**Preserving Time Series Features for Forecasting with Protected Data**

**Abstract**

While data protection can enable access to otherwise restricted time series data, privacy methods alter time series values and can significantly change forecasts. Existing decentralized privacy solutions are complex to implement and only enable model parameters or forecasts to be shared with forecasters while existing centralized privacy methods are poorly suited for preserving forecast accuracy at meaningful levels of privacy.

We propose a matrix-based privacy method called *k*-nTS+ that uses machine learning techniques on time series features to choose values to swap between time series. We apply our method to multiple forecasting models and find that it balances the tradeoff between privacy and forecast accuracy well. Compared to unprotected data, *k*-nTS+ reduces forecast accuracy by only 14% at similar protection levels to other forms of protected data like differential privacy. We also show that the representativeness of k-nTS+ protected time series is improved, which improves trust and makes it more likely for organizations to share protected time series data.

1. **Introduction**

Personally identifiable time series data is now ubiquitous and requires protection (Boone et al., 2019). Recently, the General Data Protection Regulation (GDPR)[[1]](#footnote-1) and other privacy laws require organizations to anonymize their personal data or place strict limitations on data transfers and processing[[2]](#footnote-2). However, Gonçalves et al. (2021) showed that anonymizing time series data with differential privacy produced unusable forecasts and required different solutions such as data owners sharing forecasts only via federated learning. This paper proposes a matrix-based privacy solution for data owners to share an entire time series data set capable of producing usable forecasts.

Various data protection methods are available depending on whether time series are stored in a single data set (centralized) or spread across multiple data owners/data sets (decentralized). In the decentralized scenario, multi-party computation and federated learning enable privacy-preserving collaborative forecasting to ensure accurate forecasts while protecting sensitive data (Gonçalves et al., 2021a; Goncalves, Bessa et al., 2021b; Sommer et al., 2021). We focus on the centralized scenario where a single data owner uses privacy methods to protect a time series data set. These privacy methods directly alter the confidential data to produce protected time series. The goal is to limit the ability of a bad actor to identify data subjects (in our case, time series) and learn sensitive information about them. The concern for forecasters is that these protected time series degrade forecast accuracy to unusable levels.

Consider the example shown in Figure 1. The time series shown in the middle plot is a simulated AR(1) process with autoregressive parameter . The series on the left is the original series, with random noise added to each period that is proportional to the standard deviation of the original series. The series on the right was created by swapping the original series values with values from two other simulated AR(1) processes, both with . Estimating an ARIMA(1, 0, 0) model on the original series yields an estimate of with a standard error of 0.07, while the noised series yields an estimate of with a standard error of 0.10. The swapped series better preserves the estimated autocorrelation of the original series, with an estimate of and a standard error of 0.08. Visually, the swapped version of the original series is more representative than the noised version, but in both cases, the time series features (*e.g.*, AR (1) parameter, variance) are altered. However, without further assumptions about the swapping process and other time series available, it is not immediately clear how the changes in time series features impact forecast accuracy. The proposed data protection framework in this paper attempts to improve extant privacy methods by swapping time series values when the time series features are more likely to maintain forecast accuracy.

**Figure 1:** Comparison of protected AR(1) processes to the original AR(1) process.

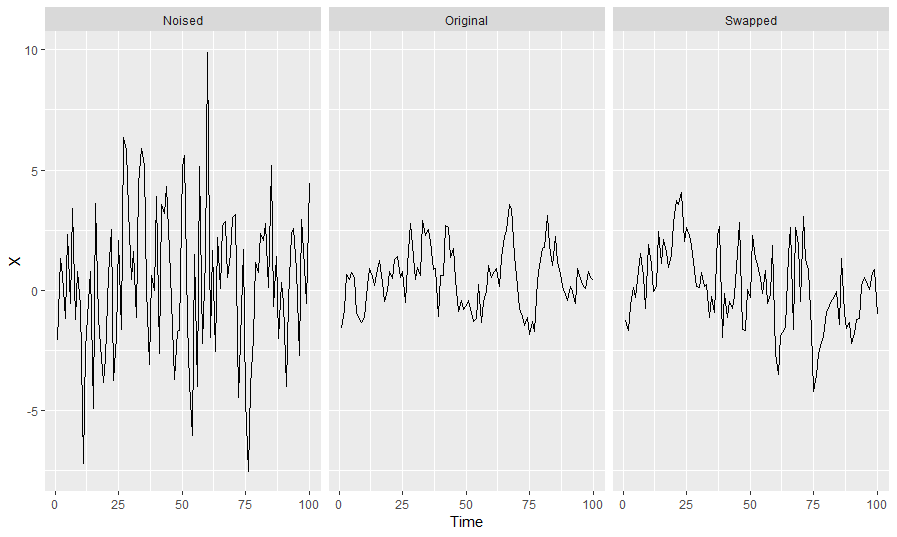


Figure 2 illustrates our k-nTS+ data protection framework. We begin our data protection framework (black arrows) by generating protected data using baseline privacy methods such as additive noise and differential privacy. Then, we generate forecasts for the original and protected series and compare their accuracies. To improve forecast accuracy (blue arrows) for our privacy method, we use a machine learning-based feedback loop (Robnik-Sikonja & Kononenko, 2003; Gregorutti et al., 2017) on the accuracy results to rank the time series features most predictive of forecast accuracy. We then compute a distance-based matrix of these features to choose the time series values to swap in *k*-nTS+. The data protection framework aims to produce protected time series that maintain the useful features for forecasting.

**Figure 2: k-nTS+ data protection framework (Blue arrows indicate the feedback loop which informs the swapping in our proposed *k*-nTS+ protection method).**

Graphical user interface, application

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Our contributions are two-fold. First, we analyze forecasts of protected data from multiple forecasting models and privacy methods. This comparison is needed because different forms of data protection produce different data points, ultimately having different forecasts than what would be produced based on the original data. While recent attention has been paid to privacy preserving collaborative forecasting, our interest is in forecasting using a single protected dataset. We extract time series features that are helpful to forecast accuracy and show how these features change after data protection. We use these changes to explain the performance of various forecasting models in our empirical application. In such a situation, our work answers how time series features and forecast accuracy change when data owners apply data protection methods in a centralized scenario.

Second, we propose a novel privacy method that preserves the usefulness of time series to forecasters. In the privacy literature, the usefulness of protected data has been often overlooked (cite here XXX) and new research (Schneider et. al., 2018, Li et. al. 2022b) demonstrates that producing useful protected data requires knowledge and craftmanship of how the data is used. To the best of our knowledge, extant privacy methods do not improve protection specifically for forecasting. We create a matrix-based k-nTS+ privacy method that uses a feedback loop based on the relationship between time series features and forecast accuracy. k-nTS+ swaps time series values with each other when they have similar features predictive of better forecast accuracy. Results show that our method provides significantly better accuracy at similar levels of privacy (identification disclosure risk) than competitor privacy methods. Furthermore, using the *performance gap* from the REP metric (Petropoulos & Siemsen, 2022), we show that k-nTS+ protected time series produced are more representative of the original series compared to the other methods, leading to improved trust in data sharing.

In Section 2, we review the relevant literature. Section 3 describes the swapping mechanism of the baseline *k*-nTS method, and Section 4 defines the time series features used for swapping in k-nTS. Section 5 details the framework for the *k*-nTS+ method from Figure 2, and Section 6 presents the results. We discuss our conclusions in Section 7, including recommendations for future research.

1. **Literature Review**
   1. *Privacy Methods*

Goncalves, Pinson, et al. (2021c) modeled a data market where data owners are compensated for sharing their time series data and purchase forecasts based on the data from other parties. This market gives data owners a monetary incentive to share their data. However, data owners may still be discouraged from sharing time series due to privacy concerns with a central party. Other privacy-preserving solutions for collaborative forecasting include secure multi-party computation, decomposition-based methods, and data transformation techniques (see Gonçalves et al. 2021a). While it has been demonstrated that differential privacy degrades forecast accuracy for VAR models and recurrent neural networks (RNNs) (Gonçalves et al., 2021a; Imtiaz et al., 2020), little work compares how different forecasting models perform on protected data.

The centralized scenario uses privacy methods that generate protected data sets for sharing. Gonçalves et al. (2021a) show that using differential privacy reduces the forecast accuracy of VAR models under very high values of the privacy parameter (weak privacy protection). Others have also studied the application of differential privacy to time series (Imtiaz et al., 2020; Liyue Fan & Li Xiong, 2014). Imtiaz et al. (2020) found that differentially private data did not always produce worse forecast accuracy when forecasting individuals' health data using a recurrent neural network. One reason could be because adding random noise to time series is a regularization technique that prevents overfitting when forecasting with neural networks (Hewamalage et al., 2021, 2022). However, Luo et al. (2018) simulated data integrity attacks and found that multiplicative noise reduces forecast accuracy by over 21% when only half the data points are altered. We note that in a privacy-preserving environment, it is common for all of the data points to be altered and forecast accuracy would be further reduced.

Another type of privacy method is generalization, where data records are aggregated or combined to make every record (or time series) identical to at least other records on a pre-determined set of attributes (or time periods). Nin & Torra (2009) evaluate the change in forecast accuracy for simple exponential smoothing, double exponential smoothing, linear regression, multiple linear regression, and polynomial regression applied to *k*-anonymized data. The authors find an overall reduction in forecast accuracy even for but do not provide the accuracy of each model individually. Top- and bottom-coding are used as another privacy method to replace the tails of distributions with a threshold value such as high-income levels. Top- and bottom-coding are likely to have an effect similar to adjusting for outliers, which improves forecast accuracy when the outliers are close to the forecast origin (Chen & Liu, 1993).

* 1. *Adjusted Forecasts*

As shown in Figure 2, privacy methods adjust forecasts by altering the underlying time series data input to a forecasting model. Similar to judgmental adjustments, this presents the forecaster with multiple possible forecasts to select from. To inform this selection, we reference the long history of judgmental forecasting (Petropoulos et al., 2022, sec. 2.11.2 and 3.7.3). However, there are two critical differences between privacy adjustments and judgmental adjustments.

First, judgmental adjustments alter forecasts after they are output from a forecasting model, and the underlying time series features are not changed. Forecasters typically discuss what types of judgmental adjustments improve forecast accuracy. Davydenko & Fildes (2013) found that both positive and negative adjustments can improve accuracy, but positive adjustments tend to give only a marginal improvement. Khosrowabadi et al. (2022) similarly found that beneficial positive adjustments tended to be small, and beneficial negative adjustments tended to be large. Fildes et al. (2019) showed that negative adjustments reduce forecast bias, whereas positive adjustments maintain bias or exacerbate it. The magnitude of judgmental adjustments is positively associated with the size of accuracy improvements, which can occur when adjusters make large adjustments based on reliable information. The accuracy improvements are more significant for time series with low volatility that are easier to forecast (Fildes et al., 2009). Compared to judgmental adjustments and the machine learning literature on how judgmental adjustments affect forecast accuracy (cite XXX), the privacy adjustment is applied directly to the data rather than the forecasts, which also alters the underlying time series features. As a result, forecasters must also consider how the forecasting model interacts with time series features to preserve forecast accuracy.

Second, the motivations for judgmental adjustments and privacy adjustments are different. For judgmental adjustments, motivations include gaining control of the forecasting process, incorporating practitioner expectations, and compensating for judgmental biases (Petropoulos et al., 2022, sec. 3.7.3). Often, the goal is to incorporate the intuition and experience of the adjuster, knowledge of special events, or insider or confidential information to add information with high diagnosticity to improve forecast accuracy (Fildes et al., 2019). Despite varying motivations, judgmental adjustments have been shown to improve forecast accuracy by 5-10% on average (Davydenko & Fildes, 2013; Khosrowabadi et al., 2022). For privacy adjustments the goal is to improve privacy while only slightly degrading forecast accuracy.

* 1. *Time Series Features and Forecasting*

Thousands of features have been used for time series classification (Fulcher & Jones, 2014) and some of those are useful for bettering forecast accuracy. Bandara et al. (2018) clustered similar time series based on eighteen interpretable features, including the mean, variance, and strength of seasonality, to improve the accuracy of RNNs between 2-11%. Also, the initial results from the M4 competition suggested that the randomness and linearity of time series were the most important determinants of forecast accuracy and that seasonal time series (typically less noisy) are easier to forecast (Makridakis et al., 2018). In a follow-up study, Spiliotis et al. (2020) used multiple linear regression to confirm the importance of randomness, linearity, and seasonal strength in predicting the MASE values of the ETS, ARIMA, Theta, and Naïve 2 (random walk applied to seasonally adjusted data) models from the M4 competition. They found that increasing the frequency, kurtosis, linearity, and seasonal strength of time series improved forecast accuracy, but increasing skewness, self-similarity, and randomness degraded forecast accuracy.

Time series features are also used for model selection and forecast combination. Qi et al. (2022) found that forecasts using the strength of trend and seasonality for exponential smoothing model selection had lower MASE, sMAPE, and MSIS than information-based selection methods for the majority of forecast horizons. Talagala et al. (2022) applied a meta-learning algorithm based on Bayesian multivariate surface regression to 37 features, including spectral entropy and the Hurst exponent, to predict the model combination that would yield the minimum forecast error for the M4 competition data. This approach achieved forecast accuracy on par with the top M4 competition methods with less computational cost. Li et al. (2022a) used features such as the first ACF value to propose an interpretable Bayesian forecast combination framework with time-varying weights. In experiments using the M3 competition data, this method reduced the average MASE by approximately 1.1% relative to the next-best forecast combination method. Petropoulos & Siemsen (2022) created a representativeness metric that selects models with trend and seasonality components when the respective signals of these components are strong. For most data frequencies, their approach produces lower average MASE on the M, M3, and M4 competition data and selects the best forecasting model approximately 3% more often than the other selection methods.

1. **The k-nearest Time Series (nTS) Swapping Method**

Let be a set of time series data (*n*-vectors). Using standard privacy methods such as differential privacy, a data provider releases protected data point for each time series based on the confidential values at time and before. These methods choose protected values based on predefined rules, not changes in forecast accuracy. The goal of the data provider should be to change to with minimal reductions in forecast accuracy while increasing privacy to an acceptable level.

We solve the data protection problem for the data provider using a matrix-based k-nTS (k-nearest time series) swapping method, where the data provider releases a set of protected time series where is based on , the confidential values of all series through time . To create a protected series , the *k*-nTS method finds the k most similar time series to where similarity is based on the time series features. For each period *t*, it randomly chooses one of the k similar series to and replaces with the confidential value from period *t* from the randomly chosen series.

Depending on the quantity of available data, *k*-nTS can use rolling windows of data that adjust for dynamic changes in time series features. For example, if we choose a rolling window of size *n*, then where . Protection in subsequent periods from to rolls forward from to , respectively. We label the time series features for the current window as which we refer to as the feature vector for time series *j* in time period  
*t* based on the *n* values in . Li et al. (2022a) also compute time series features over rolling, fixed-length windows. For simplicity, we omit the *t* subscript for the feature vectors and write .

For each time series , the data provider computes the feature vector . This vector can contain any single-valued feature calculated from the values in , such as the strength of the trend and seasonality, the spectral entropy, or the mean value of the current window. Let be the set of *m*-vectors containing the features from each of the *J* time series windows. For each , the data provider computes a set of squared distances of the elements of . We define as the distance between and , i.e., the feature vectors corresponding to two distinct time series from . Without loss of generality, we use the Euclidean norm, or ℓ2-norm, as a distance metric[[3]](#footnote-3). Since our case is multivariate and partially ordered, we can get a totally ordered set based on the Euclidean distance.

We define as the *k*th nearest neighbor of , with the corresponding feature vector . Then, for a time series , we have such that for any integers where . Note that and the superscript means the *i*th order statistic of the related Euclidean distances of all from . Thus, for a given time series vector , its *k*-nearest time series can be represented as the set based on or an ordered set .

For more efficient computation, we introduce a symmetric distance matrix containing the squared distances between time series feature vectors. The squared distance between and is given by and is the (*i, j*)th entry of (also note that ). Suppose we have a confidential data matrix , where (i.e., We can write as the following:

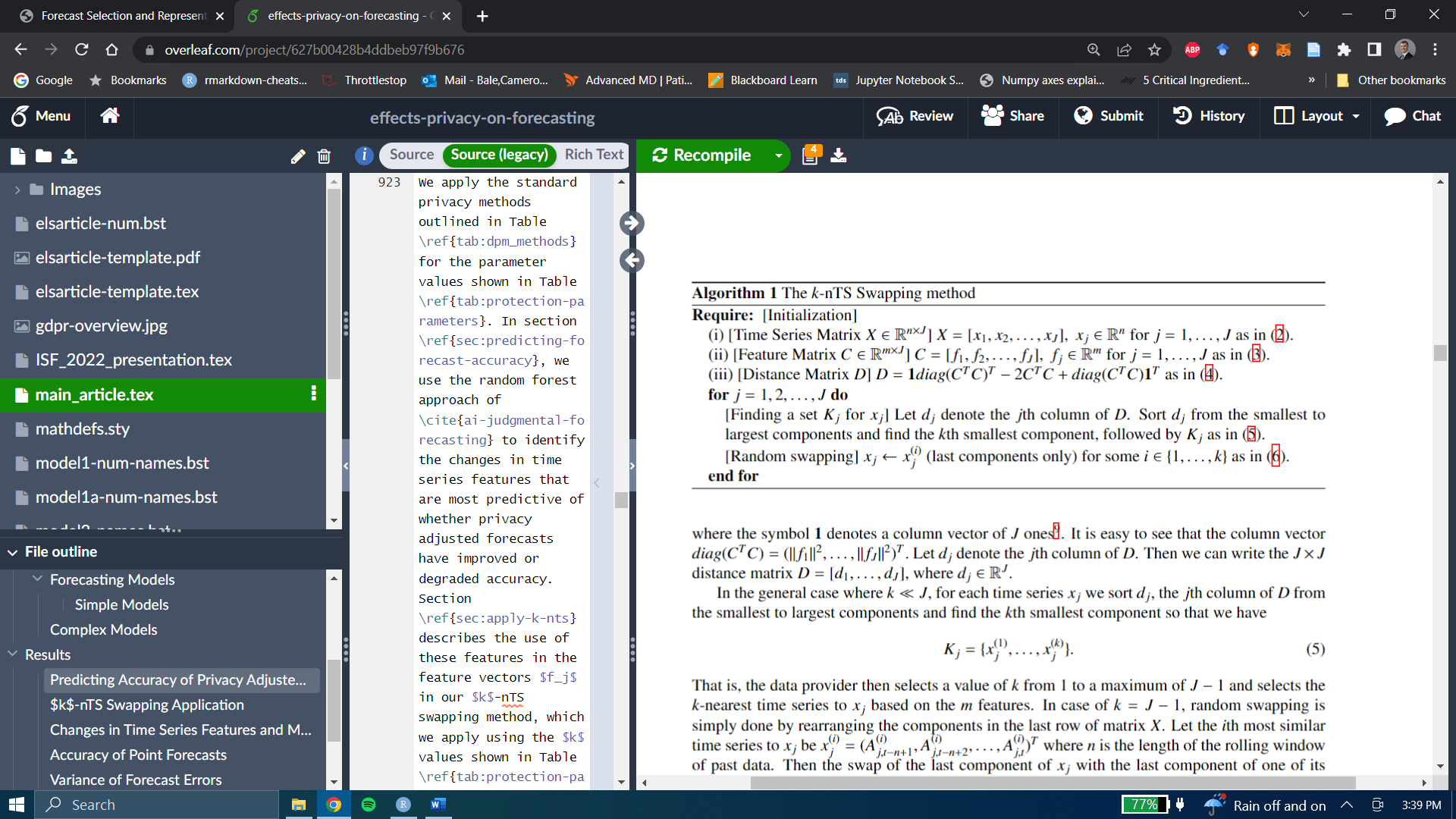
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where , , and We calculate the desired features based on each and construct a feature matrix (where ) as follows:

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We calculate the matrix using the fact that , which can be written as the following:

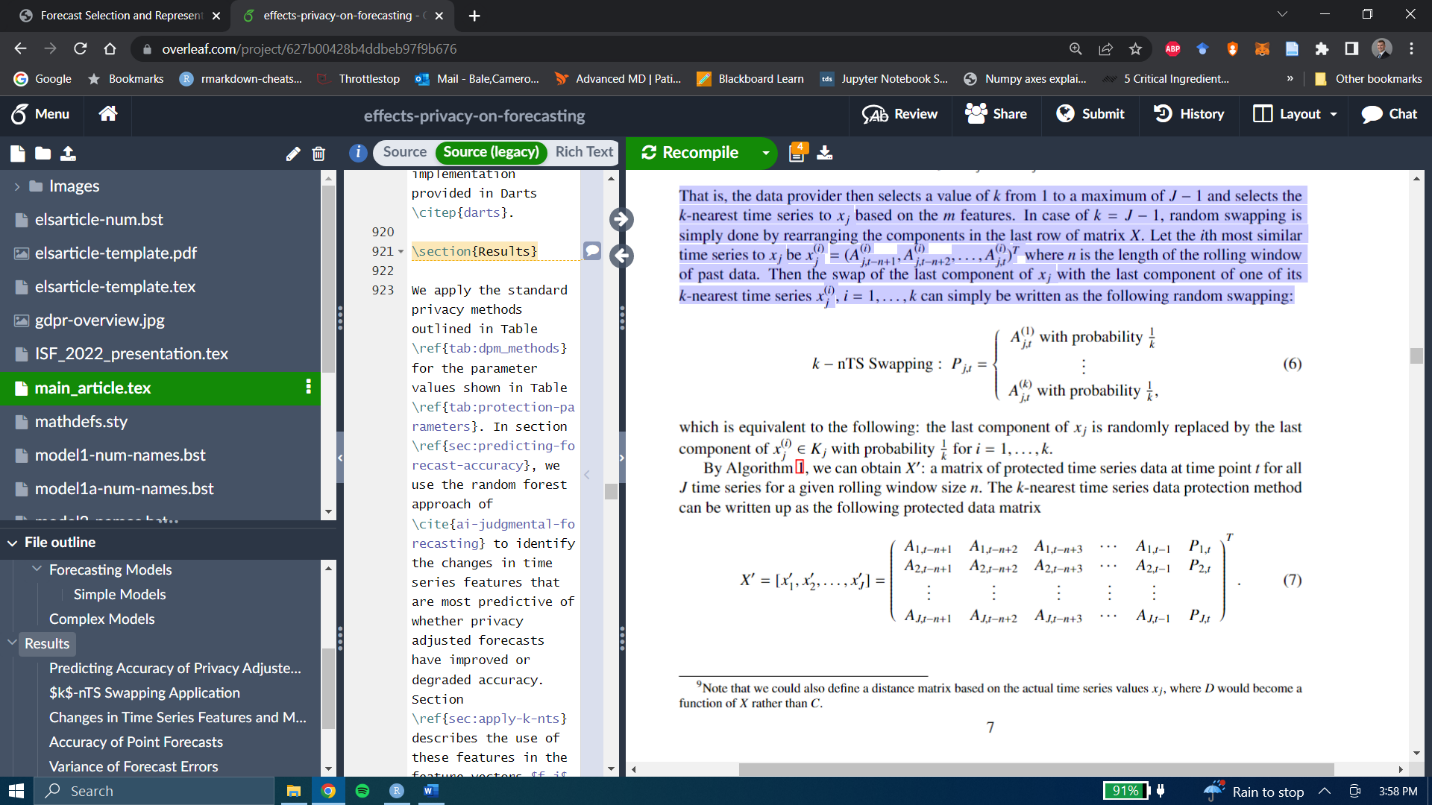
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where **1** denotes a column vector of ones[[4]](#footnote-4). It is easy to see that the column vector Let denote the *j*th column of . Then we can write the distance matrix where

In the general case where , for each time series we sort and take the *k* smallest components so that we have

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That is, the data provider selects a value of *k* from 1 to a maximum of and selects the *k*-nearest time series to based on the *m* features. When , swapping is performed by rearranging the components in the last row of the matrix . Let the *i*th most similar time series to be where *n* is the length of the rolling window of past data. Swapping the last component of with the last component of one of its *k*-nearest time series , is written as:



which is equivalent to the last component of being replaced by the last component of with probability for

By Algorithm 1, we can obtain : a matrix of protected time series data through time *t* for all *J* time series for a rolling window size *n*. The *k*-nTS privacy method can be written as the following protected data matrix,

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As an example, consider a time series where its last component is replaced by the  
last component of which was randomly selected among the 10-nearest time series (based on features) to . Using our notation, we can write

We can represent each time series as a vector, and put them in a graph , which consists of a set *V* of vertices (or nodes) and a set *E* of undirected edges. In our case, we can use weighted edges to represent the Euclidean distance between time series feature vectors: . If we put all the nodes on the graph and assign weight on every edge (every pair of nodes), e.g., for all then we will have a complete graph. The *k*-nearest time series swapping method is a random edge selection problem of the graph. Figure 3 depicts the case of for the *k*-nearest time series of , where the last component of is swapped with that of .

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**Fig 3**: Random edge selection for k-nTS Swapping.

* 1. **The *k*-nearest Time Series + (nTS+) Swapping Method**

The k-nTS+ privacy method adds a feature selection process to k-nTS which selects features that are good predictor of forecast errors. Preserving the values of these features in the protected data ensures that forecast accuracy is maintained while the privacy of the data increases.

For the first step, the data controller generates forecasts for period for the original data and the data protected using baseline privacy methods such as differential privacy and additive noise. The data controller measures the forecast errors and time series features for each original and protected time series. Next, similar to Li et al. (2022a), the RreliefF algorithm (Robnik-Sikonja & Kononenko, 2003) is used as an initial screening process to weight each feature on whether it discriminates between time series with similar features and different forecast errors.

One issue with the above approach is that Relief-based algorithms do not remove redundant features. Urbanowicz et. al. (2018) explains that there is no straightforward method for choosing the number of features to keep if the algorithm is applied. Including all of the features with large RReliefF weights in *k*-nTS would significantly increase the dimensionality and reduce the efficiency of the swapping process. To address this problem, we apply a random forest-based recursive feature elimination (RFE) algorithm to the features selected by RReliefF. Prior work has shown that random forest-based RFE is efficient when applied to sets of highly correlated features (Gregorutti et al. 2017).

Our interest is in obtaining a small set of features that predict forecast accuracy well. When we know the true ranking of time series features for predicting forecast errors, we can use a random forest to predict the forecast errors using the most highly ranked features that produce the minimum out-of-bag (OOB) mean-squared error (MSE). Let denote that minimum MSE. Our desired number of features, , is defined as follows,

,

where is the maximum percentage difference between the minimum MSE and the MSE from predicting the forecast errors using only the most important features. The number of chosen features is the smallest number that offers similar predictive accuracy to the number of features with the best accuracy.

However, the true ranking of the features is unknown and we must estimate it based on the average elimination order of the features across repetitions of the RFE algorithm. In each iteration of the RFE algorithm, a random forest is used to predict forecast errors using the current subset of time series features. The OOB MSE and permutation-based feature importance values are saved, the least important feature is removed, and the model is retrained for the next iteration. These steps repeat until one feature remains. The value is calculated using the averages of the OOB MSE across the repetitions for each . We include the features with the highest average ranks in the *k*-nTS+ swapping method.

**Algorithm 2: RFE for *k*-nTS+ Using Random Forest**

**for** i:

* Train random forest
* Calculate MSE of OOB predictions
* Calculate permutation-based feature importance

**for** subset size :

* + Keep the most important features
  + Retrain random forest using only the most important features
  + Calculate MSE of OOB predictions
  + Calculate permutation-based feature importance

**end**

**end**

Calculate the average MSE for each subset size across all iterations.

Calculate the rank of each feature as the average of the elimination orders from each iteration.

Calculate the desired number of features as the minimum number of features with an average prediction error within of the minimum average prediction error.

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The *k*-nTS+ algorithm can be used collaboratively between the data controller and the forecaster. If, for example, the forecaster specifies their preferred forecasting model, the data controller can apply the model to the original and protected data up through time period *T – 1*, assess which features are most predictive of accuracy for the specified model, and release data to the forecaster using *k*-nTS+ based on these features up through time period .

1. **Empirical Application**
   1. *Data*

The organizers of the early M competitions did not disclose the identity of the business and economic time series used in their competitions (Makridakis & Hibon, 2000). For our application, this provides a natural connection to privacy because we can compute the identity disclosure risk of each protected time series. Identify disclosure risk involves computing the probability of matching a protected time series to its true time series in the confidential data set (see subsection XXXX for details). Good privacy implies the identity disclosure risk is low or similar to random guessing.

Furthermore, recent work by Spiliotis et al. (2020) showed that the M3 competition data are representative of the real world based on time series features which makes it suitable for our k-nTS+ swapping method. We use the monthly micro dataset from the M3 competition, which includes 474 strictly positive time series with values ranging from 120 to 18,100. Of the 474 series, 18 consist of 67 time periods, 259 consist of 68 time periods, and 197 consist of 125 time periods.

* 1. *Time Series Features for the k-nTS+ Swapping Method*

In section 2.3, we reviewed the time series features that had a relationship with forecast accuracy. Below, we define these features in detail for our k-nTS swapping method and include additional features found to affect forecast accuracy in the M3 Competition (Spiliotis et al., 2020). These include skewness, kurtosis, linearity, and strength of trend and seasonality. We omit stability and non-linearity since these features had little to no effect on accuracy, and frequency, since none of the privacy methods we consider affect frequency. We note that higher frequencies are associated with improved forecast accuracy, but our application only uses monthly data.

* + 1. *Spectral Entropy*

Suppose is a univariate stationary time series with a finite mean and constant variance. The spectral density of is estimated as the scaled Fourier transform of the autocovariance function of . The spectral density can be thought of as the probability density function of a random variable on the unit circle (Goerg, 2013), where for a non-zero integer , when , the spectral density will have a peak at the corresponding frequency . The forecastability, or spectral entropy, of is measured using the Shannon entropy of , given by

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where the maximum entropy occurs when . In practice, estimates of , where high values represent a low signal-to-noise ratio, indicating that is difficult to forecast (Kang et al., 2017).

* + 1. *Hurst*

Next, we consider the self-similarity feature quantified using the Hurst parameter (Wang et al., 2006), which measures the long-range dependence of a time series and has a large effect on forecast accuracy (Spiliotis et al. 2020). We use the definition of self-similarity of a time series described by (Willinger et al., 1998). Suppose that is the increment process of , *i.e.*, . An aggregate sequence, denoted , is created by averaging over non-overlapping blocks of size , where

and indexes the block. If is a self-similar time series, then

for all integers . We use the definition of second-order self-similarity, where s exactly second-order self-similar if has the same variance and autocorrelation as for all values of , or is asymptotically second-order self-similar if this holds as (Rose, 1996). The parameter is the Hurst exponent, which is estimated using the differencing term from a fractional ARIMA model, i.e., FARIMA(0, , 0) (Wang et al., 2006; Hyndman et al., 2022), where

5.

Estimates of range from 0 to 1, where corresponds to a random walk (Sobolev, 2017), corresponds to anti-persistent or mean-reverting series, and corresponds to persistent time series that are more likely to maintain their current trend.

* + 1. *Skewness*

Skewness, which we denote , measures the lack of symmetry in the distribution of the values of (Wang et al., 2006), where positive (negative) values are associated with a right- (left-) skewed data distribution:

* + 1. *Kurtosis*

We measure Kurtosis relative to the standard normal distribution (Wang et al., 2006). Positive kurtosis corresponds to distributions that tend to have a distinct peak near the mean with heavy tails, whereas negative kurtosis corresponds to distributions that are relatively flat near the mean,

where 3 is the kurtosis of the standard normal distribution.

* + 1. *Extended Autocorrelation Function (E\_acf)*

Next, we perform STL decomposition (Cleveland et al. 1990) to obtain the trend, seasonal, and remainder components of . We use the approach of Hyndman et al. (2019) to obtain

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where , , and are the trend, seasonal, and remainder components, respectively.

We extract the first-order autocorrelation coefficient of the detrended and deseasonalized series, referred to as 'linearity' by (Spiliotis et al. 2018):

*E\_acf* .

E\_acf measures time series forecastability after the trend and seasonality have been accounted for via decomposition.

* + 1. *Trend and Seasonality*

We also compute the strength of trend and strength of the seasonal component as follows,

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and

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In practice, the values of and range from 0 to 1 (Hyndman 2022).

* + 1. *Mean and Variance*

The final two features are the mean and variance, also used by Bandara et al. (2018) to cluster similar time series for forecasting. The idea is to swap values between series with similar characteristics such as first-order autocorrelation parameters, whose values have similar magnitudes. We write the mean and the variance of as follows,

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* + 1. *Other Features*

**Table 4: Names and descriptions of features selected for *k*-nTS+.**

|  |  |
| --- | --- |
| **Feature Name** | **Description** |
| trend | Strength of trend. |
| spike | The variance of the leave-one-out variances of the remainder component of the decomposed series. |
| max\_var\_shift | The largest variance shift between two consecutive sliding windows. |
| series\_variance | The variance of the series. |
| max\_level\_shift | The largest mean shift between two consecutive sliding windows. |
| series\_mean | The mean of the series. |

* + 1. *Illustration of Time Series Features*

Figure 3 illustrates two monthly time series with desirable and undesirable features . The corresponding values of the time series features are compared in Table 1. The low spectral entropy and high Hurst coefficient values of the desirable time series indicate good forecastability. The undesirable series is essentially a random walk as indicated by the 0.50 value of the Hurst coefficient (Rose XXXX). Furthermore, the undesirable series has a spectral entropy of one indicating a low signal-to-noise ratio. Comparatively, the variance of the desirable series is due to a forecastable trend, whereas the variance of the undesirable series is due to randomness. The undesirable serieshas low *Kurtosis* with a light tailed distribution compared to the undesirable series.

**Fig 3: Comparison of time series with desirable and undesirable features.**

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**Table 1: Values of desirable and undesirable time series features**

|  |  |  |
| --- | --- | --- |
| **Feature** | **Desirable Features (left Fig. 1)** | **Undesirable Features (right Fig. 1)** |
| *SpecEntropy* | 0.07 | 1.00 |
| *Hurst* | 1.00 | 0.50 |
| *Skewness* | -0.42 | -0.57 |
| *Kurtosis* | -1.24 | 1.16 |
| *E\_acf* | -0.09 | -0.19 |
| *Trend* | 0.97 | 0.12 |
| *Seasonality* | 0.16 | 0.23 |
| *SeriesMean* | 7.96 | 7.01 |
| *SeriesVariance* | 0.29 | 0.65 |

* 1. *Tuning of Privacy Methods*

Table 2 shows the tuning parameter values for the baseline privacy methods (additive noise, diferential privacy) and our proposed privacy methods.

**Table 2: Baseline and Proposed Privacy Methods**

|  |  |  |
| --- | --- | --- |
| ***Privacy Method*** | ***Parameter*** | ***Values*** |
| Additive Noise |  | 0.25, 0.50, 1.0, 1.5, 2.0 |
| Differential Privacy |  | 20.0, 10.0, 4.6, 1.0, 0.1 |
| *k*-nTS |  | 3, 5, 7, 10, 15 |
| *k*-nTS+ |  | 3, 5, 7, 10, 15 |

* + 1. *Differential Privacy*

A mechanism satisfies -differential privacy by guaranteeing that, for every output of and every pair of serie and ,

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A differentially private time series can be created using a randomized mechanism that adds Laplace random noise with scale parameter to a confidential time series x. The sensitivity is determined as the maximum absolute difference between two time series and which differ in at most one observation, where . We follow XXXX (XXXX 2021) for our implementation.

* + 1. *Additive Noise*

Additive noise adds a normally distributed random number with mean zero and standard deviation to each value in a confidential time series . Protected values can be written , where and . The protection parameter denotes the number of standard deviations of that define the standard deviation of the sampling distribution of .

* + 1. *k-nTS and k-nTS+*

We use the nine features described above for k-nTS. To perform feature selection for *k*-nTS+, we create protected versions of the original data using additive noise and differential privacy for all parameter values shown in Table 2 (*i.e.*, ). Then, we follow the process for our k-nTS+ privacy method in Figure 2. We generate forecasts for each of the protected data sets for time period and compute the absolute error of each forecast for each series. In order to detect the variation in forecast accuracy due to changes in time series features and not the forecasting model, we apply the *k*-nTS+ selection method for each forecasting model separately. We then select the most important features across all models.

For our empirical application, we select so the features are within 5% of the minimum average prediction error. We selected the six features with the highest average rank across the RFE iterations for all forecasting models.

* + 1. *Idenification Disclosure Risk*

As previously mentioned, the participants of the M3 competition were not intended to identify the original time series. For our privacy metric,, we assess the ability of each privacy method to protect against *identification disclosure*, which occurs when a third party correctly predicts the identity of a protected time series. Each protected dataset consists of the protected series along with a pseudo identifier, i.e., . The pseudo identifier in our application is the `Series` column from the original M3 data, which contains a PID for each time series, e.g., `N1402`. Identification disclosure would occur if a participant (or any other third party) correctly predicted the identity of one or more of the time series in the M3 data set based on the protected time series and some original time series values which the third party possesses. For example, identification disclosure would occur if a third party correctly stated, "Series N1402 comes from [retailer specific name]." For simplicity, we assume the third party does not know which privacy method was applied to the data, but knows that the time series of interest is contained in the protected data set.[[5]](#footnote-5)

The metric we use to measure the risk of identification disclosure, , gives the average proportion of the time series which are correctly identified across simulated privacy attacks:

where is the third party's prediction of the identity of the th protected time series, and identification disclosure occurs when the predicted identity is equal to the true identity . We refer the reader to the Appendix for added mathematical details.

* 1. *Results*

For all privacy methods, forecasts are produced using models readily available to implement in R or Python. Table 3 lists forecasting models into univariate models trained to forecast one series at a time and multivariate models trained to generate forecasts for multiple series. We perform minimal data pre-processing and use the standard settings in the off-the-shelf packages.

**Table 3: Univariate and Multivariate Forecast Models**

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Model Name** | **Variant** | **Global (Yes/No)** |
| Univariate Models | SES | - | N |
|  | DES | Additive trend | N |
|  | TES | Additive trend/seasonality | N |
|  | Auto-ARIMA | Seasonal | N |
| Multivariate Models | VAR | - | N |
|  | LGBM | - | Y |
|  | RNN | LSTM | Y |

**Table 5** displays the average MAE of one-step ahead point forecasts across all models and series, the identification disclosure metric , and the average performance gap across all series. The percentages in parentheses are the increase in average MAE relative to the average MAE from the original data. The average across models for additive noise and differential privacy excludes the VAR model error for AN (s = 1) and DP (), as the errors in these cases were over 1000% larger than the error of any other model. These errors are due to noisy values at or near the forecast origin of some time series, causing the VAR forecasts to explode. This problem did not occur for the other forecasting models, which did a better job smoothing out the random noise.

For identification disclosure risk, we used random samples of external data with ten values from each time series. The results show an inverse relationship between forecast accuracy and the strength of privacy protection. While strong differential privacy provides the lowest risk of identification (1.85%), it nearly quadruples (+383%) the average forecast error relative to the original data resulting in unusable forecasts. Under weak differential privacy with , over 49% of series are identified correctly on average, which is poor identification disclosure risk.

Protection against identification disclosure is better under additive noise with where 22.5% of series are correctly identified on average. However, this comes at a cost to forecast accuracy, which degrades by nearly 45%.

Standard *k*-nTS with *k* = 3 offers a good identification disclosure risk of 2.1%, but forecast accuracy degrades to 39.6%. Our proposed method of *k-*nTS+ swapping with offers similar levels of protection against reidentification (3.3%) with a reduction in forecast accuracy of only 13.9%. Part of this improvement in forecast accuracy at a minimal tradeoff to identification disclosure risk is due to the incorporation of the accuracy feedback loop for time series features. Thus, we recommend data owners to use our k-nTS+ swapping method (k=3) with the relevant time series features to balance the tradeoff between privacy and and forecast accuracy.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Privacy Method** | ***k*-nTS+** | **15** | 2.73% | 839.78  (+22.47%) | 90 |
| **7** | 3.50% | 822.26  (+19.91%) | 82 |
| **3** | 3.26% | 781.02  (+13.90%) | 77 |
| ***k*-nTS** | **15** | 1.57% | 1066.16  (+55.48%) | 127 |
| **7** | 2.12% | 987.04  (+43.94%) | 120 |
| **3** | 2.05% | 956.89  (+39.55%) | 112 |
| **Differential Privacy** | **1.0** | 1.85% | 3310.34  (+382.76%) | 1,930,653 |
| **4.6** | 13.55% | 1400.95  (+104.31%) | 305,396 |
| **10** | 49.03% | 899.38  (+31.16%) | 73,803 |
| **Additive Noise** | **2.0** | 5.84% | 1821.38  (+165.62%) | 489,840 |
| **1.5** | 10.35% | 1343.29  (+95.90%) | 304,482 |
| **1.0** | 22.51% | 993.95  (+44.95%) | 142,095 |
|  |  | **Type** | Privacy (Average Proportion of Identified Series) | Accuracy (Average MAE) | Representativeness (Average Performance Gap) |

Forecasters also prefer representative forecasts that look like the time series used to produce the forecasts (Petropoulos & Siemsen, 2022). Representativeness improves trust between data owners and forecasters, and makes it more likely for data owners to use protected data if it’s representative. We use the *performance gap* portion of the REP metric of Petropoulos & Siemsen (2022) to measure the distance between the protected and original time series values,

Performance gap = ,

which is calculated after applying a Box-Cox transformation and scaling the original and protected series. The performance gap values in Petropoulos & Siemsen (2022) are calculated using the fitted values of forecasting models relative to the training data. Compared to alternative privacy methods, the results show that k-nts+ and *k*-nTS produce protected time series with the smallest performance gaps by a large margin. Organizations need to trust privacy methods before implementing them and k-nTS+ produces more plausible time series than competiting privacy methods. However, we note that these performance gap values are significantly smaller than the fitted values from a forecasting model.

Past research on the M3 competition also found that complex forecasting models forecast more accurately than simple models using the monthly micro data (Koning et al., 2005). Table 5 displaysthe ranks of the MAE, forecast error varance, and average rank across all forecasting models using k-nTS+ swapping. The results show that k-nTS+ preserves the ranking of the best and worst models on MAE for the monthly micro data in the M3 Comeptition. Univarite models (SES and DES) moved up in the ranking and more complex models (Auto-ARIMA and RNN) moved down.

**Table 5:** Ranks of MAE, forecast error variance for the original data and the k-nTS+ swapping (*k*=3) data.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | MAE Ranks | | Forecast Error Variance Ranks | |
| Model | Original | Protected | Original | Protected |
| TES | 1 | 1 | 2 | 4 |
| ARIMA | 2 | 4 | 1 | 1 |
| RNN | 3 | 5 | 5 | 5 |
| DES | 4 | 2 | 3 | 2 |
| SES | 5 | 3 | 4 | 3 |
| LGBM | 6 | 6 | 7 | 6 |
| VAR | 7 | 7 | 6 | 7 |

* 1. *Analysis of Time Series Features*

For k-nTS+ (k=3) swapping which balanced the tradeoff between identification disclosure risk and forecast accuracy well, we computed 39 time series features using the tsfeatures package in R, including the nine features in Table 1. We used RReliefF to predict the absolute forecast errors for each model for each series across the original and protected data sets. Figure 4 shows the RReliefF weights for each of the 39 features averaged across forecasting models. Surprisingly, the hurst parameter and spectral entropy had negative weights which implied they were not useful to improve forecast accuracy for swapping in the protected data. On the other hand, the variance and E\_acf were very important to improve forecast accuracy.

**Figure 4: Average RReliefF weights across the results for each forecasting model.**

**Chart

Description automatically generated**

The average Out-of-Bag (OOB) Mean Square Error (MSE) across feature subset sizes and models is shown in Figure 5. We eliminated features with negative weights since these are poor predictors of forecast error. We applied the RFE algorithm for each forecasting model for iterations. We find that most of the reduction in OOB MSE occurs using five or fewer features for all forecasting models.

**Fig. 5: Average OOB MSE across feature subset sizes when predicting the MAE of each forecasting model.**

Graphical user interface

Description automatically generated

Figure 6 shows the permutation-based importance values for each forecasting model's eight most highly ranked features. Some features, such as *spike,* *max\_var\_shift, max\_level\_shift, series\_mean,* and *series\_variance,* are highly ranked across most or all forecasting models. Other features appear to be highly important only for specific models. Examples include *trend*, which is highly important for DES and TES, *seasonal\_strength*, which is highly important for TES, and *unitroot\_pp,* which is important for Auto-ARIMA and SES.

**Fig 6: Permutation importance for each model's eight most important features (based on RFE elimination rank).**

Graphical user interface, chart

Description automatically generated

Finally, we replicate the desirable and undesirable time series from Figure XXX and apply the privacy methods to the original time series. Figure 7 illustrates the results and

**Figure 7: Comparison of original, AN (s = 1), and *k*-nTS+ (*k* = 7) protected series with desirable and undesirable features.**

Chart

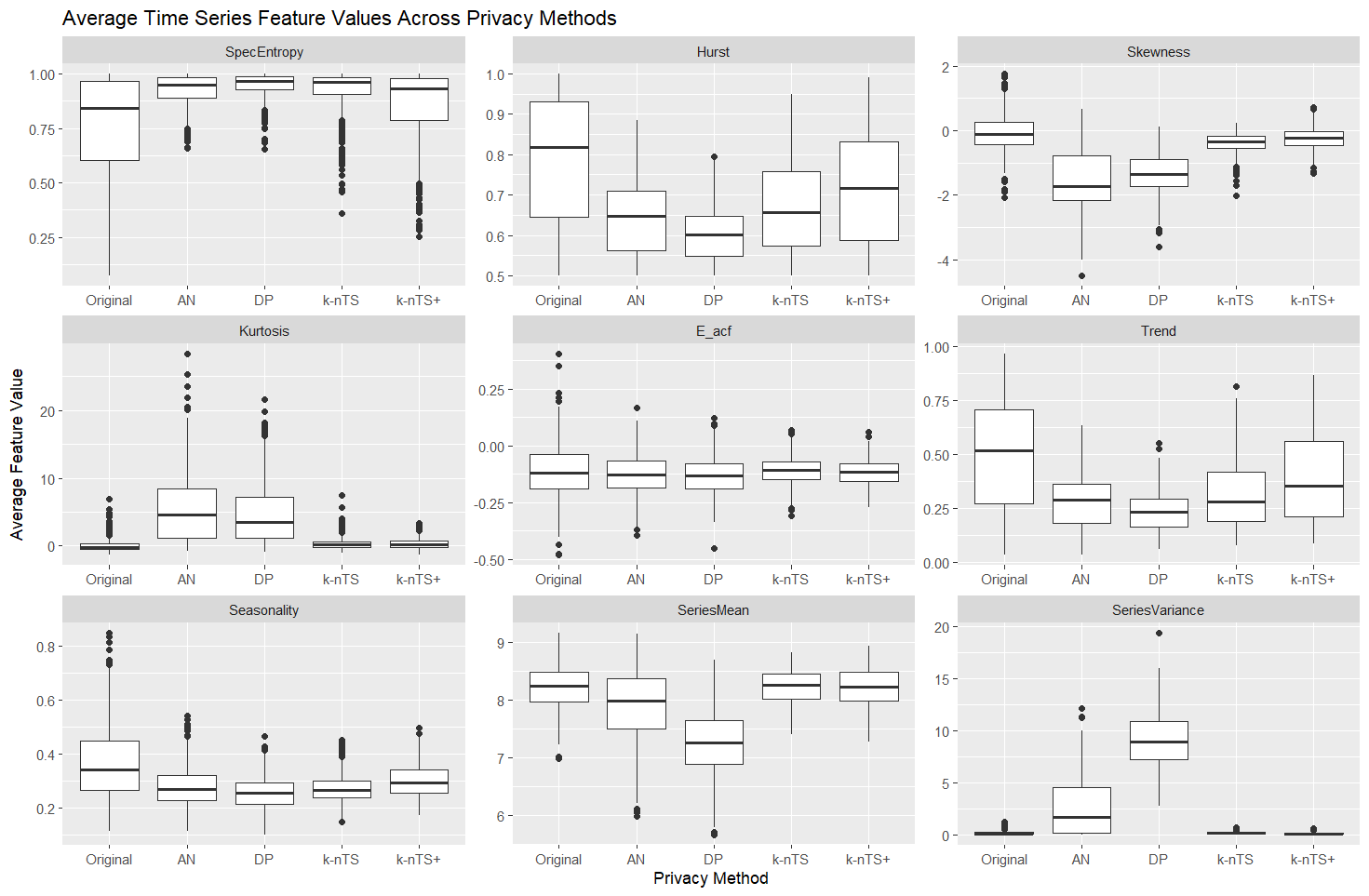
Description automatically generated

In **Figure 8**, we calculate the average feature value for each series across the protected datasets for each privacy method. We plot these distributions next to the distribution of each feature from the original data.

Figure 8 displays boxplots of the time series feature values for all of the privacy methods across the 9 time series features in Table 1. Random noise protection (AN and DP) methods increase the randomness of time series and significantly change distributional characteristics, leading to poor forecast accuracy. Random noise also produces a negative bias in the means of the protected series and significantly increases the variance. On the other hand, the *k*-nTS and *k*-nTS+ methods increase the randomness of time series but better preserve their feature distributions. Figure 8 shows that the feature distributions of *k*-nTS+ are closer to the original distributions than for any other privacy method, which led to improved forecast accuracy results.

We note that while the base *k*-nTS method performed swapping based on the values of *SpecEntropy, Hurst*, and *Seasonality*, it does not perform as well as *k*-nTS+ at preserving the distributions of these features, leading to poorer forecast accuracy. *k*-nTS+ did not explicitly swap based on the values of these features. Even though they correlted with forecast accuracy, *SpecEntropy* and *Hurst* were eliminated in the first stage of the *k*-nTS+ feature selection process using RReliefF. Instead, k-nTS+ achieved better feature preservation (and forecast accuracy)y by swapping using the time series features which are correlated with overall measures of forecastability.

**Fig 8: Distributions of time series features for each privacy method.**



1. **Conclusions**

This paper has examined forecasting using protected data. A substantial portion of the privacy literature is focused on theoretical privacy guarantees, i.e., differential privacy. Our findings agree with past research (Goncalves et al. 2021a) and show that differential privacy (and additive noise) lead forecasting models to generate unusable forecasts at meaningful levels of privacy. This undesirable privacy-utility tradeoff under differential privacy has also been demonstrated in other contexts. A recent paper by Blanco-Justicia et al. (2022) found that much of the work on differential privacy and deep learning utilized relaxed versions of differential privacy with values of that theoretically do not provide meaningful levels of privacy protection. Their experiments found that model regularization (e.g., L2-regularization) provided comparable privacy protection with better accuracy and lower model learning cost than differential privacy.

Rather than adding random noise to time series, our proposed *k*-nTS+ privacy method uses time series features to swap the values between time series. We demonstrated the effectiveness of our protection approach using data from a well-known forecasting competition where the identities of the time series needed to be kept confidential. The proposed method limited the reduction in average forecast accuracy to 14% of the original accuracy. Research in other domains found that a 10-15% loss in usability is often a best-case scenario under privacy protection (Schneider et al. 2018). Our method preserved the ranking of the best and worst forecasting models and provides comparable levels of privacy to differential privacy at meaningful levels of while enabling models to produce usable forecasts.

Decentralization based methods, such as those in (Gonçalves et al., 2021a; Goncalves, Bessa, et al., 2021b; Sommer et al., 2021) are effective but require a complicated decentralized framework. Our proposed k-nTS+ method reduces the frictions of implementing privacy protection since organizations need only select an appropriate value for k and apply the method to their data. Further, our method enables shared data sets containing protected time series rather than just parameter estimates or forecasts.

While we showed that *k*-nTS+ preserves data utility for forecasting, future work should examine the utility of protected time series for other use cases, such as classification. Future work should also assess forecasting with protected data using multiple forecast horizons. A theoretical examination of forecasting model performance on protected data would help us further understand which models will perform well under different conditions and could improve model selection procedures when forecasting with protected data. Future work should also assess whether forecast combinations, which tend to improve accuracy on unprotected data (Makridakis et al. 2018), are beneficial when forecasting with protected data.

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1. **Appendix**

**Mathematical Details of Identification and Attribute Disclosure**

To perform identification disclosure, we assume a third party possesses some original data on a unit of interest in the protected dataset. For the above example, this would be some sequence of original daily sales quantities for a known retailer. Denote this original data which contains a direct identifier (e.g., the identity of retailer ) and original data which contains a sequence of values that are components of the original time series .

We let denote the random variable (from the perspective of the third party) that indicates the corresponding for , i.e., when the values in are components of the original version of the protected series . Since the true value is unknown, the third party predicts the value of to be the series with the highest match probability, conditional on the known values, as follows

, (1)

where identification disclosure occurs when The probability is calculated as follows. Let denote the protected values of each time series that occur in the same time periods as . The third party computes the similarity between and the protected values using the Euclidean distance,

.

Using these similarities, the third party builds a probability mass function for over all protected series in as

,

and predicts as in (1).

To estimate the risk of identification disclosure, we perform simulations in which we sample sequential values from each original time series , and we measure the average proportion of series which are identified. The sampled values are denoted . Each of the vectors corresponds to one of the original time series, and we compute conditional on the sampled from series . We repeat this simulation times to obtain , and computethe average proportion of correctly identified time series across all external data samples and original time series,

where [.] are Iverson brackets.

These simulations assume that the third party in possession of predicts the match for each vector independently of the predicted matches for other vectors. The risk estimate from a given simulation is equivalent to the identification risk when independent third parties are each in possession of one of the vectors and each attempts identification risk as described above. Overall, multiple vectors may be matched to the same protected time series.

1. For examples in the United States, see [this](https://iapp.org/resources/article/us-state-privacy-legislation-tracker/) map. [↑](#footnote-ref-1)
2. See articles 6, 45, and 46 of the GDPR. [↑](#footnote-ref-2)
3. All norms on are equivalent to the Euclidean norm. [↑](#footnote-ref-3)
4. Note that we could also define a distance matrix based on the actual time series values , where would become a function of rather than . [↑](#footnote-ref-4)
5. We note that there are other privacy leaks such as attribute disclosure (Li et al. 2007) or membership inference (Shokri et al. 2017). Identification disclosure is the most applicable for our data, and we consider this a steppingstone to additional privacy leaks – e.g., identifying a time series within a protected data set enables a third party to learn unknown information with greater certainty. [↑](#footnote-ref-5)