Schema Refinement and Normal Forms

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The Evils of Redundancy

- *Redundancy* is at the root of several problems associated with relational schemas:
  - redundant storage, insert/delete/update anomalies
- Integrity constraints, in particular *functional dependencies*, can be used to identify schemas with such problems and to suggest refinements.
- Main refinement technique: *decomposition* (replacing ABCD with, say, AB and BCD, or ACD and ABD).
- Decomposition should be used judiciously:
  - Is there reason to decompose a relation?
  - What problems (if any) does the decomposition cause?

Functional Dependencies (FDs)

- A <u>functional dependency</u> X $\rightarrow$ Y holds over relation R if, for every instance *r* of R:
  - $t1 \in r,\ t2 \in r,\ \pi_X(t1) = \pi_X(t2)$ implies $\pi_Y(t1) = \pi_Y(t2)$
  - i.e., given two tuples in *r*, if the X values agree, then the Y values must also agree. (X and Y are *sets* of attributes.)
- K is a candidate key for R means that K $\rightarrow$ R
  - However, K $\rightarrow$ R does not require K to be *minimal*!
- X $\rightarrow$ Y; read
  - X functionally determines Y, or
  - Y functionally depends on X

Functional Dependencies (cont'd)

- Reminders:
  - FDs are assertions about the real world.
  - FDs are statement about *all* allowable instances of a relation schema.

- *Example*: Consider relation R = {A, B, C}
- Can we deduce that $A \rightarrow C$ holds?
- Can we deduce that $A \rightarrow B$ does not hold?

A	B	C
1	2	3
4	5	4
1	3	3

Example: Constraints on Entity Set

- Consider relation obtained from Hourly_Emps:
  - Hourly_Emps (<u>*ssn*</u>, *name, lot, rating, hrly_wages, hrs_worked*)
- <u>*Notation*</u>: We will denote this relation schema by listing the attributes: SNLRWH
  - This is really the *set* of attributes {S,N,L,R,W,H}.
  - Sometimes, we will refer to all attributes of a relation by using the relation name. (e.g., Hourly_Emps for SNLRWH)
- Some FDs on Hourly_Emps:
  - *ssn* is the key: $S \rightarrow SNLRWH$
  - *rating* determines *hrly_wages*: $R \rightarrow W$

Example (Contd.)

- Problems due to R $\longrightarrow$ W :
  - *Update anomaly*: Can we change W in just the 1st tuple of SNLRWH?
  - *Insertion anomaly*: What if we want to insert an employee and don't know the hourly wage for his rating?
  - *Deletion anomaly*: If we delete all employees with rating 5, we lose the information about the wage for rating 5!

Will 2 smaller tables be better?

Wages

R	W
8	10
5	7

Hourly_Emps2

S	N	L	R	H
123-22-3666	Attishoo	48	8	40
231-31-5368	Smiley	22	8	30
131-24-3650	Smethurst	35	5	30
434-26-3751	Guldu	35	5	32
612-67-4134	Madayan	35	8	40

S	N	L	R	W	H
123-22-3666	Attishoo	48	8	10	40
231-31-5368	Smiley	22	8	10	30
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Reasoning About FDs

- Given some FDs, we can usually infer additional FDs; e.g.:
  - $ssn \rightarrow did$, $did \rightarrow lot$ implies $ssn \rightarrow lot$
- F^+ = *closure of F* is the set of all FDs that are implied by *F*.
- Armstrong's Axioms (X, Y, Z are sets of attributes):
  - *Reflexivity*: If $Y \subseteq X$, then $X \rightarrow Y$
  - *Augmentation*: If $X \rightarrow Y$, then $XZ \rightarrow YZ$ for any Z
    (Note: the opposite is not true!)
  - *Transitivity*: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$

Reasoning About FDs (Contd.)

- Couple of additional rules (that follow from AA):
  - *Union*: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$
  - *Decomposition*: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$
- $X \rightarrow Y$ is *trivial* if it is true for any X and Y of any relation, regardless of the X and Y semantics. Example:
  - $A \rightarrow A$
  - In general, if $Y \subseteq X$ then $X \rightarrow Y$ is trivial; e.g.,
    - $ABC \rightarrow BC$ and
    - $ABC \rightarrow AC$ and … every subset of ABC

Example – Deriving FDs

- *Contracts*(*cid,sid,jid,did,pid,qty,value*)
  We denote *Contracts* as CSJDPQV:
  C is an agreement that *S* will supply *Q* items of part *P* to project *J* associated with *D* at a cost *V*, and the following ICs are known to hold:
  - C is a key: $C \rightarrow CSJDPQV$
  - Project purchases each part using single contract: $JP \rightarrow C$
  - Dept purchases at most one part from a supplier: $SD \rightarrow P$
- $JP \rightarrow C$, $C \rightarrow CSJDPQV$ imply $JP \rightarrow CSJDPQV$
- $SD \rightarrow P$ implies $SDJ \rightarrow JP$
- $SDJ \rightarrow JP$, $JP \rightarrow CSJDPQV$ imply $SDJ \rightarrow CSJDPQV$

Example – Proving an FD holds

- A relation holding addresses:
  R = {numstr, city, zip}
- Known functional dependencies:
  F = {{numstr, city} → zip, zip → city}
- Prove: {numstr, zip} → city
- Proof
  - From zip → city and the augmentation rule
    - {numstr, zip} → {numstr, city}
  - Since city is a subset of {city, numstr}
    - {numstr, zip} → {numstr, city} → city
  - Therefore (transitivity): numstr, zip → city

The Closure of a Set of Attributes X^+

- Computing the closure of a set of FDs can be expensive. (Size of closure is exponential in # attrs!)
- Typically, we just want to check if a given FD $X \rightarrow Y$ is in the closure of a set of FDs F. An efficient check:
  - Compute *attribute closure* of X (denoted X^+) wrt *F:*
    - Set $X^+ = X$;
    - While (X^+ keeps changing) do
      - **For each $Y \rightarrow Z$ in *F*, if Y in X^+ then $X^+ = X^+ \cup Z$;**
- Does $F = \{A \rightarrow B,\ B \rightarrow C,\ C\,D \rightarrow E\}$ imply $A \rightarrow E$?
  - i.e, is $A \rightarrow E$ in the closure F^+? Equivalently, is E in A^+ ?

Example – Closure of Attributes

R = {A, B, C, D, E} with the following FDs:

$C \rightarrow A$ (1), $AB \rightarrow C$ (2), $BC \rightarrow D$ (3),

$A \rightarrow BE$ (4), $B \rightarrow E$ (5)

Then:

- $A^+ = A$, $A^+ =_4 ABE$, $A^+ =_2 ABCE$, $A^+ =_3 ABCDE$
- $B^+ = B$, $B^+ =_5 BE$
- $C^+ = C$, $C^+ =_1 AC$, $C^+ =_4 ABCE$, $C^+ =_3 ABCDE$
- $D^+ = D$
- $E^+ = E$

Notice the identification of keys: $A \rightarrow R$ and $C \rightarrow R$.

Normal Forms & Decomposition

- Returning to the issue of schema refinement, the first question to ask is whether any refinement is needed!
- If a relation is in a certain *normal form* (BCNF, 3NF etc.), it is known that certain kinds of problems are avoided/minimized. This can be used to help us decide whether decomposing the relation will help.
- Role of FDs in detecting redundancy:
  - Consider a relation R with 3 attributes, ABC.
    - No FDs hold: There is no redundancy here.
    - Given $A \rightarrow B$: Several tuples could have the same A value, and if so, they'll all have the same B value!

Boyce-Codd Normal Form (BCNF)

- Reln R with FDs *F* is in BCNF if, for all $X \rightarrow A$ in F^+
  - $A \in X$ (called a *trivial* FD), or
  - X contains a key for R (i.e., X is a superkey)
- In other words, R is in BCNF if the only non-trivial FDs that hold over R are key constraints.

Third Normal Form (3NF)

- Reln R with FDs *F* is in 3NF if, for all $X \rightarrow A$ in F^+
  - $A \in X$ (called a *trivial* FD), or
  - X contains a key for R, or
  - A is part of some key for R.
- *Minimality* of a key is crucial in third condition above!
- If R is in BCNF, obviously in 3NF.
- If R is in 3NF, some redundancy is possible. It is a compromise, used when BCNF not achievable (e.g., no ``good'' decomp, or performance considerations).
  - *Lossless-join, dependency-preserving decomposition of R into a collection of 3NF relations always possible.*

What Does 3NF Achieve?

- If 3NF violated by $X \rightarrow A$, one of the following holds:
  - X is a subset of some key K
    - We store (X, A) pairs redundantly.
  - X is not a proper subset of any key.
    - There is a chain of FDs $K \rightarrow X \rightarrow A$, which means that we cannot associate an X value with a K value unless we also associate an A value with an X value.
- Thus, 3NF is a compromise relative to BCNF.

Decomposition of a Relation Scheme

- Suppose that relation R contains attributes *A1 … An*. A ***decomposition*** of R consists of replacing R by two or more relations such that:
  - Each new relation scheme contains a subset of the attributes of R (and no attributes that do not appear in R), and
  - Every attribute of R appears as an attribute in one of the new relations.

Example Decomposition

- Hourly_Emps (*<u>ssn</u>, name, lot, rating, hrly_wages, hrs_worked*)
- SNLRWH has FDs S → SNLRWH and R → W
  - Second FD causes violation of 3NF (W is not part of a key - W values repeatedly associated with R values). Easiest way to fix this is to create a relation RW to store these associations, and to remove W from the main schema:
    - i.e., we decompose SNLRWH into SNLRH and RW
- Are there any potential problems that we should be aware of?

Problems with Decompositions

- There are 3 potential problems to consider:
  1. Some queries become more expensive.
     - e.g., How much did sailor Joe earn? (salary = W*H)
  2. Lossless-join: Given instances of the decomposed relations, we may not be able to reconstruct the corresponding instance of the original relation!
     - Fortunately, not in the SNLRWH example.
  3. Dependency preserving: Checking some dependencies may require joining the instances of the decomposed relations.
     - Fortunately, not in the SNLRWH example.
- *Tradeoff*: Must consider these issues vs. redundancy.

Lossless Join Decompositions

- Decomposition of R into X and Y is ***lossless-join*** w.r.t. a set of FDs F if, for every instance *r* that satisfies F:
  - $\pi_X(r) \bowtie \pi_Y(r) = r$
- It is always true that $r \subseteq \pi_X(r) \bowtie \pi_Y(r)$
  - In general, the other direction does not hold! If it does, the decomposition is lossless-join.
- Definition extended to decomposition into 3 or more relations in a straightforward way.
- *It is essential that all decompositions used to deal with redundancy be lossless!*

Testing for Lossless Join

- The decomposition of R into R1 and R2 is lossless-join wrt F if and only if the closure of F (F^+) contains at least one of:
  - $X \cap Y \rightarrow X$, or
  - $X \cap Y \rightarrow Y$
- In particular, if $X \rightarrow Y$ holds over R and $X \cap Y$ is empty, the decomposition of R into R – Y and XY is lossless-join.

A	B	C
1	2	3
4	5	6
7	2	8

A	B
1	2
4	5
7	2

B	C
2	3
5	6
2	8

A	B	C
1	2	3
4	5	6
7	2	8
1	2	8
7	2	3

Dependency Preserving Decomposition

- Consider CSJDPQV, C is key, JP $\rightarrow$ C and SD $\rightarrow$ P.
  - BCNF decomposition: CSJDQV and SDP
  - Problem: Checking JP $\rightarrow$ C requires a join!
- Dependency preserving decomposition (Intuitive):
  - If R is decomposed into X, Y and Z, and we enforce the FDs that hold on X, on Y and on Z, then all FDs that were given to hold on R must also hold.
- *Projection of set of FDs F*: If R is decomposed into X, ... projection of F onto X (denoted F_X) is the set of FDs U $\rightarrow$ V in F^+ (*closure of F*) such that U, V are in X.

Dependency Preserving Decompositions (Contd.)

- Decomposition of R into X and Y is ***dependency preserving*** if $(F_X \cup F_Y)^+ = F^+$
  - i.e., if we consider only dependencies in the closure F^+ that can be checked in X without considering Y, and in Y without considering X, these imply all dependencies in F^+.
- Important to consider F^+, not F, in this definition:
  - ABC, $A \rightarrow B$, $B \rightarrow C$, $C \rightarrow A$, decomposed into AB and BC.
  - Is this dependency preserving? Is $C \rightarrow A$ preserved?????
- Dependency preserving does not imply lossless join:
  - ABC, $A \rightarrow B$, decomposed into AB and BC.
- And vice-versa.

Decomposition into BCNF

- Consider relation R with FDs F. If X→ Y violates BCNF, decompose R into R - Y and XY.
  - Repeated application of this idea will give us a collection of relations that are in BCNF; lossless join decomposition, and guaranteed to terminate.
  - e.g., CSJDPQV, key C, JP→ C, SD→ P, J→ S
  - To deal with SD→P, decompose into SDP, CSJDQV.
  - To deal with J→ S, decompose CSJDQV into JS and CJDQV
- In general, several dependencies may cause violation of BCNF. The order in which we \`\`deal with'' them could lead to very different sets of relations!

BCNF and Dependency Preservation

- In general, there may not be a dependency preserving decomposition into BCNF.
  - e.g., CSZ, CS $\rightarrow$ Z, Z $\rightarrow$ C
  - Can't decompose while preserving 1st FD; not in BCNF.
- Similarly, decomposition of CSJDQV into SDP, JS and CJDQV is not dependency preserving (w.r.t. the FDs JP $\rightarrow$ C, SD $\rightarrow$ P and J $\rightarrow$ S).
  - However, it is a lossless join decomposition.
  - In this case, adding JPC to the collection of relations gives us a dependency preserving decomposition.
    - JPC tuples stored only for checking FD! (*Redundancy!*)

Decomposition into 3NF

- Obviously, the algorithm for lossless join decomp into BCNF can be used to obtain a lossless join decomp into 3NF (typically, can stop earlier).
- To ensure dependency preservation, one idea:
  - If $X \rightarrow Y$ is not preserved, add relation XY.
  - Problem is that XY may violate 3NF! e.g., consider the addition of CJP to `preserve' $JP \rightarrow C$. What if we also have $J \rightarrow C$?
- Refinement: Instead of the given set of FDs F, use a *minimal cover for F*.

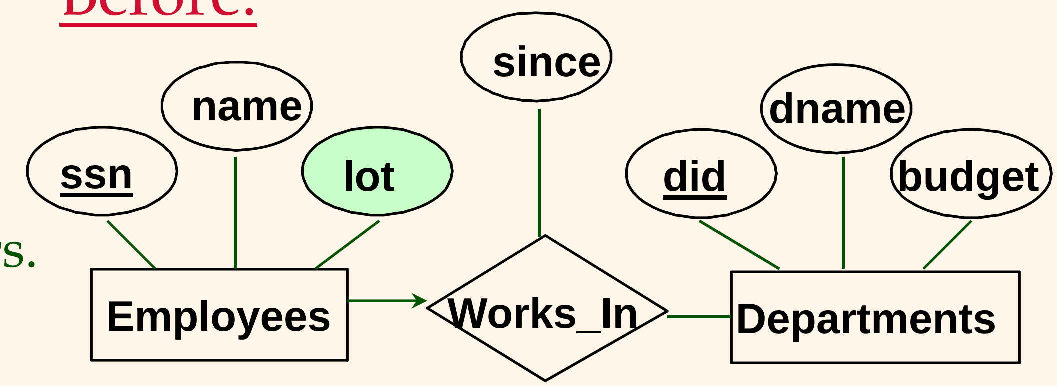
Minimal Cover for a Set of FDs

- *Minimal cover* G for a set of FDs F:
  - Closure of F = closure of G.
  - Right hand side of each FD in G is a single attribute.
  - If we modify G by deleting an FD or by deleting attributes from an FD in G, the closure changes.
- Intuitively, every FD in G is needed, and ``*as small as possible*'' in order to get the same closure as F.
- e.g., $A \rightarrow B$, $ABCD \rightarrow E$, $EF \rightarrow GH$, $ACDF \rightarrow EG$ has the following minimal cover:
  - $A \rightarrow B$, $ACD \rightarrow E$, $EF \rightarrow G$ and $EF \rightarrow H$
- M.C. $\rightarrow$ Lossless-Join, Dep. Pres. Decomp!!! (in book)

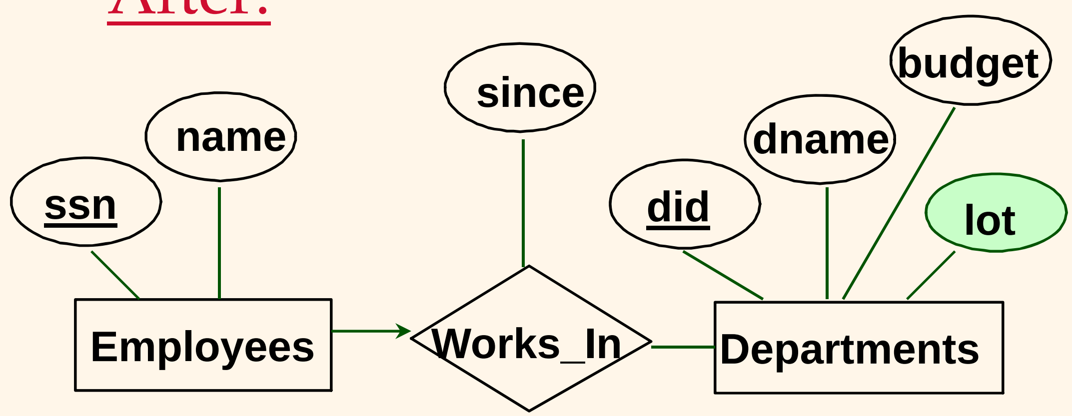
Refining an ER Diagram

- 1st diagram translated:
  Workers(S,N,L,D,S)
  Departments(D,M,B)
  - Lots associated with workers.
- Suppose all workers in a dept are assigned the same lot: $D \rightarrow L$
- Redundancy; fixed by:
  Workers2(S,N,D,S)
  Dept_Lots(D,L)
- Can fine-tune this:
  Workers2(S,N,D,S)
  Departments(D,M,B,L)

Before:



After:



Summary of Schema Refinement

- If a relation is in BCNF, it is free of redundancies that can be detected using FDs. Thus, trying to ensure that all relations are in BCNF is a good heuristic.
- If a relation is not in BCNF, we can try to decompose it into a collection of BCNF relations.
  - Must consider whether all FDs are preserved. If a lossless-join, dependency preserving decomposition into BCNF is not possible (or unsuitable, given typical queries), should consider decomposition into 3NF.
  - Decompositions should be carried out and/or re-examined while keeping *performance requirements* in mind.