University of London

Assessment Coversheet

Complete this coversheet and read the instructions below carefully.

Candidate Number:
LX0303

Degree Title:
BSc Computer Science

Course/Module Title:
Computational Mathematics

Course/Module Code:
CM1015

Enter the numbers, and sub-sections, of the questions in the order in which you have attempted them:

Q2: (a) i., (a) ii., (b), (c) i., (c) ii., (d), (e)
Q4: (a) i., (a) ii., (a) iii., (b) i., (b) ii., (c), (d) i., (d) ii., (e)

Date: Sep. 7th, 2021

Instructions to Candidates

1. Complete this coversheet and begin typing your answers on the page below, or, submit the coversheet with your handwritten answers (where handwritten answers are permitted or required as part of your online timed assessment).
2. Clearly state the question number, and any sub-sections, at the beginning of each answer and also note them in the space provided above.
3. For typed answers, use a plain font such as Arial or Calibri and font size 11 or larger.
4. Where permission has been given in advance, handwritten answers (including diagrams or mathematical formulae) must be done on light coloured paper using blue or black ink.
5. Reference your diagrams in your typed answers. Label diagrams clearly.

Section B

Question 2

(a) Show whether the following series are convergent or divergent. If the series is convergent determine the value of it.

i. $s_n = \dfrac{6+7n^2}{5-3n^2}$

Ans.

$s_0 = \dfrac{6}{5} = 1.2,\ s_{10} = \dfrac{706}{-295} \approx -2.4,\ s_{1000} = \dfrac{700006}{-299995} \approx -2.3.$

Convergent to 2.3 ■

ii. $\Sigma_{n=1}^{\infty}(-1)^n \cos\left(\dfrac{1}{n}\right)$

Ans.

$n = 10 \to 1 \cdot \cos\left(\dfrac{1}{10}\right) = 0.995\ldots$

$n = 100 \to 1 \cdot \cos\left(\dfrac{1}{10}\right) = 0.99995\ldots$

$n = 9 \to 1 \cdot \cos\left(\dfrac{1}{10}\right) = -0.99\ldots$

$n = 99 \to 1 \cdot \cos\left(\dfrac{1}{10}\right) = -0.9999\ldots$

Doesn't converge, hence divergent. ■

(b) Suppose that we have a die which is rolled and a coin which is tossed. What is the probability that the die shows an odd number and the coin shows a head.

Ans.

The probability of rolling an odd number on the die is $\dfrac{3}{6}$ since there are three odd numbers out of the six possible ones.

The probability of tossing a head on a coin is $\dfrac{1}{2}$ since there are two possible outcomes of a coin.

Two events are independent hence the result has to be multiplied.

$\dfrac{3}{6} \cdot \dfrac{1}{2} = \dfrac{1}{2} \cdot \dfrac{1}{2} = \dfrac{1}{4}.$ ■

(c) Find the value of the following, put it in its simplest form.

i. $\cos \dfrac{\pi}{12}$

Ans.

$$\begin{aligned}
\cos \frac{\pi}{12} &= \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) \\
&= \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right) \\
&= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} \\
&= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \\
&= \frac{\sqrt{2}+\sqrt{6}}{4}
\end{aligned}$$

■

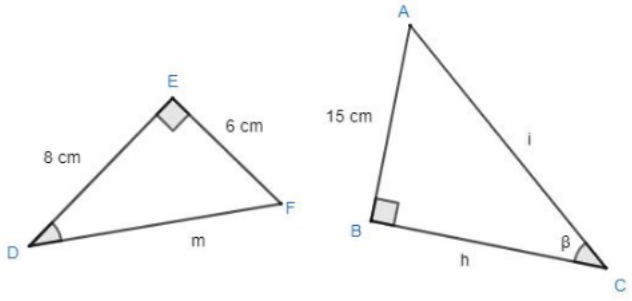
ii. $\log_{x^2} x^3$

Ans.
Let a be the answer.
$(x^2)^a = x^3 \to x^{2a} = x^3 \to 2a = 3 \to a = \dfrac{3}{2}$
■

(d) Consider that the position of an object is given by the following equation: $s(t) = 2te^t$. Will this object stop moving? If so, at which value of t the object will stop moving.

Ans.
No, it just keeps getting faster. ■

(e) If the two triangles given below are similar. Find the length of the side i.



Ans.

$$\frac{6}{15} = \frac{8}{h} = \frac{m}{i}$$
$$\frac{2}{5} = \frac{8}{h} = \frac{10}{i} \quad m = 10 \text{ since } \triangle\text{DEF is a right triangle}$$
$$\frac{2}{5} = \frac{8}{20} = \frac{10}{25} \quad \text{hence } i = 25 \quad \blacksquare$$

Question 4

(a) Point P moves along the x-axis in such a way that its position at time t/s is given by $x = 2t^3 - 15t^2 + 24t$ ft.

i. Find the velocity and acceleration of P at time t

Ans.
$v = x' = 6t^2 - 30t + 24 = 6(t^2 - 5t + 4) = 6(t-4)(t-1)$
$a = x'' = 12t - 30 = 6(2t - 5)$ ■

ii. In which direction and how fast is P moving at $t = 2s$? Is it speeding up or slowing down at that time?

Ans.
$v = 6(-2)(1) = -12$ ft/s, moving at -12 ft/s, negative direction.
$a = 6(2 \cdot 2 - 5) = -6$ ft/s^2, the speed is slowing down. ■

iii. When is P instantaneously at rest? When is its speed instantaneously not changing?

Ans.
At rest means the velocity is zero, if take the velocity equation, we can see that when $t = 1$s and $t = 4$s, the velocity is zero, therefore the answer is $t = 1, 4$s.
Speed not changing means the acceleration is zero, if we take the acceleration equation, we can see that when $t = 2.5$s, the acceleration is zero, therefore the answer is $t = 2.5$s. ■

(b) You want to invite your friends to have a party. You have 15 friends but you have only 7 chairs in your garden.

i. How many different ways do you have for which 7 friends to invite?

Ans.
$$C_7^{15} = \frac{15!}{8!7!} = \frac{15 * 14 * 13 * 12 * 11 * 10 * 9}{2 * 3 * 4 * 5 * 6 * 7} = 6435 \text{ ways } \blacksquare$$

ii. What if you decided not only which friends to invite but also where to seat them along your table? How many different ways do you have?

Ans.
$P_7^{15} = \frac{15!}{8!} = 15 * 14 * 13 * 12 * 11 * 10 * 9 = 32432400$ ways ■

(c) Solve the equation $3\log(x+5) = 2\log(7-x)$.

Ans.

$$(x+5)^3 = (7-x)^2$$
$$x^3 + 15x^2 + 75x + 125 = x^2 - 14x + 49$$
$$x^3 + 14x^2 + 89x + 76 = 0$$

$x = 0 \to 76,\ x = 1 \to 180,\ x = -1 \to 0,\ x = -2 \to -54$
$x = -1$ ■

(d) Find the derivatives of the following:

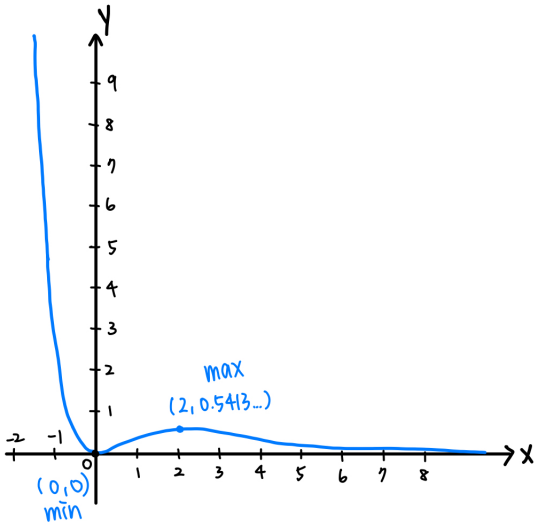
i. $y = e^{x^3 - 3x^2}$

Ans.
$(3x^2 - 6x)e^{x^3-3x^2}$

ii. $y = 10^{5x}$

Ans.
$5 \cdot 10^{5x} \cdot \ln(10)$

(e) Find the critical points (maximum and minimum values) of the function $f(x) = x^2 e^{-x}$. Sketch the graph.



x	0	1	2	3	4
f(x)	0	e^{-1}	$4e^{-2}$	0.4481...	0.2930...
		0.3678...	0.5413...		

$f'(x) = x^2 \cdot (-1)e^{-x} + 2xe^{-x}$
$= e^{-x}(-x^2+2x)$
$= xe^{-x}(-x+2)$

$x = 2 \to$ critical point (max)
$x = 0 \to$ critical point (min)