Derivation

The equation of a plane is: $ax + by + cz = d$

If we choose our error function to be the mean-squared distance we get:

$$E = \sum_{i=1}^{n} (ax_i + by_i + cz_i - d)^2$$

We can differentiate E with respect to d and get: $d = a\bar{x} + b\bar{y} + c\bar{z}$

Where $\bar{x}, \bar{y}, and\ \bar{z}$ are the mean valuea of x, y, and z. If we substitute this for d we get:

$$E = \sum_{i=1}^{n} (a(x_i - \bar{x}) + b(y_i - \bar{y}) + c(z_i - \bar{z}))^2 = \|An\|^2\ where\ A = \begin{bmatrix} x_1 - \bar{x} & y_1 - \bar{y} & z_1 - \bar{z} \\ \vdots & \vdots & \vdots \\ x_n - \bar{x} & y_n - \bar{y} & z_n - \bar{z} \end{bmatrix}$$

The solution to this is found by taking the eigenvectors of A^TA and using the eigenvector associated to the smallest eigenvalue. This will give values for a, b, and c. d can be easily found by the third equation above.

Results

I tested this by producing 1000 data points on the actual plane, then added random noise with a variance of 1 and mean of zero. The plots show the data points with the calculated plane.

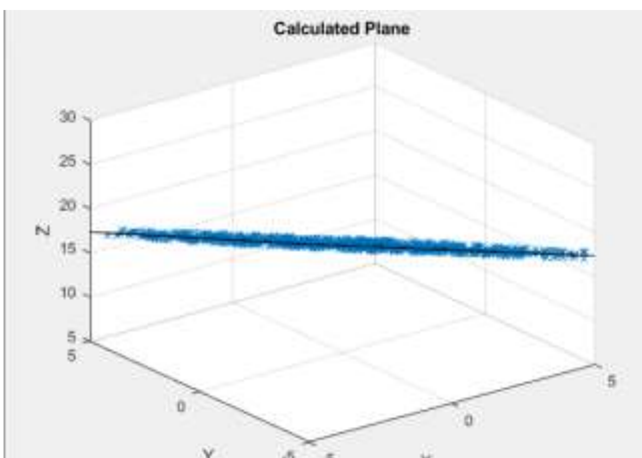


Figure 1: Actual: **0.5774x + 0.5774y + 0.5774z = 10**, Calculated: **0.5766x + 0.5783y + 0.5772z = 9.9529**

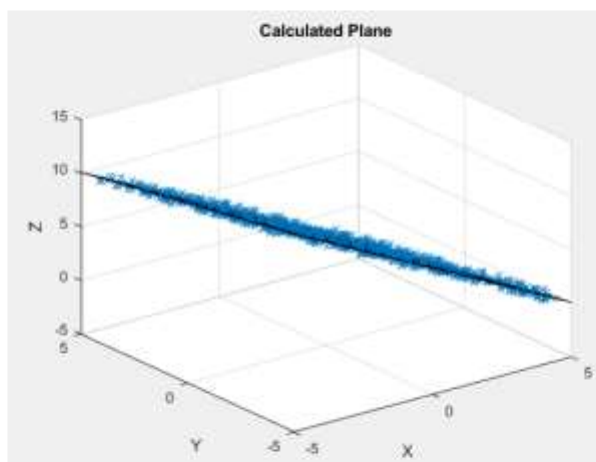
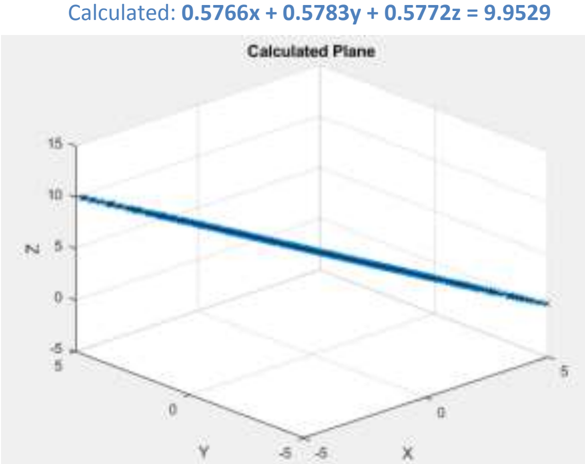


Figure 2: Actual: **0.7845x + 0.1961y + 0.5883z = 3**, Calculated: **0.7846x + 0.1990y + 0.5872z = 2.9942**



I ran a final test to make sure that, without any added noise, the smallest eigenvalue is 0. The figure on the left shows this case, with an eigenvalue of -1.02×10^{-12}. **Actual**: 0.7845x + 0.1961y + 0.5883z = 3**, Calculated:** 0.7845x + 0.1961y + 0.5883z = 3