



# FFORMPP: Feature-based forecast model performance prediction



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## ABSTRACT

This paper introduces a novel meta-learning algorithm for time series forecast model performance prediction. We model the forecast error as a function of time series features calculated from historical time series with an efficient Bayesian multivariate surface regression approach. The minimum predicted forecast error is then used to identify an individual model or a combination of models to produce the final forecasts. It is well known that the performance of most meta-learning models depends on the representativeness of the reference dataset used for training. In such circumstances, we augment the reference dataset with a feature-based time series simulation approach, namely GRATIS, to generate a rich and representative time series collection. The proposed framework is tested using the M4 competition data and is compared against commonly used forecasting approaches. Our approach provides comparable performance to other model selection and combination approaches but at a lower computational cost and a higher degree of interpretability, which is important for supporting decisions. We also provide useful insights regarding which forecasting models are expected to work better for particular types of time series, the intrinsic mechanisms of the meta-learners, and how the forecasting performance is affected by various factors.

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## 1. Introduction

Forecasting is an important aspect of every business operation. The selection of a suitable forecast model or a combination of models to use in forecasting is at the heart of the forecasting process (Tashman & Leach, 2013). However, this selection and combination process is challenging in the context of large-scale time series forecasting for several reasons: (i) there is no universal method that performs best for all kinds of forecasting problems, (ii) a trial-and-error process of model selection would increase the computational time as well as the computational cost

significantly, (iii) it is not possible to derive a typical algebraic expression for choosing the best-performing model or models out of a portfolio of algorithms, and (iv) the traditional expert's judgement cannot be quickly scaled to forecast a large number of series problems, due to the computational cost and time constraints. Therefore, automatic forecast model selection is essential and challenging in business today. Automatic forecast model selection approaches often test multiple forecasting algorithms and select the most appropriate one according to some criterion, such as an information criterion (Hyndman & Khandakar, 2008) or out-of-sample tests (Tashman, 2000). Meta-learning serves as a promising alternative to solve this problem. Petropoulos et al. (2018) state that there are model, parameter, and data uncertainties in forecasting. That means that even if the above-mentioned challenges could be effectively addressed, there

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would still be different forms of uncertainties, which meta-learning could help to mitigate by learning across different time series and datasets.

The idea of using meta-learning to select the best forecasting model for a given time series has been explored by many researchers in the forecasting community (Adya et al., 2001; Collopy & Armstrong, 1992; Petropoulos et al., 2014; Shah, 1997; Wang et al., 2009). This approach is also known as the algorithm selection problem and can be expressed firmly using Rice's framework for algorithm selection (Rice, 1976). More recently, Petropoulos et al. (2014) pointed out that there are "horses for courses", which also supports the idea of meta-learning. Further evidence in favour of this idea is given in Talagala et al. (2018). In all these cases, a vector of features computed from time series is used as an input to train a meta-learner. The output and the objective function used in the meta-learner are approached differently. For example, Shah (1997) uses the best forecast model as the output label and applies discriminant analysis to predict the forecast model that is expected to perform best on a given time series, where the objective function is to minimise the classification error. Also, Prudêncio and Ludermir (2004b) use neural network approaches to identify weights for the best linear combination of methods to improve forecast accuracy; thus, the objective function is to minimise the forecast error. Petropoulos et al. (2014) translate seven time series features and the forecasting horizon into forecasting model selection, directly linking the forecasting horizon to short- and long-term forecast tasks.

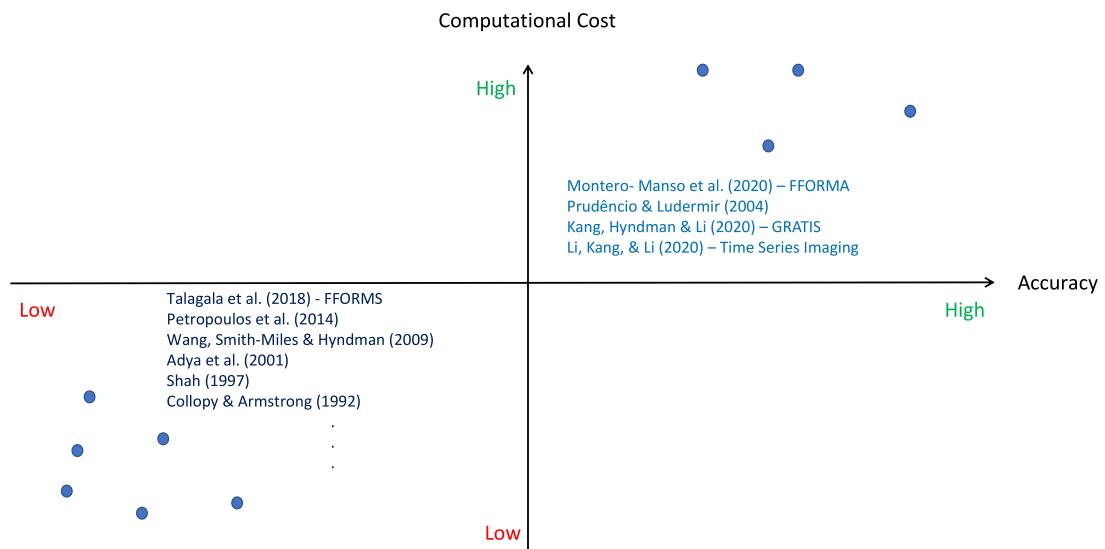
Although many researchers have highlighted the usefulness of the meta-learning approach to compute forecasts, few studies could conclude that their approach is always superior to simple benchmarks and commonly used forecasting approaches. For example, Meade (2000) shows that the summary statistics are useful in selecting a suitable forecasting method, but are not necessarily the best. Three possible reasons for the infeasibility of the selection process are the use of inadequate features, the improper choice of meta-learning algorithms, and having training time series data that are not sufficiently diverse to predict different forecast model performance (Kang et al., 2020). Recently, several studies have considered these matters. For example, the results of the recent large-scale M4 competition highlight features-based algorithms as a promising solution for time series forecasting. Li et al. (2020) use time series imaging to extract a comprehensive set of time series features as the input of the meta-learning framework. Wang et al. (2021), for the first time, investigate feature-based interval forecasting. Therefore, developing new or improving existing feature-based forecasting approaches might be a worthwhile endeavour.

We address the aforementioned issues by developing meta-learning frameworks for forecast model selection based on features computed from the time series. Our first attempt to develop a framework for forecast model selection is described in Talagala et al. (2018). The first framework is called FFORMS: Feature-based FOREcast Model Selection, in which we use a classification algorithm (random forests) to predict the "best forecast model" on a

given time series. In our follow-up essay—Montero-Manso et al. (2020)—rather than mapping time series to a single forecast model, we use a gradient boosting algorithm to obtain the weights for forecast combinations. The second framework is named FFORMA: Feature-based FOREcast Model Averaging. FFORMA placed second in the M4 competition (Makridakis et al., 2018). In this paper, which is the third in this series, we consider the correlation structure of algorithm performance in the model training process. Having revisited the literature, we found that, to the best of our knowledge, few of these studies have tackled forecast model performance prediction with meta-learning. In a conventional univariate supervised meta-learning problem, merely labelling the instances using the "best forecast model" makes it difficult for the meta-learner to correctly retrieve the interconnections between individual forecast models. The current paper further extends the idea of meta-learning by making the dependent variables a multivariate forecast errors vector. Hence, the meta-model is trained to learn the interrelationship between the forecast errors across different class labels.

The existing meta-learning algorithms for forecasting are divided into two categories: (1) selecting the "best forecast model" (the model with the smallest forecast error) out of a pool of forecast models, and (2) identifying suitable weights to combine forecasts from all available models in the pool. Based on this, meta-learning algorithms can be further classified into two extremes in terms of forecasting accuracy and computational cost: (1) low computational cost and low accuracy, and (2) high computational cost and high accuracy. There is a continuum between these two extremes. All existing approaches fall at one extreme of the spectrum: (1) low-low: forecasting based on a single best model, and (2) high-high: forecasting based on all available models in the pool. In this paper, we treat the forecast model selection problem as a ranking problem. We propose an algorithm to rank forecast models by simultaneously predicting forecast errors. The rankings allow the user to identify a subset of forecasting models. This way, we can gain a good balance between accuracy and computational cost, which allows the algorithm to move between the two extremes of the spectrum. Therefore, we refer to our new framework as FFORMPP: Feature-based FOREcast Model Performance Prediction. A schematic illustration is given in Fig. 1. Table 1 provides an overview of some of the meta-learners available in the literature.

The proposed algorithm is closely related to the concept of forecast pooling, i.e. selecting and combining from a pool of suitable models instead of all models available in the model. For example, Aiolfi and Timmermann (2006) propose a conditional forecast combination approach that clusters the candidate models, pools forecasts within each cluster, and then estimates their optimal weights. Matysyura et al. (2018) combine expert forecasts by forecast pooling. Kourentzes et al. (2019) introduce the forecast island approach for forecast selection and combination. The authors investigate the use of quantiles to form groups and propose a heuristic to automatically identify the forecast pool rather than using a model to decide which forecasts should be included in the combination.



**Fig. 1.** Classification of existing meta-learning algorithms based on computational cost and accuracy.

**Table 1**  
Overview of literature on time series forecasting with meta-learning approaches.

Authors (Year)	Type of meta-learner (selection, combination, both)	Output	Type of forecasts	Meta-learning algorithm	Number of features used	Number of forecasting methods
Arinze et al. (1997)	Both	Error	Point	Rule-based induction	6	6
Shah (1997)	Selection	Label	Point	Discriminant analysis	26	3
Venkatachalam and Sohl (1999)	Selection	Error	Point	Two-stage neural network	6	9
Adya et al. (2000)	Both	Error	Point	Rule-based (judgemental)	18	3
Prudêncio and Ludermir (2004a): study 1	Both	Error	Point	C4.5 decision tree in (Hall et al., 2009)	14	2
Prudêncio and Ludermir (2004a): study 2	Both	Ranking based on error	Point	Neural network classifier	16	3
Wang et al. (2006)	Selection	Error	Point	C4.5 decision tree	13	4
Wang et al. (2009)	Selection	Error	Point	SOM clustering and DT algorithm	9	4
Lemke and Gabrys (2010)	Both	Error	Point	(i) A feed-forward neural network, (ii) decision tree, (iii) support vector machine	24	15
Widodo and Budi (2013)	Selection	Label	Point	K-Nearest Neighbor	13 features by Wang et al. (2009)	6
Petropoulos et al. (2014)	Selection	Error	Point	Multiple regression	14	7
Kück et al. (2016)	Selection	Label	Point	Neural network	127	4
Talagala et al. (2018)	Selection	Label	Point	Random forest algorithm	Yearly - 26 , quarterly - 30 , monthly - 30	Yearly - 9 , quarterly - 14 , monthly - 14
Montero-Manso et al. (2020)	Combination	Weights for forecasting combination	Point	XGBoost	42	9
Li et al. (2020)	Combination	Error	Point	XGBoost	Time series imaging features	9
Wang et al. (2021)	Both	Error	Interval	GAM models	40	8

In our approach, we utilise the multivariate structure in the response variables to simultaneously model the correlation structure between different forecast model

errors using the time series features. The Bayesian multivariate surface regression approach proposed by Li and Villani (2013) is used to estimate the forecast error for

each model in the pool. This allows the forecast models to be ranked with respect to their forecast errors and the evaluation of their relative forecast performance without calculating forecasts from all available individual models in the pool. Our framework makes the following contributions to the forecasting domain.

1. The diversity of features in the collection of time series used to train a meta-learner plays a critical role in training a meta-learning model. Often, time series with the required amount of feature diversity and quality might not be available (Kang et al., 2020, 2017). To this end, we augment the original reference dataset with GRATIS, proposed by Kang et al. (2020), to obtain a diverse collection of time series. We explore whether enriching the diversity of the training set helps to increase the accuracy of forecasts.
2. One noticeable gap in the meta-learning literature is that little attempt has been made to identify the relationships learned by the meta-learning algorithm. To this end, we visualise the relationships learned by the meta-learning algorithm. The visualisation approach involves mapping each time series as a point in a two-dimensional instance space given by the features and exploring the relationship between the features and forecast models selected. This helps to gain insights into why specific models perform better on certain types of time series, which is also useful for understanding the strengths and weaknesses of the meta-learning algorithm.
3. We utilise an efficient Bayesian multivariate surface regression model to train the meta-learner. Compared with other meta-learning algorithms, our approach treats the features as covariates in the regression form for improving the meta-learning interpretability, because linear and nonlinear effects can be separated with stochastic knots. Unlike univariate regression models that do not take into account the relationships between features and their correlations to the classification between forecasting models, the multivariate surface regression model jointly models the relationship between forecast errors and time series features for multiple forecasting models.

The remainder of the paper is organised as follows: Section 2 introduces the methodology, including the methodology for simulating time series to augment the reference set to train a model and the efficient Bayesian multivariate regression approach. Section 3 discusses the results in an application to the M4 competition data. Section 5 concludes the paper.

## 2. Methodology

Our FFORMPP framework (Fig. 2) consists of two phases: an offline phase and an online phase. We treat the forecast model selection problem as a multi-class ranking problem, where predictors are features, and the outcome consists of forecast errors (the mean absolute scaled error, MASE) calculated over all available models. In particular,

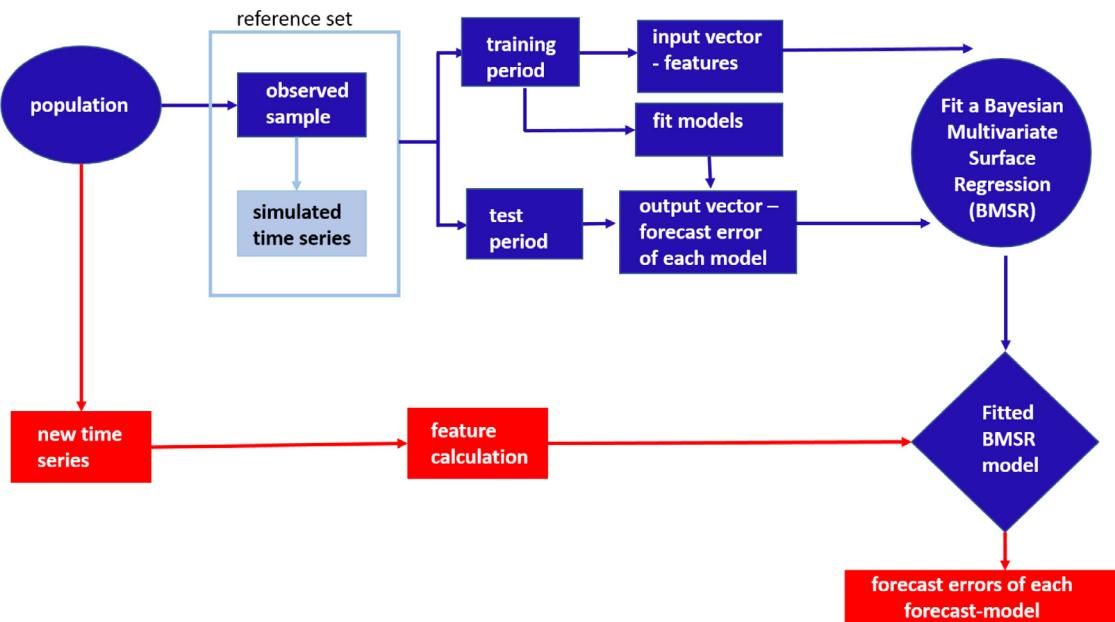
an instance  $i$  is a tuple  $(f_{1i}, f_{2i}, \dots, f_{mi}, e_{1i}, e_{2i}, \dots, e_{ni})$ , where predictors  $(f_{1i}, f_{2i}, \dots, f_{mi})$  form a vector of  $m$  features, which is the input to our algorithm, and the outcome  $(e_{1i}, e_{2i}, \dots, e_{ni})$  is a vector of MASE values of all  $n$  forecast models. A description of the offline and online phases is as follows.

- *Offline phase:*

1. **Reference set and its augmentation with GRATIS.** We take an observed sample of time series from the population of interest. In addition to the observed time series, we augment the reference set by adding simulated time series. This data simulation part is an optional component in our framework. This is needed only if the practitioners have a limited observed sample. If the observed sample is sufficiently large and diverse, there is no need to add simulated time series.
2. **Splitting the training and testing periods.** We split all time series in the reference set into a training period and a test period once we created the reference set. For each series in the reference set, we compute features based on the training period. The calculated features become the input vector to our meta-learner.
3. **Time series features and model class labels.** The output vector for each series consists of the MASE of all available candidate models. To compute the MASE, all candidate models are estimated using the training period, and forecasts are generated for the whole of the test period (based on “a fixed origin” evaluation). Subsequently, the MASE for each model is computed over the test period. This completes the data processing part of our algorithm.
4. **Meta-learner training.** Finally, an efficient Bayesian multivariate surface regression (EBMSR) approach, proposed by Li and Villani (2013), is used to model forecast error measures (which in our case is the MASE) as a function of features calculated from the time series. This produces a meta-learner to be used in the online phase.

- *Online phase:* The online phase requires only the calculation of a simple vector of features for any newly given time series. It uses the pre-trained classifier to estimate the forecast error for each candidate model in the pool. Finally, the forecast models are ranked based on the predicted errors. The top-ranking models are used to produce forecasts.

Most of the expensive computations for processing data and training a meta-learner are performed in the offline phase. In the following sections, each component in the offline phase is explained in detail with an application to forecasting M4 competition data. In our application, we treat M4 competition data as the new time series to be forecast in the online phase.



**Fig. 2.** FFORMPP (Feature-based FOREcast Model Performance Prediction) framework. The offline phase is shown in blue and the online phase in red.

### 2.1. Reference set and its augmentation with GRATIS

We use the time series of the M1 and M3 competitions as the observed sample for yearly, quarterly, and monthly time series. We also simulate 10,000 time series with GRATIS<sup>1</sup> (Kang et al., 2020) to augment the reference set. The observed time series and the simulated time series form the reference set to fit the model. For weekly, daily, and hourly frequencies, only the simulated time series are used to create the reference set because the previous M-competitions do not contain the time series corresponding to weekly, daily, and hourly frequencies.

The augmentation with simulated time series helps obtain a more heterogeneous collection of time series for training a meta-learner, which helps prevent overfitting to a relatively homogeneous set of data and increases the generalisability of the model when applied to new time series with different conditions. Furthermore, the augmentation is particularly beneficial when there is no observed set of time series available to train a model in the offline phase, in judgemental forecasts, for instance, or for business forecasting with new products. Although there is no standard process for simulating time series, the most common approach involves simulation based on some data-generating processes (DGPs) such as exponential smoothing and ARIMA models. Instead of relying on a set of DGPs to generate time series, the GRATIS algorithm simulates time series with diverse time series features using mixture autoregressive (MAR) models. The algorithm can also allow a user to set controllable features for simulating time series.

**Table 2** lists the choice of values for the parameters used to simulate the time series in each frequency category. We use frequencies of 1, 4, 12, and 52 to generate yearly, quarterly, monthly, and weekly series, respectively. Daily and hourly time series with a long history often show multiple seasonal patterns. Hence, for daily series, frequencies are set to 7 (time-of-week patterns) and 365.25 (annual seasonality), and hourly series frequencies are set to 24 (time-of-day patterns) and 168 (time-of-week patterns). None of the time series in our hourly test dataset is longer than 8760. Hence, time-of-year patterns ( $365 \times 24 = 8760$ ) are not considered. Except for the hourly series, the length of the time series is randomly chosen from the uniform distribution. The minimum and maximum values of the distributions are selected based on lower ( $Q1 - 1.5 \times IQR$ ) and upper ( $Q3 + 1.5 \times IQR$ ) edges of the box-and-whisker plots.

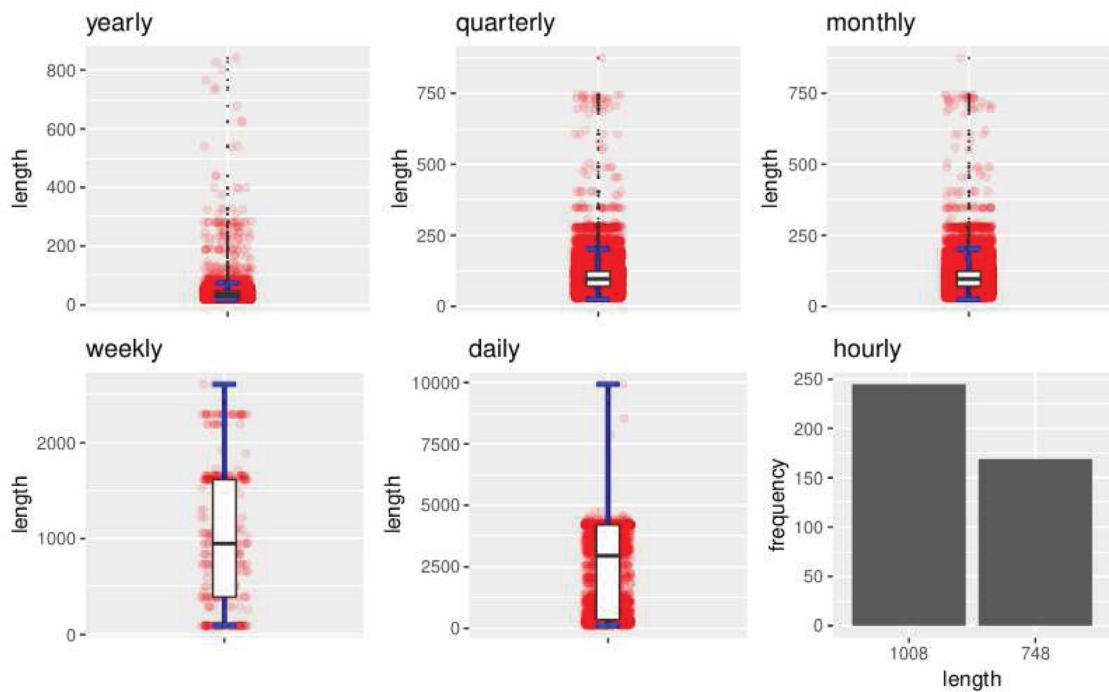
We use the M4 competition data to compute the associated statistics for length. The corresponding distributions are shown in Fig. 3. The reason for this choice is that we use the length of the time series as a feature in our meta-learning framework. Hence, the lengths of the series in our reference set should cover all or the majority of time series we need to forecast. In practice, it might be difficult to obtain a reference set covering the whole length of the new time series for two main reasons: (i) information regarding the range of length is not available at the offline stage (although, a rough idea about the distribution of the majority can be obtained), and (ii) the range of the length is very wide, owing to “outlying” observations (for example, in the quarterly and monthly series shown in Fig. 3). In such circumstances, a reference set covering most of the lengths of future time series is a reasonable approach. Further, for each series we randomly select a number of mixing components from

<sup>1</sup> The R package *gratis* accompanies this work and is publicly available at <https://CRAN.R-project.org/package=gratis>.

**Table 2**

Choice of values for parameters for simulating time series using GRATIS.

Parameter	Description	Value
Frequency	Seasonal period	Yearly 1
		Quarterly 4
		Monthly 12
		Weekly 52
		Daily (7, 365)
		Hourly (24, 168)
Length	Length of time series	Yearly U(19, 75)
		Quarterly U(24, 202)
		Monthly U(60, 660)
		Weekly U(93, 2610)
		Daily U(107, 9933)
		Hourly 40.8% with a length of 748 and 59.2% with a length of 1008
k	Number of mixing components	For each series randomly chosen from {1, 2, 3, 4, 5}
$\alpha_k$	Weights of mixture components	$\alpha_k = \beta_k / \sum_{i=1}^K \beta_i$ , where $\beta_i \sim U(0, 1)$
$\theta_{ki}$	Coefficients of the AR part	$N(0, 0.5)$
$\Theta_{kj}$	Coefficients of the seasonal AR parts	$N(0, 0.5)$
$d_k$	Number of differences in each component	Bernoulli(0.9)
$D_k$	Number of seasonal differences in each component	Bernoulli(0.4)

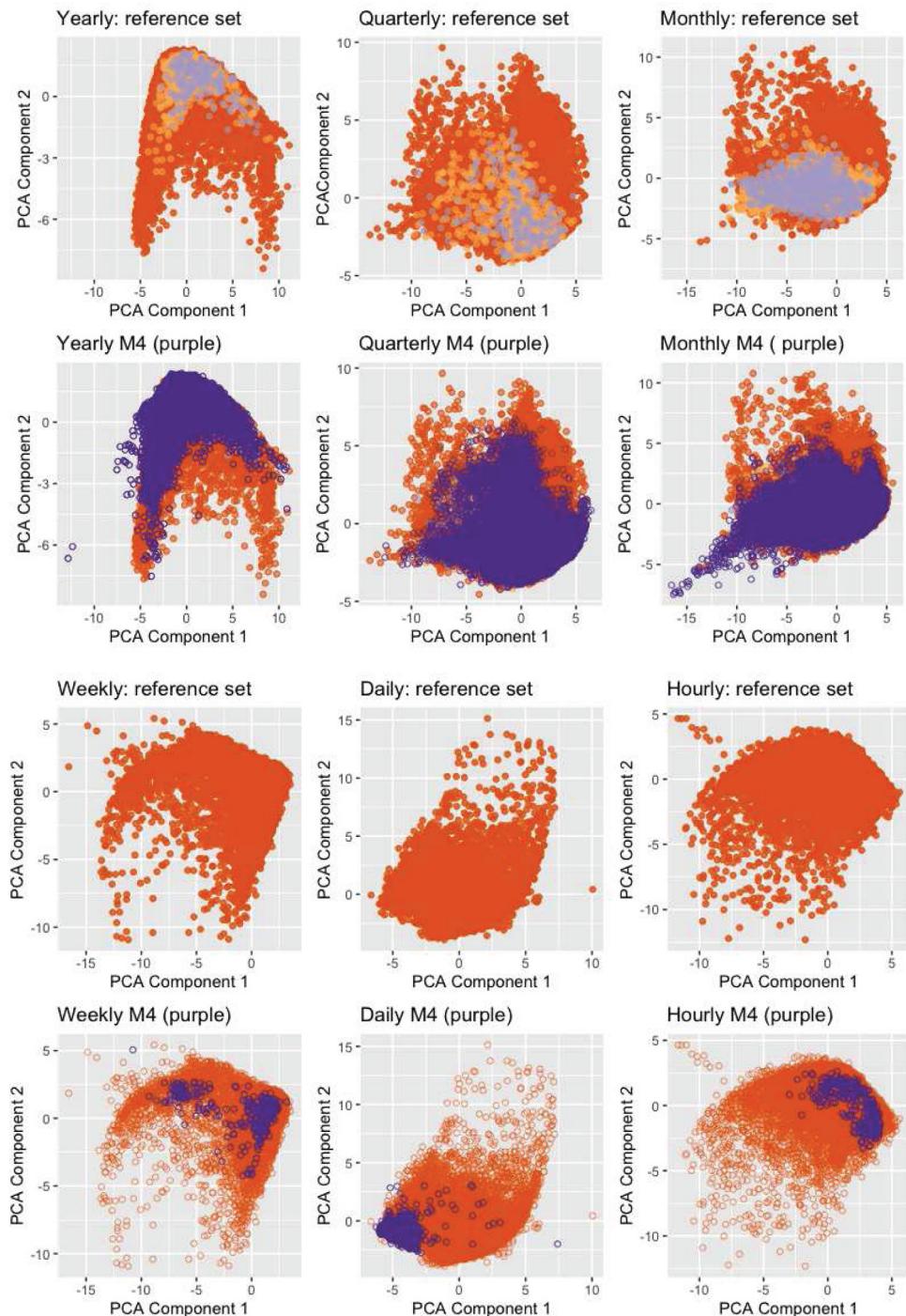


**Fig. 3.** Distribution of the time series lengths of M4 data across different frequency categories. The data points are shown in red. The horizontal blue solid line shows the lower and the upper whisker borders of each boxplot. Some outliers can be observed in the yearly, quarterly, and monthly categories. There are only two distinct levels of length for the hourly series.

{1, 2, 3, 4, 5}. This is because Li et al. (2010) and Villani et al. (2009) point out that, for mixture models with comprehensive mean structures, at most five components are sufficient in order to capture widely the shapes of the data. Other parameters are set with random values from certain distributions, as used in GRATIS. These are analogous to non-informative priors in Bayesian contexts. That is, the diversity of the generated time series should not rely on the parameter settings. Having each parameter set as in Table 2, we generate 10,000 time series from each

frequency category. This is to reduce the training time of the efficient Bayesian multivariate surface regression model while maintaining the diversity of the reference set.

As shown in Fig. 4, the use of simulated data generated by GRATIS along with M1 and M3 competition data allows us to examine whether the simulated data generated based on the GRATIS approach are actually representative of real data. Note that training the algorithms only using simulated time series could be dangerous in cases where



**Fig. 4.** Distribution of yearly, quarterly, monthly, weekly, daily, and hourly time series on the PCA space. On each subfigure, dark orange represents the simulated series and purple denotes new time series (M4 data). In the first row, observed time series in the reference are highlighted in light orange and light purple. The PCA space is computed based on the time series in the reference set, and then the M4 competition series (new series) are projected into the two-dimensional PCA space. Except for a few series, the majority of the new time series we need to forecast fall within the space of the reference set. Note that for yearly, quarterly, and monthly series the size of the test set is much larger than the size of the corresponding reference set.

the simulated series may not be a representative sample of the population of interest. However, if the user does not have the option to obtain the required data, then the only

option is to use simulated data. In such circumstances, extra care needs to be taken when setting parameter values for the GRATIS algorithm.

**Table 3**

Composition of the time series in the reference set and collection of new time series.

Frequency	Reference set			New series
	M1	M3	Augmented	
Yearly	181	645	10,000	23,000
Quarterly	203	756	10,000	24,000
Monthly	617	1,428	10,000	48,000
Weekly	–	–	10,000	359
Daily	–	–	10,000	4227
Hourly	–	–	10,000	414

## 2.2. Splitting the training and test periods

Once we form the reference set, we split each time series in the reference set into a training period and test period. Features are calculated based on the training period of each series, and the forecast error measure is calculated over the test period of each time series. Table 3 summarises the number of time series in the reference set and the new time series in each frequency category. Note that for yearly, quarterly, and monthly time series, the reference set used to train a meta-learner is much smaller than the test set evaluating the meta-learner. This is often the case when applying the meta-learner in practice during the online phase.

## 2.3. Time series features and model class labels

For each series, a set of features computed from the training period of each series comprising an input vector and the mean absolute scaled error (MASE, Hyndman & Koehler, 2006) values for each method give the output vector to train a model. In this paper, we use the MASE as a measure of out-of-sample forecasting accuracy because it is less dependent on the data scales. Hence, the MASE can be used to compare forecasts across time series with different scales. Furthermore, Hyndman and Koehler (2006) describe the MASE as a “generally applicable measurement of forecasting accuracy without the problems seen in the other measurements”. Besides, MASE is one of the main measurements of forecasting accuracy used in forecasting competitions. For a time series  $y_t$ ,  $t = 1, 2, \dots, T$ , the MASE is defined by

$$\text{MASE} = \frac{\frac{1}{H} \sum_{h=1}^H |y_{T+h} - \hat{y}_{T+h}|}{\frac{1}{n-m} \sum_{t=m+1}^T |y_t - y_{t-m}|}, \quad (1)$$

where  $H$  is the length of the forecast horizon,  $T$  is the length of the training period of the time series, and  $m$  is the frequency of the time series.  $\hat{y}_{T+h}$  is the  $h$ -step point forecast. A description of the features calculated in each frequency category is given in Table 4. We analyse yearly, quarterly, monthly, weekly, daily, and hourly series separately. Table 5 shows the forecast models we consider within each frequency category.

## 2.4. Training the meta-learner

We use a Bayesian surface spline regression to train the meta-learner. The most commonly used additive spline

regression assumes additivity in the regressor; that is,  $E(y|x_1, \dots, x_q) = \sum_{j=1}^q f_j(x_j)$ , where  $f_j(x_j)$  is a spline regressor of the  $j$ th regressor. Even though the assumption of additivity simplifies the model, it is quite a restrictive assumption. This problem has motivated research on surface models with interactions between regressors (Li & Villani, 2013). Li and Villani (2013) proposed a general Bayesian approach for fitting surface models for a continuous multivariate response by combining additive splines and interactive splines. The proposed modelling is called efficient Bayesian multivariate surface regression.

The reason for using multivariate analysis is that univariate analysis does not take into account the relationships between variables and their correlations to the classification between groups. The motivation for the use of efficient Bayesian multivariate surface regression is due to its flexibility. The main challenge in spline regression is the choice of knot locations.

The number of knots has an important influence on the resulting fit of spline regression: without enough knots, the regression is underfitted, and with too many knots, it is overfitted. Choosing the locations of knots is also a challenge. This becomes even more difficult for surface regression than it is for additive models, because any feasible set of  $q$ -dimensional knots is necessarily sparse in  $R^q$  when the number of regressors,  $q$ , is moderate or large. This causes the curse of dimensionality. The most common approach in the literature is to use a fixed set of knot locations, and most of these algorithms place the knots at the centroids of the clusters computed based on regressor observations. Li and Villani (2013) point out that this is impractical when estimating a surface with several regressors. Hence, the authors use a computationally efficient Markov chain Monte Carlo (MCMC) algorithm for the Gaussian multivariate surface regression to update the locations of the knots jointly. Instead of a fixed set of locations, the authors introduce moving knots that allow the location of the knots to move freely in the regressor space.

The proposed Gaussian multivariate regression model can be written as follows:

$$\mathbf{Y} = \mathbf{X}_0 \mathbf{B}_0 + \mathbf{X}_a(\xi_a) \mathbf{B}_a + \mathbf{X}_s(\xi_s) \mathbf{B}_s + \mathbf{E}, \quad (2)$$

where  $\mathbf{Y}$  is a matrix of  $n$  observations and  $p$  response variables. The rows of  $\mathbf{E}$  are error vectors assumed to be independent and identically distributed as  $N_p(0, \Sigma)$ . The proposed efficient Bayesian multivariate surface regression model contains three components:

1. **Linear component:** The linear component contains the original covariates, including the constant term. This enters the model in linear form. The matrix  $\mathbf{X}_0$  is an  $n \times q_0$  vector in which the first column contains ones for the intercept. The corresponding regression coefficients are in  $\mathbf{B}_0$ .

2. **Additive component:** The second component of the model contains additive spline basis functions of the covariates in  $\mathbf{X}_0$ . This is represented by  $\mathbf{X}_a(\xi_a)$ , where  $\xi_a$  represents the knots. It is important to note that the knots in the additive part of the model are scalars and this model allows for an unequal number of knots for different covariates. The

**Table 4**

Time series features and availabilities for yearly (Y), quarterly (Q), monthly (M), weekly (W), daily (D), and hourly (H) data.

Index	Feature	Description	Y	Q/M	W	D/H
1	T	Length of time series	✓	✓	✓	✓
2	trend	Strength of trend	✓	✓	✓	✓
3	seasonality_q	Strength of quarterly seasonality	–	✓	–	–
4	seasonality_m	Strength of monthly seasonality	–	✓	–	–
5	seasonality_w	Strength of weekly seasonality	–	–	✓	✓
6	seasonality_d	Strength of daily seasonality	–	–	–	✓
7	seasonality_y	Strength of yearly seasonality	–	–	–	✓
8	linearity	Linearity	✓	✓	✓	✓
9	curvature	Curvature	✓	✓	✓	✓
10	spikiness	Spikiness	✓	✓	✓	✓
11	e_acf1	First ACF value of remainder series	✓	✓	✓	✓
12	stability	Stability	✓	✓	✓	✓
13	lumpiness	Lumpiness	✓	✓	✓	✓
14	entropy	Spectral entropy	✓	✓	✓	✓
15	hurst	Hurst exponent	✓	✓	✓	✓
16	nonlinearity	Nonlinearity	✓	✓	✓	✓
17	alpha	ETS(A,A,N) $\hat{\alpha}$	✓	✓	✓	–
18	beta	ETS(A,A,N) $\hat{\beta}$	✓	✓	✓	–
19	hwalpha	ETS(A,A,A) $\hat{\alpha}$	–	✓	–	–
20	hwbeta	ETS(A,A,A) $\hat{\beta}$	–	✓	–	–
21	hwgamma	ETS(A,A,A) $\hat{\gamma}$	–	✓	–	–
22	ur_pp	Test statistic based on Phillips-Perron test	✓	–	–	–
23	ur_kpss	Test statistic based on KPSS test	✓	–	–	–
24	y_acf1	First ACF value of the original series	✓	✓	✓	✓
25	diff1y_acf1	First ACF value of the differenced series	✓	✓	✓	✓
26	diff2y_acf1	First ACF value of the twice-differenced series	✓	✓	✓	✓
27	y_acf5	Sum of squares of first 5 ACF values of original series	✓	✓	✓	✓
28	diff1y_acf5	Sum of squares of first 5 ACF values of differenced series	✓	✓	✓	✓
29	diff2y_acf5	Sum of squares of first 5 ACF values of twice-differenced series	✓	✓	✓	✓
30	sediff_acf1	ACF value at the first lag of seasonally differenced series	–	✓	✓	✓
31	sediff_seacf1	ACF value at the first seasonal lag of seasonally differenced series	–	✓	✓	✓
32	sediff_acf5	Sum of squares of first 5 autocorrelation coefficients of seasonally differenced series	–	✓	✓	✓
33	seas_pacf	Partial autocorrelation coefficient at first seasonal lag	–	✓	✓	✓
34	lmres_acf1	First ACF value of residual series of linear trend model	✓	–	–	–
35	y_pacf5	Sum of squares of first 5 PACF values of original series	✓	✓	✓	✓
36	diff1y_pacf5	Sum of squares of first 5 PACF values of differenced series	✓	✓	✓	✓
37	diff2y_pacf5	Sum of squares of first 5 PACF values of twice-differenced series	✓	✓	✓	✓

**Table 5**

Forecasting model class label and availabilities for yearly (Y), quarterly (Q), monthly (M), weekly (W), daily (D), and hourly (H) data.

Model class label	Description	Y	Q/M	W	D/H
WN	White noise process	✓	✓	✓	✓
auto.arima	Autoregressive integrated moving average model with automatic lag selections.	✓	✓	✓	–
ets	Exponential smoothing state space model.	✓	✓	–	–
rw	Random walk	✓	✓	✓	✓
rwd	Random walk with drift.	✓	✓	✓	✓
theta	Decomposition forecasting model by modifying the local curvature of the time series through a coefficient “Theta” that is applied directly to the second differences of the data.	✓	✓	✓	✓
stlar	STL decomposition with AR modelling of the seasonally adjusted series.	–	✓	✓	✓
snaive	Seasonal naïve method, which forecasts using the most recent values of the same season.	–	✓	✓	✓
tbats	Exponential smoothing state space model with a Box-Cox transformation, ARMA errors, trend, and seasonal components	–	✓	✓	✓
nn	Neural network time series forecasts.	✓	✓	✓	✓
mstlets	Multiple seasonal time series forecasts with ETS.	–	–	✓	✓
mstlarima	Multiple seasonal time series forecasts with ARIMA.	–	–	–	✓

matrix  $\mathbf{B}_a$  contains the regression coefficients corresponding to the additive component. The additive component of the model captures the nonlinear relationship between features and the response  $\mathbf{Y}$ .

**3. Surface component:** The surface component of the model contains a radial basis function for capturing the remaining part of the surface and interactions. This is denoted by  $\mathbf{X}_s(\xi_s)$ . Note that the  $\xi_s$  is a

$q_0$ -dimensional vector. The matrix  $\mathbf{B}_s$  contains the regression coefficients corresponding to the surface component, and  $\xi_s$  represents the surface knots.

For notational convenience, Eq. (2) can be written as

$$\mathbf{Y} = \mathbf{X}\mathbf{B} + \mathbf{E}, \quad (3)$$

where  $\mathbf{X} = [\mathbf{X}_0, \mathbf{X}_a, \mathbf{X}_s]$  is the  $n \times q$  design matrix ( $q = q_0 + q_a + q_s$ ) and  $\mathbf{B} = [\mathbf{B}_0, \mathbf{B}_a, \mathbf{B}_s]$ . For a given set of fixed knot locations, the model in Eq. (2) is linear in regression parameters. The over-parameterised problem is addressed by using shrinkage priors to shrink small regression coefficients towards zero. The shrinkage parameters and knot locations of  $\xi_a$  and  $\xi_s$  are treated as unknown parameters to be estimated. Li and Villani (2013) proposed a computationally efficient MCMC algorithm to estimate shrinkage parameters and update the locations of the knots of additive and surface components jointly. This approach allows for an unequal number of knots in the different covariates. In addition, separate shrinkage parameters for the linear, additive, and surface parts of the model are allowed. Further, this approach permits separate shrinkage parameters for the  $p$  responses within each of the three model parts. In contrast to other spline-based models, this approach allows the knot locations to move freely in the regressor space, and thus fewer knots are usually required. The estimation and computation details can be found in Li and Villani (2013).

### 3. Application to the M4 competition data

#### 3.1. Dissimilarity between different datasets

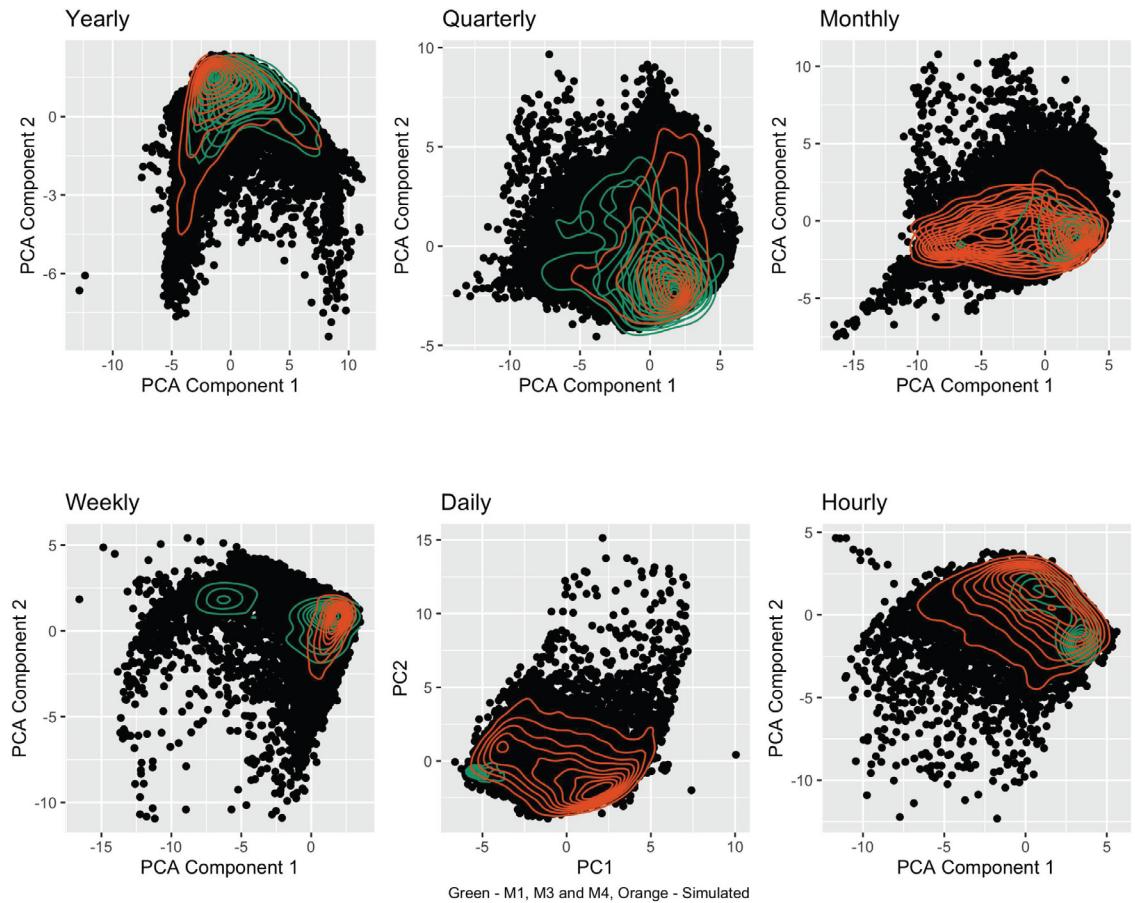
Principal component analysis (PCA) preserves the dissimilarity between widely separated data points rather than the similarity between nearby data points. This feature is useful for improved confidence in simulated series' representativeness of real time series. For example, if the simulated time series results in isolated clusters or highly dense clusters far apart from the real-world time series, it may indicate a poor representation of the real data.

We use PCA to visualise dissimilarity between the datasets: observed time series (M1 and M3), simulated series, and new time series (M4) in the two-dimensional instance space. The data are normalised using the Z-scale transformation before applying PCA. The purpose of projecting M4 on the PCA space created by the reference time series (M1, M3, and GRATIS) is to explore the differences in feature distribution characteristics between the training set (reference set) time series and the test set time series (M4). Since the meta-learner is trained on the reference data, the new time series to be forecast must lie within the space of the reference data. Therefore, we first compute the principal component projection using the features in the reference set, and then project the new time series to the same low-dimensional feature space. The coverage of the reference set over the new time series in the space provides an initial idea about the suitability of the meta-learner in forecasting new time series. For example, if many series fall outside of the space covered by the reference set, a new meta-learner must be trained.

The projection also helps us to gain an idea about the global structure of the location of the different collections. The results are shown in Fig. 4. The associated density plots are shown in Fig. 5. We compute principal components using the time series in the reference set (observed series and simulated series) and project new series (M4) into the two-dimensional space spanned by the first two eigenvectors. The first two principal components explain 51.3%, 48.7%, and 48.3% of the total variation in the yearly, quarterly, and monthly data, respectively. We see that the distribution of the simulated time series (represented by the dark orange dots) clearly nests and fills in the instance space, although the simulated data are independent from the observed series, except that we use the length distribution of the M4 series as a reference for that of the simulated series lengths, which is consistent with our theoretical expectation. Further, we can see that the projection of M4 series falls within the space created by the series in the reference set (M1, M3, and simulated series). This is very important because our FFORMPP framework is trained based on the series in the reference set; hence, the model is valid over the space of the reference set. A few M4 time series in the monthly frequency category fall outside the space covered by the reference set in the first two principal components owing to very high lengths. Note that for yearly, quarterly, and monthly series, the size of the M4 time series collection is much greater than the size of the corresponding collection of simulated series. For yearly and quarterly data, the number of time series in the M4 collection is about twice as large as the simulated series, and for monthly data, the M4 competition collection is four times larger than the simulated series. This shows the efficiency of the GRATIS simulation approach in increasing the diversity of feature space without having many time series similar in size to the new time series collection from which we wish to produce forecasts.

For weekly, daily, and hourly time series the first two principal components explain 48.7%, 43.4%, and 45.4% of the total variance of weekly, daily, and hourly series, respectively. In all three frequency categories, M4 competition data fall within the feature space of simulated series. For weekly, daily, and hourly frequency categories, the simulated time series successfully fill in the space of the new time series. This shows the efficiency of the GRATIS approach in generating a realistic set of representative time series. However, the daily series of the M4 competition series are clustered into a single location of the feature space. This is due to quality issues with the M4 competition data. Daily series in the M4 competition appear very similar to each other, owing to data leakages, as pointed out by Ingel et al. (2020). Examples of data leakage include the use of different segments of time series to create a new time series, and the addition of a constant value to create new time series. Another reason that the series are clustered in one place is that the selected features are not sufficiently diverse to capture the subtle changes of those time series.

Fig. 6 visualises the reasons why some test series of M4 fall outside the space of the training set. For yearly series, the reason is high length values compared to the



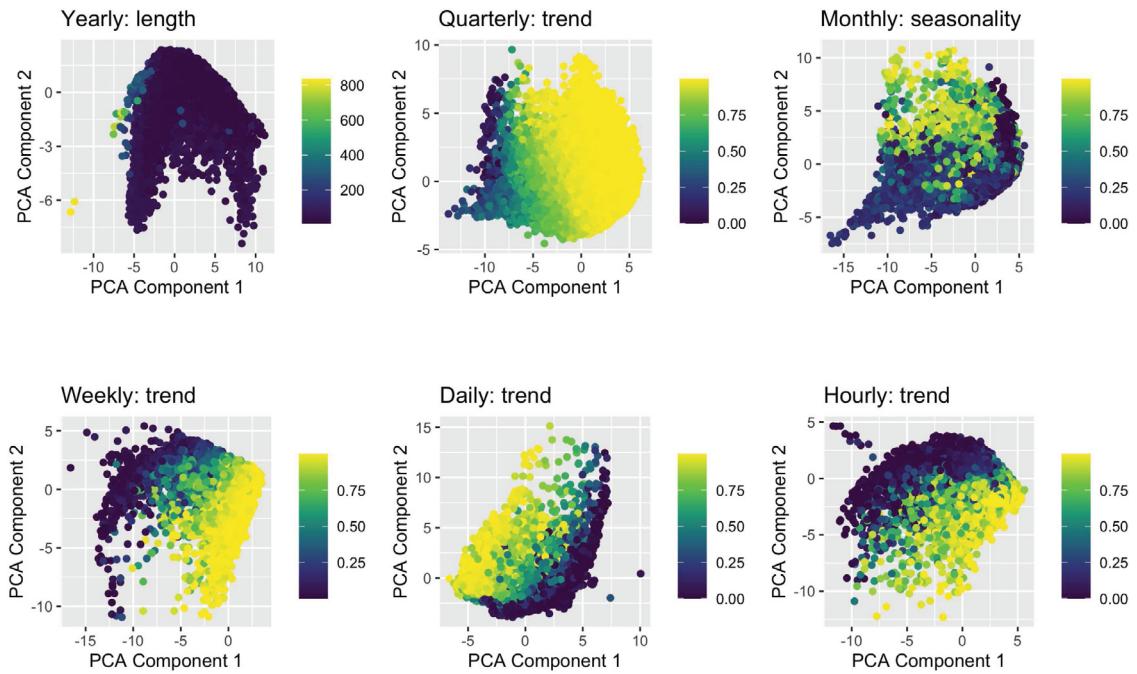
**Fig. 5.** Visualisation of densities of yearly, quarterly, monthly, weekly, daily, and hourly time series on the PCA space. On each graph, dark orange represents the simulated series and green denotes new time series (M4 data). Note that for yearly, quarterly, and monthly series the size of the test set is much larger than the size of the corresponding reference set.

training set. For quarterly series, the reason is high trend values, and for monthly series, the reason is low seasonality. For weekly, daily, and quarterly series, the reason is the heterogeneity in trend values. These associated features are identified by reviewing the coefficients of the PCA components. Fig. 6 is obtained by colouring the PCA feature space using the corresponding features. The information revealed from Fig. 6 can be taken into account in the offline phase of the algorithm to make the simulated space even more representative of the M4 test set. For example, a few yearly M4 test series fall outside the training space, due to high length values. We can use the GRATIS approach to simulate time series that cover the full length range of the test data. Furthermore, for monthly series, the values for the strength of the seasonality of the training series are very high compared to actual ones. By generating series with strength of seasonality varying between 0.1 to 0.3, our training dataset will be more representative of the test set.

### 3.2. Forecasting results

We evaluate the out-of-sample performance of our proposed meta-learning framework against the benchmarks based on the M4 competition series. The MASE

is used to evaluate forecast accuracy. We compute both individual and combination forecasts for the M4 competition series. Individual forecasts, denoted as “FFORMPP-selection”, are computed based on the model corresponding to the minimum predicted MASE. Combination forecasts, denoted as “FFORMPP-combination-4”, are computed by taking the median of individual forecasts corresponding to the four models with the minimum predicted MASE values. The reason for using the average of the best four models is that, as pointed by (Lichten-dahl Jr et al., 2013; Petropoulos & Svetunkov, 2020), the simple average combination is as accurate as complex approaches and is a robust combination method that is hard to beat. Also, using the median of an even number of forecasts actually makes the final forecasts a combination of two individual forecasting methods, which simplifies the computation. The overall accuracy of each approach is computed using the weighted average based on the number of series involved in each data frequency. We also include the median forecast calculations based on 2, 4, 6, 8, and 10 models in order to show how forecasting accuracy is affected when the number of models used increases. According to Table 6, we can see the forecasting accuracy based on the median of the four models. The



**Fig. 6.** Visualisation of reasons why some test series of M4 fall outside the space of the training set. For yearly series, the reason is high length values compared to the training set. For the quarterly series, the reason is high trend values, and for monthly series, the reason is low seasonality. For weekly, daily, and quarterly series, the reason is heterogeneity in the trend values. The legends to the right of the graphs show the values of the corresponding time series features.

minimum predicted MASE is higher than the forecasting accuracy based on the median of the two models with the minimum MASE or the median of the 10 models with the minimum MASE. There is no significant difference in overall forecast accuracy when we increase the number of models used from four to six or eight. Hence, to reduce the computational cost, our final combination forecasts are computed based on the median of the four models with the minimum MASE.

We also compare our results with individual forecasting models, with their simple combinations, including SA-median-12 (simple averaging using the median of all 12 models available) and SA-mean-12 (simple averaging using the mean of all 12 models), and with FFORMS. FFORMS is trained based on the random forests algorithm. FFORMS outperforms FFORMPP-selection forecasts. However, FFORMPP-combination-4 outperforms FFORMS. One could argue that, in the FFORMS algorithm, the random forest probability scores could be used for weighting alternative forecast models, constructing a robust combination scheme, or selecting a set of models for combination forecast. However, this approach did not bring any improvement in performance. Furthermore, with some series, it degraded the performance. The reason for this is that, in FFORMS, the forecast model selection problem is treated as a classification problem. Hence, in this case, the weights are mostly close to either 0 and 1; the best model has a weight close to 1 and others have a weight very close to 0. For example, suppose for a given series that the forecast error vector for the random walk, the random walk with drift, and a white noise process is [1.31, 1.30, 3.4]. Then, ideally, the vector of class weight we expect

from FFORMS is [0, 1, 0]. That is, the best model is given class probability 1 and others 0. This limitation is another motivation for us to introduce FFORMPP.

Further, we compare our results with those of the top three places of the M4 competition. The winning method of the competition is based on the hybrid approach: a combination of exponential smoothing models and neural network approaches (Makridakis et al., 2018). The second and third approaches are based on combination forecasts computed based on nine and seven individual forecast models, respectively. According to the results of Table 6, we can see that our approach achieved comparable results in a much more cost- and time-effective manner because our combination forecasts are calculated based on four individual models. The performance of the proposed method is comparable to that of the top-ranked forecasting methods of the competition.

In addition to that, we also compare our results with the most recent feature-based meta-learning algorithms introduced by Li et al. (2020) and Wang et al. (2021). Li et al. (2020) proposed a meta-learning algorithm based on imaging, and Wang et al. (2021) proposed a feature-based algorithm based on generalised additive models (GAMs). We can see from Table 6 that FFORMPP yields comparable accuracy results.

To conclude, the results in Table 6 suggest that the forecast combination obtained through the FFORMPP framework outperforms the corresponding benchmarks and other commonly used methods of forecasting. Note that in this study, all evaluations are made out of sample with a fixed forecast origin and forecast horizon. The reason for evaluating the performance through a fixed

**Table 6**

MASE values calculated over the M4 competition data. The “FFORMPP-selection” results are computed based on the model corresponding to the minimum predicted MASE. The “FFORMPP-combination-4” results are computed by taking the median of individual forecasts corresponding to the four models with the minimum predicted MASE values. The median forecast calculations based on 2, 4, 6, 8, and 10 models are also included. “SA-median-12” means simple averaging using the median of all 12 models available, while “SA-mean-12” demonstrates simple averaging using the mean of all 12 models.

	Yearly	Quarterly	Monthly	Weekly	Daily	Hourly	Overall <sup>a</sup>
<b>FFORMPP-selection</b>	3.37	1.17	1.05	2.53	4.26	1.06	1.75
<b>FFORMPP-combination-4</b>	3.07	1.13	<b>0.89</b>	2.46	3.62	0.96	1.57
FFORMPP-combination-2	3.25	1.15	0.96	<b>2.38</b>	4.18	1.09	1.67
FFORMPP-combination-6	3.14	<b>1.12</b>	0.90	2.51	3.35	1.00	1.57
FFORMPP-combination-8	<b>3.06</b>	1.16	0.92	2.54	3.30	1.11	1.57
FFORMPP-combination-10	3.28	1.22	0.94	2.57	3.52	1.33	1.66
auto.arima	3.40	1.17	0.93	2.55	—	—	1.59
ets	3.44	1.16	0.95	—	—	—	1.60
theta	3.37	1.24	0.97	2.64	3.33	1.59	1.69
rwd	3.07	1.33	1.18	2.68	<b>3.25</b>	11.45	1.78
rw	3.97	1.48	1.21	2.78	3.27	11.60	2.04
nn	4.06	1.55	1.14	4.04	3.90	1.09	2.03
stlar	—	2.02	1.33	3.15	4.49	1.49	1.72
snaive	—	1.66	1.26	2.78	24.46	2.86	2.66
tbats	—	1.19	1.05	2.49	3.27	1.30	1.22
wn	13.42	6.50	4.11	49.91	38.07	11.68	8.45
mstlarima	—	—	—	—	3.84	1.12	3.59
mstlets	—	—	—	—	3.73	1.23	3.50
SA-median-12	3.29	1.22	0.95	2.57	3.52	1.33	1.66
SA-mean-12	4.09	1.58	1.16	6.96	7.94	3.93	2.25
FFORMS (random forest)	3.17	1.20	0.98	2.31	3.57	<b>0.84</b>	1.65
M4 competition top three places							
M4-1st	2.98	1.12	0.88	2.36	3.45	0.89	1.53
M4-2nd	3.06	1.11	0.89	2.11	3.34	0.81	1.54
M4-3rd	3.13	1.12	0.91	2.16	2.64	0.87	1.54
Meta-learning approaches							
Image-based features (Li et al., 2020)	3.13	1.12	0.90	2.26	3.46	0.84	1.57
FUMA - features and GAM models (Wang et al., 2021)	3.01	1.14	0.91	NA	NA	NA	1.47

<sup>a</sup>Weighted average based on the number of series for yearly, quarterly, monthly, weekly, daily, and hourly frequencies.

forecast origin is that most of the research results in this area, and the results of the M4 forecasting competition, are evaluated based on a fixed origin. Hence, using a fixed forecast origin makes it easy to compare our results with those in published work. Furthermore, the out-of-sample accuracy of each forecasting method can also be evaluated using point forecasts generated using a rolling forecast origin in future research comparisons.

#### 4. Understanding the FFORMPP meta-learner

##### 4.1. Performance impact factors

To evaluate whether the GRATIS simulation approach leads to a better forecast, we also trained a meta learning model using series originating only from the M1 and M3 competitions and compared results against model training, including the GRATIS-based simulated time series. In the experiments, we keep the training set as independent as possible of any knowledge of the time series patterns of the test set, since the independence of the test set to the training set is of fundamental importance to determine a model's out-of-sample predictive ability. The time series generated from the GRATIS approach are independent of test set (M4 data) except that the length distribution of the M4 series is used as a reference to that of the simulated series lengths.

We run four experiments to evaluate which part (addition of simulated time series or the EBMSR approach) of the algorithm helps improve the forecasting performance. Their details are as follows.

- **Experiment 1.** We use only M1 and M3 competition data as our reference set. Then, similar to the approach used by Petropoulos et al. (2014), we use a multiple linear regression approach to model the relationship between forecast errors and features. Separate regression models are fitted to model the relationship between features and errors for different forecasting methods.
- **Experiment 2.** In addition to M1 and M3 competition data, we also use a simulated time series. Multiple linear regression is used to model the relationship between and features. The only difference between the first and the second experiment is the addition of the simulated time series.
- **Experiment 3.** We use M1 and M3 as our reference set. The EBMSR approach is used to model the forecast errors simultaneously as a function of features.
- **Experiment 4.** It corresponds to our FFORMPP approach. We use M1, M3, and the simulated time series as our reference set. The EBMSR approach is used to model the forecast errors simultaneously as a function of features.

**Table 7**

MASE over the M4 competition data. The same experimental set-up introduced in Section 3 is used for producing the results, with the only difference being the reference set considered for training the meta-learner algorithms.

	Reference set	Yearly	Quarterly	Monthly	Weekly	Daily	Hourly	Overall <sup>a</sup>
Multiple linear regression (MLR)								
Experiment 1	M1 & M3	3.38	1.60	1.12	–	–	–	1.75
Experiment 2	M1, M3 & Simulated	3.38	1.18	1.05	2.61	3.32	1.19	1.68
Efficient Bayesian multivariate surface regression (EBMSR)								
Experiment 3	M1 & M3	3.44	1.18	1.47	–	–	–	1.87
Experiment 4 (FFORMPP)	M1, M3 & Simulated	3.07	1.13	0.89	2.46	3.62	0.96	1.47

<sup>a</sup>Weighted average based on the number of series for yearly, quarterly, and monthly frequencies.

**Table 8**

Computational time for producing forecasts based on 100 randomly selected series from each frequency category of the M4 dataset. The reported values are median elapsed time of 100 replicates. The corresponding interquartile range (IQRs) is given in parentheses.

	Computational time for producing forecasts in seconds (IQR)					
	Yearly	Quarterly	Monthly	Weekly	Daily	Hourly
FFORMPP	5.31(0.21)	30.45 (1.44)	190.05 (5.15)	183.36 (6.78)	93.76 (0.34)	56.14 (11.67)
auto.arima	5.91 (0.05)	42.11 (2.15)	448.41 (1.95)	584.98 (2.35)	–	–
ets	1.14 (0.02)	16.92 (0.09)	115.50 (0.17)	–	–	–
theta	2.74 (2.48)	10.15 (11.45)	29.13 (1.15)	96.06 (0.42)	83.77 (2.67)	54.32 (2.32)
rwd	0.29 (5.42)	0.29 (8.20)	0.33 (15.57)	0.34 (21.78)	0.41 (33.72)	0.37 (26.46)
rw	0.16 (4.65)	0.20 (6.67)	0.26 (15.68)	0.22 (17.16)	0.27 (19.56)	0.24 (9.58)
nn	2.32 (0.14)	6.54 (0.23)	32.78(0.25)	281.68 (0.61)	424.28 (1.71)	354.97 (3.61)
stlar	–	0.83 (17.97)	0.94 (12.03)	0.90 (10.11)	2.21 (0.12)	1.70 (0.01)
snaive	–	0.18 (4.51)	0.30 (3.12)	0.20 (1.44)	0.32 (0.34)	0.44 (2.12)
tbats	–	20.16 (6.98)	38.12 (2.44)	40.16 (3.36)	68.73 (0.65)	49.52 (2.98)
wn	0.19 (2.65)	0.20 (4.30)	0.23 (0.08)	0.19 (4.51)	0.26 (1.00)	0.22 (0.05)
mstlarima	–	–	–	–	86.92 (0.52)	30.60 (0.17)
mstlets	–	–	–	–	19.79 (0.09)	10.13 (0.48)

The accuracy of the four experiments is evaluated by comparing the out-of-sample MASE. The results of the experiments are shown in Table 7. The overall accuracy is the weighted average based on the number of series involved in yearly, quarterly, and monthly series. It can be seen that the accuracy in Experiment 2 is higher than that in Experiment 1. Furthermore, the accuracy of the results in Experiment 4 (FFORMPP) is higher than the accuracy of the results in Experiment 3. This suggests that the addition of the simulated time series helps to improve the accuracy of the results. According to the results of Experiment 1 and Experiment 3, using EBMSR over MLR when considering only the M1 and M3 data has a negative effect (reducing the forecasting accuracy). However, according to the results of Experiment 4, combining EBMSR with the simulated data improves the forecasting accuracy. Hence, from the four experiments implemented, we find that the effect of GRATIS depends on the algorithm used for employing the meta-learner, and using GRATIS and EBMSR separately does not always guarantee improvements in forecasting accuracy. It could be the case that EBMSR requires a large collection of series to identify the relationship between independent and dependent variables and that, when this is not the case, simpler meta-learning algorithms may lead to better results. The overall results indicate that both simulated series and the EBMSR approach contribute to an improvement in forecasting accuracy.

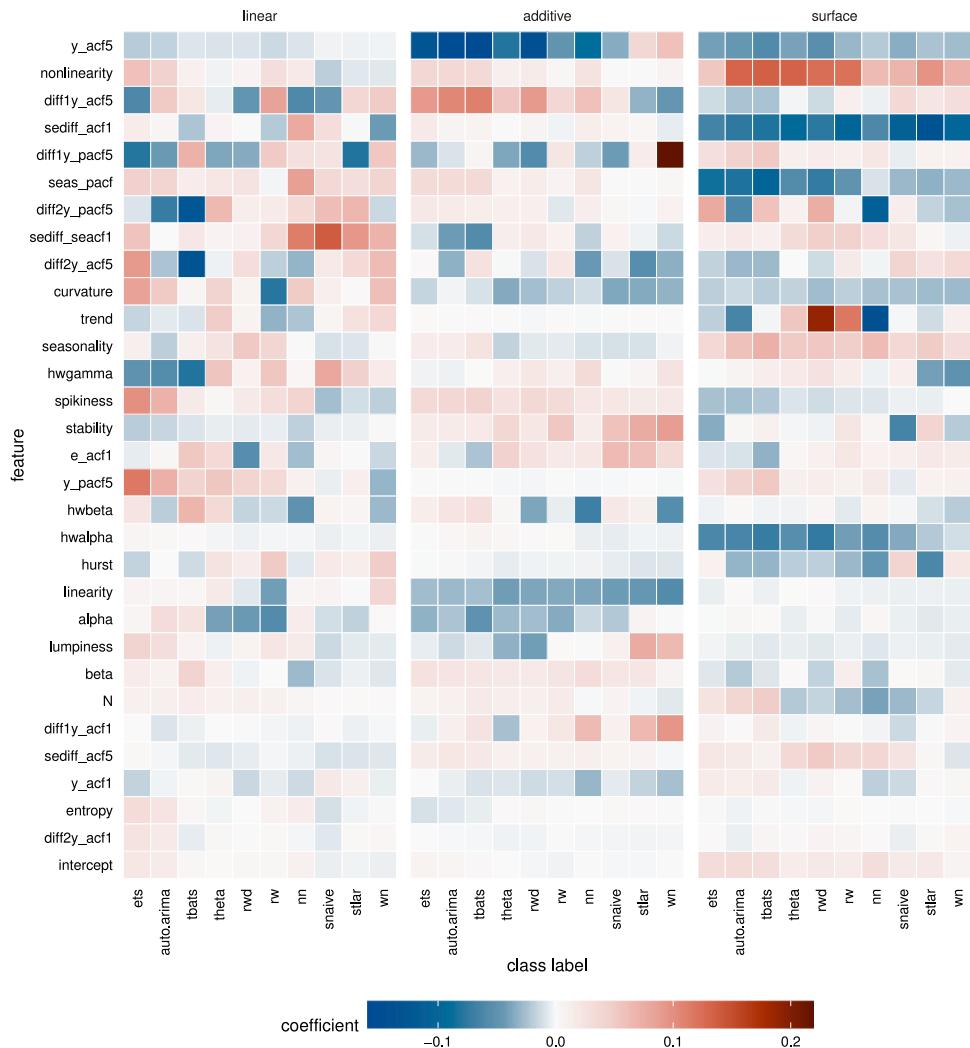
To ensure a fair comparison, the computational time for producing forecasts based on 100 randomly selected

series from each frequency category of the M4 competition dataset is given in Table 8. The reported values are the median elapsed time of 100 replicates. The corresponding interquartile range (IQR) is given in parentheses. We can see that FFORMPP is significantly faster than the total computational time of the individual methods used for forecast combination. The computational time was measured using the R package *microbenchmark* (Mersmann, 2019) on 24 core Xeon-E5 2.50 GHz servers.

#### 4.2. Visualising patterns learned by the meta-learner

We first visualise the estimated model coefficients for the linear component and two nonlinear components (additive and surface) of the model. Fig. 7 shows a heat map of estimated model coefficients for the quarterly series. The corresponding visualisations for yearly, monthly, weekly, daily, and hourly frequencies are given in the Appendix.

According to Fig. 7, the estimated coefficients of features related to autocorrelation and partial autocorrelation coefficients drastically change across class labels and within the three different components of the model. The estimated coefficients related to the trend, Hurst exponent, nonlinearity, length, and strength of seasonality vary across class labels within the additive component of the model. Furthermore, Fig. 7 shows that different features play a different role within the linear, additive, and surface components of the model. Zooming into individual cells, conditional on other components and features, we



**Fig. 7.** Visualisation of estimated model coefficients for the quarterly series. The y-axis represents features, the x-axis represents class labels, and the heat map cells represent estimated model coefficients. The class labels are ordered with their descending ranks.

can see that within the additive component, the feature  $y\_acf5$  increases as the estimated MASE value of `auto.arima` decreases, while for `wn`,  $y\_acf5$  increases as the estimated MASE increases. This indicates that for series with a high  $y\_acf5$  value, the `auto.arima` algorithm gives low MASE values, while the white noise process gives high MASE values. The other coefficients can be interpreted similarly. For general guidance of interpreting semiparametric models such as the ones we used in the paper, we refer the reader to Ruppert et al. (2003) and Li and Villani (2013).

We also report the relative frequencies with which each forecast model is selected as a component of the calculation of combination forecasts based on FFORMPP. Further, to gain an idea of the different types of model combinations used to compute the forecast, we cluster time series based on the models that are used to calculate the FFORMPP combination. For this purpose, we first create a design matrix of ones and zeroes. The columns of the design matrix correspond to each forecast model,

and rows correspond to each time series. The cell values of the matrix are ones and zeroes, where one is assigned if a corresponding model is used for combination forecast, and zero is assigned otherwise. Then, hierarchical clustering is done by using a binary distance metric and ward clustering method. A cluster analysis is performed separately for each frequency category.

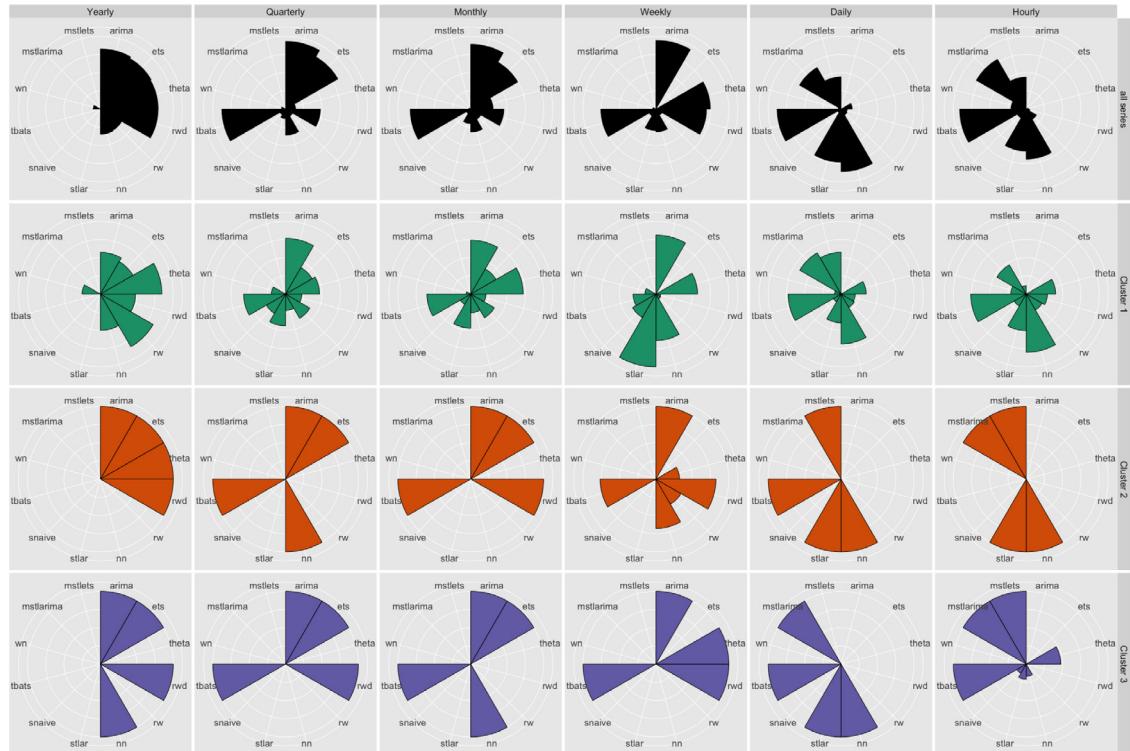
For each frequency category, we can identify three main clusters. The results are shown in Table 9, and the associated graphical representation is shown in Fig. 8. According to the results of Table 6, for yearly data we can see that `auto.arima`, `ets`, `theta`, and `rwd` give the best individual forecasts. From Table 9, we can see that those four models are most frequently selected to the FFORMPP combination forecast. Similarly, from Table 6 we can see that for the quarterly and monthly series, `auto.arima`, `ets`, and `tbats` provide the best individual forecasts, and according to Table 9, we can see that those models are selected most often (approximately greater than 75%) for the quarterly and monthly series. Similarly,

**Table 9**

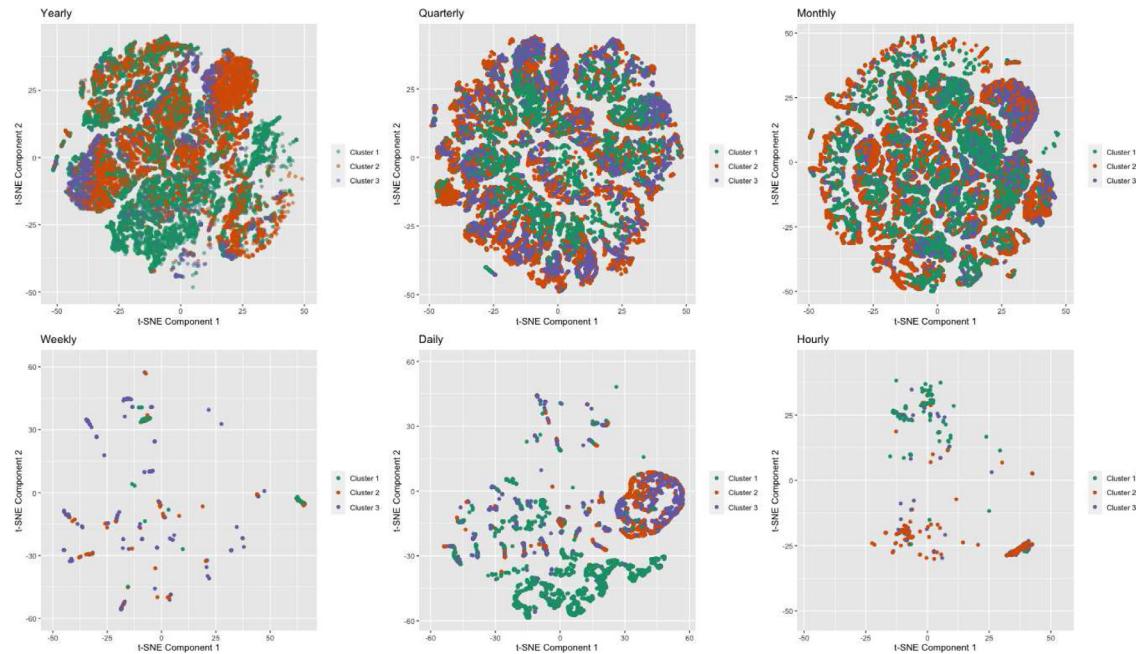
Relative frequencies that each forecast model was selected as a component for the calculation of combination forecasts based on FFORMPP (All values are shown in percentages).

Source	No. of series	auto.arima	ets	theta	rwd	rw	nn	stlar	snaive	tbats	wn	mstlarima	mstlets
Yearly	23,000	<b>82.7</b>	<b>80.4</b>	<b>79.2</b>	<b>79.1</b>	33.7	34.6	—	—	—	10.2	—	—
Cluster 1	9,287	57.2	51.5	84.3	48.3	83.5	49.9	—	—	—	25.3	—	—
Cluster 2	10,391	100	100	100	100	0	0	—	—	—	0	—	—
Cluster 3	3,322	100	100	0	100	0	100	—	—	—	0	—	—
Quarterly	24,000	<b>93.1</b>	<b>82.8</b>	13.8	47.8	11.4	35.8	12.9	9.1	<b>87.4</b>	5.8	—	—
Cluster 1	7,126	76.6	42.1	46.6	22.5	38.4	22.6	43.7	30.5	57.6	19.5	—	—
Cluster 2	6,994	100	100	0	0	0	100	0	0	100	0	—	—
Cluster 3	9,880	100	100	0	100	0	0	0	0	100	0	—	—
Monthly <sup>a</sup>	48,000	<b>88.9</b>	<b>74.4</b>	30.6	45.4	16.1	31.8	19.9	7.1	<b>83.1</b>	2.8	—	—
Cluster 1	12,615	74.1	40.0	72.0	20.9	37.5	25.5	46.8	16.5	60.1	6.5	—	—
Cluster 2	11,084	100	100	0	100	0	0	0	0	100	0	—	—
Cluster 3	6,301	100	100	0	0	0	100	0	0	100	0	—	—
Weekly	359	<b>94.4</b>	—	<b>74.7</b>	69.4	9.8	31.5	29.5	10.9	<b>75.5</b>	4.5	—	—
Cluster 1	105	80.9	—	57.1	6.7	76.2	63.8	100	37.1	31.4	15.2	—	—
Cluster 2	68	100	—	32.4	82.4	39.7	67.6	1.5	0	76.5	0	—	—
Cluster 3	186	100	—	100	100	0	0	0	0	100	0	—	—
Daily	4,227	—	—	18.4	8.7	9.5	<b>85.9</b>	<b>72.9</b>	5.63	<b>87.7</b>	4.1	65.8	44.3
Cluster 1	1,891	—	—	34.5	19.5	21.2	68.5	39.9	12.6	72.6	9.1	65.4	52.3
Cluster 2	791	—	—	0	0	0	100	100	0	100	0	0	100
Cluster 3	1,545	—	—	0	0	0	100	100	0	100	0	100	0
Hourly	414	—	—	2.9	4.8	16.2	68.8	57.7	14.3	<b>91.3</b>	21.0	<b>78.9</b>	43.9
Cluster 1	151	—	—	40.4	29.1	25.2	80.1	50.3	19.2	76.2	21.2	46.4	11.9
Cluster 2	162	—	—	0	0	0	100	100	0	0	0	100	100
Cluster 3	101	—	—	47.5	0	0	16.8	20.8	14.9	100	0	100	100

<sup>a</sup>Note: Cluster analysis is based on 30,000 series.



**Fig. 8.** Visual representation of relative frequencies that each forecast model was selected as a component for the calculation of combination forecasts based on FFORMPP. Each polar coordinate shows the relative frequency that each forecast model was selected as a component for the calculation of combination forecasts.



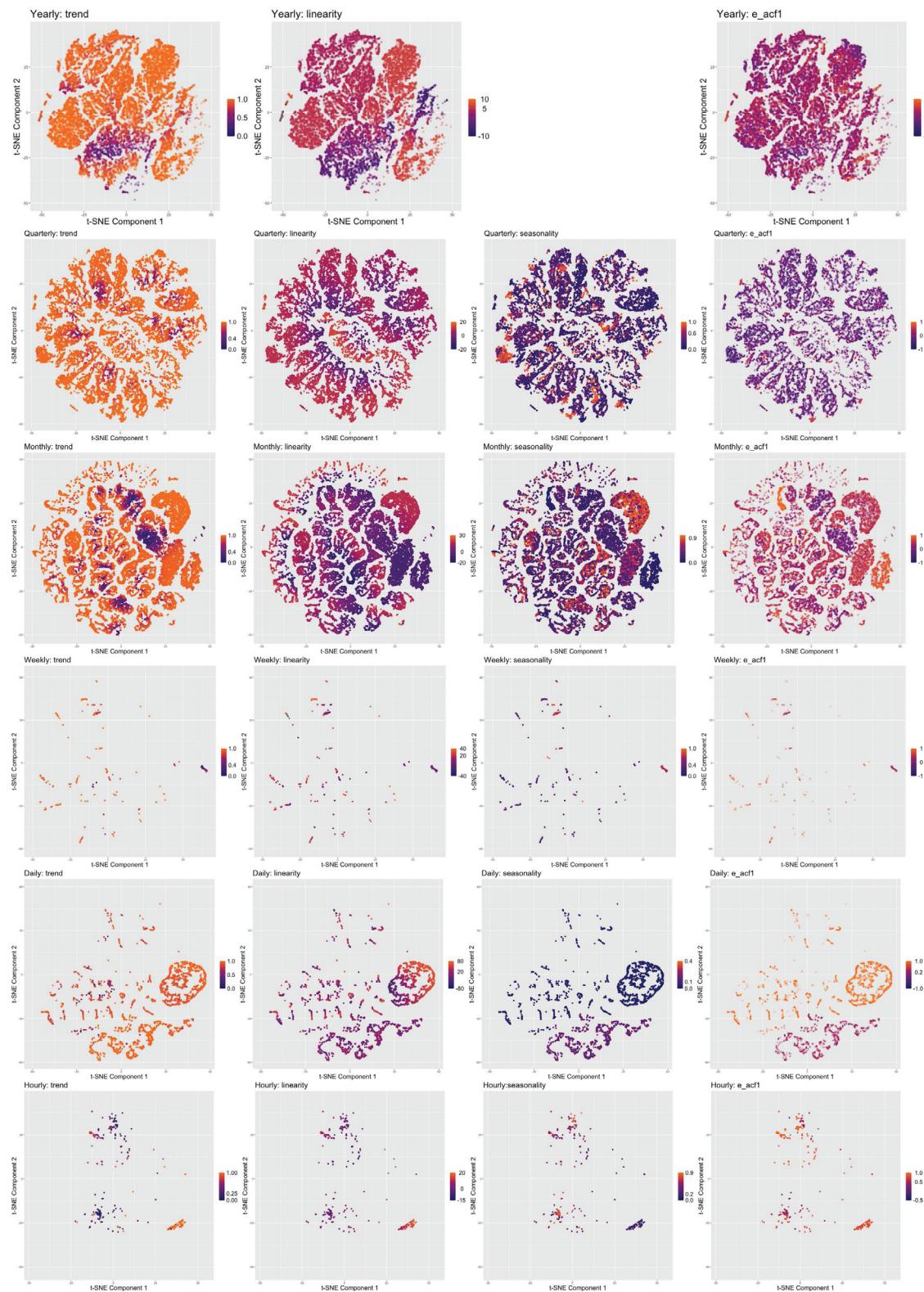
**Fig. 9.** Location of the three clusters in the instance space.

we can interpret the results for weekly, daily, and hourly series. Cluster 3 in the yearly series is very similar to cluster 2, the only difference being that cluster 3 uses nn instead of theta model. The series in the first cluster uses a different combination of models from that which we considered for the yearly series, apart from the two combinations used in cluster 2 and cluster 3, respectively. For the quarterly series, the biggest cluster is cluster 3, in which auto.arima, ets, rwd, and tbats are used to calculate combination forecasts. Similar to the results of quarterly series, for monthly and daily series we observe two clusters that are homogeneous in terms of the models used to compute combination forecasts. In terms of weekly data, most of the series use auto.arima, theta, rwd, and tbats for combination forecasts. For daily and hourly series, it is interesting to observe that all the series in cluster 2 and cluster 3 use at least one of the models, mstlarima or mstlets, which handle multiple seasonality, as a component in calculating combination forecasts.

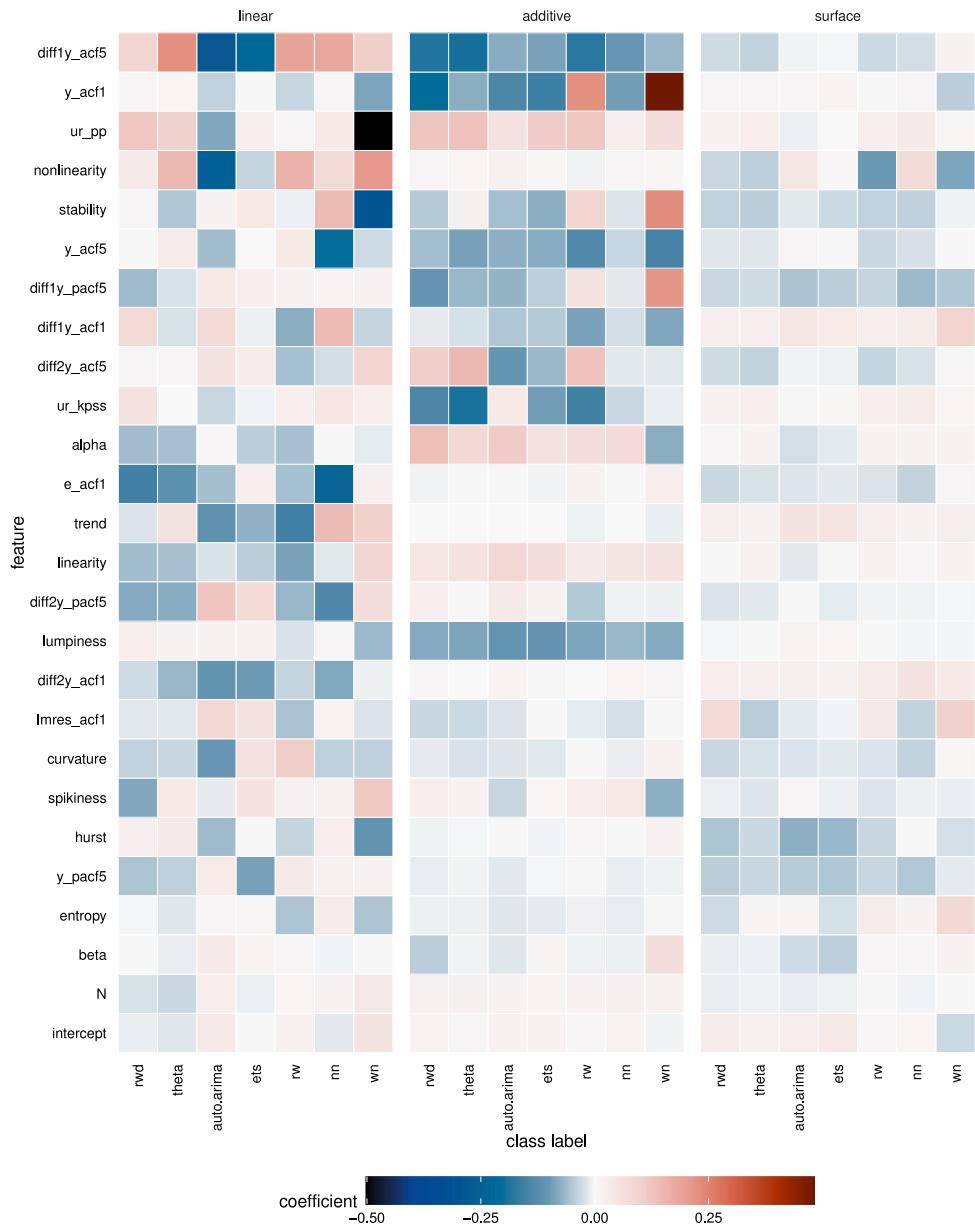
We now examine the locations of different clusters of M4 competition series in the instance space defined by features. Fig. 9 shows the locations of the three clusters in the instance space computed based on the t-SNE approach (Maaten & Hinton, 2008). For yearly series, we can see that most of the series in cluster 1 fall just below the diagonal, while series in clusters 2 and 3 fall within the upper triangular region. Further, clusters 2 and 3 are similar, and the series corresponding to clusters 2 and 3 stay close together. For the quarterly series, an interesting pattern of clusters can be observed. The time series in cluster 1 fall within the inner circle, while the series in clusters 2 and 3 fall within the second and third outer rings of the circle. This pattern of clusters is called non-spherical. For monthly series, the grouping of the clusters

is not obvious. However, a close inspection of the instance space shows a spiral pattern of clusters. Further, cluster 3 is preferred by the series in the upper-right corner. For weekly series, clusters are more dispersed across the instance space. For daily and hourly series, a clear separation of cluster 1 from clusters 2 and 3 can be observed. Clearly, within each frequency category, similar clusters (for example, clusters 2 and 3 for yearly, quarterly, and monthly series) are located close together in the instance space, ensuring the similarity of the features of those time series.

The useful description provided by Fig. 9 further prompts us to consider the challenge of identifying how the features of time series influence the grouping of these clusters. We now consider how different features vary across the instance space to understand how the locations of different time series reveal the relationship between the features and cluster separation. The preliminary results of the M4 competition (Makridakis et al., 2018) show that the randomness of time series is the most critical factor influencing forecast accuracy, followed by linearity. Further, in their follow-up paper Spiliotis et al. (2019) point out that highly trended and seasonal time series tend to be easier to forecast. The information about the remaining series is useful for gaining an idea of the random variation not explained by the trend and seasonality of the series. Hence, we explore the instance space corresponding to the features, the strength of the trend (trend), the extent of linearity (linearity), the strength of seasonality (seasonality), and the first autocorrelation coefficient of the remaining series after applying STL decomposition on the time series (e\_acf1) (Cleveland et al., 1990). The results are shown in Fig. 10 for yearly, quarterly, monthly, weekly, daily, and hourly series.



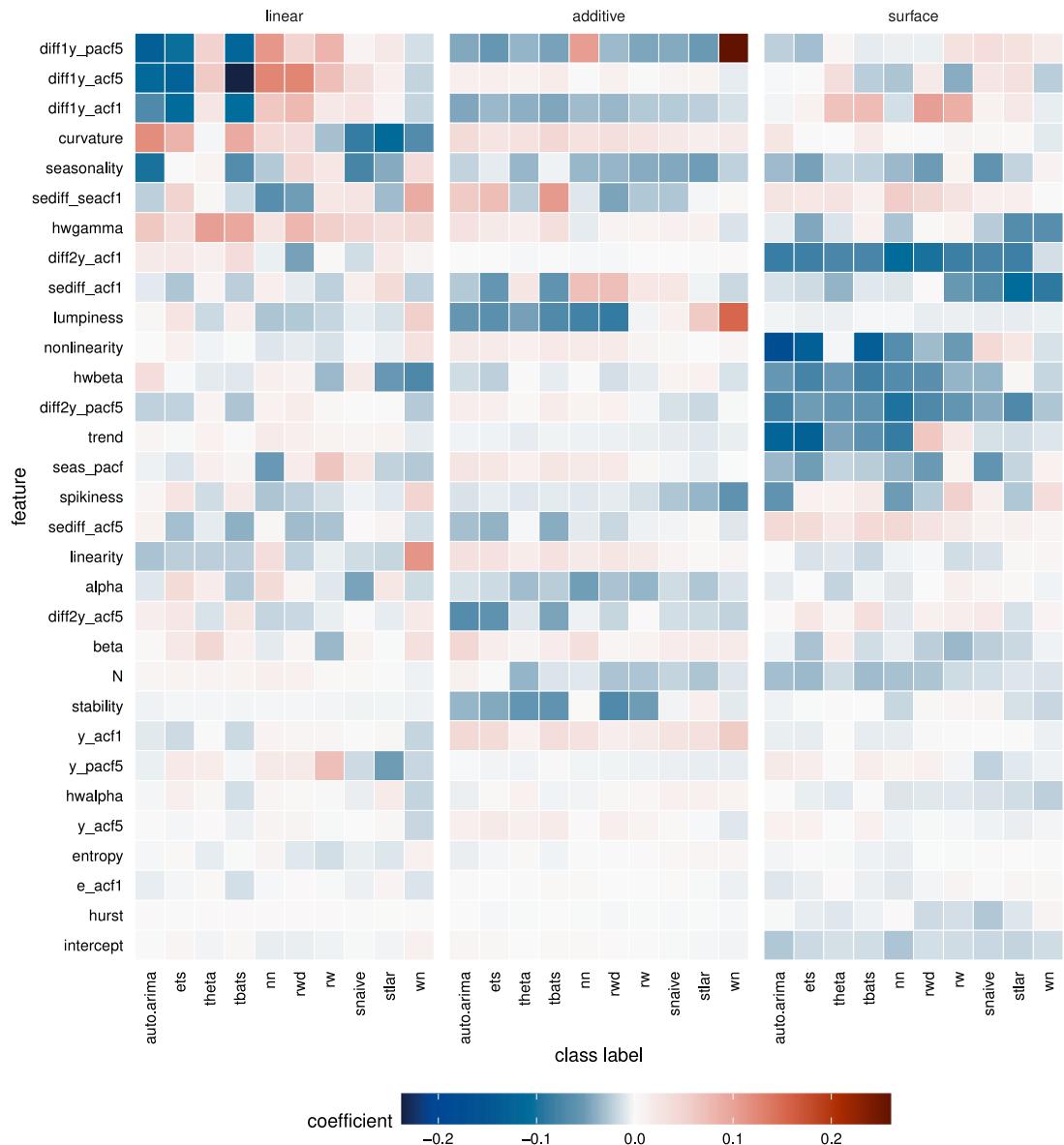
**Fig. 10.** Distribution of trend, linearity, seasonality, and e\_acf1 features across the instance space of the M4 competition data. The instances in the lower-right corner of the instance space are highly trended and less seasonal. Note that the seasonality feature is not available for the yearly data.



**Fig. 11.** Visualisation of estimated model coefficients for yearly series. The y-axis represents features, the x-axis represents class labels, and the heat map cells represent estimated model coefficients. The class labels are ordered with their descending ranks.

Combining the views of the instance spaces in Figs. 9 and 10, we have a picture of how the features contribute to the differences in clusters. According to Fig. 10, for yearly, quarterly, monthly, and weekly series, highly trended series are more likely to fall within clusters 2 and 3. For yearly, quarterly, and monthly series, most of the time series in the green cluster (in Fig. 9) are less trended with low linearity (in Fig. 10). Hence, those time series can be considered hard or challenging series to forecast. This is further confirmed from the results of Table 6 and Fig. 9: cluster 1 is more heterogeneous according to the way the models are selected to com-

pute combination forecasts (because they use different combinations of individual forecast models), in contrast to clusters 2 and 3. Furthermore, according to Fig. 10, we can see that for yearly, quarterly, and monthly series, the instance space coloured by linearity exhibits a similar structure to the corresponding cluster distribution across the instance space shown in Fig. 9. For daily series, the features seasonality and e\_acf1 clearly separate cluster 1 from the rest. Fig. 10 shows that, for hourly time series, features appear to separate the instance space to the left and right.



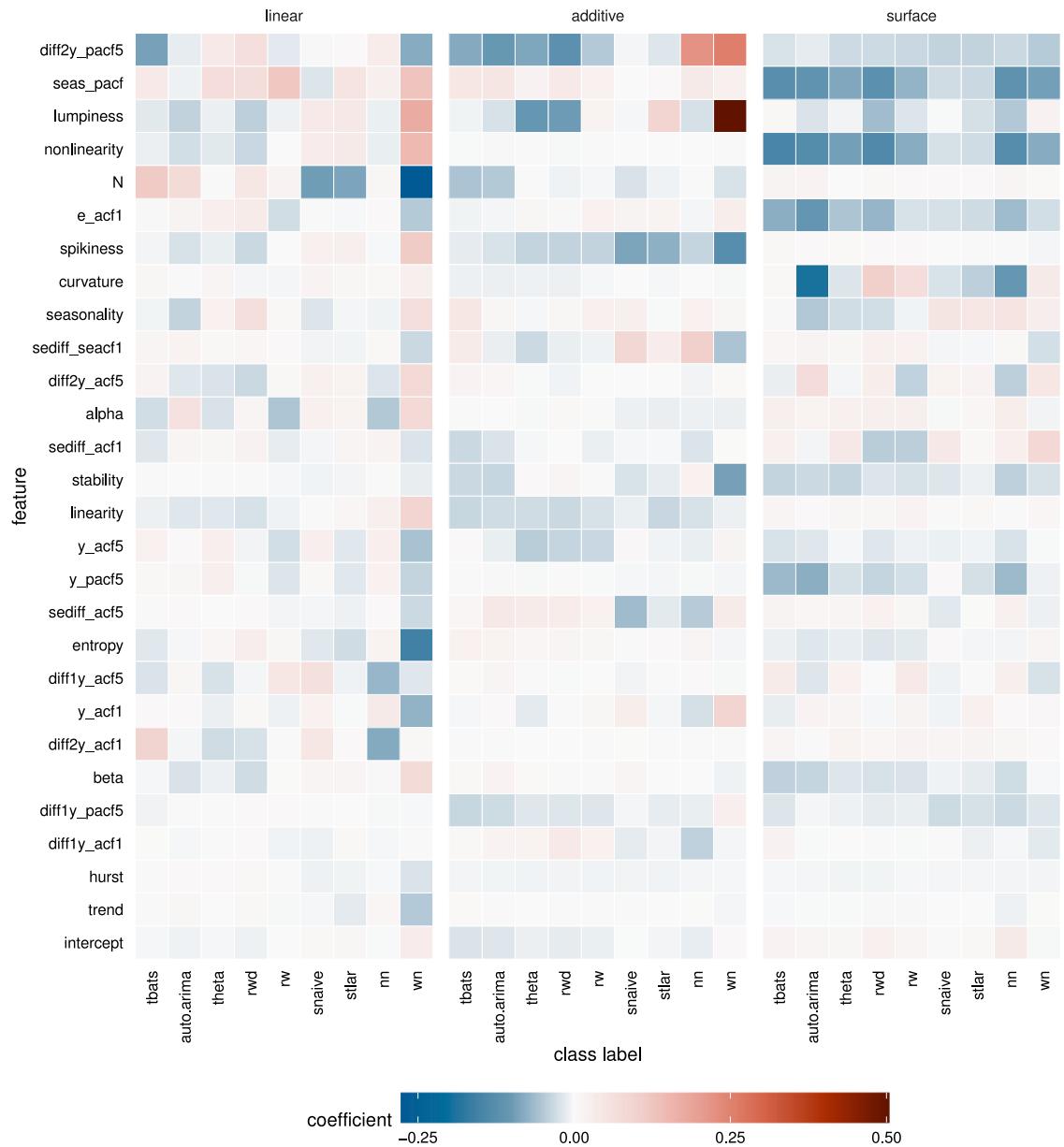
**Fig. 12.** Visualisation of estimated model coefficients for monthly series. The y-axis represents features, the x-axis represents class labels, and the heat map cells represent estimated model coefficients. The class labels are ordered with their descending ranks.

## 5. Conclusions

This paper proposed a new meta-learning framework for large-scale time series forecasting. The proposed framework can be used to compute both individual forecasts and combination forecasts, and it can be easily generalised to allow for forecast selection, pooling, or combination depending on the requirements and objectives of the forecasters. Moreover, the results provide useful insights regarding which forecasting models are expected to be better for particular types of series. The proposed algorithm provides comparable accuracy results to those of the top-ranked forecasting methods of the competition at lower computational cost, and can be easily parallelisable for use with large datasets for a given computing budget,

which is important for supporting decisions. Furthermore, we present a visual guide to exploring the use of meta-learning for forecast model selection and combination and how meta-learners are affected by various factors, such as the size of the selected method pool, and the size and diversity of the training samples.

Our results show that features of time series are useful for selecting an optimal subset of models from all individual models without the need to run all possible combinations of individual models during the online phase. Apart from the obvious utility of this approach for forecast model selection, the ranking of models provides an alternative solution to practitioners who may wish to incorporate their own judgements or expertise into the

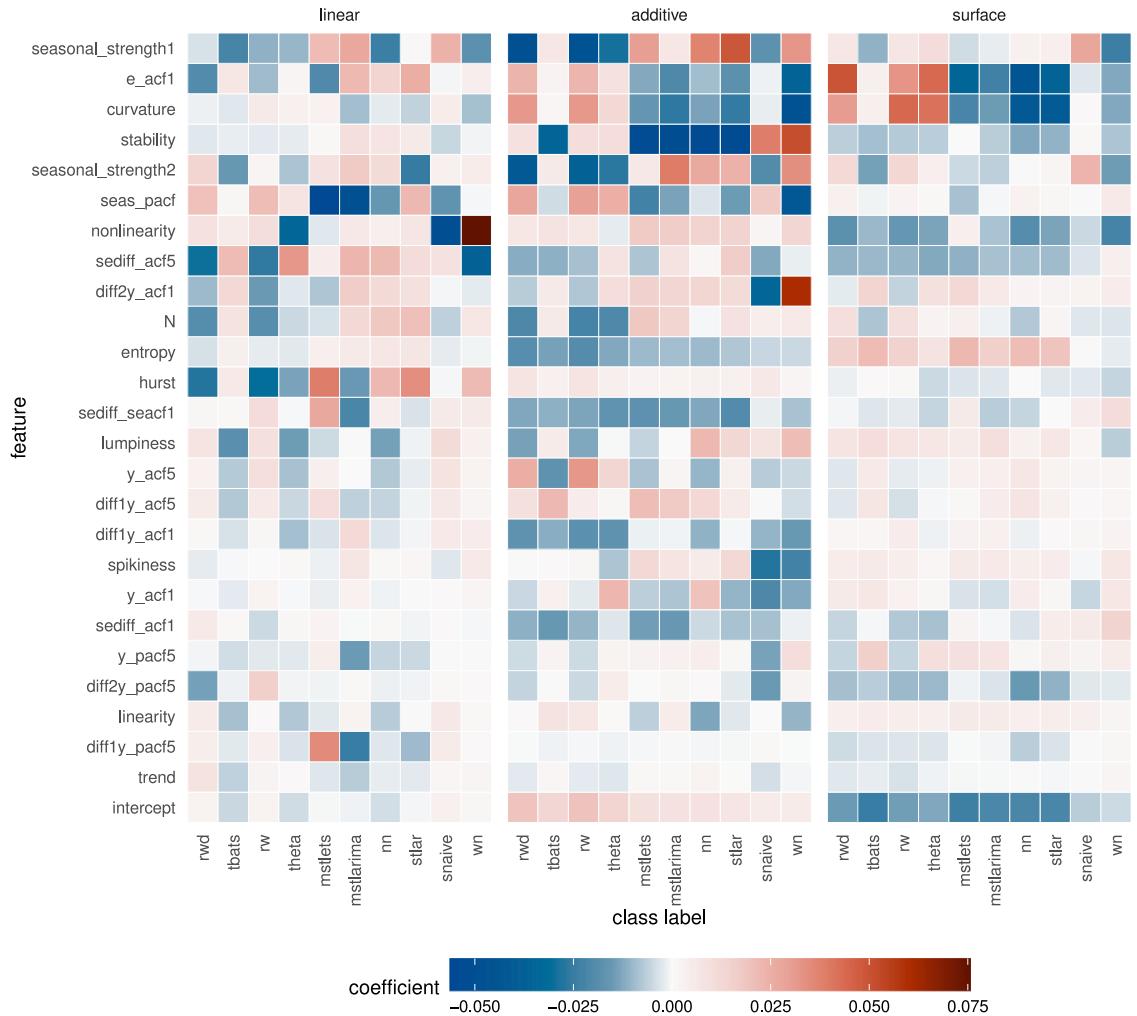


**Fig. 13.** Visualisation of estimated model coefficients for weekly series. The y-axis represents features, the x-axis represents class labels, and the heat map cells represent estimated model coefficients. The class labels are ordered with their descending ranks.

forecasting decision process. We further explored the instance space defined by features to understand how certain features of the time series influence the forecast model selection. A further contribution of the paper is the provision of empirical support for the findings of the M4 competition (Makridakis et al., 2018) that hold that combination forecasts, in general, outperform the best individual forecasts. An interesting future extension of this framework would be to apply this methodology to produce probabilistic forecasts.

In general, the performance of any model strongly depends on the dataset (reference set) used to train the model. We investigated the feasibility of using the GRATIS approach to increase the diversity of the reference set.

This approach is beneficial when researchers have a small sample with which to build a reliable classifier, or when no sample is available due to data privacy issues. However, one could argue there can be parts of the simulation that are not very representative of reality, and that learning models might pay a cost for fitting this unnecessary part of the model. The GRATIS approach simulates a realistic set of time series by taking into account the natural boundaries of the features due to constraints on combinations of features. For example, it is not possible to have a time series with very low spectral entropy and a very low trend. By using GRATIS, we eliminate the possibility of generating time series that are not representative of reality.



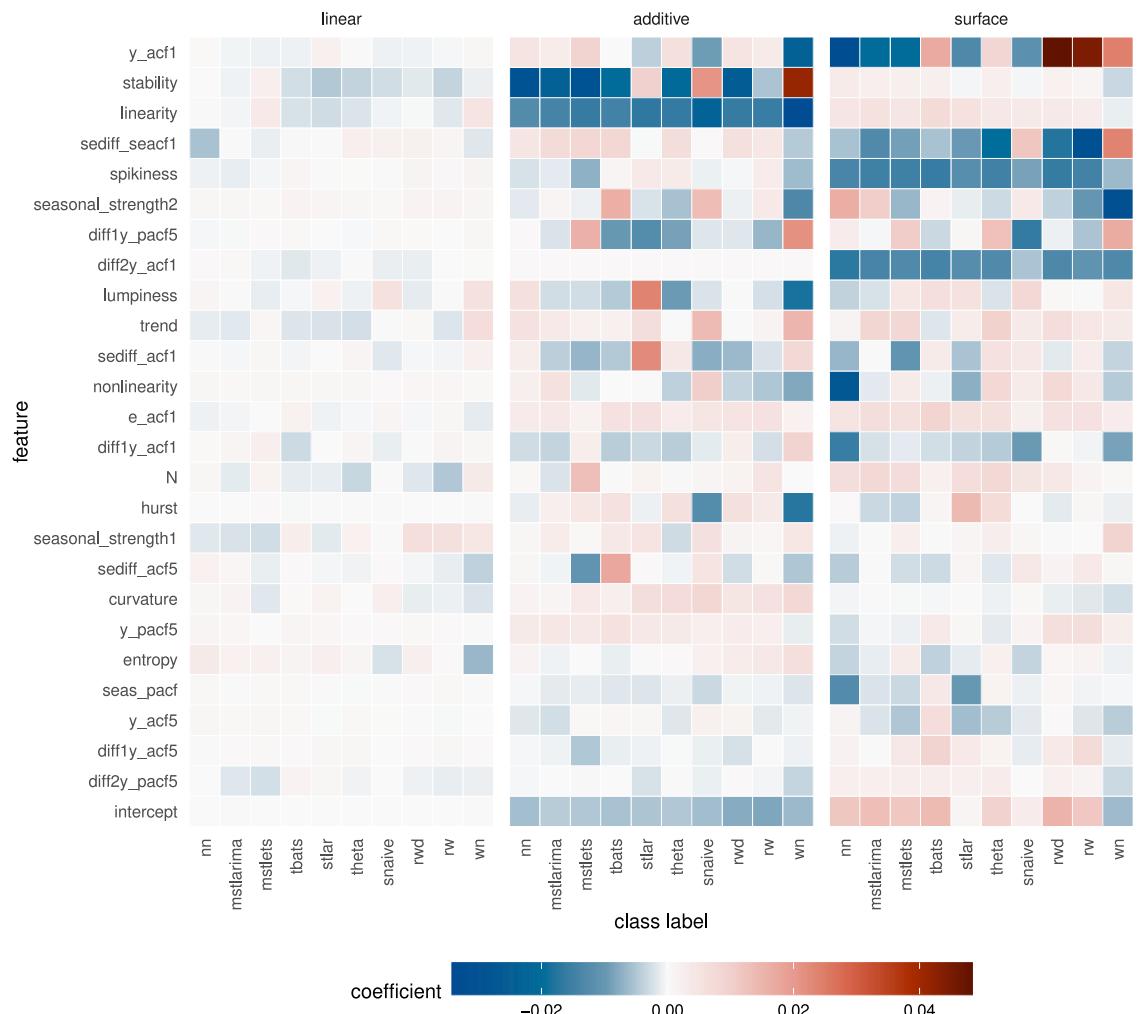
**Fig. 14.** Visualisation of estimated model coefficients for daily series. The y-axis represents features, the x-axis represents class labels, and the heat map cells represent estimated model coefficients. The class labels are ordered with their descending ranks.

The FFORMPP individual model selection results are less accurate in yearly and monthly time series than naively picking a single method that is known to work well *a priori*, such as a random walk with drift for yearly time series or auto.arima/ets/theta for monthly series. The possible reasons for this could be: i) the lack of diversity in the M4 competition's yearly and monthly time series, ii) that the features may not be sufficiently discriminative to reveal the uniqueness of the time series models, and (iii) that the efficient Bayesian multivariate surface regression model is not suitable for identifying the best forecast model. This highlights the need for the following further research.

- The current applications are limited to the M1, M3, and M4 competition datasets. Therefore, the applicability of the feature space and algorithm space introduced in this paper is limited to the collection of time series of features similar to the data in the M-competitions. An important research direction is to expand the frameworks to other datasets that come

from different application domains, such as forecasting stock return data, electricity demand data, irregular time series, etc. When adapting the frameworks to other applications, the feature space should be revised with appropriate features that measure the characteristics of interest. The algorithm space may also be revised with suitable forecast models. Strictly speaking, any proposed approach after the availability of post-sample data, which may allow for testing and hyper-parametrisation, is not directly comparable with the methods submitted in the M4 competition, and this is the case with our method as well.

- A vital component of a meta-learning framework is the construction of an engine that maps an input space composed of features to an output space consisting of forecast model performance. In fact, Bayesian spline regression with a shrinkage estimator or a spike and slab prior (standard setting) can be considered analogous to LASSO with regard to regression problems. A future research direction to investigate could be to replace the training algorithm



**Fig. 15.** Visualisation of estimated model coefficients for hourly series. The y-axis represents features, the x-axis represents class labels, and the heat map cells represent estimated model coefficients. The class labels are ordered with their descending ranks.

with other alternatives such as deep-learning architectures, machine learning algorithms (e.g. random forests and XGBoost), etc., and then test whether these approaches outperform the results given by Bayesian multivariate surface regression. The FFORMPP framework is implemented in an R package `fformpp`, which can be downloaded from <https://github.com/thiyangt/fformpp>.

#### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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#### Appendix. Visualisations of the meta-learner coefficients

This section further visualises the model coefficients for the yearly (Fig. 11), monthly (Fig. 12), weekly (Fig. 13),

daily (Fig. 14), and hourly (Fig. 15) series. The heat maps depict strong linear and additive nonlinear, but mild interactive (surface) nonlinear relationships between features and the MASE vectors for the yearly data. For the monthly and daily series, all three components are vital to the MASE vectors. Nonetheless, the hourly series exhibit strong nonlinear characteristics. The contribution of features for each the linear or nonlinear components is also observable row-wise in the heat maps.

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