Ch6.3 Permutations and Combinations

Permutations

1 **THEOREM 1** If n is a positive integer and r is an integer with $1 \le r \le n$, then there are $P(n,r) = n(n-1)(n-2)\cdots(n-r+1)$ r-permutations of a set with n distinct elements.

1 **COROLLARY 1** If n and r are integers with $0 \le r \le n$, then $P(n,r) = \dfrac{n!}{(n-r)!}$.

▼ Example 5

Suppose there are 8 runners in a race. The winner receives a gold medal, the second-place finisher receives a silver medal, and the third-place finisher receives a bronze medal. How many different ways are there to award these medals, if all possible outcomes of the race can occur and there ar

Combinations

2 **THEOREM 2** The number of r-combinations of a set with n elements, where n is a nonnegative integer and r is an integer with is an integer with $0 \le r \le n$, equals $C(n,r) = \dfrac{n!}{r!\,(n-r)!}$.

2 **COROLLARY 2** Let n and r be nonnegative integers with $r \le n$. Then $C(n,r) = C(n, n-r)$.

▼ Example 11

How many poker hands of give cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select 47 cards from a standard deck of 52 cards?

Solution: Because the order in which the five cards are dealt from a deck of 52 cards does not matter, there are $C(52,5) = \dfrac{52!}{5!47!} = \dfrac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 26 \cdot 17 \cdot 10 \cdot 49 \cdot 12 = 2,598,960$ different hands of five cards that can be dealt. Consequently, there are $C(52,47) = \dfrac{52!}{47!5!} = 2,598,960$ different poker hands of give cards that can be dealt from a standard deck of 52 cards.

1 A *combinatorial proof* of an identity is a proof that uses counting arguments to prove that both sides of the identity count the same objects but in different ways or a proof that is based on showing that there is a bijection between the sets of objects counted by the two sides of the identity. These two types of proofs are called *double counting proofs* and *bijective proofs*, respectively.