Ch 1.3 & 1.6 (Week 8)

Propositional Equivalences

Introduction

Compound proposition

- An expression formed from propositional variables using logical operators

> 1 A compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it, is called a *tautology*. A compound proposition that is always false is called a *contradiction*. A compound proposition that is neither a tautology nor a contradiction is called a *contingency*.

Logical Equivalences

- Have the same truth values in all possible cases

> 2 The compound propositions p and q are called *logically equivalent* if $p \leftrightarrow q$ is a tautology. The notation $p \equiv q$ denotes that p and q are logically equivalent.

- $\Leftrightarrow$ is sometimes used instead of $\equiv$

Rules of Inference

Introduction

Argument : A sequence of statements that end with a conclusion.

Valid : The conclusion, or final statement of the argument, must follow from the truth of the preceding statements, or premises, of the argument.

Valid Arguments in Propositional Logic

> 1 An *argument* in propositional logic is a sequence of propositions. All but the final proposition in the argument are called *premises* and the final proposition is called the *conclusion*. An argument is *valid* if the truth of all its premises implies that the conclusion is true.
> An *argument form* in propositional logic is a sequence of compound propositions involving propositional variables. An argument form is *valid* no matter which particular propositions are substituted for the propositional variables in its premises, the conclusion is true if the premises are all true.

Rules of Inference for Propositional Logic

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \rightarrow q$ $\therefore q$	$(p \land (p \rightarrow q)) \rightarrow q$	Modus ponens
$\neg q$ $p \rightarrow q$ $\therefore \neg p$	$(\neg q \land (p \rightarrow q)) \rightarrow \neg p$	Modus tollens
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	$((p \rightarrow q) \land (q \rightarrow r)) \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$((p \lor q) \land \neg p) \rightarrow q$	Disjunctive syllogism
p $\therefore p \lor q$	$p \rightarrow (p \lor q)$	Addition
$p \land q$ $\therefore p$	$(p \land q) \rightarrow p$	Simplification
p q $\therefore p \land q$	$((p) \land (q)) \rightarrow (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$((p \lor q) \land (\neg p \lor r)) \rightarrow (q \lor r)$	Resolution

▼ Example 3

"It is below freezing now. Therefore, it is either below freezing or raining now."

Solution:
p is "It is below freezing now", and q is "It is raining now."

$$\frac{p}{\therefore\ p \lor q}$$

This is an argument that uses the *addition rule*.

▼ Example 4

"It is below freezing and raining now. Therefore, it is below freezing now."

Solution:
p is "It is below freezing now", and q is "It is raining now."

$$\frac{p \land q}{\therefore\ p}$$

This argument uses the *simplification rule*.

▼ Example 5

"If is rains today, then we will not have a barbecue today. If we do not have a barbecue today, then we will have a barbecue tomorrow. Therefore, if it rains today, we will have a barbecue tomorrow."

Solution:
p is "It is raining today," q is "We will not have a barbecue today," and r is "We will have a barbecue tomorrow."

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore\ p \rightarrow r \end{array}$$

Hence, this argument is a *hypothetical syllogism*.

Using Rules of Inference to Build Arguments

▼ Example 7

"If you send me an email message, then I will finish writing the program," "If you do not send me an email message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed."

Solution:
p is "You sent me an email message", q is "I will finish writing the program", r is "I will go to sleep early", and s is "I will wake up feeling refreshed".
The premises are $p \rightarrow q$, $\neg p \rightarrow r$, and $r \rightarrow s$. The desired conclusion is $\neg q \rightarrow s$.

Step	**Reason**
1. $p \rightarrow q$	Premise
2. $\neg q \rightarrow \neg p$	Contrapositive of (1)
3. $\neg p \rightarrow r$	Premise
4. $\neg q \rightarrow r$	Hypothetical syllogism using (2) and (3)
5. $r \rightarrow s$	Premise
6. $\neg q \rightarrow s$	Hypothetical syllogism using (4) and (5)

Resolution

Resolution → $((p \lor q) \land (\neg p \lor r)) \rightarrow (q \lor r)$

Resolvent → $(q \lor r)$

Clauses → A disjunction of variables or negations of these variables.

Fallacies

$((p \rightarrow q) \wedge q) \rightarrow p$ is not a tautology ⇒ fallacy of affirming the conclusion

$((p \rightarrow q) \wedge \neg p) \rightarrow \neg q$ is not a tautology ⇒ fallacy of denying the hypothesis

Rules of Inference for Quantified Statements

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	*Name*
$\forall x P(x)$ $\therefore P(c)$	Universal instantiation
$P(c)$ for an arbitrary c $\therefore \forall x P(x)$	Universal generalization
$\exists x P(x)$ $\therefore P(c)$ for some element c	Existential instantiation
$P(c)$ for some element c $\therefore \exists x P(x)$	Existential generalization

Universal instantiation

$P(c)$ is true, where c is a particular member of the domain, given the premise $\forall x P(x)$.

Universal generalization

$\forall x P(x)$ is true, given the premise that $P(c)$ is true for all elements c in the domain.

Existential instantiation

There is an element c in the domain for which $P(c)$ is true if we know that $\exists x P(x)$ is true.

Existential generalization

$\exists x P(x)$ is true when a particular element c with $P(c)$ true is known.

Combining Rules of Inference for Propositions and Quantified Statements

Universal modus ponens ⇒ universal instantiation + modus penens

if $\forall x(P(x) \rightarrow Q(x))$ is true, and if $P(a)$ is true for a particular element a in the domain of the universal quantifier, then $Q(a)$ must also be true.

Universal modus tollens ⇒ univesal instantiation + modus tollens

if $\forall x(P(x) \rightarrow Q(x))$ is true, and $\neg Q(a)$ is true where a is a particular element in the domain, then $\neg P(a)$ must be true.