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Forecasting Time Series with Outliers

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ABSTRACT

Time-series data are often contaminated with outliers due to the influence of unusual and non-repetitive events. Forecast accuracy in such situations is reduced due to (1) a carry-over effect of the outlier on the point forecast and (2) a bias in the estimates of model parameters. Hillmer (1984) and Ledolter (1989) studied the effect of additive outliers on forecasts. It was found that forecast intervals are quite sensitive to additive outliers, but that point forecasts are largely unaffected unless the outlier occurs near the forecast origin. In such a situation the carry-over effect of the outlier can be quite substantial. In this study, we investigate the issues of forecasting when outliers occur near or at the forecast origin. We propose a strategy which first estimates the model parameters and outlier effects using the procedure of Chen and Liu (1993) to reduce the bias in the parameter estimates, and then uses a lower critical value to detect outliers near the forecast origin in the forecasting stage. One aspect of this study is on the carry-over effects of outliers on forecasts. Four types of outliers are considered: innovational outlier, additive outlier, temporary change, and level shift. The effects due to a misidentification of an outlier type are examined. The performance of the outlier detection procedure is studied for cases where outliers are near the end of the series. In such cases, we demonstrate that statistical procedures may not be able to effectively determine the outlier types due to insufficient information. Some strategies are recommended to reduce potential difficulties caused by incorrectly detected outlier types. These findings may serve as a justification for forecasting in conjunction with judgment. Two real examples are employed to illustrate the issues discussed.

KEY WORDS Misidentification Forecast errors Innovation outlier Additive outlier Temporary change Level shift Outlier test statistics Critical value

INTRODUCTION

Time-series data are often contaminated with outliers. The impact of outliers on the estimates of model parameters and innovation variance has been extensively studied since Fox (1972) proposed the concepts of additive outliers (AO) and innovational outliers (IO) in time-series

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modeling. More details on other developments in the detection and estimation of outliers can be found in Guttman and Tiao (1978), Abraham and Box (1979), Hillmer *et al.* (1983), Hillmer (1984), Chang *et al.* (1988), Tsay (1989), Ledolter (1989), Chen and Tiao (1990), Chen and Liu (1993), and others. These procedures follow the line of a two-step operation: identifying the locations and the types of outliers, and then adjusting the effects of the outliers for the purpose of model parameter estimation.

To alleviate the influence of outliers in time-series modeling, Martin (1981) considered a robust procedure for the estimation of model parameters in the presence of outliers. In this approach, outlying observations are down-weighted through various types of a ψ-function in the estimation process. However, all types of outliers are treated in the same fashion. This partially explains why some robust methods perform well in some cases and perform poorly in others, as reported in a recent simulation study conducted by Chuang and Abraham (1989). Another disadvantage of this approach is that no information may be obtained regarding the nature of an individual outlier. This may cause substantial loss of forecast accuracy since we cannot effectively incorporate the impact of outliers on forecasts.

On the other hand, Smith and West (1983) proposed an on-line monitoring procedure to detect the presence of abrupt level changes, slope changes and (additive) outliers within a state-space model formulation. They classified each observation of a time series into one of four states (steady state, change in level, change in slope, and outlier) in terms of the patterns of three variances in the state-space model. To apply their proposed procedure, users need to specify the variances and the prior probabilities of each state as the input information. Inference on the outliers and structural changes is based on the posterior probabilities of various states estimated by usual Bayesian approaches. The approach of Smith and West is appropriate when sufficient prior knowledge and experiences about the data and potential structural changes are available.

The main goal of this study is to investigate the outlier issues in time-series forecasting using a detection and adjustment approach. Unlike outliers in time-series modeling, the study of outliers in time-series forecasting is much less extensive and a number of issues still need to be addressed. Hillmer (1984) studied the monitoring and adjustment of forecasts in the presence of outliers under ARIMA models. Ledolter (1989) considered the effect of additive outliers on the forecasts from ARIMA models. Both Hillmer (1984) and Ledolter (1989) concluded that forecast intervals are quite sensitive to additive outliers. However, they found that point forecasts are largely unaffected unless the outlier occurs near the forecast origin. In general, forecasts are affected by outliers through (1) a carry-over effect of the outlier on the point forecast and (2) a bias in the estimates of model parameters. Our study focuses on forecasting issues when the procedure of Chen and Liu (1993) is used for the steps of parameter estimation and outlier detection. In particular, we examine the situations that outliers occur near or at the forecast origin.

In the studies by Hillmer (1984) and Ledolter (1989) only additive and innovational outliers are considered. Since innovational outliers do not affect forecasts (see below), their primary interest is on the effect of additive outliers. In this paper we also consider two other types of outliers, namely, temporary change (TC) and level shift (LS). Our interest is to determine the effects of these outliers on forecasts when they occur at the end of a time series (i.e. at the forecast origin), or near the end of a time series. We shall also explore the loss of accuracy in forecasts when the type of an outlier is misidentified or mis-specified. In studying the accuracy of forecasts, we find that it is quite misleading to simply employ the overall post-sample accuracy measures (e.g. root mean square error, or absolute average percentage error), and more informative measures should be considered. By separating forecast errors during the post-sample period according to the occurrences of outliers, new insights into the comparison of forecast performance are revealed.

As mentioned earlier, the type of an outlier that occurs at the end of a series cannot be determined empirically. Therefore, some judgment on the nature of the outlier is needed. Our study also indicates that the loss of forecast accuracy due to undetected outliers near the forecast origin may be substantial. To forecast in the presence of outliers, one needs to combine the forecast of the uncontaminated series and the forecast of the outlier effects. We propose a strategy which first jointly estimates model parameters and outlier effects using the procedure of Chen and Liu (1993) and then uses a lower critical value to detect outliers near the forecast origin in the forecasting stage. To improve the effectiveness of outlier detection near the end of a time series, we also find it is helpful to restrict the types of outliers for the observations near the forecasting origin.

This paper is organized as follows. In the next section we present the models and the types of outliers considered in this study. In the third section we derive forecast errors and mean square errors of forecasts if outliers that occur near or at the end of a time series are misidentified or mis-specified. Simulation studies are conducted in the fourth section to study the performance of the outlier-detection procedure when an outlier occurs near the end of the series. Two examples are employed in the fifth section to illustrate forecasting time series with outliers proposed in this paper. A summary and conclusions of this paper are provided in the final section.

ARIMA MODELS WITH OUTLIERS

In this paper we shall assume that a time series can be modeled by an autoregressive-integrated moving average, ARIMA(p, d, q), model (Box and Jenkins, 1970):

$$\phi(B)(1-B)^d Z_t = \theta(B)a_t, \qquad t = 1, 2, \ldots, n \tag{1}$$

where n is the number of observations for the series, B is the back shift operator such that $B^i Z_t = Z_{t-i}$, $\phi(B) = 1 - \phi_q B - \cdots - \phi_p B^p$ and $\theta(B) = 1 - \theta_1 B - \cdots - \theta_q B^q$ are two polynomials in B with orders p and q, d is a non-negative integer and a_t's are independently and identically distributed with mean zero and constant variance σ_a^2. We also assume that $\phi(B)$ and $\theta(B)$ have no common factors and that all of their roots lie outside the unit circle. If the time series is seasonal with a periodicity s, we shall use the usual multiplicative ARIMA$(p, d, q) \times (P, D, Q)$ model as the underlying model:

$$\phi(B)\Phi(B^s)(1-B)^d(1-B^s)^D Z_t = \theta(B)\theta(B^s)a_t, \qquad t = 1, 2, \ldots, n \tag{2}$$

where $\Phi(B^s) = 1 - \Phi_1 B^s - \cdots - \Phi_P B^{Ps}$ and $\theta(B^s) = 1 - \theta_1 B^s - \cdots - \theta_Q B^{Qs}$ satisfy certain conditions similar to those of $\phi(B)$ and $\theta(B)$ defined in equation (1) and D is a non-negative integer. Both models in equations (1) and (2) may include a constant term on the right-hand side. To simplify the presentation, however, only models without a constant term will be employed.

There are several ways in which an outlier can affect a time series. In this paper, four types of outliers shall be considered. They are innovational outlier (IO), additive outlier (AO), level shift (LS) and temporary change (TC). Assuming only one outlier occurred in the series at $t = t_1$ and $I_t(t_1) = 1$ when $t = t_1$, $I_t(t_1) = 0$ otherwise, the observed series $\{Y_t\}$ can be expressed as follows:

$$Y_t = Z_t + \omega L(B) I_t(t_1) \tag{3}$$

with

$$L(B) = \frac{\theta(B)}{(1-B)^d \phi(B)} \quad \text{for IO} \tag{4}$$

$$L(B) = 1 \quad \text{for AO} \tag{5}$$

$$L(B) = \frac{1}{(1-B)} \quad \text{for LS} \tag{6}$$

and

$$L(B) = \frac{1}{(1-\delta B)} \quad \text{for TC } (0 < \delta < 1) \tag{7}$$

For a seasonal time series, the $L(B)$ for an innovational outlier needs to modified according to equation (2).

It is useful to note that, except for the case of an IO, the effect of an outlier on the observed series is independent of the model. Also, the AO and LS are the boundary cases of a TC, in which $\delta = 0$ and $\delta = 1$ respectively. For the case of a TC, the outlier produces an initial effect ω at time t_1 and this effect dies out gradually with time. The parameter δ is designed to model the pace of the dynamic dampening effect. In practice, the value of δ can be specified by the analyst. To properly distinguish the nature of a TC from that of an AO or LS, we recommend that $\delta = 0.7$ be used to identify a TC. The performance of such a specification can be found in Chen and Liu (1993). In the case of an AO, the outlier causes an immediate and one-shot effect on the observed series. An LS produces an abrupt and permanent step change in the series.

The effect of an IO on a time series is more intricate than those of the other types of outliers. According to equation (4), we see the effect of an IO is dependent upon the ψ_j weights of the ARIMA model of Z_t. For a stationary series, an IO will produce a temporary effect since the ψ_j's decay to zero exponentially. However, the pattern of $\{\psi_j\}$ for a nonstationary series can be quite different. Depending on the model of Z_t, an IO may produce

(1) An initial effect at the time of its occurrence and a level shift from the following period when Z_t follows an ARIMA(0, 1, 1) model;
(2) An initial effect at the time of its occurrence which will gradually converge to a permanent level shift when Z_t follows an ARIMA(1, 1, 1) model;
(3) A seasonal level shift when Z_t follows a pure seasonal ARIMA(0, 1, 1) model, e.g. a level shift at every January of each year and so on; and
(4) An annual trend change when Z_t follows a multiplicative seasonal ARIMA$(0, 1, 1) \times (0, 1, 1)$ model.

This suggests that the effects of LS and TC outliers may sometimes be approximated by an IO effect.

Effects of an outlier on the forecasts from ARIMA models with known coefficients

To better understand the effect of an outlier on forecasts, we first consider model (3) and assume that the coefficients of the ARIMA model and the outlier effect are known. The l-step-ahead minimum mean square error (MMSE) forecast at time origin t = n can be obtained by combining the forecasts of the two components

$$Y_n(l) = Z_n(l) + \omega L(B) I_{n+l}(t_1) \tag{8}$$

where $Z_n(l)$ is the l-step-ahead MMSE forecast of Z_{n+l} at time origin n. In general, this forecast is

$$Z_n(l) = \pi_1^{(l)} Z_n + \pi_2^{(l)} Z_{n-1} + \cdots \tag{9}$$

where Z_t's can be obtained based on the observations Y_t's, the model parameters and the outlier effects. The π-weights in the above computation are obtained from the following recursive formula (see Box and Jenkins, 1970, p. 142):

$$\pi_j^{(l)} = \pi_{j+l-1} + \sum_{h=1}^{l-1} \pi_h \pi_j^{(l<h)}, \qquad j = 1, 2, \ldots \tag{10}$$

and $\pi_j^{(1)} = \pi_j$ are the π-weights obtained from models (1) or (2). Alternatively, the forecast $Z_n(l)$ can also be computed using

$$Z_n(l) = \psi_l a_n + \psi_{l+1} a_{n-1} + \cdots \tag{11}$$

where ψ_j's are the w-weights obtained from models (1) or (2) (see Box and Jenkins, 1970, p. 128). Under the above ideal situation (i.e. both ARIMA coefficients and outlier effects are known), the l-step-ahead forecast error is

$$e_n(l) = Y_{n+l} - Y_n(l) = Z_{n+l} - Z_n(l) \tag{12}$$

Typically, information concerning the presence or the type of outliers is unknown. One approach to address this issue is to apply the procedure developed by Chen and Liu (1993) to detect the outlier and to obtain the model parameter estimates. The series adjusted for outlier effects can then be regarded as an uncontaminated series and its forecast can be calculated based on the usual recursive formula associated with the ARIMA model. In this case, the l-step-ahead MMSE forecasts at time origin $t = n$ can be derived based on the empirically detected outlier as

$$\hat{Y}_n(l) = \hat{Z}_n(l) + \hat{\omega}\hat{L}(B) I_{n+l}(\hat{t}_1) \tag{13}$$

where $\hat{Z}_n(l)$ is the l-step-ahead MMSE forecast based on the adjusted observations and $\hat{\omega}$, $\hat{L}(B)$ and $\hat{t}_1$ are the estimated outlier effect, type and location, respectively. In other words, forecasts in the presence of outliers can be obtained by adding the forecast of the uncontaminated series (i.e. the outlier-adjusted series) and the extrapolation of the outlier effect. It is important to emphasize that $\hat{Z}_n(l)$ represents the forecast of the uncontaminated series (i.e. the adjusted series). In general, this uncontaminated series is obtained via outlier adjustment based on the location, type and size of the detected outlier. The empirically determined outlier type and location, $\hat{L}(B)$ and $\hat{t}_1$, are the same as $L(B)$ and t_1 if the type of outlier and its location are detected correctly, otherwise they can be different. Thus, the forecast error in this case is

$$\begin{aligned}\hat{e}_n(l) &= Y_{n+l} - \hat{Y}_n(l) \\ &= Z_{n+l} - \hat{Z}_n(l) + (\omega L(B) I_{n+l}(t_1) - \hat{\omega}\hat{L}(B) I_{n+l}(\hat{t}_1))\end{aligned} \tag{14}$$

As discussed in Chen and Liu (1993), an outlier occurred in the middle of the series usually can be detected accurately. However, an outlier near the end of the series is difficult to identify due to insufficient data. In particular, when an outlier occurs at the last period of the series, the test statistics used to determine the outlier type are identical for the four types of outliers in equations (4) to (7) and hence the outlier type cannot be empirically determined in this case. Therefore, forecasters are required to make a judgmental decision as to the nature of the outlier at the forecast origin. In other situations, an outlier may occur at one or two periods before the forecast origin. In such cases, an outlier-detection procedure may not be able to identify the type of the outlier correctly. In the following section we investigate the behavior of forecast errors when an outlier occurs near or at the forecast origin assuming that a detection procedure is applied. The cases to be considered include situations that the outliers may or may not be identified; and if identified, the types may or may not be correct.

FORECAST ERRORS WHEN AN OUTLIER OCCURS NEAR OR AT THE FORECAST ORIGIN

In this section we study the forecast errors when an outlier occurs near or at the forecast origin. We first investigate how forecasts should be calculated when an outlier occurs. We then study the forecast errors and loss of efficiency when misidentification takes place.

The forecast origin is identified as an outlier

Assuming the coefficients of the ARIMA models are known, the effect of an outlier described in equation (3) can be computed based on the filtered series $\hat{u}_t$, where

$$\hat{u}_t = \pi(B)Y_t \tag{15}$$

and $\pi(B)$ is the π-polynomial obtained based on models (1) or (2) (Box and Jenkins, 1970). At the forecast origin $t = n$, the estimate of the outlier effect (for all types) and its test statistic are

$$\hat{\omega} = \hat{u}_n \tag{16}$$

and

$$\hat{\tau} = \hat{\omega}/\hat{\sigma}_a \tag{17}$$

where $\hat{\sigma}_a$ is a robust estimate of σ_a. More details on the estimation of outlier effects can be found in Chen and Liu (1993). In this situation, even though we can test if the observation at $t = n$ is an outlier, a judgmental decision is required to determine the type of the outlier. Depending upon the choice of the outlier type, the l-step-ahead forecast $\hat{Y}_n(l)$ can be computed as follows based on equation (13):

$$\begin{array}{lll} (1) & \text{Not an outlier} & \hat{Y}_n(l)_{\mathrm{NO}} = \hat{Z}_n(l)_{\mathrm{NO}} \\ (2) & \text{IO} & \hat{Y}_n(l)_{\mathrm{IO}} = \hat{Z}_n(l)_{\mathrm{IO}} + \hat{\omega}\psi_l \\ (3) & \text{AO} & \hat{Y}_n(l)_{\mathrm{AO}} = \hat{Z}_n(l)_{\mathrm{AO}} \\ (4) & \text{TC} & \hat{Y}_n(l)_{\mathrm{TC}} = \hat{Z}_n(l)_{\mathrm{TC}} + \hat{\omega}\,\delta^l \\ (5) & \text{LS} & \hat{Y}_n(l)_{\mathrm{LS}} = \hat{Z}_n(l)_{\mathrm{LS}} + \hat{\omega} \end{array} \tag{18}$$

The notation $\hat{Z}_n(l)_{\mathrm{NO}}$ (which is equal to $\hat{Y}_n(l)_{\mathrm{NO}}$) represents the l-step-ahead MMSE forecast of Y_{n+l} without any outlier adjustment, and $\hat{Z}_n(l)_{\mathrm{TP}}$ is the l-step-ahead MMSE forecast of Z_{n+l} based on the adjusted observations if an outlier exists. In the case that an outlier occurs at the forecast origin, the estimated outlier effect for any outlier type is the same ($\hat{\omega}$), and its impact can only be observed at $t = n$. Therefore, a judgmental decision of the outlier type in this case only affects the extrapolation of the outlier effects to the future observations, and has no effect on the adjusted observation at the forecast origin.

The results of equation (18) show that the outlier-adjusted forecasts consist of the sum of the forecasts of the uncontaminated series and the outlier effects on future observations. The former component represents the homogeneous behavior of the series and the latter denotes the impact of the outlier on the future observations. When the forecast origin is detected as an outlier, typical outlier detection procedures are able to provide an estimate for the first component, but may have difficulty to provide complete information for the second.

To obtain a better understanding of the effect of an outlier adjustment on forecasts, we express the outlier-adjusted l-step-ahead forecast $\hat{Y}_n(l)$ relative to typical forecasts without any outlier adjustment:

$$\begin{array}{lll} (1) & \text{Not an outlier} & \hat{Y}_n(l)_{\text{NO}} = \hat{Z}_n(l)_{\text{NO}} \\ (2) & \text{IO} & \hat{Y}_n(l)_{\text{IO}} = \hat{Y}_n(l)_{\text{NO}} \\ (3) & \text{AO} & \hat{Y}_n(l)_{\text{AO}} = \hat{Y}_n(l)_{\text{NO}} - \hat{\omega}\psi_l \\ (4) & \text{TC} & \hat{Y}_n(l)_{\text{TC}} = \hat{Y}_n(l)_{\text{NO}} + \hat{\omega}(\delta^l - \psi_l) \\ (5) & \text{LS} & \hat{Y}_n(l)_{\text{LS}} = \hat{Y}_n(l)_{\text{NO}} + \hat{\omega}(1 - \psi_l) \end{array} \tag{19}$$

The above expression can be easily obtained using results of equations (10), (11) and (16).

The results of equation (19) show the differences among forecasts with outlier adjustment and those if outliers are ignored. It is useful to note that under the IO situation, the l-step-ahead forecast is identical to the forecast if no outlier adjustment is employed. This finding is in congruence with that obtained by Ledolter (1989), and is an alternative way of showing the adaptive nature of the ARIMA forecasts. That is, when an outlier produces an effect that follows the pattern of the ψ-weight memory there is no need to conduct any adjustment for the outlier to obtain a more accurate forecast. This is part of the reason that traditional ARIMA models may still perform well in forecasting even if the time series is subjected to outliers and interventions.

To obtain the statistical properties of the forecast errors, we need to express the forecast of the uncontaminated series in terms of the forecast of the underlying process as described in equation (9). The adjusted series can be obtained by substituting the detected outliers into equation (3). Hence it can be shown that

$$\hat{Z}_n(l)_{\text{TP}} = \begin{cases} Z_n(l) + \psi_l(\omega - \hat{\omega}) & \text{when an outlier occurs at } t = n \\ Z_n(l) - \psi_l\hat{\omega} & \text{when no outlier occurs at } t = n \end{cases} \tag{20}$$

with TP = AO, TC, IO, LS and

$$\hat{Z}_n(l)_{\text{NO}} = \begin{cases} Z_n(l) + \psi_l\omega & \text{when an outlier occurs at } t = n \\ Z_n(l) & \text{when no outlier occurs at } t = n \end{cases} \tag{21}$$

where $Z_n(l)$ is defined in equation (9). Based on equations (3), (17) and (21), we can derive the forecast errors contingent upon the specification of outlier type in terms of $e_n(l)$ defined in equation (12) and the outlier effect. These forecast errors are listed in Table I. In this table the diagonal terms provide the forecast error when the outlier type is correctly specified, and the off-diagonal terms provide the forecast error when the outlier type is mis-specified. In particular, the first row gives the forecast errors when forecasts are made with no adjustment for outliers. It then provides the bias due to an undetected outlier at the forecast origin. Except for the LS outlier, the major effect for an outlier is on short-term forecasting, i.e. when l is small. The effect associated with a mis-specified outlier type depends on the underlying memory pattern $\{\psi_l\}$ and the actual outlier type.

Using the fact that

$$\hat{\omega} = \begin{cases} \omega + a_n & \text{when an outlier occurs at } t = n \\ a_n & \text{when no outlier occurs at } t = n \end{cases} \tag{22}$$

Table I. Forecast error when an outlier occurs at the forecast origin

Forecast	Actual: No	IO	AO	TC	LS
No	$e_n(l)$	$e_n(l)$	$e_n(l) - \omega\psi_l$	$e_n(l) - \omega(\psi_l - \delta^l)$	$e_n(l) - \omega(\psi_l - 1)$
IO	$e_n(l)$	$e_n(l)$	$e_n(l) - \omega\psi_l$	$e_n(l) - \omega(\psi_l - \delta^l)$	$e_n(l) - \omega(\psi_l - 1)$
AO	$e_n(l) + \hat{\omega}\psi_l$	$e_n(l) + \hat{\omega}\psi_l$	$e_n(l) + (\hat{\omega} - \omega)\psi_l$	$e_n(l) + (\hat{\omega} - \omega)\psi_l + \omega\,\delta^l$	$e_n(l) + (\hat{\omega} - \omega)\psi_l + \omega$
TC	$e_n(l) + \hat{\omega}(\psi_l - \delta^l)$	$e_n(l) + \hat{\omega}(\psi_l - \delta^l)$	$e_n(l) + (\hat{\omega} - \omega)\psi_l - \hat{\omega}\,\delta^l$	$e_n(l) + (\hat{\omega} - \omega)(\psi_l - \delta^l)$	$e_n(l) + (\hat{\omega} - \omega)\psi_l + (\omega - \hat{\omega}\,\delta^l)$
LS	$e_n(l) + \hat{\omega}(\psi_l - 1)$	$e_n(l) + \hat{\omega}(\psi_l - 1)$	$e_n(l) + (\hat{\omega} - \omega)\psi_l - \hat{\omega}$	$e_n(l) + (\hat{\omega} - \omega)\psi_l - (\hat{\omega} - \omega\,\delta^l)$	$e_n(l) + (\hat{\omega} - \omega)(\psi_l - 1)$

Table II. MSE of forecasts when an outlier occurs at the forecast origin

Forecast	Actual: No	IO	AO	TC	LS
No	$\sigma_e^2(l)$	$\sigma_e^2(l)$	$\sigma_e^2(l) + \psi_l^2\omega^2$	$\sigma_e^2(l) + (\psi_l - \delta^l)^2\omega^2$	$\sigma_e^2(l) + (\psi_l - 1)^2\omega^2$
IO	$\sigma_e^2(l)$	$\sigma_e^2(l)$	$\sigma_e^2(l) + \psi_l^2\omega^2$	$\sigma_e^2(l) + (\psi_l - \delta^l)^2\omega^2$	$\sigma_e^2(l) + (\psi_l - 1)^2\omega^2$
AO	$\sigma_e^2(l) + \psi_l^2\sigma_a^2$	$\sigma_e^2(l) + \psi_l^2(\sigma_a^2 + \omega^2)$	$\sigma_e^2(l) + \psi_l^2\sigma_a^2$	$\sigma_e^2(l) + \psi_l^2\sigma_a^2 + \omega^2\,\delta^{2l}$	$\sigma_e^2(l) + \psi_l^2\sigma_a^2 + \omega^2$
TC	$\sigma_e^2(l) + (\psi_l - \delta^l)^2\sigma_a^2$	$\sigma_e^2(l) + (\psi_l - \delta^l)^2(\sigma_a^2 + \omega^2)$	$\sigma_e^2(l) + (\psi_l - \delta^l)^2\sigma_a^2 + \omega^2\,\delta^{2l}$	$\sigma_e^2(l) + (\psi_l - \delta^l)^2\sigma_a^2$	$\sigma_e^2(l) + (\psi_l - \delta^l)^2\sigma_a^2 + \omega^2(1 - \delta^l)^2$
LS	$\sigma_e^2(l) + (\psi_l - 1)^2\sigma_a^2$	$\sigma_e^2(l) + (\psi_l - 1)^2(\sigma_a^2 + \omega^2)$	$\sigma_e^2(l) + (\psi_l - 1)^2\sigma_a^2 + \omega^2$	$\sigma_e^2(l) + (\psi_l - 1)^2\sigma_a^2 + \omega^2(1 - \delta^l)^2$	$\sigma_e^2(l) + (\psi_l - 1)^2\sigma_a^2$

we can easily obtain the mean square errors (MSE) of the forecast in correspondence to those shown in Table I. The first row of this table shows the effects of various outliers when outlier detection and adjustment is not employed. We observe that except for IO, the MSE of forecasts increases if an outlier occurs at the end of a series.

By examining Table II row by row it is easy to see that correct outlier detection produces the smallest mean square error. For a given row, the ratio of MSEs to the smallest MSE of the row provides information on the loss of efficiency due to an incorrect judgmental decision for using a specific outlier type. The columns of Table II list the MSE of forecasts under five different conditions. For the cases of IO and no outlier, if there is a mis-specification of an AO, TC or LS, the MSE is increased. When an IO is misidentified as a non-outlier, or an observation is spuriously identified as an IO, no loss of MSE is involved. This result may provide a basis to argue that perhaps there is no need to consider detecting an IO in the context of forecasting (Ledolter, 1989). However, this may not be a sound suggestion considering that when an IO is misidentified as AO, TC or LS, there is always a loss of efficiency in the MSE.

In general, the loss of efficiency in MSE due to a mis-specified outlier type depends on (1) ψ-weights of the underlying process, (2) the size of the outlier effect ω and (3) outlier types. When an AO, TC or LS is mis-specified as an IO or no outlier, or when an AO is mis-specified as a TC or an LS, the MSE of forecasts is always greater than that of a correct detection if $|\omega| > \sigma_a$. When a TC or an LS is mis-specified as an AO, TC or LS, the MSE increases if

$$|\omega| > \sigma_a \sqrt{\frac{2(1-\psi_l)}{1-\delta^l}+1}$$

Outlier occurs k periods before the forecast origin

When the outlier occurs k periods before the forecast origin n, the observation Y_t can be expressed as

$$Y_{n-i} = \begin{cases} Z_{n-i} & i > k \\ Z_{n-i} + \omega L(B) I_{n-i}(n-k) & i \leqslant k \end{cases} \tag{23}$$

The residuals obtained by filtering the series Y_t with the known model parameters can be expressed as

$$\hat{u}_{n-i} = \begin{cases} a_{n-i} & i > k \\ a_{n-i} + \omega\pi(B) L(B) I_{n-i}(n-k) & 0 \leqslant i \leqslant k \end{cases} \tag{24}$$

A misidentified outlier may result from a correct location detection but a wrong type specification, from a correct type identification but a wrong location identification, or from both incorrect type and location. In this investigation of the effect of a misidentified outlier on the forecast, we only consider the case that the location of an outlier is correctly identified but the type of an outlier may be mis-specified. The results of other cases are complicated and do not provide further insight on the outlier effects on the forecast and hence are not presented here.

When an outlier is detected at k periods before the forecast origin n, the estimates for different types of outliers are

$$\begin{aligned}
&\hat{\omega}_{\mathrm{IO}} = \hat{u}_{n-k}, && \hat{\tau}_{\mathrm{IO}} = \hat{\omega}_{\mathrm{IO}}/\hat{\sigma}_a \\
&\hat{\omega}_{\mathrm{AO}} = (\hat{u}_{n-k} - \pi_1 \hat{u}_{n-k+1} - \cdots - \pi_k \hat{u}_n)/SS(\pi)_{\mathrm{AO}}, && \hat{\tau}_{\mathrm{AO}} = \hat{\omega}_{\mathrm{AO}}/(\hat{\sigma}_a/SS(\pi)_{\mathrm{AO}}) \\
&\hat{\omega}_{\mathrm{TC}} = \{\hat{u}_{n-k} + (\delta - \pi_1)\hat{u}_{n-k+1} + \cdots && \hat{\tau}_{\mathrm{TC}} = \hat{\omega}_{\mathrm{TC}}/(\hat{\sigma}_a/SS(\pi)_{\mathrm{TC}}) \\
&\qquad + (\delta^k - \delta^{k-1}\pi_1 - \cdots - \pi_k)\hat{u}_n\}/SS(\pi)_{\mathrm{TC}}, && \\
&\hat{\omega}_{\mathrm{LS}} = \{\hat{u}_{n-k} + (1-\pi_1)\hat{u}_{n-k+1} + \cdots + (1 - \pi_1 - \cdots - \pi_k)\hat{u}_n\}/SS(\pi)_{\mathrm{LS}}, && \hat{\tau}_{\mathrm{LS}} = \hat{\omega}_{\mathrm{LS}}/(\hat{\sigma}_a/SS(\pi)_{\mathrm{LS}})
\end{aligned} \tag{25}$$

where

$$SS(\pi)_{AO} = 1 + \pi_1^2 + \cdots + \pi_k^2$$
$$SS(\pi)_{TC} = 1 + (\delta - \pi_1)^2 + \cdots + (\delta^k - \delta^{k-1}\pi_1 - \cdots - \pi_k)^2$$
$$SS(\pi)_{LS} = 1 + (1 - \pi_1)^2 + \cdots + (1 - \pi_1 - \cdots - \pi_k)^2$$

Using equation (23), the adjusted observations according to the outlier type TP can be expressed as:

$$\hat{Z}_{n-i} = \begin{cases} Y_{n-i} & i > k \\ Y_{n-i} - \hat{\omega}_{TP}\hat{L}(B)I_{n-i}(n-k) & i \leqslant k \end{cases}$$
$$= \begin{cases} Z_{n-i} & i \gtrless k \\ Z_{n-i} + \omega L(B)I_{n-i}(n-k) - \hat{\omega}_{TP}\hat{L}(B)I_{n-i}(n-k) & i \leqslant k \end{cases} \tag{26}$$

Using the above result and equation (9), the l-step-ahead forecast of the adjusted series of the outlier type TP can be expressed as

$$\hat{Z}_n(l)_{TP} = Z_n(l) + \sum_{i=0}^{K} \pi_{i+1}^{(l)}[\omega L(B)I_{n-i}(n-k) - \hat{\omega}_{TP}\hat{L}(B)I_{n-i}(n-k)] \tag{27}$$

where $Z_n(l)$ is the l-step-ahead MMSE forecast of the underlying process $\{Z_t\}$. As in equation (13), we can express $\hat{Y}_n(l)_{TP}$ as

$$\hat{Y}_n(l)_{TP} = \hat{Z}_n(l)_{TP} + \hat{\omega}_{TP}\hat{L}(B)I_{n+l}(n-k) \tag{28}$$

Thus the forecast error defined in equation (14) can be rewritten as

$$\begin{aligned} \hat{e}_n(l) &= Y_{n+l} - \hat{Y}_n(l)_{TP} \\ &= \{Z_{n+l} - Z_n(l)\} \\ &\quad - \left\{\sum_{i=0}^{K} \pi_{i+1}^{(l)}(\omega L(B)I_{n-i}(n-k) - \hat{\omega}_{TP}\hat{L}(B)I_{n-i}(n-k)\right\} \\ &\quad + \{\omega L(B)I_{n+l}(n-k) - \hat{\omega}_{TP}\hat{L}(B)I_{n+l}(n-k)\} \end{aligned} \tag{29}$$

The three components (each enclosed in a pair of brackets) in the above expression respectively represent (1) the intrinsic uncertainty of forecast errors associated with the underlying time series process, (2) sampling errors in the adjustment of outlier effects on the observations and (3) the error associated with the extrapolation of the outlier effect. Substituting $\hat{u}_{t-i}$ of equation (24) into equations (25) and (29), we can derive the MSE of forecast errors as a function of the detected outlier and the true outlier similar to those shown in Table II.

In summary, we have explicitly demonstrated the effect of an outlier that occurs at or near the forecast origin. The potential loss of efficiency due to either ignoring the presence of an outlier or misidentifying the outlier type is also discussed. In practice, a judgmental decision is required to determine the outlier type when it occurs at the forecast origin. When outliers occur a few periods before the forecast origin, the usual outlier detection procedure may not be very effective due to limited available information. A strategy of reducing the critical value for detecting outliers at or near the end of the series is proposed in the next section. A simulation study is also conducted to demonstrate the performance of the proposed method. To reduce the probability of misidentifying outlier type near the end of a time series, it may be useful to restrict the choice of outlier types to an appropriate subset.

OUTLIER DETECTION NEAR OR AT THE FORECAST ORIGIN

As pointed out by Chen and Liu (1993), the estimates of model parameters can be affected by the choice of the critical value for outlier detection in the joint estimation of model parameters and outlier effects. If the critical value is too large, the power of outlier detection will be low. In the extreme situation that a very large critical value is used, it is equivalent to no outlier detection. When a smaller critical value is employed for outlier detection, the power of outlier detection is increased. However, the chance of Type I error (detecting spurious outliers) is also increased. As a result, the estimates of model parameters based on the joint estimation procedure may also be biased due to over-adjustment of spurious outliers. For the purpose of forecasting, we would like to obtain accurate estimates of model parameters, and also to have high power of outlier detection during the forecasting period. To achieve these two goals, we recommend a two-step strategy. (1) for model estimation, use a moderate critical value in the joint estimation procedure; and (2) for forecasting, use the model estimated in (1), but employ a smaller critical value for outlier detection during forecasting.

In this section we present a study of the performance of the above strategy with respect to a range of critical values. In addition to studying the power of outlier detection, it is also desirable to obtain empirical information concerning the nature of type misidentification of outliers. The simulation study considers four major factors: (1) the model of the underlying process, (2) the size of the outlier effect, (3) the location of the outlier, and (4) the magnitude of the critical value. For each outlier type, a time series of length 100 is generated based on one of three models: AR(1), MA(1) and IMA(1,1). The standard deviation of the innovation series is set to 1. Two outlier effect sizes ($\omega = 3$ and $\omega = 4$) and three outlier locations ($t_1 = 50$, $t_1 = 98$ and $t_1 = 100$) are considered. Assuming the model is correctly specified, the simulated series (which is contaminated with an outlier) is fitted by the joint estimation procedure of Chen and Liu (1993) with the critical value $C = 2.75$ for outlier detection. Based on the model estimated in this manner, outlier detection is applied to the same series again using six possible critical values: $C = 2.00$, $C = 2.25$, $C = 2.50$, $C = 2.75$, $C = 3.00$ and $C = 3.25$. For each specification of outlier type, underlying model, outlier effect size and outlier location, the above procedure is repeated 500 times. The main goal of this simulation is to examine the performance of the outlier detection under various situations. The results of this simulation study are summarized in three tables. Table III presents the probabilities of the correct detection of the outlier location. Tables IV and V provide more detailed classification of the types of outlier detected.

Power of outlier detection and its implication

The results in Table III are organized into six groups according to the critical value for outlier detection. For each outlier type, the first row shows the probability of correct detection of location (regardless of type) when the outlier occurred at $t_1 = 50$ and the second row gives those when $t_1 = 98$. The last row in each group presents the results when $t_1 = 100$. It is important to note that when an outlier occurs at the end of the series ($t_1 = 100$) there is no statistical information to determine the outlier type. Except for the case of a level shift, the power of detection is not greatly affected by the underlying models. As expected, the power for the cases of the smaller outlier ($\omega = 3$) is lower than that of the larger outlier ($\omega = 4$). Another study based on a multiplicative seasonal model (the airline model) gives very similar results. The power of detecting an outlier is slightly lower than that of the IMA(0, 1, 1) case, but very close to that of the MA(1). With respect to the location of outliers, we find no major differences when an outlier occurs in the middle of the series ($t_1 = 50$) or near the end ($t_1 = 98$) based on

Table III. Detection power of outlier location

	AR(1)	MA(1) $\omega = 3$	IMA(1, 1)	AR(1)	MA(1) $\omega = 4$	IMA(1, 1)		AR(1)	MA(1) $\omega = 3$	IMA(1, 1)	AR(1)	MA(1) $\omega = 4$	IMA(1, 1)
			$C = 2.00$							$C = 2.75$			
IO	0.90	0.85	0.88	0.98	0.96	0.97	IO	0.69	0.61	0.73	0.93	0.88	0.91
	0.84	0.83	0.87	0.97	0.97	0.98		0.68	0.62	0.72	0.92	0.89	0.92
AO	0.91	0.82	0.91	0.99	0.95	0.98	AO	0.78	0.70	0.78	0.97	0.92	0.94
	0.91	0.81	0.89	0.98	0.94	0.99		0.78	0.71	0.73	0.95	0.92	0.95
TC	0.90	0.84	0.89	0.97	0.97	0.98	TC	0.68	0.63	0.78	0.92	0.92	0.95
	0.84	0.84	0.86	0.97	0.98	0.97		0.67	0.76	0.74	0.93	0.92	0.92
LS	0.78	0.64	0.88	0.91	0.75	0.98	LS	0.62	0.60	0.79	0.84	0.72	0.96
	0.84	0.81	0.86	0.97	0.95	0.96		0.73	0.64	0.78	0.94	0.90	0.95
$T = 100$	0.77	0.77	0.79	0.94	0.91	0.97	$T = 100$	0.58	0.55	0.61	0.89	0.83	0.91
			$C = 2.25$							$C = 3.00$			
IO	0.88	0.84	0.88	0.98	0.96	0.97	IO	0.58	0.54	0.61	0.89	0.84	0.88
	0.83	0.81	0.87	0.97	0.95	0.98		0.61	0.52	0.63	0.89	0.84	0.87
AO	0.90	0.80	0.90	0.99	0.95	0.98	AO	0.72	0.67	0.69	0.96	0.90	0.92
	0.90	0.80	0.88	0.98	0.93	0.99		0.71	0.67	0.64	0.92	0.91	0.91
TC	0.87	0.81	0.89	0.97	0.97	0.98	TC	0.60	0.57	0.68	0.89	0.89	0.92
	0.82	0.83	0.86	0.97	0.97	0.97		0.56	0.75	0.67	0.89	0.90	0.88
LS	0.78	0.62	0.88	0.91	0.73	0.98	LS	0.58	0.61	0.73	0.84	0.72	0.94
	0.83	0.80	0.86	0.97	0.94	0.96		0.67	0.60	0.72	0.93	0.83	0.94
$T = 100$	0.75	0.74	0.79	0.94	0.90	0.97	$T = 100$	0.50	0.48	0.50	0.83	0.79	0.86
			$C = 2.50$							$C = 3.25$			
IO	0.82	0.73	0.82	0.96	0.93	0.95	IO	0.47	0.45	0.52	0.82	0.76	0.83
	0.76	0.73	0.82	0.96	0.93	0.97		0.52	0.41	0.51	0.85	0.77	0.82
AO	0.87	0.77	0.86	0.99	0.95	0.97	AO	0.62	0.64	0.60	0.93	0.89	0.88
	0.85	0.77	0.82	0.97	0.93	0.98		0.59	0.61	0.52	0.87	0.89	0.85
TC	0.78	0.75	0.86	0.95	0.96	0.97	TC	0.48	0.46	0.59	0.81	0.83	0.89
	0.78	0.80	0.82	0.96	0.95	0.95		0.47	0.73	0.56	0.84	0.86	0.82
LS	0.71	0.60	0.86	0.89	0.72	0.97	LS	0.54	0.60	0.67	0.82	0.73	0.94
	0.79	0.71	0.84	0.96	0.92	0.96		0.59	0.57	0.65	0.89	0.77	0.91
$T = 100$	0.69	0.65	0.71	0.92	0.87	0.95	$T = 100$	0.38	0.38	0.40	0.78	0.71	0.79

For IO, AO, TC, and LS cases, first row $t = 50$, second row $t = 98$.

Table IV. Power of outlier detection: a simulation study when an outlier occurs at $t_I = 50$ and $n = 100$ (The first number in each cell represents the probability associated with $\omega = 3$ and the second number associated with $\omega = 4$)

		AR(1) model ($\phi = 0.6$)				MA(1) model ($\theta = 0.6$)				IMA(1,1) model ($\theta = 0.6$)			
		Detected type				Detected type				Detected type			
Actual type		IO	AO	TC	LS	IO	AO	TC	LS	IO	AO	TC	LS
$C = 2.00$	IO	0.348	0.156	0.394	0.000	0.752	0.100	0.000	0.000	0.414	0.202	0.136	0.132
		0.414	0.130	0.438	0.000	0.842	0.120	0.002	0.000	0.562	0.170	0.140	0.094
	AO	0.148	0.686	0.074	0.000	0.162	0.630	0.026	0.000	0.222	0.604	0.066	0.014
		0.106	0.854	0.032	0.000	0.098	0.828	0.028	0.000	0.216	0.744	0.024	0.000
	TC	0.294	0.120	0.482	0.000	0.264	0.120	0.456	0.000	0.138	0.096	0.576	0.080
		0.332	0.060	0.580	0.002	0.029	0.062	0.622	0.000	0.128	0.032	0.776	0.044
	LS	0.110	0.060	0.058	0.552	0.016	0.076	0.044	0.504	0.104	0.016	0.082	0.678
		0.138	0.058	0.046	0.672	0.014	0.094	0.044	0.594	0.064	0.000	0.036	0.878
$C = 2.25$	IO	0.344	0.148	0.390	0.000	0.736	0.102	0.000	0.000	0.410	0.200	0.134	0.132
		0.414	0.130	0.438	0.000	0.836	0.120	0.002	0.000	0.562	0.170	0.140	0.094
	AO	0.148	0.684	0.072	0.000	0.148	0.624	0.028	0.000	0.216	0.602	0.064	0.014
		0.106	0.854	0.032	0.000	0.088	0.832	0.034	0.000	0.216	0.744	0.024	0.000
	TC	0.284	0.118	0.470	0.000	0.252	0.116	0.446	0.000	0.138	0.096	0.576	0.078
		0.332	0.058	0.580	0.002	0.290	0.062	0.622	0.000	0.128	0.032	0.776	0.044
	LS	0.108	0.060	0.054	0.554	0.012	0.064	0.038	0.504	0.102	0.016	0.082	0.678
		0.138	0.056	0.046	0.674	0.004	0.086	0.038	0.604	0.062	0.000	0.036	0.878
$C = 2.50$	IO	0.314	0.136	0.372	0.000	0.636	0.092	0.000	0.000	0.382	0.188	0.124	0.126
		0.408	0.126	0.426	0.000	0.810	0.120	0.002	0.000	0.554	0.160	0.134	0.104
	AO	0.142	0.662	0.064	0.000	0.122	0.608	0.036	0.000	0.208	0.580	0.062	0.012
		0.106	0.850	0.030	0.000	0.082	0.828	0.038	0.000	0.214	0.736	0.024	0.000
	TC	0.264	0.092	0.428	0.000	0.228	0.108	0.410	0.000	0.130	0.088	0.564	0.078
		0.326	0.058	0.568	0.000	0.280	0.062	0.618	0.000	0.126	0.032	0.774	0.042
	LS	0.094	0.050	0.048	0.522	0.004	0.046	0.032	0.514	0.092	0.014	0.078	0.674
		0.130	0.042	0.042	0.678	0.002	0.076	0.030	0.608	0.058	0.000	0.030	0.878
$C = 2.75$	IO	0.272	0.092	0.324	0.000	0.536	0.076	0.000	0.000	0.340	0.162	0.110	0.120
		0.388	0.122	0.416	0.000	0.762	0.116	0.000	0.000	0.534	0.140	0.132	0.108
	AO	0.124	0.610	0.048	0.000	0.104	0.566	0.032	0.000	0.186	0.524	0.060	0.008
		0.102	0.840	0.028	0.002	0.064	0.816	0.038	0.000	0.204	0.718	0.022	0.000
	TC	0.228	0.068	0.378	0.002	0.188	0.088	0.352	0.000	0.108	0.068	0.528	0.074
		0.314	0.056	0.546	0.000	0.266	0.060	0.598	0.000	0.122	0.026	0.764	0.042
	LS	0.082	0.036	0.038	0.468	0.002	0.038	0.022	0.536	0.082	0.004	0.072	0.628
		0.128	0.038	0.038	0.638	0.004	0.076	0.018	0.620	0.058	0.000	0.024	0.878
$C = 3.00$	IO	0.226	0.068	0.290	0.000	0.468	0.074	0.000	0.000	0.294	0.118	0.096	0.106
		0.374	0.116	0.404	0.000	0.724	0.118	0.000	0.000	0.516	0.132	0.128	0.106
	AO	0.110	0.564	0.044	0.000	0.086	0.552	0.036	0.000	0.160	0.474	0.054	0.006
		0.100	0.828	0.026	0.002	0.056	0.810	0.038	0.000	0.200	0.696	0.022	0.002
	TC	0.204	0.058	0.336	0.002	0.166	0.070	0.330	0.000	0.090	0.050	0.480	0.058
		0.308	0.048	0.534	0.000	0.256	0.060	0.574	0.002	0.112	0.024	0.748	0.038
	LS	0.074	0.024	0.028	0.452	0.000	0.032	0.020	0.562	0.070	0.000	0.062	0.600
		0.122	0.034	0.034	0.646	0.002	0.070	0.018	0.626	0.050	0.000	0.026	0.868
$C = 3.25$	IO	0.176	0.046	0.244	0.000	0.388	0.064	0.000	0.000	0.252	0.100	0.080	0.090
		0.346	0.098	0.380	0.000	0.646	0.114	0.000	0.000	0.488	0.126	0.124	0.090
	AO	0.088	0.498	0.038	0.000	0.076	0.524	0.036	0.000	0.132	0.412	0.050	0.006
		0.096	0.804	0.026	0.002	0.054	0.796	0.038	0.000	0.192	0.668	0.022	0.002
	TC	0.156	0.048	0.278	0.002	0.128	0.060	0.276	0.000	0.082	0.046	0.412	0.054
		0.290	0.040	0.484	0.000	0.234	0.054	0.540	0.002	0.104	0.020	0.726	0.038
	LS	0.066	0.010	0.022	0.440	0.000	0.026	0.014	0.564	0.058	0.000	0.050	0.558
		0.120	0.026	0.028	0.642	0.000	0.064	0.018	0.652	0.046	0.000	0.026	0.864

Table V. Power of outlier detection: a simulation study when an outlier occurs at $t_1 = 98$ and $n = 100$ (The first number in each cell represents the probability associated with $\omega = 3$ and the second number associated with $\omega = 4$)

		AR(1) model ($\phi = 0.6$)				MA(1) model ($\theta = 0.6$)				IMA(1,1) model ($\theta = 0.6$)			
		Detected type				Detected type				Detected type			
Actual type		IO	AO	TC	LS	IO	AO	TC	LS	IO	AO	TC	LS
$C = 2.00$	IO	0.328	0.176	0.254	0.084	0.680	0.138	0.006	0.002	0.306	0.178	0.292	0.092
		0.390	0.134	0.358	0.092	0.778	0.186	0.002	0.002	0.470	0.180	0.278	0.052
	AO	0.144	0.698	0.052	0.012	0.104	0.612	0.078	0.014	0.198	0.636	0.046	0.008
		0.136	0.828	0.012	0.004	0.084	0.758	0.092	0.004	0.196	0.768	0.028	0.00
	TC	0.238	0.116	0.302	0.184	0.112	0.102	0.474	0.148	0.178	0.060	0.442	0.202
		0.302	0.058	0.446	0.168	0.226	0.034	0.572	0.144	0.230	0.024	0.554	0.164
	LS	0.056	0.028	0.192	0.560	0.288	0.032	0.178	0.308	0.066	0.004	0.154	0.638
		0.034	0.010	0.138	0.790	0.474	0.028	0.142	0.310	0.054	0.004	0.134	0.766
$C = 2.25$	IO	0.322	0.174	0.254	0.082	0.666	0.136	0.004	0.002	0.306	0.178	0.292	0.092
		0.390	0.134	0.358	0.092	0.762	0.186	0.002	0.002	0.470	0.178	0.278	0.052
	AO	0.140	0.694	0.052	0.010	0.098	0.610	0.080	0.014	0.194	0.632	0.046	0.008
		0.134	0.828	0.012	0.004	0.080	0.758	0.092	0.004	0.196	0.768	0.028	0.000
	TC	0.230	0.112	0.298	0.178	0.100	0.102	0.476	0.148	0.176	0.058	0.420	0.202
		0.302	0.056	0.446	0.168	0.222	0.034	0.572	0.144	0.230	0.024	0.554	0.164
	LS	0.056	0.026	0.190	0.558	0.280	0.032	0.178	0.308	0.066	0.004	0.154	0.638
		0.034	0.008	0.138	0.790	0.464	0.028	0.144	0.308	0.054	0.004	0.132	0.766
$C = 2.50$	IO	0.298	0.146	0.242	0.076	0.604	0.126	0.000	0.002	0.290	0.164	0.284	0.086
		0.382	0.130	0.354	0.092	0.744	0.188	0.002	0.000	0.468	0.176	0.270	0.052
	AO	0.128	0.666	0.050	0.010	0.082	0.596	0.078	0.014	0.184	0.590	0.042	0.006
		0.128	0.826	0.012	0.004	0.074	0.758	0.092	0.004	0.196	0.760	0.026	0.000
	TC	0.220	0.100	0.288	0.170	0.074	0.094	0.480	0.148	0.166	0.048	0.406	0.202
		0.296	0.054	0.440	0.166	0.196	0.034	0.576	0.144	0.224	0.018	0.544	0.164
	LS	0.050	0.016	0.184	0.540	0.198	0.026	0.178	0.312	0.062	0.002	0.150	0.626
		0.032	0.000	0.142	0.788	0.442	0.022	0.144	0.312	0.032	0.000	0.142	0.788
$C = 2.75$	IO	0.266	0.128	0.218	0.070	0.512	0.102	0.000	0.002	0.260	0.132	0.256	0.074
		0.370	0.122	0.344	0.086	0.700	0.188	0.002	0.000	0.462	0.156	0.258	0.048
	AO	0.116	0.610	0.048	0.010	0.068	0.562	0.066	0.010	0.162	0.534	0.032	0.006
		0.122	0.816	0.012	0.002	0.064	0.756	0.092	0.004	0.186	0.742	0.026	0.000
	TC	0.180	0.078	0.256	0.156	0.044	0.090	0.480	0.148	0.148	0.042	0.364	0.188
		0.290	0.046	0.430	0.162	0.168	0.034	0.574	0.146	0.220	0.014	0.536	0.154
	LS	0.042	0.014	0.170	0.504	0.130	0.024	0.178	0.312	0.056	0.000	0.138	0.582
		0.030	0.004	0.130	0.776	0.420	0.024	0.144	0.312	0.046	0.000	0.134	0.768
$C = 3.00$	IO	0.238	0.104	0.206	0.064	0.424	0.092	0.000	0.000	0.226	0.112	0.226	0.068
		0.366	0.114	0.336	0.078	0.652	0.182	0.002	0.000	0.448	0.134	0.250	0.042
	AO	0.098	0.558	0.042	0.010	0.060	0.540	0.062	0.010	0.136	0.472	0.030	0.006
		0.120	0.790	0.012	0.000	0.060	0.752	0.090	0.006	0.180	0.706	0.026	0.000
	TC	0.148	0.066	0.210	0.134	0.030	0.088	0.480	0.150	0.126	0.036	0.338	0.170
		0.280	0.036	0.422	0.152	0.144	0.034	0.574	0.146	0.208	0.014	0.508	0.150
	LS	0.038	0.010	0.158	0.462	0.088	0.022	0.178	0.312	0.050	0.000	0.126	0.546
		0.028	0.002	0.126	0.772	0.358	0.020	0.144	0.312	0.044	0.000	0.132	0.764
$C = 3.25$	IO	0.196	0.080	0.180	0.060	0.338	0.072	0.000	0.000	0.192	0.076	0.186	0.058
		0.348	0.106	0.324	0.070	0.608	0.162	0.002	0.000	0.412	0.124	0.242	0.040
	AO	0.084	0.466	0.034	0.010	0.046	0.494	0.060	0.008	0.122	0.372	0.022	0.006
		0.110	0.754	0.010	0.000	0.060	0.742	0.086	0.006	0.168	0.662	0.024	0.000
	TC	0.136	0.048	0.182	0.104	0.014	0.086	0.480	0.152	0.098	0.026	0.286	0.150
		0.266	0.032	0.398	0.142	0.108	0.032	0.574	0.146	0.194	0.012	0.472	0.144
	LS	0.030	0.004	0.140	0.416	0.062	0.018	0.178	0.312	0.046	0.000	0.112	0.494
		0.022	0.002	0.124	0.742	0.294	0.016	0.144	0.312	0.040	0.000	0.126	0.748

the models we explored. However, this does not imply that the same result holds true for all models. For more complicated models, we expect that the power will be reduced if the outlier occurs near the end of a series.

In the case when an outlier occurs at the end of the series we observe a noticeable power reduction. This is more pronounced for the cases of the smaller outlier effect ($\omega = 3$) than that of the larger effect ($\omega = 4$). These findings suggest that (1) the proposed procedure is quite effective for reasonably large outliers ($\omega = 4$) even if the outlier occurs near or at the end of the series; and (2) a smaller critical value may be needed to detect outliers of smaller sizes ($\omega \leqslant 3$), particularly when the outlier is at the end of the series. Based on the results in Table III, we find that when the size of the outlier effect is 3 ($\omega = 3$), the probability to detect the outlier at the end of a series is only about 50%. The probability increases to about 70% if the critical value is reduced to $C = 2.50$. Furthermore, based on the results discussed in the previous section and summarized in Table II, we find that if the effect of an outlier is greater than the standard deviation of the innovation series (σ_a), then the MSE of the forecasts will be improved if the outlier can be correctly detected. In practice, we may only be interested in outliers at least greater than $2\sigma_a$. With these considerations in mind, a smaller critical value (say, between 2.0 and 2.5) is desirable for outlier detection near the forecast origin.

There are a few points worth further explanation regarding the results in Table III. In the case of a level shift, two unusual patterns are observed: (1) the power of detection in the MA(1) case is lower, and (2) the power of detection for outliers in the middle of the series is lower than those with outliers near the end of the series. The former result is mainly due to the strong implication of a level shift on parameter estimates when its effect is not adjusted. In the latter case, a more detailed examination of the detected results shows that there is a higher tendency to obtain a detection of a level shift missed by one time period when it occurs in the middle of the series. By including the detected outliers that miss by one time period, we find the similar pattern that the power of detecting a level shift is again higher in the middle of the series than near the end.

Misidentification of outlier type and its implication

To take full advantage of outlier adjustment in forecasting, we need to identify both the location and the type correctly. Tables IV and V summarize the results of more detailed classification of outliers detected. Table IV presents the results for an outlier that occurs in the middle of a time series ($t_1 = 50$) and Table V those for an outlier that occurs near the end of the series ($t_1 = 98$). In each panel, the results are displayed into eighteen groups associated with six critical values and three models. Within each group, we present the frequency of the detected outlier type for each case of actual outlier type. For instance, in the first group of Table IV the first row and the first column gives the probability that an IO is identified as an IO with critical value $C = 2.00$ and an AR(1) underlying model. Generally, the following observations are obtained:

(1) The probability of type misidentification is greater when the critical value is small;
(2) The probability of misidentification increases when an outlier occurs near the end of the series;
(3) AO has lower chance to be misidentified in comparison with IO, TC or LS;
(4) When an AO is misidentified, it is typically classified as an IO (depending on the underlying model, misidentification ranges from 10% to 20%);
(5) LS has a low chance to be misidentified if the outlier occurred in the middle of the series, and has a higher probability to be misidentified as IO or TC if the outlier occurs near the end of the series;

(6) TC and IO are not clearly distinguishable, particularly for AR(1) and IMA(1,1) models; and
(7) The power of detection is a decreasing function of the critical value and an increasing function of the size of the outlier effect.

The above findings indicate the usefulness as well as some potential limitations in the application of outlier adjustment in forecasting. In addition, it provides information for the strategy of forecasting time series with outlier adjustment. Since the probability of misidentification of outlier types increases when the critical value decreases, particularly if the outliers occur closer to the end of the series, it is more advantageous if the types of outliers under detection are restricted to a more limited subset. Since the gain of forecast efficiency is high for an AO and an AO is less likely to be misidentified, it is advisable that AO is always considered. As discussed in the third section, IO has an adaptive nature similar to typical ARIMA innovations, thus it should also be considered. It is not advisable to always include TC and LS outliers during the forecasting process since such outliers cannot be accurately determined in terms of their types and effects when they occur near the end of a time series. However, if strong evidence exists for such outlier types, they should be incorporated into the forecasting process accordingly. The finding here provides a justification for forecasting in conjunction with judgment.

ILLUSTRATIVE EXAMPLES

In this section we use two examples to illustrate the application of outlier detection and adjustment in forecasting. The first example is the monthly totals (in thousands) of international airline passengers from January 1949 through December 1960 in Box and Jenkins (1970). The second example is the quarterly imports of goods and services of Taiwan between the first quarter of 1968 and the fourth quarter of 1990. In the analysis of these examples we employ the SCA Statistical System (Liu *et al.*, 1986, and Chen *et al.*, 1990) for joint estimation of model parameters and outlier effects. Other examples related to forecasting with outliers can be found in Hillmer (1984), Tsay (1988), Liu (1991), and Liu and Lin (1991). Before we present these two examples, we first consider the criteria for forecast performance that may be employed.

Forecast performance

In this research we are particularly interested in the improvement of forecast performance due to outlier adjustment. To evaluate forecast performance, we may employ the root mean squared error (RMSE) for the post-sample periods. Generally, the RMSE is defined as

$$\mathrm{RMSE} = \sqrt{\frac{1}{m}\sum_{t=1}^{m}(Z_t - \hat{Z}_t)^2} \tag{30}$$

where $\hat{Z}_t$ is the one-step-ahead forecast of Z_t based on an estimated model and m is the number of forecasts used in the comparison. Assuming that the estimated model is representative of the forecasting period, the post-sample RMSE should be in consonant with the sample standard deviation ($\hat{\sigma}_a$) of the estimated model. While the gross RMSE defined above is appropriate if no outliers exist, this value may be greatly increased if any outliers exist during the post-sample period. As a result, comparisons of forecast performance based on the gross RMSE are often misleading and inclusive. We shall denote the gross RMSE defined in equation

(30) as $RMSE_g$. To obtain better insights into the effects of outlier adjustment on forecasting, we consider three additional variations of RMSE.

Based on the results in the third section of this paper, it is easy to see that at a time point that an outlier occurs, the forecast cannot be improved, no matter if outlier adjustment is employed or not. However, the forecasts after such a time point can be greatly affected depending upon whether outlier adjustment is applied properly or not. In particular, the forecasts immediately following an outlier are subject to the greatest impact. With these points in mind, we consider the following variations of RMSE:

$RMSE_o$: Post-sample RMSE computed using the time periods where outliers occurred
$RMSE_n$: Post-sample RMSE computed using the time periods immediately next to the outliers
$RMSE_r$: Post-sample RMSE computed using the time periods excluding outliers and those immediately following the outliers

In the evaluation of forecast performance we shall compare the quadruplet ($RMSE_g$, $RMSE_r$, $RMSE_o$, $RMSE_n$).

Airline passenger example

In this example we consider the Series G of Box and Jenkins (1970), the monthly totals (in thousands) of international airline passengers from January 1949 through December 1960. There are 144 observations in this series. The first 132 observations are employed for model fitting and the last 12 observations are reserved for post-sample comparisons. In order to obtain a more homogeneous variance, we will use the natural logarithm of the monthly totals. This log transformed series is denoted as Z_t and is plotted in Figure 1.

Box and Jenkins (1970) considered a multiplicative seasonal model, ARIMA $(0, 1, 1) \times (0, 1, 1)_{12}$, for this time series. The model can be written as:

$$(1 - B)(1 - B^{12})Z_t = (1 - \theta_1 B)(1 - \theta_2 B^{12})a_t \tag{31}$$

Using the first 132 observations, the exact maximum likelihood estimates of model parameters in the above equation are listed in the third column of Table VI and the parameter and outlier estimates based on the joint estimation procedure of Chen and Liu (1993) are listed in the

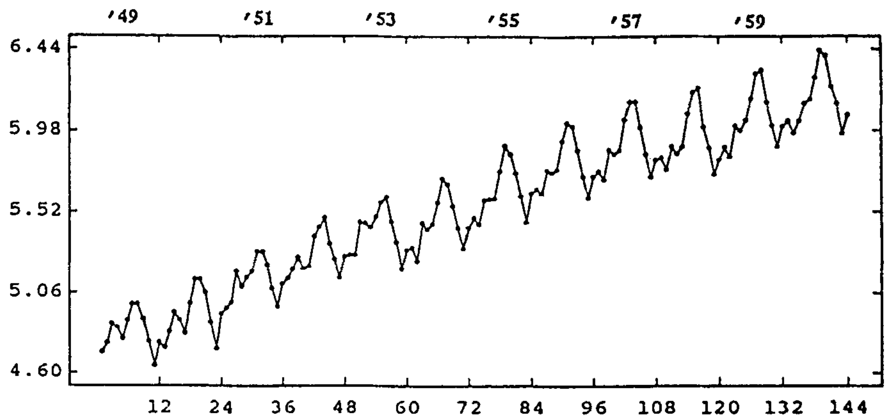

Figure 1. Airline passenger data of Box and Jenkins (1970): natural logarithm of monthly totals (in thousands) for the period 1/1949 to 12/1960

Table VI. Estimates of model parameters and outlier effects for the airline passenger data using ARIMA model (31)

Parameter	Estimates and their t-values: With outlier adjustment				Estimates and their t-values: Without outlier adjustment
θ_1	0.3180 (3.63)				0.3488 (4.05)
θ_2	0.4824 (6.24)				0.5624 (7.65)
σ_a	0.0332				0.0362
	Time	Type	Estimate	t-value	
Outliers detected	29	AO	0.095	4.08	
	54	LS	−0.097	−3.55	
	62	AO	−0.080	−3.44	

second column of the table. The critical value 3.0 is used for outlier detection in the joint estimation procedure.

As shown in Table VI, we find that three outliers are detected, none of them near the forecast origin ($t = 132$). By adjusting for these outliers, $\hat{\sigma}_a$ is reduced by about 9%. Using the estimated model shown in Table VI we can compute one-step-ahead forecasts for the forecast origins 132 through 143. During the forecasting period, the critical value 2.5 is used for outlier detection. There is only one outlier (an additive outlier at $t = 135$) detected during the post-sample period. The estimate of the outlier effect and its t-value are -0.093 and -2.79 respectively. It is important to note that this outlier will not be detected if the critical value for outlier detection is still set to 3.0 (see Hillmer, 1984). Assuming that the outlier is 'correctly' specified as an AO during forecasting, the one-step-ahead forecasts with outlier adjustment are listed in Table VII. The forecasts based on traditional ARIMA model without outlier adjustment are also listed in the Table.

Based on the results presented in Table VII, we obtain the quadruplet ($RMSE_g$, $RMSE_r$, $RMSE_o$, $RMSE_n$) for forecasting with outlier adjustment to be (0.0343, 0.0215, 0.0927,

Table VII. Forecasts of the airline data in the post-sample period

t	Actual value	Without outlier adjustment: One-step-ahead forecast	Without outlier adjustment: Forecast error	Without outlier adjustment: One-step-ahead forecast	Without outlier adjustment: Forecast error
133	6.0331	6.0410	−0.0079	6.0386	−0.0055
134	5.9687	5.9846	−0.0159	5.9851	−0.0164
135[a]	6.0379	6.1306	−0.0927	6.1311	−0.0932
136	6.1334	6.1037	0.0297	6.0440	0.0894
137	6.1570	6.1715	−0.0145	6.1429	0.0141
138	6.2823	6.3052	−0.0229	6.2971	−0.0148
139	6.4329	6.4214	0.0115	6.4160	0.0169
140	6.4069	6.4446	−0.0377	6.4397	−0.0328
141	6.2305	6.2343	−0.0038	6.2391	−0.0086
142	6.1334	5.1018	0.0316	6.1030	0.0304
143	5.9661	5.9960	−0.0299	5.9945	−0.0284
144	6.0684	6.0810	−0.0126	6.0825	−0.0141

[a] An AO occurs at $t = 135$ with $t = 2.79$.

0.0297), while that for forecasting without outlier adjustment to be (0.0416 ,0.0202, 0.0932, 0.0894). Comparing these two sets of RMSEs, we find that the gross RMSE for the forecasts with outlier adjustment is about 17.5% less than that without outlier adjustment, which is almost entirely caused by the reduction of forecast error for $t = 136$. The RMSE_r and RMSE_o for forecasting with and without adjustment are about the same in this case. From the above RMSEs, we observe that the decomposed RMSEs in general provide more informative knowledge on forecasting accuracy than the gross RMSE alone.

We also examine the impact on forecasts if the outlier type at $t = 135$ is not specified correctly. Since in this case the outlier at $t = 135$ is always detected as an AO as soon as the data at $t = 136$ are available, the mis-specification of the outlier type at $t = 135$ will only affect the one-step-ahead forecast for $t = 136$. The rest of forecasts are the same as those shown in Table VII. The one-step-ahead forecast for $t = 136$ is 6.0582 if the outlier at $t = 135$ is assumed to be a TC, 6.0404 if the outlier is assumed to be an IO (based on the parameter estimates with outlier adjustment) and 6.0110 if the outlier is assumed to be an LS. Based on these results, we find that the forecast performance is similar to typical ARIMA forecasting without outlier adjustment if the outlier type at $t = 135$ is mis-specified as a TC or an IO, and is worse than typical ARIMA forecasting if the outlier type is mis-specified as an LS. Since an LS outlier has a permanent impact on forecasts as shown in the third section of this paper, it should be used with such understanding. In general, a judgmental decision is required to specify the outlier type at the forecast origin. When the outlier type is appropriately specified, outlier adjustment will improve the accuracy of forecasts.

Quarterly total imports of Taiwan, ROC

The second example is concerned with quarterly imports of goods and services of Taiwan in constant billion New Taiwan Dollars (NT$). The data were obtained from the Directorate General of Budget, Accounting and Statistics (DGBAS) of the Republic of China in Taiwan. Figure 2 displays this series (M_t) between the first quarter of 1968 and the fourth quarter of 1990. The imports of goods and services of a country are generally influenced by a number of economic variables, in particular, the value of its currency (i.e. the exchange rate). The currency of Taiwan (NT$) appreciated greatly with respect to US dollars (US$) since 1987, and has stimulated Taiwan's imports of goods and services since then. This impact of the exchange rate on Taiwan's imports can be clearly observed in Figure 2. In the following analyses, the data between 1/68 and 4/87 are used for model building, and the data between 1/88 and 4/90 are reserved for post-sample forecasting comparison.

Following the model identification procedure outlined in Box and Jenkins (1970), the following seasonal ARIMA model appears to be appropriate for the import series

$$(1 - B)(1 - B^4)M_t = (1 - \theta B^4)a_t \tag{32}$$

Using the first 80 observations, the exact maximum likelihood estimates of model parameters of equation (32) are listed in the third column of Table VIII, and the parameter and outlier estimates based on the joint estimation procedure of Chen and Liu (1993) are listed in the second column of the table. A critical value of 2.5 is used for outlier detection in the joint estimation procedure since the length of the series is short.

Using model estimation with outlier detection, we find two outliers as shown in Table VIII. The first outlier is an AO which occurred at $t = 64$. The second is an IO at $t = 78$. Under model (32), an IO implies an annual trend change. This annual trend change was mainly caused by the drastic appreciation of NT$ versus US$. By adjusting for these outliers, $\hat{\sigma}_a$ is reduced by about 10%.

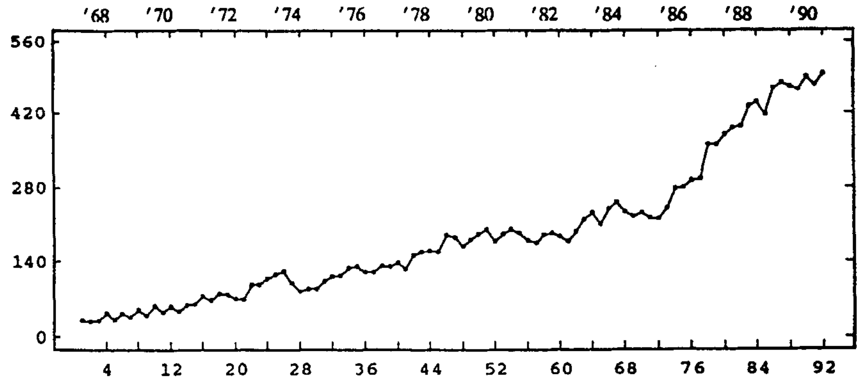

Figure 2. Quarterly total imports (in constant billion NT$) of Taiwan, ROC, for the period 1/1968 to 4/1990

Using the parameter estimates shown in Table VIII, we compute one-step-ahead forecasts for the forecast origins between $t = 81$ and $t = 91$. The critical value 2.0 is used for outlier detection during the post-sample period if outlier adjustment is employed. For forecasting with outlier adjustment, we consider the outlier types during the forecasting period can be (1) AO and IO only; (2) AO, IO and TC; (3) AO, IO and LS; and (4) AO, IO, TC and LS. The results of one-step-ahead forecasts under each scenario are listed in Table IX. In the process of forecasting, we first assume that outliers occurred at the forecast origins are all additive outliers. Such a choice is considered to be conservative in the sense that we do not expect an outlier to affect future observations. The results obtained by assuming innovational outliers at the forecast origins are discussed later.

When only AO and IO are allowed during forecasting period, we find that the forecast origins at $t = 82$ and $t = 85$ are identified as outliers. These two outliers are both identified as additive outliers when the entire series is estimated with outlier adjustment. Hence in the computation of RMSE_o and RMSE_n for different models and outlier types, we treat these two points as true outliers. The results of RMSEs for different scenarios are also listed in Table IX. From these results, we find that RMSEs of one-step-ahead forecasts with outlier adjustment are generally smaller than those generated by typical ARIMA forecasting. The only exception is the case that all outlier types (AO, IO, TC and LS) are allowed during the forecasting period.

Table VIII. Estimates of model parameters and outlier effects for Taiwan import data using ARIMA model (32)

	Estimates and their t-values				
Parameter	With outlier adjustment				Without outlier adjustment
θ	0.7394 (9.38)				0.6879 (6.96)
σ_a	11.4540				12.7124
	Time	Type	Estimate	t-value	
Outliers detected	64	AO	20.733	2.72	
	78	IO	42.819	3.74	

Table IX. One-step-ahead forecasts of Taiwain import data in the post-sample period

Time	Actual value	No outliers allowed	Forecasts with outlier adjustment			
			AO, IO	AO, IO, TC	AO, IO, LS	AO, IO, TC, LS
81	392.060	381.2888	382.8683	382.8683	382.8683	382.8683
82	394.993	428.3065	425.3440[b]	425.3440[b]	425.3440[b]	425.3440[b]
83	432.538	396.6221	427.2278[a]	397.1408	396.8768	396.8768
84	441.280	439.0435	435.7460	435.7460	435.7460	435.7460
85	416.387	445.6038	446.2357	446.2357	446.2357[b]	446.2357[b]
86	468.695	442.2381	480.2113[a]	480.2113	470.9777	427.1635
87	479.167	481.5315	471.4624	471.4624	471.4624	471.4624
88	472.004	486.3705	483.8389	488.6172	483.8389	488.6172
89	466.250	467.2108	475.5242	475.5242	475.5242	475.5242
90	489.856	500.3570	498.6801	498.6801	498.6801	498.6801
91	475.215	501.9548	494.9600	494.9600	494.9600	494.9600
92	495.769	477.9352	476.8002	476.8002	476.8002	476.8002
$RMSE_g$		21.273	16.333	19.497	18.967	22.680
$RMSE_r$		13.589	12.390	13.058	12.390	13.058
$RMSE_o$		31.332	30.101	30.101	30.101	30.101
$RMSE_n$		31.543	8.967	26.321	25.268	38.708

[a] The one-step-ahead forecast for $t = 83$ is 396.8767, and that for $t = 86$ is 450.3643, if the outliers at $t = 82$ and $t = 85$ are assumed to be IO.

[b] The corresponding observations are not identified as outliers when they are used as forecast origins, but are identified as outliers later.

In the case when only AO and IO are allowed during the forecasting period and all outliers at the forecasting origins are treated as AOs we obtain the most impressive improvement for all types of RMSEs, particularly $RMSE_n$. If we specified the outlier type at $t = 82$ and $t = 85$ incorrectly as an IO (rather than an AO), then $RMSE_n$ and $RMSE_g$ are increased to 28.353 and 19.681, respectively, and $RMSE_r$ and $RMSE_o$ remain the same. This result indicates the importance of specifying a correct outlier type when an outlier occurs at the forecast origin. However, in this case, forecasting with outlier adjustment generally produces better results than typical ARIMA forecasting even if the outliers at the forecast origins are specified incorrectly.

The inferior results produced by other cases of forecasting with outlier adjustment are mainly caused by the difficulty in determining the locations and the types of outliers. As a result, the outliers at $t = 82$ or $t = 85$ may not be detected and the forecasts for $t = 83$ and $t = 86$ are not improved. In addition, the locations and the types of outliers often change as the forecast origin increases. These results are consistent with our findings in the simulation studies of the fourth section of this paper. Some of the difficulties arise since (1) a TC or LS outlier needs more data for the determination of its type and estimate, and cannot be obtained accurately at the end of the series; and (2) depending upon the model, a TC or LS may not be distinguishable from an IO. As a result of these complications, it is important to limit the types of outliers allowed during the forecasting period.

SUMMARY AND CONCLUSIONS

In this paper we investigate the effects of outliers on forecasting. Four types of outliers are considered. It has been shown that an outlier occurring at the forecast origin has the greatest impact on forecasts. As an outlier occurs further away from the forecast origin, its effect on forecasts becomes smaller. We have also shown analytically that outlier adjustment can improve forecast accuracy if the effect of the outlier is greater than one standard deviation of the innovation series. Furthermore, we have demonstrated through a simulation study that the power of outlier detection is reduced when the outlier occurs at the end of the series (i.e. the forecast origin). Based on these results, we have proposed a strategy for forecasting time series with outliers.

We have also investigated the potential loss of forecast accuracy due to the misidentification of an outlier type. Generally, forecasting with outlier adjustment seems to perform better than forecasting without outlier adjustment even though sometimes outlier type may be misidentified. Based on the simulation study, we find some potential difficulty in the identification of outlier types at the end of a series. Further study is needed in this area.

We have proposed four different post-sample RMSEs to reflect the forecast performance of a model with and without the consideration of outlier adjustment. These measures provide us with more informative ways to evaluate the forecasting performance of a model. The proposed forecasting approach is applied to two real examples in which some practical forecasting issues are discussed and the usefulness of the proposed approach is demonstrated.

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