

FCS Midterm Question

1. Without using the truth table show the following statements are true. Explain your reasoning.

(a) $(p \rightarrow q) \vee (p \rightarrow r) \equiv \neg r \rightarrow (\neg p \vee q)$ [3 marks]

(b) $(p \rightarrow \neg q) \wedge \neg p \equiv p \rightarrow (\neg q \wedge \neg p)$ [3 marks]

(c) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$ [3 marks]

2. Prove the following statement by induction.

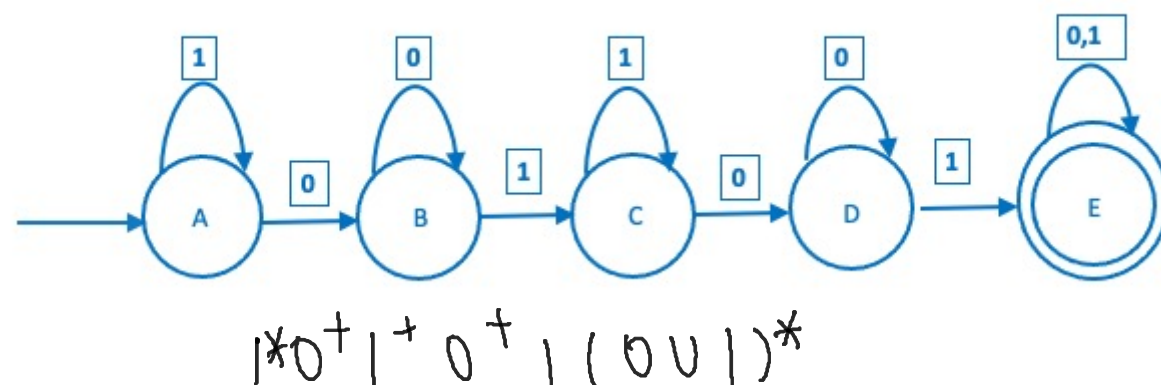
For all nonnegative integers n , 3 divides $n^3 + 2n + 3$. State the mathematical induction and show your work clearly. [9 marks]

3. Students are required to create **6-character** long passwords to access the library. The letters must be from lowercase letters or digits. Each password must contain **at least two** lowercase-letters, **at least one** digit and contains **no repeated digits**. How many valid passwords are there? You are required to show your work step-by-step. [9 marks]

Note:

1hfgt1 is invalid because 1 appears more than once. 134ggg is valid because there are 3 lower-case letters and digits are not repeated.

4. Consider the following automaton.



$$1^*0^+1^+0^+(0 \vee 1)^*$$

a) Give an example of a string containing 11 that is **accepted** by the following automaton. [2 marks]

b) Give an example of a string of length 8 that is **rejected** by the following automaton. [2 marks]

c) Describe the language of this automaton in plain English. [4 marks]

d) Describe the language of this automaton using Regular expression. [3 marks]

5. Given $R = (0^*10^+)^+1^*$ and $S = (1^*01^+)^*$

a) Give an example of a string that is neither in the language of R nor in S. [2marks] 0^+

b) Give an example of a string that is in the language of S but not R. [2 marks]

c) Give an example of a string that is in the language of R but not S. [2 marks]

d) Give an example of a string that is in the language of R and S. [2 marks]

e) Design a regular expression that accepts the language of all binary strings with no occurrences of bab [4 marks]

$$a^*b^*a^*$$