Circuit Analysis and Design

Academic Year 2025/2026 – Semester 1

Lecture 12 – RC Circuits

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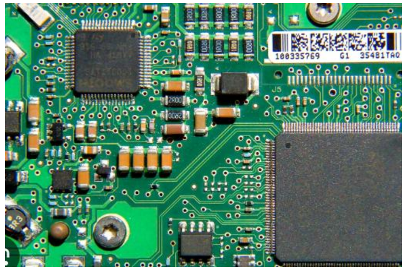
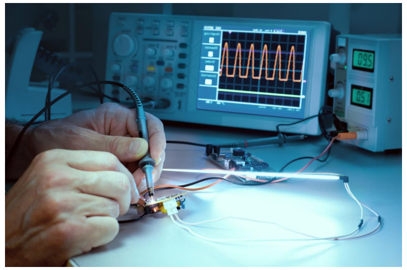




Agenda

- **Natural response of RC circuit**
- **Step response of RC circuit**

Introduction

- **When a capacitor or an inductor possesses initial energy, the circuit responds to the initial energy until all the energy is spent, even when there is no input signal. The response of a circuit due to initial energy only is called a natural response (also transient response, zero input response, and source-free response).**

- **The response of a circuit to a dc input signal (step input) is called a step response. The step response includes the response due to the initial energy stored in the capacitor or inductor.**

Natural Response of RC Circuit

- **The switch in Figure 7.1 has been in position *a* for a long time before it is moved to position *b* at $t = 0$. At t = 0, the voltage across the capacitor is equal to the voltage of the source V_S; that is, $v(0) = V_0 = V_S$. For $t \geq 0$, the circuit shown in Figure 7.1 becomes the circuit shown in Figure 7.2, with initial voltage of $v(0) = V_0 = V_S$.**

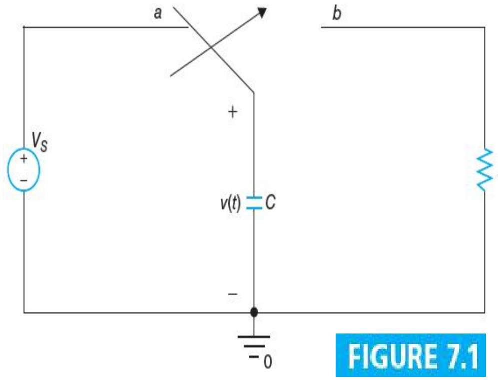


FIGURE 7.1

- **Summing the currents leaving node 1, we obtain**

$$C\frac{dv(t)}{dt}+\frac{v(t)}{R}=0 \Rightarrow \frac{dv(t)}{dt}=-\frac{v(t)}{RC} \Rightarrow \frac{\frac{dv(t)}{dt}}{v(t)}=-\frac{1}{RC} \Rightarrow \frac{d}{dt}\ln[v(t)]=-\frac{1}{RC} \quad (1)$$

- **Integrating on both sides of the last expression of (1), we get**

$$\ln[v(t)]=-\int_0^t \frac{1}{RC}dt+K \quad (2)$$

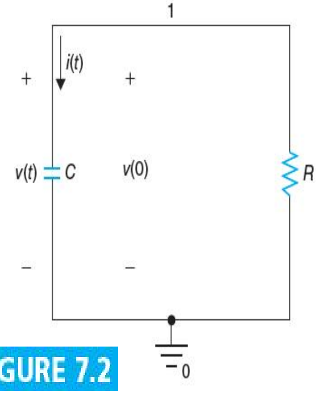


FIGURE 7.2

- **Exponentiation on both sides of Equation (2), we obtain**

$$e^{\ln[v(t)]}=v(t)=e^{\left(-\frac{t}{RC}+K\right)}=e^{K}e^{-\frac{t}{RC}}=Ae^{-\frac{t}{RC}} \quad (3)$$

Natural Response of RC Circuit (Continued)

- At $t = 0$, Equation (3) becomes $$v(0) = Ae^{-\frac{0}{RC}} = A$$

- Thus, $A = v(0) = V_0$. The voltage across the capacitor (also resistor) is given by

$$v(t) = v(0)e^{-\frac{t}{RC}}u(t) = V_0 e^{-\frac{t}{RC}}u(t) \quad (4)$$

where $u(t) = 1$ for $t \geq 0$ and $u(t) = 0$ for $t < 0$. $u(t)$ is called unit step function.

- The current through the capacitor is given by

$$i(t) = C\frac{dv(t)}{dt} = C\left(-\frac{1}{RC}\right)v(0)e^{-\frac{t}{RC}} = -\frac{v(0)}{R}e^{-\frac{t}{RC}} = -\frac{V_0}{R}e^{-\frac{t}{RC}}u(t)$$

- The instantaneous power on the capacitor is given by

$$p_C(t) = v(t)i(t) = -\frac{v^2(0)}{R}e^{-\frac{2t}{RC}}u(t) \quad \text{(power is released)}$$

- The energy absorbed by the resistor is given by $$w_R(t) = \int_0^t p_R(\lambda)d\lambda = \frac{Cv^2(0)}{2}\left(1 - e^{-\frac{2t}{RC}}\right)u(t)$$
- At $t = \infty$, $w(\infty) = 0.5CV_0^2$.

EXAMPLE 7.1

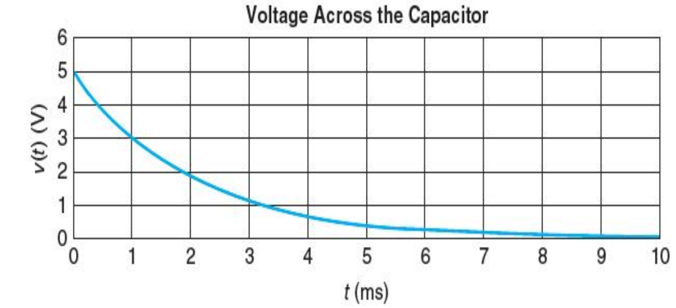
- **Let** $v(0) = V_S = 5\ V$**,** $R = 2\ k\Omega$**,** $C = 1\ \mu F$ **in the circuit shown in Figure 7.2. Find the expression of** $v(t)$, $i(t)$, $p_C(t)$ **and plot** $v(t)$, $i(t)$, **and** $p_C(t)$ **for** $t \geq 0$**.**

EXAMPLE 7.1

- **Let** $v(0) = V_S = 5$ V, $R = 2$ kΩ, $C = 1$ μF **in the circuit shown in Figure 7.2. Find the expression of** $v(t)$, $i(t)$, $p_C(t)$ **and plot** $v(t)$, $i(t)$, **and** $p_C(t)$ **for** $t \geq 0$.

- $RC = 2\times10^{-3} = 2$ ms, $1/(RC) = 500$ (1/s)

$$v(t)=V_0e^{-\frac{t}{RC}}u(t)=5e^{-500t}u(t)\,\mathrm{V}, i(t)=\frac{-V_0}{R}e^{-\frac{t}{RC}}u(t)=-2.5e^{-500t}u(t)\,\mathrm{mA}, p_C(t)=\frac{-V_0^2}{R}e^{-\frac{2t}{RC}}u(t)=-12.5e^{-1000t}u(t)\,\mathrm{mW}$$



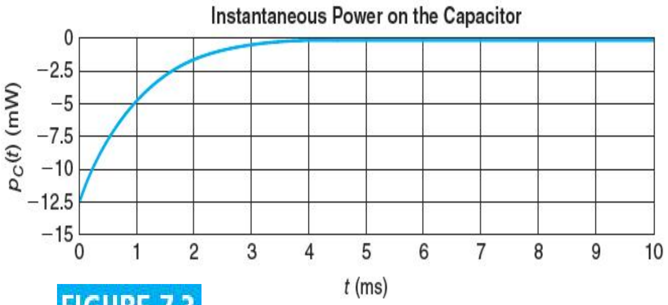


FIGURE 7.3

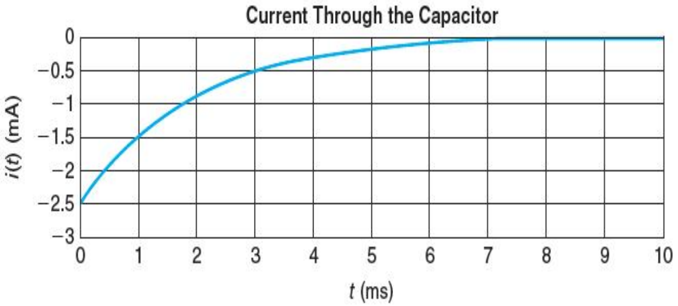
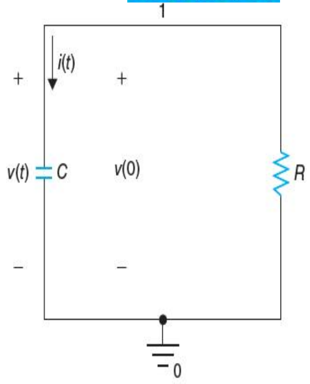


FIGURE 7.2



Time Constant of RC Circuit

- **The product of R and C, RC, is measured in seconds and is called a time constant, denoted by τ. Thus,** τ = RC

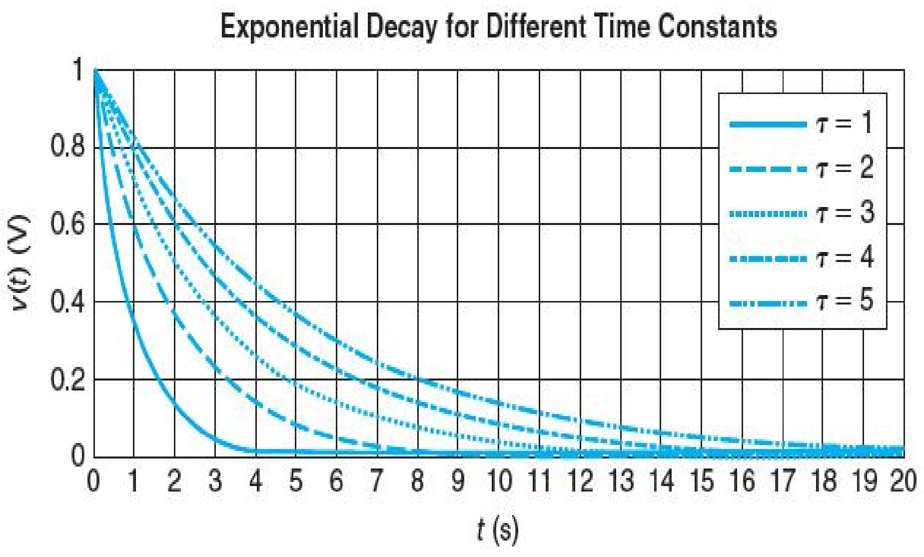
- **In terms of τ,** *v(t), i(t),* **and** $p_C(t)$ **for the circuit shown in Figure 7.2 become, respectively**

$$v(t) = V(0)e^{-\frac{t}{\tau}}u(t), i(t) = \frac{-V(0)}{R}e^{-\frac{t}{\tau}}u(t), p_C(t) = \frac{-V^2(0)}{R}e^{-\frac{2t}{\tau}}u(t)$$

- **Figure 7.4 shows v(t) for** τ = *1, 2, 3, 4*, **and** *5* **[(***v(0)* = *1 V***].**

- **As the time constant τ increases, it takes longer time for the voltage across the capacitor to decay. The speed at which the charges stored on the capacitor plates are discharged is controlled by the time constant τ.**

FIGURE 7.4



Time Constant of RC Circuit (Continued)

- **At time zero (t = 0), the voltage is at its peak value v(0). At time t = τ, the voltage is**

$$v(\tau) = v(0)e^{-\frac{\tau}{\tau}} = v(0)e^{-1} = 0.3678794412\, v(0)$$

- **At time** t = τ**, the voltage across the capacitor drops to** *36.788%* **of the initial value at** *t = 0*. **For** $t = 2\tau, 3\tau, 4\tau, 5\tau, \ldots, 10\tau$**, we have the values shown in Table 7.1.**

TABLE 7.1 Voltage Across the Capacitor Normalized to V_0 at $t = n\tau$.

n	exp(−n)
0	1.000000000000000
1.000000000000000	0.367879441171442
2.000000000000000	0.135335283236613
3.000000000000000	0.049787068367864
4.000000000000000	0.018315638888734
5.000000000000000	0.006737946999085
6.000000000000000	0.002478752176666
7.000000000000000	0.000911881965555
8.000000000000000	0.000335462627903
9.000000000000000	0.000123409804087
10.000000000000000	0.000045399929762

Time Constant of RC Circuit (Continued)

- **At five times the time constant, the voltage across the capacitor due to initial energy on the capacitor is less than** *1%* **of the initial voltage (***0.6738%***). For all practical purposes, the transient response can be ignored after about five times the time constant.**
- **The time derivative of the voltage across the capacitor is given by**

$$\frac{dv(t)}{dt} = v(0)\frac{d}{dt}e^{-\frac{t}{\tau}} = -\frac{v(0)}{\tau}e^{-\frac{t}{\tau}}$$

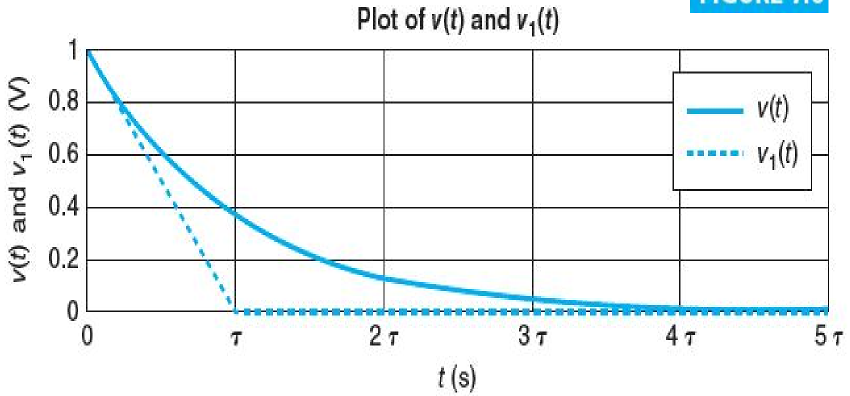
- **The rate of decay of the voltage across the capacitor is at its maximum at t = 0 and slows down as time progresses. The rate of decay at** $t = 0$ **is** $-v(0)/\tau$**.**

  If the voltage decreases at this rate, $v(t)$ **is given by**

$$v_1(t) - v(0) = -\frac{v(0)}{\tau}(t - 0)$$

- **Figure 7.6 shows** $v(t)$ **and** $v_1(t)$**.**

FIGURE 7.6



Finding the Time Constant

- If there is one resistor with resistance *R* and one capacitor with capacitance *C*, as in the circuit shown in Figure 7.2, the time constant is the product of the *R* and *C*; that is, $\tau = RC$.

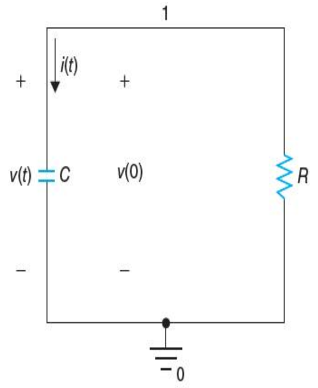
- If there is one capacitor with capacitance *C* and more than one resistor in the circuit, find the equivalent resistance R_{eq} of all the resistors in the circuit that connects in parallel to the capacitor. Then, the circuit reduces to one capacitor with capacitance *C* and one resistor with resistance R_{eq} connected in parallel. The time constant is given by $\tau = R_{eq}C$.

- The voltage across the capacitor is given by

$$v(t) = v(0) e^{-\frac{t}{\tau}} u(t) \text{ V}$$

- Finding the voltage *v(t)* across the capacitor for the given circuit involves finding the initial voltage *v(0)* across the capacitor, finding the equivalent resistance R_{eq}, and finding the time constant $\tau = R_{eq}C$.

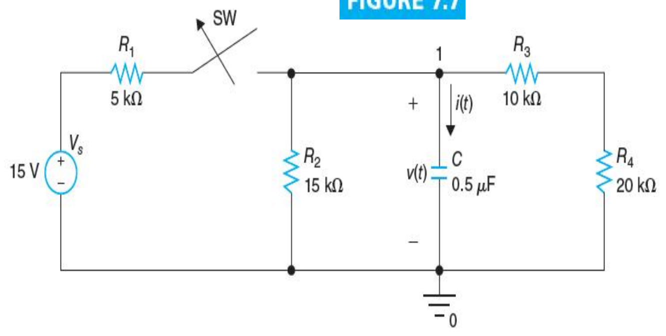
FIGURE 7.2



EXAMPLE 7.2

- In the circuit shown in Figure 7.7, the switch has been closed for a long time before it is opened at $t = 0$. Find the voltage $v(t)$ across the capacitor and the current $i(t)$ through the capacitor for $t \geq 0$ and plot $v(t)$ and $i(t)$ for $t \geq 0$. Before the switch is opened, the capacitor can be treated as an open circuit.

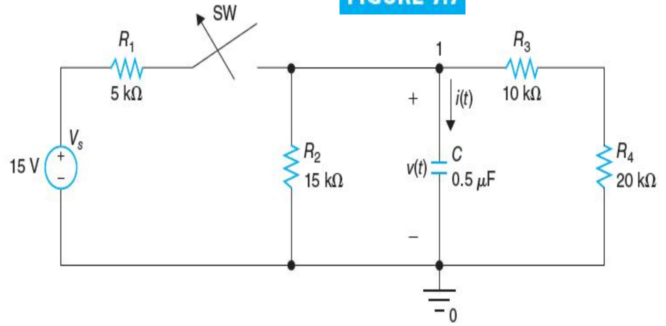
FIGURE 7.7



EXAMPLE 7.2

- **In the circuit shown in Figure 7.7, the switch has been closed for a long time before it is opened at** $t = 0$**. Find the voltage** $v(t)$ **across the capacitor and the current** $i(t)$ **through the capacitor for** $t \geq 0$ **and plot** $v(t)$ **and** $i(t)$ **for** $t \geq 0$**. Before the switch is opened, the capacitor can be treated as an open circuit.**
- $R_2 \parallel (R_3 + R_4) = 15 \times 30/(15 + 30)\ k\Omega = 10\ k\Omega$
- **Voltage divider rule:** $v(0) = V_0 = 15\ V \times 10\ k\Omega / 15\ k\Omega = 10\ V$
- $R_{eq} = R_2 \parallel (R_3 + R_4) = 15 \times 30/(15 + 30)\ k\Omega = 10\ k\Omega$
- $\tau = R_{eq}C = 10^4 \times 5 \times 10^{-7} = 5 \times 10^{-3} = 5\ ms,\ 1/\tau = 200\ (1/s)$

FIGURE 7.7



EXAMPLE 7.2

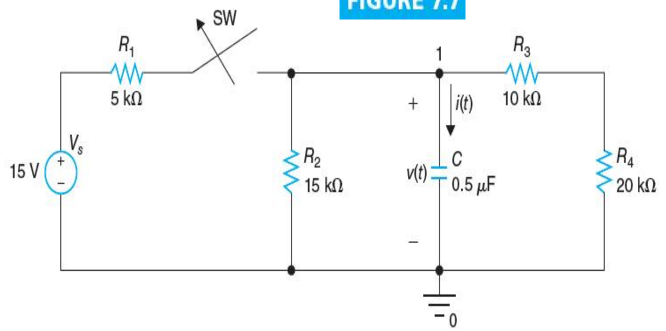
- **In the circuit shown in Figure 7.7, the switch has been closed for a long time before it is opened at** $t = 0$**. Find the voltage** $v(t)$ **across the capacitor and the current** $i(t)$ **through the capacitor for** $t \geq 0$ **and plot** $v(t)$ **and** $i(t)$ **for** $t \geq 0$**. Before the switch is opened, the capacitor can be treated as an open circuit.**
- $R_2 \,||\, (R_3 + R_4) = 15\times30/(15 + 30)\ k\Omega = 10\ k\Omega$
- **Voltage divider rule:** $v(0) = V_0 = 15\ V\times10\ k\Omega/15\ k\Omega = 10\ V$
- $R_{eq} = R_2 \,||\, (R_3 + R_4) = 15\times30/(15 + 30)\ k\Omega = 10\ k\Omega$
- $\tau = R_{eq}C = 10^4\times5\times10^{-7} = 5\times10^{-3} = 5\ ms,\ 1/\tau = 200\ (1/s)$
- **The voltage across the capacitor is given by**

$$v(t) = V(0)e^{-\frac{t}{\tau}}u(t) = 10e^{-200t}u(t)\ \mathrm{V}$$

- **The current through the capacitor is**

$$i(t) = C\frac{dv(t)}{dt} = 5\times10^{-7}\times10\times(-200)e^{-200t}u(t) = -e^{-200t}u(t)\ \mathrm{mA}$$

FIGURE 7.7



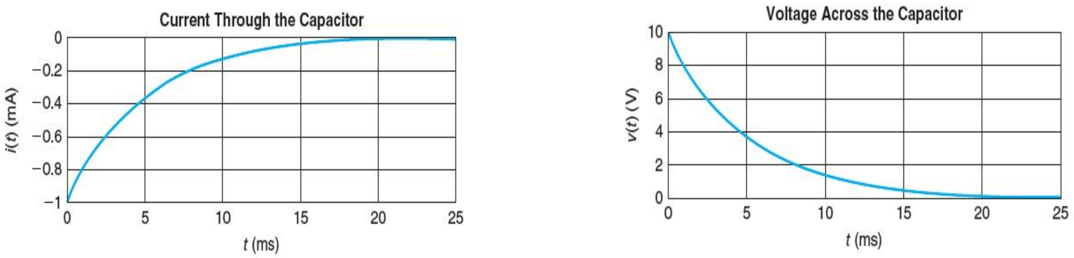


FIGURE 7.10

EXAMPLE 7.3

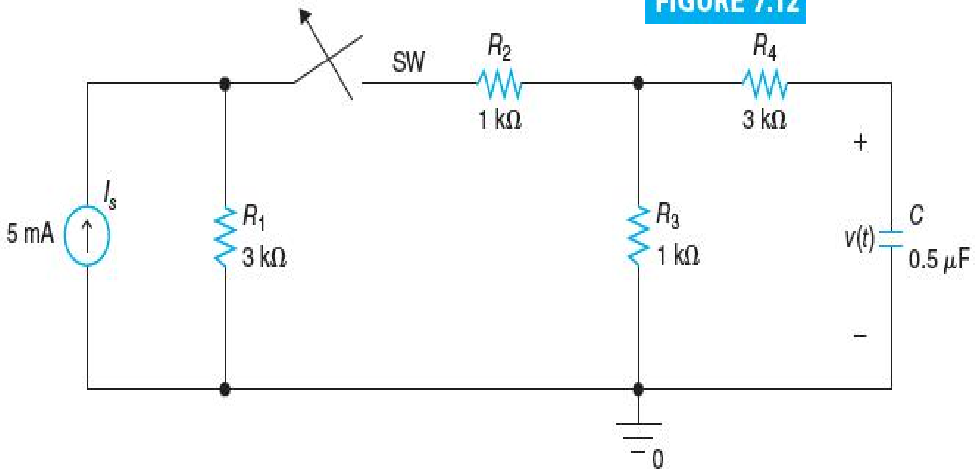
- In the circuit shown in Figure 7.12, the switch has been closed for a long time before it is opened at $t = 0$. Find the voltage v(t) across the capacitor for $t \geq 0$. Before the switch is opened, the capacitor can be treated as an open circuit. The current through R_4 is zero.

- From the current divider rule, the current through R_3 is given by $5\ mA \times 3\ k\Omega / 5\ k\Omega = 3\ mA$
- Ohm's law: $v(0) = V_0 = 1\ k\Omega \times 3\ mA = 3\ V$

- $R_{eq} = R_3 + R_4 = 4\ k\Omega$
- $\tau = R_{eq}C = 4 \times 10^3 \times 5 \times 10^{-7} = 2 \times 10^{-3} = 2\ ms,\ 1/\tau = 500\ (1/s)$

- The voltage across the capacitor is given by $$v(t) = V(0)e^{-\frac{t}{\tau}}u(t) = 3e^{-500t}u(t)\ \text{V}$$

FIGURE 7.12



Step Response of RC Circuit

- The switch in the circuit shown in Figure 7.15 is closed at $t = 0$. At $t = 0$, the voltage across the capacitor is $v(0) = V_0$.

- Collecting the voltage drops around the mesh in the clockwise direction for $t \geq 0$, we have
  $- V_S + R\, i(t) + v(t) = 0 \quad (1)$

- The current through the capacitor is given by $i(t) = C\dfrac{dv(t)}{dt} \quad (2)$

- Substitution of Equation (2) into Equation (1) yields

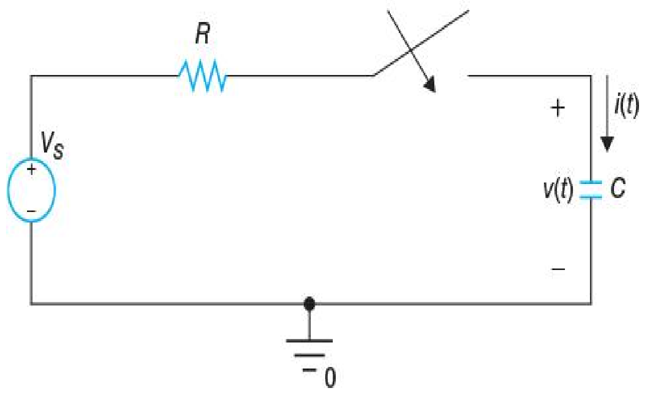
$$-V_S + RC\frac{dv(t)}{dt} + v(t) = 0 \Rightarrow \frac{dv(t)}{dt} = \frac{-1}{RC}\left[v(t) - V_S\right] \quad (3)$$

- Dividing by $v(t) - V_S$ on both sides, we obtain

$$\frac{\frac{dv(t)}{dt}}{v(t) - V_S} = \frac{-1}{RC} \Rightarrow \frac{d}{dt}\left[\ln\left|v(t) - V_S\right|\right] = \frac{-1}{RC} \quad (4)$$

FIGURE 7.15

RC circuit with step input.



Step Response of RC Circuit (Continued)

- **Integrating on both sides of Equation (4), we obtain**

$$\ln\left|v(t)-V_S\right|=\int_0^t \frac{-1}{RC}dt+K=\frac{-t}{RC}+K \quad (5)$$

- **Exponentiation on both sides of Equation (5) yields**

$$e^{\ln\left|v(t)-V_S\right|}=\left|v(t)-V_S\right|=e^K e^{-\frac{t}{RC}} \Rightarrow v(t)-V_S=\pm e^K e^{-\frac{t}{RC}} \quad (6)$$

- **Equation (6) can be rewritten as**

$$v(t)=V_S+Ae^{-\frac{t}{RC}} \quad (7)$$

- **The constant A can be found by applying the initial condition:**

  $v(0) = V_0 = V_S + A \Rightarrow A = V_0 - V_S$

Step Response of RC Circuit (Continued)

- The voltage across the capacitor can be written as

$$v(t) = V_S + \left(V_0 - V_S\right) e^{-\frac{t}{RC}} \quad (8)$$

- This solution is valid for $t \geq 0$. At t = 0, the voltage is $v(0) = V_0$, and at $t = \infty$, the voltage is $v(\infty) = V_S$. The voltage across the capacitor changes from the initial value of $v(0) = V_0$ at $t = 0$ to the final value of $v(\infty) = V_S$ at $t = \infty$. The final value of $v(\infty) = V_S$ can be obtained from the circuit shown in Figure 7.15. At $t = \infty$, the capacitor can be treated as an open circuit. The current through the resistor is zero and the voltage drop across the resistor is zero. The voltage across the capacitor is V_S.

- Equation (8) can be rewritten as ($\tau = RC$)

$$v(t) = v(\infty) + \left[v(0) - v(\infty)\right] e^{-\frac{t}{\tau}} = (\text{Final Value}) + \left[(\text{Initial Value}) - (\text{Final Value})\right] e^{-\frac{t}{(\text{Time Constant})}} \quad (9)$$

- If there is a time delay t_d, replace t by $t - t_d$.

Step Response of RC Circuit (Continued)

- **Equation (9) is the solution to a differential equation given by the first equation in Equation (3):**

$$-V_S + RC\frac{dv(t)}{dt} + v(t) = 0 \Rightarrow \frac{dv(t)}{dt} + \frac{1}{RC}v(t) = \frac{1}{RC}V_S \Rightarrow \frac{dv(t)}{dt} + \frac{1}{\tau}v(t) = \frac{1}{\tau}V_S \quad (10)$$

- **In the steady state at** $t = \infty$**, since** $dv(t)/dt = 0$**, Equation (10) becomes**

$$\frac{1}{\tau}v(\infty) = \frac{1}{\tau}V_S$$

Thus, $v(\infty) = V_S$**.**

- **If there is a time delay** t_d**, replace** t **by** $t - t_d$ **in Equation (9).**

Time Constant

•For RC circuits with one capacitor in the circuit, the time constant is given by $\tau = R_{eq}C$ where R_{eq} is the equivalent resistance seen from the capacitor.

• The equivalent resistance R_{eq} is the Thévenin equivalent resistance when the rest of the circuit (excluding the capacitor) is converted to the Thévenin equivalent circuit.

•In general, R_{eq} can be found by deactivating independent sources (short-circuit voltage sources and open-circuit current sources) and finding the equivalent resistance seen from the capacitor. Other methods, such as test voltage and test current, can also be used.

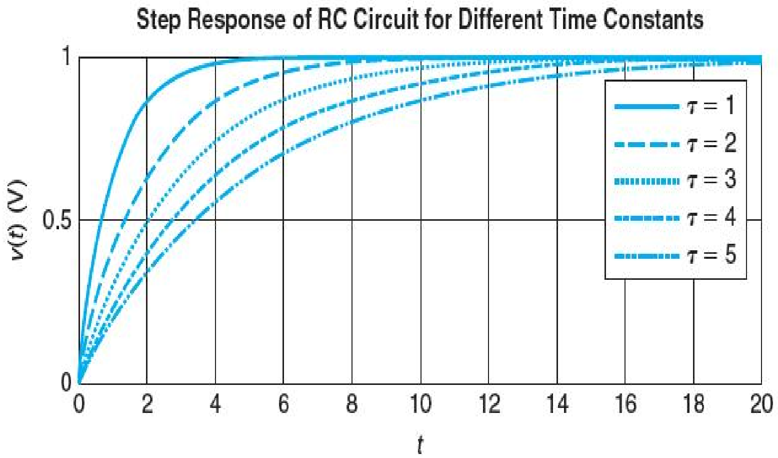
•Figure 7.16 shows $v(t)$ given by Equation (9) for $V_S = 1\ V$, $V_0 = 0\ V$, and five different values of τ.

•At $t = \tau$, $v(\tau) = 0.63212\ V_S$. At $t = \tau$, the voltage reaches *63.212%* of

•the final value.

•At $t = 5\tau$, the voltage reaches *99.3262%* of the final value. Refer to Table 7.2 in the text.

FIGURE 7.16



EXAMPLE 7.4

- **Let** $V_S = 1\ V,\ R = 4\ k\Omega,\ C = 1\ \mu F,\ v(0) = 0\ V$ **for the circuit shown in Figure 7.15. Find the voltage** $v(t)$ **across the capacitor and the current i(t) through the capacitor for** $t \geq 0$**. Plot** $v(t)$ **and i(t) for** $t \geq 0$**.**

- $\tau = RC = 4000\times1\times10^{-6} = 4\times10^{-3} = 4\ ms,\ 1/\tau = 250\ (1/s),\ v(\infty) = V_S = 1\ V.$

- **The voltage across the capacitor is given by**

$$v(t)=V_S+\left(V_0-V_S\right)e^{-\frac{t}{RC}}=1+(0-1)e^{-250t}=\left(1-e^{-250t}\right)u(t)\ \mathrm{V}$$

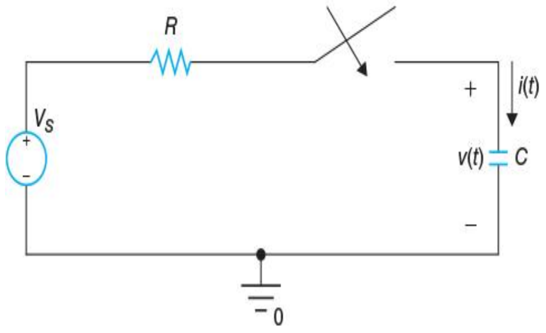
- **The current through the capacitor is given by**

$$i(t)=C\frac{dv(t)}{dt}=CV_S\frac{1}{RC}e^{-\frac{t}{RC}}=\frac{V_S}{R}e^{-\frac{t}{RC}}=250e^{-250t}u(t)\ \mu\mathrm{A}$$

- **Figure 7.19 shows** $v(t)$ **and** $i(t)$**.**

FIGURE 7.15

RC circuit with step input.



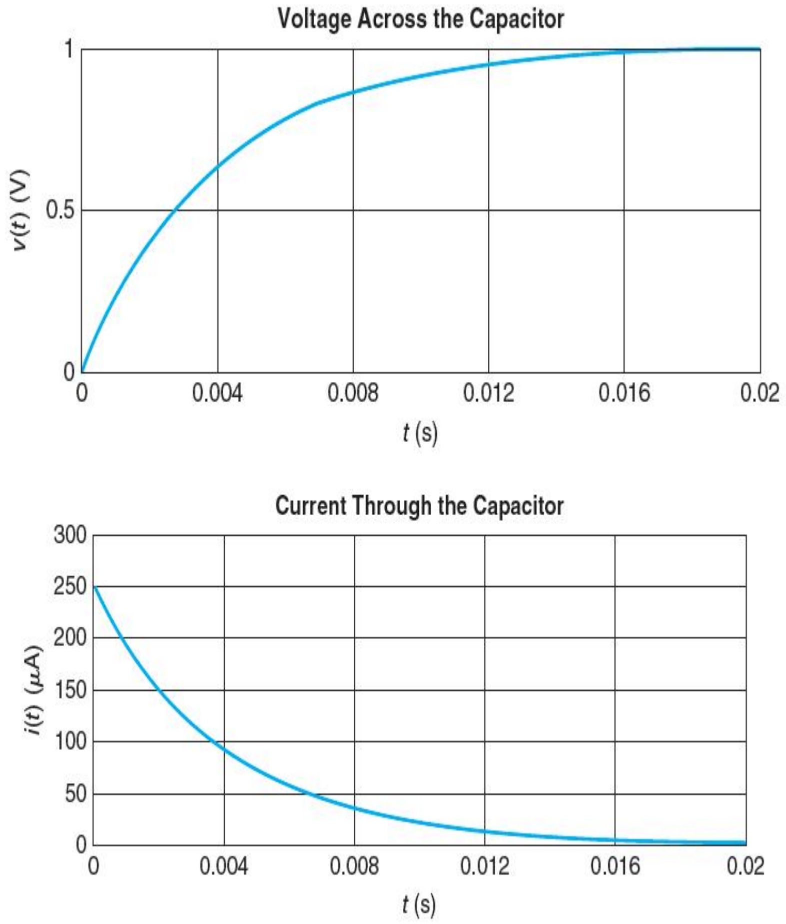


FIGURE 7.19

EXAMPLE 7.5

- **The switch in the circuit shown in Figure 7.20 is closed at *t = 0*. At *t = 0*, the voltage across the capacitor is *v(0)* = V_0. Find the voltage *v(t)* across the capacitor and the current *i(t)* through the capacitor.**

- **Applying the source transformation to the circuit shown in Figure 7.20, we obtain the circuit shown in Figure 7.21.**

- *Final value* = $v(\infty) = I_S R$, *time constant* = $\tau = RC$

- **The voltage across the capacitor is given by**

$$v(t) = I_s R + (V_0 - I_S R) e^{-\frac{t}{RC}}$$

- **The current through the capacitor is given by**

$$i(t) = C\frac{dv(t)}{dt} = \frac{-C}{RC}(V_0 - I_s R) e^{-\frac{t}{RC}} = \left(I_s - \frac{V_0}{R}\right) e^{-\frac{t}{RC}} u(t) \text{ A}$$

FIGURE 7.20

An *RC* circuit.

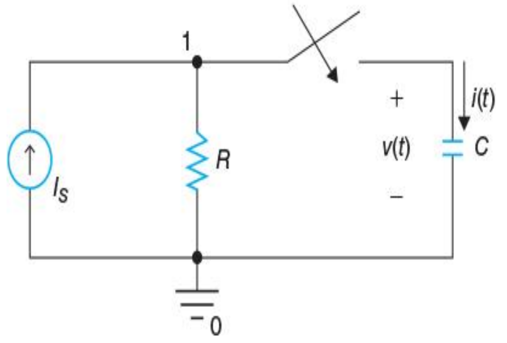
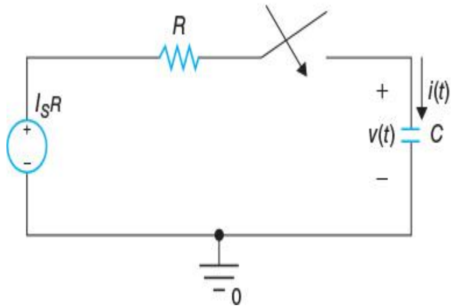


FIGURE 7.21

Thévenin equivalent circuit.



EXAMPLE 7.6

- **Switch 1 is opened at $t = 0$ and switch 2 is closed at $t = 0.5\ ms$ in the circuit shown in Figure 7.22.**

  Find the voltage $v(t)$ across the capacitor for $t \geq 0$.

- **The initial voltage at $t = 0$ across the capacitor is $v(0) = 10\ V \times 4\ k\Omega / 10\ k\Omega = 4\ V$.**

- **For $0 \leq t \leq 0.5\ ms$, $\tau = R_2C = 4000 \times 0.25 \times 10^{-6} = 1\ ms$, $1/\tau = 1000$. $v(t) = 4\ exp(-1000t)\ u(t)\ V$.**

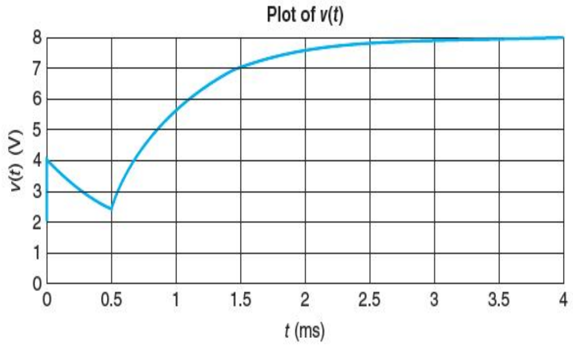
- **At $t = 0.5\ ms$, $V_1 = v(0.5\ ms) = 4\ exp(-0.5) = 2.4261\ V$**

- **For $0.5\ ms \leq t$, the final value is $v(\infty) = 20\ V \times 4\ k\Omega / 10\ k\Omega = 8\ V$. $R_{eq} = R_2 \,||\, R_3 = 2.4\ k\Omega$**

- **For $0.5\ ms \leq t$, $\tau = R_{eq}C = 2400 \times 0.25 \times 10^{-6} = 0.6\ ms$, $1/\tau = 1666.6667\ (1/s)$.**

- **For $0.5\ ms \leq t$, $v(t) = [8 + (2.4261 - 8)exp(-1666.6667(t - 0.0005))]$**

  $= [8 - 5.5739exp(-1666.6667(t - 0.0005))]\ u(t)\ V$



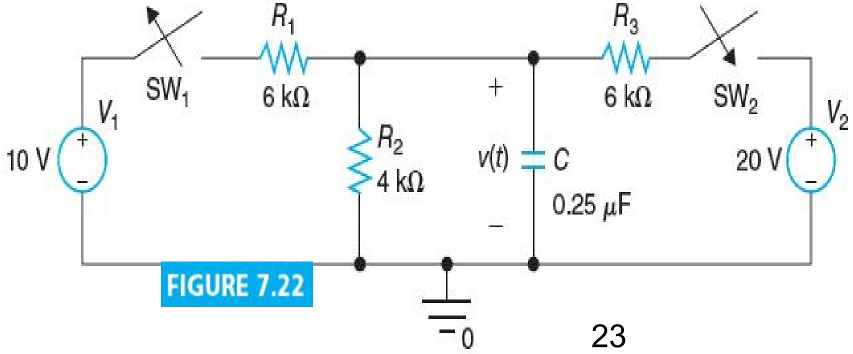


FIGURE 7.22

EXAMPLE 7.7

- **Switch 1 is opened at** $t = 0$ **and switch 2 is closed at** $t = 0$ **in the circuit shown in Figure 7.25.**

  Find the voltage $v(t)$ **across the capacitor and voltage** $v_a(t)$ **at node** ***a*** **for** $t \geq 0$**.**

- **For** $t \leq 0$**, the current through R_3-C path is zero. Voltage divider rule:** $v(0) = 5\ V \times 20/50 = 2\ V$**.**

- **At** $t = \infty$**, the current through R_3-C path is zero. Current through R_4-R_2 path is** $i_2 = 0.2\ mA$**:**

  Current divider rule: $0.5\ mA \times 20/(20 + 10 + 20) = 0.2\ mA,\ v(\infty) = R_2 i_2 = 20\ k\Omega \times 0.2\ mA = 4\ V$

- **For** $0 \leq t,\ R_{eq} = R_3 + [R_2 \,||\, (R4 + R5)] = 8\ k\Omega + [20\ k\Omega \,||\, 30\ k\Omega] = 20\ k\Omega$

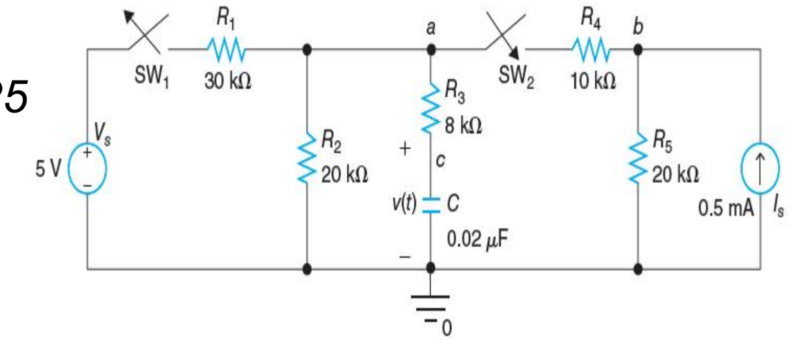
- **For** $0 \leq t,\ \tau = R_{eq}C = 20{,}000 \times 2 \times 10^{-8} = 0.4\ ms,\ 1/\tau = 2500\ (1/s)$

- **For** $0 \leq t,\ v(t) = [4 + (2 - 4)\exp(-2500t)]\ u(t)\ V = [4 - 2\exp(-25$

- **For** $0 \leq t,\ i(t) = C\ dv(t)/dt = 0.1\exp(-2500t)]\ u(t)\ mA$

  $v_a(t) = R_3 i + v(t) = [4 - 1.2\exp(-2500t)]\ u(t)\ V$

FIGURE 7.25



EXAMPLE 7.8

- **Find the voltage *v(t)* across the capacitor in the circuit shown in Figure 7.27. The initial voltage across the capacitor is *v(0) = 0 V*.**
- **The voltage v_a can be obtained by applying the voltage divider rule:**

$v_a = V_s \times R_2/(R_1 + R_2) = 0.5\ V \times 2/5 = 0.2\ V$

- **Summing the currents leaving node *b*, we obtain**

$$0.01\times 0.2+\frac{v(t)}{1000}+1\times 10^{-6}\frac{dv(t)}{dt}=0\Rightarrow\frac{dv(t)}{dt}+1000v(t)=-2000$$

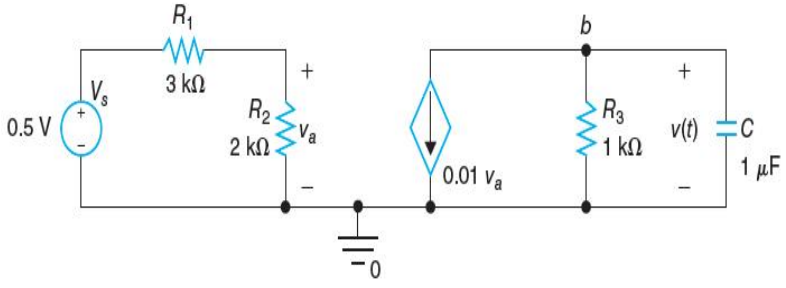


FIGURE 7.27

- **At** $t = \infty$, $dv(t)/dt = 0$, $1000v(\infty) = -2000 \Rightarrow v(\infty) = -2\ V$
- $1/\tau = 1000\ (1/s)$, $\tau = 1/1000 = 1\ ms$
- **The voltage across the capacitor is**

$$v(t)=\left[v(\infty)+\left(v(0)-v(\infty)\right)e^{-\frac{t}{\tau}}\right]u(t)=\left[-2+2e^{-1000t}\right]u(t)\ \mathrm{V}$$

Summary

- Figure 7.2 shows a circuit with one capacitor with capacitance *C* and one resistor with resistance *R* connected in parallel. Let the initial voltage across the capacitor at *t = 0* be *v(0)*. Then, the voltage *v(t)* across the capacitor for $t \geq 0$ is given by

$$v(t) = v(0) e^{-\frac{t}{\tau}} u(t) \text{ V}$$

where the time constant τ is given by $\tau = RC$.

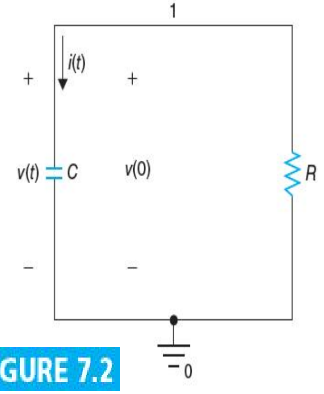
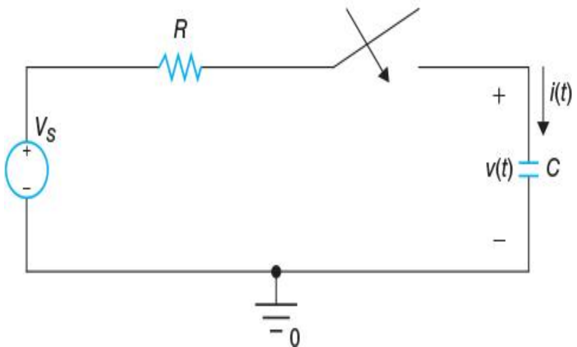


FIGURE 7.2

- Figure 7.15 shows a circuit with a voltage source with voltage V_S, a resistor with resistance *R*, and a capacitor with capacitance *C* connected in series. Let the initial voltage across the capacitor at t = 0 be $v(0) = V_0$. Then, the voltage *v(t)* across the capacitor for $t \geq 0$ is given by

FIGURE 7.15

RC circuit with step input.



where the time constant τ is given by $\tau = RC$.

$$v(t) = V_S + (V_0 - V_S) e^{-\frac{t}{\tau}}$$