Ch1.4 (Week 7)

Predicate

Propositional function P

- A statement of the form $P(x_1, x_2, \ldots, x_n)$, also called n-**place predicate** or a n-**ary predicate**.

Preconditions & Postconditions

- Valid input & satisfied output

▼ EX.7

`temp := x` `x := y` `y := temp` Find the predicate that can use as the precondition and postcondition.

Precondition → $P(x, y)$ is "$x = a$ and $y = b$"
Postcondition → $Q(x, y)$ is "$x = b$ and $y = a$"

To Verify:
Assume the precondition holds, go through the first step `temp := x`, then $x = a$, $temp = a$, and $y = b$. After the second step `x := y`, then $x = b$, $temp = a$, and $y = b$. Finally, after the third step `y := temp`, then $x = b$, $temp = a$, and $y = a$. After the program is run, the postcondition $Q(x, y)$ holds.

Quantifiers

Predicate Calculus

- The area of logic that deals with predicates and quantifiers

Domain | | Domain of Discourse

- a property is true for all values of a variable in a particular domain

1 The *universal quantification* of $P(x)$ is the statement
"$P(x)$ for all values of x in the domain."
The notation $\forall x P(x)$ denotes the universal quantification of $P(x)$. Here $\forall$ is called the universal quantifier. We read $\forall x P(x)$ as "for all $x P(x)$" or "for every $x P(x)$." An element for which $P(x)$ is false is called a counterexample of $\forall x P(x)$.

- "all of", "for each", "given any", "for arbitrary", and "for any"

TABLE 1 Quantifiers.

Statement	*When True?*	*When False?*
$\forall x P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false.
$\exists x P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x.

2 The *existential quantification* of $P(x)$ is the proposition
"There exists an element x in the domain such that $P(x)$."
We use the notation $\exists x P(x)$ for the existential quantification of $P(x)$. Here $\exists$ is called the existential quantifier.

- "for some", "for at least one", or "there is"
- Without specifying the domain, the statement $\exists x P(x)$ has no meaning

The Uniqueness Quantifier

- denoted by $\exists!$ or $\exists_1$

  "There exist a unique x such that $P(x)$ is true.", "there is exactly one" and "there is one and only one"

Quantifiers and Restricted Domains

- $\forall x < 0\ (x^2 > 0)$ is the same as $\forall x(x < 0 \to x^2 > 0)$ [Conditional Statement]
- $\exists z > 0\ (x^2 = 2)$ is the same as $\exists z\ (x > 0 \wedge z^2 = 2)$ [Conjunction]

Precedence of Quantifiers

- $\forall$ and $\exists$ have higher precedence than all logical operators
  $\forall x P(x) \vee Q(x)$ means $(\forall x\ P(x)) \vee Q(x)$ rather than $\forall x(P(x) \vee Q(x))$

Binding Variables

Scope

- the part of a logical expression that a quantifier is applied
- free variable: outside the scope

▼ EX.18

In the statement $\exists x\ (x + y = 1)$, the variable x is bound by the existential quantification $\exists x$, but the variable y is free because it is not bound by a quantifier and no value is assigned.
⇒ x is bound, but y is free.

Logical Equivalences Involving Quantifiers

3 Statements involving predicates and quantifiers are *logically equivalent* if and only if they have the same truth value no matter which predicates are substituted into these statements and which domain of discourse is used for the variables in these propositional functions. We use the notation $S \equiv T$ to indicate that two statements S and T involving predicates and quantifiers are logically equivalent.

Negating Quantified Expressions

▼ example

$P(x)$ is "x has taken a course in calculus", domain → students in your class.
Negation → "It is not the case that every student in your class has taken a course in calculus."
Equivalent to → "There is a student in your class who has not taken a course in calculus."

- $\neg\forall x\ P(x) \equiv \exists x\ \neg P(x)$
- $\neg\exists x\ Q(x) \equiv\ \forall x\ \neg Q(x)$

TABLE 2 De Morgan's Laws for Quantifiers.

Negation	*Equivalent Statement*	*When Is Negation True?*	*When False?*
$\neg\exists x P(x)$	$\forall x \neg P(x)$	For every x, $P(x)$ is false.	There is an x for which $P(x)$ is true.
$\neg\forall x P(x)$	$\exists x \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x.