Ch 24 Trigonometrical identities and equations (C)

Common Trigonometrical identities

$$\frac{\sin A}{\cos A} = \tan A$$

$$\begin{aligned}\sin(A+B) &= \sin A\cos B + \sin B\cos A\\ \sin(A-B) &= \sin A\cos B - \sin B\cos A\\ \sin 2A &= 2\sin A\cos A\end{aligned}$$

$$\begin{aligned}\cos(A+B) &= \cos A\cos B - \sin A\sin B\\ \cos(A-B) &= \cos A\cos B + \sin A\sin B\\ \cos 2A &= (\cos A)^2 - (\sin A)^2 = \cos^2 A - \sin^2 A\end{aligned}$$

$$\begin{aligned}\tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A\tan B}\\ \tan(A-B) &= \frac{\tan A - \tan B}{1 + \tan A\tan B}\end{aligned}$$

$$\begin{aligned}\sin A + \sin B &= 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\\ \sin A - \sin B &= 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)\\ \cos A + \cos B &= 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\\ \cos A - \cos B &= -2\sin\left(\frac{A-B}{2}\right)\sin\left(\frac{A+B}{2}\right)\end{aligned}$$

$$\begin{aligned}\sin\theta &= \sin(180^\circ - \theta)\\ &= -\sin(\theta - 180^\circ)\\ &= -\sin(360^\circ - \theta)\end{aligned}$$

$$\begin{aligned}\cos\theta &= -\cos(180^\circ - \theta)\\ &= -\cos(\theta - 180^\circ)\\ &= \cos(360^\circ - \theta)\end{aligned}$$

$$\begin{aligned}\tan\theta &= -\tan(180^\circ - \theta)\\ &= \tan(\theta - 180^\circ)\\ &= -\tan(360^\circ - \theta)\end{aligned}$$

$$\begin{aligned}\sin A &= -\sin(-A)\\ \cos A &= \cos(-A)\\ \tan A &= -\tan(-A)\end{aligned}$$

▼ Solve $\tan(2\theta + 20^\circ) = 0.3 \quad 0^\circ \leq \theta \leq 360^\circ$

Let $z = 2\theta + 20^\circ$. As $0^\circ \leq \theta \leq 360^\circ$ then $20^\circ \leq z \leq 740^\circ$.

First we solve $\tan z = 0.3 \quad 0^\circ \leq z \leq 360^\circ$

This leads to $z = 16.7^\circ + 360^\circ,\ 196.7^\circ + 360^\circ = 376.7^\circ,\ 556.7^\circ$.

By adding a further 360° values of z in the range 720° to 1080° are found. These are $z = 736.7^\circ,\ 916.7^\circ$.

Hence values of z in the range $0^\circ - 1080^\circ$ are $z = 16.7^\circ,\ 196.7^\circ,\ 376.7^\circ,\ 556.7^\circ, 736.7^\circ,\ 916.7^\circ$.

Values of z in the range $20^\circ - 740^\circ$ are thus $z = 196.7^\circ,\ 376.7^\circ,\ 556.7^\circ,\ 736.7^\circ$.

The values of θ in the range 0-360 are found using $\theta = (z - 20^\circ)/2$:

$$\theta = \frac{z - 20^\circ}{2} = 88.35^\circ,\ 178.35^\circ,\ 268.35^\circ,\ 358.35^\circ.$$