Circuit Analysis and Design

Academic Year 2025/2026 – Semester 1

Lecture 13 – RL Circuits

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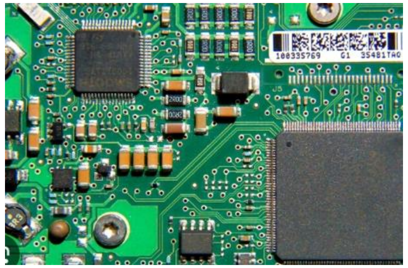
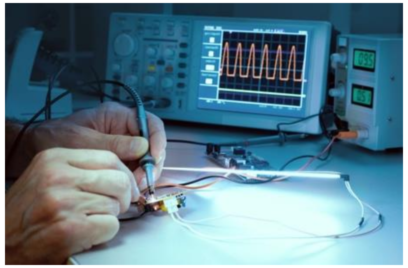




Agenda

- Natural response of RL circuit
- Step response of RL circuit

Introduction

- When a capacitor or an inductor possesses initial energy, the circuit responds to the initial energy until all the energy is spent, even when there is no input signal.

- The response of a circuit due to initial energy only is called a natural response (also transient response, zero input response, and source-free response).

- The response of a circuit to a dc input signal (step input) is called a step response. The step response includes the response due to the initial energy stored in the capacitor or inductor.

Natural Response of RL Circuit

- The switch in Figure 7.29 has been closed for a long time before it is opened at $t = 0$. At $t = 0$, the current through the inductor is equal to the current from the source I_S; that is, $i(0) = I_0 = I_S$. For $t \geq 0$, the circuit shown in Figure 7.29 becomes the circuit shown in Figure 7.30, with initial current of $i(0) = I_0 = I_S$.
- Summing the currents leaving node 1, we obtain

$$i(t)+\frac{L}{R}\frac{di(t)}{dt}=0 \Rightarrow \frac{di(t)}{dt}=-\frac{R}{L}i(t) \Rightarrow \frac{\frac{di(t)}{dt}}{i(t)}=-\frac{R}{L} \Rightarrow \frac{d}{dt}\ln\left[i(t)\right]=-\frac{R}{L} \quad (1)$$

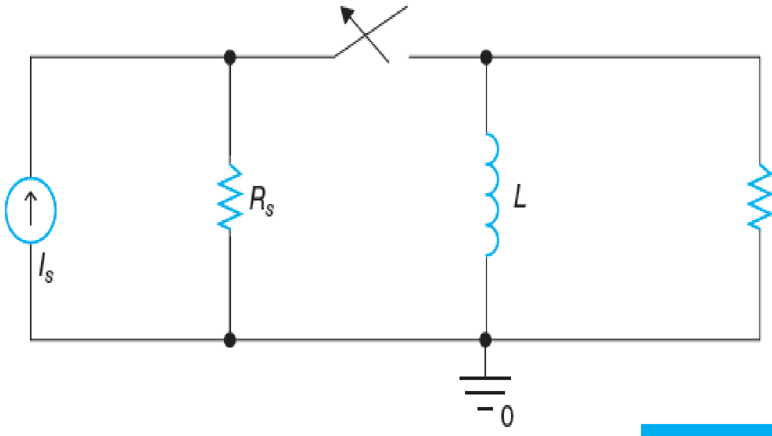


FIGURE 7.29

- Integrating on both sides of the last expression of (1), we get

$$\ln\left[i(t)\right]=-\int_0^t \frac{R}{L}dt+K \quad (2)$$

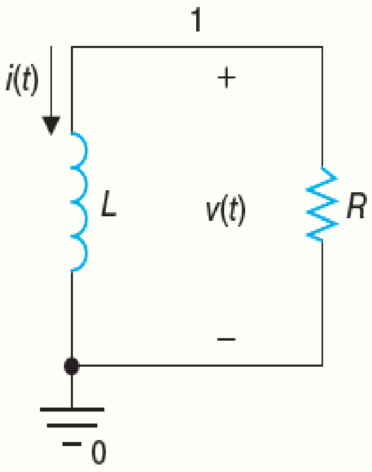


FIGURE 7.30

- Exponentiation on both sides of Equation (2), we obtain

$$e^{\ln\left[i(t)\right]}=i(t)=e^{\left(-\frac{R}{L}t+K\right)}=e^{K}e^{-\frac{R}{L}t}=Ae^{-\frac{R}{L}t} \quad (3)$$

Natural Response of RL Circuit (Continued)

- At t = 0, Equation (3) becomes : $$i(0) = Ae^{-\frac{R}{L}0} = A$$
- Since, $A = i(0) = I_0$, therefore,

  the current through the inductor (also resistor) is given by: $$i(t) = i(0)e^{-\frac{t}{L/R}}u(t) = I_0 e^{-\frac{t}{L/R}}u(t) \quad (4)$$

  where $u(t) = 1$ for $t \geq 0$ and $u(t) = 0$ for $t < 0$. $u(t)$ is called unit step function.
- The voltage across the inductor is given by :

$$v(t) = L\frac{di(t)}{dt} = L\left(-\frac{R}{L}\right)i(0)e^{-\frac{t}{L/R}} = -Ri(0)e^{-\frac{t}{L/R}} = -RI_0 e^{-\frac{t}{L/R}}u(t)$$

- The instantaneous power on the inductor is given by:

$$p(t) = v(t)i(t) = -I_0^2 Re^{-\frac{2t}{L/R}}u(t) \quad \text{(power is released)}$$

- The energy on the resistor is given by:

$$w(t) = \int_0^t p(\lambda)d\lambda = \frac{1}{2}LI_0^2\left(1 - e^{-\frac{2t}{L/R}}\right)u(t)$$

  - At $t = \infty$, $w(\infty) = 0.5LI_0^2$.

Time Constant of an RL Circuit

- The ratio of L over R, L/R, has a unit of seconds and is called a time constant of the RL circuit. The time constant is denoted by τ. Thus, $\tau = L/R$.
- In terms of τ, $i(t)$, $v(t)$, and $p_L(t)$ for the circuit shown in Figure 7.30 become, respectively

$$i(t) = I_0 e^{-\frac{t}{\tau}} u(t), v(t) = -RI_0 e^{-\frac{t}{\tau}} u(t), p_L(t) = -I_0^2 R e^{-\frac{2t}{\tau}} u(t)$$

- Figure 7.31 shows $i(t)$ for $\tau = 1, 2, 3, 4$, and 5 [($i(0) = 1\ A$)].
- As the time constant τ increases, it takes longer time for the current through the inductor to decay.

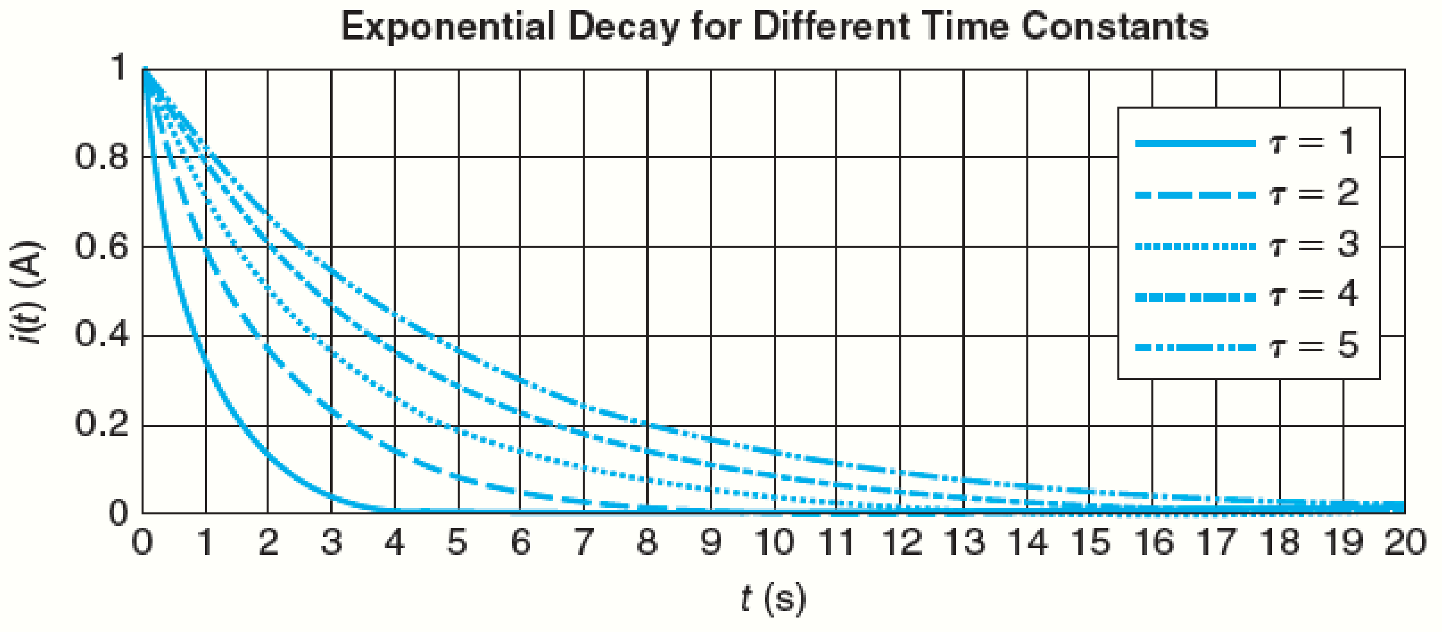


FIGURE 7.31

Time Constant of an RL Circuit (Continued)

- At t = 0, the current is at its peak value $i(0)$. At time $t = \tau$, the current is

$$i(\tau) = i(0)e^{-\frac{\tau}{\tau}} = i(0)e^{-1} = 0.3678794412\, i(0)$$

- At $t = \tau$, the current through the inductor drops to 36.788% of the initial value at $t = 0$. *For* $t = 2\tau, 3\tau, 4\tau, 5\tau, \ldots, 10\tau$, we have the values shown in Table 7.3.

n	$\exp(-n)$
0	1.000000000000000
1.000000000000000	0.367879441171442
2.000000000000000	0.135335283236613
3.000000000000000	0.049787068367864
4.000000000000000	0.018315638888734
5.000000000000000	0.006737946999085
6.000000000000000	0.002478752176666
7.000000000000000	0.000911881965555
8.000000000000000	0.000335462627903
9.000000000000000	0.000123409804087
10.000000000000000	0.000045399929762

TABLE 7.3

Current Through the Inductor Normalized to I_0 at $t = n\tau$.

Time Constant of an RL Circuit (Continued)

- **At five times the time constant, the current through the inductor due to initial energy on the inductor is less than *1%* of the initial voltage (*0.6738%*). For all practical purposes, the transient response can be ignored after about five times the time constant.**
- **The time derivative of the current through the inductor is given by**

$$\frac{di(t)}{dt} = I_0 \frac{d}{dt} e^{-\frac{t}{\tau}} = -\frac{I_0}{\tau} e^{-\frac{t}{\tau}} = -\frac{RI_0}{L} e^{-\frac{t}{\tau}}$$

- **The rate of decay of the current through the inductor is at its maximum at $t = 0$ and slows down as time progresses. The rate of decay at $t = 0$ is $-I_0/\tau$.**

- **If the current decreases at this rate, *i(t)* is given by**

$$i_1(t) - I_0 = -\frac{I_0}{\tau}(t - 0)$$

- **Figure 7.33 shows *i(t)* and $i_1(t)$.**

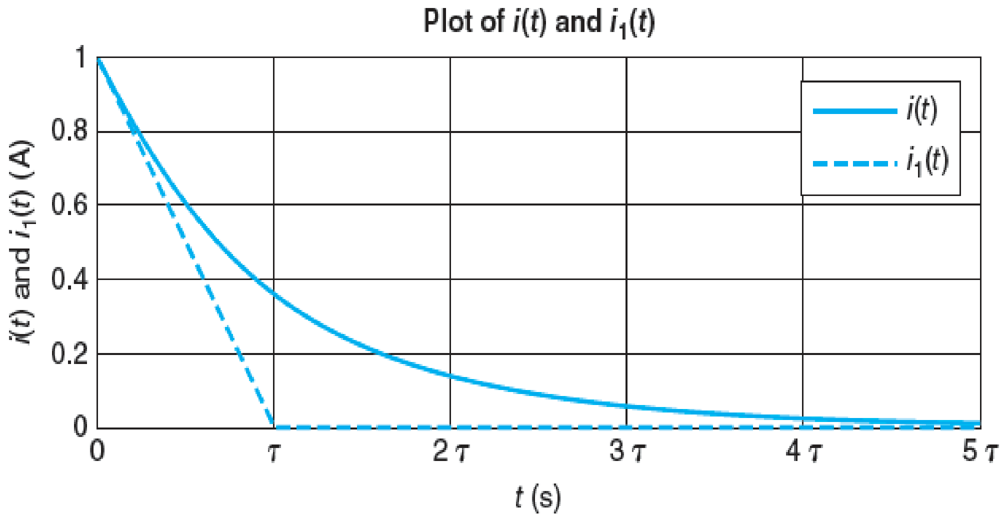


FIGURE 7.33

Finding the Time Constant

- If there is one resistor with resistance R and one inductor with inductance L, as in the circuit shown in Figure 7.30, the time constant is the ratio of L over R; that is, $\tau = L/R$.

- If there is one inductor with inductance L and more than one resistor in the circuit,
  1. Find the equivalent resistance R_{eq} of all the resistors in the circuit seen from the inductor (Thévenin equivalent resistance).
  2. Then, the circuit reduces to one inductor with inductance L and one resistor with resistance R_{eq}. The time constant is given by $\tau = L/R_{eq}$.

- The current through the inductor is given by : $i(t) = I_0 e^{-\frac{t}{\tau}} u(t)$ A

- Finding the current $i(t)$ through the inductor for the given circuit involves
  1. finding the initial current $i(0)$ through the inductor,
  2. finding the equivalent resistance R_{eq}, and
  3. finding the time constant $\tau = L/R_{eq}$.

EXAMPLE 7.9

- The switch in the circuit shown in Figure 7.34 has been closed for a long time before it is opened at $t = 0$. Find the current i(t) through the inductor and voltage v(t) across the inductor for $t \geq 0$. Also, plot $i(t)$ and $v(t)$ for $t \geq 0$.

- For $t \leq 0$, the inductor can be treated as a short circuit. $i(0) = I_0 = V_s/R_1 = 12\ V/4\ k\Omega = 3\ mA$
- For $t \geq 0$, the time constant is $\tau = L/R = 0.5/100 = 0.005\ s = 5\ ms$. $1/\tau = 200\ (1/s)$
- For $t \geq 0$, $i(t) = I_0 exp(-t/\tau) = 3\ exp(-200t)\ u(t)\ mA$.
- For $t \geq 0$, $v(t) = L\ di(t)/dt = 0.5 \times 3 \times 10^{-3} \times (-200)\ exp(-200t)\ u(t) = -0.3\ exp(-200t)\ u(t) V$

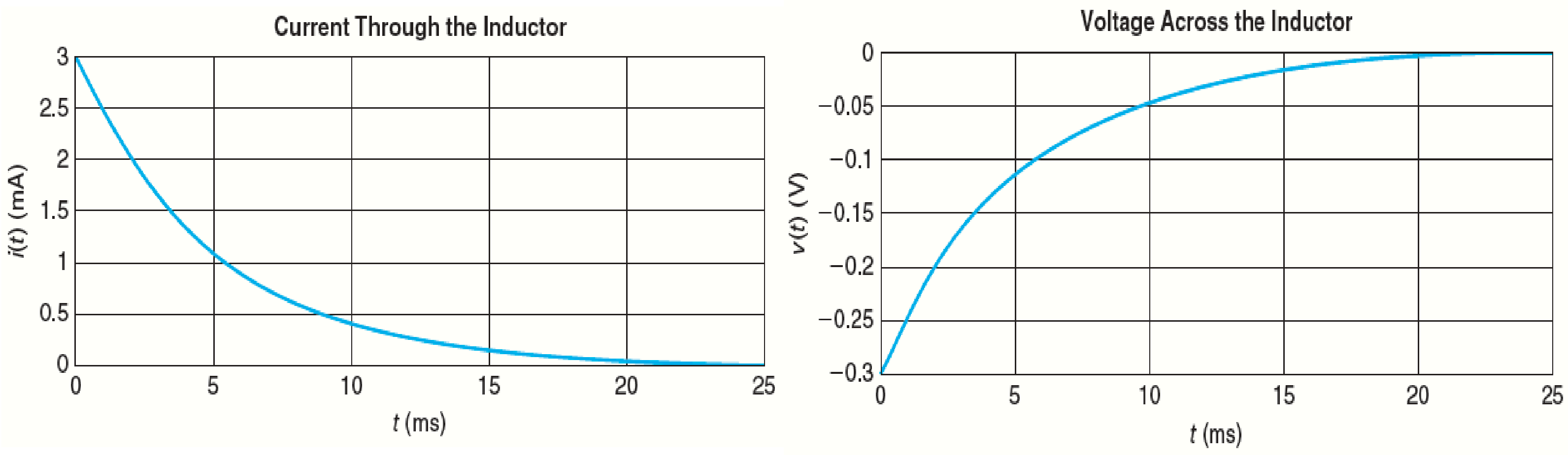


FIGURE 7.35

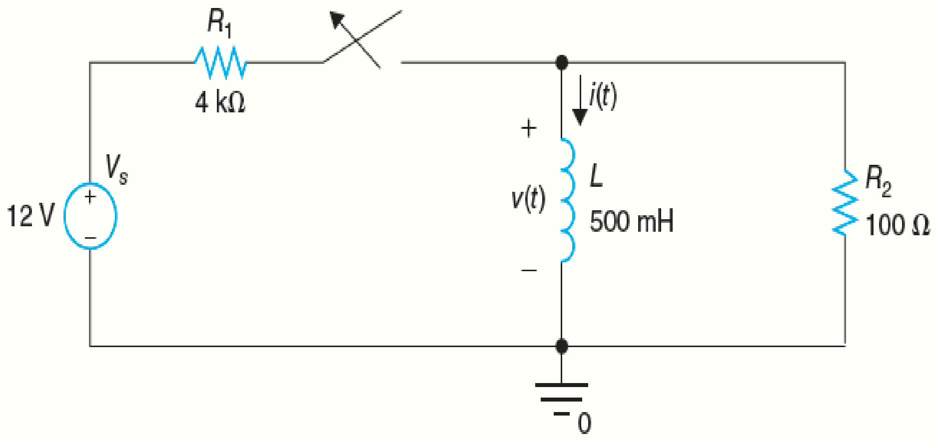


FIGURE 7.34

EXAMPLE 7.10

- **The switch in the circuit shown in Figure 7.36 has been closed for a long time before it is opened at $t = 0$. Find the current $i(t)$ through the inductor for $t \geq 0$.**

- **For $t < 0$, the inductor can be treated as a short circuit. From the current divider rule, we get**
  $i(0) = I_0 = I_s \times R_1/(R_1 + R_2) = 20\ mA \times 2\ k\Omega/(2\ k\Omega + 3\ k\Omega) = 8\ mA$
- **For $t \geq 0$, $R_{eq} = R_3 \,||\, (R_4 + R_5) = 6 \times 30/(6 + 30)\ k\Omega = 5\ k\Omega$**
- **For $t \geq 0$, the time constant is $\tau = L/R_{eq} = 0.01/5000 = 2\ \mu s.\ 1/\tau = 500{,}000\ (1/s)$**
- **For $t \geq 0$, $i(t) = I_0 exp(-t/\tau) = 8\ exp(-500{,}000t)\ u(t)\ mA$.**

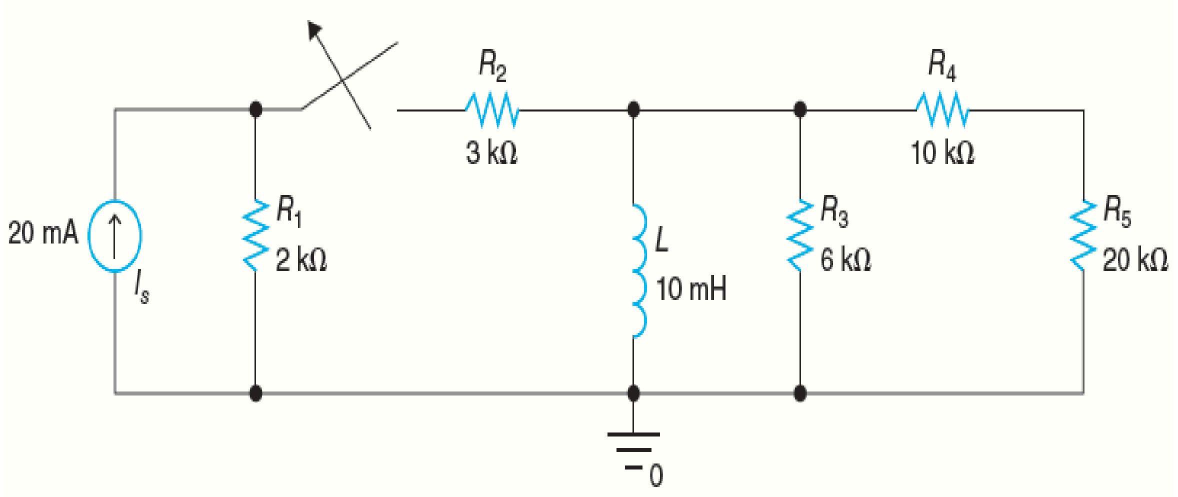
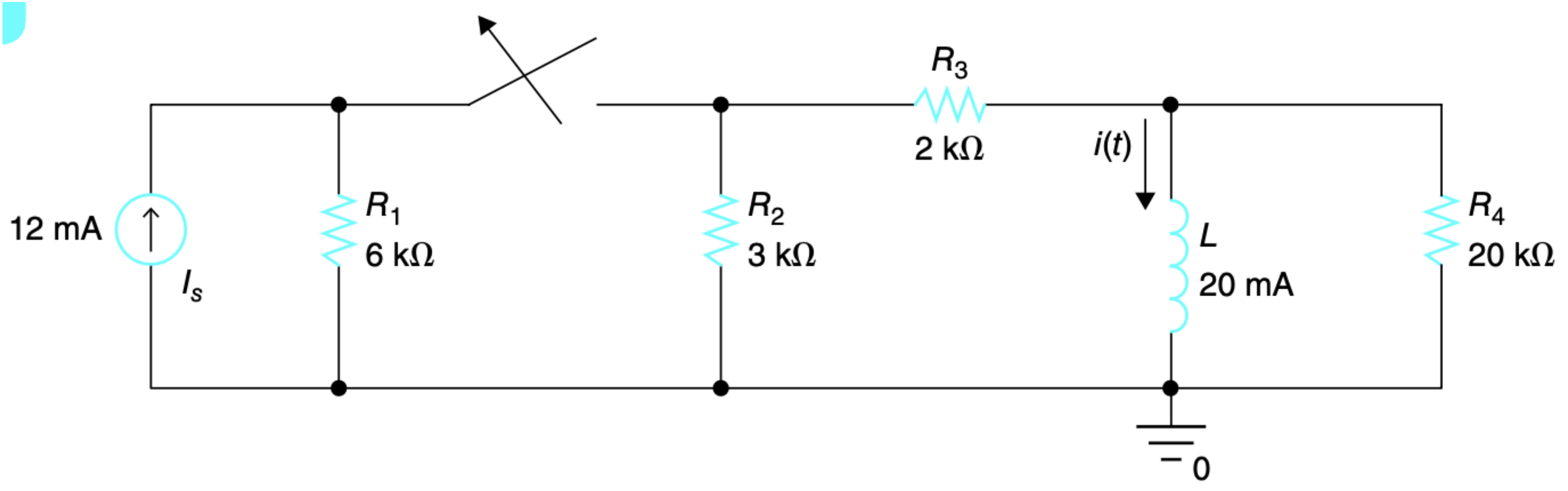


FIGURE 7.36

Class Quiz

- The switch in the circuit has been closed for a long time before it is opened at t>0. Find the $i(0) = I_0$, the current at t=0 and τ through the inductor for t > 0.



A. I_0 = 3 mA and $\tau = 500x10^{-6}$ s
B. I_0 = 6 mA and $\tau = 400x10^{-6}$ s
C. I_0 = 6 mA and $\tau = 500x10^{-6}$ s

EXAMPLE 7.11

- The switch in the circuit shown in Figure 7.39 has been closed for a long time before it is opened at $t = 0$. Find the current $i(t)$ through the inductor for $t \geq 0$.

- For $t < 0$, the inductor can be treated as a short circuit.
- Let $R_a = R_3 \,||\, R_4$. Then, $R_a = 12\times4/(12 + 4)\ k\Omega = 48/16\ k\Omega = 3\ k\Omega$.
- Let $R_b = R_a \,||\, R_5$. Then, $R_b = 3\times3/(3 + 3)\ k\Omega = 9/6\ k\Omega = 1.5\ k\Omega$.
- Let $R_c = R_1 + R_2 + R_b$. Then, $R_c = 4.5\ k\Omega$. $I_{R1} = V_s/R_c = 2\ mA$, $V_a = V_s - 3\ k\Omega \times 2\ mA = 3\ V$
- $i(0) = I_0 = V_a/R_5 = 3\ V/3\ k\Omega = 1\ mA$
- For $t \geq 0$, $R_{eq} = R_5 + (R_3 \,||\, R_4) = 3\ k\Omega + 3\ k\Omega = 6\ k\Omega$
- $\tau = L/R_{eq} = 0.048/6000 = 8\ \mu s$. $1/\tau = 125{,}000\ (1/s)$
- For $t \geq 0$, $i(t) = I_0 \exp(-t/\tau) = \exp(-125{,}000t)\ u(t)\ mA$.

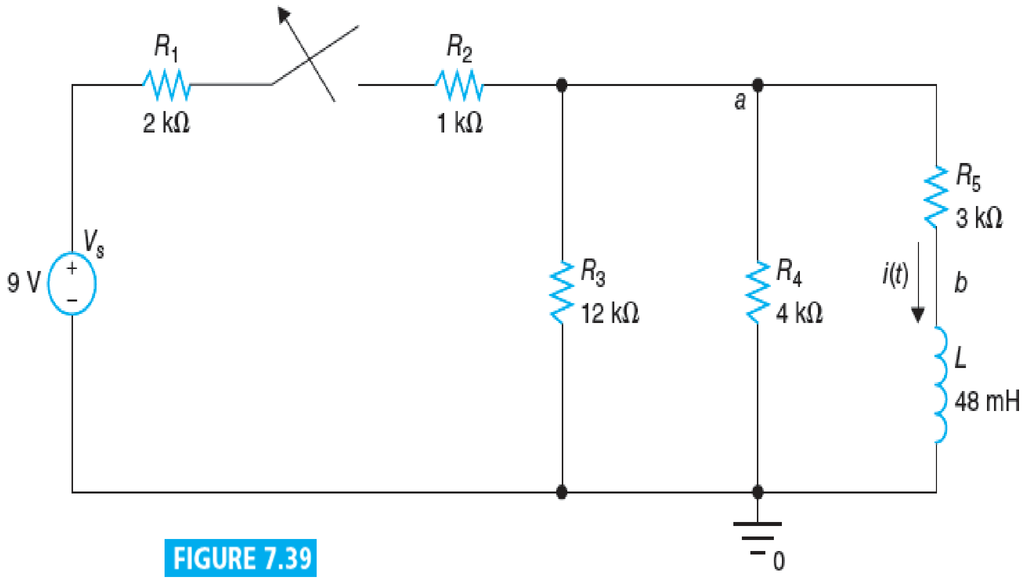


FIGURE 7.39

EXAMPLE 7.12

- The switch in the circuit shown in Figure 7.41 has been closed for a long time before it is opened at $t = 0$. Find the current $i(t)$ through the inductor and the voltage $v(t)$ across the inductor for $t \geq 0$. $R_a = R_2 + (R_3 \| R_4) + R_5 = 8\ k\Omega$
- From the current divider rule, the current through R_a is

$$I_{R_a} = I_s \times \frac{\frac{1}{R_a}}{\frac{1}{R_1}+\frac{1}{R_a}+\frac{1}{R_6}} = 11\,\text{mA} \times \frac{\frac{1}{8}}{\frac{1}{12}+\frac{1}{8}+\frac{1}{4}} = 11\,\text{mA} \times \frac{3}{11} = 3\,\text{mA}$$

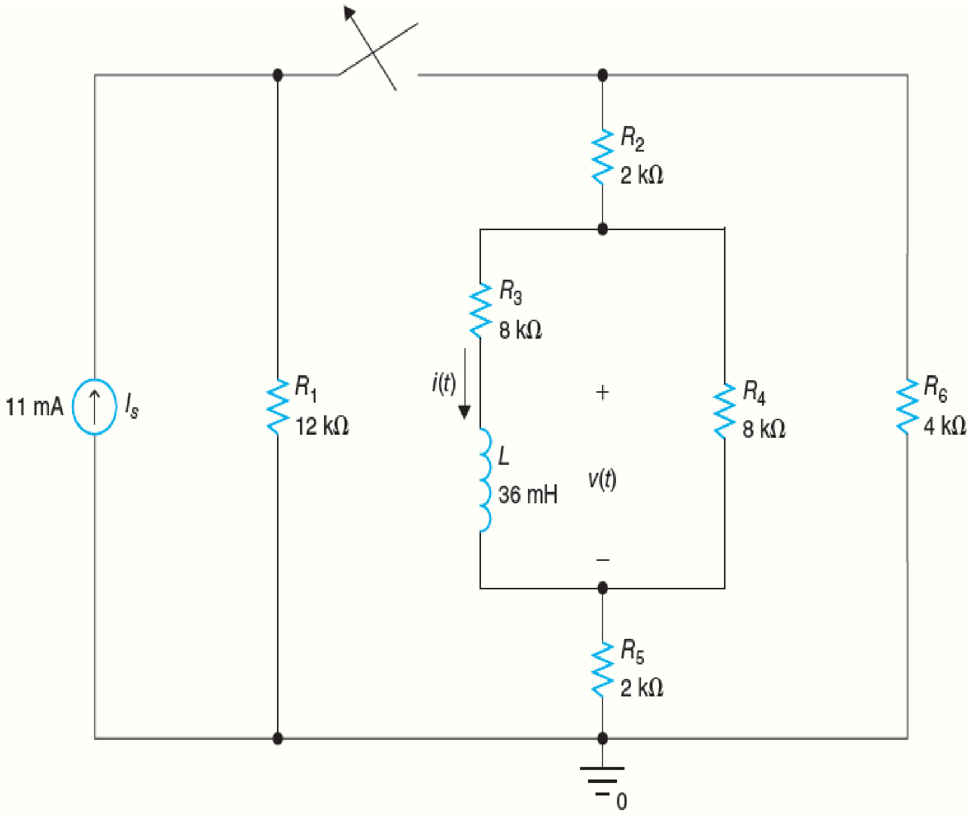


FIGURE 7.41

- $i(0) = I_0 = I_{Ra}/2 = 1.5\ mA$
- $R_{eq} = R_3 + [R_4 \| (R_2 + R_6 + R_5)] = 12\ k\Omega$
- $\tau = L/R_{eq} = 0.036/12000 = 3\ \mu s$. $1/\tau = 333{,}333\ (1/s)$
- $i(t) = I_0 \exp(-t/\tau) = 1.5 \exp(-333{,}333t)\ u(t)$ mA.
- $v(t) = L\, di(t)/dt = -18 \exp(-333{,}333t)\ u(t)$ V.

Step Response of RL Circuit

- The switch in the circuit shown in Figure 7.43 is closed at $t = 0$. At $t = 0$, the current through the inductor is $i(0) = I_0$. For $t \geq 0$, summing the currents leaving node 1, we obtain

$$-I_S + \frac{L}{R}\frac{di(t)}{dt} + i(t) = 0 \Rightarrow \frac{di(t)}{dt} = -\frac{R}{L}\left[i(t) - I_S\right] \Rightarrow \frac{\frac{di(t)}{dt}}{i(t) - I_S} = -\frac{R}{L} \Rightarrow \frac{d}{dt}\ln\left[\left|i(t) - I_S\right|\right] = -\frac{R}{L} \quad (1)$$

- Integrating on both sides of Equation (1), we obtain

$$\ln\left|i(t) - I_S\right| = \int_0^t \frac{-R}{L}dt = \frac{-Rt}{L} + K \quad (2)$$

- Exponentiation on both sides of Equation (2) yields

$$e^{\ln\left|i(t)-I_S\right|} = \left|i(t) - I_S\right| = e^K e^{-\frac{Rt}{L}} \Rightarrow i(t) - I_S = \pm e^K e^{-\frac{Rt}{L}} \quad (3)$$

- Let $A = \pm e^K$

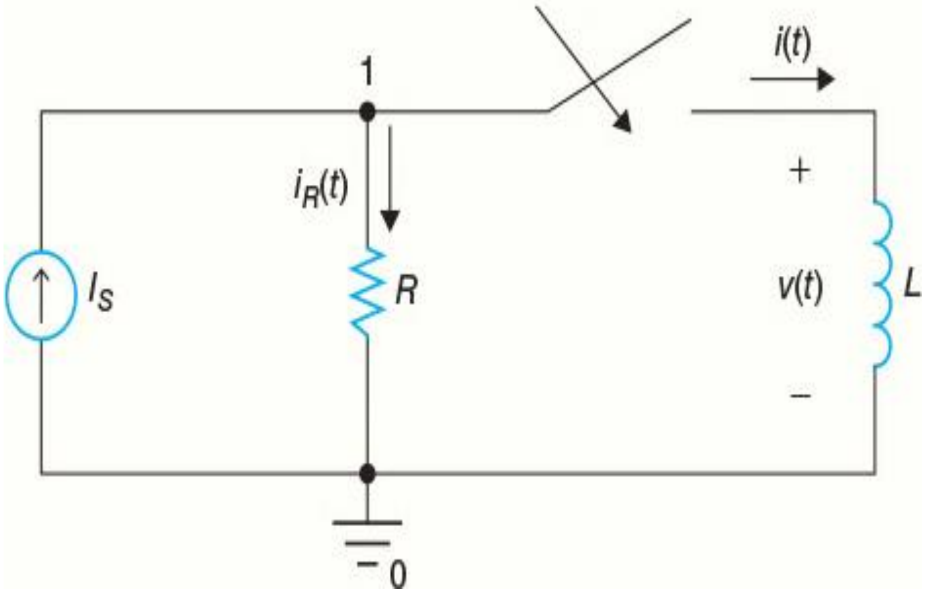


FIGURE 7.43

RL circuit with dc input.

Step Response of RL Circuit (Continued)

- Then, Equation (3) can be rewritten as

$$i(t) = I_S + Ae^{-\frac{t}{L/R}} \quad (4)$$

- The constant A can be found by applying the initial condition:

$$i(0) = I_0 = I_S + A \Rightarrow A = I_0 - I_S$$

- The current through the inductor can be written as ($\tau = L/R$)

$$i(t) = I_S + (I_0 - I_S)e^{-\frac{t}{L/R}} = I_S + (I_0 - I_S)e^{-\frac{t}{\tau}} \quad (5)$$

- This solution is valid for $t \geq 0$.
  - At $t = 0$, the current is $i(0) = I_0$, and at $t = \infty$, the current is $i(\infty) = I_S$. The current through the inductor changes from the initial value of $i(0) = I_0$ at $t = 0$ to the final value of $i(\infty) = I_S$ at $t = \infty$.

Step Response of RL Circuit (Continued)

- The final value of $i(\infty) = I_S$ can be obtained from the circuit shown in Figure 7.43. At $t = \infty$, the inductor can be treated as a short circuit. The current through the inductor is I_S.
- Equation (5) can be rewritten as ($\tau = L/R$)

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right]e^{-\frac{t}{\tau}} = (\text{Final Value}) + \left[(\text{Initial Value}) - (\text{Final Value})\right]e^{-\frac{t}{(\text{Time Constant})}} \quad (6)$$

- Equation (6) is the solution to a differential equation given by the first equation in Equation (1):

$$\frac{di(t)}{dt} + \frac{1}{L/R}i(t) = \frac{1}{L/R}I_S \Rightarrow \frac{di(t)}{dt} + \frac{1}{\tau}i(t) = \frac{1}{\tau}I_S \quad (7)$$

- In the steady state at $t = \infty$, since $di(t)/dt = 0$, Equation (7) becomes $\frac{1}{\tau}i(\infty) = \frac{1}{\tau}I_S$
- Thus, $i(\infty) = I_S$.
- If there is a time delay t_d, replace t by $t - t_d$ in Equation (6).

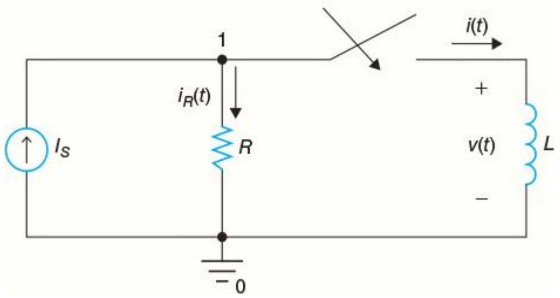


FIGURE 7.43
RL circuit with dc input.

Time Constant

- For RL circuits with one inductor in the circuit, the time constant is given *by* $\tau = L/R_{eq}$ where R_{eq} is the equivalent resistance seen from the inductor.
  - The equivalent resistance R_{eq} is the Thévenin equivalent resistance when the rest of the circuit (excluding the inductor) is converted to the Thévenin equivalent circuit.
  - In general, R_{eq} can be found by deactivating independent sources (short-circuit current sources and open-circuit voltage sources) and finding the equivalent resistance seen from the inductor. Other methods, such as test voltage and test current, can also be used.

- Figure 7.44 shows *i(t)* given by Equation (5) for $I_S = 1\ A$, $I_0 = 0\ A$, and five different values of τ.
- At $t = \tau$, $i(\tau) = 0.63212\ I_S$. At $t = \tau$, the current reaches *63.212%* of the final value.
- At $t = 5\tau$, the current reaches *99.3262%* of the final value.

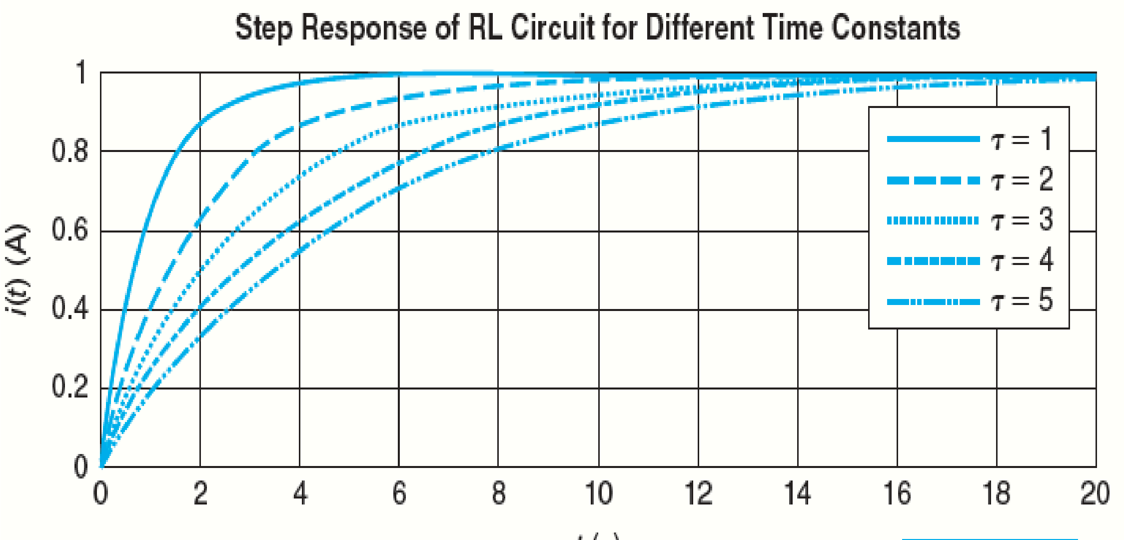
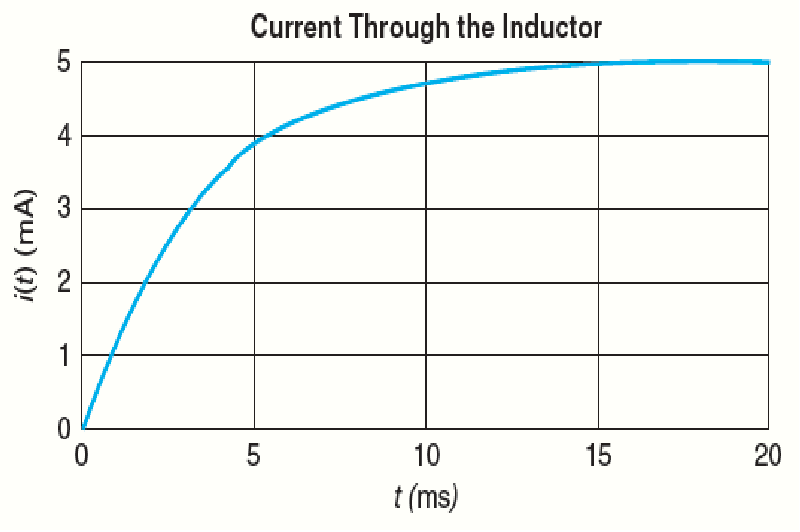
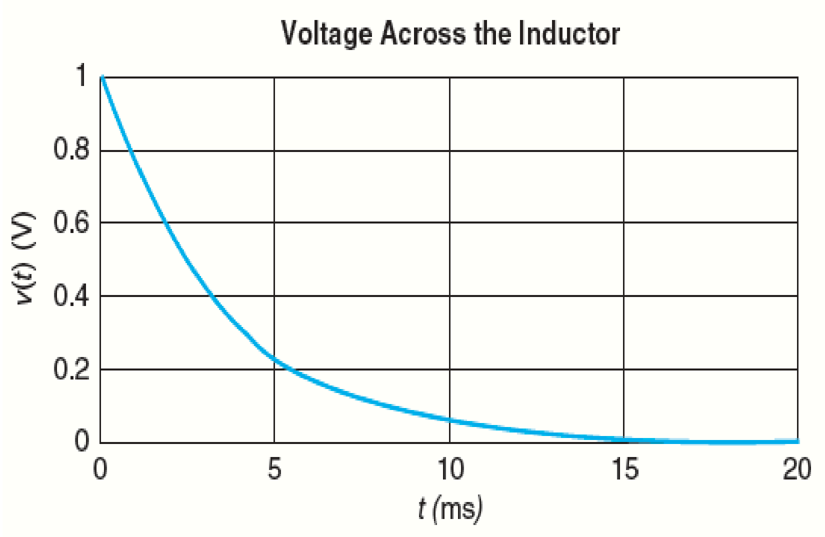


FIGURE 7.44

EXAMPLE 7.13

- **Let** $L = 700\ mH$, $R = 200\ \Omega$, $I_S = 5\ mA$, **and** $I_0 = 0\ A$ **in the circuit shown in Figure 7.43. Find the current** $i(t)$ **through the inductor and voltage** $v(t)$ **across the inductor for** $t \geq 0$, **and plot** $i(t)$ **and** $v(t)$**.**
  - **Final value:** $i(\infty) = I_S = 5\ mA$
  - **Time constant:** $\tau = L/R = 0.7/200 = 3.5\ ms$, $1/\tau = 285.7143\ (1/s)$
  - $i(t) = [I_S + (I_0 - I_S)exp(-285.7143t)]\ u(t) = 5[1 - exp(-285.7143t)]\ u(t)\ mA$
  - $v(t) = L\ di(t)/dt = 0.7 \times (-0.005) \times (-285.7143)\ exp(-285.7143t)\ u(t) = \ exp(-285.7143t)]\ u(t)\ \ V$





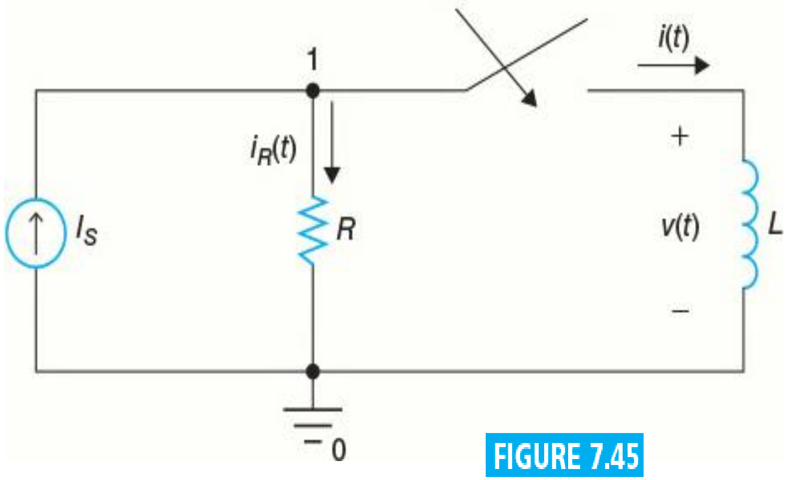


FIGURE 7.45

EXAMPLE 7.14

- The switch in the circuit shown in Figure 7.46 is closed at $t = 0$. The initial current through the inductor is $i(0) = I_0$. Find the current $i(t)$ through the inductor and the voltage $v(t)$ across the inductor.
- Sum the voltage drops around the mesh: $$-V_s + Ri(t) + L\frac{di(t)}{dt} = 0 \Rightarrow \frac{di(t)}{dt} + \frac{R}{L}i(t) = \frac{1}{L}V_s$$
- Final value: $$\frac{R}{L}i(t) = \frac{1}{L}V_s \Rightarrow i(\infty) = \frac{V_s}{R}$$
- Time constant: $\tau = L/R$
- Current: $$i(t) = \left[\frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-\frac{t}{L/R}}\right]u(t)\ \mathrm{A}$$
- Voltage: $$v(t) = \left(V_s - RI_0\right)e^{-\frac{t}{L/R}}u(t)\ \mathrm{V}$$

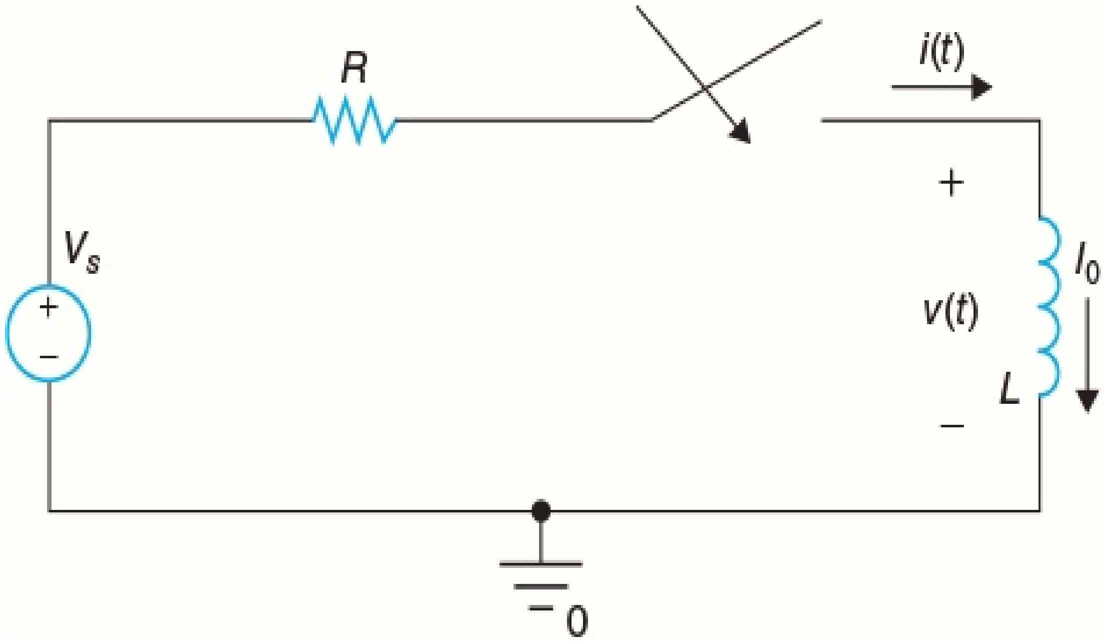


FIGURE 7.46

An *RL* circuit for EXAMPLE 7.14.

EXAMPLE 7.15

- In the circuit shown in Figure 7.48, switch 1 has been closed for a long time before it is opened at t = 0. Switch 2 is closed at $t = 12\ \mu s$. Find the current $i(t)$ through the inductor for $t \geq 0$. Initial value: from the current divider rule, $i(0) = 9\ mA \times 2/(2 + 1) = 6\ mA$
- For $0 \leq t \leq 12\ \mu s$, $R_{eq} = R_3 + R_4 = 4\ k\Omega$, $\tau = L/R = 6\ \mu s$, $1/\tau = 166{,}667\ (1/s)$
- For $0 \leq t \leq 12\ \mu s$, $i(t) = 6\ \exp(-166{,}667t)[u(t) - u(t - 12\times10^{-6})]\ mA$, $i(12\times10^{-6}) = 0.8120\ mA$
- At $t = \infty$, $V_{R4} = 18\ V \times 0.75/(6 + 0.75) = 2\ V$, $i(\infty) = V_{R4}/R_3 = 2\ mA$
- For $12\ \mu s \leq t$, $R_{eq} = R_3 + (R_4 || R_5) = 3\ k\Omega$, $\tau = L/R_{eq} = 8\ \mu s$, $1/\tau = 125{,}000\ (1/s)$
- $i(t) = [2 + (0.8120 - 2)\exp(-125{,}000(t - 12\times10^{-6})]\ u(t - 12\times10^{-6})\ mA$
- $= [2 - 1.188\ \exp(-125{,}000(t - 12\times10^{-6}))]\ u(t - 12\times10^{-6})\ mA$

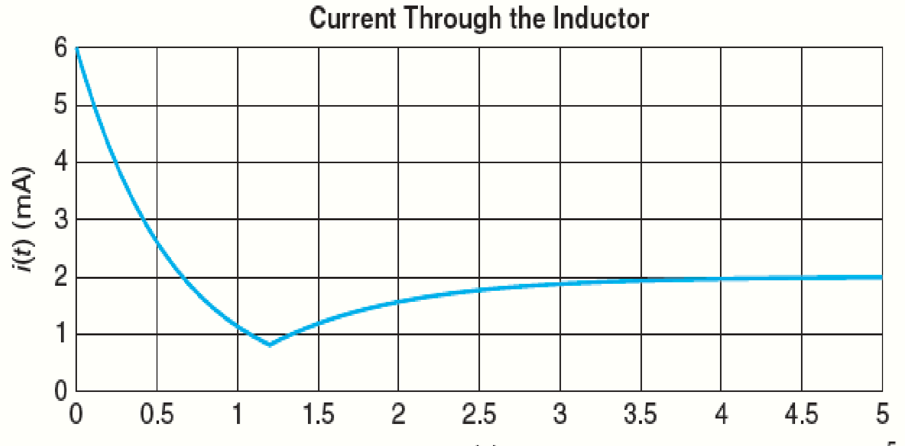
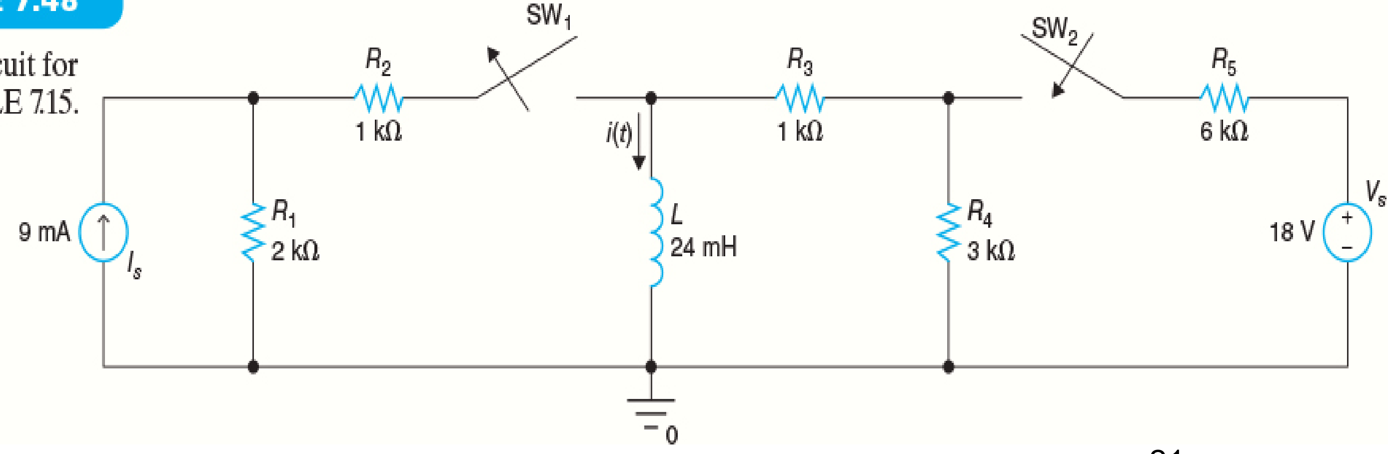


FIGURE 7.50

FIGURE 7.48

Circuit for EXAMPLE 7.15.

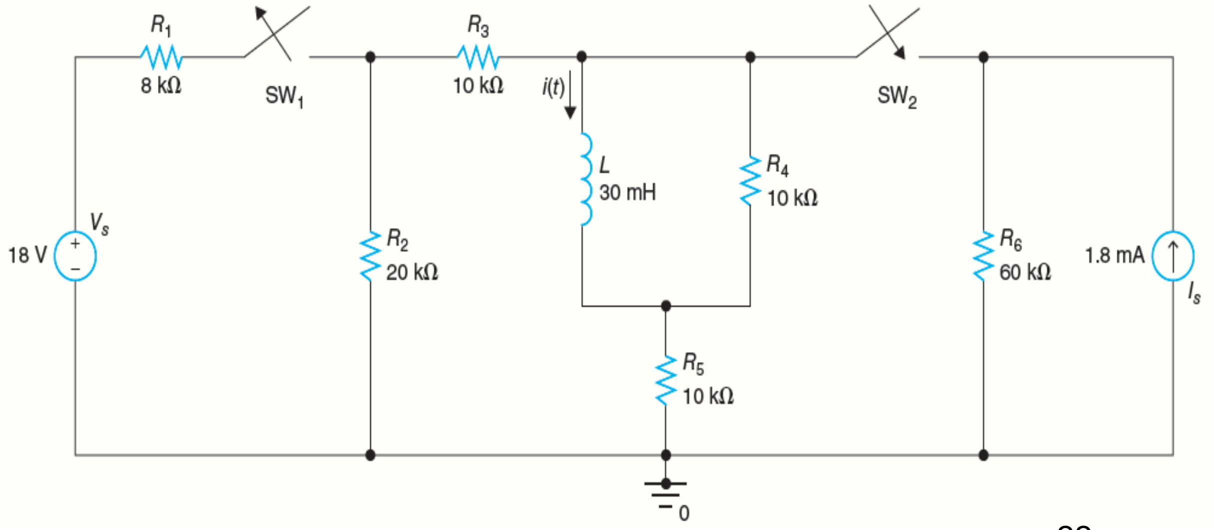


EXAMPLE 7.16

- In the circuit shown in Figure 7.51, switch 1 has been closed for a long time before it is opened at t = 0. Switch 2 is closed at $t = 2\ \mu s$. Find the current $i(t)$ through the inductor for $t \geq 0$. **Initial value:** $R_2 \,||\, (R_3 + R_5) = 10\ k\Omega$, $I_{R1} = 18\ V/18\ k\Omega = 1\ mA$, $i(0) = 1\ mA/2 = 0.5\ mA$
- **For** $0 \leq t \leq 2\ \mu s$, $R_{eq1} = R_4 \,||\, (R_3 + R_2 + R_5) = 8\ k\Omega$, $\tau = L/R_{eq1} = 3.75\ \mu s$, $1/\tau = 266{,}667\ (1/s)$
- **For** $0 \leq t \leq 2\ \mu s$, $i(t) = 0.5 \exp(-266{,}667t)[u(t) - u(t - 2\times10^{-6})]\ mA$, $i(2\times10^{-6}) = 0.2933\ mA$
- **At** $t = \infty$, $R_a = R_6 \,||\, (R_3 + R_2) = 20\ k\Omega$, $i(\infty) = I_s\times R_a/(R_5 + R_a) = 1.2\ mA$
- **For** $2\ \mu s \leq t$, $R_{eq2} = R_4||(R_a + R_5) = 7.5\ k\Omega$, $\tau = L/R_{eq2} = 4\ \mu s$, $1/\tau = 250{,}000\ (1/s)$
- $i(t) = [1.2 + (0.2933 - 1.2)\exp(-250{,}000(t - 2\times10^{-6})]\ u(t - 2\times10^{-6})\ mA$

$= [1.2 - 0.9067 \exp(-250{,}000(t - 2\times10^{-6}))]\ \times u(t - 2\times10^{-6})\ mA$

FIGURE 7.51



EXAMPLE 7.17

- **In the circuit shown in Figure 7.55, the switch has been closed for a long time before it is opened at $t = 0$. Find the current i(t) through the inductor for $t \geq 0$.**

- $i(0)$ **from** V_s: $R_a = (R_1 + R_2)||R_3 = 254.5455\ \Omega$, $V_{a1} = V_s \times R_a/(R_a + R_4) = 8.4$ V, $i_1(0) = V_{a1}/R_3 = 0.021$ A
- $i(0)$ **from** I_s: $R_b = R_3||R_4 = 133.3333\ \Omega$, $R_c = R_1||(R_2 + R_b) = 200\ \Omega$, $V_{R1} = R_c \times I_s = 11$ V
- $V_{a2} = V_{R1} \times R_b/(R_b + R_2) = 4.4$ V, $i_2(0) = V_{a2}/R_3 = 0.011$ A
- $i(0) = i_1(0) + i_2(0) = 32$ mA
- $i(\infty) = I_s \times R_1/(R_1 + R_2 + R_3) = 25$ mA
- $R_{eq} = R_2 + R_1 + R_3 = 1100\ \Omega$
- $\tau = L/R_{eq} = 2 \times 10^{-4}$ s $= 0.2$ ms, $1/\tau = 5000$ (1/s)
- $i(t) = [25 + (32 - 25)\exp(-5000t)]\, u(t)$ mA
- $i(t) = [25 + 7\exp(-5000t)]\, u(t)$ mA

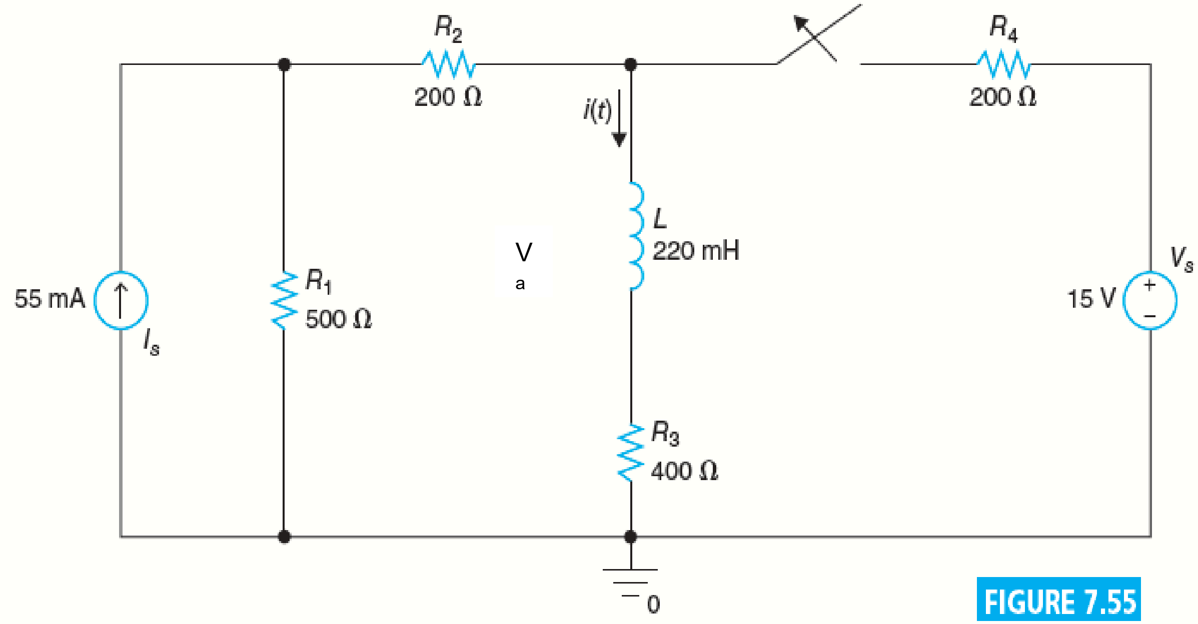


FIGURE 7.55

EXAMPLE 7.18

- The initial current through the inductor is $i(0) = 1\ A$ in the circuit shown in Figure 7.59. Find the current $i_o(t)$ through R_2 for $t \geq 0$.
- Sum the voltage drops around the mesh in the left side:

$$-9 + 3i(t) + 5\frac{di(t)}{dt} = 0 \Rightarrow \frac{di(t)}{dt} + 0.6\,i(t) = 1.8$$

- $0.6\ i(\infty) = 1.8 \Rightarrow i(\infty) = 3\ A$
- $1/\tau = 0.6 \Rightarrow \tau = 1/0.6 = 5/3 = 1.6667$
- $i(t) = [3 + (1 - 3)\exp(-0.6t)]\ u(t)\ A$
- $i(t) = [3 - 2\exp(-0.6t)]\ u(t)\ A$
- $di(t)/dt = 1.2\ \exp(-0.6t)\ u(t)\ A$
- $I_o(t) = (2/6) \times di(t)/dt = 0.4\ \exp(-0.6t)\ u(t)\ A$

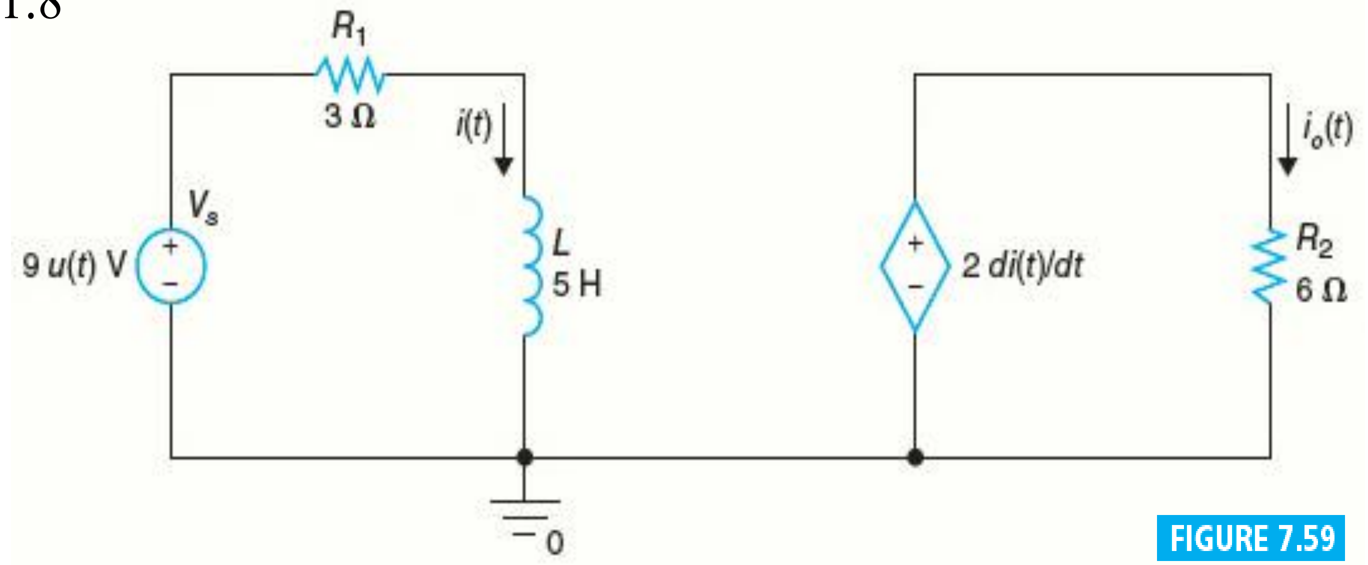


FIGURE 7.59

Summary

- Figure 7.30 shows a circuit with one inductor with inductance *L* and one resistor with resistance *R* connected in parallel. Let the initial current through the inductor at $t = 0$ be $i(0)$. Then, the current $i(t)$ through the inductor for $t \geq 0$ is given by

$$i(t) = i(0)e^{-\frac{t}{\tau}}u(t) \text{ A}$$

where the time constant τ is given by $\tau = L/R$.

- Figure 7.43 shows a circuit with a current source with current I_S, a resistor with resistance *R*, and an inductor with inductance *L* connected in parallel. Let the initial current through the inductor at $t = 0$ be $i(0) = I_0$. Then, the current $i(t)$ through the inductor for $t \geq 0$ is given by

$$i(t) = I_S + \left(I_0 - I_S\right)e^{-\frac{t}{\tau}}$$

where the time constant τ is given by $\tau = L/R$.