FCS Midterm Answer Q1

1. (a)

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\begin{aligned}
(p \to q) \lor (p \to r) &\equiv \neg r \to (\neg p \lor q) && \\
p \to (q \lor r) &\equiv \neg r \to (\neg p \lor q) && \text{LHS: by } (p \to q) \lor (p \to r) \equiv p \to (q \lor r) \\
\neg p \lor (q \lor r) &\equiv \neg r \to (\neg p \lor q) && \text{LHS: by } p \to q \equiv \neg p \lor q \\
(\neg p \lor q) \lor r &\equiv \neg r \to (\neg p \lor q) && \text{LHS: by Associative Law} \\
(p \to q) \lor r &\equiv \neg r \to (\neg p \lor q) && \text{LHS: by } p \to q \equiv \neg p \lor q \\
(p \to q) \lor r &\equiv \neg r \to (p \to q) && \text{RHS: by } p \to q \equiv \neg p \lor q \\
(p \to q) \lor r &\equiv r \lor (p \to q) && \text{RHS: by } p \to q \equiv \neg p \lor q \\
(p \to q) \lor r &\equiv (p \to q) \lor r && \text{RHS: by Commutative Law}
\end{aligned}
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(b)

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\begin{aligned}
(p \to \neg q) \land \neg p &\equiv p \to (\neg q \land \neg p) && \\
(\neg p \lor \neg q) \land \neg p &\equiv p \to (\neg q \land \neg p) && \text{LHS: by } p \to q \equiv \neg p \lor q \\
(\neg p \land \neg p) \lor (\neg q \land \neg p) &\equiv p \to (\neg q \land \neg p) && \text{LHS: by Distributive Law} \\
\neg p \lor (\neg q \land \neg p) &\equiv p \to (\neg q \land \neg p) && \text{LHS: by Idempotent Law} \\
\neg p \lor \neg (q \lor p) &\equiv p \to (\neg q \land \neg p) && \text{LHS: by De Morgan's Law} \\
\neg (p \land (q \lor p)) &\equiv p \to (\neg q \land \neg p) && \text{LHS: by De Morgan's Law} \\
\neg p &\equiv p \to (\neg q \land \neg p) && \text{LHS: by Absorption Law} \\
\neg p &\equiv p \to \neg (q \lor p) && \text{RHS: by De Morgan's Law} \\
\neg p &\equiv \neg p \lor \neg (q \lor p) && \text{RHS: by } p \to q \equiv \neg p \lor q \\
\neg p &\equiv \neg (p \land (q \lor p)) && \text{RHS: by De Morgan's Law} \\
\neg p &\equiv \neg p && \text{RHS: by Absorption Law}
\end{aligned}
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(c)

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\begin{aligned}
\neg p \to (q \to r) &\equiv q \to (p \lor r) && \\
p \lor (q \to r) &\equiv q \to (p \lor r) && \text{LHS: by } p \lor q \equiv \neg p \to q \\
p \lor (\neg q \lor r) &\equiv q \to (p \lor r) && \text{LHS: by } p \to q \equiv \neg p \lor q \\
p \lor (\neg q \lor r) &\equiv \neg q \lor (p \lor r) && \text{RHS: by } p \to q \equiv \neg p \lor q \\
p \lor (\neg q \lor r) &\equiv p \lor (\neg q \lor r) && \text{RHS: by Associative Law}
\end{aligned}
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