

# Ch1.1 (Week 5)

#### Introduction

• The rules of logic are used to distinuish between valid and invalid mathematical arguments.

#### **Propositions**

• A **proposition** is a sentence that declares a fact, that is either true or false, but not both.

Let p be a proposition. The *negation* of p, denoted by  $\neg p$  (also denoted by  $\overline{p}$ ), is the statement "It is not the case that p."

The proposition  $\neg p$  is read "not p." The truth value of the negation of p,  $\neg p$ , is the opposite of the truth value p.

TABLE 1 The Truth Table for the Negation of a Proposition.						
p	p - p					
T F						
F T						

Let p and q be propositions. The *conjunction* of p and q, denoted by  $p \wedge q$ , is the proposition "p and q." The conjunction  $p \wedge q$  is true when both p and q are true and is false otherwise.

TABLE 2 The Truth Table for the Conjunction of Two Propositions.							
p	$p \qquad q \qquad p \wedge q$						
T	T	T					
T	F	F					
F T F							
F F F							

• In logic, the word "but" sometimes is used instead of "and" in a conjunction.

Let p and q be propositions. The *disjunction* of p and q, denoted by  $p \vee q$ , is the proposition "p or q." The conjunction  $p \vee q$  is true when both p and q are false and is true otherwise.

TABLE 3 The Truth Table for the Disjunction of Two Propositions.					
$p \qquad q \qquad p \lor q$					
T	T	T			
T	T F T				
F	T	T			
F F F					

Let p and q be propositions. The *exclusive or* of p and q, denoted by  $p \oplus q$ , is the proposition that is true when exactly one of p and q is true and is false otherwise.

TABLE 4 The Truth Table for the Exclusive Or of Two Propositions.					
$p \qquad q \qquad p \oplus q$					
T	F				
F	T				
F T T					
F F F					
	rsive Or of ons.  q  T  F				

#### **Conditional Statements (Implication)**

Let p and q be propositions. The *conditional statement* p o q is the proposition "if p, then q." The conditional statement p o q is false when p is true and q is false, and true otherwise.

In the conditional statement  $p \to q$ , p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

<b>TABLE</b> 5 The Truth Table for the Conditional Statement $p \rightarrow q$ .			
p	q	$p \rightarrow q$	
T	T	T	
T	F	F	
F	T	T	
F	F	T	

- Common ways to express conditional statement:
  - "if p, then q." "if p, q." "q if p" "q when p" "p implies q" "p only if q"

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#### Converse, Contrapositive, and Inverse

$$egin{array}{ll} p 
ightarrow q \ q 
ightarrow p & converse \ 
eg q 
ightarrow 
eg p & contrapositive \ 
eg p 
ightarrow 
eg q & inverse \end{array}$$

Equivalent (truth value)
 Origin == Contrapositive
 Converse == Inverse

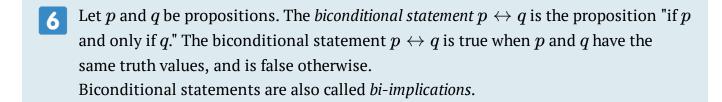


TABLE 6 The Truth Table for the Biconditional $p \leftrightarrow q$ .						
p	$p \qquad q \qquad p \leftrightarrow q$					
T	T	T				
T	F	F				
F	T	F				
F	F	T				

- Common ways to express biconditional statement:
  - "p iff q."
- Same truth value as  $(p o q)\wedge (q o p)$

## **Truth Tables and Compound Propositions**

TABI	<b>TABLE</b> 7 The Truth Table of $(p \lor \neg q) \to (p \land q)$ .					
p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \to (p \wedge q)$	
T	T	F	Т	Т	Т	
T	F	T	Т	F	F	
F	T	F	F	F	Т	
F	F	T	Т	F	F	

### **Precedence of Logical Operators**

TABLE 8 Precedence of Logical Operators.			
Operator Precedence			
_	1		
^ V	2 3		
$\begin{array}{c} \rightarrow \\ \leftrightarrow \end{array}$	4 5		

## **Logic and Bit Operations**

Truth Value	Bit
Т	1
F	0

TABLE 9 Table for the Bit Operators <i>OR</i> , <i>AND</i> , and <i>XOR</i> .						
X	$y \qquad x \lor y \qquad x \land y \qquad x \oplus y$					
0	0	0	0	0		
0	1	1	0	1		
1	0	1	0	1		
1	1	1	1	0		

- A bit strings is a sequence of zero or more bits. The length of this string is the number of bits in the string.
- bitwise notation:
  - OR:  $\vee$ , AND:  $\wedge$ , XOR:  $\oplus$