University of London

Assessment Coversheet

Complete this coversheet and read the instructions below carefully.

Candidate Number:
LX0303

Degree Title:
BSc Computer Science

Course/Module Title:
Fundamentals of Computer Science

Course/Module Code:
CM1025

Enter the numbers, and sub-sections, of the questions in the order in which you have attempted them:

Q2(a), Q2(b), Q3(c), Q2(d), Q2(e)
Q4(a), Q4(b), Q4(c), Q4(d), Q4(e), Q4(f)

Date: Sep. 8th, 2021

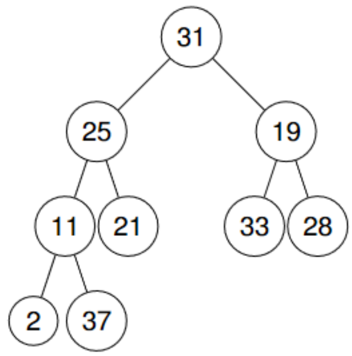
Instructions to Candidates

1. Complete this coversheet and begin typing your answers on the page below, or, submit the coversheet with your handwritten answers (where handwritten answers are permitted or required as part of your online timed assessment).
2. Clearly state the question number, and any sub-sections, at the beginning of each answer and also note them in the space provided above.
3. For typed answers, use a plain font such as Arial or Calibri and font size 11 or larger.
4. Where permission has been given in advance, handwritten answers (including diagrams or mathematical formulae) must be done on light coloured paper using blue or black ink.
5. Reference your diagrams in your typed answers. Label diagrams clearly.

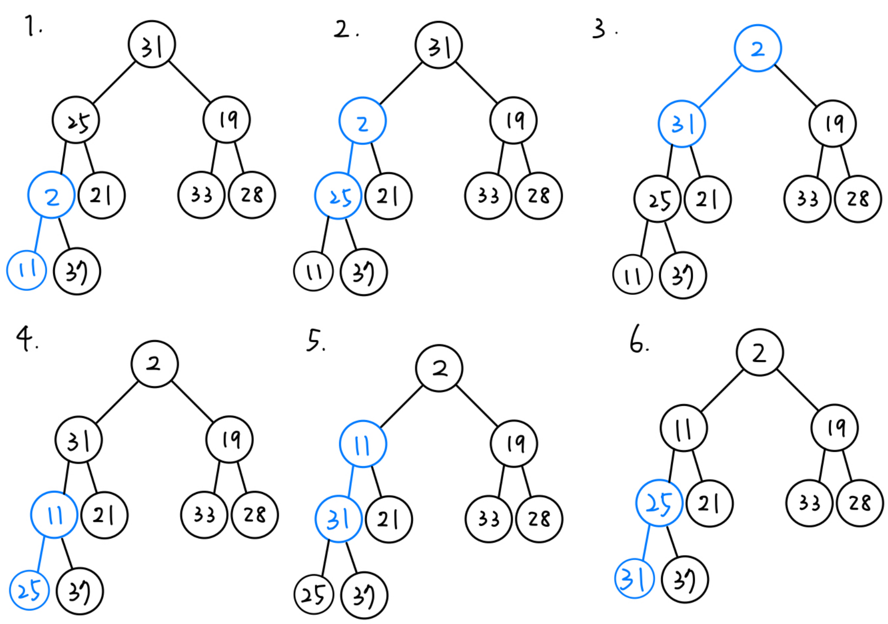
Section B

Question 2

(a) Heapify the following tree, make every step clear. (Min heap)



Ans.



$\blacksquare$

(b) Given $R = 1^*0^+1^+0^+1\Sigma^*$ and $S = (1^+0^*)^*1$ where $\Sigma = \{0, 1\}$

- Given an example of a string that is neither in the language of R nor in S.

  Ans.
  0

■

- Given an example of a string that is in the language of S but not R.

  Ans.
  11
  ■

- Given an example of a string that is in the language of R but not S.

  Ans.
  01010
  ■

- Given an example of a string that is in the language R and S.

  Ans.
  10101
  ■

- Design a regular expression that accepts the language of all binary strings with no occurrences of 1001

  Ans.
  0*1*1*0*
  ■

(c) Answer the following for the context-free grammar $G: \; S \rightarrow 0S1S \mid 1S0S \mid \epsilon$

- Given two non-empty strings that can be generated from G, show the derivations.

  Ans.
  $S \rightarrow 0S1S \rightarrow 01 \Rightarrow 01$
  $S \rightarrow 1S0S \rightarrow 10 \Rightarrow 10$
  ■

- Given two strings that cannot be generated from the context-free grammar G.

  Ans.
  $0000, 1111$
  ■

- Can 110 be generated by G? Justify your reasoning.

  Ans.
  $S \rightarrow 1S0S \rightarrow 11S0S0S \rightarrow 1100$
  No, it cannot be generated, the closest will be 1100.
  ■

- What is the language of G?

  Ans.
  Even number of strings with same amount of 1's and 0's.
  ■

(d) Use mathematical induction to prove that for all natural numbers $n > 3, 2^n < n!$. State every step of the proof.

Proof.

Let P(n) be $2^n < n!$ for $n \in \mathcal{N}, n > 3$.
Base step: P(4) is true, because $2^4 = 16 < 4! = 24$.
Inductive step:
Assume P(k) is true, that is $2^k < k!$ for $k \in \mathcal{N}, k > 3$.
Then $P(k+1) \Rightarrow 2^{k+1} < (k+1)!$
$\Rightarrow 2 \cdot 2^k < k!(k+1)$
Since we already assume $2^k < k!$, we therefore only need to analyze 2 and $k+1$ to see if this inequality is correct. Because $k > 3$, we know that $k+1 > 2$, hence the inequality holds and is true.
This completes the inductive step.
We have completed the base and inductive step, so by mathematical induction we know that $P(n)$ is true for all $\mathcal{N}$ where $n > 3$. That is, we have proven that $2^n > n!$ for $n \in \mathcal{N}, n > 3$.■

(e) State the contrapositive and use it to prove the following statement is true. If $n^2 + n - 1$ is divisible by 3 then n is divisible by 3.

Contrapositive: "If n is not divisible by 3, then $n^2 + n - 1$ is not divisible by 3."

Proof.

Let P(n) be "If n is not divisible by 3, then $n^2 + n - 1$ is not divisible by 3."
Base step: P(1) is true, because 1 is not divisible by 3, therefore $1^2 + 1 - 1 = 1$ is also not divisible by 3.
Inductive step:
Assume P(k) is true, that is "If k is not divisible by 3, then $k^2 + k - 1$ is not divisible by 3."
Then $P(k+1)$ would be "If $k+1$ is not divisible by 3, then $(k+1)^2 + (k+1) - 1$ is not divisible by 3."

$$\begin{aligned}(k+1)^2 + (k+1) - 1 &= k^2 + 2k + 1 + k + 1 - 1 \\ &= k^2 + 3k + 1 \\ &= (k^2 + k - 1) + 2(k+1)\end{aligned}$$

Given that k^2+k-1 is not divisible by 3, $(k^2+k-1)+2(k+1)$ is also not divisible by 3.

This completes the inductive step.

We have completed the base and inductive step, so by mathematical induction, we know that $P(n)$ is true. That is, we have proven that if n is not divisible by 3, then n^2+n-1 is not divisible by 3. ■

Question 4

(a) Given $\Sigma_1 = \{a, b\}$, and $\Sigma_2 = \{1, 2, 3\}$.

- What is the cardinality of Σ_2^3?

  Ans.
  $|\Sigma_2| = 3,\ |\Sigma_2^3| = 3^3 = 27.$
  ■

- List all strings of Σ_1^3.

  Ans.
  $\{aaa, aab, aba, abb, baa, bab, bba, bbb\}$.
  ■

- List three strings of $\Sigma_2^2 \circ \Sigma_1^3$.

  Ans.
  $\{11aaa, 11aab, 11aba\}$.
  ■

(b) A coin is flipped six times where each flip comes up either heads or tails. In how many possible outcomes are the number of heads and tails are not equal?

Ans.
All possible outcomes $= 2^6 = 64$, equal$\rightarrow 3H3T \Rightarrow \mathrm{C}_3^6 C_3^3 = 20 \cdot 1 = 20$.
All $-$ equal $=$ not equal $= 64 - 20 = 44$.

■

(c) Write the paths representing the parsing of the following input by the automaton depicted below, state if the input is accepted or rejected.

i. 1101

   Ans.
   $\rightarrow AAABA$ rejected
   ■

ii. 010010

   Ans.
   $\rightarrow ABABCDC$ accepted

■

- Describe the language of this automaton in plain English.

  Ans.
  The automaton starts with state A, on which it stays as long as it reads 1 from the input string.
  Once it reads 0, it moves to state B. Which will return to state A if it reads 1.
  It will move to state C if it reads 0 from the input string, which is the accepting state.
  It will move to state E if it reads another 0, on which it will stay there no matter 1 or 0 is read.
  It will move to state D if it reads 1 from state C, on which it stays as long as it reads 1 from the input string, which is another accepting state.
  Once it reads 0, it moves back to state C.

  ■

- Describe the language of this automaton using Regular expression.

  Ans.
  1*001*

  ■

(d) Use the merge sort to sort the following list in ascending order. Show your work step by step.

Ans.

split

① 4 17 15 10 6 | 2 9 3 11

② 4 17 15 | 10 6 | 2 9 | 3 11

③ 4 17 | 15 | 10 | 6 | 2 | 9 | 3 | 11

④ 4 | 17

merge

⑤ 4 17 | 15 | 6 10 | 2 9 | 3 11

⑥ 4 15 17 | 6 10 | 2 3 9 11

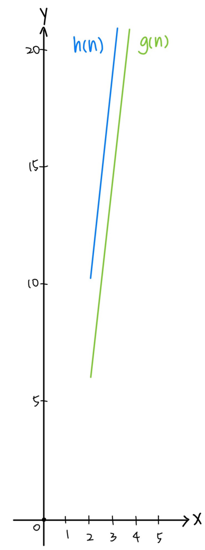
⑦ 4 6 10 15 17 | 2 3 9 11

⑧ 2 3 4 6 9 10 11 15 17

(e) Write the asymptotic functions of the following. Prove your claim: if you claim $f(n) = O(g(n)$ you need to show there exist c, k such that $f(x) \leq c \cdot g(x)$ for all $x > k$.

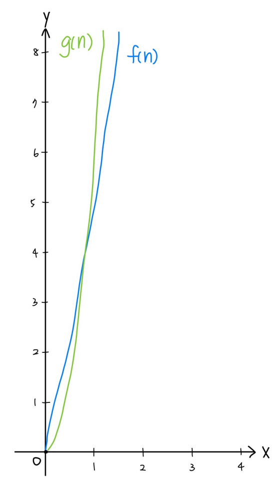
- $h(n) = 3n + 7n \log n$

  Ans.
  $h(n) = O(n \log n)$
  $n < n^2$ if $n > 1(k)$, so $4n + n^2 < 4n^2 + n^2 = 5n^2$
  i.e. $g(n) = c \cdot n^2,\ f(n) = 4n + n^2,\ c = 6,\ k = 1$
  ■



- $l(n) = 4n + n^2$

  Ans.
  $l(n) = O(n^2)$
  $logn > 1$, if $n > 2(k)$, so $7n \log n > 7n \Rightarrow 3n + 7n \log n > 10n$
  i.e. $g(n) = c \log n,\ f(n) = 3n + 7n \log n,\ c = 10, k = 2$
  ■



(f) Design a Turing Machine that accepts all binary words in the form of a^*b^*.

Ans.