Management of interacting species: regulation under nonlinearities and hysteresis[☆]

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Abstract

We consider the optimal regulation of an open access fishery when biomass dynamics are combined with fishing effort dynamics. The interaction of inelastic demand and backward bending nature of the ecological supply curve introduce nonlinearities that could lead to multiple bionomic equilibria and hysteresis effects. We obtain convergence results for general bionomic models and indicate cases where hysteretic regulation failure could be a problem when economic dynamics are slow enough relative to biological dynamics. We show that when biological dynamics are slow enough relative to the economic dynamics, the social optimum can be achieved by a time dependent landing fee.
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1. Introduction

The management of renewable resources under open access conditions is fairly well developed, mostly in the single but also in the multiple species cases.[1] Starting with Gordon (1954) the theory of open access resource exploitation predicts excess effort and rent dissipation. Moreover, Smith (1968) explicitly models the dynamics of effort expansion by a fishing industry in an open access situation along with the fishery growth dynamics, while

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in a recent paper Sanchirico and Wilen (1999) extend this idea to a model incorporating spatial movements.

In this paper we consider the optimal regulation of an open access fishery when species interact and biomass dynamics are combined with fishing effort dynamics. In our model the interaction of inelastic demand and backward bending nature of the ecological supply curve introduce nonlinearities that could lead to multiple bionomic equilibria and hysteresis effects.

The paper develops as follows. Section 2 analyzes management under two regimes: (i) socially-optimal, and (ii) rational expectations competitive equilibrium with full property rights. We obtain equivalence of competitive equilibria under full property rights with the social optimum.

Under appropriate interpretation, the models of interacting species developed in Section 2 of the paper as well as many of the techniques used in the analysis of these models can make use of well-known core results of resource economics and bioeconomics (cf. Clark, 1990; Dasgupta and Heal, 1979). It is useful to exploit this well-known material to build a platform to launch probes into less familiar territory. Thus, given the importance of the socially-optimal solution for regulatory purposes, we develop in Section 3 an extension of well-known value loss arguments to extend the turnpike theory reviewed in Carlson et al. (1991) to quite general bionomic models.[2]

We provide examples where optimal decentralized regulation by incentive instruments applied at the individual agent level of open access bionomic equilibria fails due to hysteresis and multiple equilibria. This is familiar from work such as Clark (1990). We indicate cases where hysteretic regulation failure is likely to be a problem, when the relative speed of adjustment of economic dynamics is slow enough relative to biological dynamics. Section 4 concentrates on developing these results mainly for the case of one species, which provides the clearest picture of difficulties encountered by standard regulatory instruments in the presence of hysteresis and many species. For the multi-species case the interaction between demand and biology in this context can lead to complicated patterns of equilibrium and hysteretic relaxation dynamics under open access exploitation.

Our results in Section 4 on decentralized regulation rely on the assumption that the biological dynamics are fast enough relative to the economic dynamics. In Section 5 we examine the opposite case in which the biological dynamics are slow enough relative to the economic dynamics. We show that the social optimum can be achieved by a time dependent landing fee equal to the shadow value of the biomass. Finally, Section 6 provides some concluding remarks.

2. Welfare optimum and rational expectations equilibria for competing populations

We consider a general model of competition among several species. In the sea, for example, competition takes place among many species or trophic levels, where food flows from

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[2] The methods we develop to extend value loss arguments suggest the potential of extending our methods to models of optimal control of partial differential equation models of multiple species on a continuum of patches. Brock (2000) suggests this possibility but the value loss argument developed here does not appear in that paper.

one species to the other, and where many species at one level may prey upon species below it. Assuming logistic growth to simplify things, the equations of interspecies competition can be written as

$$\dot{x}_i = F(\mathbf{x}) = x_i \left(s_i - \sum_{j=1}^{n} r_{ij} x_j \right), \quad i = 1, \ldots, n \tag{1}$$

where $\mathbf{x} = (x_1, \ldots, x_n)$ denotes species biomasses and r_{ij} are the competition coefficients.[3] To introduce harvesting $\mathbf{H} = (H_1, \ldots, H_n)$ into the model, we follow Clark's (1990) notation, to let q_i, E_i, p_i, Y_i, c_i denote, respectively, catchability coefficient of species i, effort spent on capturing species i, unit price of species i, total number of units captured of species i, and cost per unit effort spent on species i, with $H_i = Y_i = q_i x_i E_i$.

The flow of economic rent, i.e. profit, generated by species i is given by

$$R_i = (p_i q_i x_i - c_i) E_i \tag{2}$$

For the discussion of bionomic equilibrium at this stage of the analysis, effort dynamics—which are also present—are assumed to equilibrate. If $p_i = P_i(\mathbf{Y})$, $\mathbf{Y} = (Y_1, \ldots, Y_n)$ is the demand curve for species i, then the steady state *interior* bionomic equilibrium with all species present is given by[4]

$$p_i q_i x_i = c_i, \quad p_i = P_i(Y_1, \ldots, Y_n) \tag{3}$$

$$Y_i = q_i E_i x_i = F(\mathbf{x}), \quad i = 1, 2, \ldots, n. \tag{4}$$

For the case $n = 1$, the systems (3) and (4) is thoroughly analyzed by Clark (1990), not only for the linear Schaefer type of cost structure, but also for various generalizations of the cost structure.[5] In order to make a clearer presentation of the welfare optimum and the rational expectation equilibrium (REE), we focus on a single species case.

2.1. Equilibria for a single species case

We investigate two regimes. First, we investigate the socially-optimal management problem (SOMP), where the regulator maximizes producer and consumer surplus subject to biomass dynamics. Second, we show that if we have N (without loss of generality put $N = 1$ for a "stand-in" fishery) identical fisheries with the same dynamics and there is no mixing or diffusion of fish across any of the N fisheries, then a version of the standard

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[3] This model can be regarded as a generalized Lotka–Volterra predator–prey system, by a partition of the set on n species to predators and prey and by a suitable change of the signs of the parameters s_i and r_{ij} (Murray, 1993, p. 69). The model can also be regarded as a multi-species version of the Kolmogorov model (Murray, 1993, p. 85).

[4] Bionomic equilibria may be noninterior and have some species with zero price, if the unit cost of effort is too high relative to demand and if the catchability coefficient is too small. To allow for these boundary equilibria, replace (4) with

$$Y_i < F(\mathbf{x}) \Rightarrow p_i = 0, \quad p_i > 0 \Rightarrow Y_i = F(\mathbf{x})$$

[5] See Clark (1990, Figures 5.9–5.11, and surrounding text).

equivalence theorem of capital theory (Becker and Boyd, 1996) applies for rational point expectations equilibrium. That is, the solution of the REE problem is the same as for the SOMP. This is so because each of the N fisheries is operated taking the path of price as parametric to maximize capitalized profits, but the dynamics of x, which generate spillovers, are taken into account. Hence this form of intertemporal competitive equilibrium is efficient. A REE may emerge if diffusion rates of the resource are localized enough so the fishing space can be subdivided into areas that are large enough so that diffusion across them is quite small, but still small enough so that Ostrom (1990) type institutions can be constructed to induce individual fishers within an area to act approximately like the rational expectations optimizers of (ii). Then approximate efficiency could be achieved. Ostrom's case studies as well as theory suggest that if the number of fishers is small enough and the interactive relationships are long term, then reasonably efficient common property management institutions can emerge and be sustained.

In the models discussed above, if there is perfect mixing or perfect diffusion of fish across the boundaries of the N fisheries, and if we assume that each fishery is operated without regard to the effect on the dynamics of x, then bionomic equilibrium is inefficient relative to the SOMP and the REE, for the same reasons discussed by Clark (1990). That is to say, because each fishery indulges in "scramble" competition to get their fish first before someone else does. This deviation implies that there is a need for a regulatory framework in order to achieve the socially-optimal solution.

2.1.1. The welfare optimum

For the SOMP we define welfare derived by catch $Y = qEx$, by the sum of consumer and producer surplus or $U(Y) = \tilde{S}(Y) - cE$, where $\tilde{S}(Y) = \int_0^Y P(u)\,du$ is the area under the demand curve $p = P(Y)$, up to $Y = qEx$ and $\tilde{S}'(Y) = P(Y) = p$. The socially-optimal management problem is defined as

$$\max_{\{E(t)\}} \int_0^\infty e^{-\rho t}[\tilde{S}(qEx) - cE]\,dt \tag{5}$$

subject to $\dot{x} = F(x) - H = x(s - rx) - qEx, \quad x(0) = x_0 > 0$ (6)

The current value Hamiltonian for this problem is given by

$$H = S(qEx) - cE + \mu x(s - rx - qE)$$

From the maximum principle, the first-order necessary conditions (FONC) for optimality are given by

$$\frac{\partial H}{\partial E} = 0 \quad \text{or} \quad \mu = P(Y) - \frac{c}{qx} \tag{7}$$

$$\dot{\mu} = \left(\rho - \frac{\partial F}{\partial x} + qE\right)\mu - qEP(Y), \quad F(x) = x(s - rx) \tag{8}$$

$$\dot{x} = x(s - rx - qE) \tag{9}$$

2.1.2. Rational expectations equilibrium

Let us now consider REE. Here the stand-in fishery takes the price function $p(t)$, $t \geq 0$ as parametric, and chooses E to maximize discounted profits $pqEx - cE$ subject to the biomass dynamics (6). So the REE solves

$$\max_{\{E(t)\}} \int_0^\infty e^{-\rho t}[pqEx - cE]\, dt,$$
$$\text{subject to (6) and } 0 \leq E \leq E_{\max} \tag{10}$$

The current value Hamiltonian for this problem[6] is given by

$$H = pqEx - cE + \mu x(s - rx - qE)$$

The FONC for a singular solution are given by

$$\mu = P(Y) - \frac{c}{qx} \tag{11}$$

where $p(t) = P(Y(t))$, along with (7) and (8). Let $\{E^*, x^*\}$ be optimal solutions and let $p^* = P(qE^*x^*)$. Then REE is defined by the requirement that $p = p^*$ for all dates t. By comparing the optimality conditions of the SOMP and the REE problems, it is clear that the FONC are the same for both problems. Hence if there is only one solution for the socially-optimal problem, REE replicates it and the equivalence between the two problems holds.

2.2. The multi-species case

The result can easily be extended to the general multi-species case. The socially-optimal problem can be written as

$$\max_{\{E_1(t),\ldots,E_n(t)\}} \int_0^\infty e^{-\rho t}\left[\tilde{S}(q_1 E_1 x_1, \ldots, q_n E_n x_n) - \sum_{i=1}^N c_i E_i \right] dt \tag{12}$$

$$\text{s.t. } \dot{x}_i = F_i(\mathbf{x}) - H_i, \quad i = 1, \ldots, n \tag{13}$$

$$H_i = Y_i = q_i E_i x_i, \qquad \frac{\partial \tilde{S}}{\partial Y_i} = p_i, \quad p_i = P_i(Y_1, \ldots, Y_n) = P_i(\mathbf{Y})$$

For the REE, each "stand-in" fishery takes the price vector $\mathbf{p}(t) = (p_1(t), \ldots, p_n(t))$, $t \geq 0$ as parametric, and chooses the effort vector $\mathbf{E}(t) = (E_1(t), \ldots, E_n(t))$ to maximize *joint* discounted profits $\sum_{i=1}^n (p_i q_i E_i x_i - c_i E_i)$ subject to the biomass dynamics. So the REE solves

$$\max_{\{\mathbf{E}(t)\}} \int_0^\infty e^{-\rho t} \sum_{i=1}^n (p_i q_i E_i x_i - c_i E_i)\, dt,$$
$$\text{subject to (13) and } 0 \leq E \leq E_{\max}$$

where $p_i(t) = P_i(\mathbf{Y}(t))$. It can easily be seen by comparing the optimality conditions of the SOMP and the REE problems that the FONC are the same for both problems. Hence if there

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[6] The additional constraint $0 \leq E \leq E_{\max}$ is required in order to make the REE problem well posed, given its linear structure.

is only one solution for the socially-optimal problem, REE replicates it and the equivalence between the two problems holds.

3. Sufficient conditions for convergence of the SOMP

The characterization of the SOMP and the deviations between the SOMP solution and the bionomic equilibrium suggest that the SOMP solution should be used as a yardstick for regulation purposes. Given, however, the complexity of the dynamic model describing the ecosystem, which turns out not to satisfy the usual concavity requirements, it is of interest to examine under what conditions the SOMP for the general model converges to a steady state.

Consider the following problem with $\rho = 0$:

$$\max \int_0^\infty S(\mathbf{H}) \, \mathrm{d}t, \qquad \text{s.t. } \dot{x}_i = F_i(\mathbf{x}) - H_i, \quad i = 1, 2, \ldots, n \tag{14}$$

where $S(\mathbf{H})$ denotes net current benefits from harvesting $\mathbf{H}$, and maximization is in the overtaking sense (Carlson et al., 1991). The standard value loss arguments (cf. Carlson et al., 1991) may be adapted to produce the new value loss argument for the following SOMP model. Let $(\mathbf{H}^*, \mathbf{x}^*)$ solve the optimal steady state (OSS), SOMP problem

$$\max S(\mathbf{H}), \qquad \text{s.t. } \mathbf{H} = \mathbf{F}(\mathbf{x}), \quad \mathbf{F}(\mathbf{x}) = (F_1(\mathbf{x}), \ldots, F_n(\mathbf{x})) \tag{15}$$

with associated Lagrangian

$$\mathcal{L} = S(\mathbf{H}) - \mathbf{p} \cdot [\mathbf{F}(\mathbf{x}) - \mathbf{H}]$$

where $\mathbf{p} = (p_1, \ldots, p_n)$ is a $1 \times n$ row vector of Lagrangian multipliers, $[\mathbf{F}(\mathbf{x}) - \mathbf{H}]$ is an $n \times 1$ column vector and "·" denotes the inner product. Let $(\mathbf{x}^*, \mathbf{H}^*, \mathbf{p}^*)$ be a solution to (15). Take the integral of (15) up to horizon T and write[7]

$$\int_0^T [S(\mathbf{H}) - S(\mathbf{H}^*)] \, \mathrm{d}t = \int_0^T [\partial_H S(\mathbf{H}^*) \cdot (\mathbf{H} - \mathbf{H}^*)] \, \mathrm{d}t - D_1$$

$$= \int_0^T \mathbf{p}^* \cdot [\mathbf{F}(\mathbf{x}) - \dot{\mathbf{x}} - \mathbf{F}(\mathbf{x}^*)] \, \mathrm{d}t - D_1$$

Assumption A1. $S(\mathbf{H})$ is strictly concave and differentiable in $\mathbf{H}$.

Assumption A2. $\mathbf{x}^* = \arg\max \mathbf{p}^* \cdot \mathbf{F}(\mathbf{x})$.

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[7] ∂_x denotes the first derivative operator on a function $f(\mathbf{x})$. Thus $\partial_x f(\mathbf{x}) = (\partial f / \partial x_1, \ldots, \partial f / \partial x_n)$ the $1 \times n$ gradient vector.

Notice that the FONC for a maximizer of $\mathbf{p}^* \cdot \mathbf{F}(\mathbf{x})$ are the same as for a maximizer of the OSS objective, $S(\mathbf{F}(\mathbf{x}))$. Suppose Assumption A1 holds for our problem. If so, we may write

$$\int_0^T \mathbf{p}^* \cdot [\mathbf{F}(\mathbf{x}) - \dot{\mathbf{x}} - \mathbf{F}(\mathbf{x}^*)]\, \mathrm{d}t - D_1 = \mathbf{p}^* \cdot (\mathbf{x}_0 - \mathbf{x}(T)) - D_1 + D_2$$

where

$$D_2 = \int_0^T \mathbf{p}^* \cdot [\mathbf{F}(\mathbf{x}) - \mathbf{F}(\mathbf{x}^*)]\, \mathrm{d}t$$

The key thing to note is that under Assumption A1, the term D_1 is an integral of non-negative terms, each of which is zero if and only if $\mathbf{H} = \mathbf{H}^*$. Under Assumption A2, each term of D_2 is nonpositive. Hence under modest regularity conditions the standard value loss arguments reviewed in Carlson et al. (1991) may be adapted to prove the following theorem.

Theorem 1. *Assumptions A1 and A2 and regularity conditions. Let* $\mathbf{x}^*(t; x_0)$, $\mathbf{H}^*(t; x_0)$ *solve the SOMP with initial condition* $\mathbf{x}(0) = \mathbf{x}_0$. *Then for any initial condition, we have*

$$\mathbf{x}^*(t; \mathbf{x}_0) \to \mathbf{x}^*, \mathbf{H}^*(t; x_0) \to \mathbf{H}^* \quad as \ t \to \infty$$

There is a new element contained in this theorem. Standard value loss arguments assume concavity of F and make one more support argument like that used to obtain term D_1 in order to obtain term D_2. We use Assumption A2 here. At this level of generality the theorem holds for any $F(\mathbf{x})$. We must investigate the plausibility of Assumption A2. The first-order conditions for a maximum of $\mathbf{p}^* \cdot \mathbf{F}(\mathbf{x})$ are linear in $\mathbf{x}$, since $F_i(\mathbf{x}), i = 1, \ldots, n$ is quadratic in $\mathbf{x}$. Hence the $1 \times n$ vector defined as $\mathbf{p}^*[\partial_x \mathbf{F}]$, where $\partial_x \mathbf{F}$ is an $n \times n$ Jacobian matrix[8] and $\mathbf{p}^*$ is a $1 \times n$ vector, can be written in the form

$$\mathbf{p}^*[\partial_x \mathbf{F}] = \mathbf{L} - \mathbf{x}^{\mathrm{T}}\mathbf{J}$$

where $\mathbf{L}$ is a $1 \times n$ vector, $\mathbf{J}$ is an $n \times n$ matrix which will be nonsingular, generically and $\mathbf{x}^{\mathrm{T}}$ is the transpose of $\mathbf{x}$, a $1 \times n$ row vector. Hence $\mathbf{x}$ that solves

$$\mathbf{0} = \mathbf{L} - \mathbf{x}^{\mathrm{T}}\mathbf{J} \tag{16}$$

will be generically unique. Since $\mathbf{x}^*$ solves (16), Assumption A2 is automatically satisfied for this case provided that $\mathbf{x}^*$ is non-negative.

The main theorem above suggests that we should expect convergence of the SOMP to OSS for small ρ by a continuity argument. This kind of argument is formalized in discrete time by Scheinkman's visit lemma argument (cf. Becker and Boyd, 1997). We expect a similar result to hold in continuous time. More importantly, many generalizations of the above argument are suggested. For example suppose systems of the form $\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x})$ are placed upon sites and are coupled by diffusion across such sites as in Clark (1990). For example let us couple two such systems as follows:

$$\dot{x}_1 = F_1(x_1) - h_1 + z(x_2 - x_1), \qquad \dot{x}_2 = F_2(x_2) - h_2 + z(x_1 - x_2)$$

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[8] The notation $\partial_x \mathbf{f}$, where $\mathbf{f}$ is a vector valued function denotes the Jacobian matrix.

The coupling only adds linear terms, therefore the same argument can be applied as above to prove that the SOMP converges to a unique OSS under zero discounting of the future. Hence, no matter how complicated the coupling and no matter how many sites, so long as the diffusion terms only add linear terms, the same argument applies to obtain convergence.

3.1. Convergence of SOMP for Clark/Schaeffer models

The *value loss* arguments that we made above, which extend methods reviewed in Carlson et al. (1991), are not directly applicable to Clark/Schaeffer (cf. Clark, 1990), where harvest $H_i = q_i E_i x_i$ and resource dynamics are defined as

$$\dot{x}_i = x_i \left(s_i - \sum r_{ij} x_j \right) - q_i E_i x_i = x_i f_i(\mathbf{x}) - q_i E_i x_i,$$

$$f_i(\mathbf{x}) = \left(s_i - \sum r_{ij} x_j \right), \quad i = 1, 2, \ldots, n \tag{17}$$

Let $S(\mathbf{H})$ be social benefit, $\mathbf{p} = \mathbf{P}(\mathbf{H}) = \partial_H S(\mathbf{H})$ and assume $S(\mathbf{H})$ is concave nondecreasing in the harvest vector $\mathbf{H}$. Net benefit is given by $S(\mathbf{H}) - \mathbf{c} \cdot \mathbf{E}'$, $\mathbf{c} = (c_1, \ldots, c_n)$, $\mathbf{E} = (E_1, \ldots, E_n)$. The OSS problem for the Clark/Schaeffer model can be written as[9]

$$\max S(\mathbf{H}) - \mathbf{c} \cdot \mathbf{E},$$

$$\text{s.t. } \mathbf{0} = \mathbf{F}(\mathbf{x}) - \mathbf{H}, \quad \mathbf{F}(\mathbf{x}) = \mathbf{x} \otimes \mathbf{f}(\mathbf{x}) \tag{18}$$

Suppose we have aggregated species in the model to the extent that it is sensible to assume that all $x_i^* > 0$. Let $\mathbf{c}/\mathbf{q}$ denote the $1 \times n$ vector with ith element c_i/q_i. Notice that the social optimum problem can then be written as

$$\max_{\mathbf{x}} \ S(\mathbf{F}(\mathbf{x})) - \left(\frac{\mathbf{c}}{\mathbf{q}} \right) \cdot \mathbf{f}(\mathbf{x})$$

Let

$$\mathbf{x}^* = \arg \max S(\mathbf{F}(\mathbf{x})) - \left(\frac{\mathbf{c}}{\mathbf{q}} \right) \cdot \mathbf{f}(\mathbf{x})$$

and assume $\mathbf{x}^*$ is unique. Notice that for the case $n = 1$, problem (18) is a concave problem because $S(\mathbf{H})$ is concave and nondecreasing in $\mathbf{H}$, $F(\mathbf{x})$ is concave in x and $f(\mathbf{x})$ is linear in $\mathbf{x}$. In the general model, however, because of species interactions coefficients r_{ij} in (17), problem (18) will *not* be a concave programming problem even though $F_i(\mathbf{x})$ is quadratic and $f_i(\mathbf{x})$ is linear. However, for $r_{ij} = 0$ for $i \neq j$, it *is* a concave programming problem. Hence there will be an open set of $\mathbf{A}$ matrices for which (18) is a concave problem. The ith equation of (17) can be solved for E_i to obtain

$$E_i = \left(\frac{1}{q_i} \right) \left[f_i(\mathbf{x}) - \left(\frac{\mathrm{d}x_i/\mathrm{d}t}{x_i} \right) \right]$$

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[9] The notation $\mathbf{z} = \mathbf{x} \otimes \mathbf{y}$ stands for the vector $\mathbf{z}$ with ith element, $z_i = x_i y_i$. That is, it is a product which is formed by multiplying each element of the first vector $\mathbf{x}$ by the corresponding element of the second vector $\mathbf{y}$. In our case $\mathbf{x} \otimes \mathbf{f}(\mathbf{x})$ is an $n \times 1$ vector.

Notice the appearance of the derivative of the natural logarithm of x_i in this equation. This will be important in what follows. Now consider the following:

$$W(T) - W^*(T) = \int_0^T [S(\mathbf{H}) - \mathbf{c} \cdot \mathbf{E} - S(\mathbf{H}^*) + \mathbf{c} \cdot \mathbf{E}^*] \, dt$$

$$= \int_0^T \{\mathbf{p}^* \cdot [\mathbf{F}(\mathbf{x}) - \dot{\mathbf{x}} - \mathbf{F}(\mathbf{x}^*)] - \mathbf{c} \cdot (\mathbf{E} - \mathbf{E}^*)\} \, dt - D_1$$

The term $\int_0^T -\mathbf{c} \cdot (\mathbf{E} - \mathbf{E}^*) \, dt$ can be written as

$$\int_0^T -\mathbf{c} \cdot (\mathbf{E} - \mathbf{E}^*) \, dt = \int_0^T -\left(\frac{\mathbf{c}}{\mathbf{q}}\right) \cdot [\mathbf{f} - \mathbf{f}^*] \, dt + \int_0^T \sum_{i=1}^n \left(\frac{c_i}{q_i}\right) \frac{d \ln(x_i)}{dt} \, dt$$

Putting it all together we finally obtain

$$W(T) - W^*(T) = \mathbf{p}^* \cdot [\mathbf{x}_0 - \mathbf{x}(T)] + \sum_{i=1}^n \left(\frac{c_i}{q_i}\right) [\ln(\mathbf{x}_i(T)) - \ln(\mathbf{x}_i(0))] - D_1$$

$$+ \int_0^T \left[\mathbf{p}^* \cdot \mathbf{F}(\mathbf{x}) - \left(\frac{\mathbf{c}}{\mathbf{q}}\right) \cdot \mathbf{f}(\mathbf{x})\right] - \left[\mathbf{p}^* \cdot \mathbf{F}(\mathbf{x}^*) - \left(\frac{\mathbf{c}}{\mathbf{q}}\right) \cdot \mathbf{f}(\mathbf{x})^*)\right] \, dt$$

Finally recall that $\mathbf{F}(\mathbf{x})$ is quadratic and $\mathbf{f}(\mathbf{x})$ is linear. Therefore the programming problem

$$\max \left[\mathbf{p}^* \cdot \mathbf{F}(\mathbf{x}) - \left(\frac{\mathbf{c}}{\mathbf{q}}\right) \cdot \mathbf{f}(\mathbf{x})\right] \tag{19}$$

is a quadratic programming problem with FONC for optimal $\mathbf{x}$ which are linear in $\mathbf{x}$. Furthermore, the FONC for optimal $\mathbf{x} = \mathbf{x}^*$ that solve (18) are given by

$$[\partial_{\mathbf{x}} S][\partial_x \mathbf{F}] - \left(\frac{\mathbf{c}}{\mathbf{q}}\right) [\partial_x \mathbf{f}] = \mathbf{0} \tag{20}$$

where $[\partial_{\mathbf{x}} S][\partial_x \mathbf{F}]$ and $(\mathbf{c}/\mathbf{q})[\partial_x \mathbf{f}]$ are $1 \times n$ vectors.[10] The FONC for (19) are given by

$$\mathbf{p}^*[\partial_x \mathbf{F}] - \left(\frac{\mathbf{c}}{\mathbf{q}}\right) [\partial_x \mathbf{f}] = \mathbf{0} \tag{21}$$

Since $\mathbf{p}^* = \partial_{\mathbf{x}} S(\mathbf{F}(\mathbf{x}^*))$ then $\mathbf{x}^*$ solves (21) since it solves (20) with $\partial_x S = \mathbf{p}^*$.

Since (21) is a linear set of equations, generically $\mathbf{x}^*$ is the one and only solution to (21). Consequently, if the optimum to (19) is interior, which we shall assume, then the optimum is $\mathbf{x}^*$ by the argument above. Hence we have the value loss statement

$$\mathbf{p}^* \cdot \mathbf{F}(\mathbf{x}) - \left(\frac{\mathbf{c}}{\mathbf{q}}\right) \cdot \mathbf{f}(\mathbf{x}) \le \mathbf{p}^* \cdot \mathbf{F}(\mathbf{x}^*) - \left(\frac{\mathbf{c}}{\mathbf{q}}\right) \cdot \mathbf{f}(\mathbf{x}^*) \quad \forall \mathbf{x} \ge 0.$$

Thus we may write

$$-D_2 = \int_0^T \left[\mathbf{p}^* \cdot \mathbf{F}(\mathbf{x}) - \left(\frac{\mathbf{c}}{\mathbf{q}}\right) \cdot \mathbf{f}(\mathbf{x})\right] - \left[\mathbf{p}^* \cdot \mathbf{F}(\mathbf{x}^*) - \left(\frac{\mathbf{c}}{\mathbf{q}}\right) \cdot \mathbf{f}(\mathbf{x}^*)\right] \, dt$$

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[10] $[\partial_x \mathbf{F}]$ is the Jacobian matrix of $\mathbf{F}$ and $[\partial_x \mathbf{f}]$ the Jacobian matrix of $\mathbf{f}$.

Now that this preliminary work is done, an obvious adaptation of value loss arguments reviewed in Carlson et al. (1991) gives us the following theorem.

Theorem 2. *Let* $\mathbf{x}^*(t|\mathbf{x}_0)$, $\mathbf{E}^*(t|\mathbf{x}_0)$ *solve the generalized Clark/Schaeffer model*

$$\max \int_0^T [\mathbf{S(H)} - \mathbf{c} \cdot \mathbf{E}] \, dt,$$
subject to (17)

Then if the OSS $\mathbf{x}^*$, $\mathbf{E}^*$ *is unique, we have*

$$(\mathbf{x}^*(t|\mathbf{x}_0), \mathbf{E}^*(t|\mathbf{x}_0)) \to (\mathbf{x}^*, \mathbf{E}^*) \quad as \ t \to \infty$$

4. Regulatory approaches

Having characterized the steady state and its stability properties at the SOMP, we turn now to an analysis of regulatory approaches that would direct a harvested system towards the socially-optimal steady state. In this section we consider the case where species biomass representing the natural system, converges fast to the steady state, relative to the effort variable which represents the economic system. For an open access fishery this means that equilibrium effort is determined by industry dynamics associated with expansion or contraction of effort, for a *given* bionomic equilibrium biomass. In Section 5 this assumption is reversed and effort is the variable that relaxes fast to its steady state.

4.1. The single species case

We start by considering the case of a single species model because of the insight that it provides into the more general models. In the open access situation (Gordon, 1954), $p(t)$ and $x(t)$ are taken as parameters and the flow of economic rent is defined as $(pqx - c)E$, $p = P(qEx)$. Thus the conditions for bionomic equilibrium for $x > 0$ are:

$$0 = x(s - rx - qE), \quad x = \frac{s - qE}{r}, \quad P(qEx)qx = c \quad \text{or} \quad J_1(E) = c,$$

$$J_1(E) = P(aE(b - E))a(b - E), \quad a = \frac{q^2}{r}, \quad b = \frac{s}{q} \quad \text{with}$$

$$J_1'(E) = aP\left[-1 + (b - E)\left(\frac{P'}{P}\right)a(b - 2E) \right]$$

Since $J_1(E)$ need not be monotonic in E, there could be multiple bionomic equilibria. Suppose that under open access fishing, the industry expands effort when profits are positive and shrinks it when profits are negative. That is, suppose the adjustment mechanism

$$\dot{E} = \varphi[(J_1(E) - c)], \quad \varphi > 0 \tag{22}$$

describes the industry dynamics. Since $(J_1(E) - c)E$ is short-term equilibrium profits when the industry is putting out effort E, one would expect positive profits to attract more effort

into the industry and vice versa for negative profits. Thus

$$\dot{E} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \ \text{as} \ J_1(E) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} c$$

It is natural to assume that demand price is higher than c for small $Y = qEx$. Thus demand price is higher than c for small E and large E slightly below s/q. Therefore, the smallest steady state of (22) will be locally stable under (22). If there are only two steady states, only the smallest will be stable. If there are three steady states, the middle one will be unstable.

The socially-optimal effort, E^* for this case, is defined by the OSS conditions for the SOMP with $\rho = 0$, or

$$\begin{aligned} &\max_E \ S(qEx) - cE, \\ &\text{s.t.} \ 0 = x(s - rx - qE) \end{aligned} \tag{23}$$

The FONC of (23) imply that E^* is obtained as the solution of

$$J_2(E) = c, \qquad J_2(E) = p\left(\frac{q}{r}\right)(s - 2qE) = P(aE(b - E))a(b - 2E)$$

We consider a tax per unit effort as a regulatory instrument. Taxes for fishery management have been proposed by economists in the form of landing taxes or fees, because of the advantages they have in decentralized decision making and the ability to maintain a fishery at the social optimum. Fishermen, however, oppose such taxes so that this type of regulation has not been regarded as a serious market-based instrument for fishery management relative to individual transferable quotas.[11] As is well known, under certainty there is an equivalence between landing taxes and transferable quotas.[12] Taxes could, however, be preferable to quotas on distributional grounds. As noted by Weitzman (2002) for the Icelandic fishery where individual transferable quotas were given free, the emerging rents from the fishery have created undesirable distributional impacts. In this case a landing fee with a transfer of the receipts could alleviate the distributional problems without impeding economic efficiency. To capture these ideas we consider a situation where the fishermen own catch quotas which are transferable and a tax τ per unit effort, E, is imposed. The tax receipts are to be redistributed lump sum to the fishermen by the regulator. An optimal tax per unit effort τ^* should be such that the bionomic equilibrium effort for the regulated fishery is E^*.

Since, however, bionomic equilibrium is determined by the steady states of (22), while the socially-optimal equilibrium is determined by $E^* : J_2(E^*) = c$, we characterize the optimal tax per unit effort τ^* in terms of the structure of bionomic and socially-optimal equilibria.

We examine first the curve $J_1(E)$. Two cases can be distinguished:

1. $J_1(E)$ is downward sloping on $(0, b)$;

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[11] A fishery resource landing tax exists in Alaska and is based on the unprocessed value of the resource. Transferable quotas programs have been used in countries such as Iceland, New Zealand, Australia and Canada.
[12] In a recent paper, Weitzman (2002) shows that under fish stock uncertainty landing fees are superior to quotas.

2. $J_1(E)$ initially decreases, then increases, then decreases on $(0, b)$, so that multiple bio-nomic equilibria exist.

Proposition 1. *For $J_1(E)$ to initially decrease, then increase, then decrease again on $(0, b)$, so that multiple bionomic equilibria exist, it is necessary that $|\varepsilon_P| < 1$ somewhere in the domain of E, where ε_P is the price elasticity of demand.*

For proof, see Appendix A.

Thus we see that demand elasticity must be small enough relative to biological and economic parameters for a solution of $J_1'(E) = 0$ to exist in $(0, b)$. When such a solution exists, J_1 initially decreases, then increases, then decreases again on $(0, b)$. Furthermore, as can be seen from the definition of J_1' above, $J_1' < 0$ for $E \in [0, b/2]$, since $(P'/P) < 0$. Thus the J_1 curve has the shape presented in Figs. 2 and 3. In general we could have one or an odd number of equilibria for (22), with the direction of the flow as shown in Figs. 1–3. A locally unstable equilibrium is between a low effort locally stable equilibrium and a high effort locally stable equilibrium.

For example with a linear demand function $P(Y) = A - BY$, J_1 is given by $J_1(E) = A(1 - \beta a(b - E)E)a(b - E)$, $\beta = B/A$, assuming that the market is large enough so that
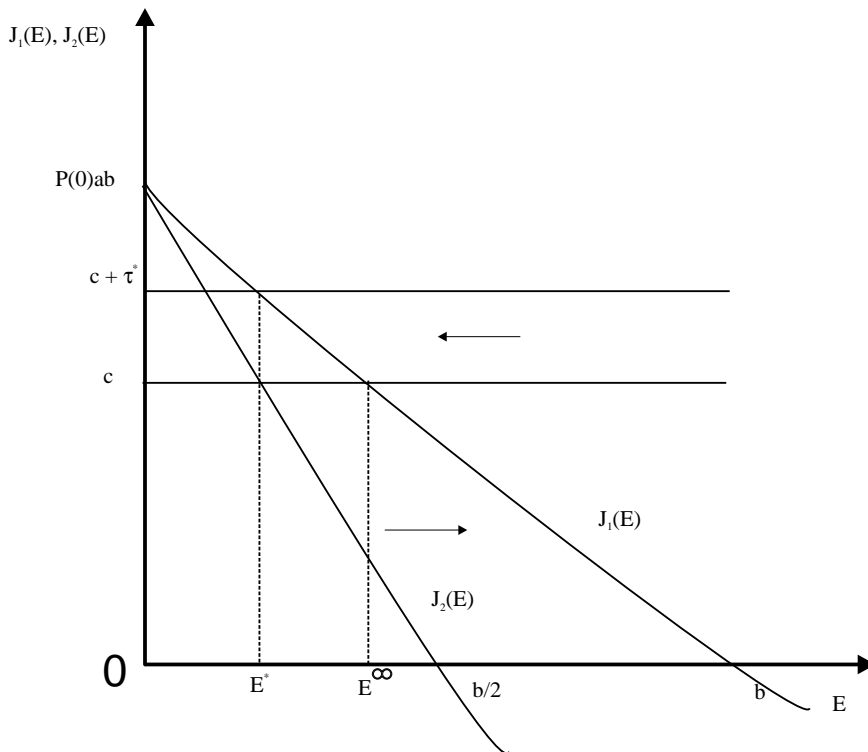

Fig. 1. Social optimum and bionomic equilibrium for a single species, one equilibrium.
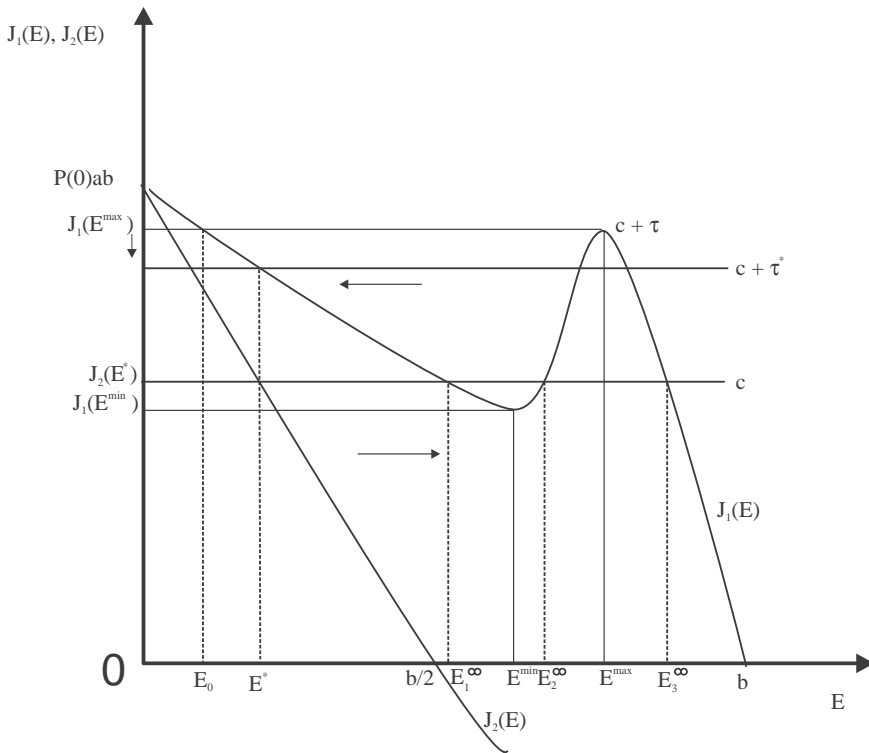
Fig. 2. Social optimum and bionomic equilibrium for a single species, three equilibria and hysteresis.

$1/\beta > \max\{a(b-E)E\}$. In this case $J_1(E)$ is a cubic function, that in general provides three equilibria as solutions to the cubic equation $(1 - Ba(b-E)E)a(b-E) - c' = 0$, $c' = c/A$.[13] For the three equilibria linear demand model, the curve $J_1(E)$ is decreasing in $[0, b/2]$ and has a local minimum and a local maximum in $(b/2, b)$. The local extrema can be determined

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[13] The roots of this equation are given by

$$E_1 = \frac{1}{6} \frac{K_1^2 + 4\beta a(\beta ab^2 - 3 + bK_1)}{\beta a K_1},$$

$$E_2 = -\frac{1}{12} \frac{K_1^2 + 4\beta a(\beta ab^2 - 3 + 2bK_1) + i\sqrt{3}(-K_1^2 + 4\beta^2 a^2 b^2 - 12\beta a)}{\beta a K_1},$$

$$E_3 = -\frac{1}{12} \frac{K_1^2 + 4\beta a(\beta ab^2 - 3 + 2bK_1) - i\sqrt{3}(-K_1^2 + 4\beta^2 a^2 b^2 - 12\beta a)}{\beta a K_1}$$

where

$$K_1 = \sqrt[3]{-4\left(2b^3\beta a^2 - 9ab + 27c' - 3\sqrt{3}\sqrt{\left(\frac{-\beta a^2 b^2 + 4a + 4b^3\beta^2 a^2 c' - 18\beta abc' + 27\beta(c')^2}{\beta}\right)}\right)\beta^2 a}$$

Fig. 3. Social optimum and bionomic equilibrium for a single species, three equilibria without hysteresis trap.

as follows. The first and second derivatives of $J_1(E)$ are

$$\frac{\mathrm{d}J_1(E)}{\mathrm{d}E} = 4\beta a^2 Eb - 3\beta a^2 E^2 - \beta a^2 b^2 - a, \qquad \frac{\mathrm{d}^2 J_1(E)}{\mathrm{d}E^2} = 4\beta a^2 b - 6\beta a^2 E$$

From $\mathrm{d}J_1(E)/\mathrm{d}E = 0$ we obtain for $\beta ab^2 - 3 > 0$ the two real roots $E^{\max} = (1/3)[2\beta ab + \sqrt{\beta}\sqrt{a}\sqrt{(\beta ab^2 - 3)}]/\beta a$, $E^{\min} = (1/3)[2\beta ab - \sqrt{\beta}\sqrt{a}\sqrt{(\beta ab^2 - 3)}]/\beta a$. It can easily be seen that $\mathrm{d}^2 J_1(E^{\max})/\mathrm{d}E^2 < 0$, $\mathrm{d}^2 J_1(E^{\min})/\mathrm{d}E^2 > 0$. Thus $E^{\max}$ is the local maximum and $E^{\min}$ the local minimum. In the (a, b) space the graph of the function $b = \sqrt{3/\beta a}$ is a bifurcation curve. When the biological parameters cross the curve in the sense that they move to the subspace where $b > \sqrt{3/\beta a}$, then one equilibrium (the single root of $J_1(E) = c$) becomes three equilibria (the three roots of $J_1(E) = c$). Furthermore, the depth of the bump in $J_1(E)$ is

$$\Delta = J_1(E^{\max}) - J_1(E^{\min}) = \frac{4}{27\sqrt{\beta}}\sqrt{a}(\beta ab^2 - 3)^{3/2} \tag{24}$$

Therefore, multiple equilibria will occur if unit harvesting cost c and market size A are such that $J_1(E^{\min}) < c/A < J_1(E^{\max})$. The width $E^{\max} - E^{\min}$ is determined as $(2/3\sqrt{\beta}\sqrt{a})\sqrt{(\beta ab^2 - 3)}$.

The presence of multiple equilibria and hysteresis requires inelastic demand for a fishery. In reality inelastic demand estimates have been obtained, for example for pelagic fish (herring, mackerel, sardine, anchovies and tuna) in the EU (Nielsen, 1999), inelastic demand for fish have been reported by Lechene (2001) in the UK, while inelastic demand for the groups of lobster-shrimp-crab and cuttlefish-squid-octopus have also been obtained for Japanese demand for fish (Eales et al., 1997). On the other hand, as shown by (24) the depth of the bump in $J_1(E)$ that determines the hysteresis effects is positively related to $a = q^2/r$ and $b = s/q$, since the logistic growth equation can be written as $sx(1 - (r/s)x)$. Thus we can define $r/s = 1/k$, where k is the carrying capacity of the environment. It follows then from (24) that $\partial \Delta / \partial k > 0$. Therefore the hysteresis effect is more likely to emerge, the larger is the carrying capacity of the environment.

On the other hand, since the J_2 curve is downward sloping on $(0, b/2)$ as shown below, a unique socially-optimal effort level exists.

Proposition 2. *When demand price is higher than c for small E, a unique socially-optimal effort level exists on* $(0, b/2)$.

For proof, see Appendix A.

Thus while under inelastic demand the open access bionomic equilibrium could produce three equilibria, the social optimum results in a unique stable equilibrium. Given these discrepancies we examine specific regulatory approaches.

4.1.1. Taxation on unit effort

To determine the tax per unit effort that can attain the socially-optimal effort E^*, we consider the following two cases.

Case I. Both J_1 and J_2 are downward sloping on $(0, b)$, $(0, b/2)$, respectively.

In this case if E^* is the unique solution of $J_2(E) = c$, we can find τ^* such that $J_1(E) = c + \tau^*$ implies $E = E^*$. This case is depicted in Fig. 1.

Case II. Function J_1 initially decreases, then increases, then decreases on $(0, b)$, while J_2 decreases on $(0, b/2)$.

Let $J_1(E^{\min})$ denote the local minimum and $J_1(E^{\max})$ the local maximum of J_1 on $(0, b)$. The following cases can be considered:

1. $J_2(E^*) \in (J_1(E^{\min}), J_1(E^{\max}))$ as shown in Fig. 2. There are three bionomic equilibria, two locally stable (E_1^{∞} and E_3^{∞}), and one (E_2^{∞}) which is locally unstable. Bionomic equilibrium depends on initial conditions:
   (i) Bionomic equilibrium is at E_1^{∞}. Then a tax τ^* can be defined as in Case I to attain the socially-optimal effort E^*.
   (ii) Bionomic equilibrium is at E_3^{∞}. Application of the same tax will not bring down effort to E^* since the locally stable equilibria are located on the right side of the graph of $J_1(E)$ against E. As τ continues to increase, there will be a hard loss of

stability at $c + \tau$ equal to the local maximum of J_1 on $(0, b)$, which moves the system to the left side of the graph at $E_0 < E^*$. Thus the regulator's effort is below the social optimum. To steer the system to the socially-optimal effort, the tax should be reduced to τ^*, in order to implement E^*. This is the hysteresis effect. The system is attracted to the high effort basin of attraction, while regulation seeks to move the system to the low effort basin of attraction. The hysteresis effect emerging from the nonlinearity, requires that in order for the system to break through towards the low effort basin of attraction the tax should be increased above the socially-optimal level. Once the system has moved to the low effort basin, the tax can be reduced to the socially-optimal level.[14] However, the initial excessive increase of the tax might create serious implementation problems, given the reluctance of fishing communities to accept taxation as the appropriate management tool.

2. $J_2(E^*) > J_1(E^{\max})$ as shown in Fig. 3. This case is equivalent to Case I where both J_1 and J_2 are downward sloping and E^* can be implemented by a tax per unit effort.

The above results indicate that if demand is elastic enough ($J_1' < 0$ on $(0, b)$) then we may control a bionomic open access equilibrium with taxation of effort.[15] On the other hand, if demand is inelastic so that multiple open access bionomic equilibria emerge, then there are cases where the implementation of the socially-optimal effort requires first increasing and then decreasing the tax on effort, in order to counterbalance the effects of hysteresis.

4.2. The multiple species case

We briefly consider regulation of the general multi-species case, where $i = 1, \ldots, n$. The conditions for bionomic equilibrium for $\mathbf{x} > \mathbf{0}$ are

$$\mathbf{0} = \mathbf{s} - \mathbf{R}\mathbf{x} - \mathbf{q} \otimes \mathbf{E}, \quad \mathbf{R} = [r_{ij}], \quad i, j = 1, \ldots, n,$$
$$\mathbf{P}(\mathbf{q} \otimes \mathbf{E} \otimes \mathbf{x}) \otimes \mathbf{q} \otimes \mathbf{x} = \mathbf{c}, \quad \text{where} \quad \mathbf{x} = \mathbf{R}^{-1}(\mathbf{s} - \mathbf{q} \otimes \mathbf{E}) \qquad (25)$$

Thus the equilibrium effort vector $\mathbf{E}$ is the solution of

$$\mathbf{J}_1(\mathbf{E}) = \mathbf{c}, \qquad \mathbf{J}_1(\mathbf{E}) = \mathbf{P}(\mathbf{q} \otimes \mathbf{E} \otimes \mathbf{R}^{-1}(\mathbf{s} - \mathbf{q} \otimes \mathbf{E})) \otimes \mathbf{q} \otimes \mathbf{R}^{-1}(\mathbf{s} - \mathbf{q} \otimes \mathbf{E})$$
$$(26)$$

Provided that a solution to (26) exists, then since $\mathbf{J}_1(\mathbf{E})$ need not be monotonic in $\mathbf{E}$, there could be multiple bionomic equilibria as in the single species case. In this case we seek equilibrium solutions that are locally stable under the industry dynamics

$$\dot{\mathbf{E}} = \boldsymbol{\phi}(\mathbf{J}_1(\mathbf{E}) - \mathbf{c})$$

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[14] The same effect appears in lake management problems studied by Carpenter et al. (1999), Brock and Starrett (1999), Dechert and Brock (2000) and Maler et al. (2000).

[15] Of course this conclusion ignores other problems with taxation of effort such as monitoring, measurement, and policing of other mechanisms of evasion.

On the other hand, the socially-optimal effort level in equilibrium is defined by the OSS conditions for the SOMP with $\rho = 0$ as the vector $\mathbf{E}^*$ that solves the problem

$$\max_{\mathbf{E}} \ \tilde{S}(\mathbf{q} \otimes \mathbf{E} \otimes \mathbf{x}) - \mathbf{cE}$$

with FONC

$$\mathbf{J}_2(\mathbf{E}) = \mathbf{c}, \qquad \mathbf{J}_2(\mathbf{E}) = \mathbf{P}(\mathbf{q} \otimes \mathbf{E} \otimes \mathbf{x})\partial_{\mathbf{E}}[\mathbf{q} \otimes \mathbf{E} \otimes \mathbf{x}], \quad \mathbf{x} = \mathbf{R}^{-1}(\mathbf{s} - \mathbf{q} \otimes \mathbf{E})$$

A regulatory approach with taxes per unit effort can be used to determine taxes (or subsidies) per unit effort $\boldsymbol{\tau}^*$ such that

$$\mathbf{J}_1(\mathbf{E}) = \mathbf{c} + \boldsymbol{\tau}^* \Rightarrow \mathbf{E} = \mathbf{E}^*$$

It is clear that taxes per unit effort face the same difficulties regarding the attainment of the social optimum as in the case of a single species because of hysteresis. The problem could be even worse given the increase in the number of possible equilibria.

5. A role of fast and slow time scales in decentralization possibilities

We pointed out in the sections above a difficulty in using effort taxes to steer an open access fishery to the social optimum. However, we assumed that the biomass relaxes fast to the steady state relative to the economic variables in that treatment. Let us bring issues into bold relief by treating the polar opposite case here. Let "$*$" denote the socially-optimal solution, i.e. of problem (5) and (6).

At date t, let profits for an individual representative fishery with output Y_t taxed at μ_t^* per unit be given by

$$p_t Y_t - cE_t - \mu_t^* Y_t$$

where μ_t^* is the socially-optimal solution for the costate variable associated with the Hamiltonian function of problems (5) and (6). Recall that $Y = qEx$, so assume that effort expands so rapidly relative to the speed of change of x that temporary economic equilibrium given x_t is determined at date t (dropping t subscripts to ease notation) by

$$pqx - c - \mu^*(qx) = 0 \tag{27}$$

$$\dot{x} = x(s - rx) - qEx, \quad x(0) = x_0 \ \text{given}, \tag{28}$$

$$p = P(Y), \qquad Y = qEx \tag{29}$$

If it is assumed that economic equilibration is fast relative to the rate of change of biomass $\dot{x}$, then (27)–(29) is a reasonable abstraction.

Here is the key question for decentralized regulation: will the function $\mu^*(\cdot)$ cause the system (27)–(29) to produce the socially-optimal path $\{x^*(\cdot), E^*(\cdot), p^*(\cdot)\}$, where $p^* = P(Y^*)$, $Y^* = qE^*x^*$?

Clearly, the starred solution *is* a solution of (27)–(29) as can be seen by inspection of the FONC (7)–(9) for the socially-optimal problem. Assume the demand function $P(\cdot)$ maps

the positive real line *onto* itself. Then, Eqs. (27) and (29) can be solved, using (7), to give the solution

$$Y_t = qE_t x_t = P^{-1} \left[\frac{c}{q x_t} + \mu_t^* \right]$$

at each date *t*. Hence we may rewrite (28) as a time dependent differential equation as follows:

$$\dot{x} = x(s - rx) - P^{-1} \left[\frac{c}{q x_t} + \mu_t^* \right] = g(x, t), \quad x(0) = x_0 \text{ given}$$

The key observation is this. The function $x^*(t)$ solves the *same* differential equation. Hence, since $x^*(0) = x(0) = x_0$, the basic uniqueness theorem of solutions of ordinary differential equations delivers (under modest regularity conditions) the result that $x^*(t) = x(t)$ for all non-negative t.[16] This is a powerful result. It says that all we must do is to impose an output tax equal to the social shadow price of a unit of biomass, $\boldsymbol{\mu}^*(t)$, at each date t and let free entry of effort and market equilibration deliver the social optimum. Of course this particular lunch is not free. It relies on the assumption that biomass dynamics move slowly relative to economic dynamics. Our previous result on the potential difficulties of decentralized implementation of social optimum by taxation on effort, assumed the polar opposite assumption regarding relative dynamical adjustment speeds. That is, that result assumed biomass dynamics were fast relative to economic dynamics. Surely the real world is somewhere in between. In any event, our brief treatment of the role of fast/slow dynamical speeds of adjustment of economic dynamics and biological dynamics has revealed an important consideration in the practical design of decentralized implementation of social objectives. That is to say, in systems where there may be multiple stable states of the coupled biological-economic system, it may not be possible to steer the joint dynamics into desirable parts of the state space using decentralized regulatory instruments such as linear catch taxes.

6. Concluding remarks

In this paper we have studied the optimal economic management of multi-species ecosystems by modeling both biomass dynamics and effort dynamics. We show that an equivalence result holds between the social optimum and rational expectations equilibrium and we locate sufficient conditions for infinite horizon management to drive the system to a unique steady state. This was a nontrivial generalization of existing value loss arguments in the turnpike theory literature.

We studied optimal regulation of these systems using decentralized instruments such as taxes. Novelties arose from dealing with problems caused by multiple equilibria and hierarchical time scales. For example we showed that implementation of social optimum by decentralized taxes in open access institutions is possible if biomass dynamics are slow

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[16] This argument is generalized to the case of *n* species, if global invertibility of the demand system is assumed.

enough relative to economic dynamics. But implementation through linear taxes faces difficulties if biomass dynamics are fast relative to economic dynamics because of hysteresis effects. In this paper we did not examine regulation through transferable effort quotas. In the case of multiple bionomic equilibria it could be expected that if unregulated equilibrium is at a high effort basin of attraction and the socially-optimal effort is at a low effort basin of attraction, then regulation with transferable effort quotas, might produce fluctuations in the market for quotas. Whether this type of regulation might be preferred to linear taxation under nonlinearities and hysteresis is an area in which further research could be useful.

Acknowledgements

Appendix A

Proof of Proposition 1. Since the demand price is higher than c for small E, it follows that J_1 initially decreases for small E. Then J_1 becomes negative for $E > b$. Therefore if J_1 fails to be monotone decreasing on $(0, b)$, its first derivative, J_1', must be zero at some point $\hat{E}$ in $(0, b)$. This gives the necessary condition

$$J_1' = aP\left[-1 + (b - E)\left(\frac{P'}{P}\right)a(b - 2E)\right] = 0$$

Since $\varepsilon_P = P/(QP'(Q))$, we have $P'/P = (1/\varepsilon_P)(1/Q)$. Substituting for $Q = aE(b - E)$ and inserting into the necessary condition above gives us

$$(b - E)\frac{1}{\varepsilon_P}\frac{1}{aE(b - E)}a(b - 2E) = 1$$

or $(1/\varepsilon_P)((b - 2E)/E) = 1$, i.e. $\hat{E} = b/(2 + \varepsilon_P)$. Hence for $\hat{E} < b$, we must have $b > b/(2 + \varepsilon_P)$, i.e. $|\varepsilon_P| < 1$. □

Proof of Proposition 2. The first derivative of J_2 is

$$J_2' = 2aP\left[-1 + \frac{1}{2}\left(\frac{P'}{P}\right)a(b - 2E)^2\right] < 0$$

since $P > 0$, $P' < 0$. Therefore J_2 is monotone decreasing on $(0, b/2)$ and a unique socially-optimal effort level exists. □

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