Ch 8.3 The Inverse of a Square Matrix (L)

The Inverse of a Matrix

Let A be an $n \times n$ matrix and let I_n be the $n \times n$ identity matrix. If there exists a matrix A^{-1} such that $AA^{-1} = I_n = A^{-1}A$ then A^{-1} is called the **inverse** of A.

Finding Inverse Matrices

If a matrix A has an inverse, then A is called **invertible** (or **nonsingular**).

A nonsquare matrix cannot have an inverse.

▼ Example 2 **Finding the Inverse of a Matrix**

Find the inverse of $A = \begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix}$.

Solution:

Solve the matrix equation $AX = I$ for X.

$$\begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_{11} + 4x_{21} & x_{12} + 4x_{22} \\ -x_{11} - 3x_{21} & -x_{12} - 3x_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{cases} x_{11} + 4x_{21} & = 1 \\ -x_{11} - 3x_{21} & = 0 \end{cases} \qquad \begin{cases} x_{12} + 4x_{22} & = 0 \\ -x_{12} - 3x_{22} & = 1 \end{cases}$$

$x_{11} = -3$ and $x_{21} = 1$.

$x_{12} = -4$ and $x_{22} = 1$.

So, $X = A^{-1}$

$$= \begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix}$$

The Inverse of a 2×2 Matrix

Only works for 2 x 2 matrices.

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Systems of Linear Equations

If A is an invertible matrix, then the system of linear equations represented by $AX = B$ has a unique solution given $X = A^{-1}B$.