Project 2: Illumination Cone

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The illumination cone is a cone that exists in R^p (where p is the number of pixels in an images) and can be used to generate images of objects under varying illuminations. A few assumptions are necessary to use this cone. First, the object must be convex. Second, the object must have a Lambertian surface. A minimum of 3 images of an object with unique lighting are necessary for this analysis. Additionally, while the cone exists in R^p, it has been shown to be relatively flat, allowing the use of a low dimensionality subspace to approximate it.

First, let **B** be the n-by-3 matrix where each row is the product of the albedo and the unit normal for a point on the surface of the object. Let **s** be a column vector that is the product of the light source's intensity with its direction vector. Then an image **x** is:

$$x = \max(B\boldsymbol{s}, 0)$$

Since x cannot be negative (negative values correspond to attached shadows).

We call L the space of all possible values of B**s**. The dimensionality of L will be the rank of B. Because B is rank 3 (for our example where we have 3 different surface normals), the dimensionality of L is also 3.

Finding this 3-dimension subspace is simple. First, take all images, flatten them into row vectors, and concatenate them vertically to form $\mathbf{T} \in R^{3xp}$. Next perform SVD on $\mathbf{TT}^\mathbf{T}$ and keep only the first 3 singular values and vectors**.**

$$[V, D] = SVD(TT^T)$$

Where V is the left singular values and D is a diagonal matrix of the singular values. Then the following defines the basis images in the new vector space:

$$B = T^T V D^{-\frac{1}{2}}$$

The three basis images are found below:

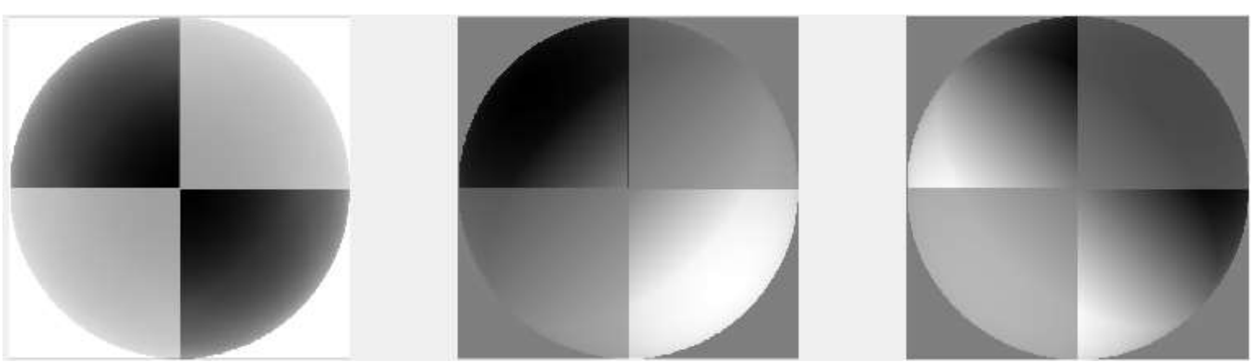

Figure 1: Basis Images for This Project

And the new image is formed by the formula for **x** defined above where:

$$s = \begin{bmatrix} cos\theta sin\varphi \\ sin\theta sin\varphi \\ cos\varphi \end{bmatrix}$$

x will of course need to be resized to the original image size since it will be a column vector after this calculation. A video of a rotating light source can be found in an attached file.

Results:

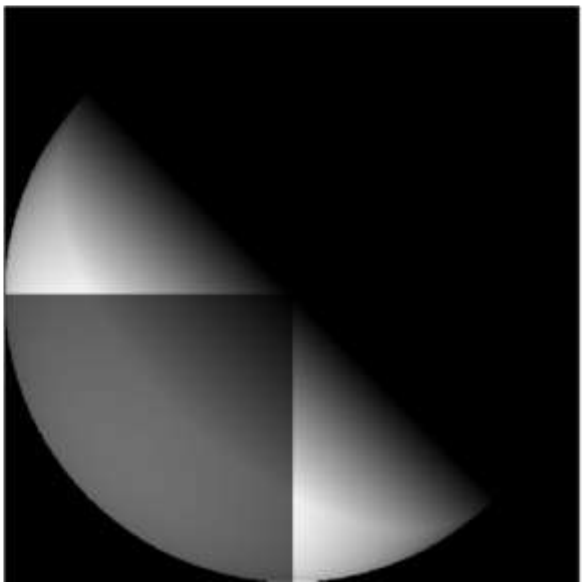

Figure 2: Phi = 0 deg, Theta = 0 deg

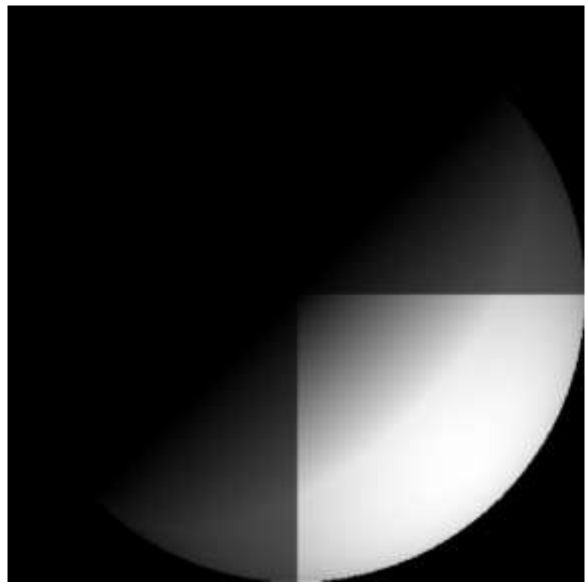

Figure 3: Phi = 90 deg, Theta = 90 deg

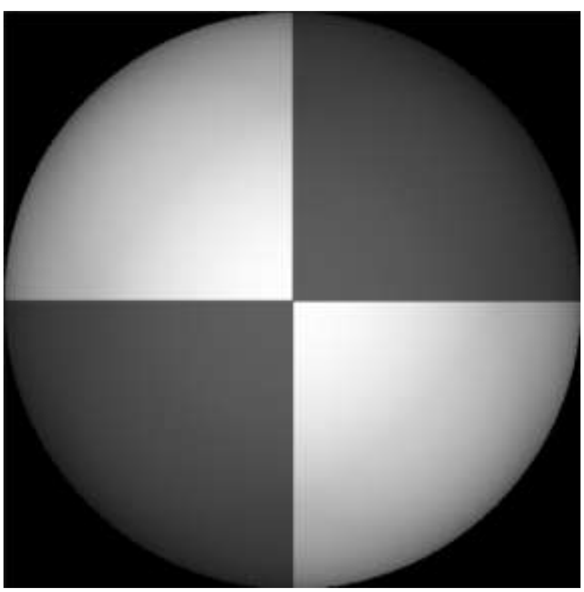

Figure 4: Phi = 270 deg, Theta = 0 deg

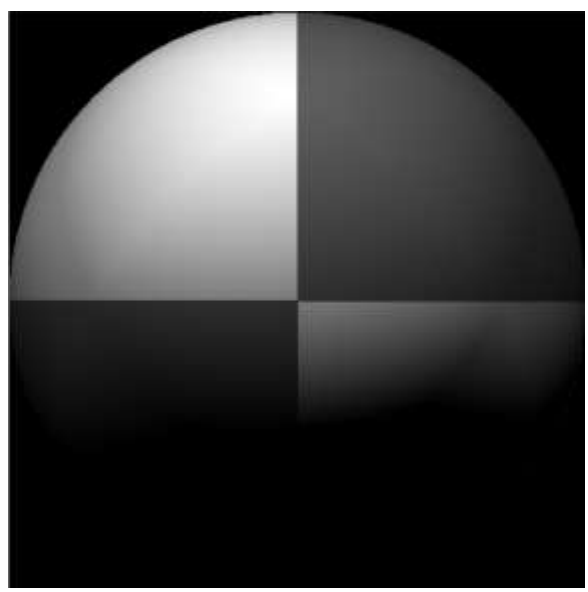

Figure 5: Phi = 270 deg, Theta = 0 deg