UNIVERSITY OF LONDON

CM1025

BSc EXAMINATION

COMPUTER SCIENCE

Fundamentals of Computer Science

Release date: Tuesday 7 September 2021 at 12:00 midday British Summer Time

Submission date: Wednesday 8 September 2021 by 12:00 midday British Summer Time

Time allowed: 24 hours to submit

INSTRUCTIONS TO CANDIDATES:

Section A of this assessment consists of a set of **10** Multiple Choice Questions (MCQs) which you will take separately from this paper. You should attempt to answer **ALL** the questions in Section A. The maximum mark for Section A is **20**.

Section A will be completed online on the VLE. You may choose to access the MCQs at any time following the release of the paper, but once you have accessed the MCQs you must submit your answers before the deadline or within **4 hours** of starting whichever occurs first.

Section B of this assessment is an online assessment to be completed within the same 24-hour window as Section A. We anticipate that approximately **1 hour** is sufficient for you to answer Section B. Candidates must answer **TWO** out of the THREE questions in Section B. The maximum mark for Section B is **80**.

Calculators are not permitted in this examination. Credit will only be given if all workings are shown.

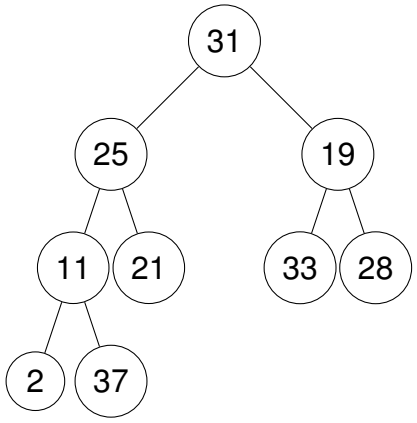
You should complete Section B of this paper and submit your answers as **one document**, if possible, in Microsoft Word or a PDF to the appropriate area on the VLE. You are permitted to upload 30 documents. However, we advise you to upload as few documents as possible. Each file uploaded must be accompanied by a coversheet containing your **candidate number**. In addition, your answers must have your candidate number written clearly at the top of the page before you upload your work. Do not write your name anywhere in your answers.

SECTION B

Candidates should answer any **TWO** questions from Section B.

Question 2

(a) Heapify the following tree, make every step clear. (Min heap) [6]



(b) Given $R = 1^*0^+1^+0^+1\Sigma^*$ and $S = (1^+0^*)^*1$ where $\Sigma = \{0, 1\}$

- Give an example of a string that is neither in the language of R nor in S. [2]
- Give an example of a string that is in the language of S but not R. [2]
- Give an example of a string that is in the language of R but not S. [2]
- Give an example of a string that is in the language of R and S. [2]
- Design a regular expression that accepts the language of all binary strings with no occurrences of 1001 [4]

(c) Answer the following for the context-free grammar

$$G:$$

$$S \to 0S1S|1S0S|\epsilon$$

- Give two non-empty strings that can be generated from G, show the derivations. [4]
- Give two strings that cannot be generated from the context-free grammar G. [2]
- Can 110 be generated by G? Justify your reasoning. [2]
- What is the language of G? [3]

(d) Use mathematical induction to prove that for all natural numbers $n > 3$, $2^n < n!$. State every step of the proof. [6]

(e) State the contrapositive and use it to prove that the following statement is true. If $n^2 + n - 1$ is divisible by 3 then n is divisible by 3. [5]

Question 3

(a) Write the negation of the following, state every rule you use:

- $\neg p \wedge q \vee p$ [4]
- $\forall x(P(x) \rightarrow Q(x))$ [4]

(b) Each student has a password, which is 5 characters long and each character is either a digit or a lowercase letter. Each password must have more letters than digits. How many possible passwords are there? [6]

(c) Using the Master theorem write the time complexity of the following, make every step clear.

- $T(n) = 16T(n/2) + O(1)$ [4]
- $T(n) = 9T(n/3) + O(n^2)$ [4]

(d) Design a context-free grammar that accepts the language of all binary strings with exactly two a's and at least one b. [5]

(e) Give a finite automaton that accepts all binary words that contains exactly two 1's and at least one 0. [7]

(f) Assume $S = \{$ NKem(N), Elena(E), Fatima(F)$\}$and $H = \{$Whittington(W), Royal Free(R), Highgate(H)$\}$. The list of the preference is as follows. Find the stable match using the Gale-Shapley algorithm. Show the steps of the algorithms clearly. [6]

Nkem	H	R	W
Elena	W	R	H
Fatima	W	H	R

Whittington (W)	N	F	E
Royal Free(R)	F	N	E
Highgate(H)	F	E	N

Question 4

(a) Given $\Sigma_1 = \{a, b\}$, and $\Sigma_2 = \{1, 2, 3\}$

- What is the cardinality of Σ_2^3? [1]
- List all strings of Σ_1^3. [3]
- List three strings of $\Sigma_2^2 \circ \Sigma_1^3$. [3]

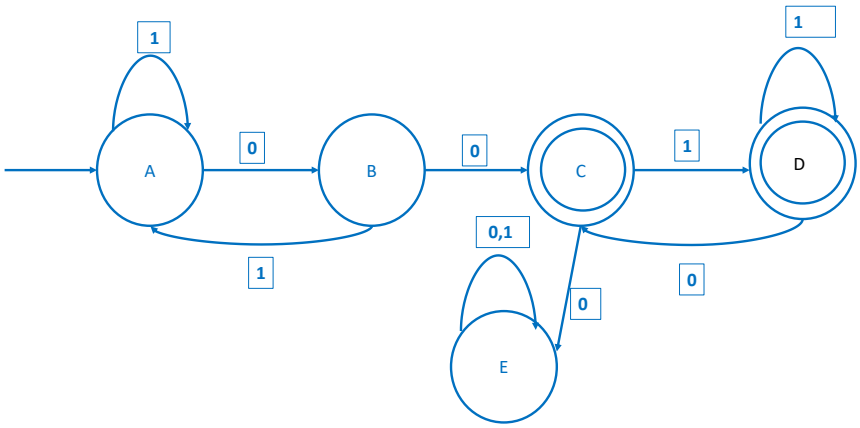
(b) A coin is flipped six times where each flip comes up either heads or tails. In how many possible outcomes are the number of heads and tails are not equal? [5]

(c) Write the paths representing the parsing of the following input by the automaton depicted below, state if the input is accepted or rejected. [2]

i. 1101

ii. 010010

- Describe the language of this automaton in plain English. [3]
- Describe the language of this automaton using Regular expression. [3]



(d) Use the merge sort to sort the following list in ascending order. Show your work step by step. [6]

4 17 15 10 6 2 9 3 11

(e) Write the asymptotic functions of the following. Prove your claim: if you claim $f(n) = O(g(n))$ you need to show there exist c, k such that $f(x) \leq c \cdot g(x)$ for all $x > k$.

- $h(n) = 3n + 7n \log n$ [4]
- $l(n) = 4n + n^2$ [4]

(f) Design a Turing Machine that accepts all binary words in the form of a^*b^*. [6]

END OF PAPER

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