Circuit Analysis and Design

Academic Year 2025/2026 – Semester 1

Lecture 8 - Circuit Theorems

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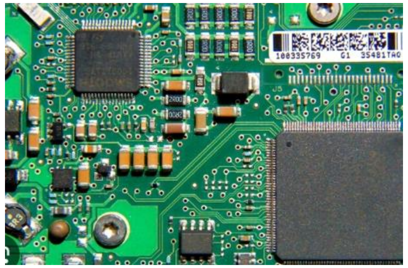

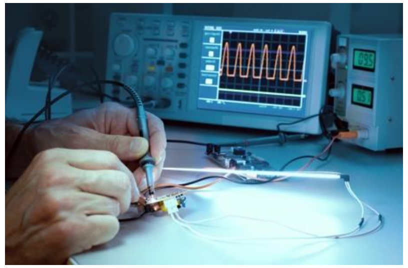

Agenda

- Thévenin’s theorem
- Summary

Thévenin's Theorem

- A circuit consisting of a voltage source V_{th} and a series resistor R_{th}, representing the original circuit looking from a pair of terminals, is called a Thévenin equivalent circuit. The voltage V_{th} is called Thévenin equivalent voltage, and the resistance R_{th} is called Thévenin equivalent resistance.

- The Thévenin equivalent circuit can be used to simplify the circuit. When a load resistor is connected between terminals *a* and *b,* we can find the effects of the circuit on the load from the Thévenin equivalent circuit.

- We do not need all the details of the original circuit to find the voltage, current, and power on the load.

- Let the voltage across terminals *a* and *b* of the Thévenin equivalent circuit be V_{oc}. This voltage is called open-circuit voltage because terminals *a* and *b* are open (with an infinite resistance between *a* and *b).*

- No current flows through the Thévenin equivalent resistor R_{th}. Thus,

  $V_{oc} = V_{th}$

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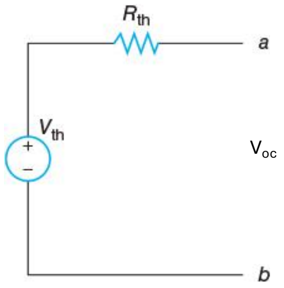
FIGURE 4.50

A Thévenin equivalent circuit.



Thévenin's Theorem (Continued)

- If the terminals *a* and *b* are short-circuited, as shown in Figure 4.51, the current through the short circuit is given by

$$I_{sc} = \frac{V_{th}}{R_{th}} = \frac{V_{oc}}{R_{th}}$$

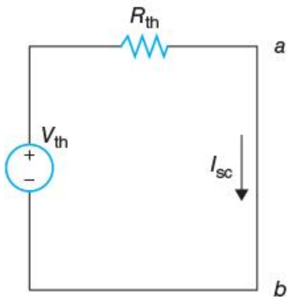
- If we solve this equation for R_{th}, we have

$$R_{th} = \frac{V_{oc}}{I_{sc}}$$

- This equation can be used to find the Thévenin equivalent resistance R_{th}.

FIGURE 4.51

Short-circuit current.



Finding the Thévenin Equivalent Resistance

Method 1:

- Deactivate all the independent sources by short-circuiting voltage sources and open-circuiting current sources.
- R_{th} is the equivalent resistance looking into the circuit from terminals *a* and *b.*
- This method cannot be used if the circuit contains dependent sources.

Method 2:

- Short-circuit terminals *a* and *b*. Find the short-circuit current I_{sc}.
- The Thévenin equivalent resistance is given by $R_{th} = V_{oc}/I_{sc} = V_{th}/I_{sc}$.

Method 3:

- Deactivate all the independent sources.
- Apply a test voltage of 1 V (or any other value) between terminals *a* and *b* with terminal *a* connected to the positive terminal of the test voltage.
- Measure the current I_t flowing out of the positive terminal of the test voltage source.
- The Thévenin equivalent resistance R_{th} is given by $R_{th} = V_t/I_t$.
- Alternatively, apply a test current between terminals *a* and *b* after deactivating the independent sources, and measure the voltage across *a* and *b* of the test current source. The Thévenin equivalent resistance R_{th} is the ratio of the voltage across *a* and *b* to the test current.

Finding V_{th} and R_{th}

- Consider the circuit shown in Figure 4.52. We are interested in finding V_{th} and R_{th} across terminals *a* and *b*.
- To find V_{th}:
  - Sum the currents leaving node 1:

$$\frac{V_1 - 5}{5000} - 0.002 + \frac{V_1}{20000} + \frac{V_1 - V_2}{5000} = 0$$

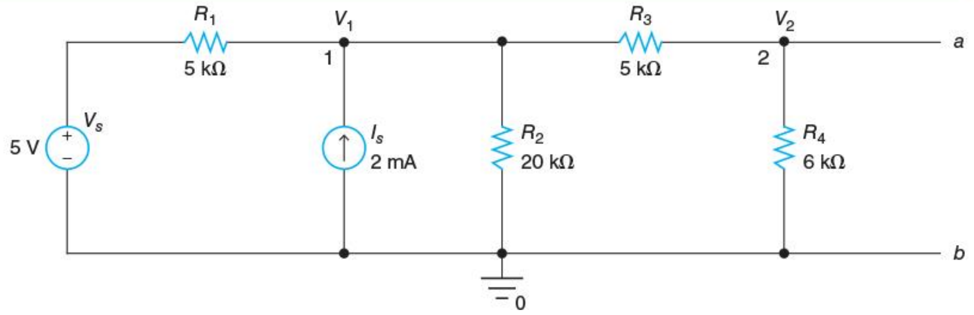
  - Multiply by 20,000:

$$4V_1 - 20 - 40 + V_1 + 4V_1 - 4V_2 = 0 \Rightarrow 9V_1 - 4V_2 = 60 \quad (1)$$

  - Sum the currents leaving node 2: $$\frac{V_2 - V_1}{5000} + \frac{V_2}{6000} = 0$$
  - Multiply by 30,000: $6V_2 - 6V_1 + 5V_2 = 0 \Rightarrow V_1 = 11/6V_2 \quad (2)$
  - Substituting (2) in (1): $V_2 = V_{th} = V_{oc} = 4.8$ V

FIGURE 4.52

A circuit with a pair of terminals.



Finding V_{th} and R_{th} (Continued)

- To find R_{th} (**Method 1**):
  - We deactivate V_s by short-circuiting it and I_s by open-circuiting it as shown in Figure 4.53, and find the equivalent resistance looking into the circuit from terminals *a* and *b.*
  - $R_a = R_1 \,||\, R_2 = 5{\times}20/(5 + 20)\ \text{k}\Omega = 100/25\ \text{k}\Omega = 4\ \text{k}\Omega$
  - $R_b = R_3 + R_a = 9\ \text{k}\Omega$
  - $R_{th} = R_4 \,||\, R_b = 6{\times}9/(6 + 9)\ \text{k}\Omega = 54/15\ \text{k}\Omega = 3.6\ \text{k}\Omega$
  - The Thévenin equivalent circuit is shown in Figure 4.57.

FIGURE 4.53

The circuit from Figure 4.52 with its sources deactivated.

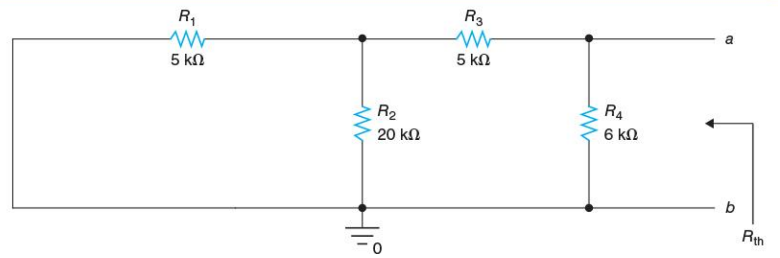
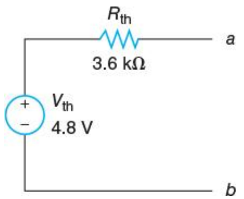


FIGURE 4.57

The Thévenin equivalent circuit.



Finding V_{th} and R_{th} (Continued)

- Consider a circuit shown in Figure 4.58. We are interested in finding V_{th} and R_{th} across terminals *a* and *b*.
- To find V_{th}:
  - Sum the currents leaving node 1:

$$\frac{V_1 - 5}{2000} + \frac{V_1}{6000} + \frac{V_1 - V_2}{1000} = 0$$

  - Multiply by 6000:

$3V_1 – 15 + V_1 + 6V_1 – 6V_2 = 0 \Rightarrow 10V_1 = 6V_2 + 15 \Rightarrow V_1 = 0.6V_2 + 1.5$

  - Sum the currents leaving node 2:

$$\frac{V_2 - V_1}{1000} + 0.0005V_1 + \frac{V_2}{10000} = 0$$

  - Multiply by 10000:

$10V_2 – 10V_1 + 5V_1 + V_2 = 0 \Rightarrow 11V_2 – 5V_1 = 0$

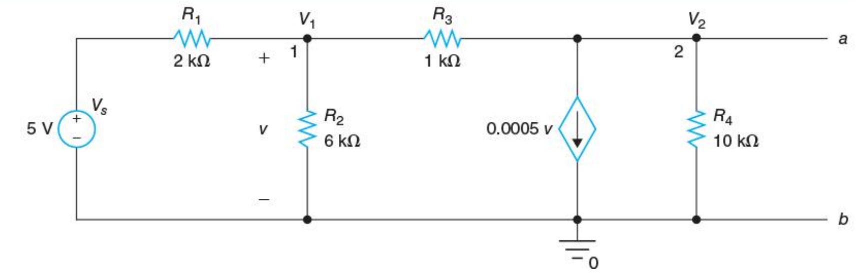
$\Rightarrow 11V_2 – 5(0.6V_2 + 1.5) = 0$

$\Rightarrow 8V_2 = 7.5$

  - $V_{th} = V_{oc} = V_2 = 7.5/8 = 0.9375$ V

FIGURE 4.58

A circuit with VCCS.



Finding V_{th} and R_{th} (Continued)

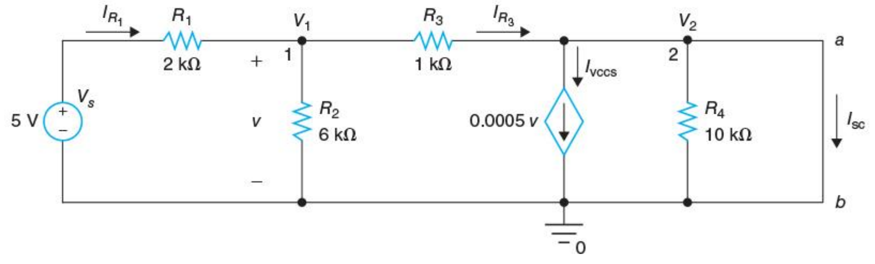
- To find R_{th} (**Method 2**):
- Since there is a dependent source, Method 1 cannot be used to find R_{th}. Either Method 2 or Method 3 can be used. We will learn Method 2.
  - Terminals *a* and *b* are short-circuited as shown in Figure 4.59. $V_2 = 0$.
  - Sum the currents leaving node 1: $$\frac{V_1 - 5}{2000} + \frac{V_1}{6000} + \frac{V_1}{1000} = 0$$
  - Multiply by 6000:

    $3V_1 - 15 + V_1 + 6V_1 = 0 \Rightarrow 10V_1 = 15 \Rightarrow V_1 = 1.5$ V, $v = V_1 = 1.5$ V
  - The current through R_3 is given by: $I_{R3} = V_1/R_3 = 1.5$ V/1 kΩ = 1.5 mA
  - The current through VCCS is given by:

    $I_{VCCS} = 0.0005V_1 = 0.0005 \times 1.5$ A = 0.75 mA
  - $I_{sc} = I_{R3} - I_{VCCS} = 1.5$ mA – 0.75 mA = 0.75 mA
  - $R_{th} = V_{th}/I_{sc} = 0.9675$ V/0.75 mA = 1.25 kΩ

FIGURE 4.59

A circuit with a short circuit between *a* and *b*.



Finding V_{th} and R_{th} (Continued)

- We wish to find V_{th} and R_{th} for the circuit shown in Figure 4.79.
- To find V_{th}:
  - Sum the currents leaving node 1: $$-0.002+\frac{V_1}{2000}+\frac{V_1-V_2}{5000}=0$$
  - Multiply by 10000:

$20 + 5V_1 + 2V_1 – 2V_2 = 0 \Rightarrow 7V_1 = 2V_2 + 20 \Rightarrow V_1 = (2/7)V_2 + 20/7$ (1)

  - Sum the currents leaving node 2: $$\frac{V_2-V_1}{5000}+2\frac{V_1}{2000}+\frac{V_2}{3000}=0$$
  - Multiply by 30000:

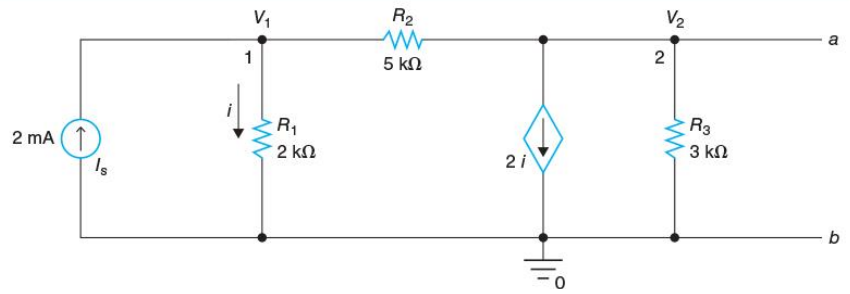
$6V_2 – 6V_1 + 30V_1 + 10V_2 = 0$ (2)

  - Substitute (1) into (2): $24[(2/7)V_2 + 20/7] + 16V_2 = 0 \Rightarrow 160V_2 = – 480$

$\Rightarrow V_{th} = V_{oc} = V_2 = – 3$ V

FIGURE 4.79

Circuit with a CCCS.



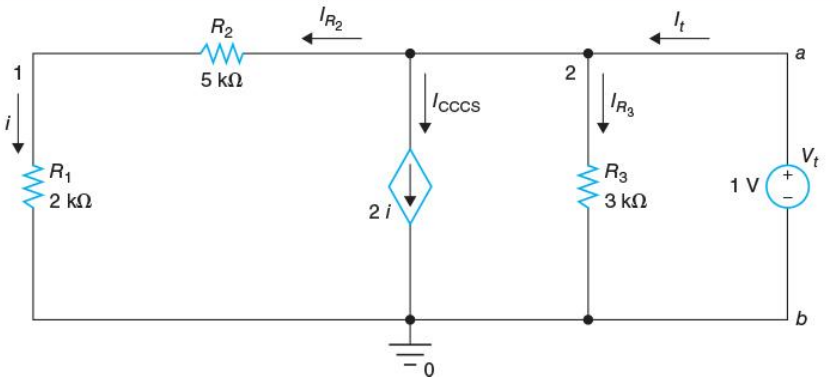
Finding V_{th} and R_{th} (Continued)

- To find R_{th}(**Method 3**):
  - R_{th} can be found using Method 2 or Method 3. We will learn Method 3.
  - I_s is open-circuited and a test voltage of 1 V is applied between *a* and *b* as shown in Figure 4.81.
  - $V_t = 1$ V
  - $i = I_{R2} = V_t/(R_1 + R_2) = (1/7)$ mA
  - $I_{CCCS} = 2i = (2/7)$ mA, $I_{R3} = V_t/R_3 = (1/3)$ mA
  - The current flowing out of the positive terminal of the test voltage source is given by:

    $I_t = I_{R2} + I_{CCCS} + I_{R3} = (3/21)$ mA $+ (6/21)$ mA $+ (7/21)$ mA $= (16/21)$ mA
  - Thévenin equivalent resistance is: $R_{th} = V_t/I_t = 21/16$ kΩ $= 1.3125$ kΩ

FIGURE 4.81

A circuit with a test voltage source.



EXAMPLE 4.8

Find V_{th} and R_{th} for the circuit shown in Figure 4.67.

- Sum the currents leaving node 1:

$$\frac{V_1 - 22.5}{15000} + \frac{V_1}{30000} + \frac{V_1 - V_2}{5000} = 0$$

- Multiply by 30,000:

$$2V_1 - 45 + V_1 + 6V_1 - 6V_2 = 0 \Rightarrow 9V_1 = 6V_2 + 45 \Rightarrow V_1 = (2/3)V_2 + 5 \quad (1)$$

- Sum the currents leaving node 2:

$$\frac{V_2 - V_1}{5000} - 0.004 + \frac{V_2}{10000} = 0$$

- Multiply by 10,000:

$$2V_2 - 2V_1 - 40 + V_2 = 0 \quad (2)$$

- Substituting (1)→(2): $3V_2 - 2[(2/3)V_2 + 5] = 40 \Rightarrow (5/3)V_2 =$

$$\Rightarrow V_{th} = V_{oc} = V_2 = 30 \text{ V}$$

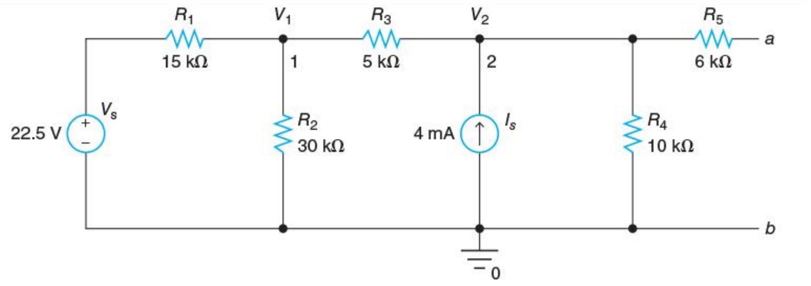


FIGURE 4.67 Circuit for EXAMPLE 4.8.

EXAMPLE 4.8 (Continued)

- To find R_{th}, V_s is short-circuited and I_s is open-circuited as shown in Figure 4.70.
- $R_a = R_1 \parallel R_2 = 15 \times 30/(15 + 30)\ \text{k}\Omega = 450/45\ \text{k}\Omega = 10\ \text{k}\Omega$
- $R_b = R_3 + R_a = 5\ \text{k}\Omega + 10\ \text{k}\Omega = 15\ \text{k}\Omega$
- $R_c = R_4 \parallel R_b = 10 \times 15/(10 + 15)\ \text{k}\Omega = 150/25\ \text{k}\Omega = 6\ \text{k}\Omega$
- $R_{th} = R_5 + R_c = 6\ \text{k}\Omega + 6\ \text{k}\Omega = 12\ \text{k}\Omega$
- The Thévenin equivalent circuit is shown in Figure 4.71.

FIGURE 4.70

The circuit shown in Figure 4.67 after deactivating the sources.

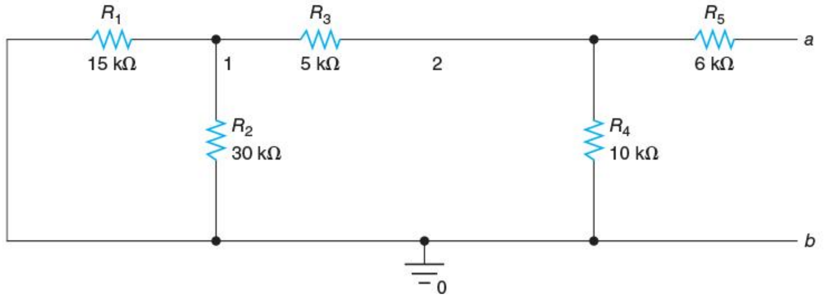
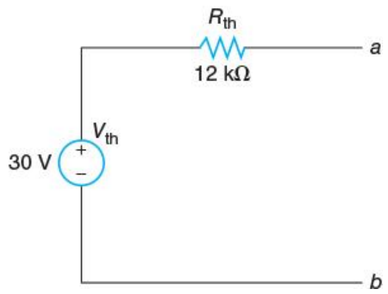


FIGURE 4.71

The Thévenin equivalent circuit.

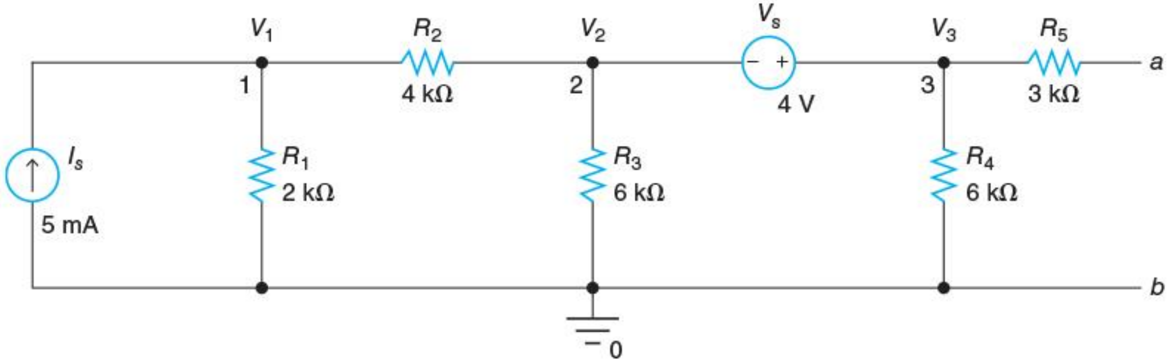


EXAMPLE 4.9

Find V_{th} and R_{th} for the circuit shown in Figure 4.73.

FIGURE 4.73

Circuit for EXAMPLE 4.9.



EXAMPLE 4.11

Find V_{th} and R_{th} for the circuit shown in Figure 4.85.

- Since $i_2 = 0$, the voltage across CCVS is zero ($2i_2 = 0$). Thus,

  $i_1 = V_s/R_1 = 10\ \text{V}/5\ \Omega = 2\ \text{A}$
- $V_{th} = V_{oc} = 3i_1 = 6\ \text{V}$

- To find R_{th}, after deactivating V_s, a test voltage of 1 V is applied across *a* and *b* as shown in Figure 4.86.
- $i_1 = -2i_2/R_1 = -2i_2/5$
- $i_2 = (V_t - 3i_1)/4 = (1 + 6i_2/5)/4$

  $\Rightarrow 14i_2 = 5 \Rightarrow i_2 = 5/14\ \text{A}$
- $R_{th} = V_t/i_2 = 14/5\ \Omega = 2.8\ \Omega$

FIGURE 4.85

Circuit for EXAMPLE 4.11.

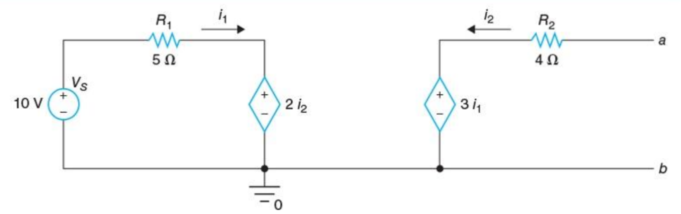
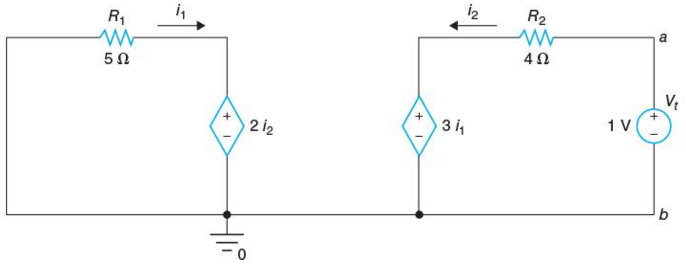


FIGURE 4.86

A circuit with a test voltage.

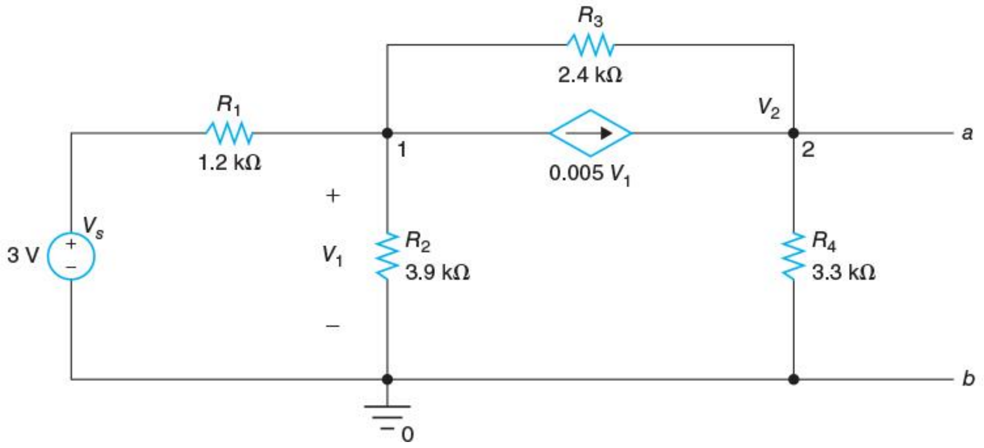


EXAMPLE 4.12

Find V_{th} and R_{th} for the circuit shown in Figure 4.89.

FIGURE 4.89

Circuit for EXAMPLE 4.12.



Summary

- Thévenin's Theorem: A circuit consisting of a voltage source V_{th} and a series resistor R_{th}, representing the original circuit looking from a pair of terminals, is called a Thévenin equivalent circuit. The voltage V_{th} is called Thévenin equivalent voltage, and the resistance R_{th} is called Thévenin equivalent resistance.
- There are three methods to find Thévenin equivalent resistance.
  - Method 1: Deactivate all the independent sources by short-circuiting voltage sources and open-circuiting current sources. Find the equivalent resistance looking into the circuit from terminals *a* and *b.*
  - Method 2: Short-circuit terminals *a* and *b*. Find the short-circuit current I_{sc}. The Thévenin equivalent resistance is given by $R_{th} = V_{oc}/I_{sc} = V_{th}/I_{sc}$.
  - Method 3: Deactivate all the independent sources. Apply a test voltage of 1 V between terminals *a* and *b* with terminal *a* connected to the positive terminal of the test voltage. Measure the current flowing out of the positive terminal of the test voltage source. The Thévenin equivalent resistance is the ratio of the voltage to current. Test current can be used also.
- What will we study in next lecture.