

Probability Cheat Sheet

1. Basic Probability Concepts

- Definition of Probability: $P(A) = \text{Number of favorable outcomes} / \text{Total number of outcomes}$
- Complement Rule: $P(A') = 1 - P(A)$
- Addition Rule for Mutually Exclusive Events: $P(A \text{ or } B) = P(A) + P(B)$
- General Addition Rule: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
- Conditional Probability: $P(A | B) = P(A \text{ and } B) / P(B)$
- Multiplication Rule for Independent Events: $P(A \text{ and } B) = P(A) * P(B)$

2. Common Probability Distributions

- Binomial: $P(X = k) = C(n, k) * p^k * (1 - p)^{(n - k)}$, where $X \sim \text{Binomial}(n, p)$
- Poisson: $P(X = k) = (\text{lambda}^k * e^{(-\text{lambda})}) / k!$, where $X \sim \text{Poisson}(\text{lambda})$
- Normal: $f(x) = (1 / \sqrt{2 * \pi * \sigma^2}) * e^{-(x - \mu)^2 / (2 * \sigma^2)}$, where $X \sim N(\mu, \sigma^2)$

3. Bayes' Theorem and Applications

- Bayes' Theorem: $P(A | B) = [P(B | A) * P(A)] / P(B)$
- Application to Diagnostic Testing: $P(\text{Disease} | \text{Positive Test}) = [P(\text{Test Positive} | \text{Disease}) * P(\text{Disease})] / P(\text{Test Positive})$

4. Counting Principles

- Permutations: $P(n, r) = n! / (n - r)!$
- Combinations: $C(n, r) = n! / [r! * (n - r)!]$

5. Expected Value and Variance

- Expected Value (Discrete): $E(X) = \text{Sum of } x * P(x)$
- Variance: $\text{Var}(X) = E(X^2) - (E(X))^2$

6. Probability in Practice

- Law of Total Probability: $P(B) = \text{Sum of } P(B | A_i) * P(A_i)$
- Markov Inequality: $P(X \geq a) \leq E(X) / a$, for $a > 0$
- Chebyshev Inequality: $P(|X - \mu| \geq k * \sigma) \leq 1 / k^2$

7. Examples

- Example 1: Coin Toss Probability: What is the probability of getting at least 2 heads in 3 coin tosses?

Use the binomial formula: $P(X \geq 2) = P(X = 2) + P(X = 3)$

- Example 2: Bayes Theorem Application: If a test has 95 percent sensitivity and 90 percent specificity, and the disease prevalence is 1 percent, what is the probability that a person who tested positive actually has the disease?

Use Bayes theorem to compute: $P(\text{Disease} | \text{Positive Test})$

8. Common Pitfalls

- Ignoring Conditional Probability: Remember that $P(A | B)$ is not equal to $P(B | A)$.
- Misunderstanding Independence: Two events are independent if $P(A \text{ and } B) = P(A) * P(B)$. Verify this before assuming independence.
- Overlooking Complement Rule: Often, it is easier to calculate $P(A')$ and use $P(A) = 1 - P(A')$.
- Mixing Up Permutations and Combinations: Use permutations when order matters, combinations when it does not.