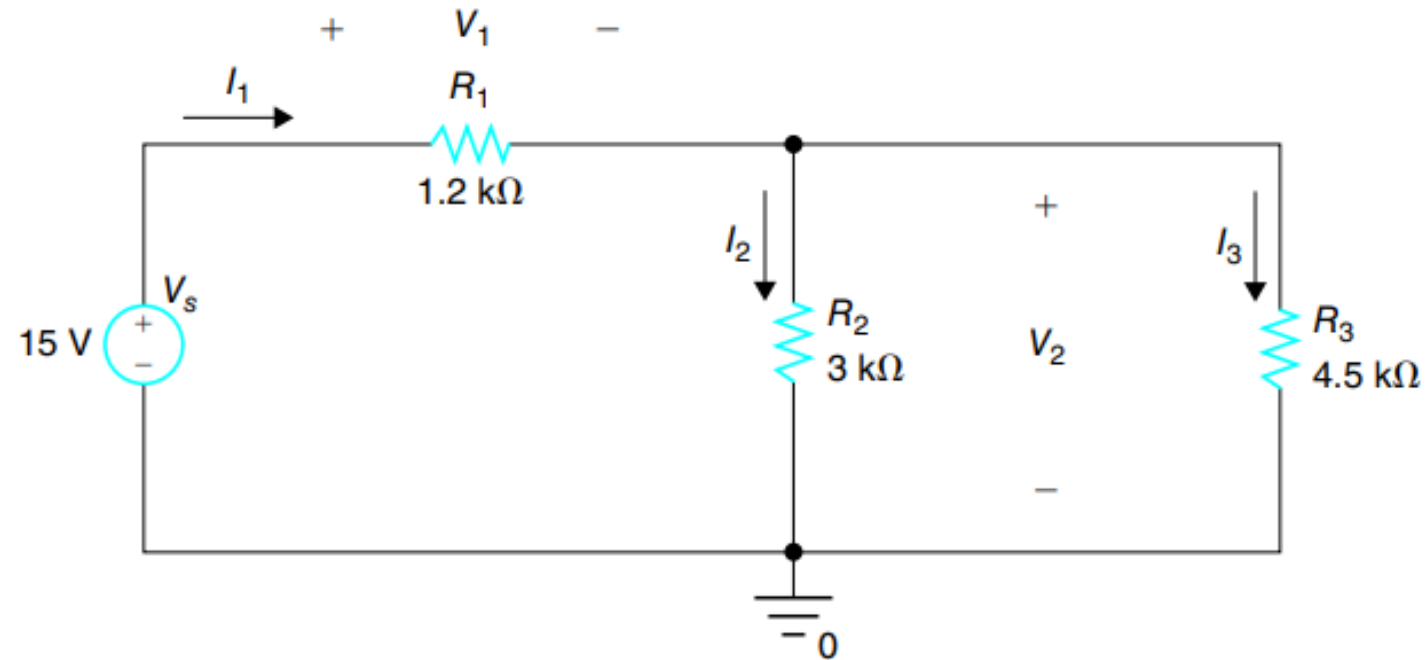
Circuit with two resistors and a voltage source.



# Ohm's Law and Power absorbed by Resistor

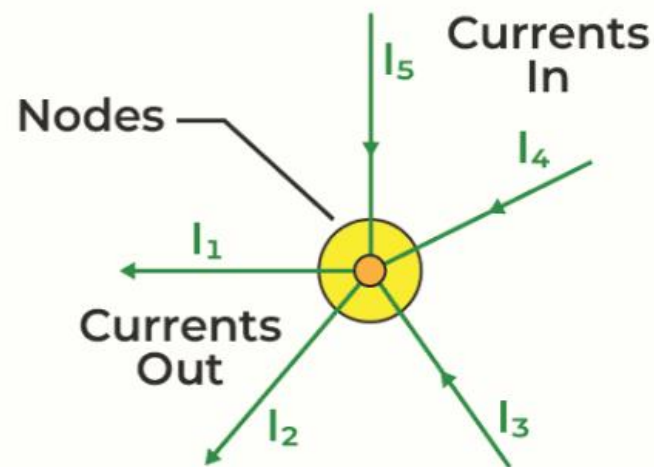
**Example:** Given  $V_2 = 9\text{ V}$ , find  $I_2$ ,  $I_3$ ,  $V_1$ ,  $I_1$ , and powers in the following circuit.

- $I_2 = V_2/R_2$   
 $I_2 = 9\text{ V}/3\text{ k}\Omega = 3\text{ mA}$
- $I_3 = V_2/R_3$   
 $= 9\text{ V}/4.5\text{ k}\Omega = 2\text{ mA}$
- $V_1 = V_s - V_2 = 15\text{ V} - 9\text{ V} = 6\text{ V}$ ,
- $I_1 = V_1/R_1 = 6\text{ V}/1.2\text{ k}\Omega = 5\text{ mA}$
- $P_{R1} = I_1 V_1 = 30\text{ mW}$ ,
- $P_{R2} = I_2 V_2 = 27\text{ mW}$ ,
- $P_{R3} = I_3 V_2 = 18\text{ mW}$ ,
- $P_{V_s} = -I_1 V_s = -75\text{ mW}$
- $P_{R1} + P_{R2} + P_{R3} + P_{V_s} = 0$



Problem P2.1 to P2.14

# Kirchhoff's Current Law (KCL)



- The sum of currents entering a node equals the sum of currents leaving the same node.

$$I_3 + I_4 + I_5 = I_1 + I_2$$

- The number of charges entering a node per second must equal the number of charges leaving the same node per second.

- The sum of currents leaving a node is zero.

$$I_1 + I_2 - I_3 - I_4 - I_5 = 0$$

- At least one of the currents leaving the node must be negative (meaning that the current actually enters the node).

- The sum of currents entering a node is zero.

$$-I_1 - I_2 + I_3 + I_4 + I_5 = 0$$

- At least one of the currents entering the node must be negative (meaning that the current actually leaves the node).

# Kirchhoff's Current Law (KCL)

- The sum of currents entering a node equals the sum of currents leaving the same node.

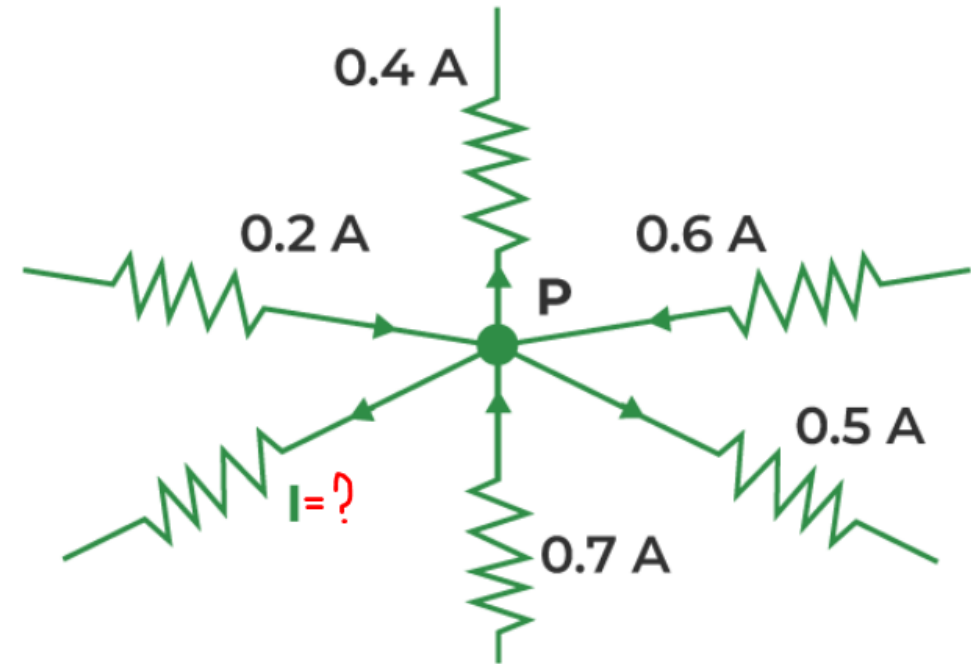
**Example : Find the value of current  $I$  from the circuit diagram.**

- Apply KCL to the **Node P** in the circuit:

- **Current entering = Current leaving**

- $0.2\text{ A} + 0.6\text{ A} + 0.7\text{ A} = 0.4\text{ A} + 0.5\text{ A} + I$
- $0.2\text{ A} - 0.4\text{ A} + 0.6\text{ A} - 0.5\text{ A} + 0.7\text{ A} - I = 0$
- $1.5\text{ A} - 0.9\text{ A} - I = 0$
- $0.6\text{ A} - I = 0$

- **$I = 0.6\text{ A}$**



# Kirchhoff's Current Law (KCL)

❑ **Example 2.1:** The sum of currents entering a node equals the sum of currents leaving the same node.

❑  $I_2 = V_s / R_1$

$$= 12 \text{ V} / 3 \text{ k}\Omega = 4 \text{ mA}$$

❑  $I_3 = V_s / R_2 = 12 \text{ V} / 4 \text{ k}\Omega = 3 \text{ mA}$

❑  $I_4 = V_s / R_3 = 12 \text{ V} / 6 \text{ k}\Omega = 2 \text{ mA}$

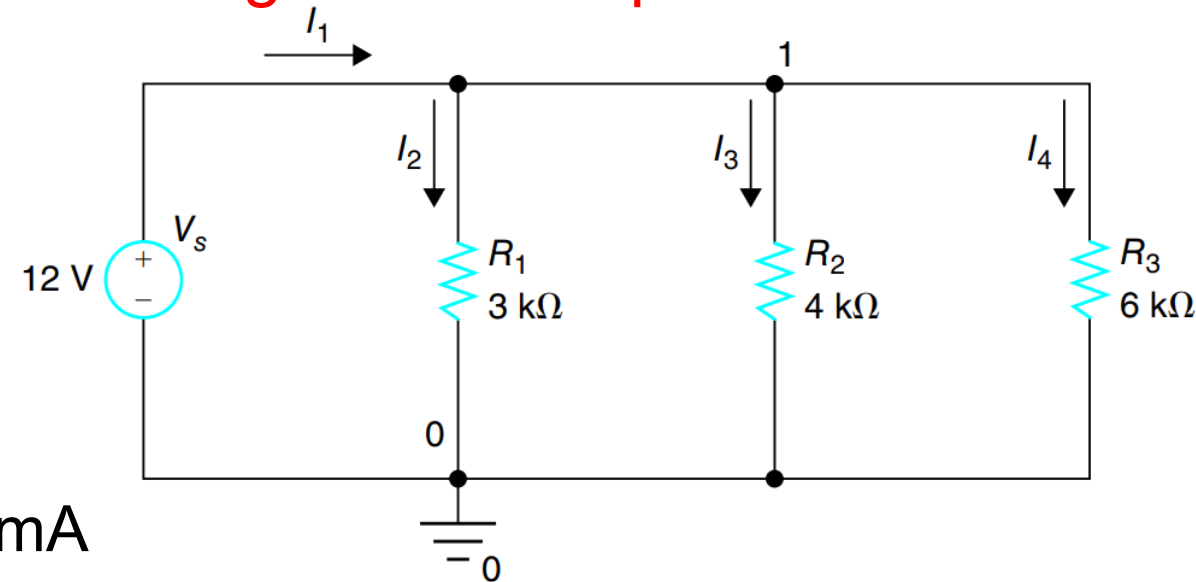
❑  $I_1 = I_2 + I_3 + I_4 = 4 \text{ mA} + 3 \text{ mA} + 2 \text{ mA} = 9 \text{ mA}$

❑ Sum of currents entering node 1 =  $I_1 = 9 \text{ mA}$

❑ Sum of currents leaving node 1 =  $I_2 + I_3 + I_4 = 9 \text{ mA}$

❑ Sum of all currents leaving node 1 =  $-I_1 + I_2 + I_3 + I_4 = -9 \text{ mA} + 4 \text{ mA} + 3 \text{ mA} + 2 \text{ mA} = 0$

❑ Sum of all currents entering node 1 =  $I_1 - I_2 - I_3 - I_4 = 9 \text{ mA} - 4 \text{ mA} - 3 \text{ mA} - 2 \text{ mA} = 0$



# Kirchhoff's Current Law (KCL)

□ **Example 2.3:** Given  $I_3 = 3 \text{ mA}$ , find  $V_3$ ,  $I_4$ ,  $I_2$ ,  $V_2$ ,  $I_1$ , and  $V_1$  in the circuit.

$$V = RI$$

$$\circ V_3 = R_3 I_3 = 2000 \times 0.003 = 6 \text{ V}$$

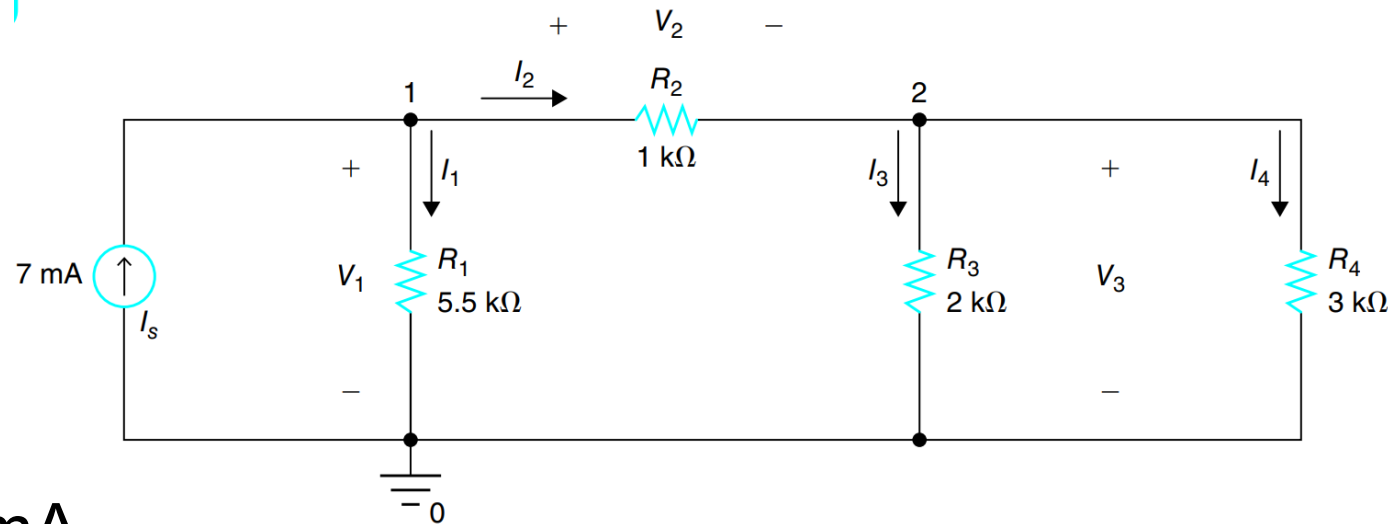
$$\circ I_4 = V_3 / R_4 = 6 / 3000 \\ = 2 \times 10^{-3} \text{ A} = 2 \text{ mA}$$

$$\circ I_2 = I_3 + I_4 = 3 \text{ mA} + 2 \text{ mA} = 5 \text{ mA}$$

$$\circ V_2 = R_2 I_2 = 1000 \times 0.005 = 5 \text{ V}$$

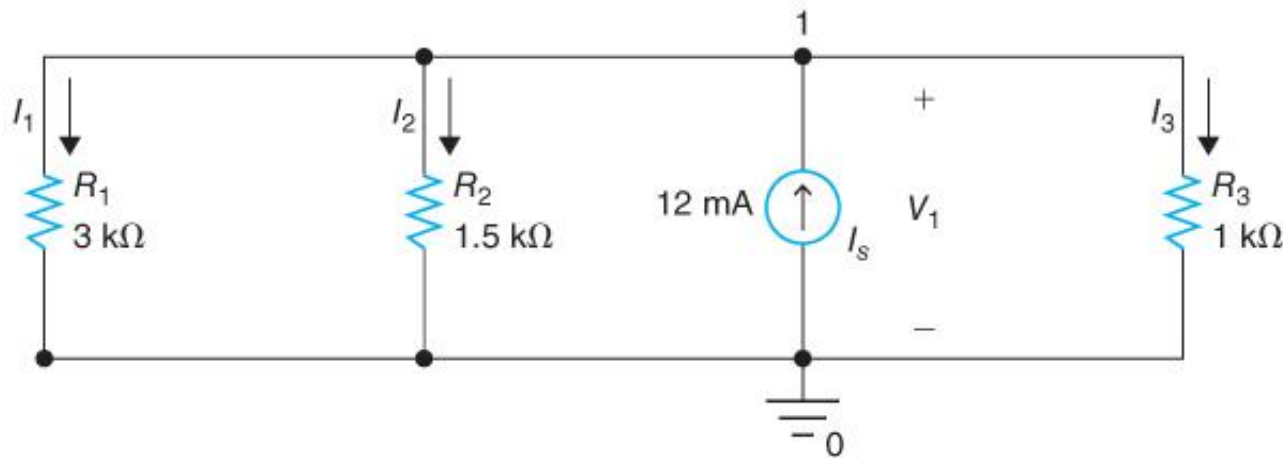
$$\circ I_1 = I_s - I_2 = 7 \text{ mA} - 5 \text{ mA} = 2 \text{ mA}$$

$$\circ V_1 = R_1 I_1 = 5500 \times 0.002 = 11 \text{ V}$$



## Example 2.4

□ Find  $V_1$ , in the circuit shown in Figure 2.13. Use KCL?



Options

- A. 4 V
- B. 6 V
- C. 12 V

★ Multiple Choice

Solution will be provided in class

# Kirchhoff's Current Law (KCL)

□ Example 2.5: Given  $I_1 = 3\text{ A}$ ,  $I_3 = 10\text{ A}$ , and  $I_6 = -8\text{ A}$ , find  $I_2$ ,  $I_4$ , and  $I_5$  in the following circuit.

□ KCL at node1: Sum of current entering Node1 = 0

$$I_2 + I_3 - I_1 = 0$$

$$I_2 = I_1 - I_3 = 3\text{ A} - 10\text{ A} = -7\text{ A}$$

□ KCL at node3: Sum of current entering Node3 = 0

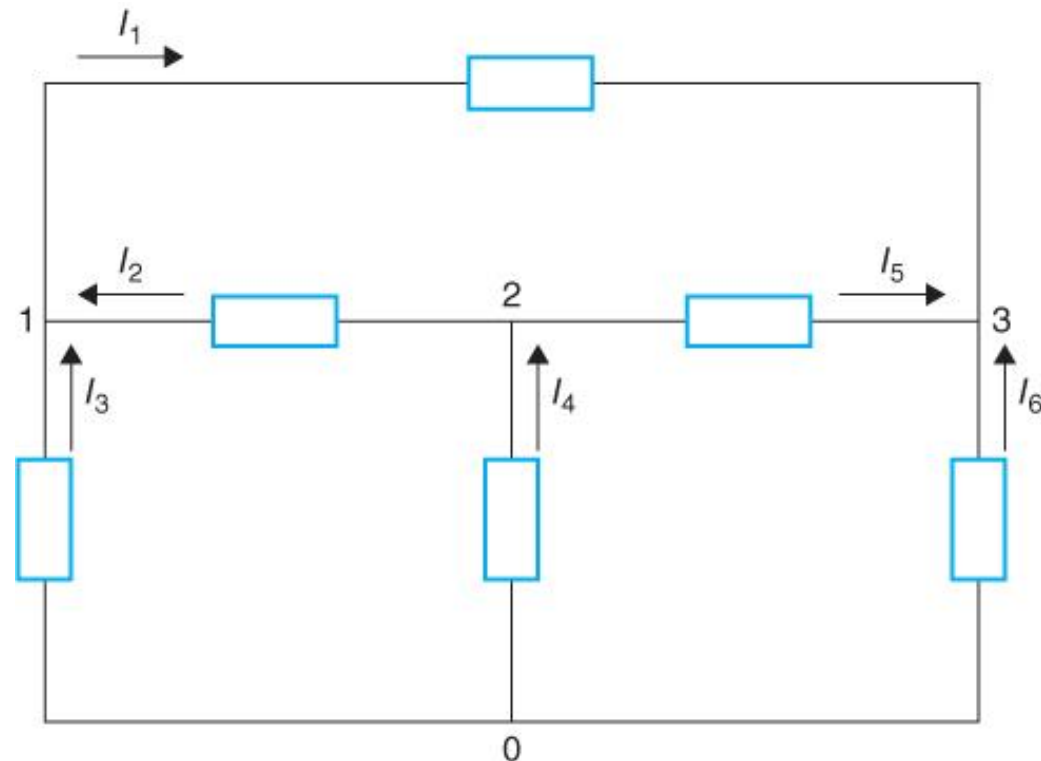
$$I_1 + I_5 + I_6 = 0$$

$$I_5 = -I_1 - I_6 = -3\text{ A} - (-8\text{ A}) = 5\text{ A}$$

○ KCL at node2: Sum of current entering Node2 = 0

$$-I_2 + I_4 - I_5 = 0$$

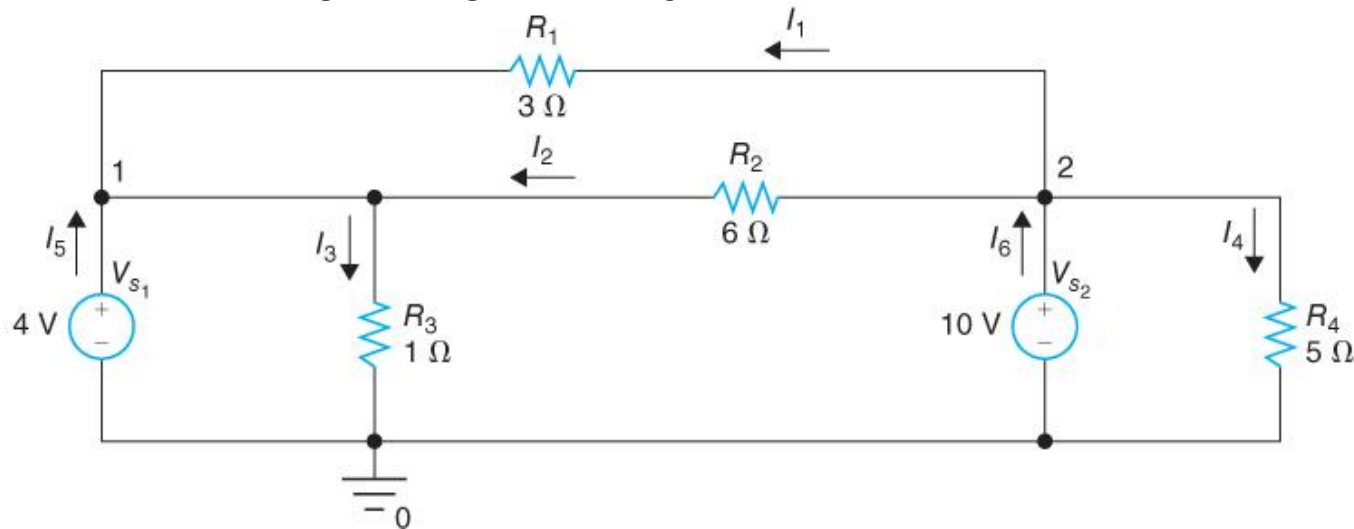
$$I_4 = I_2 + I_5 = -7\text{ A} + 5\text{ A} = -2\text{ A}$$





# Kirchhoff's Current Law (KCL)

□ **Example 2.6:** Find  $I_1$ ,  $I_2$ ,  $I_3$ ,  $I_4$ ,  $I_5$ , and  $I_6$  in the following circuit.



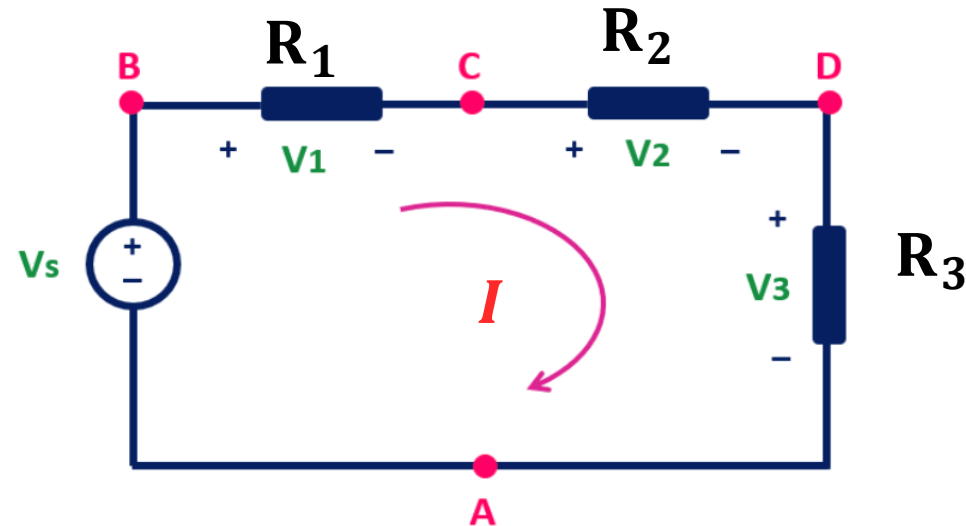
○ Ohm's law:  $I_1 = (V_{s2} - V_{s1})/R_1 = 6 \text{ V}/3 \Omega = 2 \text{ A}$ ,  $I_2 = (V_{s2} - V_{s1})/R_2 = 6 \text{ V}/6 \Omega = 1 \text{ A}$

$$I_3 = V_{s1}/R_3 = 4 \text{ V}/1 \Omega = 4 \text{ A}, I_4 = V_{s2}/R_4 = 10 \text{ V}/5 \Omega = 2 \text{ A}$$

○ KCL at node1:  $I_5 = -I_1 - I_2 + I_3 = -2 \text{ A} - 1 \text{ A} + 4 \text{ A} = 1 \text{ A}$

○ KCL at node2:  $I_6 = I_1 + I_2 + I_4 = 2 \text{ A} + 1 \text{ A} + 2 \text{ A} = 5 \text{ A}$

# Kirchhoff's Voltage Law (KVL)



□ Kirchhoff's Voltage Law (KVL) states that

Sum of Voltage Rise in a Loop = Sum of Voltage Drop in a Loop

$$V_s = V_1 + V_2 + V_3$$

□ Sign Convention of Elements: (KVL)



# Kirchhoff's Voltage Law (KVL)

- ❑ The sum of voltage drops around a loop equals the sum of voltage rises of the same loop.
- ❑ The sum of voltage drops around a loop is zero.
  - At least one of the voltage drops around the loop must be negative (meaning that the voltage actually rises on the branch).
- ❑ The sum of voltage rises around a loop is zero.
  - At least one of the voltage rises around the loop must be negative (meaning that the voltage actually drops on the branch) for this statement to be true.
- ❑ The voltage of a node must be unique, and the voltage for any node cannot have two different values.
- ❑ Since a mesh is also a loop, the KVL applies to mesh as well.

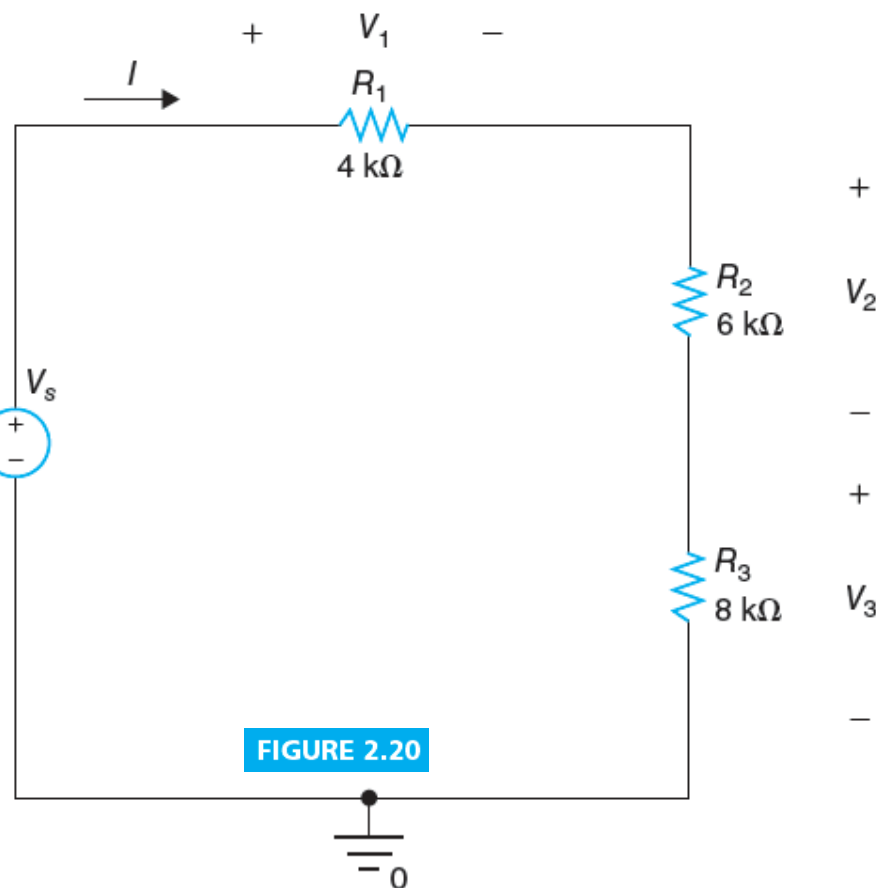
# Kirchhoff's Voltage Law (KVL)

□ Consider a circuit shown in Figure 2.20. We are interested in finding the voltages across the resistors  $R_1$ ,  $R_2$ , and  $R_3$  and the current through them.

- Ohm's law:  $V_1 = R_1 I$ ,  $V_2 = R_2 I$ ,  $V_3 = R_3 I$
- According to KVL, the sum of voltage drops around the mesh in the clockwise direction is zero::  $-V_s + R_1 I + R_2 I + R_3 I = 0$

$$I = \frac{V_s}{R_1 + R_2 + R_3} = \frac{9}{4000 + 6000 + 8000} \text{ A} = 0.5 \text{ mA}$$

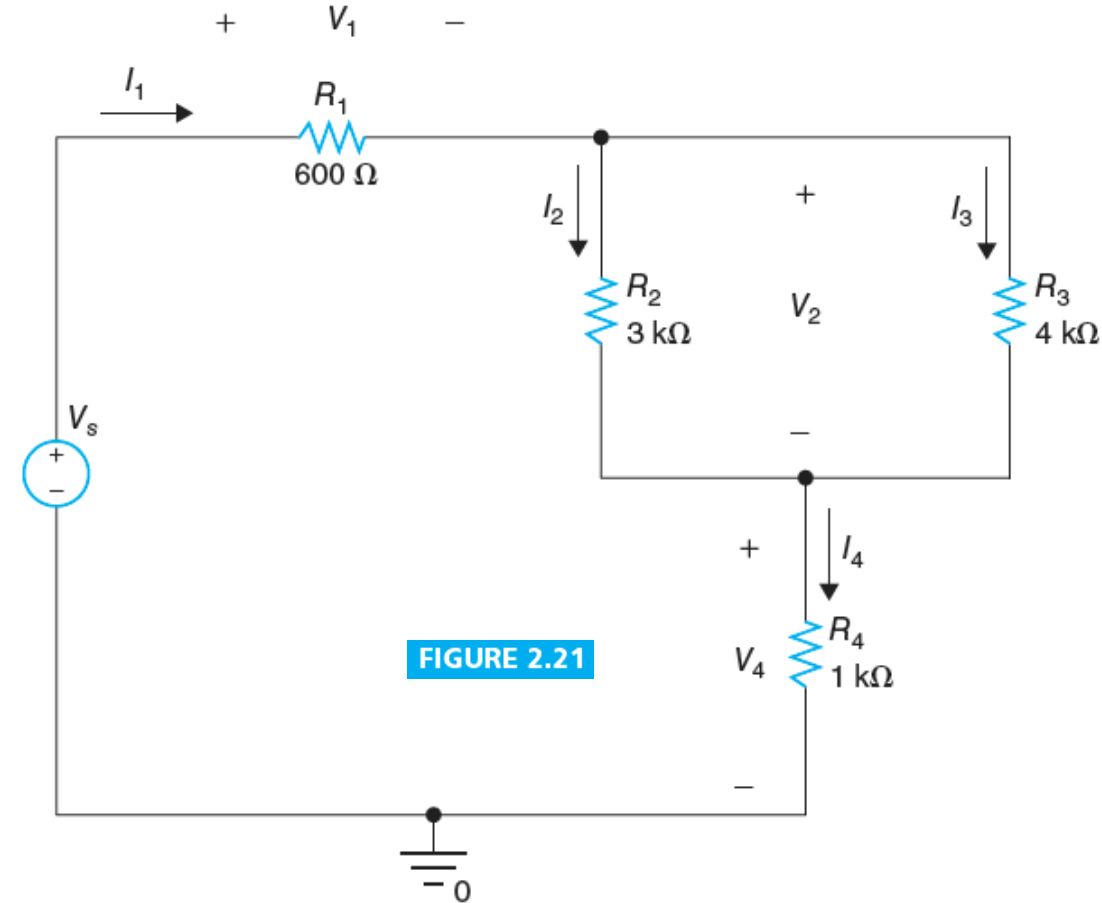
- Ohm's law:  $V_1 = R_1 I = 4000 \times 0.0005 \text{ V} = 2 \text{ V}$   
 $V_2 = R_2 I = 6000 \times 0.0005 \text{ V} = 3 \text{ V}$   
 $V_3 = R_3 I = 8000 \times 0.0005 \text{ V} = 4 \text{ V}$



# Kirchhoff's Voltage Law (KVL)

□ **Example 2.7:** Given  $V_2 = 6\text{ V}$ , find  $I_2$ ,  $I_3$ ,  $I_4$ ,  $V_4$ ,  $I_1$ ,  $V_1$ ,  $V_s$  in the circuit shown in Figure 2.21.

- Ohm's law:  $I_2 = V_2/R_2 = 6/3000\text{ A} = 2\text{ mA}$   
 $I_3 = V_2/R_3 = 6/4000\text{ A} = 1.5\text{ mA}$
- KCL:  $I_1 = I_4 = I_2 + I_3 = 2\text{mA} + 1.5\text{mA} = 3.5\text{mA}$
- Ohm's law:  $V_4 = R_4 I_4 = 1000 \times 0.0035\text{ V} = 3.5\text{V}$   
 $V_1 = R_1 I_1 = 600 \times 0.0035\text{ V} = 2.1\text{V}$
- KVL:  $-V_s + V_1 + V_2 + V_4 = 0$   
 $V_s = V_1 + V_2 + V_4 = 2.1\text{V} + 6\text{V} + 3.5\text{V} = 11.6\text{V}$



# Kirchhoff's Voltage Law (KVL)

- ❑ **Example 2.8:** Given  $V_1 = 6\text{ V}$ ,  $V_5 = 5\text{ V}$ ,  $V_6 = 3\text{ V}$ , and  $V_7 = 7\text{ V}$ , find  $V_2$ ,  $V_3$ ,  $V_4$ , and  $V_8$  in the circuit shown in Figure 2.23.

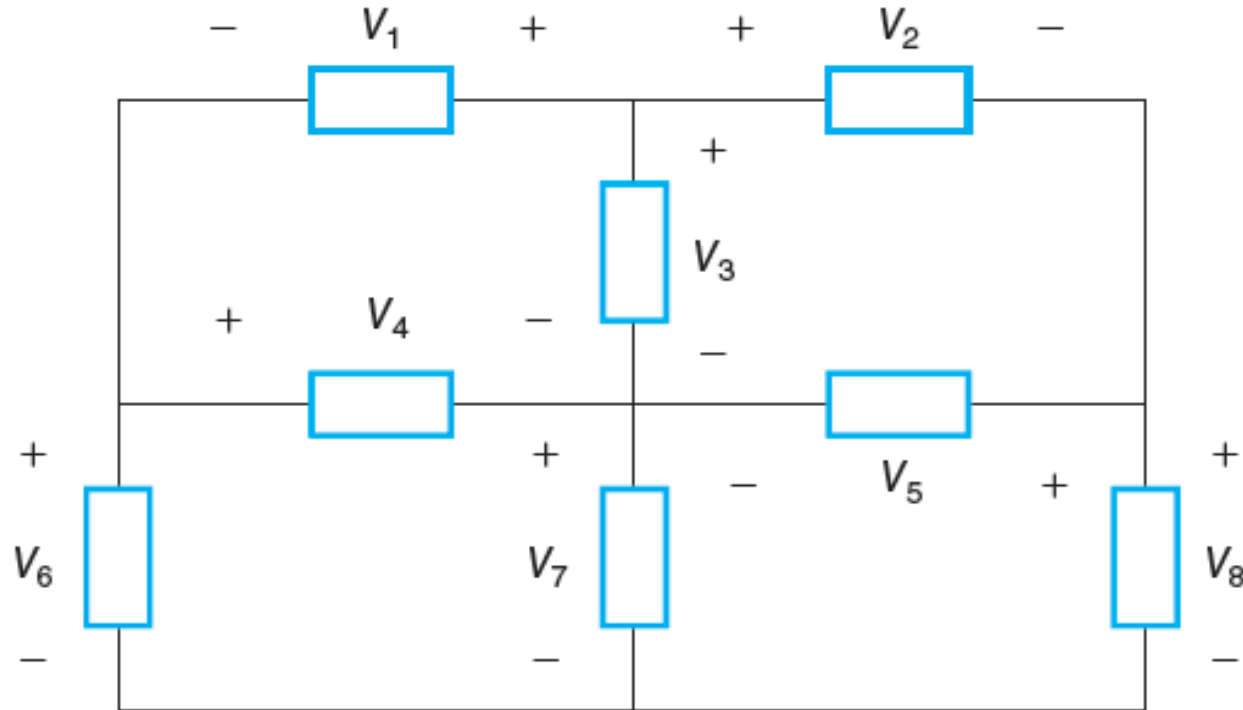


FIGURE 2.23

What is the value of  $V_4$  ?

- A.  $-3\text{ V}$
- B.  $-4\text{ V}$
- C.  $-5\text{ V}$

- ❑ The sum of voltage drops around a loop is zero.
- ❑ The sum of voltage drops around a loop equals the sum of voltage rises of the same loop.

Solution will be provided in class

# Kirchhoff's Voltage Law (KVL)

□ **Example 2.9:** Find  $i$ ,  $V_1$ ,  $V_2$ ,  $I_2$ ,  $I_3$ ,  $I_4$ ,  $I_5$  in the circuit shown in Figure 2.25.

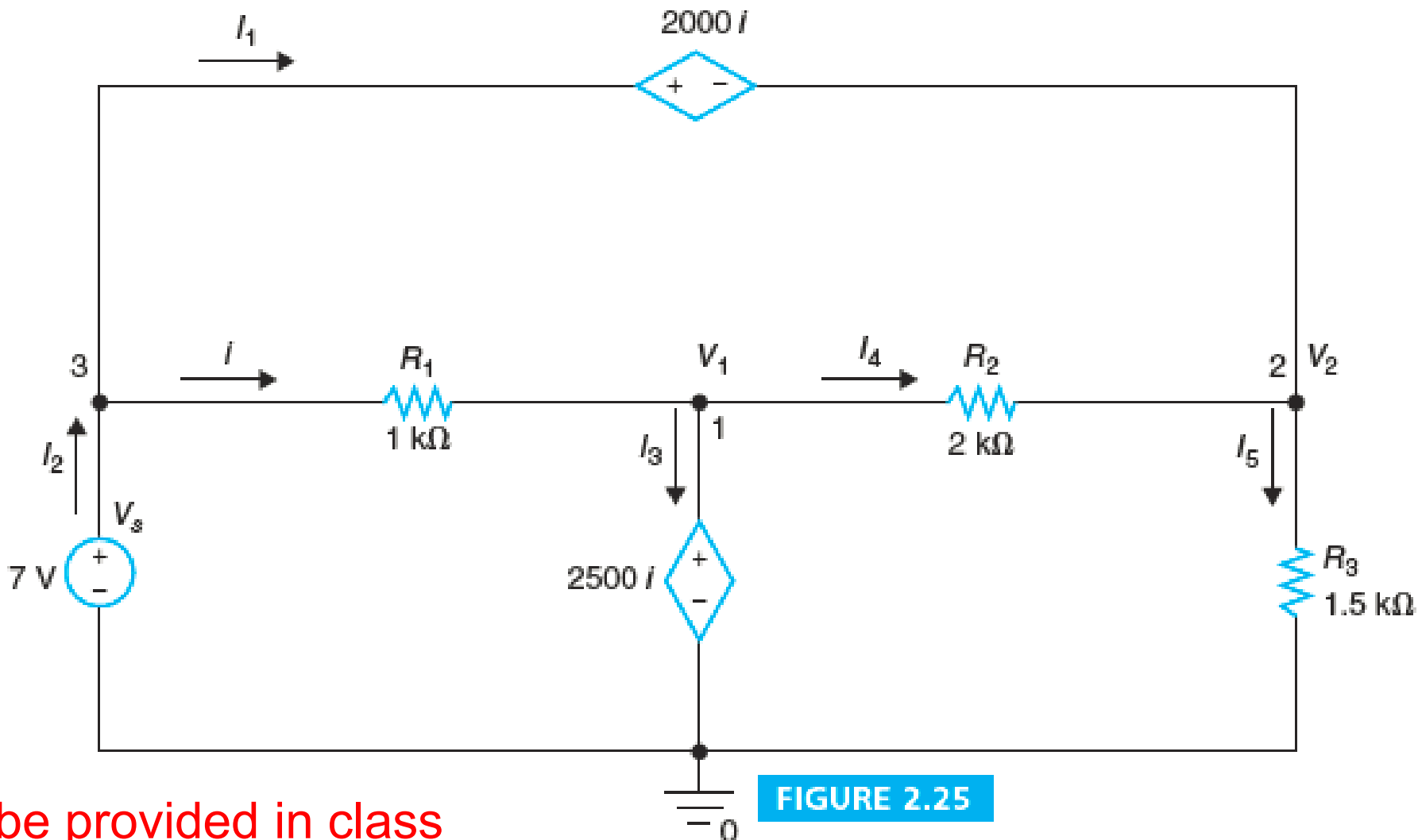


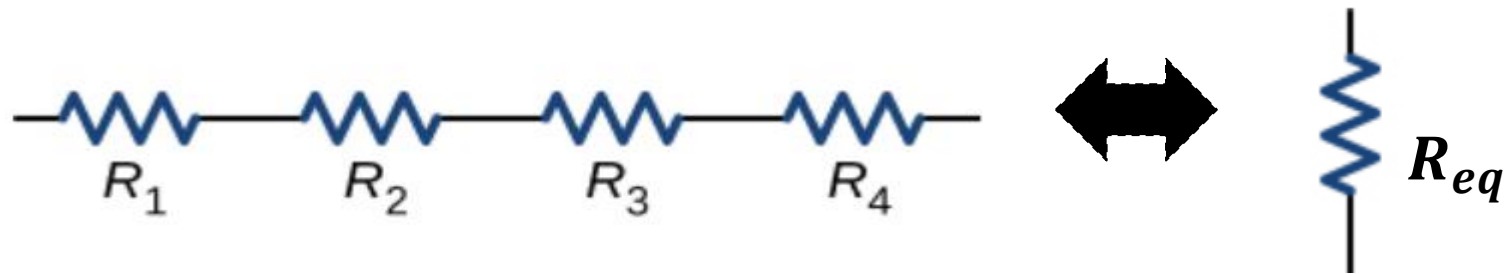
FIGURE 2.25

Solution will be provided in class

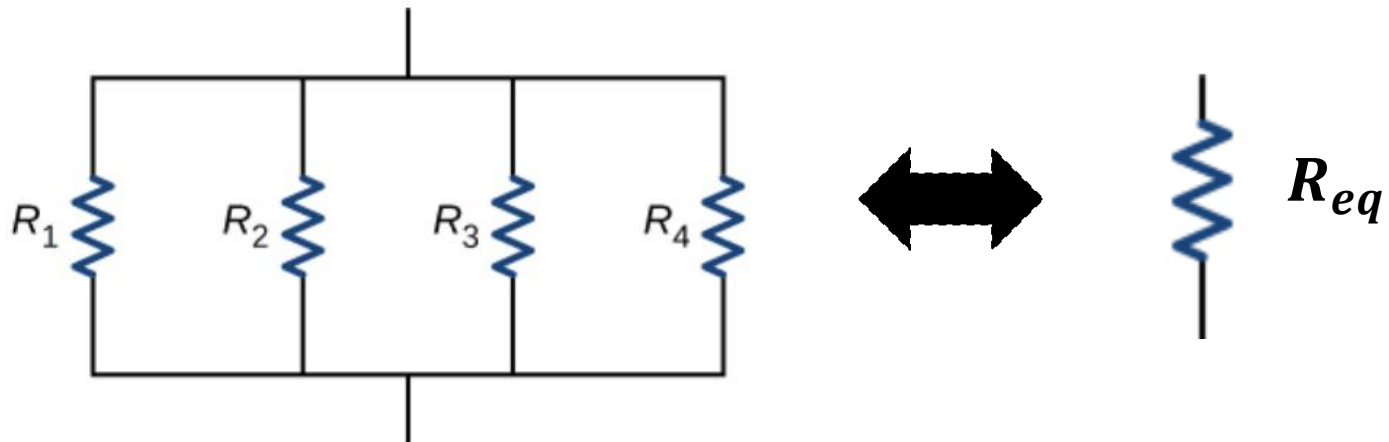
# Equivalent Resistance

□ Replacing multiple resistance with one resistor to produce the same impact

## 1. Series Equivalent resistance



## 2. Parallel Equivalent Resistance





# Equivalent Resistance in Series

❑ Two resistors with resistances  $R_1$  and  $R_2$  are connected in series in Fig.2.27(a).

❑ Ohm's law:  $V_1 = R_1 I$ ,  $V_2 = R_2 I$

❑ KVL:  $-V + V_1 + V_2 = 0$

$$V = V_1 + V_2 = R_1 I + R_2 I = (R_1 + R_2) I = R_{eq} I$$

where  $R_{eq}$  is the equivalent resistance of the series connection of  $R_1$  and  $R_2$ .

❑ The circuit shown in Fig.2.27(a) can be replaced by the circuit shown in Fig.2.27(b).

❑ If  $n$  resistors with resistances  $R_1, R_2, \dots, R_n$  are connected in series, the equivalent resistance is given by:  $R_{eq} = R_1 + R_2 + \dots + R_n$

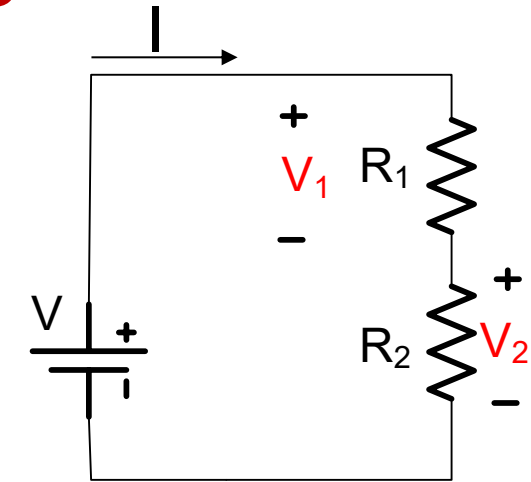


Fig.2.27(a).

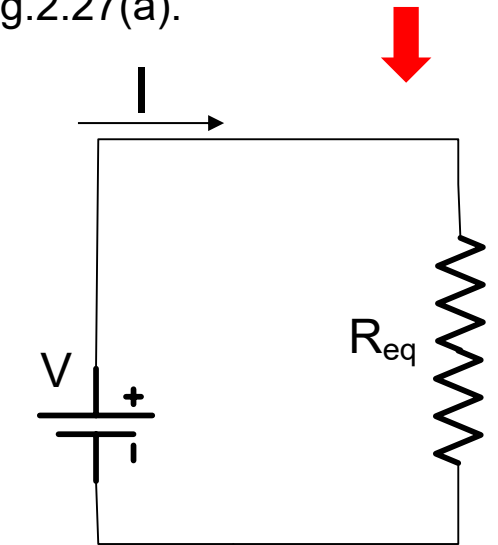
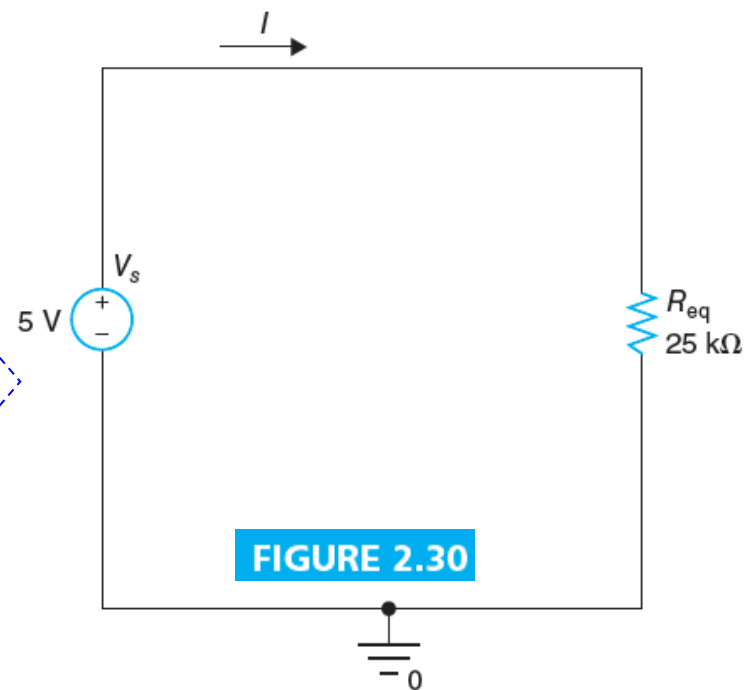
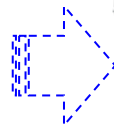
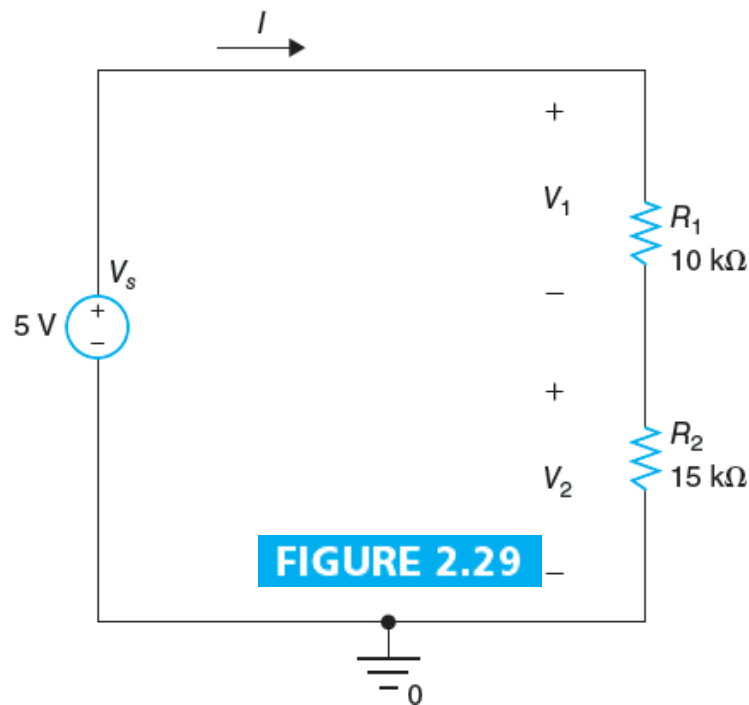


Fig.2.27(b).

# Equivalent Resistance in Series

- ❑ The equivalent resistance in the series connection of  $R_1$  and  $R_2$  in Fig.2.29 is given by:  $R_{eq} = R_1 + R_2 = 25 \text{ k}\Omega$
- ❑ When  $R_1$  and  $R_2$  are replaced by  $R_{eq}$ , we obtain the circuit shown in Figure 2.30.
- ❑ Ohm's law:  $I = V_s / R_{eq} = 5 \text{ V} / 25 \text{ k}\Omega = 0.2 \text{ mA}$
- ❑ Ohm's law:  $V_1 = R_1 I = 10 \text{ k}\Omega \times 0.2 \text{ mA} = 2 \text{ V}$ ;  $V_2 = R_2 I = 15 \text{ k}\Omega \times 0.2 \text{ mA} = 3 \text{ V}$



# Equivalent Resistance in Series

□ **Example 2.10:** Given  $I_3 = 750 \mu\text{A}$ , find  $V_a$ ,  $I_2$ ,  $I_4$ ,  $I_1$ ,  $V_s$  in the circuit shown in Figure 2.31.

- $R_a = R_2 + R_3 + R_4 = 50 \text{ k}\Omega$ ,  $R_b = R_5 + R_6 = 20 \text{ k}\Omega$ ,  $R_c = R_7 + R_8 + R_9 + R_{10} = 75 \text{ k}\Omega$
- $V_a = R_b I_3 = 15 \text{ V}$ ,  $I_2 = V_a / R_a = 0.3 \text{ mA}$ ,  $I_4 = V_a / R_c = 0.2 \text{ mA}$ ,  $I_1 = I_2 + I_3 + I_4 = 1.25 \text{ mA}$
- $V_1 = R_1 I_1 = 5 \text{ V}$ ,  $V_s = V_1 + V_a = 20 \text{ V}$

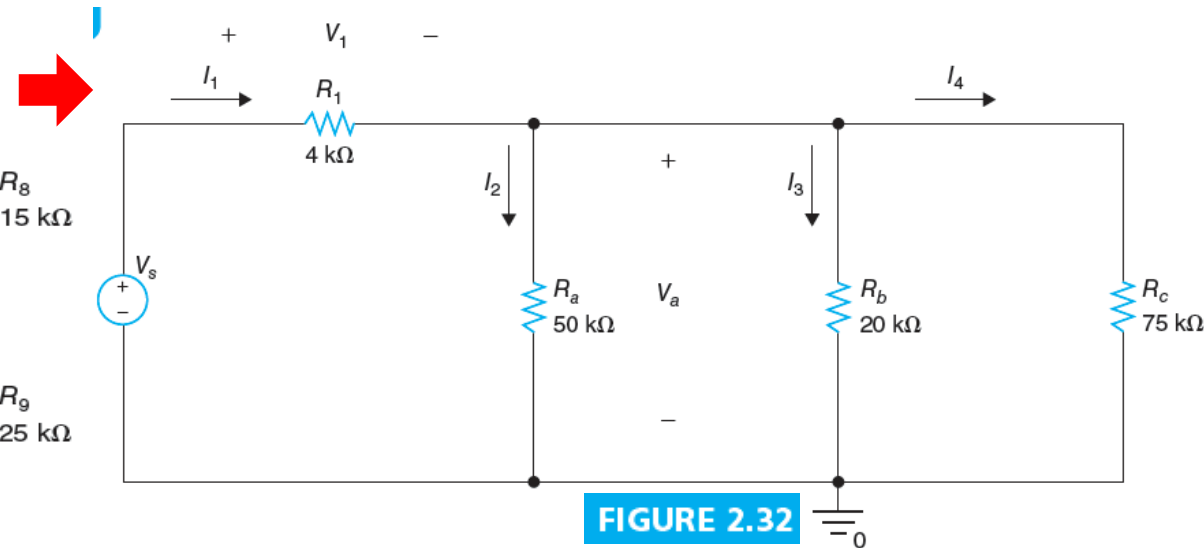
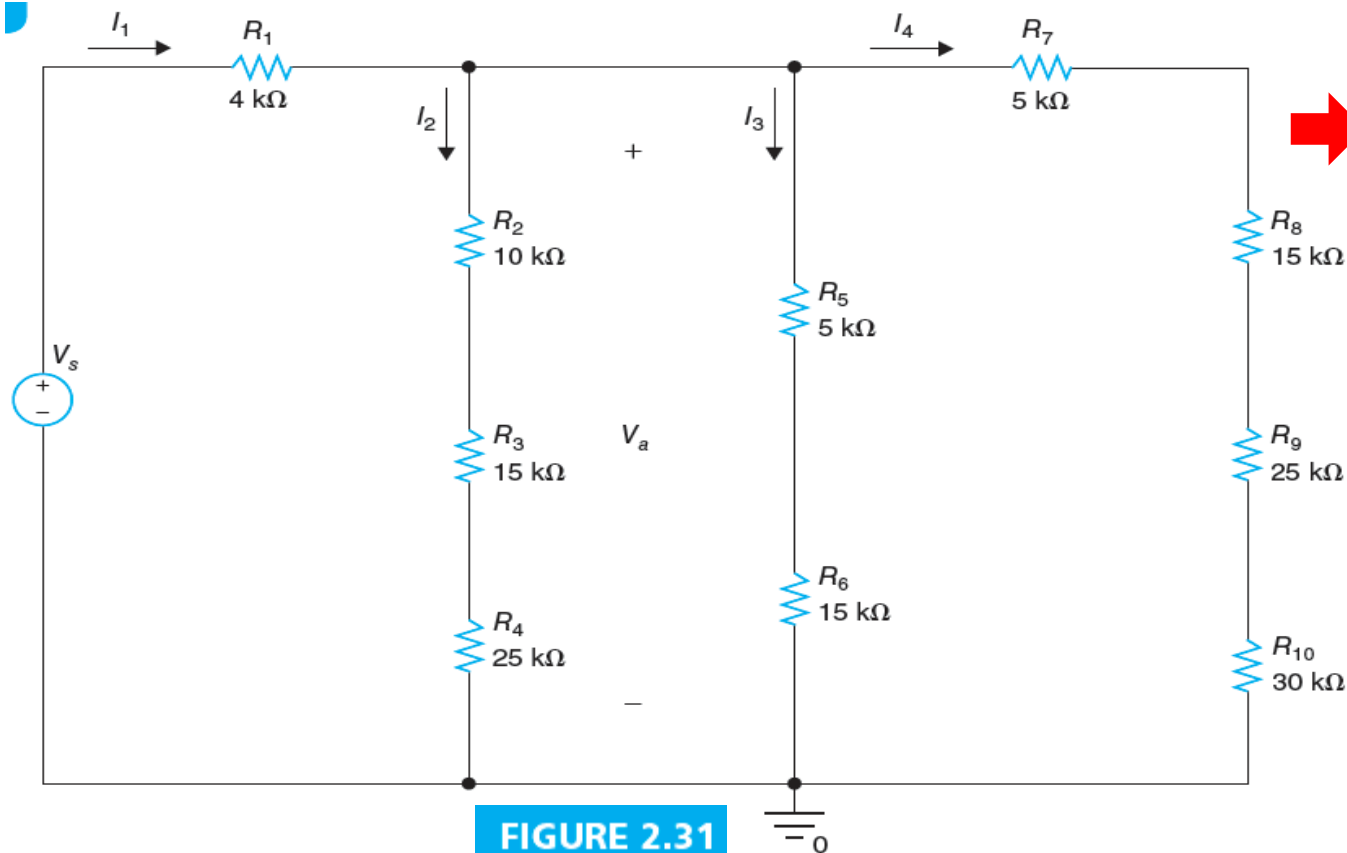


Figure 2.32 shows simplified circuit.

# Equivalent Resistance in Parallel

❑ Two resistors  $R_1$  and  $R_2$  are connected in parallel as shown in Figure 2.34(a).

❑  $I_1$  = current through  $R_1$ ,  $I_2$  = current through  $R_2$ ,  $V$  = voltage across  $R_1$  and  $R_2$

❑ KCL: 
$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = \left( \frac{1}{R_1} + \frac{1}{R_2} \right) V \Rightarrow V = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} I = R_{eq} I$$

❑ The equivalent resistance  $R_{eq}$  is given by:

$$R_{eq} = R_1 || R_2 = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

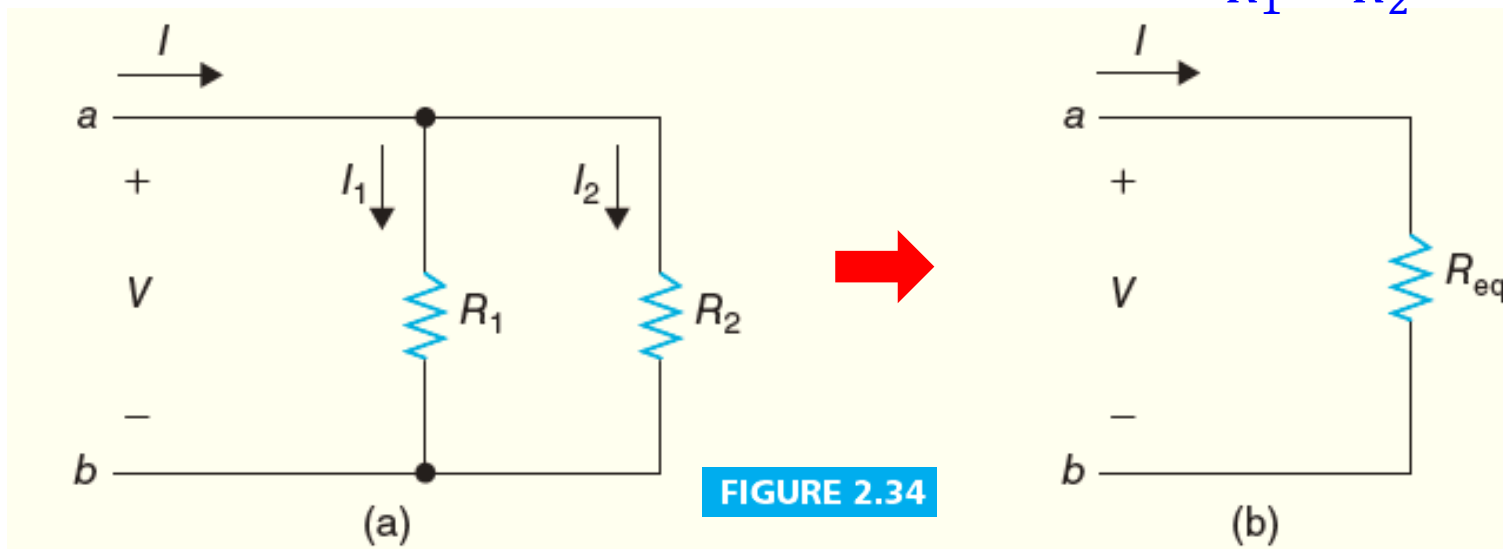


FIGURE 2.34

# Properties of $R_{eq} = R_1 \parallel R_2$

$$R_{eq} = R_1 \parallel R_2 = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

□  $R_{eq} < R_1$

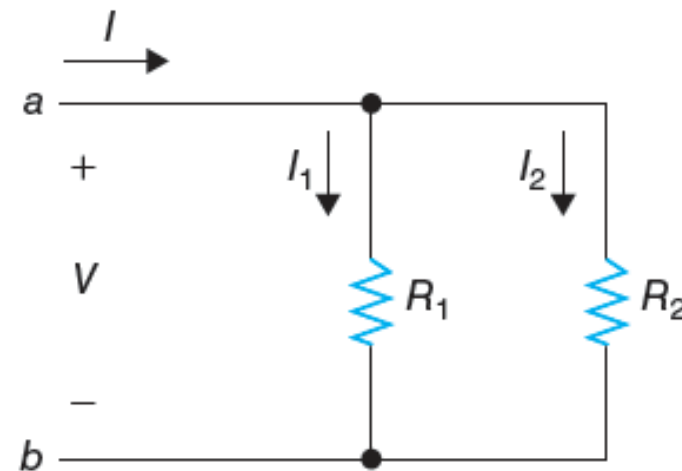
□  $R_{eq} < R_2$

□ The equivalent resistance is smaller than the smallest resistance in parallel.

□  $R_1 \parallel 0 = 0$ ,  $R_1 \parallel \infty = R_1$ .

□ If  $R_1 \ll R_2$ ,  $R_1 \parallel R_2 \approx R_1$ .

□ If  $R_1 = R_2 = R$ ,  $R_1 \parallel R_2 = R \parallel R = R/2$ .



# Equivalent Resistance of n Parallel Resistors

- n resistors  $R_1, R_2, \dots, R_n$  are connected in parallel as shown in Figure 2.35(a).
- $I_1$  = current through  $R_1$ ,  $I_2$  = current through  $R_2$ , ...,  $I_n$  = current through  $R_n$
- $V$  = voltage across  $R_1, R_2, \dots, R_n$ , equivalent circuit in Figure 2.35(b).
- KCL

$$I = I_1 + I_2 + \dots + I_n = \frac{V}{R_1} + \frac{V}{R_2} + \dots + \frac{V}{R_n} = \left( \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} \right) V$$

$$V = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}} I = R_{eq} I \quad R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}}$$

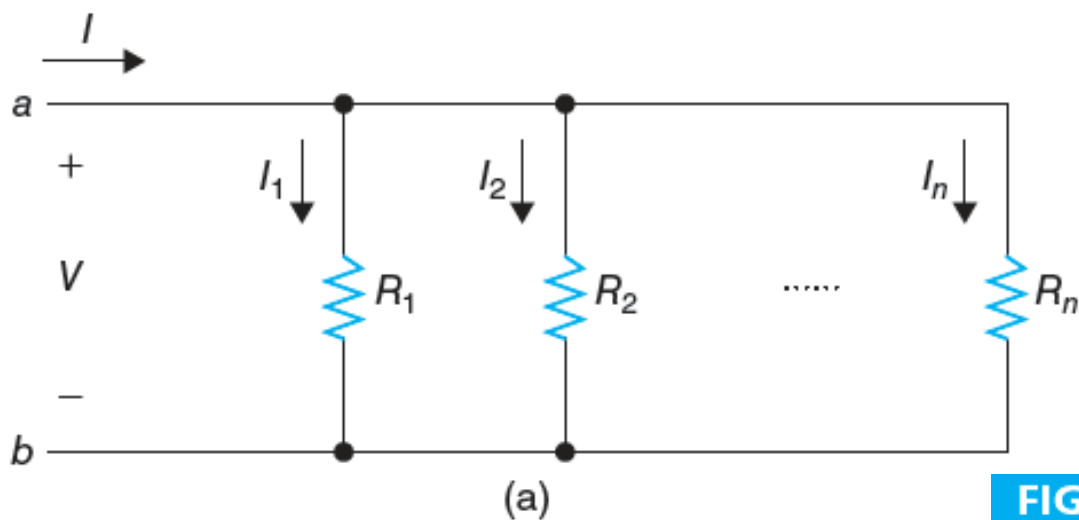
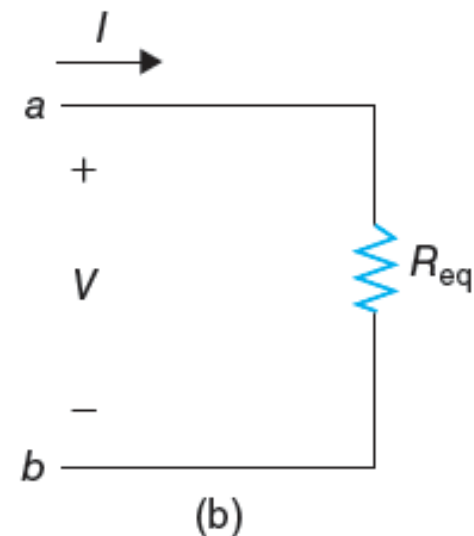


FIGURE 2.35



# Circuits with Parallel and Series Resistors

□ **Example:** Find  $I_1$ ,  $V_1$ ,  $V_2$ ,  $I_2$ ,  $I_3$ ,  $V_3$ ,  $I_4$ ,  $I_5$ , and  $I_6$  in the circuit shown in Figure 2.36.

$$R_a = R_2 || R_3 = \frac{R_2 \times R_3}{R_2 + R_3} = \frac{21k\Omega \times 28k\Omega}{21k\Omega + 28k\Omega} = \frac{588}{49} k\Omega = 12k\Omega$$

$$R_b = R_4 || R_5 || R_6 = \frac{1}{\frac{1}{33} + \frac{1}{40} + \frac{1}{88}} k\Omega = \frac{1}{0.06667} k\Omega = 15k\Omega$$

$$I_1 = \frac{V_s}{R_1 + R_a + R_b} = \frac{15V}{30k\Omega} = 0.5mA$$

- $V_1 = R_1 I_1 = 1.5 V$ ,  $V_2 = R_a I_1 = 6 V$ ,  $V_3 = R_b I_1 = 7.5 V$ ,  $I_2 = V_2 / R_2 = 0.2857 mA$ ,  
 $I_3 = V_2 / R_3 = 0.2143 mA$ ,  $I_4 = V_3 / R_4 = 0.2273mA$ ,  $I_5 = V_3 / R_5 = 0.1875 mA$ ,  $I_6 = V_3 / R_6 = 0.08523 mA$

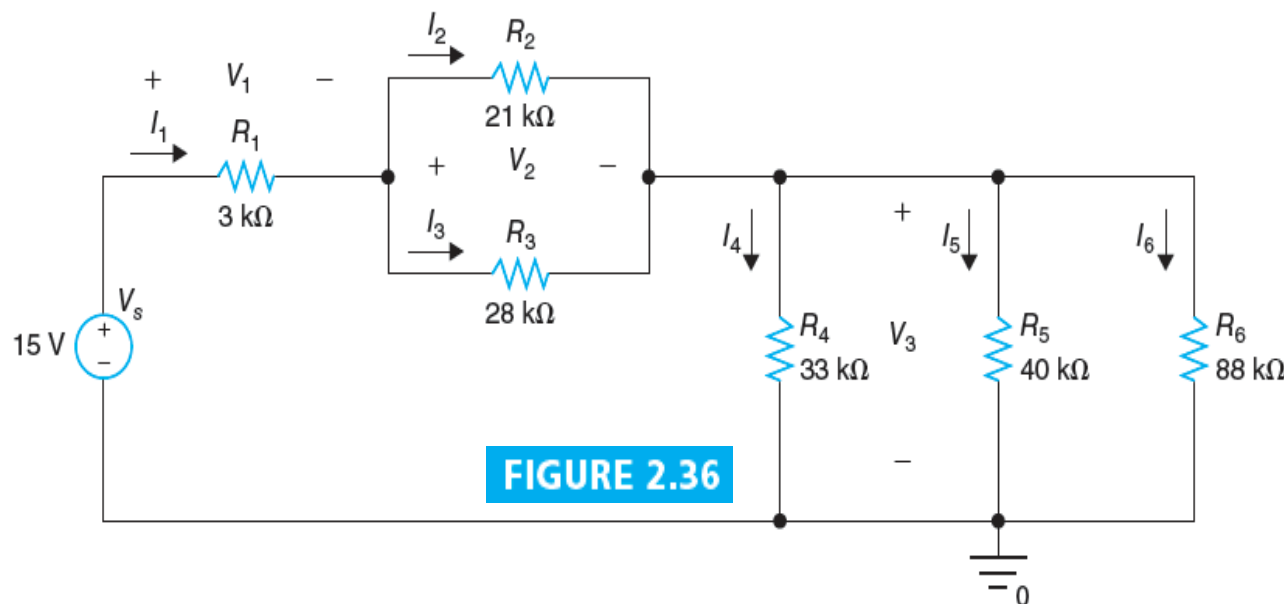
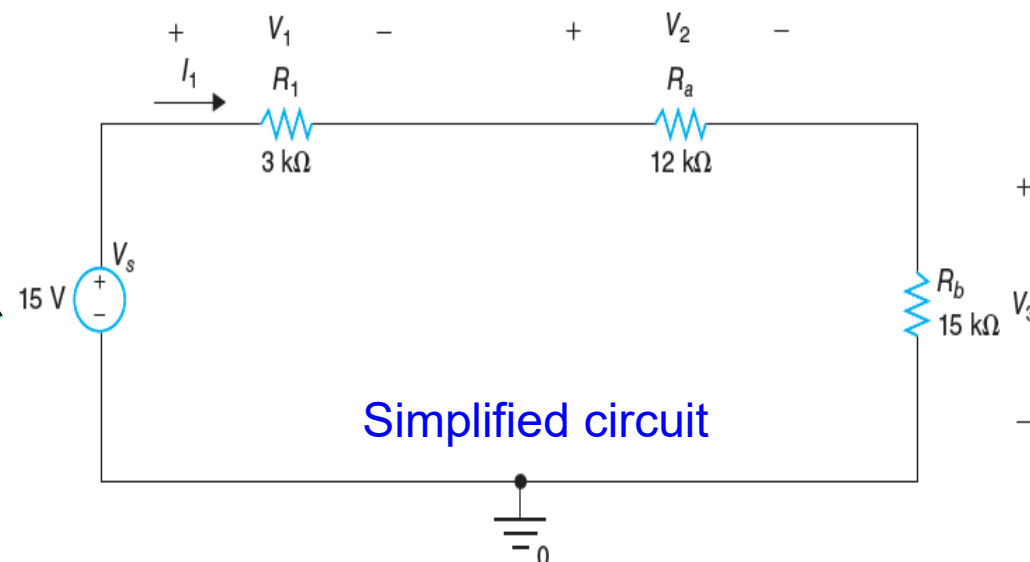


FIGURE 2.36



Simplified circuit

# Circuits with Parallel and Series Resistors

□ **Example 2.11:** Find the equivalent resistance between terminals  $a$  and  $b$  for the circuit shown in Figure 2.38.

$$R_6 = R_4 || R_5 = \frac{50 \times 75}{50 + 75} k\Omega = 30 k\Omega \quad R_7 = R_3 + R_6 = 30 + 30 k\Omega = 60 k\Omega$$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_7}} = \frac{1}{\frac{1}{45} + \frac{1}{90} + \frac{1}{60}} k\Omega = 20 k\Omega$$

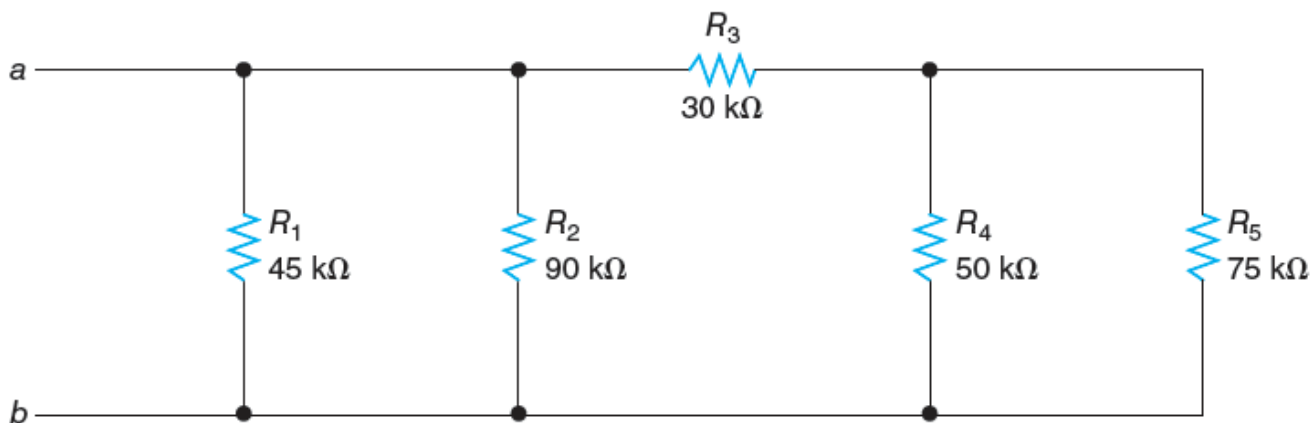


FIGURE 2.38



# Circuits with Parallel and Series Resistors

□ **Example 2.12:** Find the equivalent resistance seen from the voltage source. Also find  $I$ ,  $I_1$ ,  $I_2$ ,  $V_1$ ,  $V_2$ ,  $V_3$ , and power absorbed by resistors and power released by the voltage source.

- $R_a = R_3 \parallel R_4 = R_3 \times R_4 / (R_3 + R_4)$   
 $= 100 / 25 \text{ k}\Omega = 4 \text{ k}\Omega$
- $R_{eq} = R_1 + R_2 + R_a = 9 \text{ k}\Omega$
- $I = V_s / R_{eq} = 9 / 9000 \text{ A} = 1 \text{ mA}$
- $V_1 = R_1 I = 2 \text{ V}$ ,  $V_2 = R_2 I = 3 \text{ V}$ ,  $V_3 = R_a I = 4 \text{ V}$
- $I_1 = V_3 / R_3 = 0.8 \text{ mA}$ ,  $I_2 = V_3 / R_4 = 0.2 \text{ mA}$
- $P_{R1} = I V_1 = 2 \text{ mW}$ ,  $P_{R2} = I V_2 = 3 \text{ mW}$
- $P_{R3} = I_1 V_3 = 3.2 \text{ mW}$ ,  $P_{R4} = I_2 V_3 = 0.8 \text{ mW}$
- $P_{Vs} = -I V_s = -9 \text{ mW}$
- Power absorbed by resistors = 9 mW
- Power released by voltage source = 9 mW

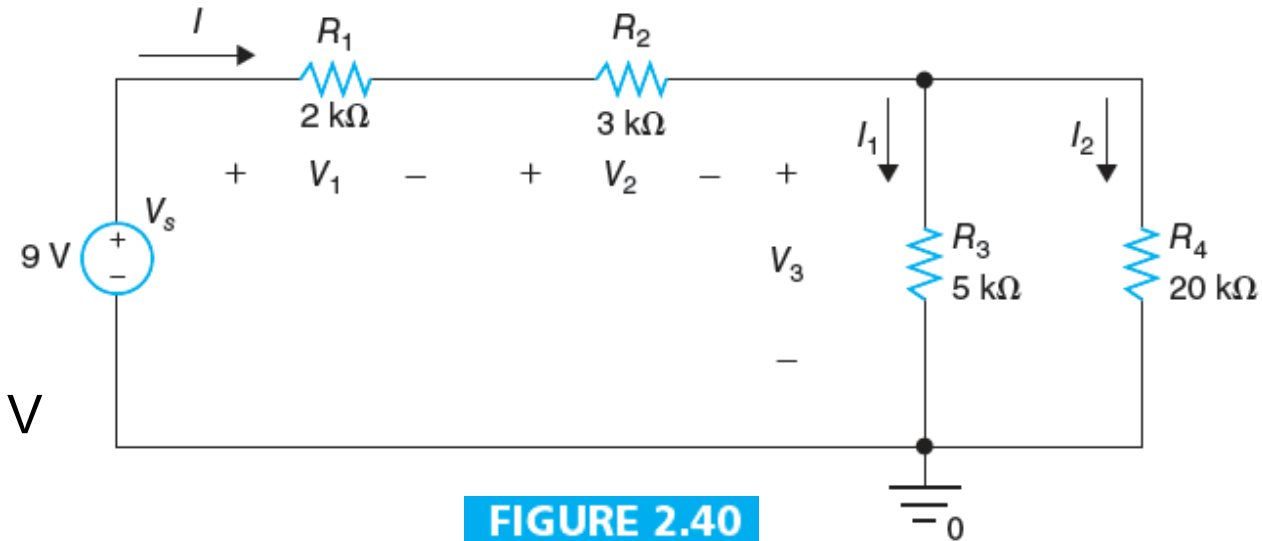


FIGURE 2.40

# Circuits with Parallel and Series Resistors

□ **Example 2.13:** Find  $R_{eq}$  seen from the voltage source in the circuit shown in Fig.2.43. Also find  $I_1$ ,  $I_2$ ,  $I_3$ ,  $I_4$ ,  $I_5$ ,  $V_1$ ,  $V_2$ ,  $V_3$ , and power absorbed by resistors and power released by the voltage source.

- $R_a = R_4 \parallel R_5 = R_4 \times R_5 / (R_4 + R_5) = 24/10 \text{ k}\Omega = 2.4 \text{ k}\Omega$ ,  $R_b = R_3 + R_a = 4 \text{ k}\Omega$
- $R_c = R_2 \parallel R_b = R_2 \times R_b / (R_2 + R_b) = 64/20 \text{ k}\Omega = 3.2 \text{ k}\Omega$ ,  $R_{eq} = R_1 + R_c = 5 \text{ k}\Omega$
- $I_1 = V_s / R_{eq} = 10/5000 \text{ A} = 2 \text{ mA}$ ,  $V_1 = V_s - R_1 I_1 = 6.4 \text{ V}$ ,  $I_2 = V_1 / R_2 = 6.4/16000 \text{ A} = 0.4 \text{ mA}$
- $I_3 = I_1 - I_2 = 1.6 \text{ mA}$ ,  $V_2 = V_1 - R_3 I_3 = 3.84 \text{ V}$ ,  $I_4 = V_2 / R_4 = 3.84/4000 \text{ A} = 0.96 \text{ mA}$ ,  $V_{R5} = V_3$
- $I_5 = V_2 / R_5 = 3.84/6000 \text{ A} = 0.64 \text{ mA}$ ,  $V_{R1} = R_1 I_1 = 3.6 \text{ V}$ ,  $V_{R3} = R_3 I_3 = 2.56 \text{ V}$ ,  $V_{R2} = V_1$ ,  $V_{R4} = V_3$
- $P_{R1} = I_1 V_{R1} = 7.2 \text{ mW}$ ,  $P_{R3} = I_3 V_{R3} = 4.096 \text{ mW}$ ,  $P_{R2} = I_2 V_{R2} = 2.56 \text{ mW}$
- $P_{R4} = I_4 V_{R4} = 3.6864 \text{ mW}$
- $P_{R5} = I_5 V_{R5} = 2.4576 \text{ mW}$
- $P_{Vs} = -I_1 V_s = -20 \text{ mW}$
- Power released = 20 mW
- Power absorbed by resistors = 20 mW

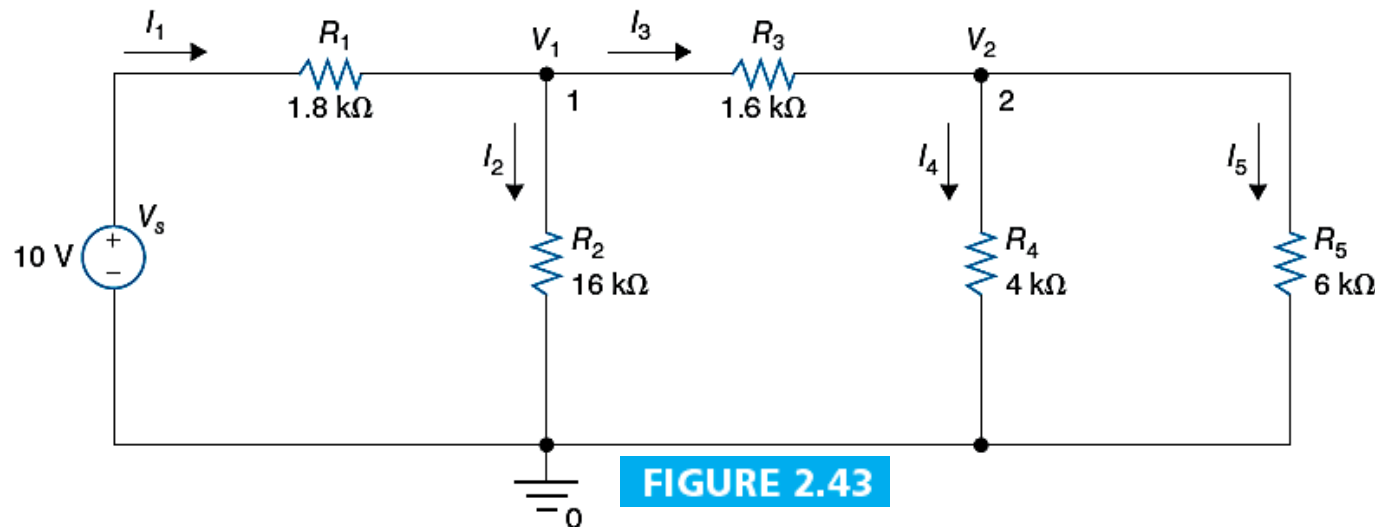
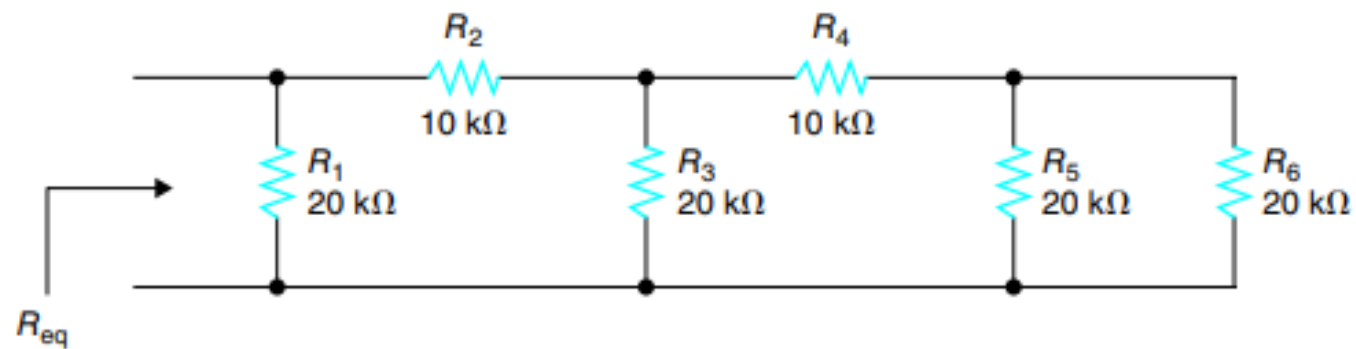


FIGURE 2.43

## Problem 2.39

FIGURE P2.39



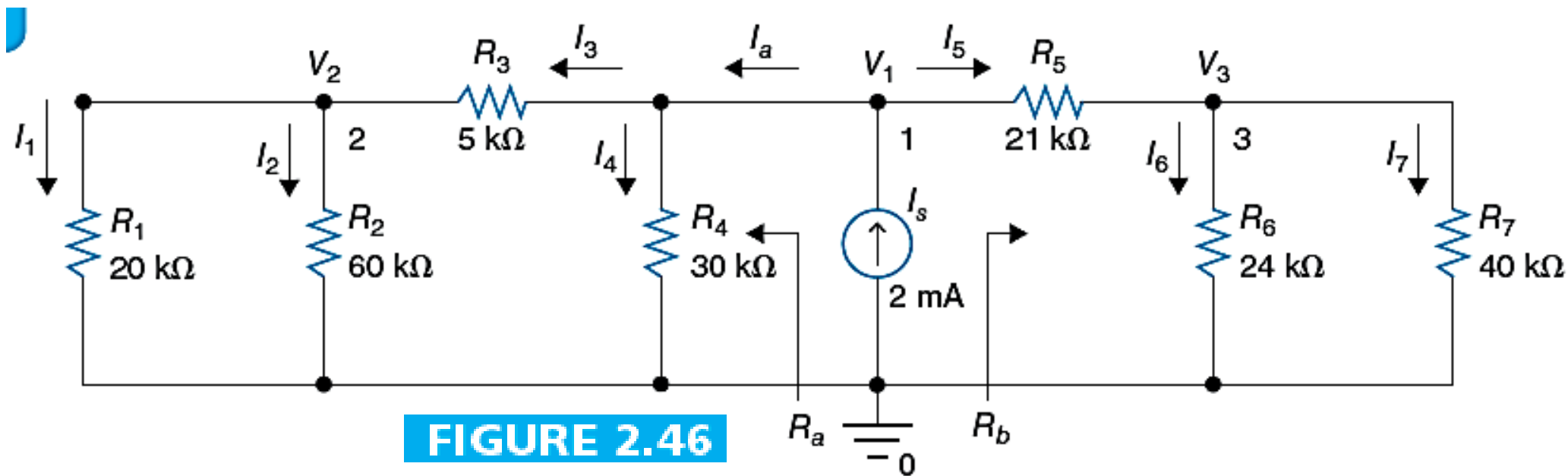
Fill in the Blanks

- What is the value of equivalent resistance -----

Solution will be provided in class

# Circuits with Parallel and Series Resistors

□ **Example 2.14:** Find the equivalent resistance seen from the current source. Also find  $I_a$ ,  $I_1$ ,  $I_2$ ,  $I_3$ ,  $I_4$ ,  $I_5$ ,  $I_6$ ,  $I_7$ ,  $V_1$ ,  $V_2$ ,  $V_3$  for the circuit shown in Figure 2.46.



Solution will be provided in class

Home practice P 2.32 to P 2.46

# Summary

- ❑ Definition of node, branch, path, loop, and mesh
- ❑ Resistor, Ohm's law
- ❑ KCL and KVL
- ❑ Equivalent resistance of series connection of resistors
- ❑ Equivalent resistance of parallel connection of resistors
- ❑ What will we study in next lecture.