Ch 3.2.2 Revision of linear combination (K)

🗣️ Let $v_1, v_2, ...$ and v_n be vectors in a vector space. If a vector x can be expressed as $x = k_1v_1 + k_2v_2 + \cdots + k_nv_n$ (where k's are scalars), then we say x is a **linear combination** of the vectors $v_1, v_2, v_3 ...$ and v_n.

▼ Example 3.8

Let $v_1 = t^2 - 1, v_2 = t^2 + 3t - 5, v_3 = t$ be vectors in P_2.

Show that the quadratic polynomial $\mathbf{x} = 7t^2 - 15$ is a linear combination of $\{v_1, v_2, v_3\}$.

Solution:

$$\begin{aligned} k_1v_1 + k_2v_2 + \cdots + k_nv_n &= k_1(t^2 - 1) + k_2(t^2 + 3t - 5) + k_3t \\ &= (k_1 + k_2)t^2 + (3k_2 + k_3)t - (k_1 + 5k_2) \\ &= 7t^2 - 15 \end{aligned}$$

$$\begin{aligned} k_1 + k_2 &= 7 \\ 3k_2 + k_3 &= 0 \\ k_1 + 5k_2 &= 15 \end{aligned} \qquad 5v_1 + 2v_2 - 6v_3 = \mathbf{x} \qquad 5(t^2 - 1) + 2(t^2 + 3t - 5) - 6t = 7t^2 - 15$$

This conclude that x is a linear combination of $\{v_1, v_2, v_3\}$.

💡 A non-empty subset S containing vectors u and v is a subspace of a vector space V ⇔ *(iff)* any linear combination $k\mathbf{u} + c\mathbf{v}$ is also in S (k and c are scalars).

▼ Example 3.9

Let S be the subset of vectors of the form $(x \;\; y \;\; 0)^T$ in the vector space $\mathbb{R}^3$. Show that S is a subspace of $\mathbb{R}^3$.

Using the above proposition, we need to show that any linear combination $k\mathbf{u} + c\mathbf{v}$ is in S for any vectors $\mathbf{u}$ and $\mathbf{v}$ in S.

It is clear that S is non-empty because the zero vector is in S. Let $u = (a \;\; b \;\; 0)^T$ and $v = (c \;\; d \;\; p)^T$ be in S. Then for real scalars k_1 and k_2 we have

$$\begin{aligned} k_1\mathbf{u} + k_2\mathbf{v} = k_1 \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} + k_2 \begin{pmatrix} c \\ d \\ 0 \end{pmatrix} \\ = \begin{pmatrix} k_1a \\ k_1b \\ 0 \end{pmatrix} + \begin{pmatrix} k_2c \\ k_2d \\ 0 \end{pmatrix} = \begin{pmatrix} k_1a + k_2c \\ k_1b + k_2d \\ 0 \end{pmatrix} \end{aligned}$$

Hence $k_1\mathbf{u} + k_2\mathbf{v}$ is also in S.