Network-Cognizant Time-Coupled Aggregate Flexibility of Distribution Systems Under Uncertainties

Bai Cui
National Renewable Energy Laboratory
in collaboration with
Ahmed Zamzam and Andrey Bernstein (NREL)

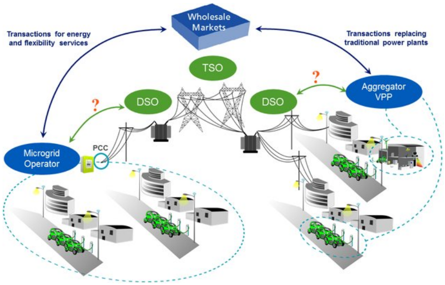
CDC Workshop on Uncertainty Management in Power System Dynamics
December 2021

Outline

Outline

Background

- Distribution systems are transitioning from passive grid elements to active virtual power plants (VPPs).
- With FERC Order 2222, regulatory barrier preventing DERs from participating in grid service is lifted.



- **Question:** As a VPP, what is the "generation capability" of a distribution system? In other words, what is the power potential of a distribution system in providing grid service?

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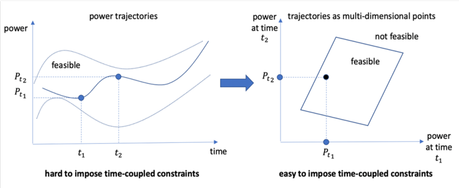
NEWS RELEASES

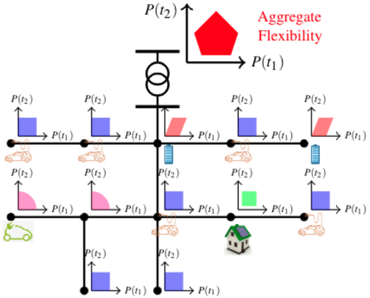
FERC Opens Wholesale Markets to Distributed Resources: Landmark Action Breaks Down Barriers to Emerging Technologies, Boosts Competition

September 17, 2020

Background

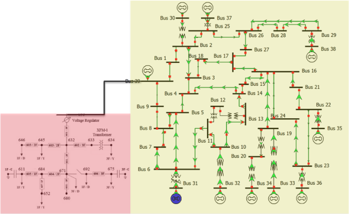
Aggregate flexibility: the *power generation shaping capability* of a collection of heterogenous DERs located on a feeder, or geographically distributed across multiple feeders over several hours to a day.



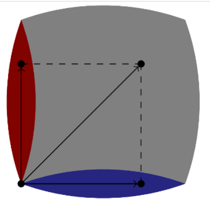


Background: Existing Approaches

- T&D co-simulation/co-optimization
  - High computational complexity
  - Requires continuous comm. between T&D
- Device-level aggregation
  - System agnostic
  - Requires homogeneity of DERs



- We need a new approach that:
  - is network-cognizant
  - is DER heterogeneity-compatible
  - is scalable
  - captures temporal correlations



Outline

DER Models with Time Coupling

Energy storage
$$\begin{cases} \underline{P}_{k,\phi}^{(e)} \le p_{k,\phi}^{(e)}(t) \le \bar{P}_{k,\phi}^{(e)}, \\ e_{k,\phi}(t) = \kappa_k e_{k,\phi}(t-1) - \tau p_{k,\phi}^{(e)}(t), \\ \underline{e}_{k,\phi} \le e_{k,\phi}(t) \le E_{k,\phi}, \end{cases}$$

HVAC system
$$\begin{cases} 0 \le p_{k,\phi}^{(h)}(t) \le \bar{P}_{k,\phi}^{(h)}(t), \\ \underline{H}_k \le H_k^{\mathrm{in}}(t) \le \bar{H}_k, \\ H_k^{\mathrm{in}}(t) = H_k^{\mathrm{in}}(t-1) + \alpha_k (H_k^{\mathrm{out}}(t) - H_k^{\mathrm{in}}(t-1)) + \tau \beta_k p_k(t)^{(h)}(t), \end{cases}$$

PV inverter $\quad 0 \le p_{k,\phi}^{(pv)}(t) \le \bar{P}_{k,\phi}^{(pv)}(t).$

Aggregate Flexibility: Model

Devices

Energy storage units, PV inverters, HVAC systems

Aggregate Flexibility: Model

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Energy storage units, PV inverters, HVAC systems

Network Model

$$
\begin{aligned}
&\mathbf{v} = \mathbf{M}_{\Delta}^{(p)}\mathbf{p}_{\Delta} + \mathbf{M}_{\Delta}^{(q)}\mathbf{q}_{\Delta} + \mathbf{M}_{Y}^{(p)}\mathbf{p}_{Y} + \mathbf{M}_{Y}^{(q)}\mathbf{q}_{Y} + \tilde{\mathbf{v}} \\
&\underline{\mathbf{v}} \leq \mathbf{v} \leq \bar{\mathbf{v}} \\
&\mathbf{p}_0 = \mathbf{G}_{\Delta}^{(p)}\mathbf{p}_{\Delta} + \mathbf{G}_{\Delta}^{(q)}\mathbf{q}_{\Delta} + \mathbf{G}_{Y}^{(p)}\mathbf{p}_{Y} + \mathbf{G}_{Y}^{(q)}\mathbf{q}_{Y} + \mathbf{c}
\end{aligned}
$$

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\end{aligned}
$$

Compact Model with Uncertainty

$$
\begin{aligned}
\mathbf{W}\mathbf{p} &\leq \mathbf{z}(\zeta) \\
\mathbf{p}_0 &= \mathbf{D}\mathbf{p} + \mathbf{b}(\zeta)
\end{aligned} \tag{$*$}
$$

Outline

Motivation for Ellipsoidal Inner Approximation

$$\begin{aligned} \mathbf{W}\mathbf{p} &\leq \mathbf{z}(\zeta) \\ \mathbf{p}_0 &= \mathbf{D}\mathbf{p} + \mathbf{b}(\zeta) \end{aligned} \qquad (*)$$

Goal

With full controllability of DERs downstream, derive the aggregate power flexibility at the feeder level ($\mathcal{F} = \{p_0 : \exists(p_0, p(p_0))$ that satisfies $(*)$ for all $\zeta\}$).

Motivation for Ellipsoidal Inner Approximation

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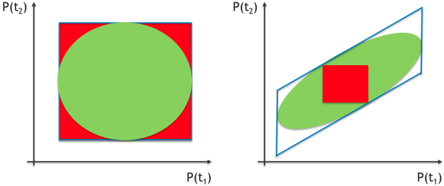

Without time coupling
With time coupling

Motivation for Ellipsoidal Inner Approximation

$$\begin{aligned} \mathbf{Wp} &\leq \mathbf{z}(\zeta) \\ \mathbf{p}_0 &= \mathbf{Dp} + \mathbf{b}(\zeta) \end{aligned} \quad (*)$$

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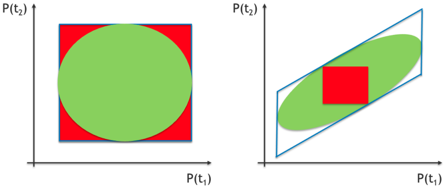

Without time coupling

With time coupling

Challenge

Finding the maximum volume inscribed ellipsoidal in a polytopic projection is nontrivial computationally.

Outline

Ellipsoidal Inner Approximation: Adaptive Robust Optimization

We want to identify the maximum volume inscribing ellipsoid (MVE)

$$\mathcal{E} = \{\mathbf{p}_0 : \ \mathbf{p}_0 = \mathbf{E}\boldsymbol{\xi} + \mathbf{e}, \|\boldsymbol{\xi}\| \leq 1\} \subset \mathbb{R}^T$$

parameterized by $\mathbf{E} \succeq 0$ and $\mathbf{e} \in \mathbb{R}^T$ such that for any $\mathbf{p}_0 \in \mathcal{E}$ and any $\|\boldsymbol{\zeta}_t\| \leq 1, \forall t$, there exists $\mathbf{p}(\mathbf{p}_0, \boldsymbol{\zeta})$ such that system constraints are satisfied.

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Initial Formulation

$$\max_{\mathbf{e}, \mathbf{E} \succeq 0} \left\{ \log \det \mathbf{E} : \ \forall \|\boldsymbol{\xi}\| \leq 1, \forall \|\boldsymbol{\zeta}_t\| \leq 1, \exists \mathbf{p} \text{ s.t. } \ \mathbf{E}\boldsymbol{\xi} + \mathbf{e} = \mathbf{D}\mathbf{p} + \mathbf{b}(\boldsymbol{\zeta}), \mathbf{W}\mathbf{p} \leq \mathbf{z}(\boldsymbol{\zeta}) \right\},$$

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Proposition

The problem can be equivalently formulated as

$$\max_{\mathbf{e}, \mathbf{E} \succeq 0} \left\{ \log\det \mathbf{E} : \ \forall \|\boldsymbol{\xi}\| \le 1, \forall \mathbf{u} \in \mathcal{U}, \exists \mathbf{y} \text{ s.t. } \mathbf{W}_1 \tilde{\mathbf{D}}^{-1}(\mathbf{E}\boldsymbol{\xi} + \mathbf{e}) + \mathbf{W}_2 \mathbf{y} \le \mathbf{u} \right\},$$

where the vector of individual DER control variables $\mathbf{p}$ can be recovered as $\mathbf{p} = \mathbf{B}_1 \tilde{\mathbf{D}}^{-1}(\mathbf{p}_0 - \mathbf{b}(\boldsymbol{\zeta})) + \mathbf{B}_2 \mathbf{y}$ given $\mathbf{p}_0$, $\mathbf{y}$, and $\boldsymbol{\zeta}$.

Policy-Based Reformulation / Safe Approximation

Quadratic and affine policies

Quadratic policy:

$$\mathbf{y} = \begin{Bmatrix} \vdots \\ \boldsymbol{\eta}^\top \mathbf{Q}_i \boldsymbol{\eta} \\ \vdots \end{Bmatrix} + \mathbf{L}\boldsymbol{\eta} + \mathbf{c},$$

where $\boldsymbol{\eta} := \begin{bmatrix} \xi \\ \zeta \end{bmatrix}$. Affine policy:

$$\mathbf{y} = \mathbf{K}\boldsymbol{\xi} + \sum_{t=1}^{T} \mathbf{L}_t \boldsymbol{\zeta}_t + \gamma,$$

Policy-Based Formulations

- Quadratic policy: SDP inner approximation with tightness factor at most $9.19\sqrt{\ln(T+1)}$.
- Affine policy: exact SOCP reformulation.

Policy-Based Safe Approximations

Quadratic policy-based approximation

Let $\boldsymbol{\rho}^i := ([\mathbf{w}_{1,i}\tilde{\mathbf{D}}^{-1}\mathbf{E},\ -\boldsymbol{\theta}_i] + \mathbf{w}_{2,i}\mathbf{L})/2$ and:

$$\mathbf{J}_i := \begin{bmatrix} \lambda_{1,i}\mathbf{I}_T & \mathbb{0} & \cdots & \mathbb{0} \\ \mathbb{0} & \lambda_{2,i}\mathbf{I}_{N_u} & \ddots & \vdots \\ \vdots & & \ddots & \mathbb{0} \\ \mathbb{0} & \cdots & & \lambda_{T+1,i}\mathbf{I}_{N_u} \end{bmatrix}$$

for $i = 1, \ldots, m$, then the problem can be approximated by

$$\begin{aligned} \max_{\substack{\mathbf{E}\succeq 0,\mathbf{e},\mathbf{c},\mathbf{L},\\ \{\mathbf{Q}_i\}_{i=1}^m,\{\lambda_{k,i}\}_{k=0,i=1}^{T+1,m}}} \quad & \log\det \mathbf{E} \\ \text{s.t.} \quad & (\forall i = 1, \ldots, m) \\ & \sum_{k=0}^{T+1} \lambda_{k,i} + \mathbf{w}_{1,i}\tilde{\mathbf{D}}^{-1}\mathbf{e} + \mathbf{w}_{2,i}\mathbf{c} - \nu_i \le 0, \\ & \begin{bmatrix} \lambda_{0,i} & \mathbb{0} \\ \mathbb{0} & \mathbf{J}_i \end{bmatrix} \succeq \begin{bmatrix} 0 & \boldsymbol{\rho}^i \\ (\boldsymbol{\rho}^i)^\top & \sum_{j=1}^{(n-1)T} W_{2,ij}\mathbf{Q}_j \end{bmatrix}, \\ & \lambda_{k,i} \ge 0, \qquad k = 0, \ldots, T+1 \end{aligned}$$

Affine policy-based approximation

$$\begin{aligned} \max_{\mathbf{E}\succeq 0,\mathbf{e},\mathbf{K},\{\mathbf{L}_t\}_{t=1}^T,\boldsymbol{\gamma},\{\alpha_i\}_{i=1}^m} \quad & \log\det \mathbf{E} \\ \text{s.t.} \quad & (\forall i = 1, \ldots, m) \\ & \|\mathbf{w}_{1,i}\tilde{\mathbf{D}}^{-1}\mathbf{E} + \mathbf{w}_{2,i}\mathbf{K}\| + \sum_{k=1}^{T} \|\mathbf{w}_{2,i}\mathbf{L}_k - \boldsymbol{\theta}_{k,i}\| \le \alpha_i, \\ & \alpha_i + \mathbf{w}_{1,i}\tilde{\mathbf{D}}^{-1}\mathbf{e} + \mathbf{w}_{2,i}\boldsymbol{\gamma} - \nu_i \le 0, \end{aligned}$$

Policy-Based Safe Approximations

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Affine policy-based approximation

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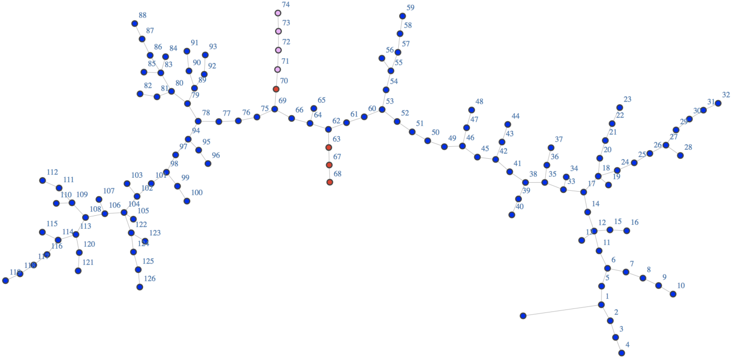
The two safe approximations are incomparable.

Outline

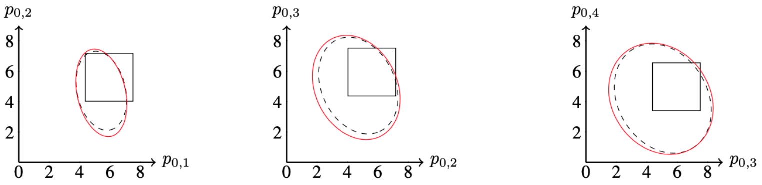
Simulation: Test System

Southern California Edison (SCE) 126-node three-phase distribution feeder.

- 27 energy storage units;
- 33 PV inverters;
- 5 HVAC systems.



Simulation Results



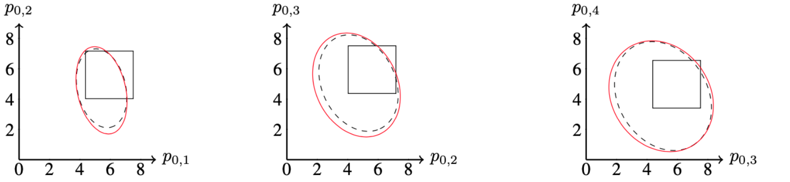
(a) Projected flexibility: 9:00–11:00. (b) Projected flexibility: 10:00–12:00. (c) Projected flexibility: 11:00–13:00.

Fig. 1: Projection of flexibility region approximations. (hyperbox: black solid line; affine policy: black dashed line; quadratic policy: red solid line.)

Table: Volumes of Flexibility Region Approximation

Method	Volume
Ellipsoid by quadratic policy	271.55
Ellipsoid by affine policy	217.57
Hyperbox approximation	96.88

Simulation Results



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Numerical simulations suggest the ellipsoidal approximations capture 75-80% of the true flexibility region.

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Conclusion

Summary

- Encapsulation of distribution system generation shaping capability by aggregate flexibility region.
- Ellipsoidal characterization of aggregate flexibility region to capture time-coupling DER flexibility.
- Tractable safe approximations of resulting ARO formulation based on quadratic and affine second-stage policies.

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- Encapsulation of distribution system generation shaping capability by aggregate flexibility region.
- Ellipsoidal characterization of aggregate flexibility region to capture time-coupling DER flexibility.
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Future directions

- Rigorous guarantees on constraint satisfaction
- Strategies for DER disaggregation

Thank you!
Questions?