College Physics (II)

- **Electromagnetism**
- **Modern physics**

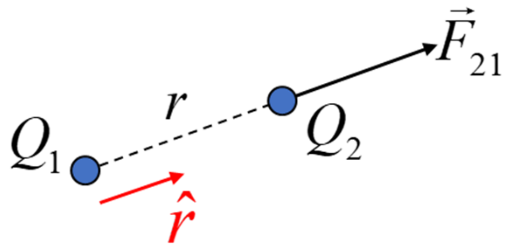
Summary-Chapter 19

1. Electric charge

Quantized: $e = 1.602 \times 10^{-19}$ C

Law of conservation of electric charge : **The net amount of electric charge produced in any process is zero,** or **no net electric charge can be created or destroyed.**

2. Coulomb's Law



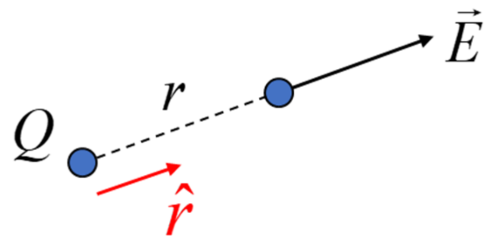
$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}\hat{r} = k\frac{Q_1 Q_2}{r^2}\hat{r}$$

Permittivity of free space:

$$\varepsilon_0 = 8.85\times10^{-12}\,C^2/N\cdot m^2 \qquad k = \frac{1}{4\pi\varepsilon_0} \approx 9.0\times10^{9}\,N\cdot m^2/C^2$$

3. Electric Field

$$\vec{E} = \frac{\vec{F}}{q} \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$



Charged particle in an electric field:

$$\vec{F} = q\vec{E}$$

Superposition principle:

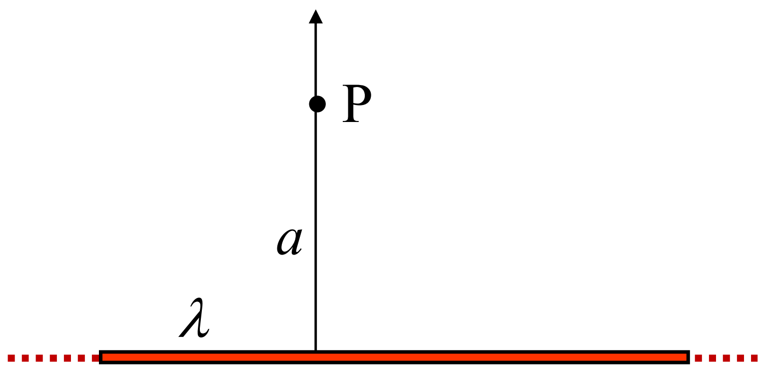
$$\vec{E} = \sum_i \vec{E}_i$$

4. Electric Field Calculations for Continuous Charge Distributions

$$\vec{E} = \int \mathrm{d}\vec{E} = \int \frac{\mathrm{d}q}{4\pi\varepsilon_0 r^2}\hat{r}$$

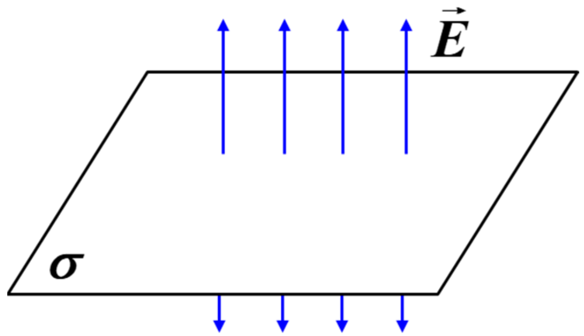
(1) infinite long charged line :

$$E_P = \frac{\lambda}{2\pi\varepsilon_0 a}$$



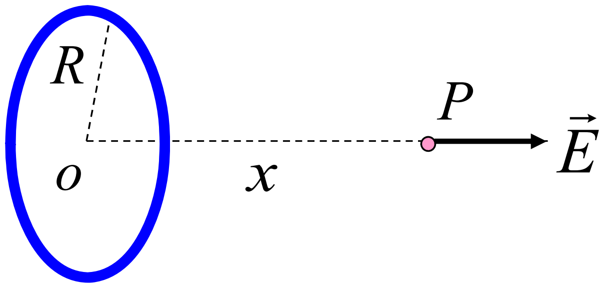
(2) infinite charged plane :

$$E_P = \frac{\sigma}{2\varepsilon_0}$$



(3) Uniformly charged ring :

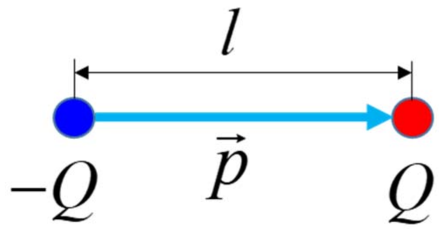
$$\boxed{E = \frac{Q \cdot x}{4\pi\varepsilon_0 (x^2 + R^2)^{3/2}}}$$



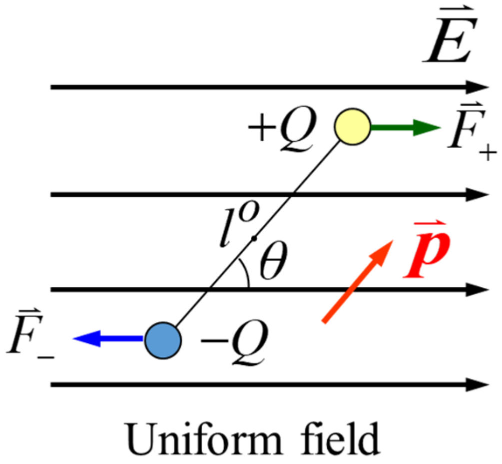
5. Electric dipole

(1) Dipole moment:

$$\vec{p} = Q\vec{l}$$



(2) In external uniform field:



The total force: $\vec{F} = \vec{F}_- + \vec{F}_+ = 0$

The total :

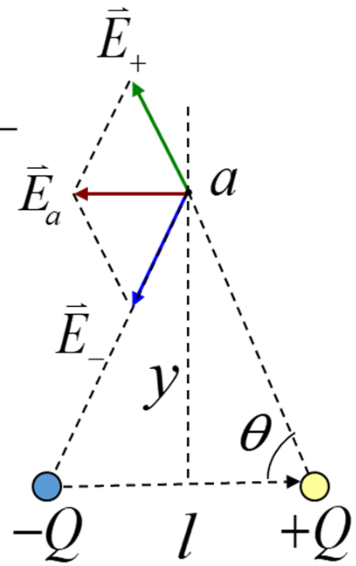
$$\vec{\tau} = \vec{p} \times \vec{E} \quad (\tau = pE\sin\theta)$$

Potential energy:

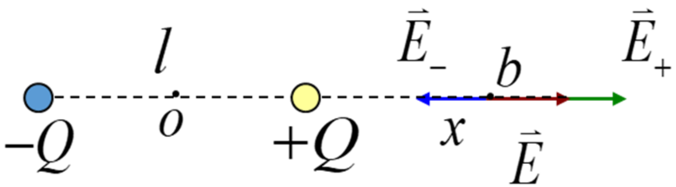
$$U = -\vec{p} \cdot \vec{E}$$

(3) Electric field produced by dipole:

$$\vec{E}_a = \vec{E}_+ + \vec{E}_-$$



$$\vec{E}_a \approx -\frac{\vec{p}}{4\pi\varepsilon_0 y^3} \qquad (y >> l)$$



$$\vec{E}_b \approx \frac{2\vec{p}}{4\pi\varepsilon_0 x^3} \qquad (x >> l)$$

$$\vec{E}_b = \vec{E}_+ + \vec{E}_-$$

Summary-Chapter 20

1. Electric Field Lines

- Direction of electric field $\vec{E}$:

  tangent to the field line at any point

- Magnitude of electric field $\vec{E}$:

  number of lines crossing unit area perpendicular to $\vec{E}$

- Field lines always start away from + charges, end on – charges
- Field lines never cross each other; and there are no closed field lines.

2. Electric Flux

Electric flux: number of electric field lines passing through an area.

Electric Flux passing through an area:

$$\Phi_E = \int_S \vec{E} \cdot d\vec{S}$$

Electric Flux passing through a closed surface:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$

- outward $\rightarrow$ positive
- inward $\rightarrow$ negative

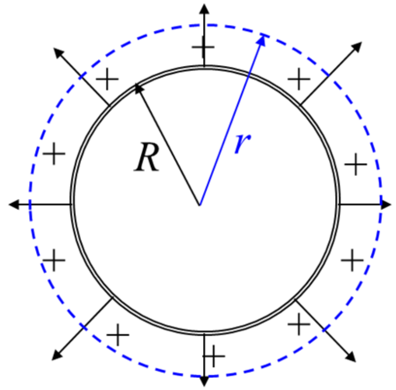
3. Gauss's Law in Vacuum

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Gauss's law in vacuum:

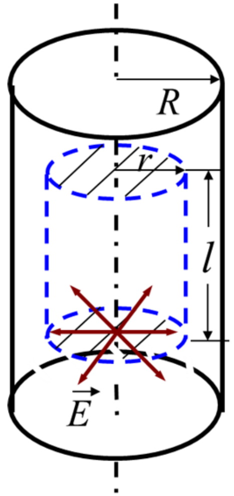
The total electric flux through a closed surface is equal to the total (net) electric charge inside the surface, divided by ε_0.

4. Application of Gauss's Law to Determine the Electric Field



(1) Charged object with spherical symmetric

$$E = \frac{Q_{encl}}{4\pi\varepsilon_0 r^2}$$

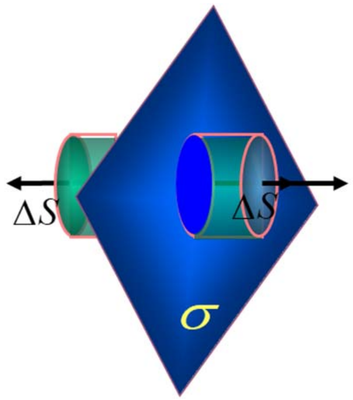


(2) Charged object with cylindrical symmetric

$$E = \frac{Q_{encl}}{2\pi\varepsilon_0 l r} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

λ (Linear charge density): charge per unit length

(3) Charged object with plane symmetric



5. Conductor in External Field

(1) Electrostatic Equilibrium

① Electric field inside at any position is 0;

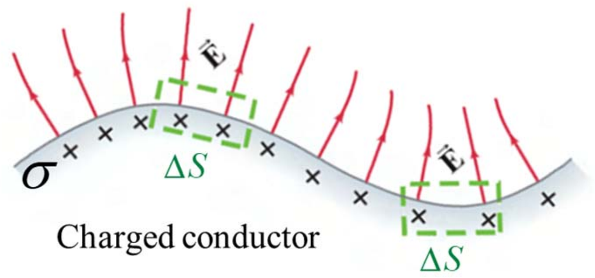
② Electric field near the out surface is perpendicular to the surface.

(2) Distribution of charges

Charges can only be distributed on the surface of conductors.

Electrostatic shielding.

(3) Electric fields outside the surface of conductors



$$E = \frac{\sigma}{\varepsilon_0}$$

Charge density per unit area $\propto$ curvature of the surface

Large curvature → Strong Field

Summary-Chapter 21

1. Static-electric field is conservative

$$\oint \vec{E} \cdot \mathrm{d}\vec{l} = 0 \quad (\nabla \times \vec{E} = 0)$$

2. Electric potential energy, electric potential, electric potential difference

$$V_a = \int_a^{V=0} \vec{E} \cdot \mathrm{d}\vec{l} \qquad V_{ab} = V_a - V_b = \int_a^b \vec{E} \cdot \mathrm{d}\vec{l}$$

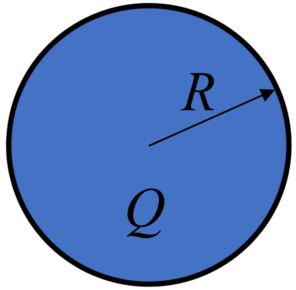
$$W_a = qV_a \qquad W_{ab} = qV_{ab} = q\left(V_a - V_b\right)$$

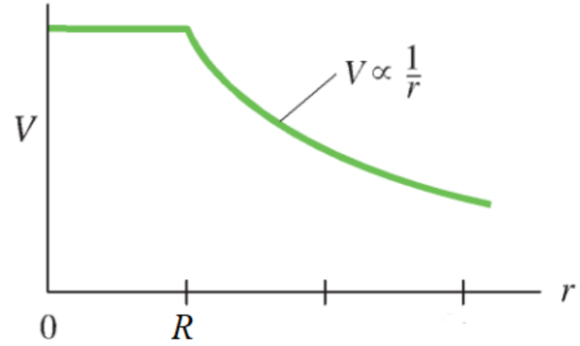
3. Electric potential due to point charges

$$V_a = \frac{Q}{4\pi\varepsilon_0 r} \qquad V_a = \sum_i \frac{q_i}{4\pi\varepsilon_0 r_i} \qquad V = \int \frac{\mathrm{d}Q}{4\pi\varepsilon_0 r}$$

4. Charged conductor sphere

$$V = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 R}, & r \le R \\ \dfrac{Q}{4\pi\varepsilon_0 r}, & r > R \end{cases}$$





5. Equipotential surface and the field lines

(1) Equipotential surfaces is orthogonal to the field lines at any point;

(2) Move a charge on the equipotential surface, no work done on the charge;

6. $\vec{E}$ is equal to the negative gradient of the electric potential V

$$\vec{E} = -\frac{\partial V}{\partial x}\vec{i} - \frac{\partial V}{\partial y}\vec{j} - \frac{\partial V}{\partial z}\vec{k} = -\nabla V$$

Summary-Chapter 22

1. Capacitance of a capacitor

$$C = \frac{Q}{V}$$

Capacitance depends only on the size, shape, and relative position of the two conductors, and also on the material that separates them.

2. Parallel-plate, cylindrical, spherical capacitor

$$C_p = \frac{\varepsilon_0 S}{d} \qquad C_c = \frac{2\pi\varepsilon_0 L}{\ln(R_2/R_1)} \qquad C_s = \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1}$$

3. Capacitors in series and parallel

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \qquad C = C_1 + C_2 + C_3$$

4. Energy stored in capacitor

$$W_e = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$

5. Electric energy density

$$w_e = \frac{1}{2}\varepsilon_0 E^2$$

6. Dielectrics

$$C = \varepsilon_r C_0 = \frac{\varepsilon_r \varepsilon_0 S}{d} = \frac{\varepsilon S}{d}$$

$$E_D = \frac{E_0}{\varepsilon_r} = \frac{\sigma}{\varepsilon_r \varepsilon_0} = \frac{\sigma}{\varepsilon}$$

$$w_e = \frac{1}{2}\varepsilon_r \varepsilon_0 E^2 = \frac{1}{2}\varepsilon E^2$$

7. Molecule description of dielectrics

Polar: $\vec{p} = Q\vec{l}\,, \quad \sum \vec{p} = 0;$

Non-polar: $\vec{p} = 0, \quad \sum \vec{p} = 0;$

8. Dielectrics in external electric fields

$$E_D = \frac{E_0}{\varepsilon_r} = E_0 - E_{ind}$$

$$E_{ind} = E_0\left(1 - \frac{1}{\varepsilon_r}\right)$$

$$\sigma_{ind} = \sigma_0\left(1 - \frac{1}{\varepsilon_r}\right)$$

Chapter 23 & 24: studied by yourself

Current: $I = \frac{\mathrm{d}q}{\mathrm{d}t}$ Ohm's law: $V = IR$

Resistance: $R = \rho \frac{L}{S} = \frac{1}{\sigma} \frac{L}{S}$ (conductivity ,resistivity)

Current density: $j = \frac{I}{S_\perp} \Leftrightarrow I = \int \vec{j} \cdot \mathrm{d}\vec{S}$

Microscopic current: $\vec{j} = -ne\vec{v}_d$

Microscopic statement of Ohm's law: $\vec{j} = \sigma \vec{E} = \frac{\vec{E}}{\rho}$

Electromotive force (EMF) of battery.

Summary-Chapter 25

1. Magnets and Magnetic Fields

(1) Magnet has N-pole and S-pole; Unlike poles attract, like poles repel; Magnetic monopoles have not been found

(2) Magnetic field lines: Form closed loops; Inside: S $\Rightarrow$ N; outside: N $\Rightarrow$ S.

2. Nature of magnetism:

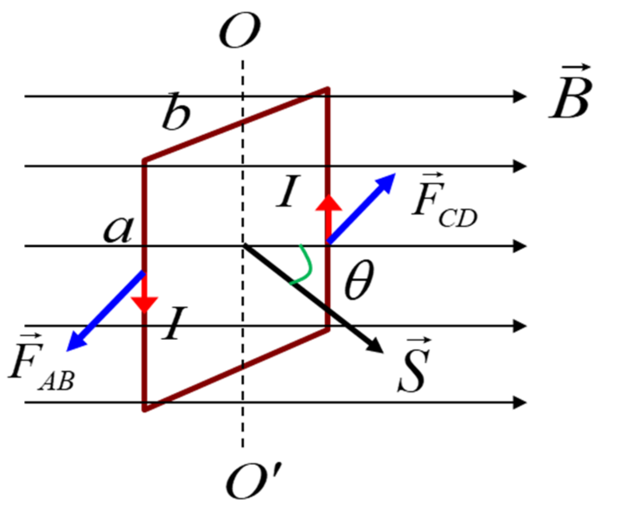
Magnetism is due to the motion of electric charges − electric currents, and can only interacts on currents or moving charges.

3. Ampère force

$$\mathrm{d}\vec{F} = I\mathrm{d}\vec{l} \times \vec{B} \qquad \vec{F} = \int I\mathrm{d}\vec{l} \times \vec{B}$$

4. Magnetic dipole moment:

$$\vec{\mu} = I\vec{S}$$



5. Torque on a Current Loop

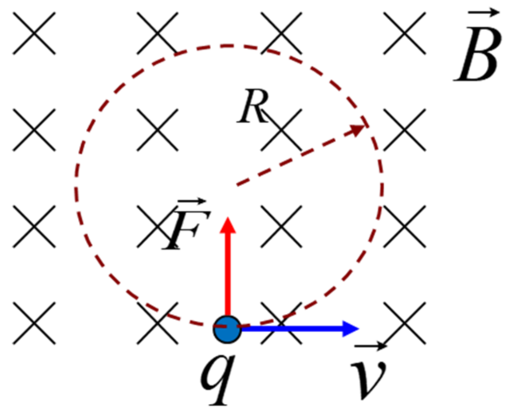
$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

6. Potential energy of a current loop

$$U = -\vec{\mu} \cdot \vec{B}$$

7. Lorentz force

$$\vec{F} = q\vec{v} \times \vec{B}$$

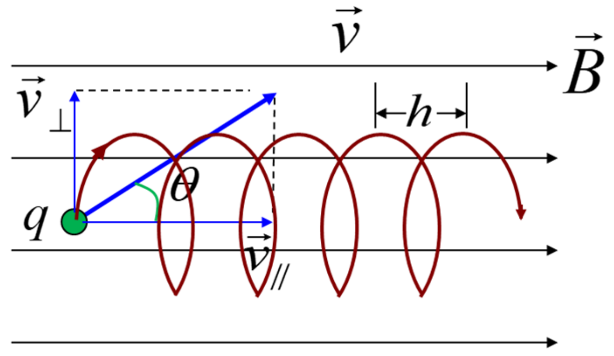


8. Motion of charged particles in a uniform magnetic field

$$R = \frac{mv}{qB} \qquad T = \frac{2\pi m}{qB}$$

$$\vec{v} = \vec{v}_{\perp} + \vec{v}_{/\!/}$$

$$R = \frac{mv\sin\theta}{qB} \qquad T = \frac{2\pi m}{qB} \qquad h = \frac{2\pi mv\cos\theta}{qB}$$



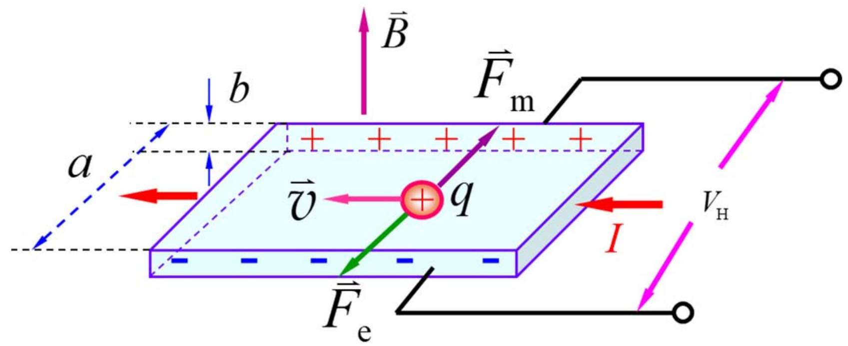
9. Hall effect:

Mechanism: Lorentz force ⟶ Hall field ⟶ equilibrium

$$qE_H = \left|\vec{F}_e\right| = \left|\vec{F}_m\right| = qvB$$

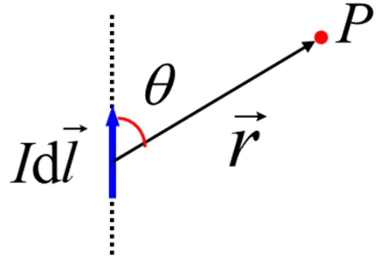
Hall voltage:

$$V_H = \frac{1}{nq}\frac{IB}{b}$$



Summary-Chapter 26

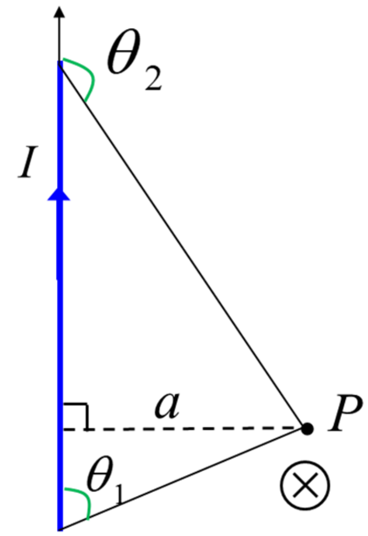
1. Biot-Savart Law



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\vec{r}}{r^3}$$

$$\mu_0 = 4\pi\times10^{-7}\ \mathrm{T\cdot m/A}$$

2. A straight current-carrying wire



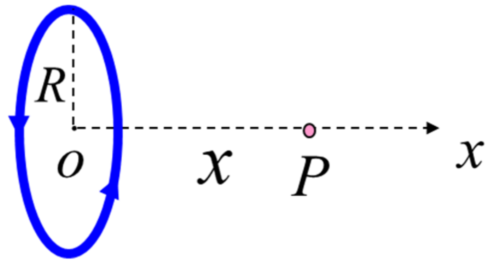
$$B = \frac{\mu_0 I}{4\pi a}\left|\cos\theta_1 - \cos\theta_2\right|$$

Infinite long current-carrying straight wire:

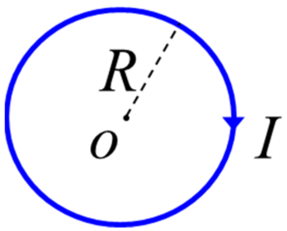
$$B = \frac{\mu_0 I}{2\pi a}$$

3. Circular current

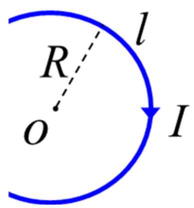
$$B=\frac{\mu_0 I R^2}{2\left(R^2+x^2\right)^{3/2}}$$



B at the center of a circular/arc current:



$$B_o=\frac{\mu_0 I}{2R}$$



$$B=\frac{\mu_0 I}{2R}\times\frac{l}{2\pi R}$$

4. Gauss's law for magnetic field

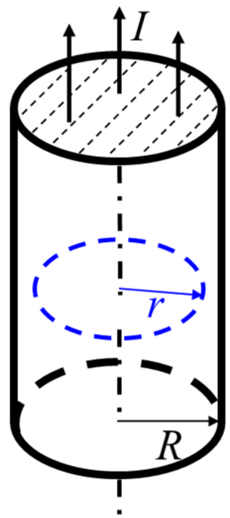
$$\oint \vec{B}\cdot d\vec{S}=0 \qquad \nabla\cdot\vec{B}=0$$

Magnetic field lines form closed loops, without start or end points.

5. Ampere's law

$$\oint_l \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I_{encl}$$

6. Cylindrical current



$$B = \frac{\mu_0 I_{encl}}{2\pi r} = \begin{cases} \dfrac{\mu_0 j}{2} r, & r < R \\ \dfrac{\mu_0 I}{2\pi r}, & r \geq R \end{cases}$$

7. Infinite solenoid

$$\begin{cases} B_{in} = \mu_0 n I \\ B_{out} = 0 \end{cases}$$

Summary-Chapter 27

1. Faraday's law of induction

$$\varepsilon = -\frac{\mathrm{d}\Phi_m}{\mathrm{d}t}$$

2. Lenz's law

An induced EMF is always in a direction that opposes the original change in flux that caused it. (effect of the result is always against the reason)

3. Motional EMF

$$\varepsilon = \int_{-}^{+} \vec{E}_k \cdot \mathrm{d}\vec{l} = \int_{-}^{+} (\vec{v} \times \vec{B}) \cdot \mathrm{d}\vec{l}$$

4. Induced (vortex) electric field

Induced electric field	Electrostatic field
nonconservative	conservative
$\oint_L \vec{E}_i \cdot \mathrm{d}\vec{l} = -\frac{\mathrm{d}\Phi_m}{\mathrm{d}t} \neq 0$	$\oint_L \vec{E}_{静} \cdot \mathrm{d}\vec{l} = 0$
Produced by changing magnetic field	Produced by charges

5. Summary of fields

Electrostatic /	induced electric /	magnetic field
$\oint \vec{E}_s \cdot \mathrm{d}\vec{l} = 0$	$\oint \vec{E}_i \cdot \mathrm{d}\vec{l} = -\frac{\mathrm{d}\Phi_m}{\mathrm{d}t}$	$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I_{encl}$
$\oint \vec{E}_s \cdot \mathrm{d}\vec{S} = \frac{Q_{encl}}{\varepsilon_0}$	$\oint \vec{E}_i \cdot \mathrm{d}\vec{S} = 0$	$\oint \vec{B} \cdot \mathrm{d}\vec{S} = 0$

Summary-Chapter 28

1. Self-Inductance

$$L = \frac{N\Phi_m}{I}, \qquad \varepsilon_L = -N\frac{\mathrm{d}\Phi_m}{\mathrm{d}t} = -L\frac{\mathrm{d}I}{\mathrm{d}t}$$

SI unit: Henry (H), 1H =1V·s/A =1Ω· s.

2. Inductance of an infinite long solenoid

$$\boldsymbol{L = \mu_0 n^2 V}$$

3. Mutual induction

$$M = \frac{N_2\Phi_{21}}{I_1} \qquad \boldsymbol{\varepsilon_{21} = -N_2\frac{\mathrm{d}\Phi_{21}}{\mathrm{d}t} = -M_{21}\frac{\mathrm{d}I_1}{\mathrm{d}t}}$$

4. mutual inductance of two coupling coils

$$M = k\cdot\sqrt{L_1 L_2}, \quad (0 \le k \le 1)$$

5. Current carrying inductor stores magnetic energy

$$U = \int_0^I LI\mathrm{d}I = \frac{1}{2}LI^2$$

6. Energy stored in a magnetic field

$$\boldsymbol{u_m} = \frac{\mathbf{1}}{\mathbf{2}}\frac{\boldsymbol{B}^2}{\boldsymbol{\mu}_0} \qquad U_m = \int_V u_m \mathrm{d}V = \frac{1}{2}LI^2$$

7. LR circuits

$$\varepsilon_L = -L\frac{\mathrm{d}I}{\mathrm{d}t} = RI \qquad I = I_0 e^{-t/\tau} \qquad \text{Time constant:} \qquad \tau = \frac{L}{R}$$

8. LC circuits

$$\frac{\mathrm{d}^2 Q}{\mathrm{d}^2 t} + \frac{Q}{LC} = 0 \qquad Q = Q_0\cos(\omega t + \varphi) \qquad I = -\frac{\mathrm{d}Q}{\mathrm{d}t} = \omega Q_0 \sin(\omega t + \varphi) \qquad \omega = \sqrt{\frac{1}{LC}}$$

LC oscillator or electromagnetic oscillation

Summary-Chapter 29

1. Displacement current

$$\vec{j}_D = \varepsilon_0 \frac{\mathrm{d}\vec{E}}{\mathrm{d}t}, \qquad I_D = \varepsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}$$

2. Generalized Ampere's law

$$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 (I_c + I_D)_{encl} = \mu_0 I_c + \mu_0 \varepsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}$$

3. Continuity of total current

$$\oint_S (\vec{j} + \vec{j}_D) \cdot \mathrm{d}\vec{S} = 0$$

4. Total electromagnetic field——two way of generating field

$$\vec{E} = \vec{E}_s + \vec{E}_i \qquad \vec{B} = \vec{B}_s + \vec{B}_i$$

5. Maxwell's equations

$$\oint \vec{E}\cdot \mathrm{d}\vec{S} = \frac{Q}{\varepsilon_0}$$

$$\oint \vec{B}\cdot \mathrm{d}\vec{S} = 0$$

$$\oint \vec{E}\cdot \mathrm{d}\vec{l} = -\frac{\mathrm{d}\Phi_m}{\mathrm{d}t}$$

$$\oint \vec{B}\cdot \mathrm{d}\vec{l} = \mu_0 I_C + \mu_0\varepsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}$$

$$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla\cdot\vec{B} = 0$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times\vec{B} = \mu_0\vec{j} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

6. Electromagnetic Waves

$$c^2\frac{\partial^2\vec{E}}{\partial x^2} = \frac{\partial^2\vec{E}}{\partial t^2}, \quad c^2\frac{\partial^2\vec{B}}{\partial x^2} = \frac{\partial^2\vec{B}}{\partial t^2}$$

$$\begin{cases} E = E_0\cos\omega(t-\dfrac{x}{c}) \\ B = B_0\cos\omega(t-\dfrac{x}{c}) \end{cases}$$

$$v = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3.0\times 10^8 m/s$$

7. Properties of EM wave

(1) EM wave is transverse wave

(2) Electric field and magnetic field are in phase, and

$$\frac{E}{B} = c$$

(3) The oscillation direction of electric and magnetic fields are perpendicular to each other:

$$\vec{E} \perp \vec{B}$$

8. Energy density of EM wave

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0} = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$$

9. Poynting vector

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}), \qquad \overline{S} = \frac{1}{2}\frac{E_0 B_0}{\mu_0}$$

10. *Radiation pressure

Absorbed: $$P = \frac{\overline{S}}{c}$$

Reflected: $$P = \frac{2\overline{S}}{c}$$

Summary-Chapter 32

1. Relativity principle

The basic laws of mechanics are the same in all inertia reference frames. or: **All inertial reference frames are equivalent for the description of mechanical phenomena.**

2. Galilean transformation

$$\begin{cases} x' = x - vt \\ y' = y \\ z' = z \\ t' = t \end{cases} \qquad \begin{cases} \upsilon'_x = \upsilon_x - v \\ \upsilon'_y = \upsilon_y \\ \upsilon'_z = \upsilon_z \end{cases} \qquad \begin{cases} a'_x = a_x \\ a'_y = a_y \\ a'_z = a_z \end{cases}$$

3. Michelson-Morley Experiment

Found no significant fringe shift, which approves that the absolute reference frame 'ether' does not exist.

4. Postulates of the Special Theory of Relativity

(1) The laws of physics have the same form in all inertial reference frames.

(2) Light propagates through empty space with a definite speed *c* independent of the speed of the source or observer.

5. Simultaneity

- Simultaneity of events depends on the observer;
- Simultaneity is not an absolute concept, but is relative;
- Time is no longer an absolute quantity, but rather related to motion.

6. Time dilation

$$\boxed{\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2 / c^2}} = \frac{\Delta t_0}{\sqrt{1 - \beta^2}} = \gamma \Delta t_0}$$

Δt_0 : Proper time (the two events occur at the same point in place)

7. Length contraction

$$\boxed{L = L_0\sqrt{1-v^2/c^2} = L_0/\gamma}$$

L_0 : Proper length (measured at rest with respect to the object)

8. Lorentz transformation

$S' \Leftarrow S$

$$\begin{cases} x' = \dfrac{x-vt}{\sqrt{1-\dfrac{v^2}{c^2}}} \\ y' = y \\ z' = z \\ t' = \dfrac{t-\dfrac{v}{c^2}x}{\sqrt{1-\dfrac{v^2}{c^2}}} \end{cases}$$

$S \Leftarrow S'$ $(v \to -v)$

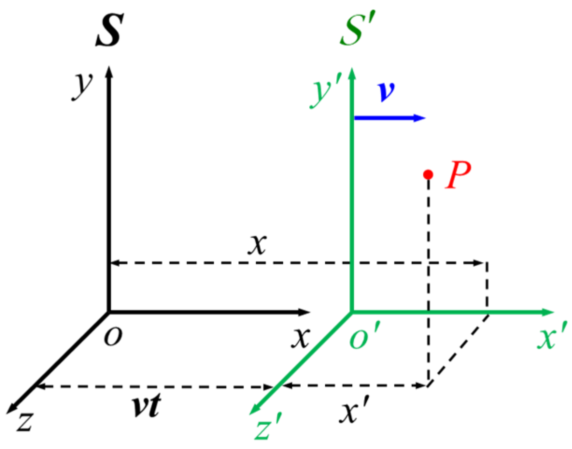
$$\begin{cases} x = \dfrac{x'+vt'}{\sqrt{1-\dfrac{v^2}{c^2}}} \\ y = y' \\ z = z' \\ t = \dfrac{t'+\dfrac{v}{c^2}x'}{\sqrt{1-\dfrac{v^2}{c^2}}} \end{cases}$$

$S' \Leftarrow S$

$$\begin{cases} u'_x = \dfrac{u_x - v}{1-\dfrac{u_x v}{c^2}} \\ u'_y = \dfrac{u_y/\gamma}{1-\dfrac{u_x v}{c^2}} \\ u'_z = \dfrac{u_z/\gamma}{1-\dfrac{u_x v}{c^2}} \end{cases}$$

$S \Leftarrow S'$ $(v \to -v)$

$$\begin{cases} u_x = \dfrac{u'_x + v}{1+\dfrac{u_x v}{c^2}} \\ u_y = \dfrac{u'_y/\gamma}{1+\dfrac{u_x v}{c^2}} \\ u_z = \dfrac{u'_z/\gamma}{1+\dfrac{u_x v}{c^2}} \end{cases}$$



(1) $v << c \to$ Galilean transformation;

(2) Light speed is independent of observer:

$$u'_x = c \quad \Rightarrow \quad u_x = c$$

9. Relativistic mass and momentum

$$\boxed{m = \frac{m_0}{\sqrt{1-v^2/c^2}} = \gamma m_0} \qquad \boxed{\vec{p} = m\vec{v} = \frac{m_0\vec{v}}{\sqrt{1-v^2/c^2}}} \qquad \boxed{\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(m\vec{v}) = m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} + \vec{v}\frac{\mathrm{d}m}{\mathrm{d}t}}$$

(1) $v \ll c$, $m = m_0$; (2) $v = c$ (photon:光子), $m_0 = 0$;

(3) For a mass object, $v \to c$, $m \to \infty$, $a = F/m \to 0$, no further acceleration, c is the ultimate speed of mass objects.

10. Relativistic energy

Kinetic energy: $E_k = mc^2 - m_0c^2$

Rest Energy: $E_0 = m_0c^2$

Total Energy: $\boxed{E = E_0 + E_k = mc^2}$

11. Relationship of quantities

$$E_k = mc^2 - m_0c^2 = m_0c^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right) = \frac{1}{2}m_0v^2 + \frac{3}{8}m_0\frac{v^4}{c^2} + \dots$$

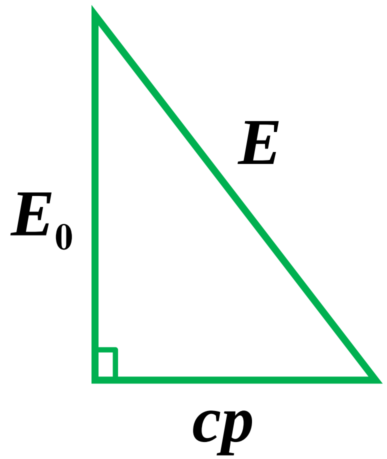
$$\left(\frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{1\cdot 3}{2\cdot 4}x^2 - \dots\right)$$

Only when $v << c$:

$$E_k = \frac{1}{2}m_0v^2$$

Energy triangle:

$$\boxed{E^2 - p^2c^2 = E_0^2}$$



Photon: $m_0 = 0 \rightarrow E = pc$

Summary-Chapter 33

1. Wein's Law:

$$\lambda_m = \frac{2.898\times 10^{-3}\ \ [\mathrm{m\cdot K}]}{T}$$

2. Planck's quantum hypothesis

$$E = n\cdot hf$$

(n : quantum number; hf : quantum of energy)

3. Photon theory of light

$$\boxed{E = hf = \frac{hc}{\lambda}} \qquad \boxed{P = \frac{h}{\lambda} = \frac{hf}{c}} \qquad \boxed{\begin{array}{l} m_0 = 0 \\ m = \dfrac{E}{c^2} = \dfrac{hf}{c^2} \end{array}}$$

4. The photoelectric effect (**photon collides inelastically with bound electron**)

$$E_{k\,\max} = \frac{1}{2}mv^2 = eV_0$$

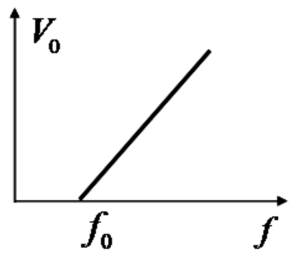
Stopping voltage: V_0

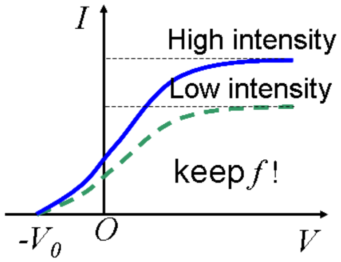
$$\boxed{hf = E_{k\,\max} + W_0}$$

Work function W_0

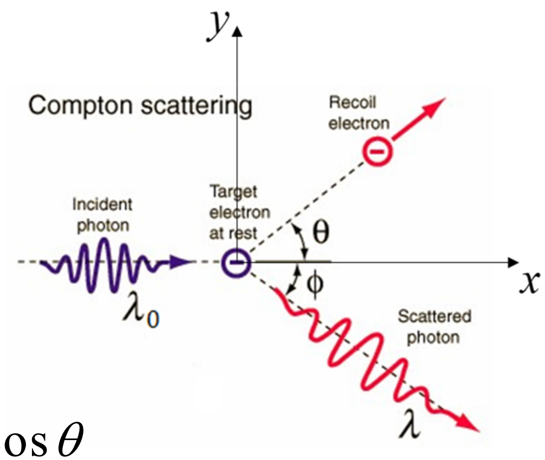
$$f_0 = \frac{W_0}{h}$$

Cutoff frequency





5. Compton effect (**photon collides elastically with free electron**)



$$\boxed{\Delta\lambda = \lambda - \lambda_0 = 2\lambda_C \sin^2\frac{\phi}{2}}$$

$$\boxed{\lambda_C = \frac{h}{m_0 c} = 2.43\times10^{-12}\,\mathrm{m}}$$

$$\boxed{\lambda_{\max} = \lambda_0 + 2\lambda_C = \lambda_0 + 4.86\times10^{-12}\,m}$$

$$\frac{hc}{\lambda_0} + m_0c^2 = \frac{hc}{\lambda} + mc^2$$

$$\begin{cases} x: & \dfrac{h}{\lambda_0} = \dfrac{h}{\lambda}\cos\phi + mv\cos\theta \\ y: & 0 = mv\sin\theta - \dfrac{h}{\lambda}\sin\phi \end{cases}$$

6. de Broglie wave

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

7. Atomic spectrum

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{k^2} - \frac{1}{n^2}\right), \quad (n = k+1,\ k+2,\ \cdots\ )}$$

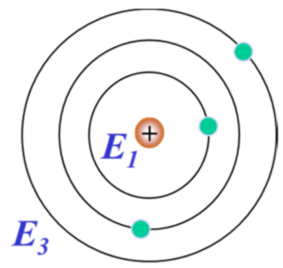
$$(Rydberg\ constant : R = 1.097 \times 10^7\,\mathrm{m}^{-1})$$

$\boldsymbol{k = 1} \longrightarrow$ Lyman series (ultraviolet)

$\boldsymbol{k = 2} \longrightarrow$ Balmer series (visible)

$\boldsymbol{k = 3} \longrightarrow$ Paschen series (infrared)   ……

8. Bohr's three postulates

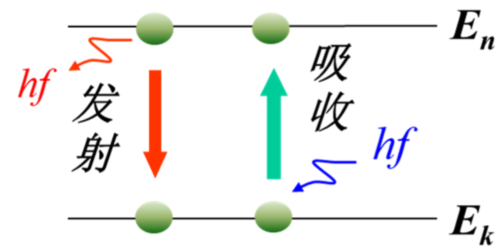


(1) Stationary states;

(2) Quantum transition: ("jump") $hf = E_n - E_k$

(3) Quantum condition: (for angular momentum)

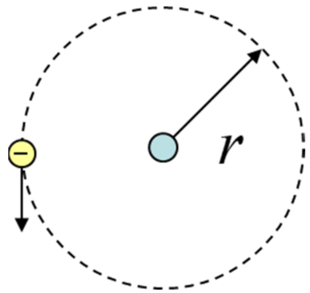
$$L = mvr = n\hbar,\ n = 1,\ 2,\ \cdots$$



9. Bohr model

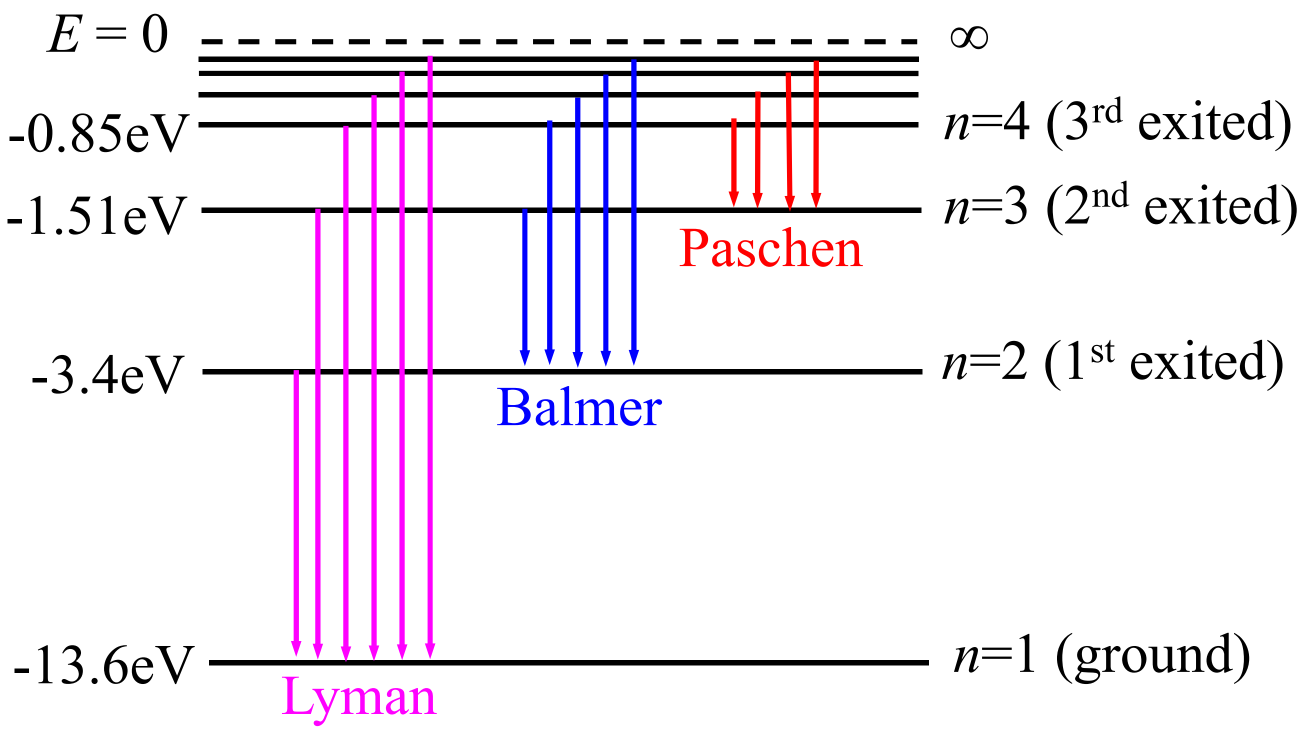
$$\left.\begin{aligned} \frac{e^2}{4\pi\varepsilon_0 r^2} &= m\frac{v^2}{r} \\ L = mvr &= \frac{nh}{2\pi} \end{aligned}\right\}$$

$$\boxed{E_n = \frac{-13.6\text{eV}}{n^2},\ n = 1,\ 2,\ \cdots}$$



10. Energy level diagram

$$E_n = \frac{-13.6\text{eV}}{n^2}, \qquad f = \frac{E_n - E_k}{h}, \qquad \frac{1}{\lambda} = R\left(\frac{1}{k^2} - \frac{1}{n^2}\right)$$



Summary-Chapter 34

1. Interpretation of Ψ(x, y, z, t)

$|\Psi(x, y, z, t)|^2$ represents the probability of finding the particle in a unit volume at the given position and time.

probability finding the particle in the region $x_a \rightarrow x_b$:

$$P_{x_a \to x_b} = \int_{x_a}^{x_b} \left|\Psi(x,t)\right|^2 \mathrm{d}x$$

2. Normalization condition

$$\boxed{\int \left|\Psi(x,y,z,t)\right|^2 \mathrm{d}V = \int \left|\Psi(x,y,z,t)\right|^2 \mathrm{d}x\mathrm{d}y\mathrm{d}z = 1}$$

3. Heisenberg uncertainty principle

$$(\Delta x)(\Delta p_x) \geq \hbar = \frac{h}{2\pi}, \quad (\Delta E)(\Delta t) \geq \hbar = \frac{h}{2\pi}, \quad (\Delta L_z)(\Delta \phi) \geq \hbar = \frac{h}{2\pi}$$

4. Schrödinger equation

1D time-dependent Schrödinger equation:

$$\boxed{i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+U(x)\Psi}$$

3D time-dependent Schrödinger equation:

$$\boxed{i\hbar\frac{\partial\Psi}{\partial t}=(-\frac{\hbar^2}{2m}\nabla^2+U)\Psi=\widehat{H}\Psi}$$

Hamilton operator:

$$\widehat{H}=-\frac{\hbar^2}{2m}\nabla^2+U$$

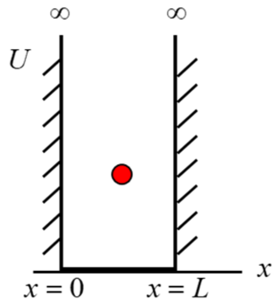
Time-independent Schrödinger equation:

$$\boxed{\left(-\frac{\hbar^2}{2m}\nabla^2+U\right)\psi=E\cdot\psi} \quad \text{or} \quad \boxed{\widehat{H}\psi=E\cdot\psi}$$

(1) E: eigen energy; (2) ψ: stationary state / eigenstate.

5. Particle in an Infinitely Deep Square Well Potential

$$U(x) = \begin{cases} 0, & 0 < x < L \\ \infty, & x \le 0 \text{ and } x \ge L \end{cases}$$



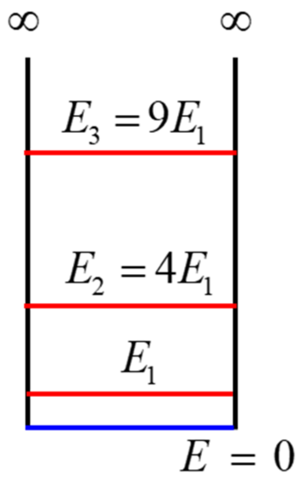
$$\psi(x) = 0, \quad (x \le 0, \ \ x \ge L)$$

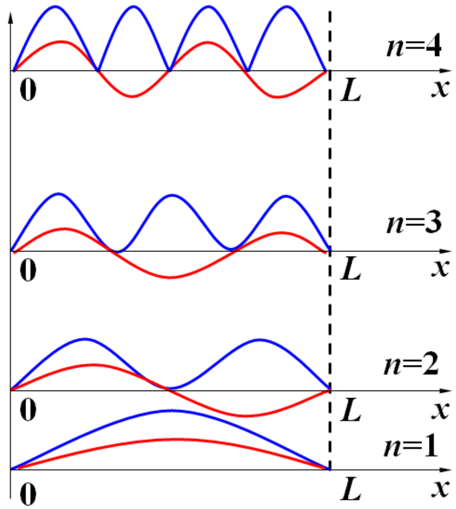
$$0 \le x \le L: \quad -\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} = E\psi, \quad or: \quad \frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + k^2\psi = 0, \quad \left(k^2 = \frac{2mE}{\hbar^2}\right)$$

$$k = \frac{n\pi}{L} \quad \Rightarrow \quad E_n = n^2\frac{\pi^2\hbar^2}{2mL^2}, \quad (n = \pm 1, \ \pm 2, \ \cdots)$$

$$E_1 = \pi^2\hbar^2 / 2mL^2 \ne 0 \quad \rightarrow$$ zero point energy

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right), \quad (n = \pm 1, \ \pm 2, \ \cdots)$$

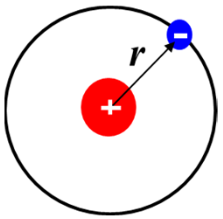




Summary-Chapter 35

1. Time-independent Schrödinger equation for hydrogen atom

$$-\frac{\hbar^2}{2m}\nabla^2\psi - \frac{e^2}{4\pi\varepsilon_0 r}\psi = E\psi$$



2. Principal quantum number n

$$E_n = \frac{-13.6eV}{n^2}, \quad n = 1,\ 2,\ \cdots$$

3. Orbital quantum number l

$$L = \sqrt{l(l+1)}\ \hbar, \quad l = 0,\ 1,\ \cdots,\ n-1$$

4. Magnetic quantum number m_l

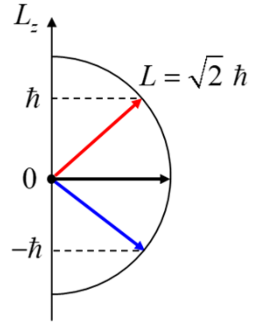
$$L_z = m_l\hbar, \quad m_l = 0,\ \pm 1,\ \cdots,\ \pm l$$

(a) l=1

$m_l = 0,\ \pm 1$

$L = \sqrt{2}\hbar$

$L_z = 0,\ \pm\hbar$

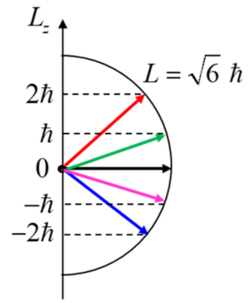


(b) l=2

$m_l = 0,\ \pm 1,\ \pm 2$

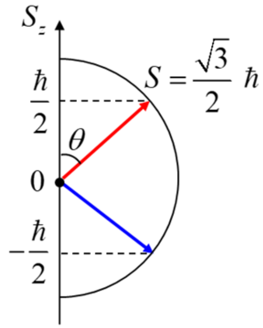
$L = \sqrt{6}\hbar$

$L_z = 0,\ \pm\hbar,\ \pm 2\hbar$



5. Spin magnetic quantum number m_s

$$s=\frac{1}{2},\quad S=\sqrt{s(s+1)}\cdot\hbar=\frac{\sqrt{3}}{2}\hbar,\quad S_z=m_s\hbar,\quad m_s=\pm\frac{1}{2}$$



$\theta=\cos^{-1}(\frac{1}{\sqrt{3}})=54.7°$

6. Radial probability distribution

$$\psi_{100}=\frac{1}{\sqrt{\pi r_0^3}}e^{-\frac{r}{r_0}}\qquad \text{Bohr radius: } r_0=\frac{h^2\varepsilon_0}{\pi m e^2}\approx 5.29\times10^{-11}\,\mathrm{m}$$

$$|\psi|^2\,\mathrm{d}V=|\psi|^2\cdot 4\pi r^2\mathrm{d}r=P_r\mathrm{d}r\quad\Rightarrow\quad P_r=4\pi r^2|\psi|^2=\frac{4r^2}{r_0^3}e^{-\frac{2r}{r_0}}$$

7. Two principles for the arrangements of electrons

(1) Lowest energy principle → ground state

(2) Pauli exclusion principle

8. Shell structure of electrons

$n =$	1,	2,	3,	4,	5,	6
	K,	L,	M,	N,	O,	P

$l =$	0,	1,	2,	3,	4
	s,	p,	d,	f,	g

9. Maximum number of electrons contained in each shells and subshells

$$N_{sub} = (2l+1)$$

$$N_{shell} = 2\sum_{l=0}^{n-1}(2l+1) = 2n^2$$

$l=$	0,	1,	2,	3,	4
	s	p	d	f	g
	2	6	10	14	18

$n=$	1,	2,	3,	4,	5
	K	L	M	N	O
	2	8	18	32	50

10. Electrons fill the shells and subshells from left to right

n \ l	s	p	d	f	g
	0	1	2	3	4
1	$1s^2$				
2	$2s^2$	$2p^6$			
3	$3s^2$	$3p^6$	$3d^{10}$		
4	$4s^2$	$4p^6$	$4d^{10}$	$4f^{14}$	
5	$5s^2$	$5p^6$	$5d^{10}$	$5f^{14}$	$5g^{18}$

Wishing you all

——the best results!