Estimation of the Hurst Parameter of Long-Range Dependent Time Series

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Research Report Series

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Report No. 137 February 1996

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Abstract

This paper is a condensed introduction to self-similarity, self-similar processes, and the estimation of the Hurst parameter in the context of time series analysis. It gives an overview of the literature on this subject and provides some assistance in implementing Hurst parameter estimators and carrying out experiments with empirical time series.

1 Introduction

The subject of self-similarity and the estimation of statistical parameters of time series in the presence of long-range dependence are becoming more and more common in several fields of science. Up to now there are only a few text books available, e.g. in [2], which give a comprehensive overview of the techniques and estimators. The intention of this paper is not to close this gap but to provide some basic information about self-similarity, self-similar processes, and estimators of the so-called Hurst parameter H. It gives a rather condensed introduction to self-similarity and contains a number of referenced papers which can be used as a starting point for a more detailed study of this subject. In addition, we provide two different estimators for the Hurst parameter of a time series: the R/S analysis and a periodogram-based method.

The paper is organized as follows. In Section 2 we define the term self-similarity and in Section 3 some of the properties of self-similar processes are presented. Section 4 introduces two important classes of self-similar processes and Section 5 is devoted to the estimation of the Hurst parameter. The paper closes with a Section on the results of estimators applied to a number of time series.

2 Definition of self-similarity

The most common way to define self-similarity of a process $X = (X_t,\ -\infty < t < \infty)$ is by means of its distribution: if (X_{at}) and $a^H(X_t)$ have identical finite-dimensional distributions for all $a > 0$ then X is self-similar with parameter H [11]. In our case, however, we need a definition which is more related to the properties of time series and which is more appropriate for the development of estimators for the self-similarity parameter H [13].

Let $X = (X_t, t = 0, 1, 2, \ldots)$ be a covariance stationary stochastic process with mean μ, variance σ^2, and autocorrelation function $r(k), k \geq 0$. In particular we assume that X has an autocorrelation function of the form

$$r(k) \sim k^{-\beta} L(k), \quad \text{as} \quad k \to \infty, \tag{1}$$

where $0 < \beta < 1$ and L is slowly varying at infinity. For simplicity, we assume that L is asymptotically constant. For each $m = 1, 2, \ldots$, let $X^{(m)} = (X_k^{(m)}, k = 1, 2, 3, \ldots)$ denote the time series obtained by averaging the original series X over non-overlapping blocks of size m, i.e. $X^{(m)}$ is given by $X_k^{(m)} = \frac{1}{m}(X_{(k-1)m} + \ldots + X_{km-1}), k \geq 1$.

Definition 1: A process X is called *(exactly) second-order self-similar* with self-similarity parameter $H = 1 - \beta/2$ if, for all $m = 1, 2, \ldots,$ $VAR[X^{(m)}] = \sigma^2 m^{-\beta}$ and

$$r^{(m)}(k) = r(k) = \frac{1}{2}((k+1)^{2H} - 2k^{2H} + |k-1|^{2H}), \quad k \geq 0, \tag{2}$$

where $r^{(m)}$ denotes the autocorrelation function of $X^{(m)}$.

Definition 2: A process X is called *(asymptotically) second-order self-similar* with self-similarity parameter $H = 1 - \beta/2$ if, for all k large enough,

$$r^{(m)}(k) \to r(k), \quad \text{as} \quad m \to \infty. \tag{3}$$

In other words, X is second-order self-similar if the corresponding aggregated processes $X^{(m)}$ are the same as X or become indistinguishable from X at least with respect to their autocorrelation functions.

3 Properties of self-similar processes

The following properties of self-similar processes are equivalent:

- *Hurst effect*: The rescaled adjusted range statistic (see Section 5.1) is characterized by a power law: $E[R(m)/S(m)] \sim a_1 m^H$ as $m \to \infty$ with $0.5 < H < 1$.

- *Slowly decaying variances*: the variances of the sample mean are decaying more slowly than the reciprocal of the sample size, i.e. $VAR[X^{(m)}] \sim a_2 m^{2H-2}$ as $m \to \infty$, with $0.5 < H < 1$. As a consequence, classical statistical tests and confidence intervals lead to wrong results.

- *Long-range dependence*: the autocorrelations decay hyperbolically rather than exponentially, implying a non-summable autocorrelation function $\sum_k r(k) = \infty$. This implies that even though the $r(k)$'s are individually small for large lags, their cumulative effect is important.

- $1/f$-*noise*: the spectral density $f(\cdot)$ obeys a power law near the origin, i.e. $f(\lambda) \sim a_3 \lambda^{1-2H}$, as $\lambda \to 0$, with $0.5 < H < 1$, where $f(\lambda) = \sum_k r(k) e^{ik\lambda}$.

The a_i are finite, positive constants, which are independent of m or λ, respectively.

In contrast, short-range dependent processes, i.e. $H = 0.5$, show the following properties:

- $VAR[X^{(m)}] \sim a_1 m^{-1}$.
- $0 < \sum_k r(k) < \infty$.
- $f(\lambda)$ at $\lambda = 0$ is positive and finite.

4 Some self-similar processes

For the analysis and simulation of systems where self-similar processes are involved, there is a need of processes which exhibit the properties described above. In this section, we give the definitions of two processes of this type, namely the *fractional Gaussian noise (FGN)* [8] and the class of *fractional autoregressive integrated moving-average (FARIMA) processes* [6].

These processes were introduced to facilitate parsimonious modeling of long-range dependent time series. Traditional models, for instance *autoregressive moving-average (ARMA)* and *autoregressive integrated moving-average (ARIMA) processes*, are only capable to model the short-range portion of the correlations of empirical data sets, and even for a large number of coefficients the long-range portion will remain unmodeled. An introduction of ARMA and ARIMA processes is beyond the scope of this paper. We refer interested readers to text books on time series analysis, e.g. [10].

FGN with parameter $H \in (0,1)$ is a stationary Gaussian process with mean μ, variance σ^2, and autocorrelation function $r(k) = \frac{1}{2}((k+1)^{2H} - 2k^{2H} + |k-1|^{2H}), \quad k > 0$. It is exactly self-similar, as long as $0.5 < H < 1$. Its spectrum is defined by

$$f(\lambda) = \frac{\sigma^2}{\pi} \sin(\pi H)\Gamma(2H+1)(1-\cos\lambda) \sum_{j=-\infty}^{\infty} |\lambda + 2\pi j|^{-2H-1}. \tag{4}$$

FGN(H) can also be defined as the process which has the same correlation function as the process of unit increments $\Delta B_H(t) = B_H(t) - B_H(t-1)$ of fractional Brownian motion with exponent H.

Instead of using the increment process of fractional Brownian motion, we can also start from Brownian motion and its discrete time analogue, the random walk. This will lead to the class of FARIMA processes, since the random walk can also be defined as an FARIMA(0,1,0) process, where for a FARIMA(p, d, q) process the orders p and q are the classical ARMA parameters and $d = H - 0.5$ is the fractional difference parameter.

With $(1-B)X_t = a_t$ being the representation of the FARIMA(0,1,0) process, where B denotes the back-shift operator and a_t a white noise process, we generalize this definition for the FARIMA$(0, d, 0)$ processes as follows:

$$(1-B)^d X_t = a_t, \quad \text{for} \quad -0.5 < d < 0.5. \tag{5}$$

The fractional difference operator $(1-B)^d$ is defined by the binomial series $(1-B)^d = \sum_{k=0}^{\infty} \binom{d}{k}(-B)^k$. The spectrum is

$$f(\lambda) = (2\sin\frac{\lambda}{2})^{-2d} = |1 - e^{i\lambda}|^{-2d}, \quad \text{for} \quad 0 < \lambda \le \pi. \tag{6}$$

Thus, $f(\lambda) \sim \lambda^{-2d} = \lambda^{1-2H}$ as $\lambda \to 0$.

If a good approximation of both the short-range and the long-range correlations is mandatory the FARIMA(p, d, q) processes with non-zero p and/or q may be used. In these cases, one may determine an estimate $\hat{d}$ of d with one of the methods described below and generate either a new sequence or a new spectrum from the original data set. To that end, the long-range correlations are removed under the assumption that $\hat{d}$ is an appropriate estimate of d of the underlying process, and the $p + q$ parameters are computed with standard ARMA techniques [10]. For instance,

$$\hat{f}_{ARMA(p,q)}(\lambda) = \hat{f}_{FARIMA(p,d,q)}(\lambda)(2\sin\frac{\lambda}{2})^{2\hat{d}}. \tag{7}$$

5 Estimation of the Hurst Parameter

The properties of self-similar processes (see Section 3) lead to the following methods to estimate H (see [13] and references therein):

1. time-domain analysis: *R/S statistics*,
2. analysis of the variances of the aggregated processes $X^{(m)}$: *variance-time plots*, and
3. periodogram-based analysis in the frequency-domain: e.g. *Whittle's Maximum Likelihood Estimator (MLE).*

In this paper we will focus on the alternatives 1 and 3.

5.1 R/S analysis

The feature that makes R/S statistics particularly attractive is its relative robustness against changes in the marginal distribution, even for long-tailed or skew distributions. On the other hand, for marginal distributions which are close to normality a dramatic loss in efficiency is reported, and, to our best knowledge, no detailed analysis of robustness of R/S statistics was carried out yet. Given an empirical time series of length N (X_k : $k = 1, \ldots, N$), the whole series is subdivided into K non-overlapping blocks. Now, we compute the rescaled adjusted range $R(t_i, d)/S(t_i, d)$ for a number of values d, where $t_i = \lfloor N/K \rfloor (i-1) + 1$ are the starting points of the blocks which satisfy $(t_i - 1) + d \leq N$.

$$R(t_i, d) = \max\{0, W(t_i, 1), \ldots, W(t_i, d)\} - \min\{0, W(t_i, 1), \ldots, W(t_i, d)\}, \tag{8}$$

where

$$W(t_i, k) = \sum_{j=1}^{k} X_{t_i+j-1} - k \cdot \left(\frac{1}{d} \sum_{j=1}^{d} X_{t_i+j-1} \right), \quad k = 1, \ldots, d. \tag{9}$$

$S^2(t_i, d)$ denotes the sample variance of $X_{t_i}, \ldots, X_{t_i+d-1}$. For each value of d one obtains a number of R/S samples. For small values of d there are K samples. The number decreases for larger values of d because of the limiting condition on the t_i values mentioned above.

One computes these samples for logarithmically spaced values of d, i.e. $d_{l+1} = m \cdot d_l$ with $m > 1$, starting with a d_0 of about 10. Plotting $\log R(t_i, d)/S(t_i, d)$ vs. $\log d$ results in the R/S plot, also known as *pox diagram*.

Next, a least squares line is fitted to the points of the R/S plot, where the R/S samples of the extremal values of d are not considered. The R/S samples of the smallest values of d are dominated by short-range correlations and samples of large values of d are statistically insignificant if the number of samples per d is less than say 5. The slope of the regression line for these R/S samples is an estimate for the Hurst parameter H. Both the number of blocks K and the number of values d should not be chosen too small. In addition, some care has to be taken when deciding about the end of the transient, i.e. which of the small values of d should not be taken into consideration for the regression line. In practice, it should be checked whether different parameter settings lead to consistent H estimates for $X^{(m)}$ with different aggregation levels m.

5.2 Periodogram-based analysis

If more information on the H-estimate, such as confidence intervals, or on the estimator itself, such as efficiency and robustness, are needed periodogram-based estimators are used. The main idea of these methods is to assume a certain, of course self-similar, process type, for instance FARIMA(p, d, q), and to fit the parameters of this process to the given empirical sample. The fitting should be optimal in the sense that the periodogram of the sample and the spectral density of the process are minimizing a given goodness-of-fit function.

As mentioned above, the spectral density of self-similar processes obeys a power law near the origin. Thus, the first idea to determine the Hurst parameter H is simply to plot the periodogram in a log-log grid, and to compute the slope of a regression line which is fitted to a number of low frequencies. This should be an estimate of $1 - 2H$. In most of the cases this will lead to a wrong estimate of H since the periodogram is *not appropriate* to estimate the spectral density [10]. More sophisticated methods have to be applied to obtain useful estimates of H.

Several periodogram-based estimators can be found in the literature. In this paper we will focus on an MLE as presented in [1, 13] which is based on Whittle's approximate MLE for Gaussian processes [12]. For Gaussian sequences this estimator is asymptotically normal and efficient [4, 3].

The spectral density of the self-similar process is denoted by $f(\lambda;\theta)$, where the parameter vector of the process $\theta = (\theta_1, \dots, \theta_M)$ is structured as follows. $\theta_1 = \sigma_\epsilon^2$ is a scale parameter, where σ_ϵ^2 is the variance of the innovation ϵ of the infinite AR-representation of the process, i.e., $X_j = \sum_{i=1}^{\infty} \alpha_i X_{j-i} + \epsilon_j$. This implies $\int_{-\pi}^{\pi} \log\{f(\lambda;(1,\theta_2,\dots,\theta_M))\}d\lambda = 0$. θ_2 denotes the Hurst parameter H. If necessary, the parameters θ_3 to θ_M describe the short-range behavior of the process. For FGN and FARIMA$(0,d,0)$, only σ_ϵ^2 and H have to be considered. With $\eta = (\theta_2, \dots, \theta_M)$, the Whittle estimator $\hat{\eta}$ of η minimizes the quality-of-fit function

$$Q(\eta) = \int_{-\pi}^{\pi} \frac{I(\lambda)}{f(\lambda;(1,\eta))} d\lambda \tag{10}$$

where $I(\cdot)$ denotes the periodogram of the given time series of length N and is defined by

$$I(\lambda) = \frac{1}{2\pi N} |\sum_{j=1}^{N} X_j e^{ij\lambda}|^2. \tag{11}$$

$\widehat{H}$ is given by $\hat{\theta}_2$ and the estimate of σ_ϵ^2 by

$$\hat{\sigma}_\epsilon^2 = \int_{-\pi}^{\pi} \frac{I(\lambda)}{f(\lambda;(1,\hat{\eta}))} d\lambda. \tag{12}$$

The approximate 95%-confidence interval of $\widehat{H}$ is given by

$$\widehat{H} \pm 1.96 \sqrt{\frac{V_{11}}{N}} \tag{13}$$

where $V = 2D^{-1}$ and the matrix D is defined by

$$D_{ij} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\partial}{\partial\theta_i} \log f(\lambda) \frac{\partial}{\partial\theta_j} \log f(\lambda) d\lambda. \tag{14}$$

For implementation details, we suggest to have a look at Chapter 12.1 of [2], where an $S+$ listing of the Whittle estimator is provided. Given some knowledge in numerical analysis, no special library functions are necessary to implement the above formulae. However, FFT, vector, and matrix functions would make the programming more convenient.

In practice, there are two problems which may have an effect on the robustness of the estimator:

- Deviations from the assumed model spectrum: i.e. deviations at higher frequencies lead to a bias in the estimate of H. One possible solution is to estimate H only from periodogram ordinates at low frequencies. For large data sets, one can also aggregate the data over non-overlapping blocks of length m and compute several $\widehat{H}^{(m)}$ for the $X^{(m)}$.

- Deviations from Gaussianity: Gaussianity can often be achieved by transforming the data but then it has to be proven that the estimates of H for the original and the transformed data sets are identical [7]. For instance, this is the case for the log-transformation $Y = \log(X)$.

5.3 Some experiments

In this section, we suggest some experiments which may be carried out in order to obtain a better understanding of the given data set.

As already mentioned above, both the R/S and the periodogram-based analysis should be applied both to the original and the aggregated data sets in order to filter out some effects of the process behavior at higher frequencies. A first check of the validity of the results is to compare the H-estimates of both estimators.

If the estimate does not clearly indicate long-range dependence, i.e. $\widehat{H}$ is considerably larger than 0.5, one should re-shuffle the data set to destroy the given correlation structure and redo the estimation. For short-range dependent data sets the estimates for the original data set and the re-shuffled data set will show only small difference, whereas for long-range dependent data sets the difference will be more obvious. Of course, this is rather an indication of long-range dependence than a proof.

If the estimate is close to 1.0 one should also apply the estimators to the differenced data

H	$\widehat{H}_{R/S}$	$\widehat{H}_{Whittle}$(FGN)	$\widehat{H}_{Whittle}$(FARIMA$(0,d,0)$)
0.5	0.51	0.50 [0.49,0.51]	0.50 [0.48,0.51]
0.6	0.61	0.59 [0.57,0.60]	0.61 [0.59,0.62]
0.7	0.67	0.67 [0.66,0.68]	0.71 [0.69,0.73]
0.8	0.77	0.75 [0.75,0.76]	0.81 [0.79,0.82]
0.9	0.87	0.84 [0.84,0.85]	0.91 [0.89,0.92]

Table 1: *H estimates for the FGN sequences*

H	$\widehat{H}_{R/S}$	$\widehat{H}_{Whittle}$(FGN)	$\widehat{H}_{Whittle}$(FARIMA$(0,d,0)$)
0.5	0.55	0.50 [0.48,0.51]	0.49 [0.48,0.51]
0.6	0.62	0.58 [0.57,0.59]	0.60 [0.59,0.62]
0.7	0.72	0.68 [0.66,0.69]	0.72 [0.70,0.73]
0.8	0.80	0.75 [0.74,0.76]	0.80 [0.79,0.82]
0.9	0.86	0.83 [0.82,0.84]	0.90 [0.88,0.91]

Table 2: *H estimates for the FARIMA(0,d,0) sequences*

sets, $Y = (1-B)^k X, \quad k = 1, 2, \ldots$, B denoting the back-shift operator, to be sure that the large H-value is not caused by instationarity.

6 Examples

In this Section, we apply the H estimators to a variety of data sets. We used the R/S analysis with $K = 8$, 30 values for d starting from $d_0 = 10$, and rejecting the d values with less than 4 R/S values for the computation of the regression line. The slope of the regression line was computed starting with d_0 to d_{14} to rule out short-range influences. The Whittle MLE of H was computed both under the assumption of an underlying FGN and a FARIMA(0,d,0) process. In the Whittle case, we also provide 95%-confidence intervals for the estimates.

First, to check their accuracy, we estimate the Hurst parameters of artificially generated fractional noise sequences. We used the algorithm of Hosking [6] to compute several FGN

m	$\widehat{H}_{R/S}$	$\widehat{H}_{Whittle}$(FGN)	$\widehat{H}_{Whittle}$(FARIMA$(0, d, 0)$)
1	0.88	1.18 [1.18,1.18]	1.30 [1.30,1.30]
100	0.81	1.06 [1.02,1.10]	1.06 [1.02,1.09]
200	0.80	0.92 [0.87,0.96]	0.97 [0.92,1.03]
300	0.83	0.93 [0.87,0.99]	0.98 [0.91,1.04]
400	0.86	0.89 [0.82,0.96]	0.93 [0.86,1.01]
500	0.86	0.85 [0.78,0.93]	0.88 [0.79,0.96]
600	0.88	0.86 [0.78,0.94]	0.88 [0.79,0.97]
700	0.86	0.81 [0.72,0.90]	0.83 [0.74,0.94]
800	0.88	0.86 [0.77,0.96]	0.89 [0.79,1.00]

Table 3: *H estimates of the aggregated I frame sequences*

m	$\widehat{H}_{R/S}$	$\widehat{H}_{Whittle}$(FGN)	$\widehat{H}_{Whittle}$(FARIMA$(0, d, 0)$)
1	0.85	1.00 [1.00,1.00]	1.11 [1.09,1.12]
10	0.84	0.92 [0.91,0.92]	0.99 [0.95,1.03]
20	0.87	0.88 [0.87,0.89]	0.95 [0.90,1.01]
30	0.90	0.90 [0.88,0.91]	0.97 [0.90,1.04]
40	0.93	0.86 [0.84,0.89]	0.93 [0.85,1.01]
50	0.93	0.91 [0.89,0.92]	0.98 [0.89,1.07]
60	0.94	0.88 [0.85,0.90]	0.94 [0.84,1.04]
70	0.97	0.90 [0.87,0.92]	0.98 [0.88,1.09]
80	0.97	0.89 [0.87,0.92]	0.99 [0.87,1.10]

Table 4: *H estimates of the aggregated GOP sequences*

and FARIMA$(0,d,0)$ samples of length 10,000 for different H values. Table 1 shows the H estimates for the FGN sequences and Table 2 for the FARIMA$(0,d,0)$ sequences.

These tables clearly show that the estimators provide a good estimate for the originally used H value. Comparing the three $\widehat{H}$ values, they are the same in tendency but in some cases the two confidence intervals do not even overlap or the $\widehat{H}_{R/S}$ value does not lie in either interval, e.g. cf. last line of Table 1. If this behavior already occurs for time series where the H value is known in advance one has to be very careful in analyzing empirical sequences.

The next two tables show the H for two different frame size traces of the compressed *Star Wars* movie. Each of the sequences contains more than 171,000 single measurements. Both sequences were log-transformed and aggregated with different levels m before the estimators were applied. The log-transform was necessary to obtain Gaussian marginals, and the aggregation was used to filter out high frequency influences. The detailed statistical description of the video sequences is beyond the scope of this paper, and can be found in [5] for the first sequence, which is named *I frame sequence* in the following, and in [9] for the second sequence, which is named *GOP sequence*.

Table 3 shows the H estimates of the *I frame sequence*. For aggregation levels larger than $m = 300$ the three estimates are consistent in the sense that the confidence intervals of the $\widehat{H}_{Whittle}$ overlap and $\widehat{H}_{R/S}$ lies in at least one of the intervals. The results indicate that the Hurst parameter of this time series is located in the interval $[0.8, 0.9]$. For modeling purposes an $\widehat{H}$ value of 0.86 is a reasonable choice.

Table 4 shows the H estimates of the *GOP sequence*. In this case, it is rather difficult to draw a conclusion from the estimates. The $\widehat{H}_{R/S}$ are increasing for increasing m from 0.85 up to 0.97 . The $\widehat{H}_{Whittle}$(FGN) lead to a rather consistent estimate of about 0.89, whereas the $\widehat{H}_{Whittle}$(FARIMA$(0,d,0)$) lead to an estimate of about 0.98 . Since the H estimates are close to 1.0 a test on instationarity should also be considered for a more detailed analysis. In addition, it should also be checked whether the high frequency behavior leads to a biased result. In this case, the experiments should be repeated with a class of processes which facilitate the modeling of the high frequency behavior, e.g. FARIMA(p,d,q) with $p > 0$ or $q > 0$.

Acknowledgments

The author would like to thank Mark Garrett (Bellcore, Morristown, NJ) for providing the *Star Wars* data set, and Frank Hübner (Bellcore, Red Bank, NJ) for fruitful discussions.

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Institut für Informatik
Universität Würzburg

Verantwortlich: Die Vorstände des Institutes für Informatik.

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