Introduction to Longitudinal Data Analysis

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Outline

What is Longitudinal Data?

- The defining characteristic of longitudinal studies is that individuals are measured repeatedly through time.
  - In contrast, cross-sectional studies involve collecting measurements on an individual only once.
- Concerned with mean change over time.
- Longitudinal studies allow examination of age and cohort effects.
  - Longitudinal studies can distinguish changes over time within individuals (aging effects) from differences among people in their baseline levels (cohort effects).

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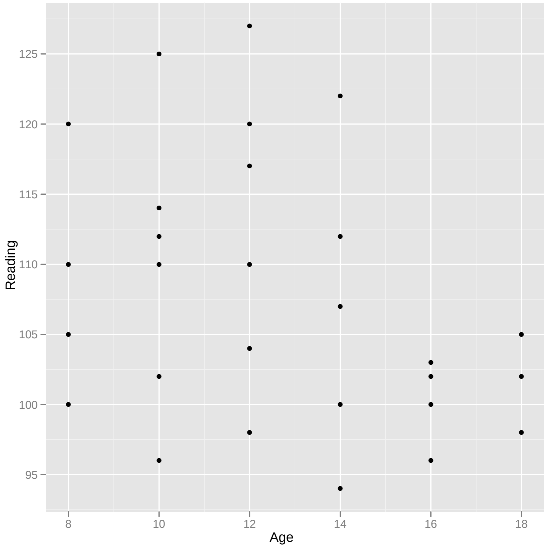
Age or cohort effects?



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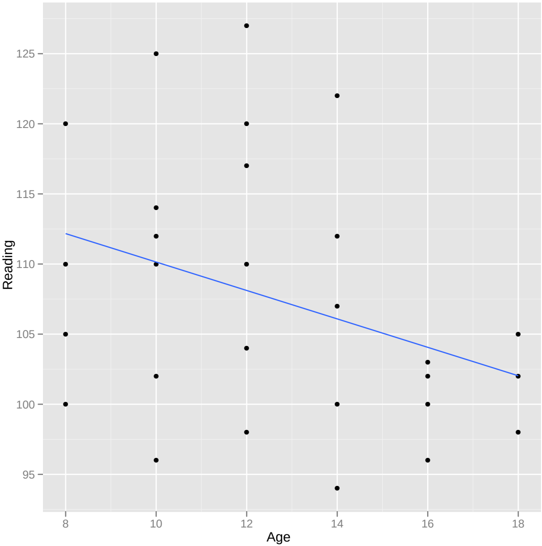

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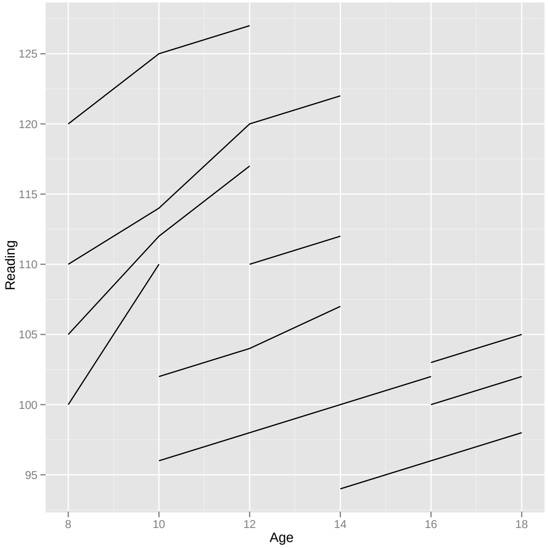

Benefits of Linear Mixed Models

- Fit intercepts and slopes for individuals
- Accounts for correlations between time points
- Accommodates missing data
- Accommodates measurements at unequal intervals
- Relatively few parameters required to account for dependency due to repeated measures
- Can handle both continuous and categorical covariates

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Correlations between time points

- Traditional regression does not account for this correlation
- This results in bogus standard error estimates and p-values.
- This is done in the covariance-variance matrix among the repeated measures known as $\mathbf{\Sigma_i}$
- Recall a correlation matrix is a standardized covariance-variance matrix.
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Accomodates missing data

- Missing data handled through robust maximum likelihood estimation
- No need for multiple imputation when using continuous variables
- Two types of Maximum Likelihood Estimates
  - Full Maximum Likelihood
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Other benefits

- Individuals can be measured at different times and do not need to measured at all times.
  - Therefore, we do not need to throw out data or impute data on individuals.
  - We throw out cases not individuals.
- Fewer parameters used in these models than ANOVA and MANOVA models. This leads to higher statistical power.
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Other names for these models

- Hierarchical linear models (HLM)
  - HLM are generally cross-sectional and involve multiple levels, such as school effects
  - In longitudinal models, measurements over time are nested within individuals rather than a school
- Linear Mixed Models
- Multilevel Models
- Mixed Effects Models

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LMM formula

Recall a simple multiple regression model is:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + ... + \beta_p X_{pi} + e_i$$

In a LMM, a regression model measured j times might become:
$Y_{ij} = \beta_0 + b_{0i} + \beta_1 X_{1i} + (\beta_2 + b_{2i}) t_{ij} + e_{ij}$

LMM formula explained

Nomenclature	Definition
Y_{ij}	refers to the outcome variable, e.g. infant's weight, for individual i at time j
β_0	refers to the intercept
b_{0i}	refers to the random intercept for individual i. This allows individuals to vary in their initial values.
$\beta_1 X_{1i}$	the parameter estimate associated with a covariate, such as primary feeding type.
β_2	the fixed effect estimate associated with time
b_{2i}	refers to the random slope for individual i.This allows individuals to vary in their slopes.
t_{ij}	time variable, months since birth.
e_{ij}	is the residual error term.

Assumptions of LMMs

- Obseravations of individuals are independent
- Random effects are normally distributed
- Random residuals are normally distributed

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Running LMMs

$$Y_{ij} = \beta_0 + b_{0i} + (\beta_1 + b_{1i})t_{ij} + e_{ij}$$

- Use the {lme4} package in **R** and the *lmer* function.
- If you can write your model as a LMM then you can program it in **R**.
- Basic **R** Formula: $m1 < -lmer(Outcome \sim 1 + Time + (1 + month|ID), data = data))$

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Running LMMs

```
> library(lme4)
> data(sleepstudy)
> m1 <- lmer(Reaction ~ 1 + Days + (1 + Days|Subject), sleepstudy)
> summary(m1)

Linear mixed model fit by REML
Formula: Reaction ~ 1 + Days + (1 + Days | Subject)
   Data: sleepstudy
  AIC  BIC logLik deviance REMLdev
 1756 1775 -871.8     1752    1744
Random effects:
 Groups   Name        Variance Std.Dev. Corr
 Subject  (Intercept) 612.090  24.7405
          Days         35.072   5.9221  0.066
 Residual             654.941  25.5918
Number of obs: 180, groups: Subject, 18

Fixed effects:
            Estimate Std. Error t value
(Intercept)  251.405      6.825   36.84
Days          10.467      1.546    6.77

Correlation of Fixed Effects:
     (Intr)
Days -0.138
```

Getting Help with LMMs and R

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