Ch 27 Matrices (C)

What is a matrix?

Element: Each number in a matrix

Size: Number of rows and number of columns in order

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 1 & 2 \\ -3 & 4 & 5 \\ \frac{1}{2} & 2 & 1 \end{bmatrix}$$

$$2 \times 3 \qquad\qquad 3 \times 1 \qquad\qquad 3 \times 3$$

Square matrix: same number of rows as columns

Diagonal matrix: a square matrix where all elements are 0 except those on the diagonal (top left to bottom right - **leading diagonal**)

Identify matrix: a diagonal matrix where all diagonal entries are 1

$\rightarrow$ when multiplying identity matrices, it leaves the matrix unchanged, just like multiply 1.

Addition, subtraction and multiplication of matrices

Add & **Subtract**: same size

Multiply & **Divide**: by a scalar (number)

Multiply: by another matrix if conditions met

Divide: never by another matrix

$+/-$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 4 & 4 \end{bmatrix}$$

$\begin{bmatrix} 1 & 2 & 9 \\ -1 & \frac{1}{2} & 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 7 \end{bmatrix}$ cannot be added nor subtracted from one another.

$\times / \div$

$$\frac{1}{4}\begin{bmatrix} 16 \\ 8 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

Multiply two matrices

If A has size $p \times q$ and B has size $q \times s$, then AB has size $p \times s$.

If $p \neq s$, then BA does not exist.

Possible

$\begin{bmatrix} 1 & 4 & 9 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 9 \\ 8 & 7 \\ -7 & 3 \end{bmatrix}$ 2 x 3 3 x 2

Not Possible

$\begin{bmatrix} 3 & 7 & 2 \\ -1 & 0 & -10 \end{bmatrix} \begin{bmatrix} 1 & -7 \\ -1 & 2 \end{bmatrix}$ 2 x 3 2 x 2

Inverse

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

- leading diagonal interchanged
- remaining elements change sign
- resulting matrix multiply by $\dfrac{1}{ad - bc}$

🗣️ $AA^{-1} = A^{-1}A = I$ (identity matrix)

🗣️ If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then its determinant is $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.

Singular matrix: its determinant is zero. Hence cannot have an inverse.

Application

🗣️ $AX = B$
$X = A^{-1}B$ provided A^{-1} exists.

▼ Example

$x + 2y = 13$

$2x - 5y = 8$

$$\begin{bmatrix} 1 & 2 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ 8 \end{bmatrix} \qquad AX = B$$

$X = A^{-1}B$

$$X = \frac{1}{-9} \begin{bmatrix} -5 & -2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 13 \\ 8 \end{bmatrix} = -\frac{1}{9} \begin{bmatrix} -81 \\ -18 \end{bmatrix} = \begin{bmatrix} 9 \\ 2 \end{bmatrix}$$

Therefore the solution is $x = 9, y = 2$.