Ch1.1 (Week 5)

Introduction

- The rules of logic are used to distinuish between valid and invalid mathematical arguments.

Propositions

- A **proposition** is a sentence that declares a fact, that is either true or false, but not both.

1 Let p be a proposition. The *negation* of p, denoted by $\neg p$ (also denoted by $\overline{p}$), is the statement "It is not the case that p."
The proposition $\neg p$ is read "not p." The truth value of the negation of p, $\neg p$, is the opposite of the truth value p.

TABLE 1 The Truth Table for the Negation of a Proposition.

p	$\neg p$
T	F
F	T

2 Let p and q be propositions. The *conjunction* of p and q, denoted by $p \wedge q$, is the proposition "p and q." The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

TABLE 2 The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- In logic, the word "but" sometimes is used instead of "and" in a conjunction.

3 Let p and q be propositions. The *disjunction* of p and q, denoted by $p \vee q$, is the proposition "p or q." The conjunction $p \vee q$ is true when both p and q are false and is true otherwise.

TABLE 3 The Truth Table for the Disjunction of Two Propositions.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

4 Let p and q be propositions. The *exclusive or* of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise.

TABLE 4 The Truth Table for the Exclusive Or of Two Propositions.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Conditional Statements (Implication)

5 Let p and q be propositions. The *conditional statement* $p \rightarrow q$ is the proposition "if p , then q." The conditional statement $p \rightarrow q$ is false when p is true and q is false, and true otherwise.
In the conditional statement $p \rightarrow q$, p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

TABLE 5 The Truth Table for the Conditional Statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

- Common ways to express conditional statement:
  - "if p, then q." "if p, q." "q if p" "q when p" "p implies q" "p only if q"

Converse, Contrapositive, and **Inverse**

$$\begin{array}{lr} p \to q & \\ q \to p & converse \\ \neg q \to \neg p & contrapositive \\ \neg p \to \neg q & inverse \end{array}$$

- *Equivalent* (truth value)
  Origin == Contrapositive
  Converse == Inverse

6 Let p and q be propositions. The *biconditional statement* $p \leftrightarrow q$ is the proposition "if p and only if q." The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise.
Biconditional statements are also called *bi-implications*.

TABLE 6 The Truth Table for the Biconditional $p \leftrightarrow q$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

- Common ways to express biconditional statement:
  - "*p iff q*."
- Same truth value as $(p \to q) \wedge (q \to p)$

Truth Tables and Compound Propositions

TABLE 7 The Truth Table of $(p \vee \neg q) \to (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \to (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Precedence of Logical Operators

TABLE 8 Precedence of Logical Operators.

Operator	*Precedence*
$\neg$	1
$\wedge$ $\vee$	2 3
$\to$ $\leftrightarrow$	4 5

Logic and Bit Operations

Truth Value	*Bit*
T	1
F	0

TABLE 9 Table for the Bit Operators *OR*, *AND*, and *XOR*.

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

7 A *bit strings* is a sequence of zero or more bits. The *length* of this string is the number of bits in the string.

- bitwise notation:
  - OR: $\vee$, AND: $\wedge$, XOR: $\oplus$