# Ch 8.2 Operations with Matrices (L)

## Equality of Matrices

Two matrices are equal when their corresponding entries are equal.

## Matrix Addition and Scalar Multiplication

> If $A = [a_{ij}]$ and $B = [b_{ij}]$ are matrices of order $m \times n$, then their sum is the $m \times n$ matrix given by $A + B = [a_{ij} + b_{ij}]$.
> The sum of two matrices of different orders is undefined.

> If $A = [a_{ij}]$ as an $m \times n$ matrix and $c$ is a scalar, then the **scalar multiple** of $A$ by $c$ is the $m \times n$ matrix given by $cA = [ca_{ij}]$.

### Properties of Matrix Addition and Scalar Multiplication

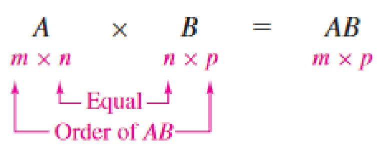
Let $A$, $B$, and $C$ be $m \times n$ matrices and let $c$ and $d$ be scalars.

1. $A + B = B + A$ — *Commutative*
2. $A + (B + C) = (A + B) + C$ — *Associative (Addition)*
3. $(cd)A = c(dA)$ — *Associative (Scalar Multiplication)*
4. $1A = A$ — *Scalar Identity*
5. $c(A + B) = cA + cB$ — *Distributive*
6. $(c + d)A = cA + dA$ — *Distributive*

| **Real Numbers (Solve for $x$.)** | **$m \times n$ Matrices (Solve for $X$.)** |
|---|---|
| $x + a = b$ | $X + A = B$ |
| $x + a + (-a) = b + (-a)$ | $X + A + (-A) = B + (-A)$ |
| $x + 0 = b - a$ | $X + O = B - A$ |
| $x = b - a$ | $X = B - A$ |

Additive identity $O$

## Matrix Multiplication




### Properties of Matrix Multiplication

Let $A$, $B$, and $C$ be matrices and let $c$ be a scalar.

1. $A(BC) = (AB)C$ — *Associative (Multiplication)*
2. $A(B + C) = AB + AC$ — *Distributive*
3. $(A + B)C = AC + BC$ — *Distributive*
4. $c(AB) = (cA)B = A(cB)$ — *Associative (Scalar Multiplication)*

**Identity Matrix ($I_n$ or $I$ )**

Consisting of 1s on its main diagonal and 0s elsewhere. $AI_n = A$ and $I_nA = A$.

## Applications

**System**

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}$$

**Matrix Equation $AX = B$**

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}}_{A} \times \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}}_{B}$$

▼ Example 12 **Solving a System of Linear Equations**

$$\begin{cases} x_1 - 2x_2 + x_3 &= -4 \\ x_2 + 2x_3 &= 4 \\ 2x_1 + 3x_2 - 2x_3 &= 2 \end{cases}$$

*Solution*:

a. In matrix form, $AX = B$, the system can be written as follows.

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 2 \\ 2 & 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 4 \\ 2 \end{bmatrix}$$

b. The augmented matrix is formed by adjoining matrix $B$ to matrix $A$.

$$[A \vdots B] = \begin{bmatrix} 1 & -2 & 1 & \vdots & -4 \\ 0 & 1 & 2 & \vdots & 4 \\ 2 & 3 & -2 & \vdots & 2 \end{bmatrix}$$

Using Gauss-Jordan elimination, you can rewrite this matrix as

$$[I \vdots X] = \begin{bmatrix} 1 & 0 & 0 & \vdots & -4 \\ 0 & 1 & 0 & \vdots & 2 \\ 0 & 0 & 1 & \vdots & 1 \end{bmatrix}$$

So, the solution of the matrix equation is

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}.$$

▼ Example 13 **Softball Team Expenses**

| Equipment | Women's Team | Men's Team |
|---|---|---|
| Bats | 12 | 15 |
| Balls | 45 | 38 |
| Gloves | 15 | 17 |

Each bat costs \$80, each ball costs \$6, and each glove costs \$60. Use matrices to find the total cost of equipment for each team.

*Solution*:

$$E = \begin{bmatrix} 12 & 15 \\ 45 & 38 \\ 15 & 17 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 80 & 6 & 60 \end{bmatrix}.$$

$$\begin{aligned} CE &= \begin{bmatrix} 80 & 6 & 60 \end{bmatrix} \begin{bmatrix} 12 & 15 \\ 45 & 38 \\ 15 & 17 \end{bmatrix} \\ &= \begin{bmatrix} 80(12) + 6(45) + 60(15) & 80(15) + 6(38) + 60(17) \end{bmatrix} \\ &= \begin{bmatrix} 2130 & 2448 \end{bmatrix} \end{aligned}$$

So, the total cost of equipment for the women's team is \$2130, and the total cost of equipment for the men's team is \$2448.