Linear-Operator-based Data-Driven Approach for Uncertainty Propagation and Reachability Analysis in Power Systems

Amarsagar Reddy Ramapuram Matavalam

Research Assistant Professor, Iowa State University, amar@iastate.edu

Collaborators : Umesh Vaidya, Venkataramana Ajjarapu

Presented at Conference on Decision and Control 2021 Workshop on Uncertainty Management in Power System Dynamics

Support from department of energy grant DE-OE-0000876 is gratefully acknowledged

Introduction

- The problem of uncertainty propagation and quantification is of interest across the various discipline of science and engineering
- Examples include power systems, fluid dynamics, robotics, and biological systems
- Typically, uncertainty arises from
  - Parametric uncertainty in the system model
  - Unknown initial state after a random disturbance
  - Uncertainty in inputs to the system
- Focus in presentation on **data-driven** uncertainty propagation and reachability analysis arising from **uncertainty in the initial condition**
- The problem of uncertainty propagation and reachability analysis is **complicated due to the nonlinear nature** of dynamics involved in these applications

Motivation for Data-driven

- It is not always possible to get the dynamical equations of an underlying system – intractable , privacy, etc.
- We can observe data from simulations/experiments non-intrusively
- Popular methods for **non-intrusive** uncertainty propagation include:
  - Monte-Carlo (MC) based methods [1]
  - Polynomial chaos (PC) based methods [2,3,4]
- These methods have drawbacks – scalability is the key issue
  - MC need many nonlinear simulations of the dynamical system
  - PC alleviates this drawback but needs <u>new simulations if uncertainty set is modified</u>

[1] P. Baraldi and E. Zio, “A combined monte carlo and possibilistic approach to uncertainty propagation in event tree analysis,” *Risk Analysis: An International Journal*, vol. 28, no. 5, pp. 1309–1326, 2008.
[2] D. Xiu and G. E. Karniadakis, “Modeling uncertainty in flow simulations via generalized polynomial chaos,” Journal of comp. physics, vol. 187, no. 1, pp. 137–167, 2003.
[3] H. N. Najm, “Uncertainty quantification and polynomial chaos techniques in computational fluid dynamics,” Annual review of fluid mechanics, vol. 41, pp. 35–52, 2009.
[4] Y. Xu, L. Mili, A. Sandu, M. R. von Spakovsky, and J. Zhao, “Propagat-ing uncertainty in power system dynamic simulations using polynomialchaos,”IEEE Transactions on Power Systems, vol. 34, no. 1, pp. 338–348, 2018..

Contribution

- Novel **data-driven** approach for uncertainty propagation and reachability analysis using **moments**
- The proposed approach relies on the **linear lifting of a nonlinear system** provided by linear **Koopman [5] and Perron-Frobenius [6] (P-F) operators**
- Demonstrate how the P-F and Koopman operators can be used for the **propagation of moments in a linear manner**
- Results are presented for
  - Simple nonlinear dynamical systems
  - Power system with complex dynamics

[5] B. Koopman and J. v. Neumann, "Dynamical systems of continuous spectra," Proceedings of the National Academy of Sciences of the United States of America, vol. 18, no. 3, p. 255, 1932
[6] C. W. Rowley, I. Mezic, S. Bagheri, P. Schlatter, and D. S. Henningson, "Spectral´ analysis of nonlinear flows," Journal of fluid mechanics, vol. 641, pp. 115–127, 2009

Operator - Definition

- An Operator is a Map from one set of functions to another set of functions
- Eg –Differentiation is an operator. We can define it from C^∞ to C^∞
- It is an infinite dimensional linear operator
- We can represent the operator by an ‘infinite’ matrix
- If we restrict our basis to polynomials in one dimension,

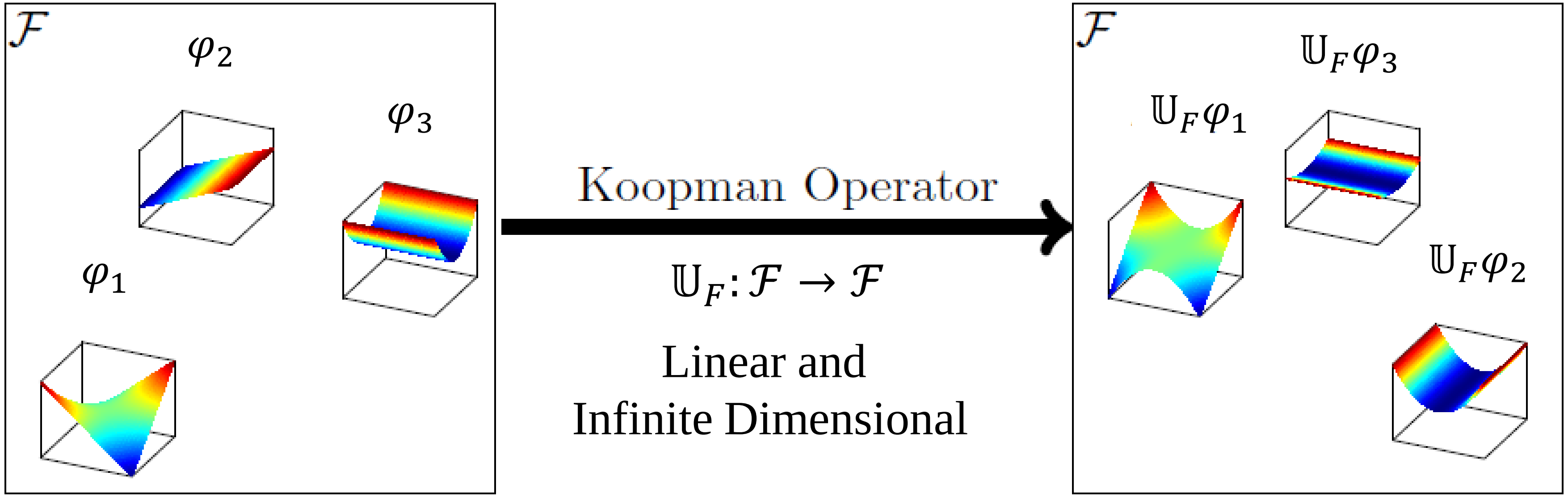
$$M = \frac{df}{dx} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & \dots \\ 0 & 0 & 2 & 0 & 0 & \dots \\ 0 & 0 & 0 & 3 & 0 & \dots \\ 0 & 0 & 0 & 0 & 4 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

Linear Operator Theoretic Framework -I

- Consider the Dynamical system:

$$x_{t+1} = F(x_t) \quad x_t \in X \subset \mathbb{R}^N$$

- The Koopman operator ($\mathcal{K}$ *or* $\mathbb{U}_F$) of F operates on functions in $\mathcal{F}$ and propagates it by one time step using the dynamics F: $[\mathbb{U}_F\, \varphi](x) = \varphi\big(F(x)\big)$



[7] M. O. Williams, I. G. Kevrekidis, and C. W. Rowley, "A data–driven approximation of the koopman operator: Extending dynamic mode decomposition," Journal of Nonlinear Science, vol. 25, no. 6, pp. 1307–1346, 2015.

Linear Operator Theoretic Framework-II

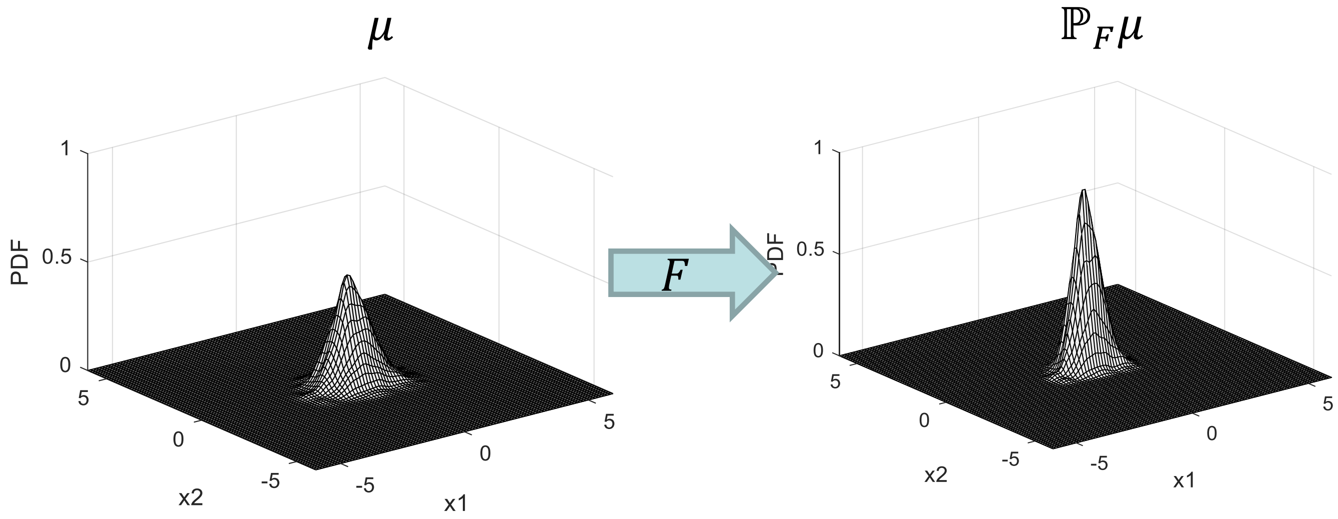
- Consider the Dynamical system:

$$x_{t+1} = F(x_t) \quad x_t \in X \subset \mathbb{R}^N$$

- The P-F operator of F operates on the space of measures

$$\mathbb{P}_F : \mathcal{M}(X) \to \mathcal{M}(X)$$

- The P-F operator propagates uncertainty in initial conditions captured through measure or probability density function.



Linear Operator Theoretic Framework - III

- P-F and Koopman operators **satisfy linearity properties** [5,6] – adding & scaling
- The P-F and Koopman operators are **infinite dimensional operators** for generic systems
- Finite dimensional approximation (matrix representations) can be estimated using data from simulations and experiments
- These finite dimension approximations can then be used for observer synthesis & control synthesis valid over large regions of state-space
- Linear observers and controls designed for the lifted model translate to **nonlinear observers and controls** in the original state space

[5] B. Koopman and J. v. Neumann, "Dynamical systems of continuous spectra," Proceedings of the National Academy of Sciences of the United States of America, vol. 18, no. 3, p. 255, 1932
[6] C. W. Rowley, I. Mezic, S. Bagheri, P. Schlatter, and D. S. Henningson, "Spectral´ analysis of nonlinear flows," Journal of fluid mechanics, vol. 641, pp. 115–127, 2009

Linear Operator Theoretic Framework -IV

- The P-F and Koopman operators are dual to each other as follows

$$\langle \mathbb{U}_F \varphi, \mu \rangle = \int_X [\mathbb{U}_F \varphi](x) d\mu(x) = \int_X \varphi(x) d[\mathbb{P}_F \mu](x) = \langle \varphi, \mathbb{P}_F \mu \rangle$$

- For an observable φ & measure μ, propagating φ and taking inner product with μ is same as propagating μ and taking the inner product with φ.
- The inner products of any function φ with a measure μ are the moments of the measure with respect to the function
- So, there is an inherent relation between the moments and these linear operators

Relation between Moments and Linear Operators

- Let $\mu_0(x)$ be the measure corresponding to the initial density function and let the moments computed w.r.t. to basis functions i.e., $\mathbf{\Psi}(x) = [\Psi_1(x), \ldots, \Psi_N(x)]$
- Moments of the initial measure $\mu_0(x)$ corresponding to these basis functions are

$$m_0^k = \int \Psi_k(x)\, d\mu_0(x) = \langle \Psi_k, \mu_0 \rangle\, , k = 1,2 \ldots, N$$

- The moments are propagated in time as follows

$$m_1^k = \langle \Psi_k, \mu_1 \rangle = \langle \Psi_k, \mathbb{P}_F \mu_0 \rangle = \langle \mathbb{U}_F \Psi_k, \mu_0 \rangle$$

$$m_{t+1}^k = \langle \Psi_k, \mu_{t+1} \rangle = \langle \Psi_k, \mathbb{P}_F \mu_t \rangle = \langle \mathbb{U}_F \Psi_k, \mu_t \rangle$$

- Using the linearity of Koopman operator the moment propagation can be expressed as

$$\mathbf{m}_{t+1} = \mathcal{K}\mathbf{m}_t$$

- Where $\mathbf{m}_t = [m_t^1, \ldots, m_t^K, \ldots]$ and $\boxed{\mathcal{K} : \mathbb{R}^\infty \to \mathbb{R}^\infty}$ → Generally infinite dimensional
Can be finite for special systems

Simple Numerical Example

- Consider the following 2-D dynamical system

$$
\begin{aligned}
x_1^{t+1} &= \rho x_1^t \\
x_2^{t+1} &= \mu x_2^t + (\rho^2 - \mu) c (x_1^t)^2
\end{aligned}
$$

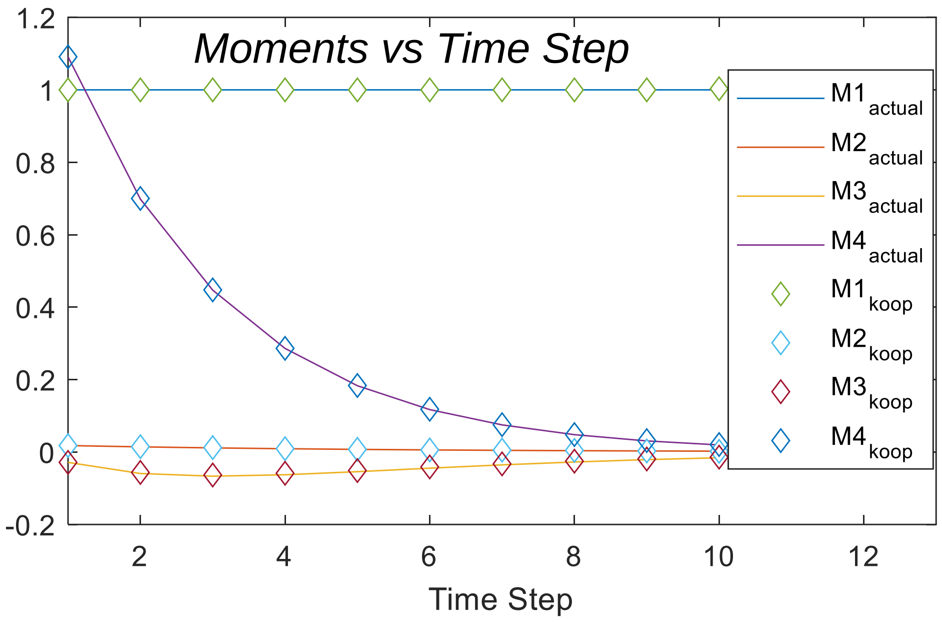
- With basis functions $\Psi = [1 \quad x_1 \quad x_2 \quad x_1^2]$ for lifting the system
- Consider the matrix **K**, We can observe that the $\Psi^{t+1} = \mathbf{K}^{\mathrm{T}} \cdot \Psi^t$

$$
\mathbf{K} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \rho & 0 & 0 \\ 0 & 0 & \mu & 0 \\ 0 & 0 & (\rho^2 - \mu)c & \rho^2 \end{bmatrix}
$$

- Thus, $\mathbf{K}^{\mathrm{T}}$ is the matrix representation of the Koopman operator and is finite for this system

Simple Numerical Example

- Consider ***any*** initial uncertainty set and estimate the uncertainty set at each time instant for the future



- Both moments are identical - MC needs explicit simulations; Koopman is matrix multiplication ; **speed up of >100x**
- PC needs new simulations (fewer than MC) if initial uncertainty set is modified while the Koopman matrix once identified can be used for ***any*** uncertainty set

Data-Driven Approximation of Koopman Operator

- Consider discrete-time system $x_{t+1} = F(x_t)$
- Define a domain of interest in the state space and randomly select initial points in the domain and record time series data

$$\{x_1, \ldots, x_t, \ldots, x_M\}, \qquad x_t \to x_{t+1}$$

- Lift the system by calculating the value of the observables Ψ(x) for each point on the recorded trajectories

$$\mathbf{\Psi}(x) = [\Psi_1(x), \ldots, \Psi_N(x)]$$

- Rewrite $y_m = x_{m+1}$ and solve the following optimization problem for the finite approximation of the Koopman operator for deterministic system with finitely many basis functions

$$\min_{\mathbf{K}} \| \mathbf{G}\mathbf{K} - \mathbf{A} \|_F, \qquad \mathbf{K}^* = \mathbf{G}^{\dagger}\mathbf{A}$$

$$\mathbf{G} = \frac{1}{M}\sum_{m=1}^{M} \mathbf{\Psi}(x_m)^{\top}\mathbf{\Psi}(x_m), \quad \mathbf{A} = \frac{1}{M}\sum_{m=1}^{M} \mathbf{\Psi}(x_m)^{\top}\mathbf{\Psi}(y_m)$$

Steps for data-driven Moment Propagation

- Identify a good Koopman approximation **K** through offline analysis
- Sample sufficient initial conditions from a given uncertainty set
- Estimate the initial moments from the initial uncertainty set
- Calculate moments at future times sequentially using $\boldsymbol{m}_{t+1} = \mathbf{K}^T \cdot \boldsymbol{m}_t$
- **Data-driven moment propagation is basically matrix multiplication – very fast**
- Contrastingly, the MC **based moment propagation is computate intensive**
- The key idea is that we use ***deterministic*** simulations to learn **K** and then use this to propagate ***uncertainty***.

A. R. Ramapuram Matavalam; U. Vaidya; V. Ajjarapu, "Data-Driven Approach for Uncertainty Propagation and Reachability Analysis in Dynamical Systems", Proceedings of American Control Conference 2020

Results – Bi-Stable Toggle

- Consider the 2-D bi-stable toggle system

$$\dot{x}_1 = \frac{1}{1+x_2^{3.55}} - 0.5x_1 \qquad \dot{x}_2 = \frac{1}{1+x_1^{3.53}} - 0.5x_2$$

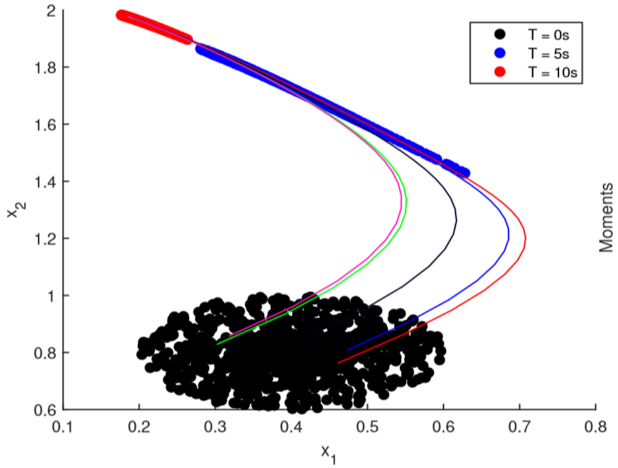
- This system has 2 equilibria - (0.16,2) & (2,0.161)
- The region of interest for this system is $(x_1, x_2) \in (0, 2.5)\times(0, 2.5)$
- Based on our experiments, using monomials with degree up to 4 (a total of 15 functions) and a scaling factor of 3 gave a good Koopman matrix, i.e.

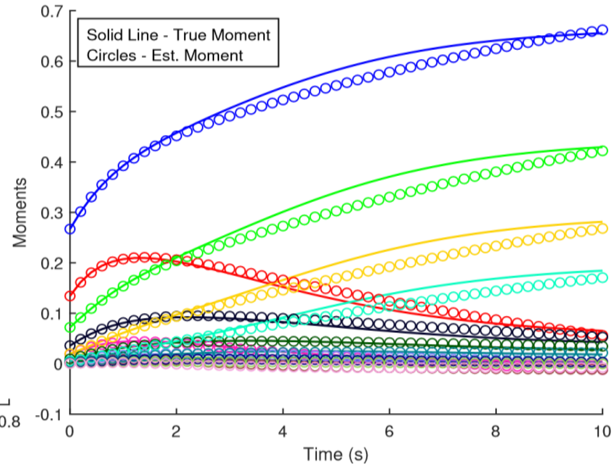
$$\Psi(x) = \left[1 \quad \frac{x_1}{3} \quad \frac{x_2}{3} \quad \frac{x_1^2}{9} \quad \frac{x_1x_2}{9} \quad \frac{x_2^2}{9} \quad \cdots \quad \frac{x_2^4}{81}\right]$$

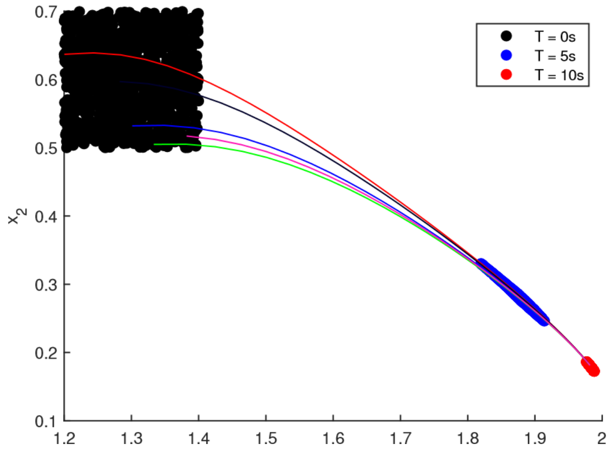
- Identification of the Koopman matrix took around 300 trajectories of 50 time steps of 0.2s

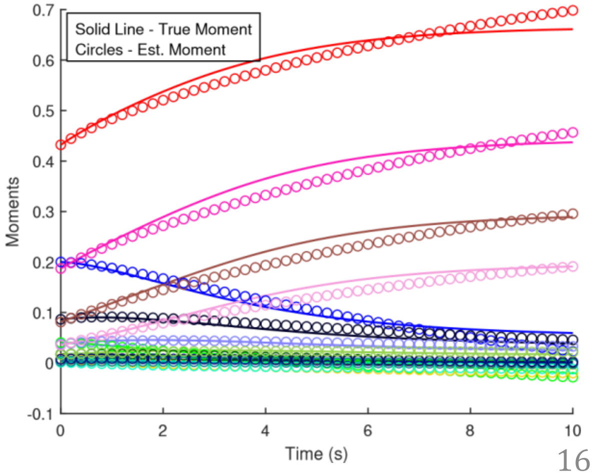
Results – Bi-Stable Toggle (cont.)

- As there are 2 equilibrium points, two different initial uncertainty sets with different shapes are used to verify the data-driven methodology
  - A circle centered at (0.4, 0.8) with radius 0.2
  - A square given by $(1.2, 1.4) \times (0.5, 0.7)$
- The **<u>same</u>** Koopman matrix is used for the moment propagation even though they are in different regions of attraction
- Comparing MC based moments and Koopman moments – time acceleration > 100x

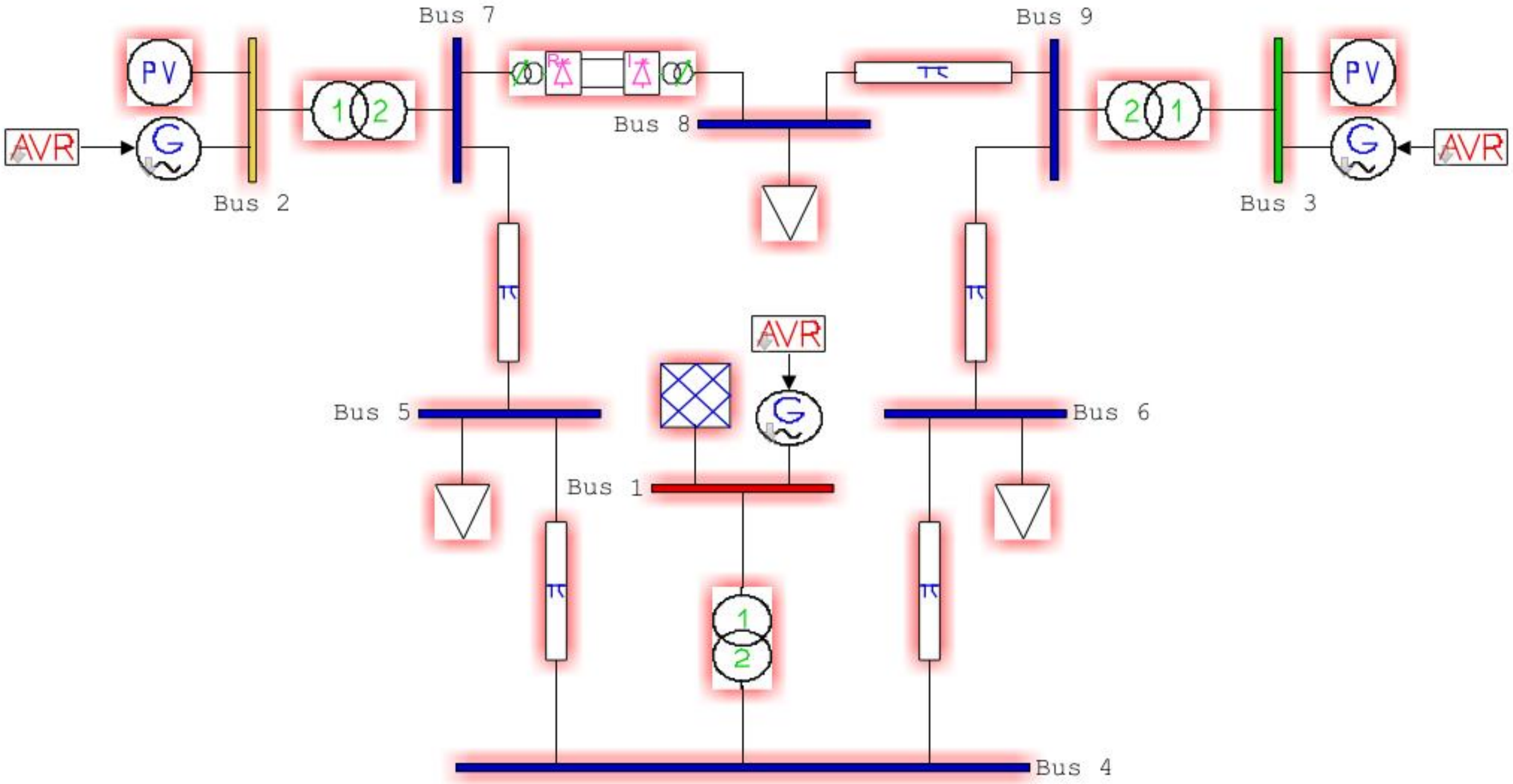






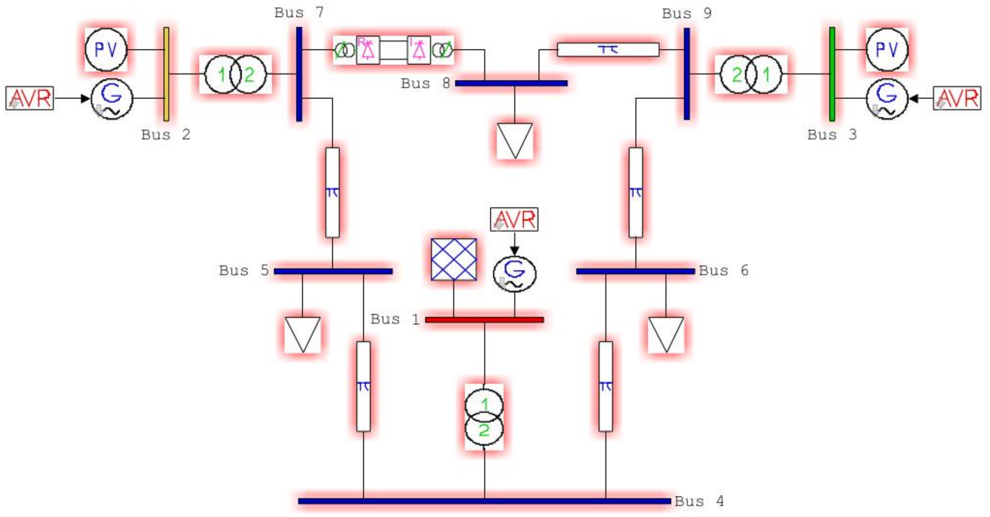


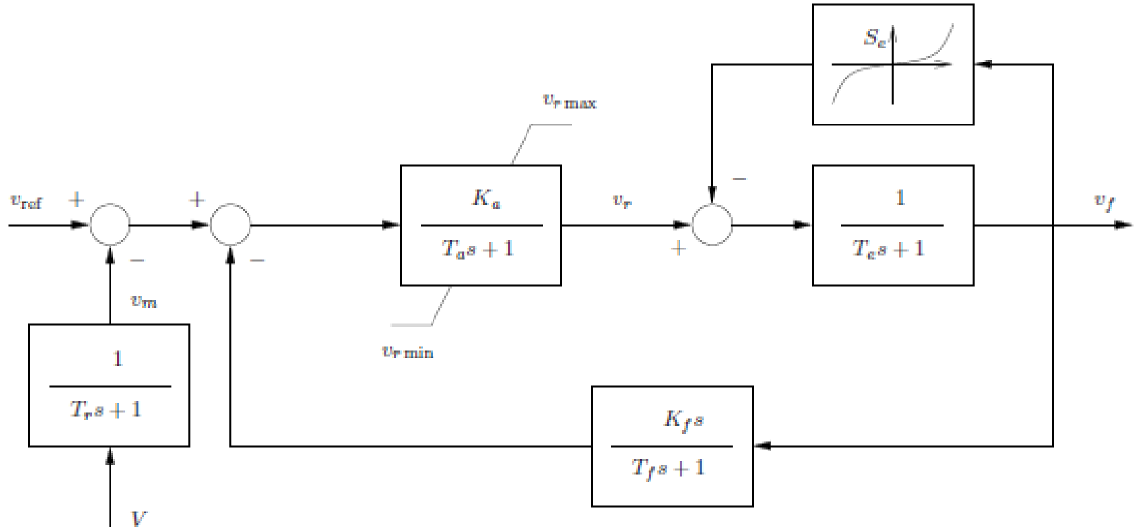
Power system example – 9 bus + HVDC



System information

- Simulated in PSAT in MATLAB – Full DAE
- The generators are 4th order machines
- The AVR is a 3rd order Type II exciter
- The HVDC is 3rd order device
- Total number of dynamic states in the system are 27
- Loads are constant power type





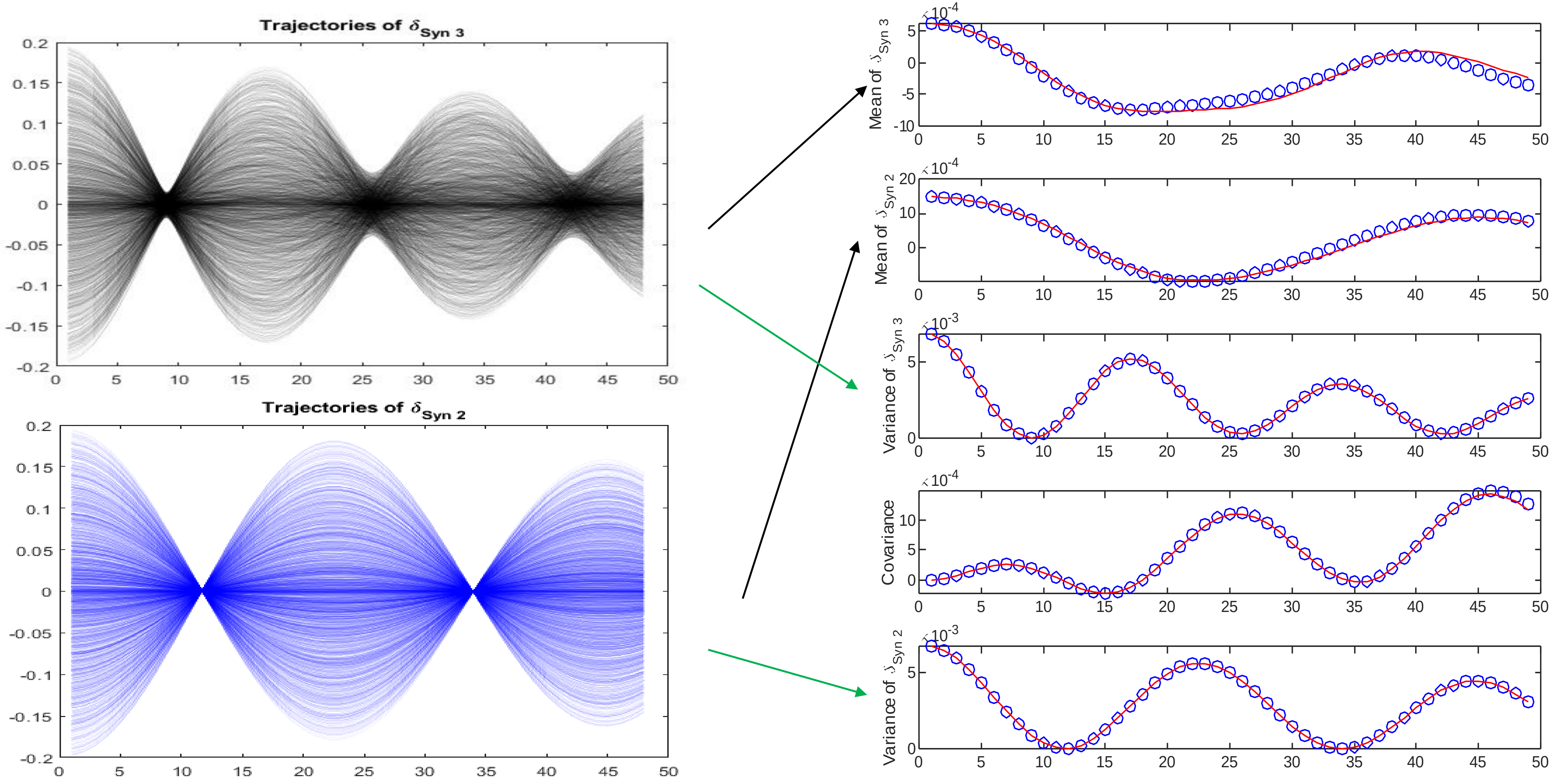
Scenarios considered

- Initial Gen 2 & 3 rotor angle states have assumed to be uncertain
- Multiple uncertainty sets are considered
  - Scenario – 1: Uniform PDF in a circle
  - Scenario – 2: Uniform PDF in a square
  - Scenario – 3: non-uniform PDF in a non-convex shape
- Uncertainty propagates to other states due to the system dynamics
- The dictionary functions are monomials with degree up to 2 – total 378 func.
  - Mean and Variance of each state
  - Covariance between each pair of states
- The Koopman matrix is estimated using 500 random initial conditions with 50 time-steps of 0.02 seconds

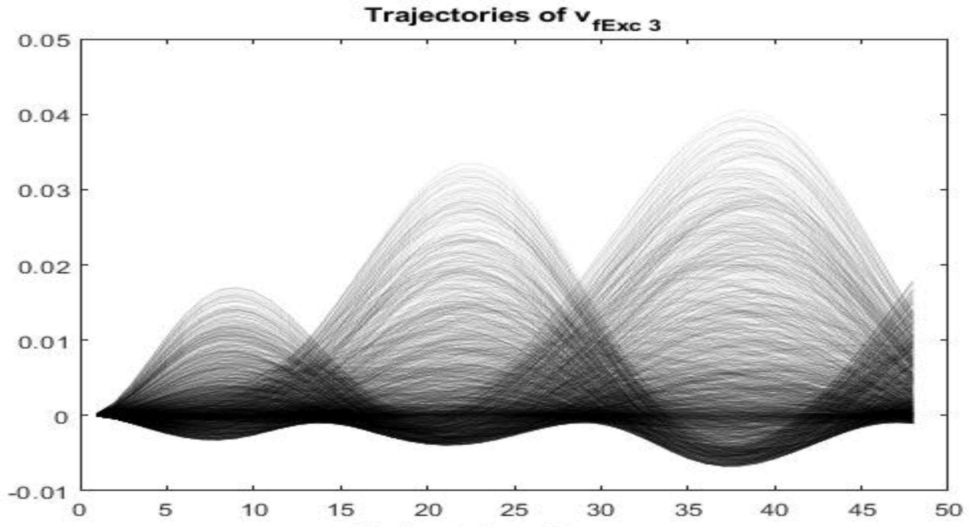
Scenarios considered

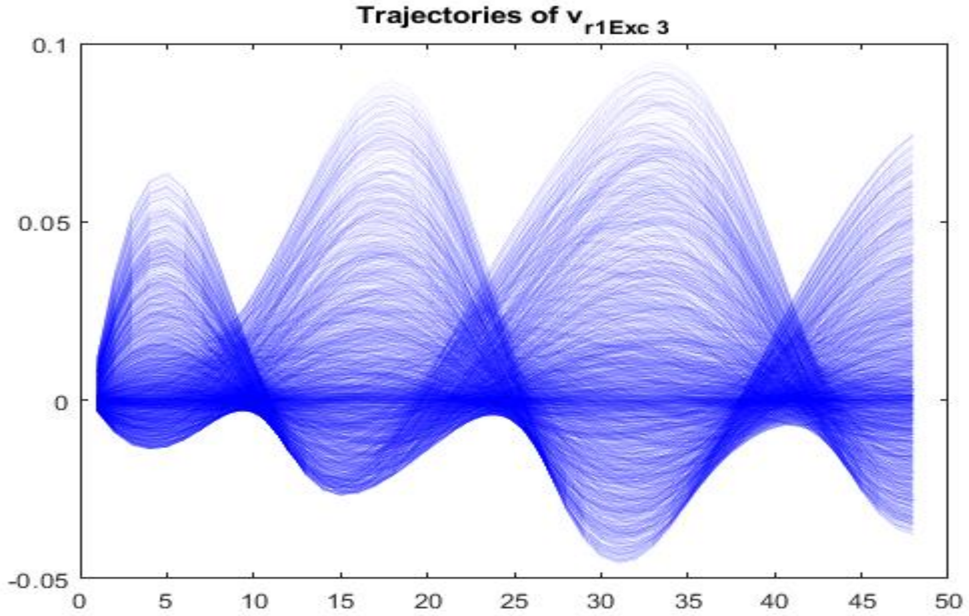
- Initial Gen 2 & 3 rotor angle states have assumed to intial uncertainty
- Multiple uncertainty sets are considered
  - Scenario – 1: Uniform PDF in a circle
  - Scenario – 2: Uniform PDF in a square
  - Scenario – 3: non-uniform PDF in a non-convex shape
- The dictionary functions are monomials with degree up to 2 – total 378 func.
- The Koopman matrix is estimated using 500 random initial conditions with 50 time-steps of 0.02 seconds

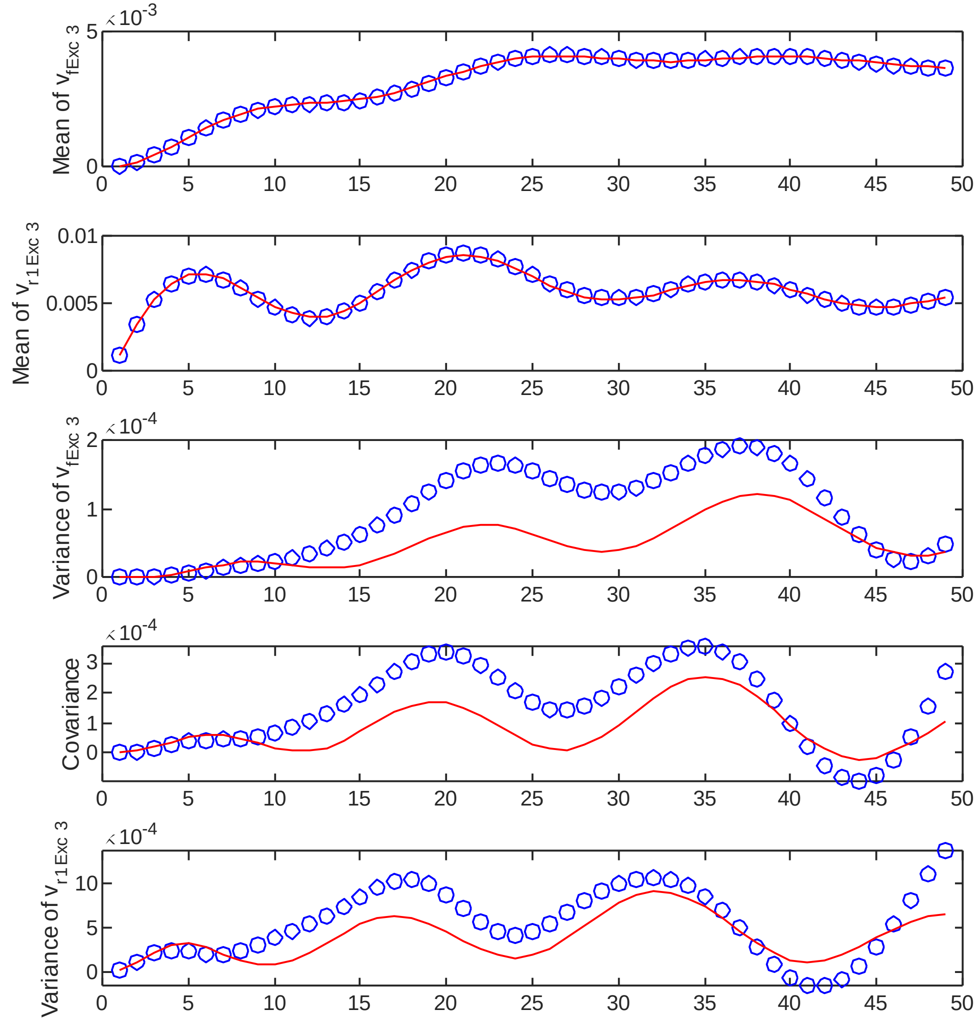
Scenario -1: Behavior of Gen 2 & 3 rotor angle



Scenario -1: Behavior of States of AVR at Gen-3

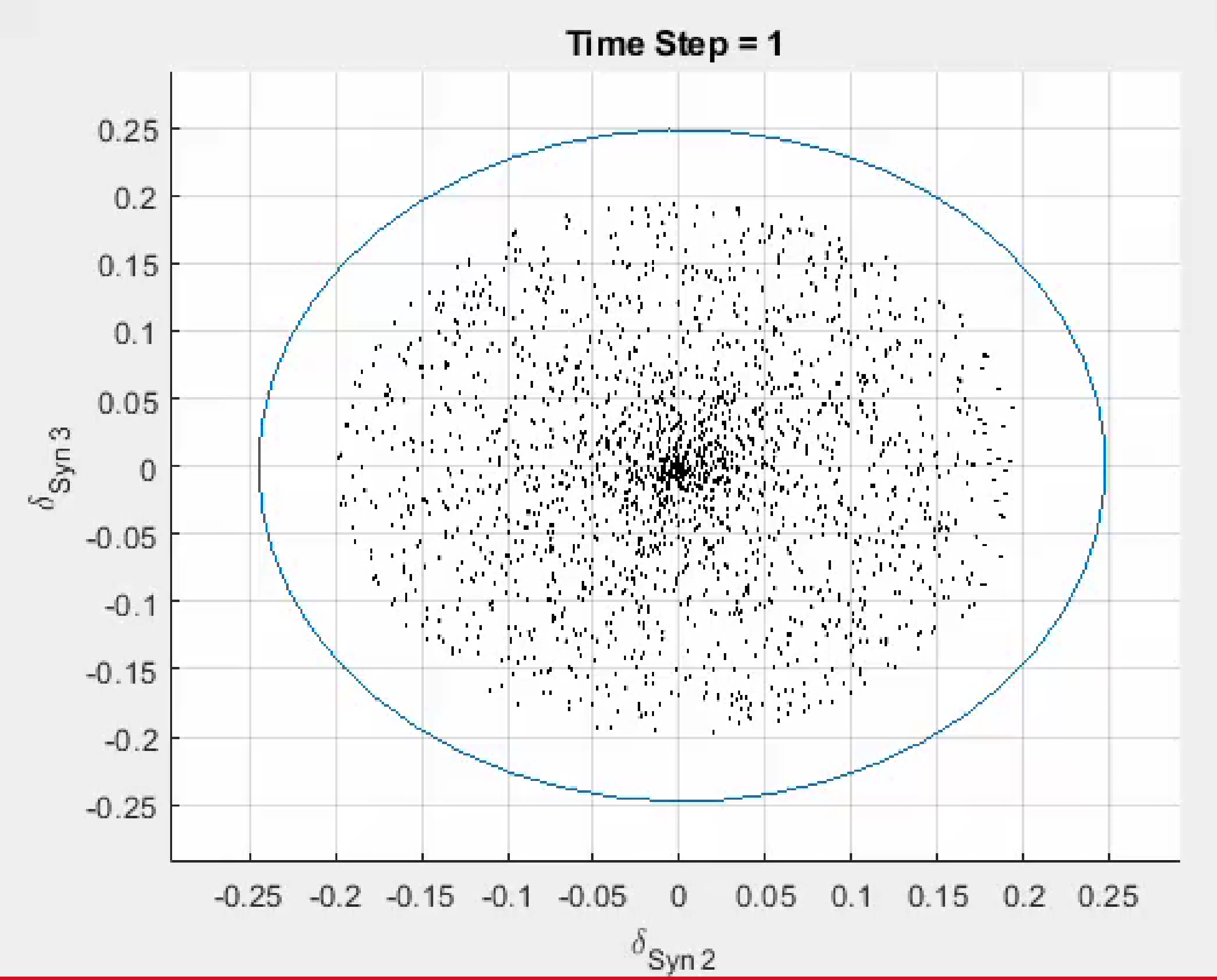




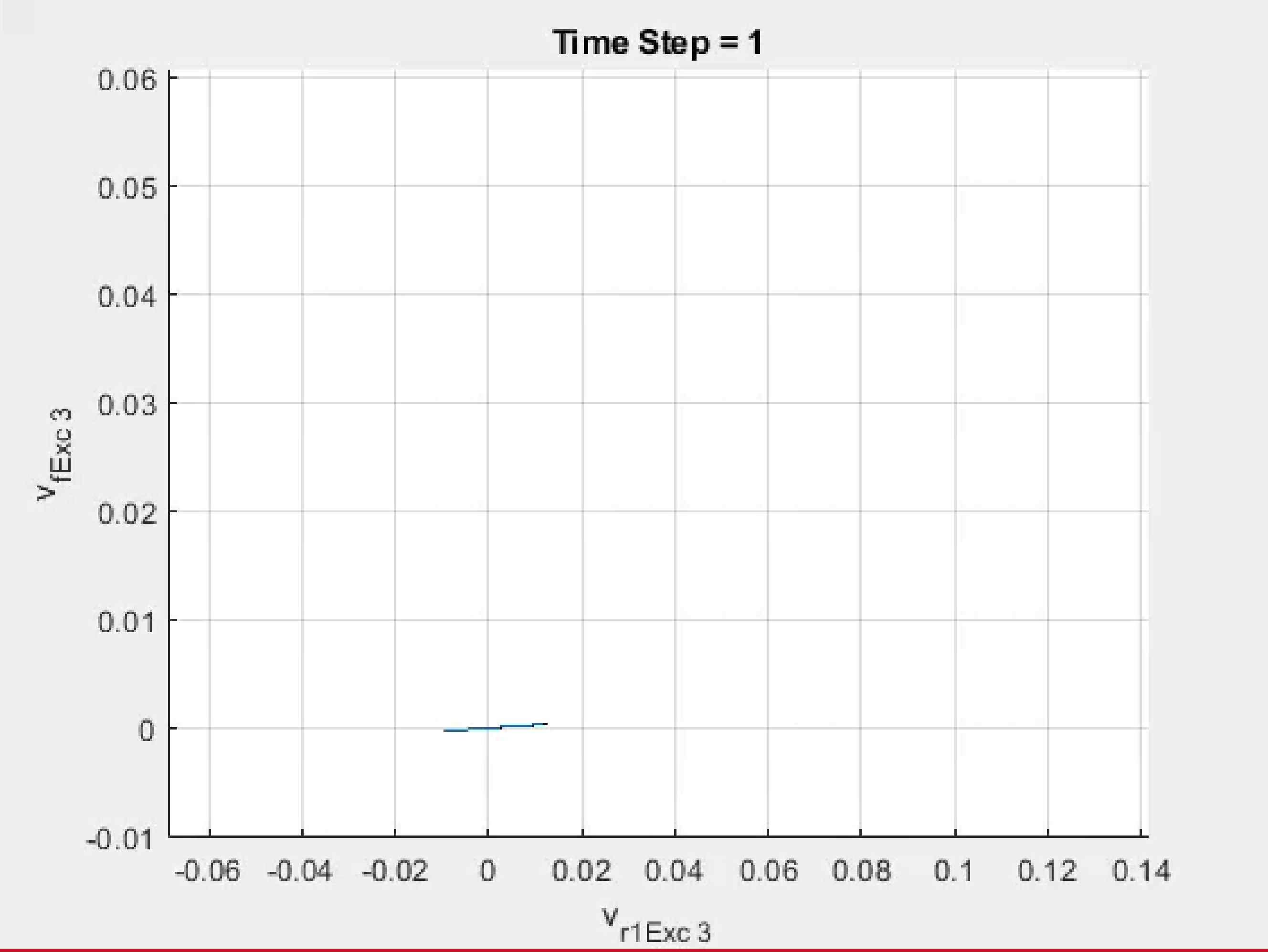


Scenario -1: Bounding Region using Est. Moments

$x - axis\colon\ \delta_{gen-2};\quad y - axis\colon\ \delta_{gen-3}$



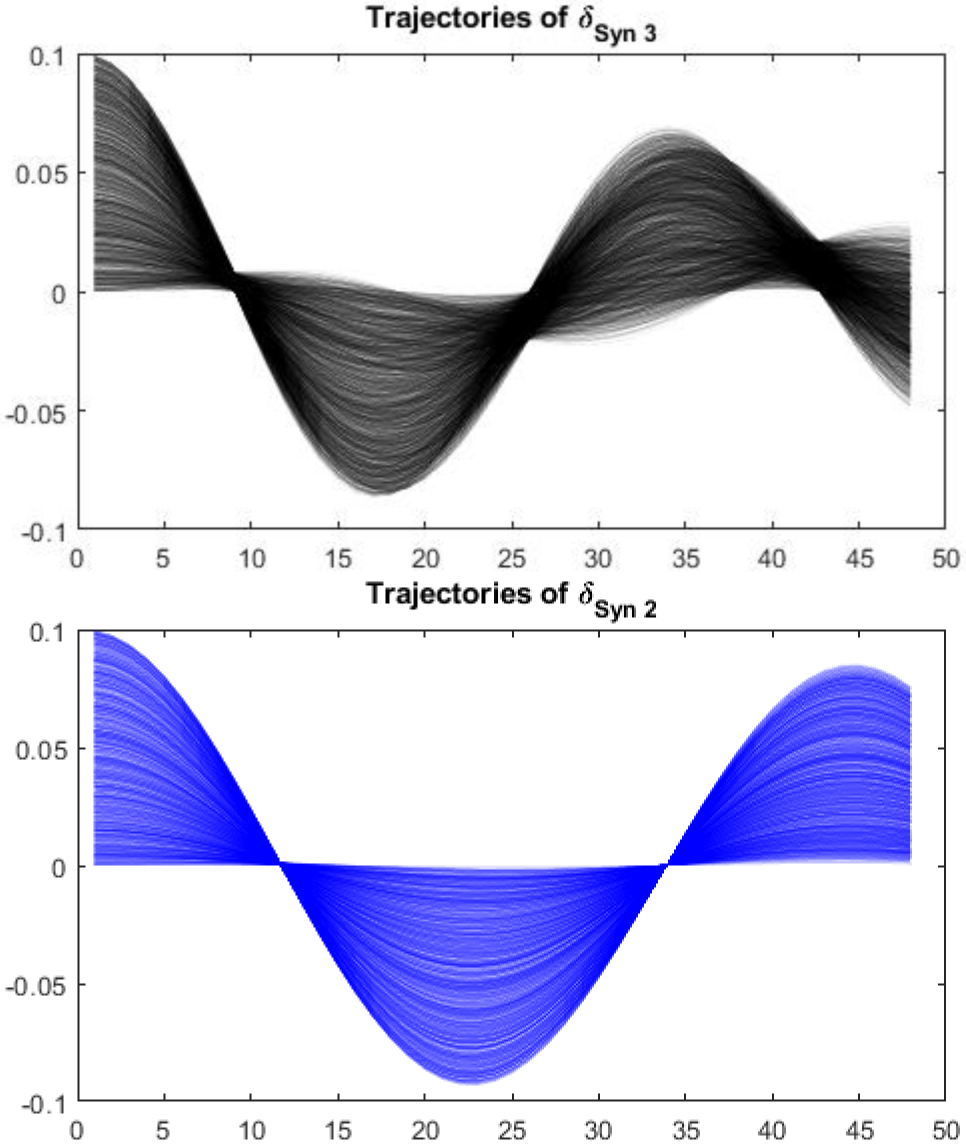
$x - axis\colon\ V_{r1_{AVR-3}};\quad y - axis\colon V_{f_{AVR-3}}$

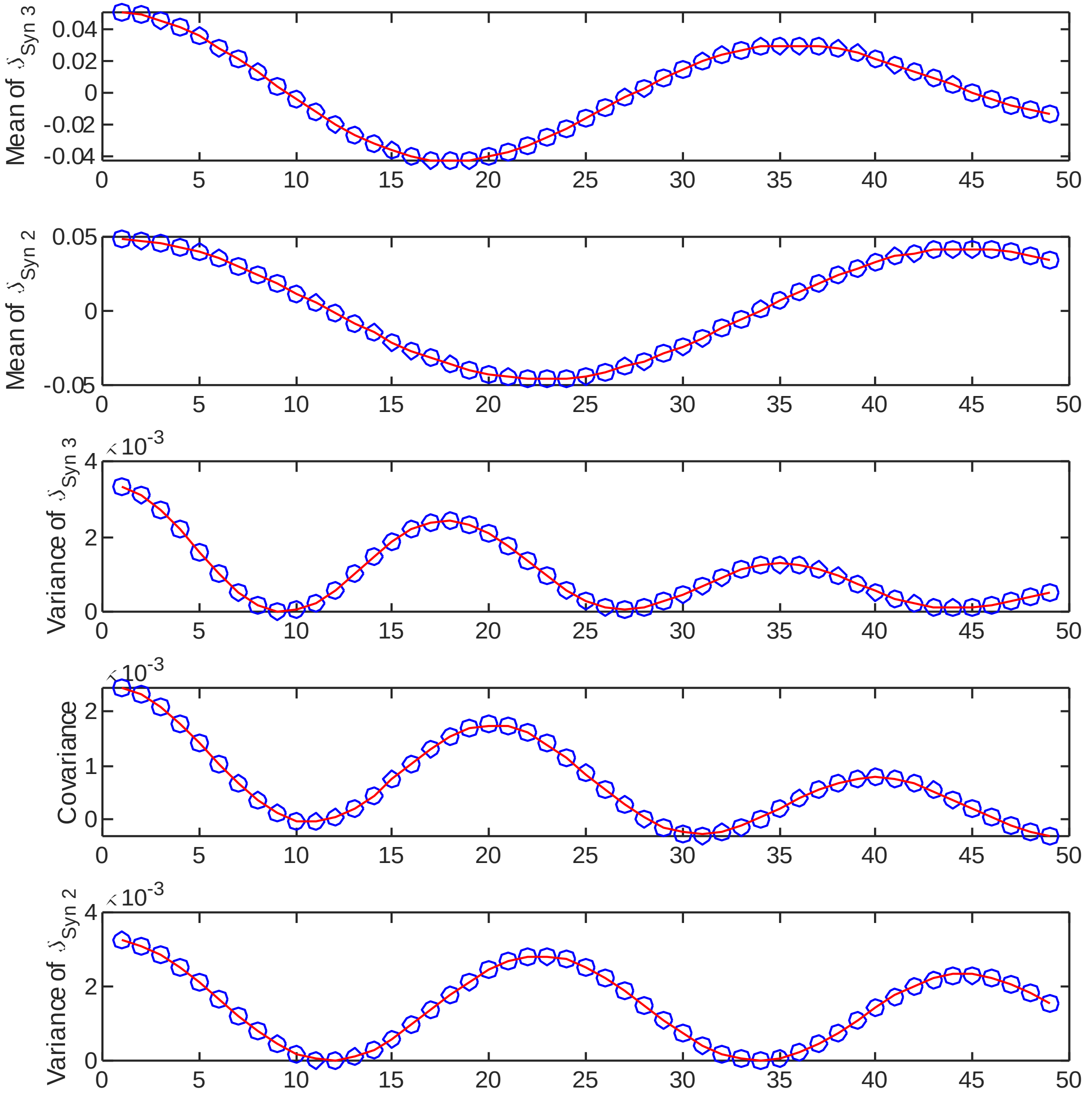


Scenarios considered

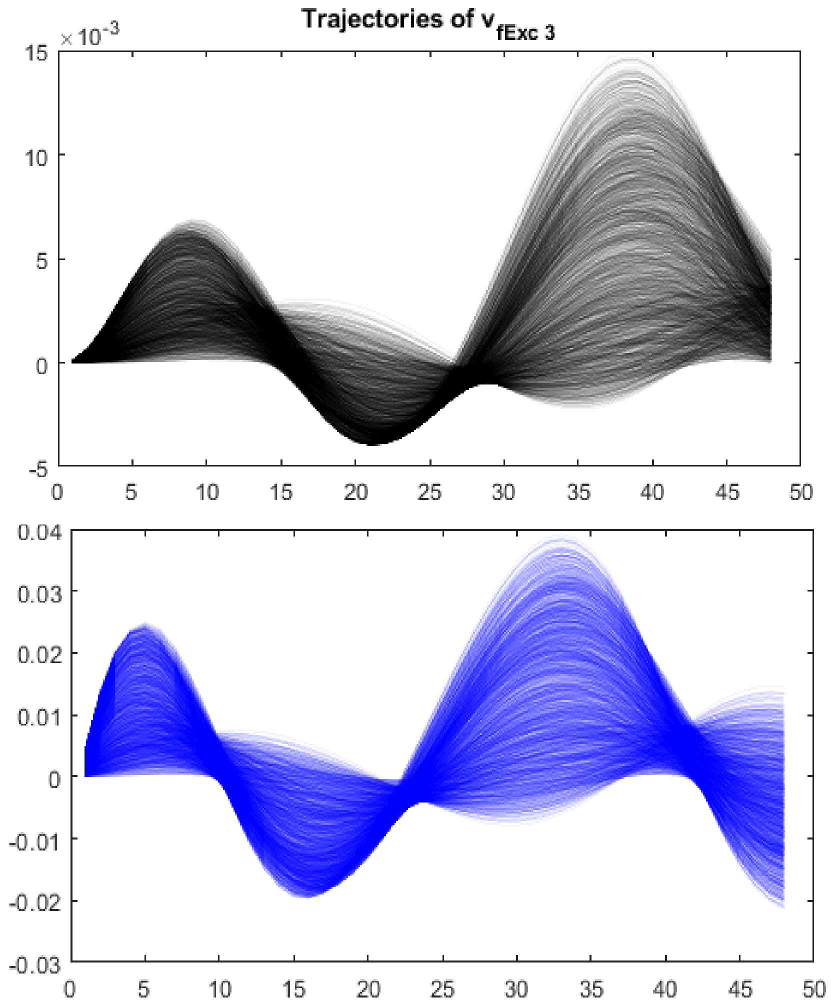
- Initial Gen 2 & 3 rotor angle states have assumed to intial uncertainty
- Multiple uncertainty sets are considered
  - Scenario – 1: Uniform PDF in a circle
  - Scenario – 2: Uniform PDF in a square
  - Scenario – 3: non-uniform PDF in a non-convex shape
- The dictionary functions are monomials with degree up to 2 – total 378 func.
- The Koopman matrix is estimated using 500 random initial conditions with 50 time-steps of 0.02 seconds

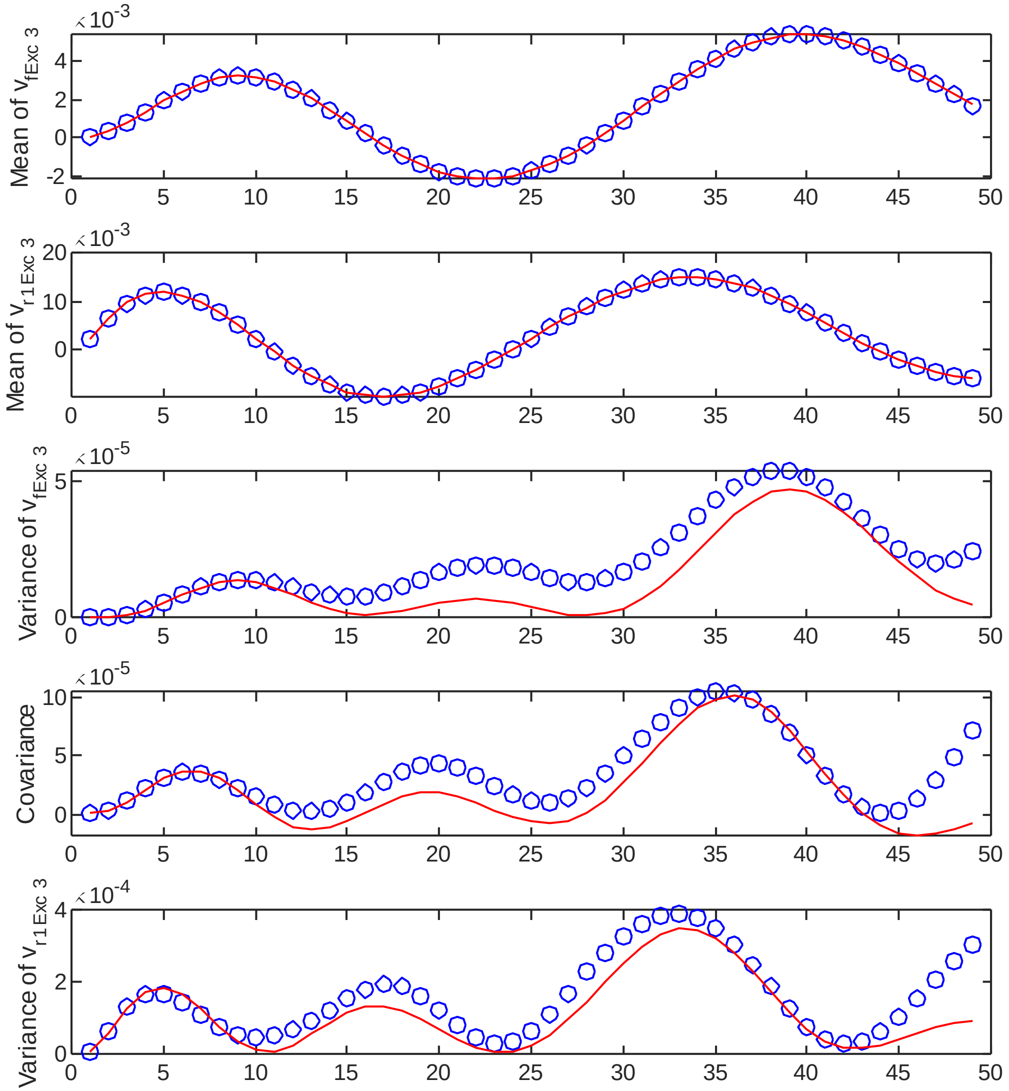
Scenario -2: Behavior of Gen 2 & 3 rotor angle





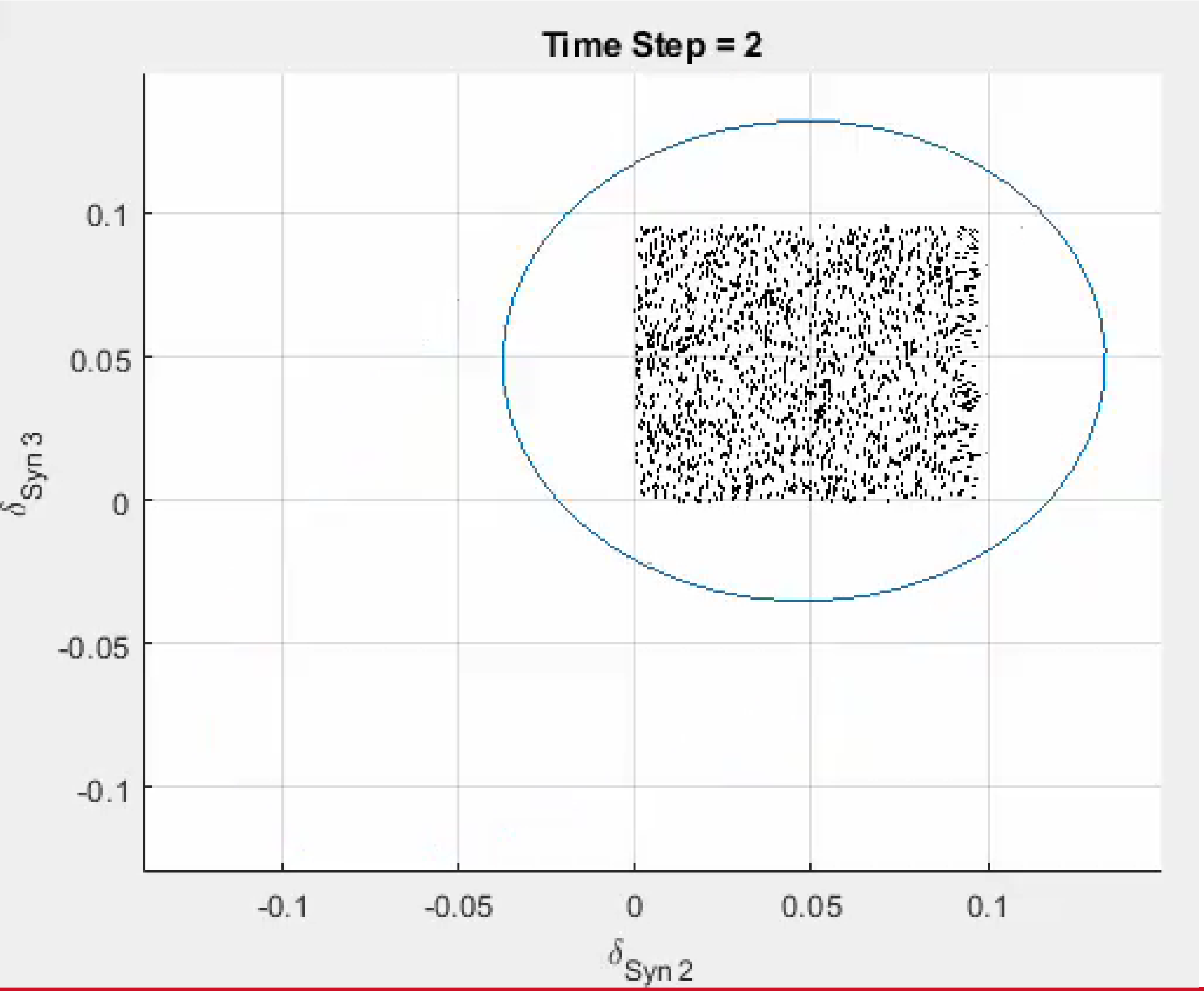
Scenario -2: Behavior of States of AVR at Gen-3



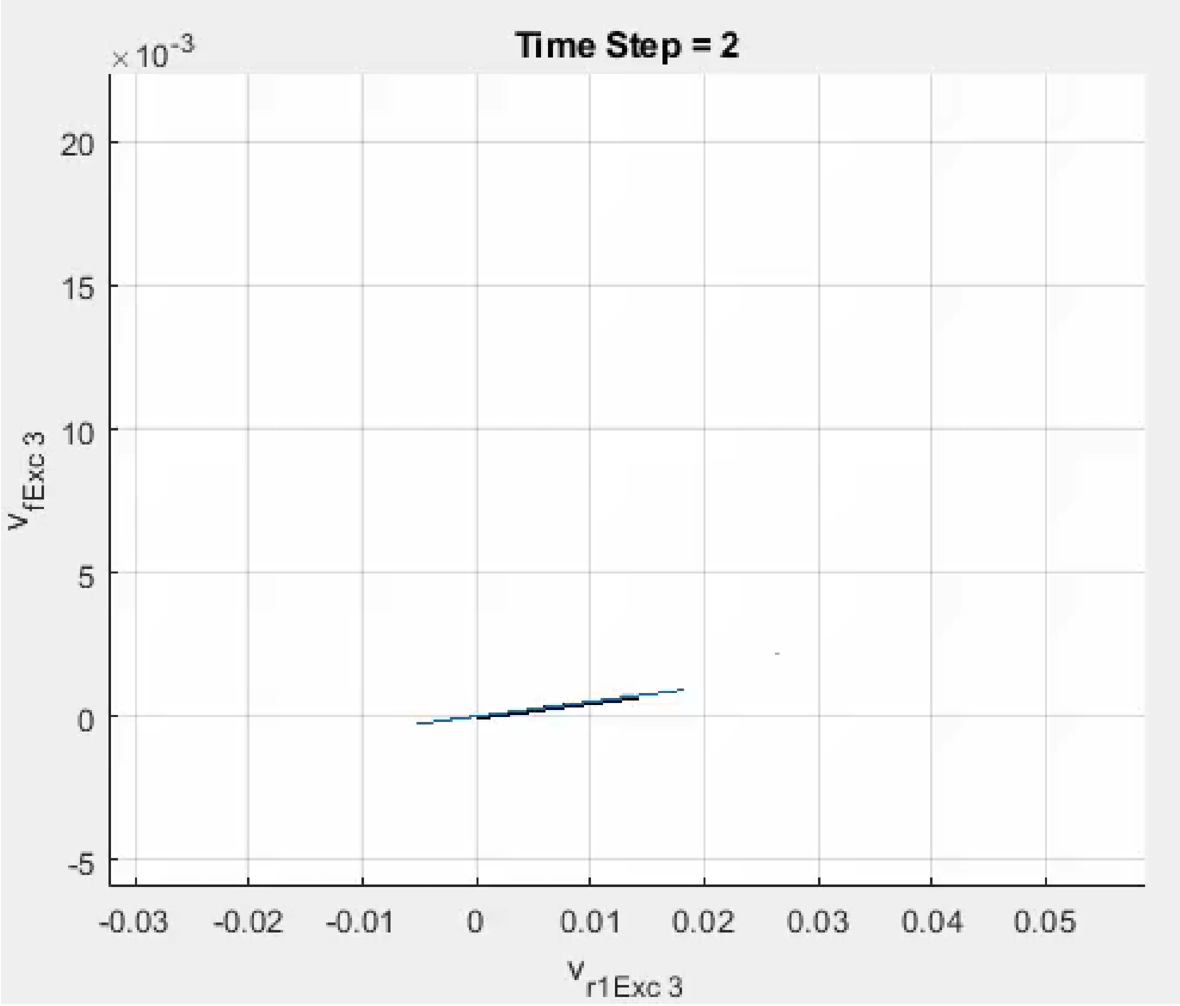


Scenario -2: Bounding Region using Est. Moments

$x - axis\text{: } \delta_{gen-2}\text{;} \quad y - axis\text{: } \delta_{gen-3}$



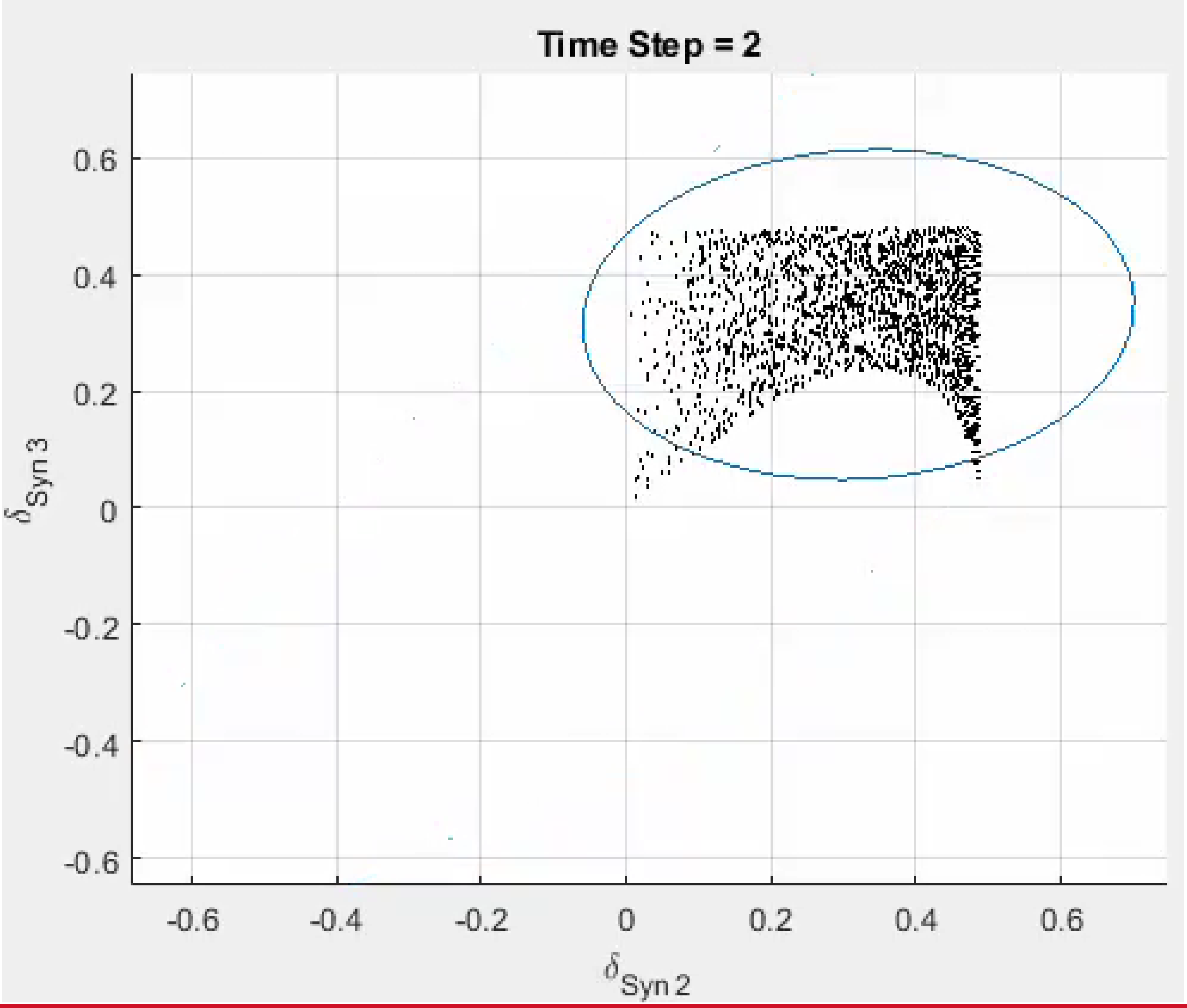
$x - axis\text{: } V_{r1_{AVR-3}}\text{;} \quad y - axis\text{:} V_{f_{AVR-3}}$

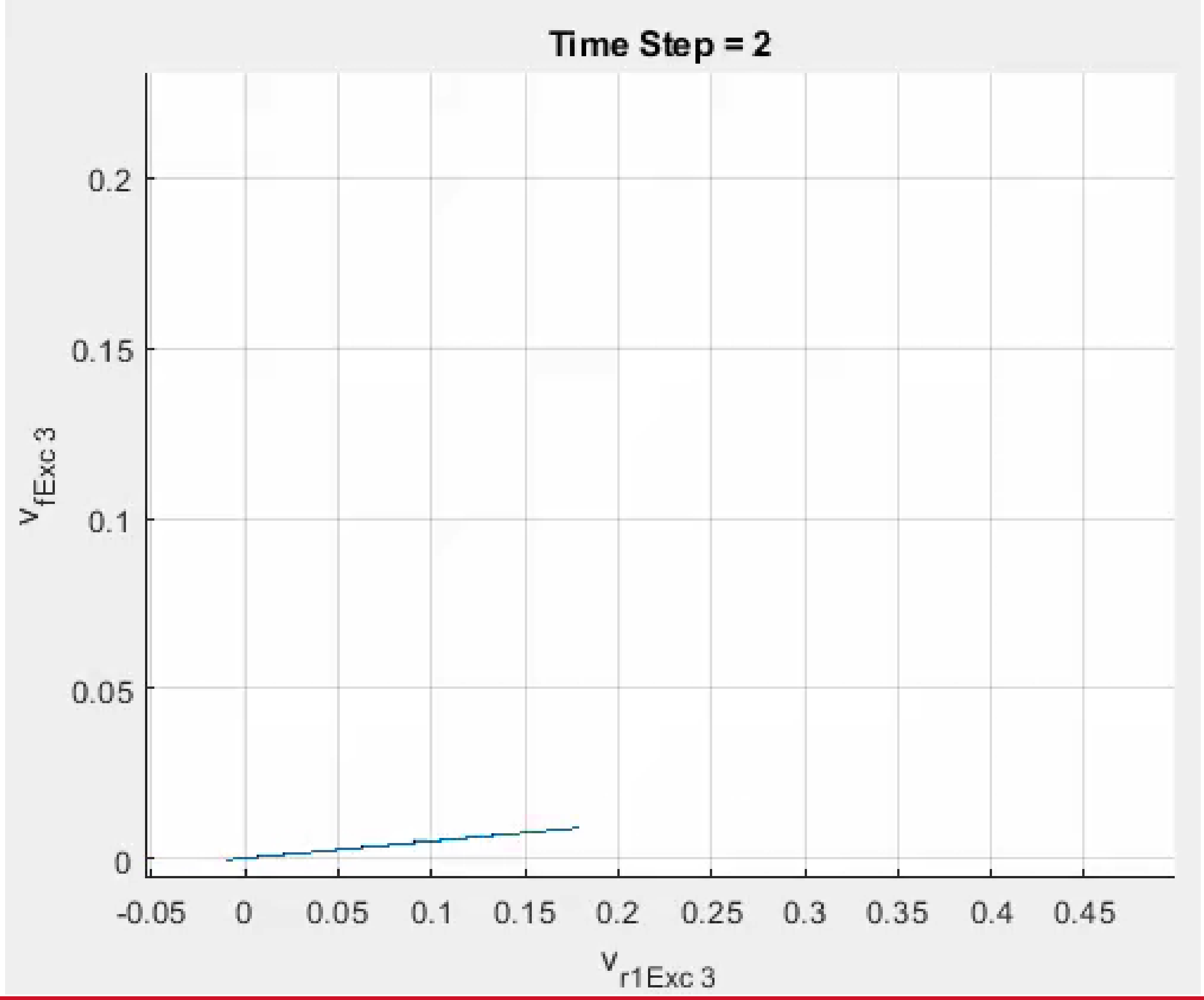


Scenarios considered

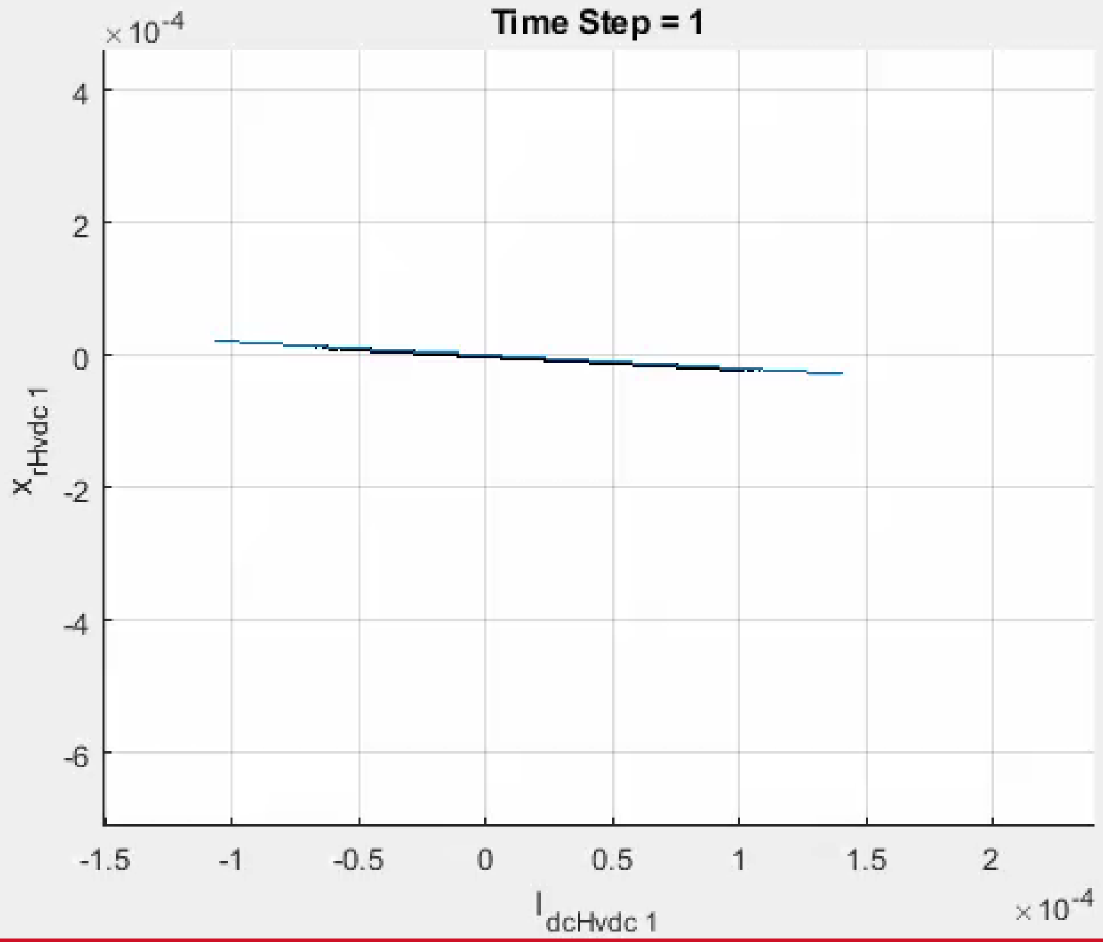
- Initial Gen 2 & 3 rotor angle states have assumed to intial uncertainty
- Multiple uncertainty sets are considered
  - Scenario – 1: Uniform PDF in a circle
  - Scenario – 2: Uniform PDF in a square
  - Scenario – 3: non-uniform PDF in a non-convex shape
- The dictionary functions are monomials with degree up to 2 – total 378 func.
- The Koopman matrix is estimated using 500 random initial conditions with 50 time-steps of 0.02 seconds

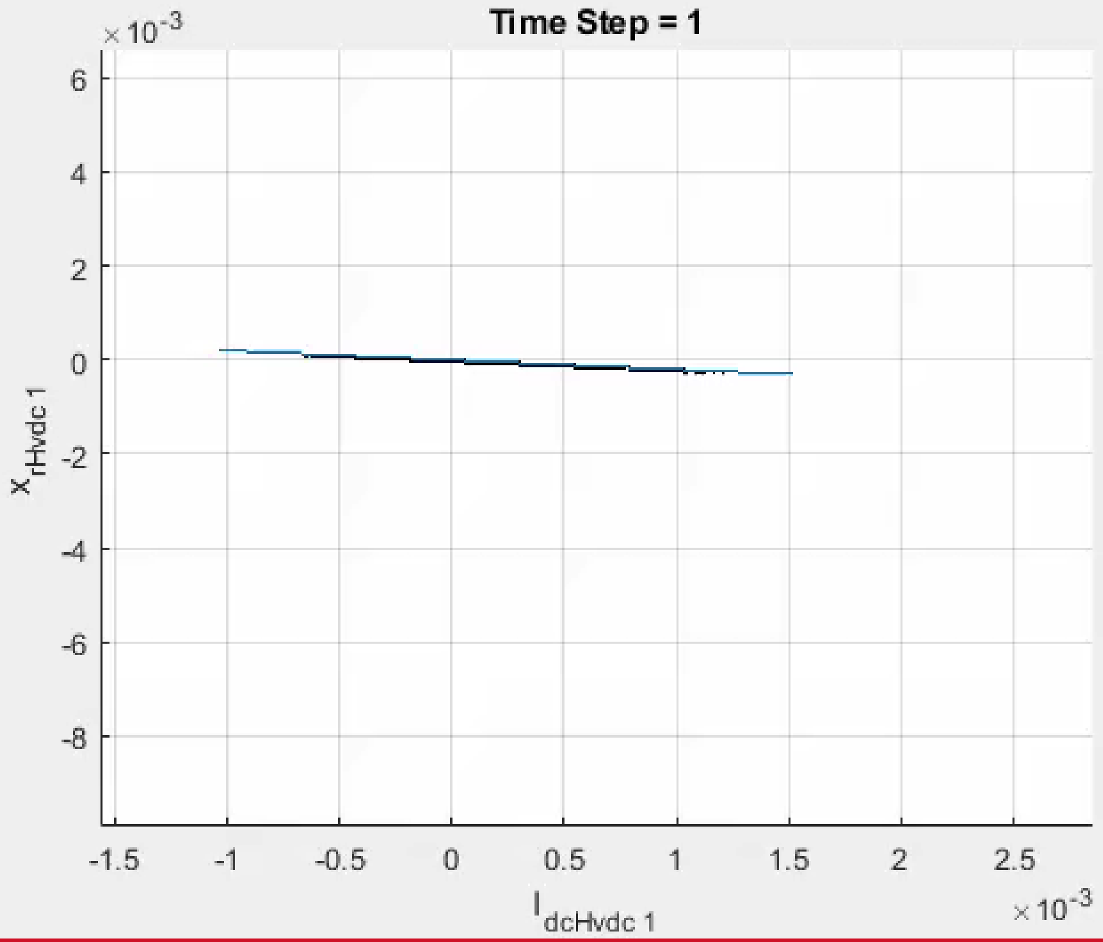
Scenario -3: Bounding Region using Est. Moments





Scenarios 2 & 3: Bounding Region using Est. Moments for two HVDC states





Reachability

- Reachability analysis is concerned with computing rigorous approximations of the set of states reachable by a dynamical system.
- In the previous results, a gaussian assumption is used to get the ellipsoid of each pair of states at 3σ standard deviation
- A different approach is to use compactly supported dictionary functions such as radial basis functions – they can be used as indicator functions

Results – Dubin Car

- Characterize the reachable set in a 3-D system where v and ω are controls given by a feedback law

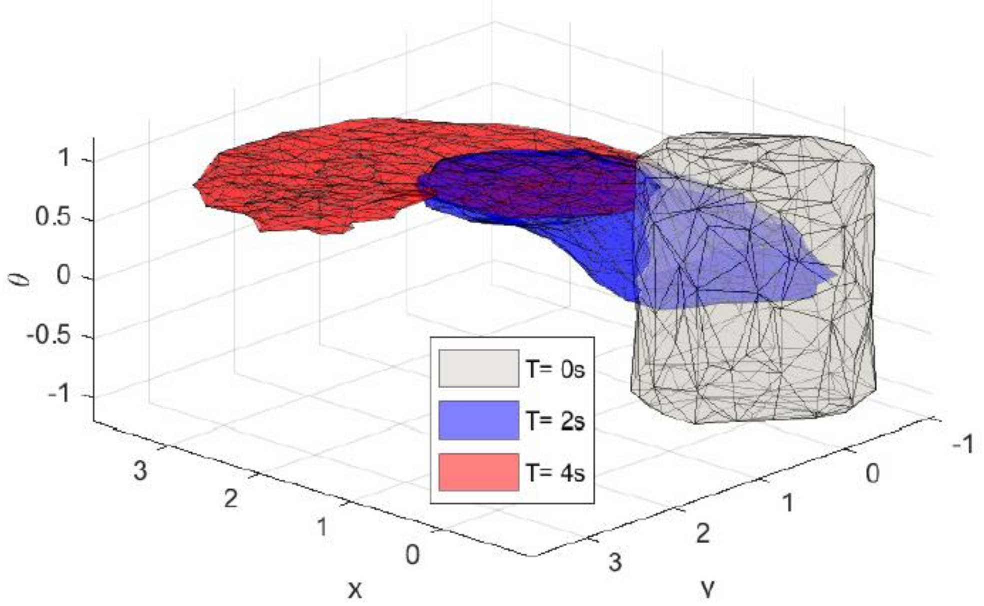
$$\begin{aligned}\dot{x} &= \nu\cos\theta \\ \dot{y} &= \nu\sin\theta \\ \dot{\theta} &= \omega\end{aligned}$$

$$\begin{bmatrix}\nu \\ \omega\end{bmatrix} = \begin{bmatrix}\nu_{dx}cos(\theta) + \nu_{dy}sin(\theta) \\ \frac{1}{b}(\nu_{dy}cos(\theta) - \nu_{dx}sin(\theta))\end{bmatrix}$$

- The region of interest is $(x_1, x_2, \theta) \in (-4,6) \times (-4,6) \times (-1.5,1.5)$
- For performing reachability analysis, 1-D Gaussian radial basis functions (RBFs) are used as dictionary functions with their centers equally spaced on the individual axis in the domain of interest
- We used 12 RBFs with their centers equally spaced along each axis (a total of 36 dictionary functions)

Results – Dubin Car (cont.)

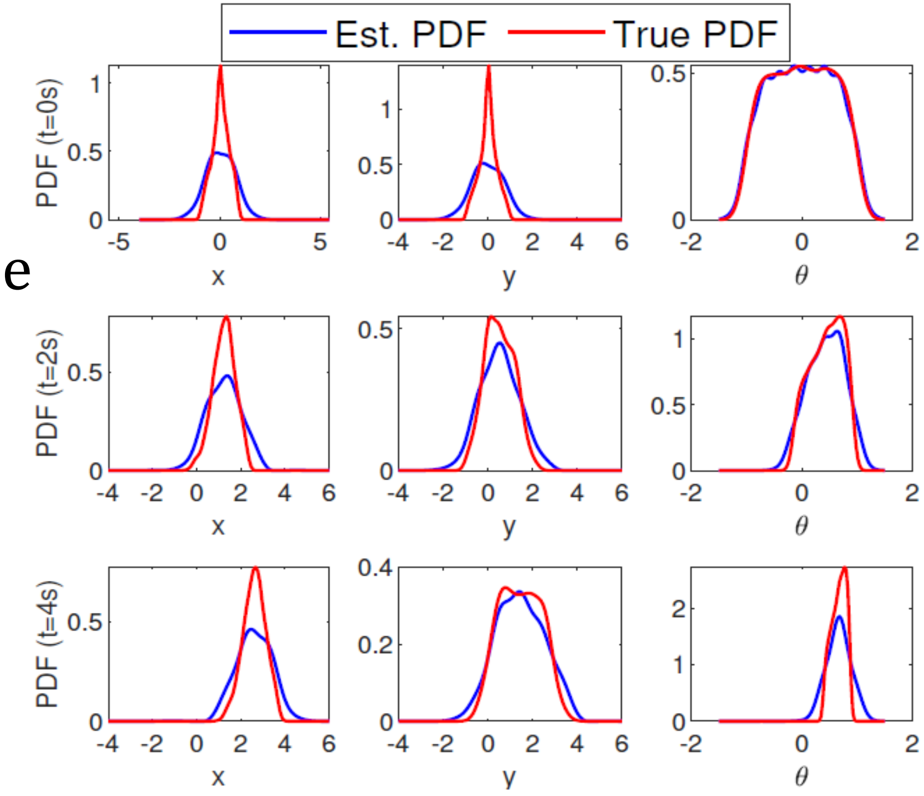
- Initial uncertainty set of a cylinder along the θ axis with a height of 2, a radius of 1 and centered at the origin
- The initial uncertainty set along with the reachable sets at $t = 2s$ & $t = 4s$ are plotted below



- Variation of θ decreases with time, variation of y increases with time and the variation of x is more or less constant with time

Results – Dubin Car (cont.)

- The pdf of the uncertainty set can be estimated from the moments as the dictionary functions are radial basis functions with less intersecting support
- The support of the estimated PDF always contains the support of the true PDF
- Thus, it seems to provide a conservative estimate of the true variation of the respective state – no correlations for now
- Comparing MC and Koopman – time acceleration > 100x
- Next steps are to extend this approach to power systems



A. R. Ramapuram Matavalam; U. Vaidya; V. Ajjarapu, "Data-Driven Approach for Uncertainty Propagation and Reachability Analysis in Dynamical Systems", Proceedings of American Control Conference 2020

Conclusion and Future Directions

- First use of linear operator theoretic framework for uncertainty propagation and reachability analysis in a dynamical system
- The proposed framework provides a systematic method to **exploit offline simulations/data to perform uncertainty propagation**
- Further, the **linearity** implies that the computation time for uncertainty propagation is much faster than MC and PC based methods (>100x-20x)
- For power systems, the generator states seem to be more 'linear' than the excitation controls and HVDC behavior
- We are in the process of extending the method to larger dimensional systems
- **Scalability remains a challenge – data required to learn K for large systems seems to increase exponentially**

Relevant Papers

- **A. R. Ramapuram Matavalam**; U. Vaidya; V. Ajjarapu, "Data-Driven Approach for Uncertainty Propagation and Reachability Analysis in Dynamical Systems", Proceedings of American Control Conference 2020
- **A. R. Ramapuram Matavalam**; U. Vaidya; V. Ajjarapu, "Propagating Uncertainty in Power System Initial Conditions using Data-Driven Linear Operators", to be Submitted to IEEE Power and Energy System Letters

Thanks for you attention!
Questions?

amar@iastate.edu