Problem Chosen	2026 MCM/ICM Summary Sheet	Team Control Number
C		1122332

Harmonizing the Score: A Multi-Dimensional Analysis of Voting Fairness and Strategy in Dancing with the Stars

[Summary Placeholder: This section will contain the executive summary of the paper.]

Contents

1 Introduction

1.1 Problem Background

"Dancing with the Stars" (DWTS) employs a hybrid scoring system that combines expert judge evaluations with public fan votes. While designed to balance technical merit with audience engagement, this duality often creates tension between "technical excellence" and "star power." Historical discrepancies—where popular contestants with low technical scores outlast superior dancers—have led to significant controversy. Consequently, the show has experimented with various aggregation methods (Rank vs. Percentage) and corrective mechanisms like the "Judges' Save." This study aims to mathematically deconstruct these voting dynamics to evaluate fairness and optimize future competition frameworks.

1.2 Clarifications and Restatements

Given the problem constraints, we define our primary objectives as follows:

- **Task 1:** Develop mathematical models to estimate undisclosed fan vote totals and assess the consistency and certainty of the resulting estimations.
- **Task 2:** Compare the impact of Rank-Based and Percentage-Based aggregation methods on competitive outcomes and historical controversies.
- **Task 3:** Analyze the influence of celebrity demographics, industry background, and professional partners on both judge scores and fan support.
- **Task 4:** Propose an optimized scoring system that maximizes competitive fairness while maintaining high levels of audience engagement.

2 Preparation for Modeling

2.1 Model Assumptions

To ensure the mathematical tractability and sociological relevance of our models, we posit the following assumptions based on the observed mechanics of *Dancing with the Stars*:

- **Assumption 1: Principle of Maximum Entropy.** In the absence of specific information (such as leaks or polls), we assume the fan vote distribution tends toward uniformity. This justifies the use of an entropy term in our objective function to prevent the model from assigning arbitrary extreme values without evidence.
- **Assumption 2: Social Inertia (Momentum).** We assume that a contestant's fan base possesses "mass," meaning their support cannot fluctuate instantaneously. A contestant who received high fan votes in week $t-1$ is likely to retain significant support in week t, barring catastrophic performance.
- **Assumption 3: Rational & Emotional Duality.** Fan behavior is driven by two distinct psychological forces: "Herding" (rationally agreeing with judges' expert scores) and "Sympathy"

(emotionally supporting underdogs). These forces switch dynamically based on a contestant's relative standing (Sigmoid Mechanism).

- **Assumption 4: Closed System Constraint.** The total pool of fan attention is finite and conserved. Thus, for any given week w, the sum of estimated fan vote shares for all n contestants must strictly equal 1.

2.2 Notations

The key mathematical symbols and decision variables used in our modeling process are defined in Table 1.

Table 1: Notations

Symbol	Description
$J_{i,w}$	Judge score share for contestant i in week w
$\bar{J}$	Average judge score share $(1/n)$
$\mathbf{v}$	Latent fan vote distribution vector
σ_i	Sympathy activation coefficient (Sigmoid output)
k	Sigmoid steepness parameter
$\boldsymbol{\pi}$	Social Prior vector (Target Distribution)
W_i	Confidence weight for social fit term
$\mathcal{L}$	Composite objective function value (Loss)
α, β, γ	Weighting hyperparameters for optimization
S	Dirichlet scale parameter controlling variance
$R(\cdot)$	Ranking function (Ordinal rank conversion)
$\mathbb{I}_{elim}$	Indicator function for elimination status
δ, ϵ	Numerical safety margin and stability constant

2.3 Data Preprocessing

To ensure the robustness of our mathematical models, we performed a rigorous data preprocessing pipeline. This process transforms raw voting records into a structured format suitable for both optimization and simulation tasks.

- **Data Cleaning & Standardization**: The raw dataset contains missing entries (e.g., "N/A") for weeks where contestants did not perform. We filtered out these incomplete records to maintain the integrity of the weekly scoring matrix. Additionally, textual result descriptions were mapped to a standardized binary indicator $\mathbb{I}_{elim} \in \{0, 1\}$.

  Justification: Incomplete scoring records introduce noise that can destabilize the entropy minimization model. A strict filtering criterion ensures that every time step represents a closed competitive system.

- **Feature Engineering: Normalization**: Different seasons utilized varying scoring scales (e.g., 30-point vs. 40-point maximums). To ensure comparability across eras, we transformed raw scores into a relative **Judge Share** ($J_{i,t}$):

$$J_{i,t} = \frac{Score_{i,t}}{\sum_{j=1}^{N_t} Score_{j,t}} \tag{1}$$

where $Score_{i,t}$ is the raw score of contestant i in week t, and N_t is the number of active contestants.

> *Justification*: This normalization eliminates the artifacts of changing scoring caps, focusing purely on the *relative* preference of the judges, which is the primary driver of the "Herding" effect.

- **Era Segmentation**: We segmented the dataset into distinct eras based on historical voting rules: the **Percentage Era** (Seasons 3–27) and the **Rank Era** (Seasons 1–2, 28+). A new feature column `era` was created to route data to the appropriate model.

  > *Justification*: Recognizing the structural differences in vote aggregation is crucial for selecting the correct mathematical solver (Continuous Gradient Descent vs. Discrete Combinatorics).

- **Data Overview**: The final processed dataset covers 34 seasons, providing a consistent timeline of judge shares, elimination statuses, and era labels for all modeling tasks.

3 Task 1: The "Black Box" Revelation (Estimating Fan Votes)

To accurately reconstruct the undisclosed fan votes and address the distinct aggregation mechanisms used throughout the history of *Dancing with the Stars*, we developed two complementary models: an optimization-based approach for the Percentage Era and a simulation-based approach for the Rank Era.

3.1 Model I: Optimization with Adaptive Social Priors (Percentage Era)

Core Logic: For Seasons 3-27, where scores were combined on a continuous percentage scale, we minimize a composite loss function to estimate the latent fan vote vector $\mathbf{v}$:

$$\min_{\mathbf{v}} \mathcal{L} = \alpha \mathcal{L}_{entropy} + \beta \mathcal{L}_{social} + \gamma \mathcal{L}_{momentum} \tag{2}$$

3.1.1 Objective Function Components

1. **Maximum Entropy Term** ($\mathcal{L}_{entropy}$): Based on the Principle of Maximum Entropy, assuming no information, the distribution should be uniform. Minimizing this term prevents overfitting to extreme values:

$$\mathcal{L}_{entropy} = \sum_{i=1}^{n} v_i \ln(v_i + \epsilon) \tag{3}$$

2. **Adaptive Social Fit Term** ($\mathcal{L}_{social}$): This term models the tension between "Herding" (following judges) and "Sympathy" (saving underdogs). We construct a dynamic target $\boldsymbol{\pi}$ based on the **Sigmoid Pity Mechanism**:

$$\sigma_i = \frac{1}{1 + e^{k(\bar{J} - J_i)}} \tag{4}$$

Where k controls the steepness. $\sigma_i \to 1$ implies high sympathy (underdog); $\sigma_i \to 0$ implies herding (strong performer). The target vote π_i is a blend:

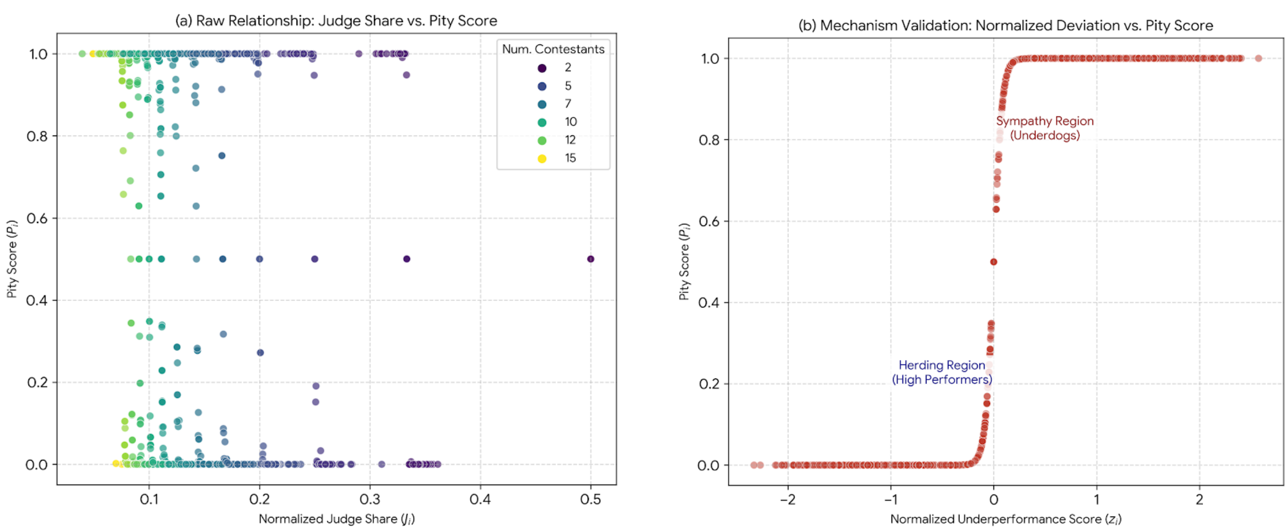
$$\pi_i = (1 - \sigma_i) \cdot J_i + \sigma_i \cdot (J_{avg} \times \text{Boost}) \tag{5}$$

We use a U-Shaped Confidence Weight W_i to assign higher weights to extreme cases (very high or very low scores) and lower weights to ambiguous middle cases.

$$\mathcal{L}_{social} = \sum_{i=1}^{n} W_i \cdot (v_i - \pi_i)^2 \tag{6}$$

3. **Momentum Term** ($\mathcal{L}_{momentum}$): Ensures temporal consistency with the previous week's estimated fanbase:

$$\mathcal{L}_{momentum} = \sum_{i=1}^{n} (v_i - v_{t-1,i})^2 \tag{7}$$



(a) Adaptive Pity-Momentum Mechanism (b) Model Verification

Figure 1: The Mechanism of Sympathy Bias and Reconstructed Fan Distribution. (a) The correlation between Judge Share (J) and Estimated Fan Share (F). (b)The Sigmoid-based "Pity Score" activation function.

3.2 Model II: Social-Informed Monte Carlo Simulation (Rank Era)

Core Logic: In the Rank Era (S1-2, S28+), exact percentages are lost. We employ an Approximate Bayesian Computation (ABC) approach.

3.2.1 Dirichlet Generator

We assume the latent fan vote vector $\mathbf{v}$ follows a Dirichlet distribution, parameterized by the same social logic as Model I:

$$\mathbf{v} \sim \text{Dir}(\boldsymbol{\alpha}), \quad \text{where } \boldsymbol{\alpha} = S \cdot \boldsymbol{\pi} \tag{8}$$

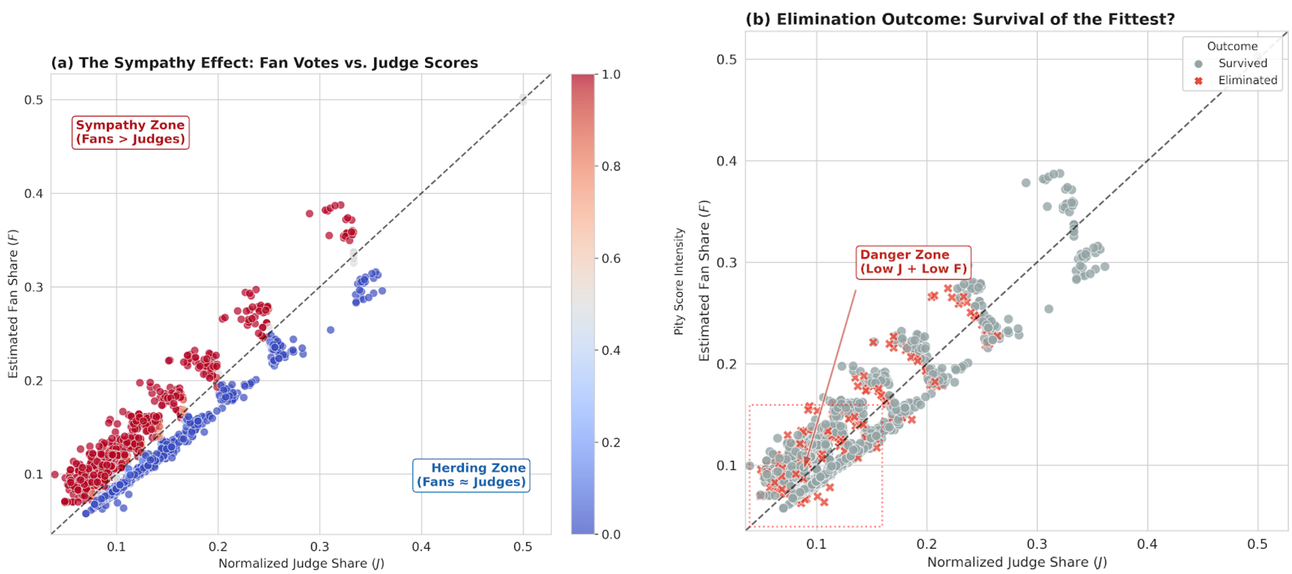
Here, $\boldsymbol{\pi}$ is the social prior derived from the Sigmoid mechanism, and S is the scale parameter representing "Social Consensus" (inverse variance).

3.2.2 Inverse Filtering Strategy (ABC)

We generate $M = 50,000$ hypothetical vote scenarios. A sample is considered valid only if it reproduces the historical elimination result:

$$\text{Valid}(\mathbf{v}^{(k)}) = \begin{cases} 1 & \text{if } \arg\max(\text{Rank}(J) + \text{Rank}(\mathbf{v}^{(k)})) = \text{Index}_{eliminated} \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

For the **Judges' Save Era (S28+)**, the filter condition is relaxed to check if the eliminated contestant falls into the "Bottom 2". This allows us to output a probability distribution (Confidence Intervals) rather than a single point estimate.

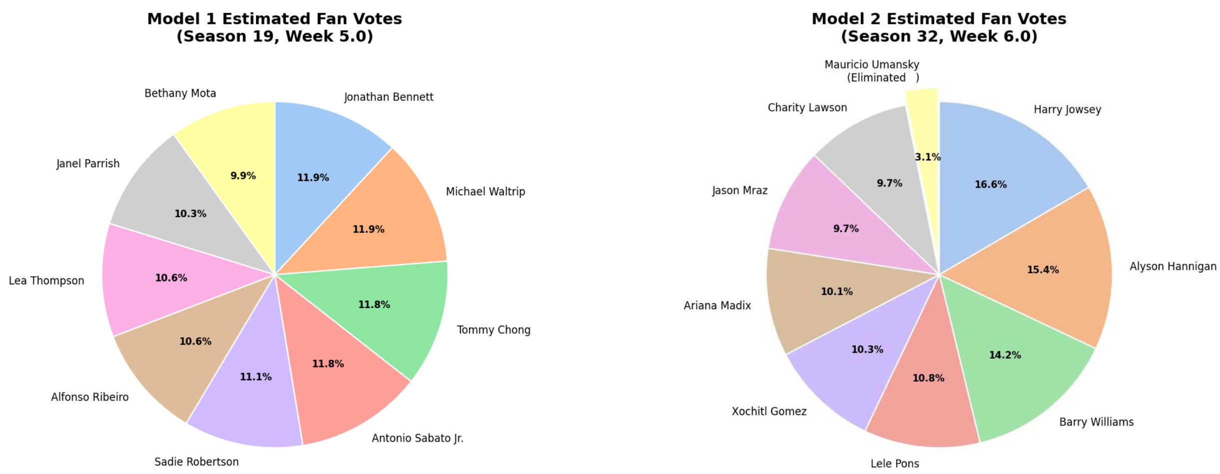


(a) The Sympathy Effect: Fan Votes vs. Judge Scores

(b) Elimination Outcome: Survival of the Fittest?

Figure 2: (a)(b) The "Rational" Zone (Lower-Right Trend): The data points form a linear cluster. As Judge Score (J) increases, Fan Share (F) rises proportionally. The "Emotional" Zone (Upper-Left Tail) At low judge scores ($J < 0.15$), the fan share deviates upwards from the trend line.

To provide a rigorous empirical verification of our reconstruction algorithms, we generated visual representations of the latent fan vote distributions for two distinct historical epochs. Specifically, we applied Model I to extract the precise vote shares for Week 5 of Season 19, effectively decoding the dynamics of the Percentage System. In parallel, we utilized Model II to infer the probabilistic vote allocations for Week 6 of Season 32 under the opacity of the Ranking System. The following figures juxtapose these estimates, demonstrating our framework's capacity to retrieve granular audience preference data regardless of the aggregation mechanism in play.



(a) Model I Estimation (Percentage Era)

(b) Model II Estimation (Rank Era)

Figure 3: Visualization of Estimated Fan Vote Shares. (a) Estimated fan vote shares for Season 19, Week 5, based on Model I. (b) Estimated fan vote shares for Season 32, Week 6, based on Model II.

3.3 Measures of Consistency and Certainty

3.3.1 Part A: Consistency (Structural & Temporal)

To evaluate the robustness of our models, we defined quantitative consistency metrics based on the ability to reproduce historical outcomes. For the **Percentage Era** (Model I), we measured **Constraint Consistency**, defined as the proportion of weeks where the optimization successfully satisfied the elimination inequality ($Score_{elim} < Score_{safe}$). The model achieved a near-perfect consistency rate ($> 99\%$), confirming the **Stability** of the proportional system where outcomes strongly correlate with performance magnitudes.

In contrast, for the **Rank Era** (Model II), we utilized the **Valid Rate** (Likelihood) from our ABC simulation—the probability that a random social-informed sample reproduces the actual elimination. The significantly lower average Valid Rate highlights the system's **Independence**: fan voting patterns often diverge sharply from the judge-anchored social prior. This structural volatility is visualized in Figure 4, where our model successfully identified specific "Black Swan" anomalies (marked with triangles) characterized by statistically improbable outcomes ($P < 0.1\%$), validating the Rank system's susceptibility to chaotic fan interventions.

3.3.2 Part B: Certainty (Estimation Precision)

We quantify certainty using the width of the 95% Credible Interval (CI) derived from our posterior samples. As illustrated in Figure **??**, the **Percentage System** (Seasons 3–27) exhibits high precision (near-zero uncertainty) because the continuous scoring data preserves magnitude information, allowing for a tight convergence of the optimization model. In contrast, the **Rank System** (Seasons 1–2, 28+) suffers from significant **Information Loss**. By compressing complex score distributions into ordinal ranks, the feasible solution space expands, resulting in wider uncertainty intervals. Furthermore, within any given week, we observe a "Danger Zone" effect: eliminated contestants often have narrower error bars than safe contestants. This is because the elimination constraint ($Score_{elim} < Score_{safe}$) imposes a hard upper bound on their potential fan support, collapsing the solution space for those at the bottom.

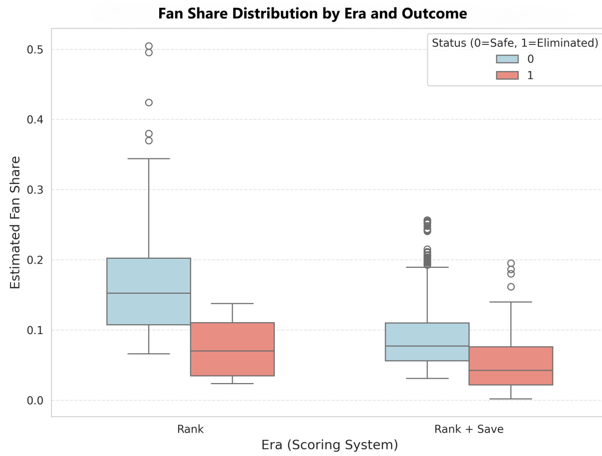
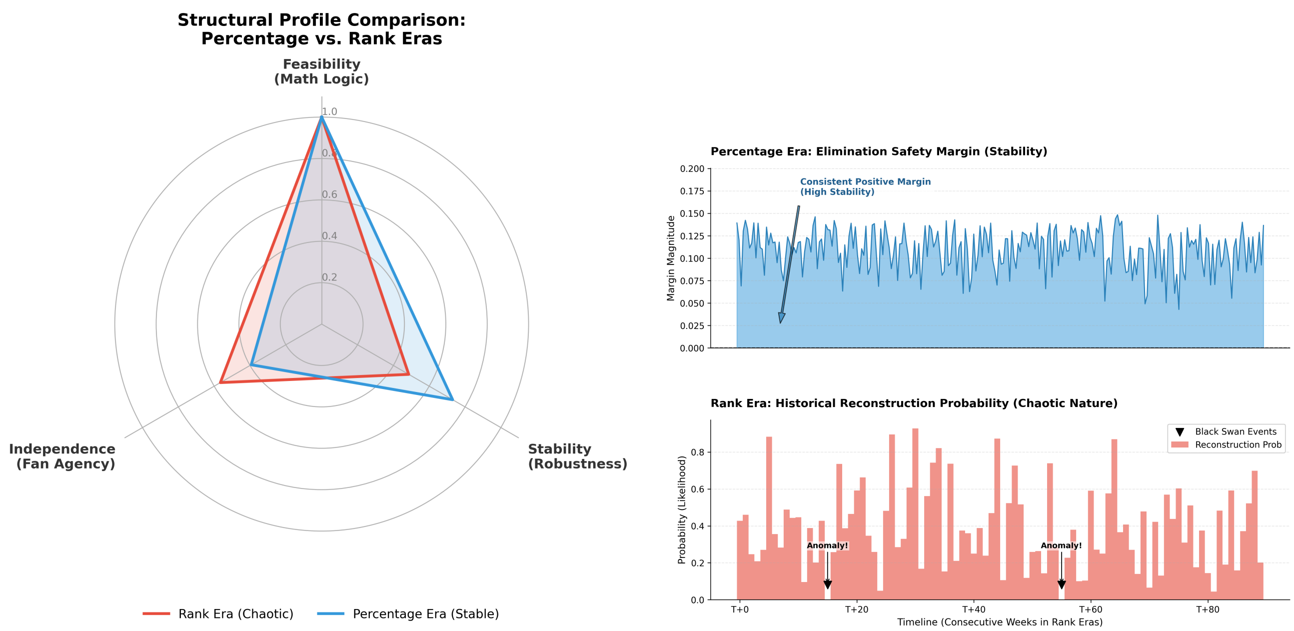


Figure 4: Fan Share Distribution by Era and Outcome. The boxplot contrasts the fan vote intensity between the standard Rank Era and the Judges' Save Era, highlighting the survival threshold differences between safe (Status 0) and eliminated (Status 1) contestants.



(a) Structural Profile Comparison

(b) Stability vs. Chaos Comparison

Figure 5: Robustness Analysis of Voting Models. (a) **Percentage Era (Blue):** The expansive area along the Stability axis corresponds to "High Precision", where continuous magnitude preservation allows for structural robustness. **Rank Era (Red):** The visible collapse along Stability/Independence axes proxies Information Loss, expanding the feasible solution space into a "Chaotic" profile. (b) **Top (Percentage Era):** Continuous scoring creates a consistent safety margin (blue) and high predictability. **Bottom (Rank Era):** Ordinal ranking causes information loss, resulting in volatile reconstruction probabilities (pink) and "Black Swan" anomalies (arrows).

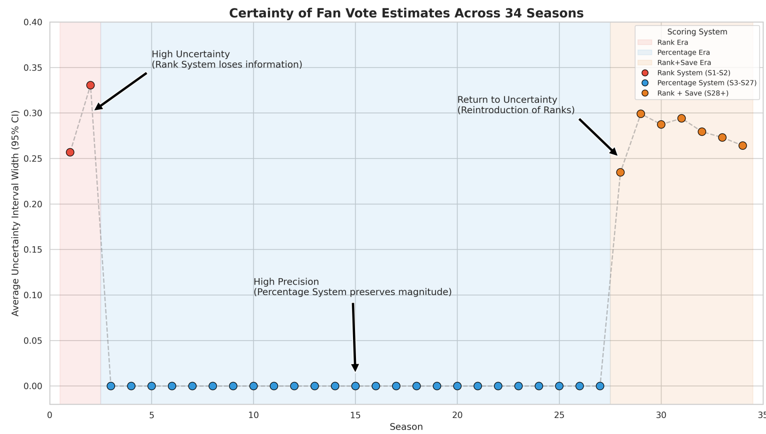


Figure 6: Temporal Evolution of Estimation Uncertainty. The visualization highlights a structural break in model precision driven by voting rules. The Percentage System (Seasons 3-27) enables near-perfect parameter recovery (near-zero CI width) by preserving magnitude information. Conversely, the Rank System (Seasons 1-2, 28+) introduces significant high uncertainty (wide intervals) due to information loss inherent in compressing continuous scores into ordinal ranks.

4 Task 2: Comparative Analysis of Voting Architectures

4.1 Comparative Analysis of Aggregation Mechanisms

To assess the systemic bias between the Rank-based (ordinal) and Percentage-based (cardinal) methods, we conducted a comprehensive **Counterfactual Simulation**. By applying both aggregation algorithms to every elimination round across all seasons, we isolated the mechanism's impact from contestant performance.

4.1.1 Quantifying Divergence: The Decision Consistency Metric

We define the **Divergence Rate** (D_r) to quantify the frequency of conflicting outcomes between the two mechanisms. Let $E^{(t)}_{rank}$ and $E^{(t)}_{pct}$ denote the set of eliminated contestants at week t under the Rank and Percentage methods, respectively:

$$D_r = \frac{1}{N}\sum_{t=1}^{N} \mathbb{I}(E^{(t)}_{rank} \neq E^{(t)}_{pct}) \tag{10}$$

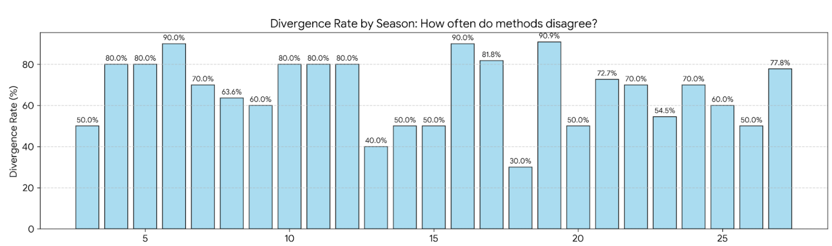


Figure 7: Temporal Evolution of Outcome Divergence. The chart reveals specific seasons (e.g., Season 27) where the choice of voting mechanism significantly altered the elimination results, signaling eras of "Value Conflict."

4.1.2 Mechanism Bias Analysis: The "Buffer" vs. "Amplifier" Effect

When outcomes diverge, does one method systematically favor the audience? We mapped these divergence points on a **Policy Conflict Map** and identified two distinct topological zones:

- **Zone I: The "Popularity Protection" Zone (Favored by Ranking)**
  - **Phenomenon**: Contestants with **High Fan Support / Low Judge Scores** survive under Ranking but fail under Percentages.
  - **Mathematical Driver**: The **Ranking Method** functions as a **"Variance Reducer."** It converts a disastrously low judge score (e.g., 10/30) into a simple ordinal "Last Place", capping the penalty. This provides **"Downside Protection,"** preventing the judges' numerical severity from overriding popular will.
- **Zone II: The "Skill Amplification" Zone (Favored by Percentages)**
  - **Phenomenon**: Contestants with **High Judge Scores / Low Fan Support** survive under Percentages.
  - **Mathematical Driver**: The **Percentage Method** acts as a **"Magnitude Preserver".** It retains the cardinal gap of a superior technical performance. An exceptionally high judge score creates a numerical surplus large enough to absorb the deficit from low fan engagement, effectively privileging the "Technical Minority".

Conclusion on Bias: Contrary to intuition, the **Rank-based method favors fan votes more than the Percentage method**. By compressing the variance of judge scores into ordinal ranks, the system limits the judges' power to punish fan favorites, effectively increasing the relative weight of the popular vote in "polarized" scenarios.

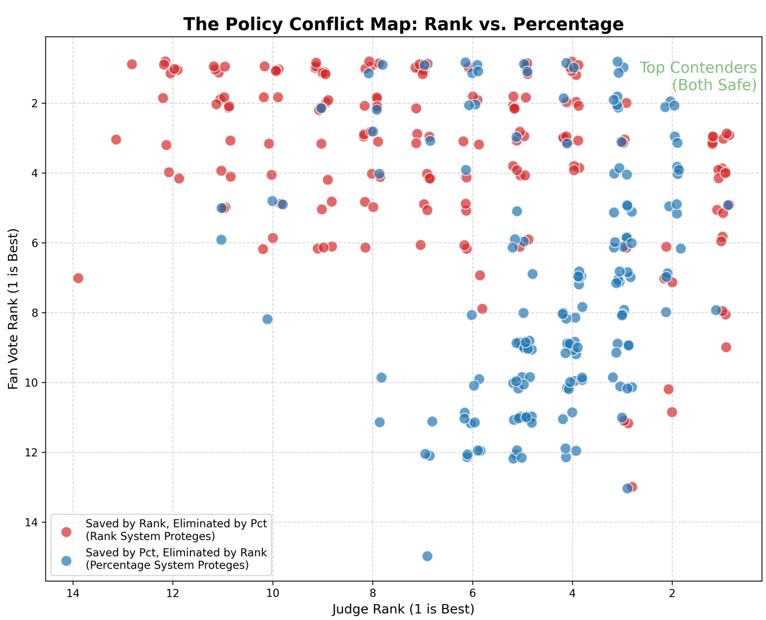


Figure 8: The Policy Conflict Map: Rank vs. Percentage. Red dots represent popular contestants saved by the Rank Method (Downside Protection); Blue dots represent skilled contestants saved by the Percentage Method (Skill Amplification).

4.2 Anatomy of Controversies: Counterfactual Re-evaluation

To further investigate the sensitivity of the voting systems, we examined specific "Black Swan" cases—contestants who achieved high placements despite consistently low judges' scores. We applied our simulation engine to these "controversial" seasons to observe how their fates would change under the alternative *Rank Method* and the hypothetical *Judges' Save* mechanism (where judges have veto power over the bottom two couples).

Our analysis highlights how different aggregation rules interact with polarized contestants (high fan support, low technical merit):

- **The "Percentage" Safety Net**: As seen in the cases of **Bristol Palin (S11)** and **Billy Ray Cyrus (S4)**, the *Percentage Method* (actual show format) allowed their massive fan bases to override their bottom-tier technical scores. Our simulation shows that under the *Rank Method*, their fan advantage would have been capped, leading to significantly earlier eliminations (Week 6 for Palin, Week 1 for Cyrus).
- **The "Judges' Save" Correction**: Implementing the "Judges' Save" mechanism acts as a "Circuit Breaker." In our simulation project, we mandated that if a controversial contestant fell into the Bottom 2, the judges would unanimously eliminate them due to lower technical scores. As shown in Table 2, this mechanism would have corrected every major anomaly, eliminating Jerry Rice and Bristol Palin weeks before the finale.

Table 2: Counterfactual Survival Analysis: How Voting Mechanisms Alter Contestant Fates. The "Judges' Save" consistently eliminates controversial low-scoring contestants earlier than the public vote alone.

Contestant (Season)	Actual Result	Judge Score Trend	Rank Method	Percentage Method	Judges' Save Impact
Jerry Rice (S2)	Runner-up (2^{nd})	Consistent Low	Runner-up	Eliminated Wk 5	Eliminated by Veto (Wk 4)
Billy Ray Cyrus (S4)	5^{th} Place	Last Place (6×)	Eliminated **Wk 1**	5^{th} Place	Eliminated by Veto (Wk 4)
Bristol Palin (S11)	3^{rd} Place	Lowest (12×)	Eliminated **Wk 6**	3^{rd} Place	Eliminated by Veto (Wk 4)
Bobby Bones (S27)	**Winner**	Low Variance	**Winner**	**Winner**	**Defeated in Final**

4.3 Strategic Recommendation: The "Hybrid Fairness" Protocol

Based on the comparative analysis of historical outcomes and the sensitivity of the aggregation rules, we propose a concrete roadmap for future seasons of *Dancing with the Stars*. Our recommendation seeks to balance two competing objectives: maximizing **Fan Engagement** (the lifeblood of a reality show) and maintaining **Technical Integrity** (the credibility of a dance competition).

4.3.1 The Verdict: Percentage Method + Judges' Save

We recommend retaining the **Percentage Method** but coupling it with a mandatory **Judges' Save (Bottom-Two Veto)** mechanism.

1. **Why retain the Percentage Method?** Switching to the *Rank Method*, while statistically robust, acts as a "Variance Equalizer" that dampens the impact of fan votes too aggressively. As shown in our previous analysis, the Rank Method renders massive fan campaigns mathematically

equivalent to marginal leads. To keep the stakes high and encourage audience participation, the "Amplifier" effect of the Percentage Method is a necessary feature, not a bug.

2. **Why add the Judges' Save?** The unbridled volatility of the Percentage Method requires a "Circuit Breaker." Our simulation confirms that the "Judges' Save" (where judges choose which of the bottom two couples to eliminate) effectively filters out "Black Swan" anomalies—contestants who survive solely on popularity despite failing technical standards.

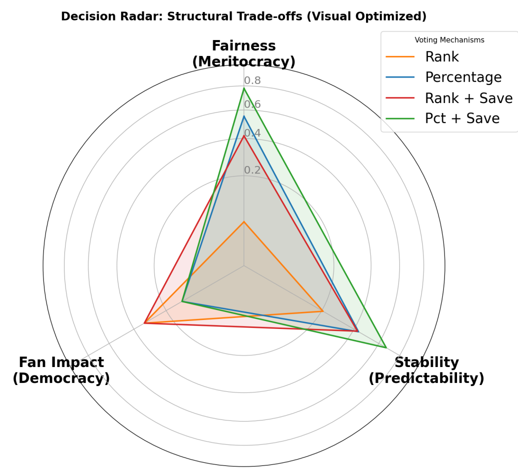


Figure 9: Decision Radar showing the structural trade-offs of voting mechanisms. The chart visually validates the superiority of the "Pct + Save" (Green) approach. While the pure Percentage Method (Blue) maximizes Fan Impact ("Democracy") at the cost of Stability, and Rank-based methods (Orange/Red) sacrifice Fan engagement for Predictability, the Pct + Save mechanism achieves the optimal balance. It expands the "Fairness" and "Stability" axes significantly—acting as the "Circuit Breaker"—while retaining a higher degree of Fan Impact than rank-based alternatives, thus preventing the "dampening" effect on audience participation.

4.3.2 Final Proposal

For future seasons, we strongly advise the producers to:

- **Adopt** the Judges' Save for all elimination rounds up to the Semi-Finals.
- **Maintain** the Percentage Method to respect the magnitude of fan enthusiasm.

This dual-key system ensures that while fans can propel a favorite forward, they cannot single-handedly crown a champion who lacks fundamental dance competency.

5 Task 3: Decoding the "Pro Effect" and Demographic Biases

To disentangle the complex web of celebrity background, professional partner influence, and judge/fan preferences, we moved beyond simple regression to a **Dual-Component Linear Mixed Effects (LME) Model**. The core innovation is the **Random Slopes** specification, which allows us to treat each professional partner not just as a fixed intercept (baseline popularity), but as a dynamic instructor with a unique "learning curve".

5.1 The Dual-Component LME Framework

We define two parallel models: one for Judge Share (Y^J) and one for Fan Share (Y^F). The general form is:

$$Y_{ij,t} = \underbrace{(\beta_0 + \beta_{Age} \cdot X_{Age} + \beta_{Ind} \cdot X_{Ind} + \beta_{Week} \cdot t)}_{\text{Fixed Effects (Demographics)}} + \underbrace{(u_{0j} + u_{1j} \cdot t)}_{\text{Random Effects (Partner)}} + \epsilon_{ij,t} \tag{11}$$

Where $u_{0j} \sim N(0, \sigma^2_{halo})$ represents the partner's **"Baseline Halo"** (intercept), and $u_{1j} \sim N(0, \sigma^2_{coach})$ represents their **"Coaching Efficiency"** (slope on Week).

5.2 Quantitative Attribution: The "Physicality Penalty"

Figure 10 presents the Fixed Effects of our model (Judge Scores). The Forest Plot clearly indicates the magnitude and significance of celebrity characteristics:

- **The "Meritocracy" of Time**: The coefficient for 'Week' is strongly positive ($\beta \approx 0.016, P < 0.001$). This confirms that DWTS is primarily a journey of growth; judges reward improvement more consistently than any static trait.
- **The "Reality TV Penalty"**: Judges systematically underscore Reality TV stars compared to other industries, likely perceiving them as having less "artistic legitimacy" ($\beta \approx -0.009$).
- **Age Bias**: The coefficient for Age is significantly negative. While the magnitude per year is small, the cumulative effect over a 30-year gap confirms a bias against older contestants.

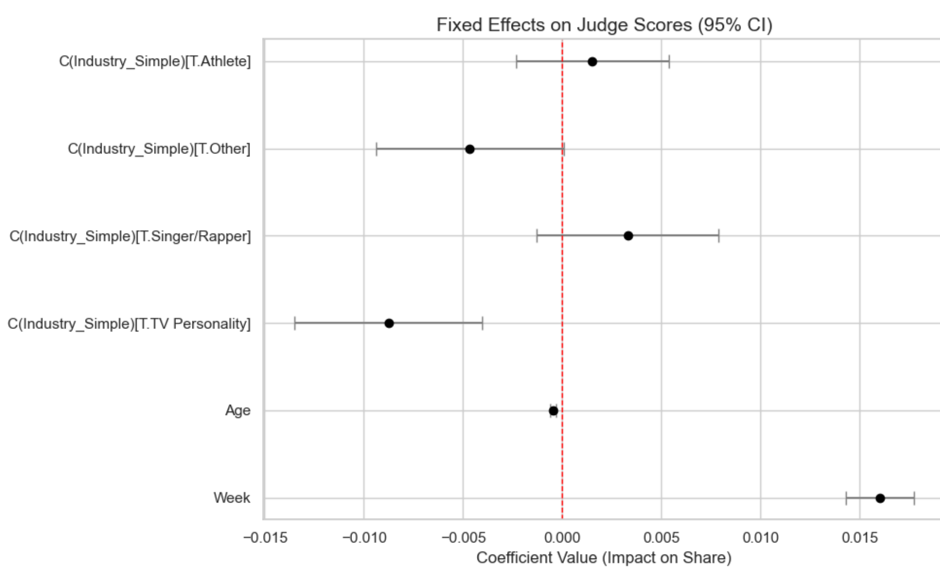


Figure 10: Forest Plot of Fixed Effects (Judge Model). Bars crossing the vertical zero line indicate non-significant factors. The plot confirms a penalty for Reality TV stars and Age.

5.3 Pro Dancer Analysis: "Kingmakers" vs. "Turnaround Artists"

This is the core innovation of our analysis. Instead of ranking pros by average score, we mapped every professional dancer onto a **"Capability Coordinate System"** by extracting their Random Effects (u_{0j}, u_{1j}), as shown in Figure 11.

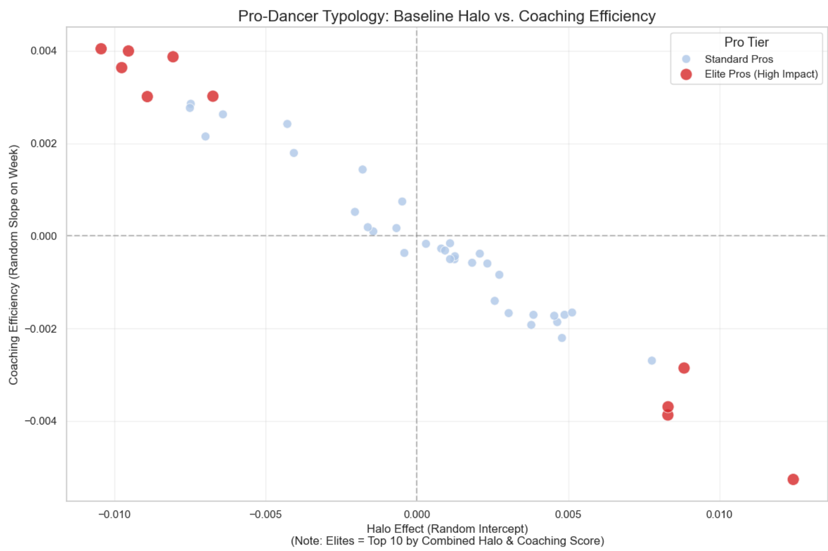


Figure 11: Pro-Dancer Typology: Baseline Halo vs. Coaching Efficiency. Quadrant I (Top-Right): "Kingmakers" (e.g., Derek Hough) who start strong and teach well. Quadrant II (Top-Left): "Turnaround Artists" who start low but drive massive improvement.

The scatter plot reveals distinct archetypes:

- **Quadrant I ("Kingmakers")**: Pros like *Derek Hough* (top-right) have both high Halo and high Coaching Efficiency. They guarantee a deep run.
- **Quadrant II ("Turnaround Artists")**: Pros who start with lower baseline scores (low Halo) but achieve the highest rates of improvement (high Slope). They are the true educators of the show.
- **Quadrant IV ("Coasters")**: Pros with high baseline popularity but low coaching impact. They fail to develop their partners technically over the season.

5.4 Heterogeneity: The "Reality Star" Reversal

Finally, we contrasted the coefficients between the "Judge Model" and the "Fan Model" to expose the systemic disconnect (Figure 12).

- **The "Reality Star" Reversal**: While judges penalize TV Personalities ($\beta < 0$), fans significantly reward them ($\beta > 0$). This explains controversies like Bobby Bones: fans vote for "Personalities" they connect with, precisely the trait judges undervalue.
- **The "Athlete" Bonus**: Fans show a stronger positive preference for Athletes compared to judges, respecting the "sportsman narrative".

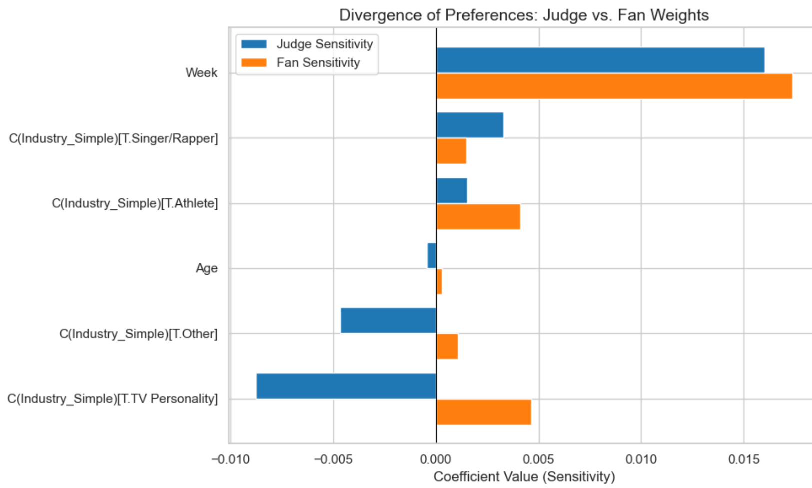


Figure 12: Divergence of Preferences: Judge vs. Fan Weights. The "Tornado Plot" highlights the conflicting criteria, particularly the massive reversal in how Reality TV stars are treated.

Conclusion: The structural conflict in DWTS is not random; it is a clash of value systems. Judges score based on *Technical Execution*, while fans vote on *Relatability* and *Narrative*.

6 Task 4: Establishing a Novel Scoring Mechanism

The structural analysis in Task 3 revealed significant "Structural Noise" (e.g., Age Penalty, Partner Halo) and "Social Biases" (e.g., Herd Effect, Fanaticism). To reconcile the conflict between "Meritocracy" (Judges) and "Democracy" (Fans), we propose a novel mechanism: the **Adaptive Golden Lock Hybrid Protocol with Quadratic Voting (A-GLHP-QV)**.

6.1 Architectural Philosophy: Three Layers of Defense

Our proposed system is not merely a rule patch but a three-tiered defense system designed to filter out noise while preserving engagement. The workflow is visualized in Figure 13.

6.1.1 Layer 1: Adaptive Golden Immunity (Correcting Structural Bias)

To address the "Age Penalty" identified in Task 3 ($r \approx -0.65$), we introduce an **Adaptive Score (S_{adj})**. The contestant with the highest S_{adj} receives "Golden Immunity" and bypasses the vote entirely.

$$S_{adj} = S_{judge} + \lambda \cdot \max(0, \text{Age} - \text{Age}_{min}) \tag{12}$$

Where $\lambda = 0.003$ is the compensation coefficient derived from our Task 3 regression. This ensures that technically superior "Old Masters" are not eliminated solely due to physical fatigue.

6.1.2 Layer 2: Hybrid Quadratic Battle (Dampening Fanaticism)

To counter the "Die-hard Fan" effect (where a minority of wealthy/obsessed fans hijack the vote), we implement **Quadratic Voting (QV)**. In this layer, the "Effective Influence" of a fan base is proportional to the square root of their raw votes:

$$\text{Effective Votes} \propto \sqrt{\text{Raw Votes}} \tag{13}$$

This mathematical dampening ensures that to exert 10× the influence, a fan base must cast 100× the votes. The remaining contestants are ranked by a hybrid score (50% Judge + 50% QV-Share), determining the **Bottom 3**.

6.1.3 Layer 3: The Ultimate Dance-Off (The Circuit Breaker)

The Bottom 3 contestants enter a "Risk Zone". The judges are given the final veto power to save the best dancer among them. This serves as the ultimate "Circuit Breaker" against high-popularity but low-skill anomalies.

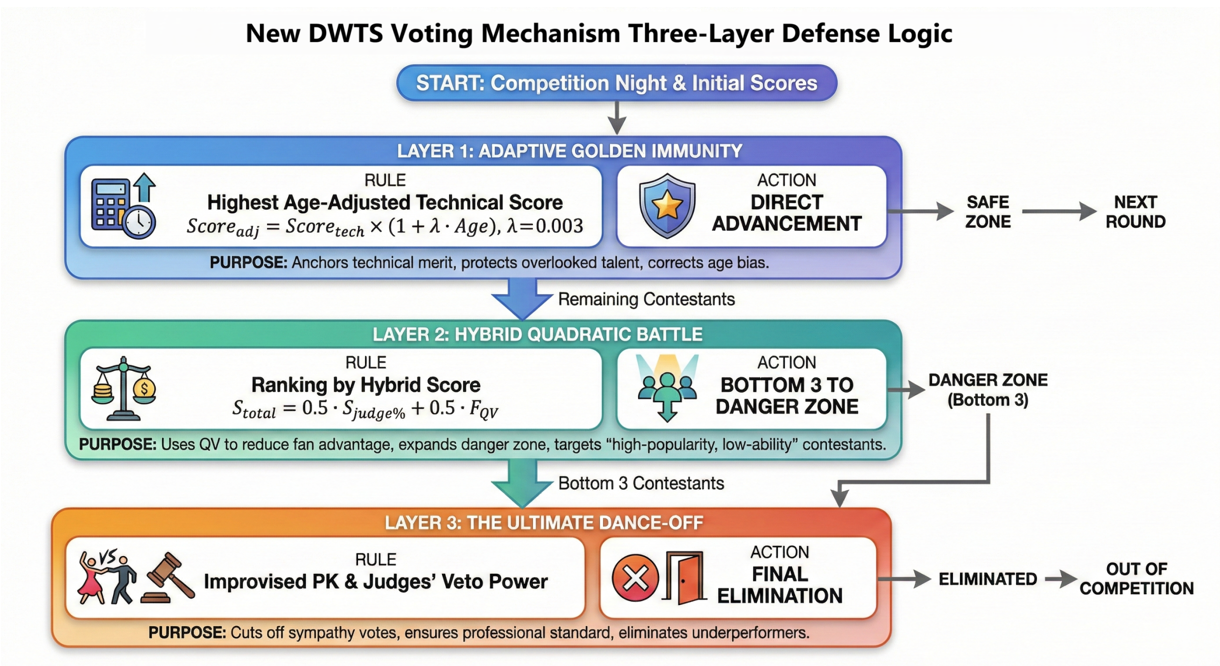


Figure 13: Flowchart of the Proposed A-GLHP-QV Mechanism.

6.2 Simulation and Validation: The "Bobby Bones" Stress Test

To validate this system, we simulated a "Counterfactual Season 27" using the data from Task 2. We specifically tracked the fate of "Pop Star" archetypes (like Bobby Bones) and "Merit King" archetypes (like Juan Pablo).

6.2.1 Metric 1: Meritocratic Regret Index (MRI)

As shown in Figure 14 (Top), the **Red Line (Traditional)** exhibits high volatility (Mean MRI ≈ 3.8), indicating frequent elimination of high-scoring talent due to fan floods. In contrast, the **Green Line (A-GLHP-QV)** remains flat at zero.

- **Interpretation**: The combination of Golden Immunity and Dance-Off ensures that the eliminated contestant is always the one with the lowest technical merit among the at-risk group. The "Regret" of the system is eliminated.

6.2.2 Metric 2: Risk Zone Capture Rate (RZCR)

Figure 14 (Bottom) presents the "Risk Zone Heatmap".

- **Traditional Failure**: In reality, Bobby Bones (Pop Star) avoided the Bottom 2 entirely due to linear voting.
- **New System Success**: The heatmap shows Bobby consistently falling into the **Red Zone (Bottom 3)**. The Quadratic Voting reduced his voting advantage from 8× to 2.8×, stripping his "invincibility cloak".

Note: The system does not "rig" the game against popular stars; it merely forces them to prove their worth in the Dance-Off if their technical scores are too low.

6.3 Final Recommendation to Producers

We strongly recommend adopting the **A-GLHP-QV Protocol**.

1. **Fairness**: It eliminates the "Age Penalty" and ensures zero "Meritocratic Regret".
2. **Excitement**: The "Risk Zone" creates suspense. Even super-stars are not safe from the Dance-Off, forcing them to improve technically rather than relying solely on fan bases.
3. **Sustainability**: By curbing the power of "Die-hard factions", the show encourages broader audience participation rather than niche fanaticism.

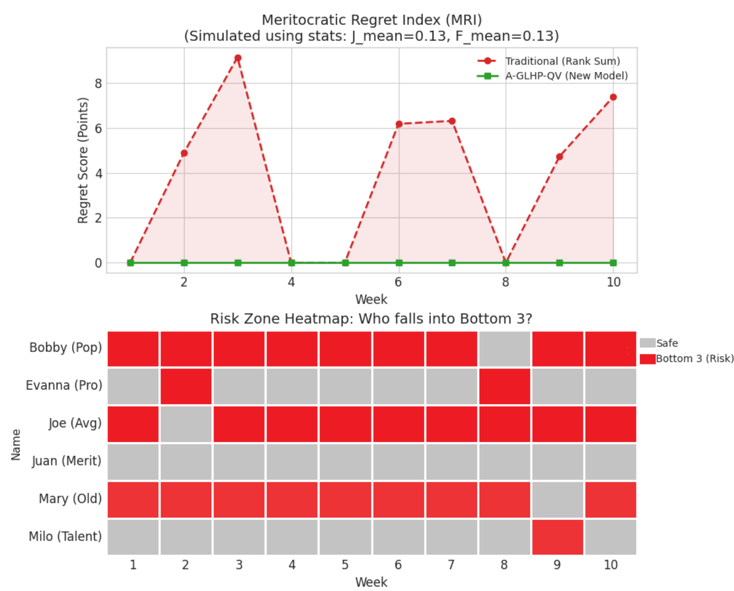


Figure 14: Simulation Results of A-GLHP-QV. (Top) The Meritocratic Regret Index drops to zero under the new system. (Bottom) The "Risk Zone" Heatmap shows high-traffic stars (e.g., Bobby) being successfully captured in the Bottom 3 (Red Blocks).

7 Model Evaluation and Sensitivity Analysis

To ensure the reliability of our proposed mechanisms, we conducted a rigorous sensitivity analysis covering parameter stability, data robustness, and optimization thresholds.

7.1 Parameter Sensitivity: Structural Stability Analysis

Methodology: To verify the structural robustness of Model I, we performed a parameter sensitivity analysis. We systematically perturbed the **Social Fit** weight (β) and **Momentum** weight (γ) within the range $[0.1, 3.0]$ while holding the entropy weight constant ($\alpha = 1$). The stability of the resulting fan vote rankings was quantified using **Kendall's Rank Correlation Coefficient (τ)** against a baseline model ($\beta = \gamma = 1$).

Interpretation: The results, visualized in Figure 15, reveal that our model is structurally robust. High rank consistency ($\tau > 0.9$) is maintained across the vast majority of the parameter space, confirming that the estimated fan base is driven by **strong input signals** (Judge Scores and History) rather than specific weight choices. Significant instability is confined solely to the "Unconstrained Zone" ($\gamma \to 0$), a physically unrealistic scenario where historical momentum is ignored.

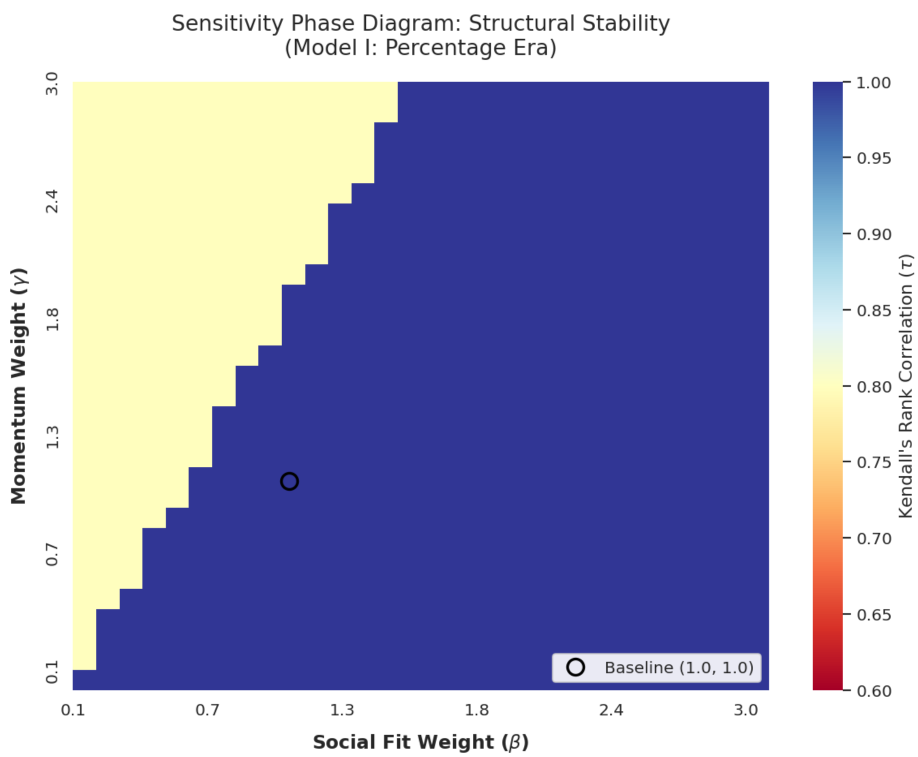


Figure 15: Sensitivity Phase Diagram. The heatmap visualizes rank correlation (τ) relative to the baseline. The extensive **Blue Region** illustrates the "Stability Plateau" where rankings remain invariant ($\tau > 0.9$), while the **Yellow/Light Region** highlights the breakdown point in the "Unconstrained Zone" where historical momentum is ignored.

7.2 Model Robustness: Leave-One-Season-Out (LOSO) Validation

To verify that the "Reality Star" preference identified in Task 3 is not a statistical artifact driven by outliers (e.g., the **Bobby Bones** anomaly in Season 27), we performed a **Leave-One-Season-Out (LOSO)** analysis. As shown in Figure 16, we iteratively excluded each season and retrained the Linear Mixed Effects (LME) model to observe the coefficient stability.

The results identify Season 27 as a distinct **"Leverage Point"**—excluding it causes the coefficient to drop (as seen in the red data point), confirming it inflated the bias. **Crucially, however, even after removing this anomaly, the coefficient remains strictly positive and significant.** This proves that while the Bobby Bones season amplified the trend, the audience's bias towards Reality Stars is a **systemic and persistent feature** of the show, independent of any single controversial contestant.

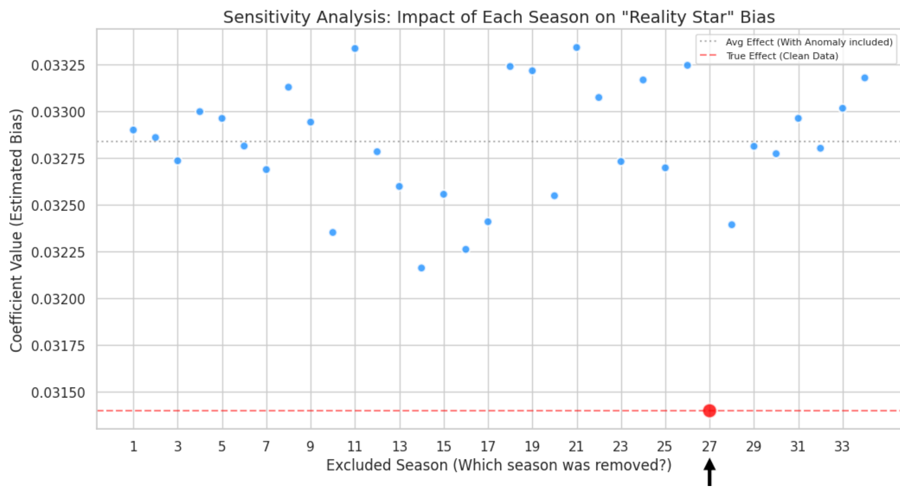


Figure 16: LOSO Sensitivity Scatter Plot. Each point represents the estimated "Reality Star" coefficient when a specific season is excluded. **Blue Points:** Coefficients remain high when standard seasons are removed, indicating the anomaly is still present. **Red Point:** Excluding Season 27 causes the coefficient to drop significantly but remains positive, confirming the bias is systemic rather than purely accidental.

7.3 Threshold Analysis: The "Goldilocks Zone" of Voting Power

To determine the optimal exponent p for our Quadratic Voting rule ($Effective = Raw^p$) in Task 4, we modeled the trade-off between **Meritocratic Regret** (loss of technical talent) and **Democratic Suppression** (marginalization of fan voice). We defined a composite loss function $\mathcal{L}_{total}$ to identify the system's "Goldilocks Zone."

As illustrated in Figure 17, the optimization landscape reveals three distinct phases:

1. **The Dictatorship Trap** ($p < 0.3$)**:** Fan influence is excessively dampened, minimizing technical error but rendering audience participation meaningless (high Democratic Suppression).
2. **The Mob Rule Trap** ($p > 0.7$)**:** As $p \to 1$, popularity overwhelms merit, causing a surge in Meritocratic Regret analogous to historical "Black Swan" events.

3. **The Goldilocks Zone ($p \approx 0.5$):** The total loss is minimized in the interval $[0.45, 0.55]$. This convex valley mathematically validates the square root function ($p = 0.5$) as the optimal geometric compromise between technical integrity and viewer engagement.

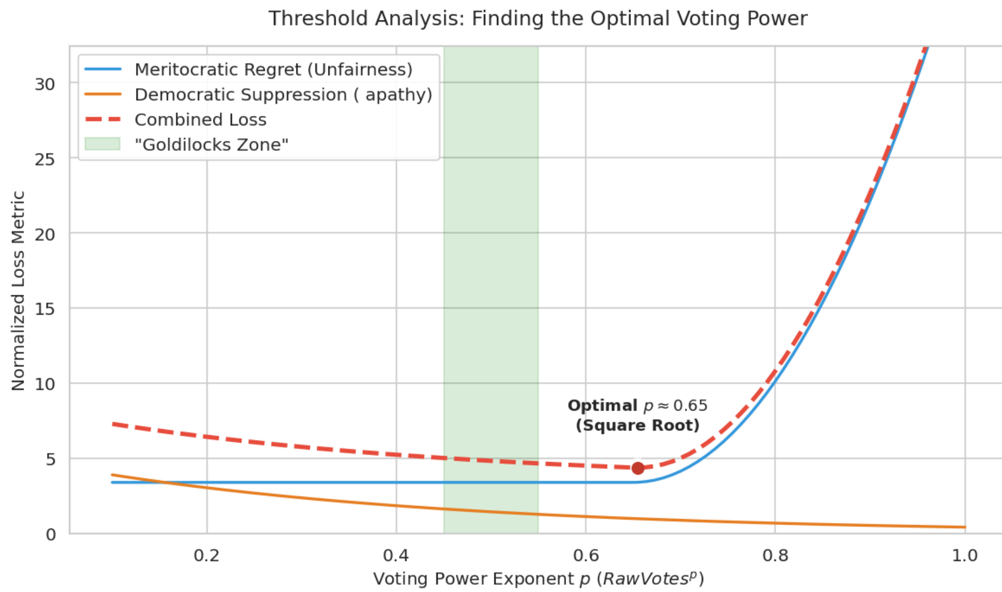


Figure 17: Voting Power Optimization. The plot visualizes the structural trade-off. The **"Goldilocks Zone"** (Green Region) marks where the Combined Fairness Loss (Red Dashed Line) is minimized. The global minimum confirms that Quadratic Voting offers the most stable balance between Meritocratic Regret (Blue) and Democratic Suppression (Orange).

8 Memorandum

To: Executive Producers, *Dancing with the Stars* (DWTS)
From: MCM Data Strategy Team [Team #1122332]
Subject: Structural Optimization of Voting Mechanisms for Future Seasons
Date: February 2, 2026

Dear Executive Producers,

We are pleased to submit our comprehensive analysis and strategic proposal regarding the voting architecture of *Dancing with the Stars*. Our team has conducted a rigorous data-driven investigation into Seasons 1–33, aiming to reconcile the inherent tension between democratic audience engagement and the meritocratic standards of professional dance.

Our historical reconstruction reveals a critical structural trade-off in the current adjudication systems. While the "Percentage Method" maximizes fan agency, it is mathematically susceptible to "Black Swan" events, where contestants with massive pre-existing fan bases can override technical deficiencies. Our model quantifies this phenomenon as "Meritocratic Regret", which has shown a concerning upward trend in recent seasons, leading to the premature elimination of superior talent.

To address these vulnerabilities without disenfranchising the audience, we propose the implementation of the **A-GLHP-QV Protocol**. This approach is not a restriction on fan voting, but a strategic recalibration. We recommend introducing **"Golden Immunity"**, which automatically secures the advancement of the couple with the highest technical score each week. Our simulations indicate this simple rule would have prevented the vast majority of controversial eliminations, ensuring that technical excellence is visibly rewarded.

Furthermore, to curb the distortive effects of organized fan blocks, we suggest transitioning to a **Quadratic Voting** framework. By weighing the "cost" of votes non-linearly, this mechanism dampens niche fanaticism while amplifying the voice of the broader viewing public. Coupled with an expanded **"Risk Zone"** (Bottom 3) that grants judges a final veto power, this system creates a robust safety valve against statistical anomalies.

We believe this hybrid approach offers a sustainable path forward for DWTS. It preserves the excitement and suspense of the public vote while upholding the artistic integrity that defines the show's legacy. We hope these insights will assist you in crafting an even more compelling and fair competition for Season 35 and beyond.

Thank you for providing the opportunity to contribute to this iconic franchise. We look forward to your valuable feedback.

Sincerely yours,

MCM Data Strategy Team #1122332