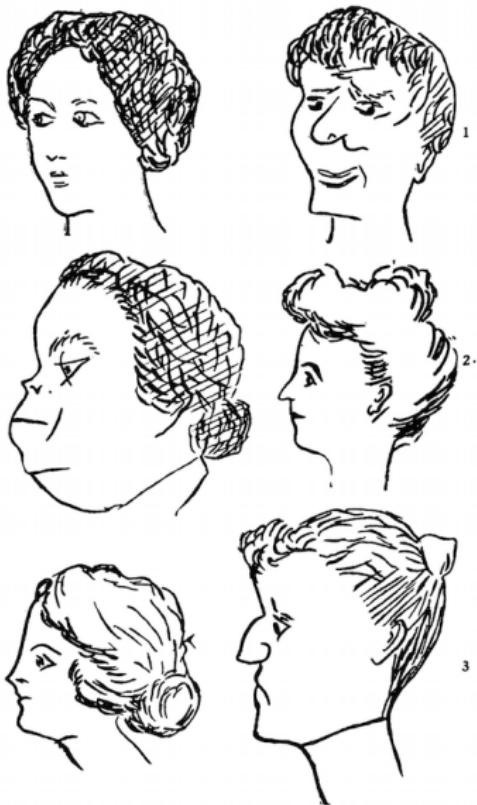


# E-411-PRMA

## Lecture 1

Christopher David Desjardins

17 August 2015



THE PSYCHOLOGICAL CLINIC is indebted for the loan of these cuts and those on p. 228 to the courtesy of Dr. Oliver P. Cornman, Associate Superintendent of Schools of Philadelphia, and Chairman of Committee on Backward Children Investigation. See Report of Committee, Dec. 31, 1910, appendix.



*Extreme desespoir.  
Eusserste Yerzweiflung*



*Colere meslée de Crainte.  
Zorn mit Furcht vermischt.*



achieve  
more®

# SAT®

## ► Topics

- ▶ Statistics, Classical Test Theory, Reliability, Validity, Item Response Theory, Generalizability Theory, Equating, and assessments/issues specific to various fields

## ► Assessments

- ▶ R computer assignments (30%)
- ▶ Item writing activity (5%)
- ▶ Midterm exam (25%)
- ▶ **Final exam (50%)**

The screenshot shows the RStudio IDE. The left pane contains an R script editor with the following code:

```
140 # view all data
141 precol
142
143 # If you want to see a specific observation, say #20, and you want to see responses from
144 # variables 2 through 3
145 precol[20,2:3]
146
147 # Or if you want to see variables 1, 3, and 6 for observations 19 through 21.
148 precol[19:21, c(1,3,6)]
149
150 # Don't worry R won't be so bad ;
151
152 # To get information about the data set and the variable types
153 str(precol)
154
155 # To get the variable labels
156 attr(precol, "variable.labels")
1:1 (Top Level) : R Script
```

The right pane shows the Environment tab with the message "Environment is empty". Below it is a search bar and a help panel for the `separate` function.

**Console (~)**

```
Copyright (C) 2015 The R Foundation for Statistical Computing
Platform: x86_64-apple-darwin13.4.0 (64-bit)

R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
Type 'license()' or 'licence()' for distribution details.

Natural language support but running in an English locale

R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.
```

>

**Help**

R: Separate one column into multiple columns.

**Description**

Given either regular expression or a vector of character positions, `separate()` turns a single character column into multiple columns.

**Usage**

```
separate(data, col, into, sep = "[[:alnum:]]+", remove = TRUE,
         convert = FALSE, extra = "error", ...)
```

**Arguments**

<code>data</code>	A data frame.
<code>col</code>	Bare column name.
<code>into</code>	Names of new variables to create as character vector.
<code>sep</code>	Separator between columns.

R: <https://www.r-project.org>  
RStudio: <https://www.rstudio.com>

# Why should I learn R?

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- ▶ High quality graphics
- ▶ Everyone is doing it
- ▶ Steep learning curve
  - ▶ Will provide nearly all the code
- ▶ No SPSS in this class

# Resources for R

- ▶ Icelandic resources

<http://kennslubanki.hi.is/search/efni/r>

<http://kennslubanki.hi.is/tolfraedi/myndbond/rrstudio-inngangur>

<http://kennslubanki.hi.is/tolfraedi/myndbond/rrstudio-fyrstu-skrefin>

- ▶ Please watch the last two videos before next class
- ▶ Please install R and RStudio before next class
- ▶ Next class will be an R workshop

# Concepts

What is **measurement**?

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**Nominal**

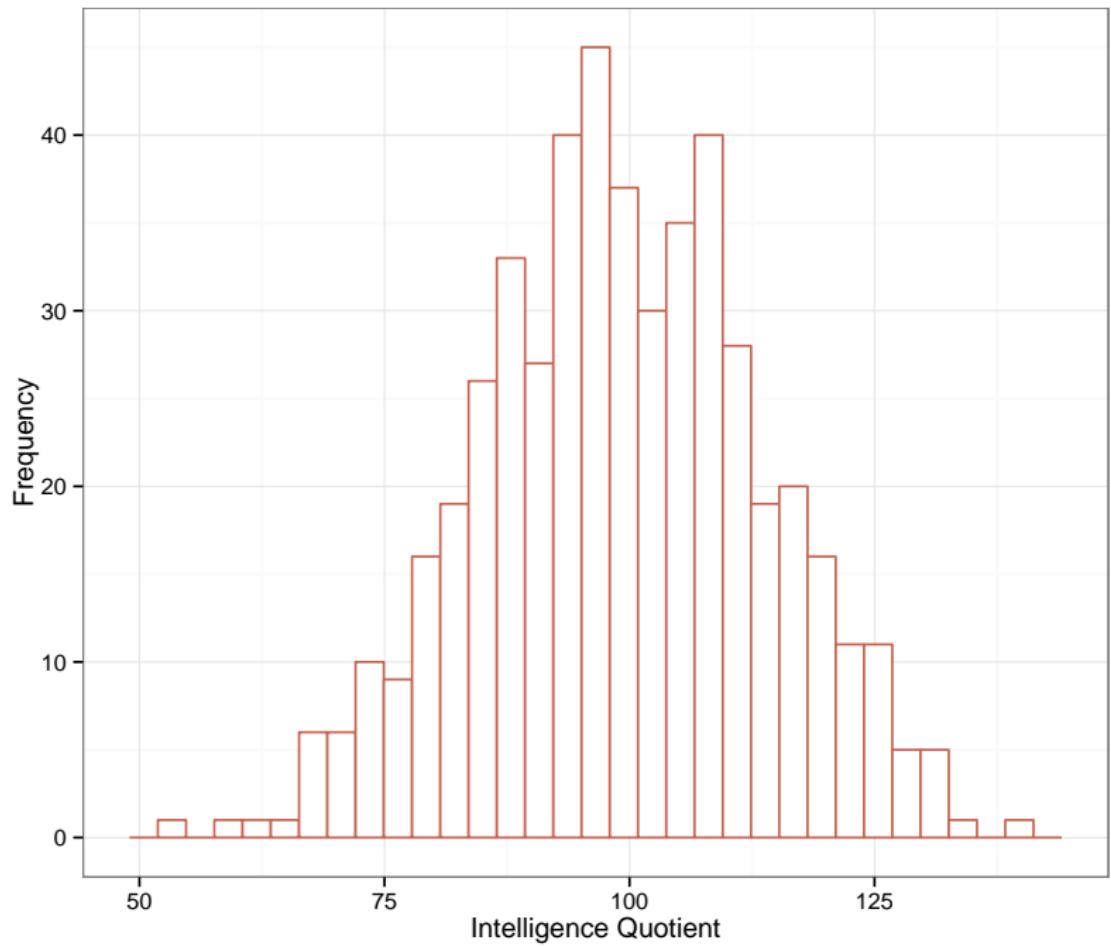
**Ordinal**

**Ratio**

**Interval**

# What kind of scales are these?

- ▶ Temperature
- ▶ Height
- ▶ Grade Point Average
- ▶ Color
- ▶ Ethnic group
- ▶ Likert-type items
- ▶ Job satisfaction

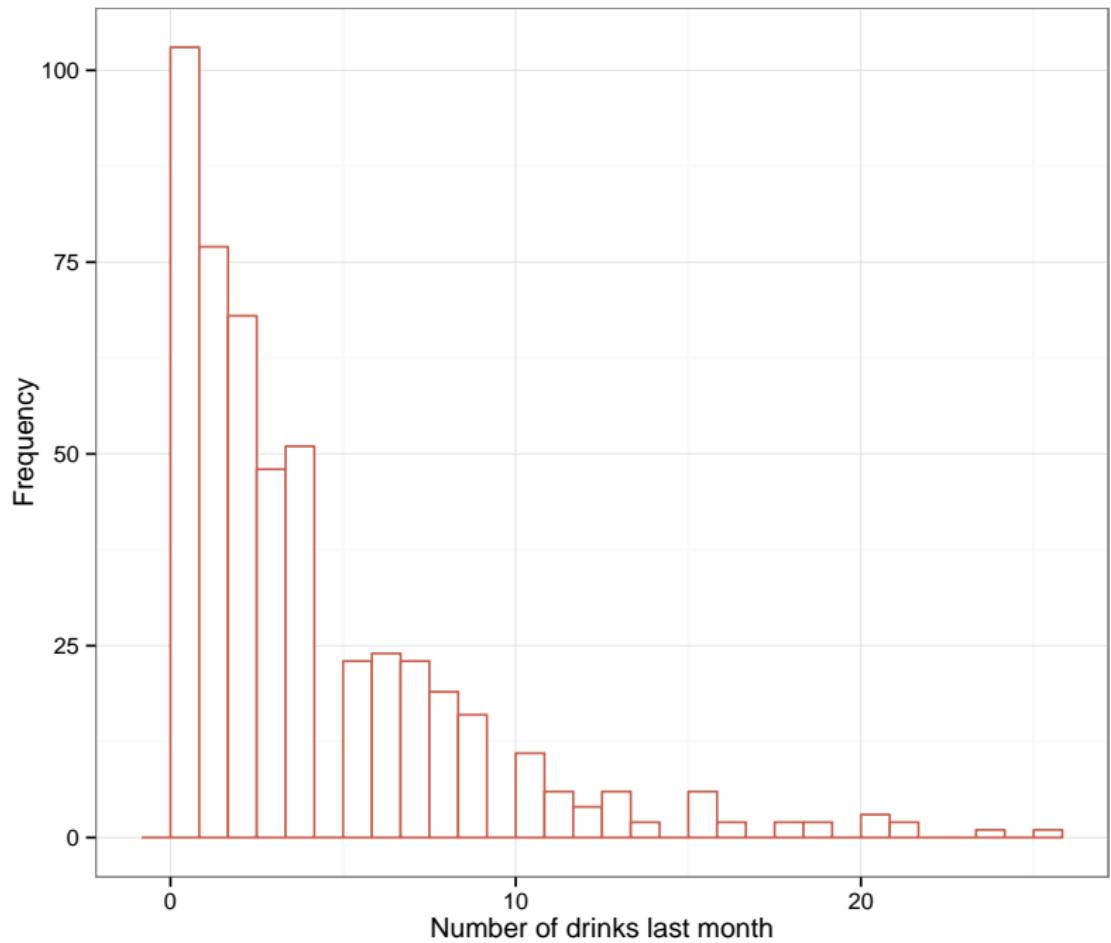


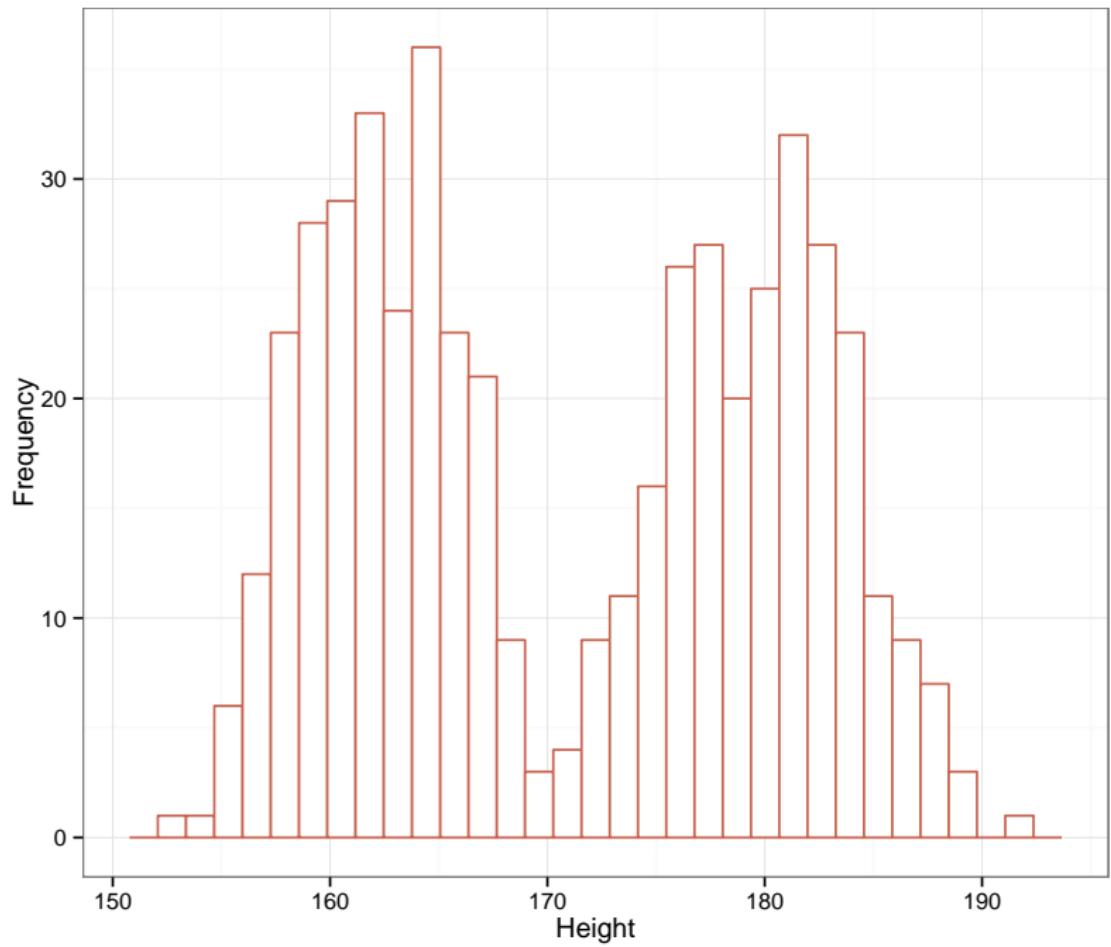
```
# Load the library
set.seed(101)
library("ggplot2")

# Set up the parameters
sample_size <- 500
mean <- 100
standard_deviation <- 15

# Generate random numbers
x <- rnorm(sample_size, mean, standard_deviation)

# Plot the data
qplot(x, fill = I("white"), color = I("#c96552")) +
  theme_bw() + xlab("Intelligence Quotient") +
  ylab("Frequency")
```





# Central Tendency Measures

## Mean

$$\bar{X} = \frac{\sum X_i}{n}$$

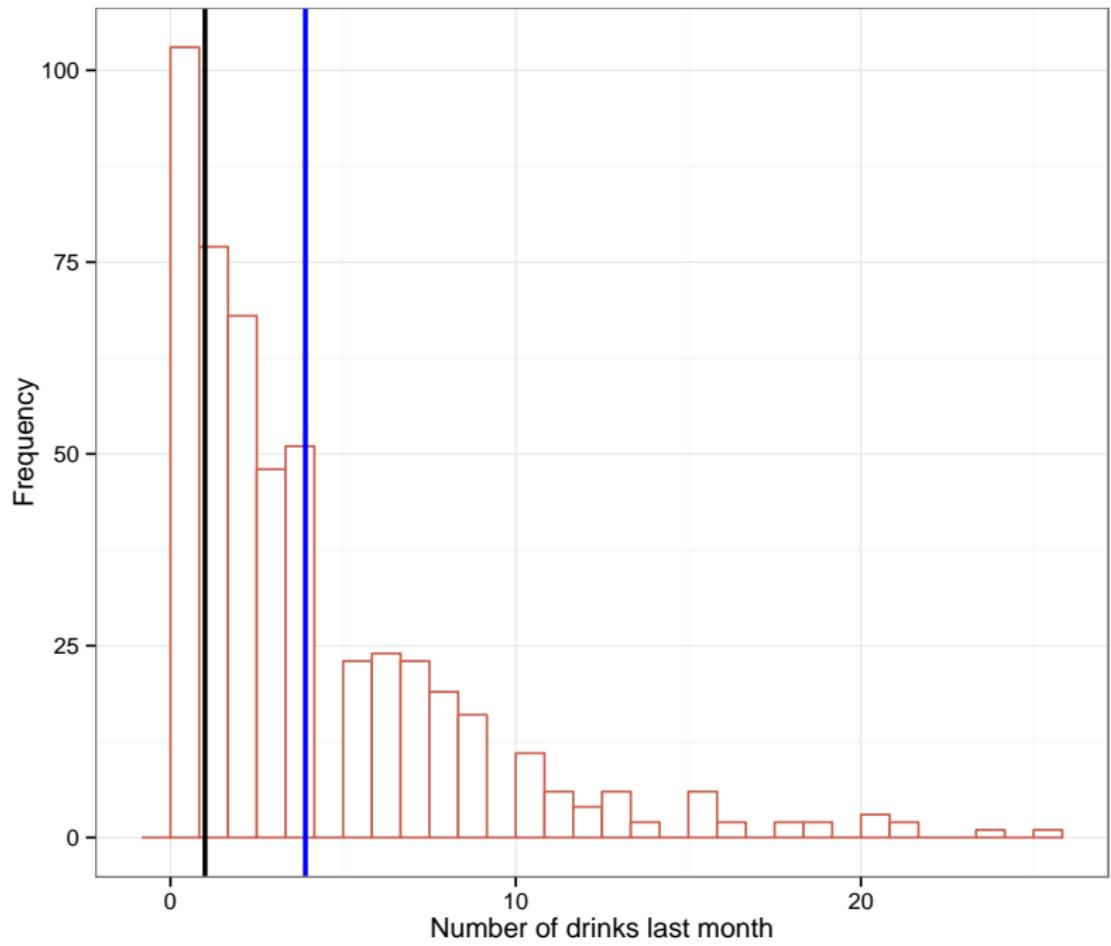
## Median

$$P(X \leq m) \geq \frac{1}{2} \text{ and } P(X \geq m) \geq \frac{1}{2}$$

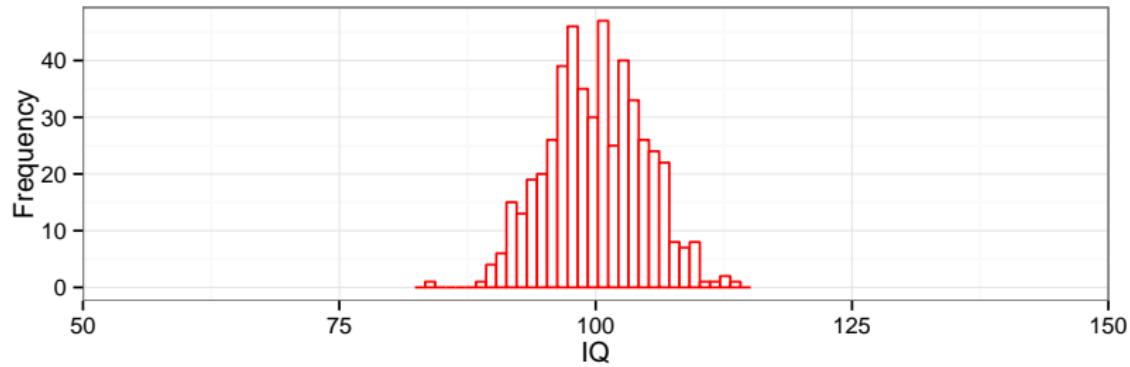
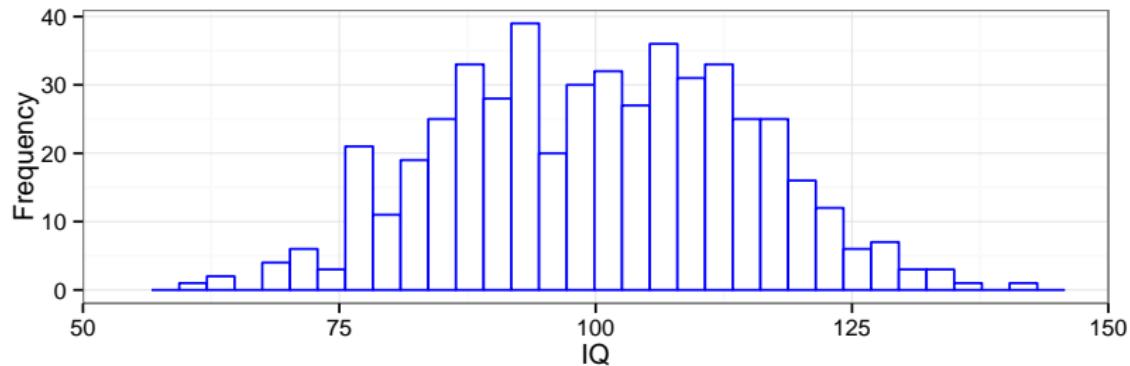
## Mode

The most frequently occurring value

Which of these statistics is most robust to outliers?



# Variability



# Measures of variability

Range

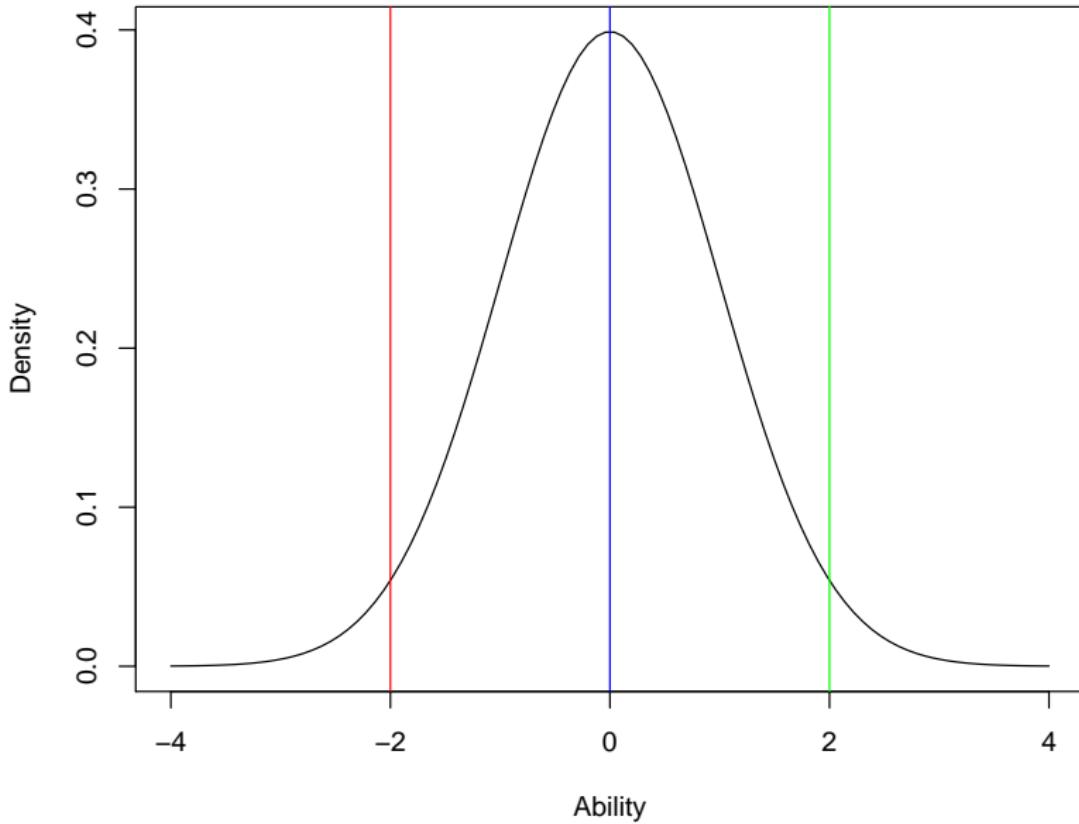
Interquartile range ( $Q_1$ ,  $Q_2$ ,  $Q_3$ )

Standard Deviation and Variance

$$s = \sqrt{\frac{\sum (X_i - \bar{X})^2}{n - 1}}$$

$$s^2 = \frac{\sum (X_i - \bar{X})^2}{n - 1}$$

## Normal Distribution

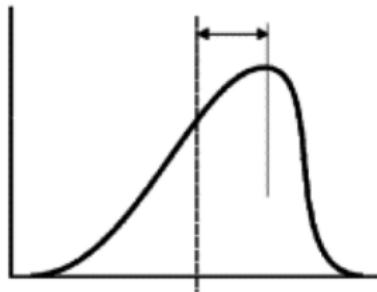


# Distributions, skewness, kurtosis

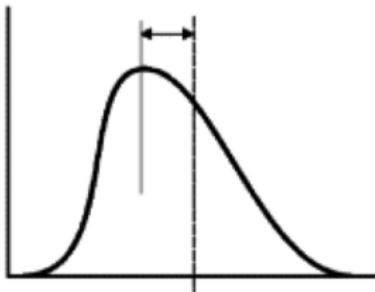
- ▶ What is a probability distribution
  - ▶ Assigns a probability, likeliness of occurrence, of a score to all possible scores
  - ▶ May be parametric or non-parametric
- ▶ What skew might you expect these outcomes to look like?
  - ▶ Reaction time in a psychological experiment
  - ▶ Number of children in a family
  - ▶ Scores on an easy test
  - ▶ Height in Iceland
- ▶ Platykurtic, mesokurtic, and leptokurtic
- ▶ Plot your data, rely less on statistics!

# Shapes of distributions

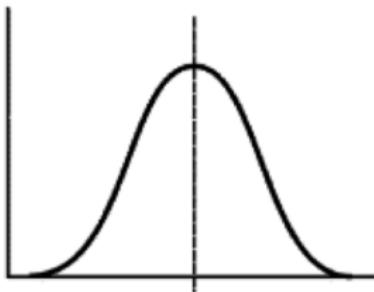
a Negatively skewed distribution



Positively skewed distribution



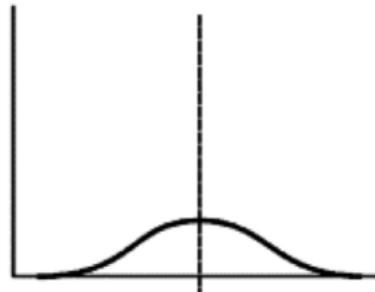
Normal distribution



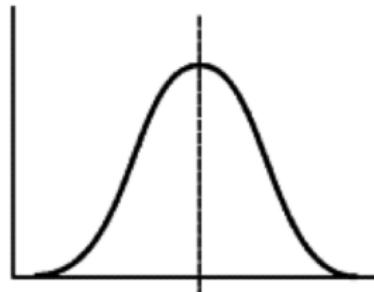
b Leptokurtic distribution



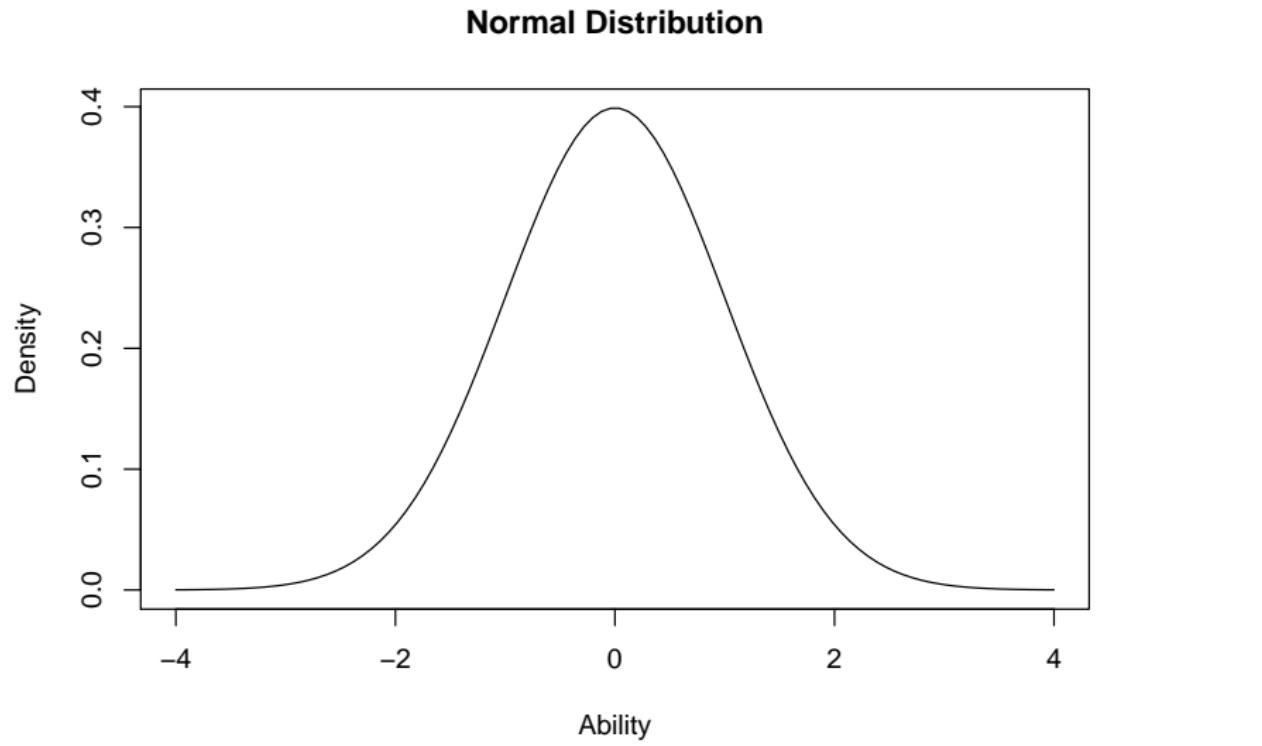
Platykurtic distribution



Normal distribution



# Normal Distribution



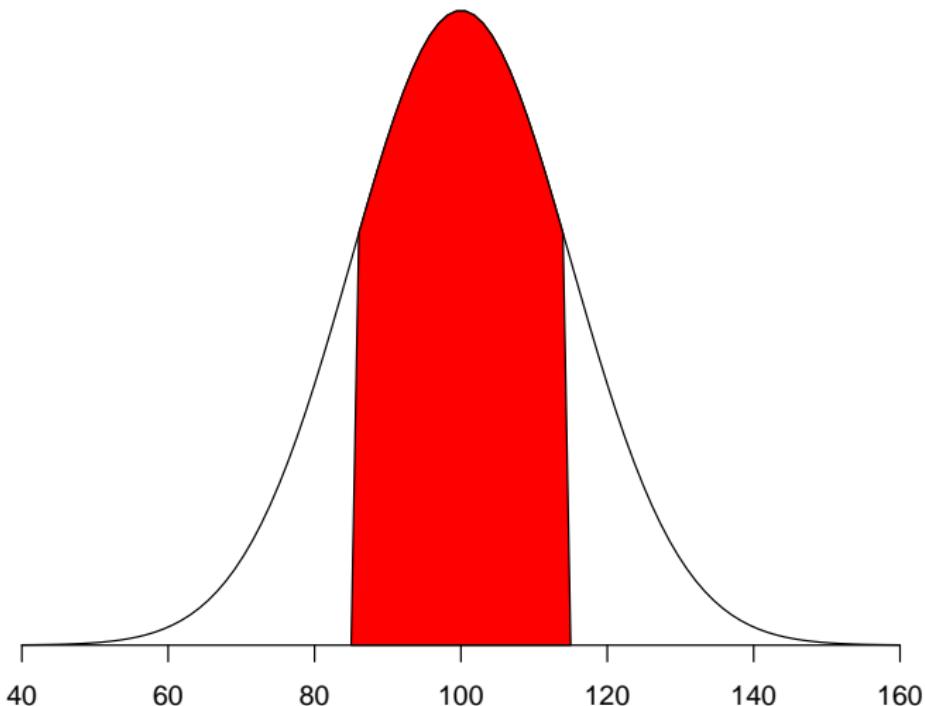
# R Normal distribution applet

1. Open RStudio
2. Open normal\_applet.R
3. Click the "Source" button

# IQ - 1 Standard Deviation

## Normal Distribution

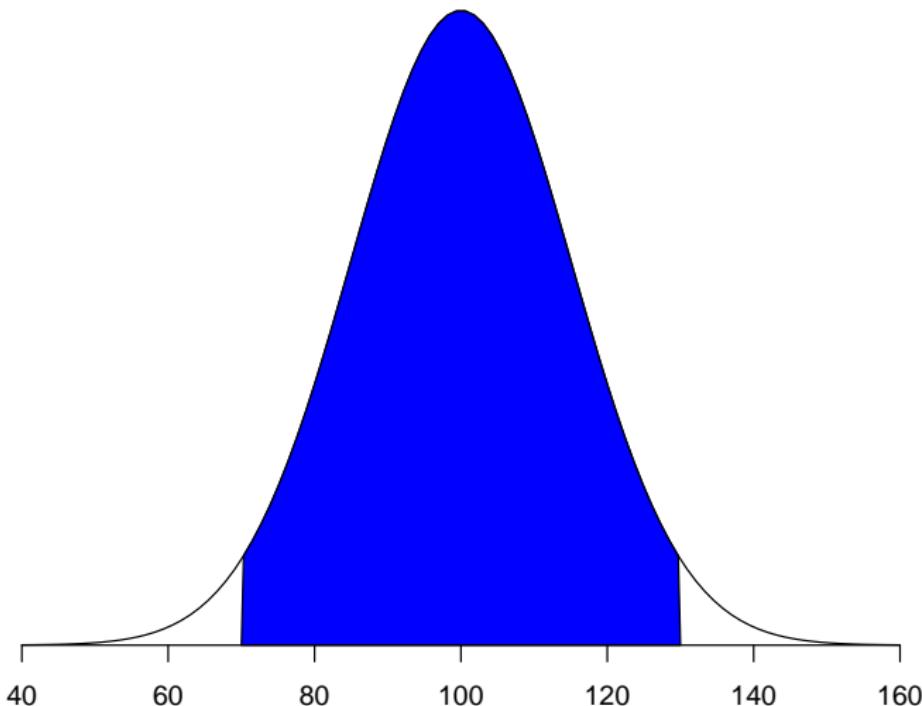
$$P( 85 < \text{IQ} < 115 ) = 0.683$$



# IQ - 2 Standard Deviation

## Normal Distribution

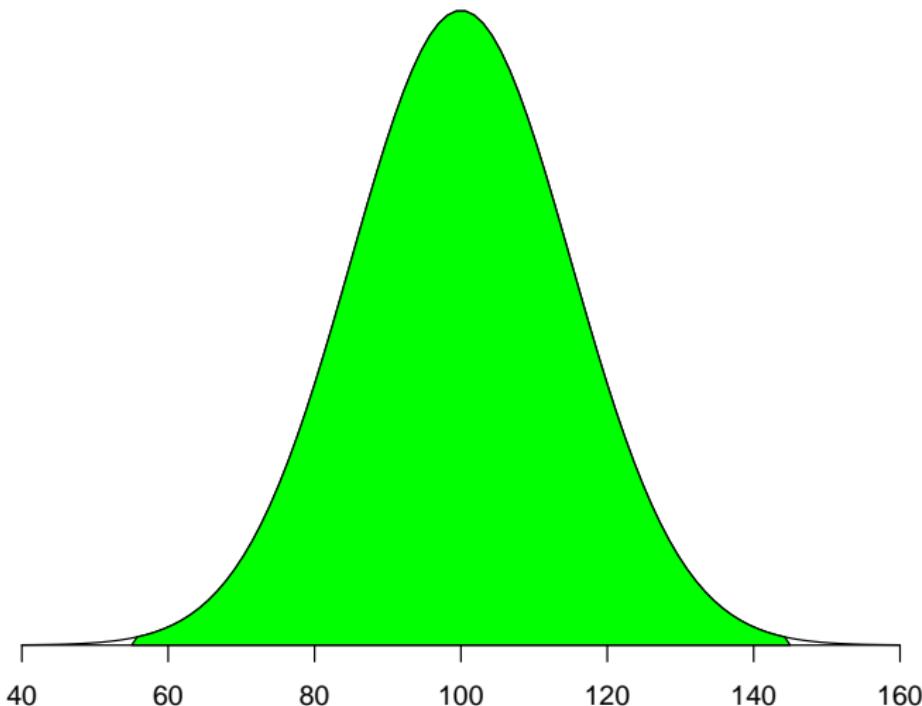
$$P( 70 < \text{IQ} < 130 ) = 0.954$$



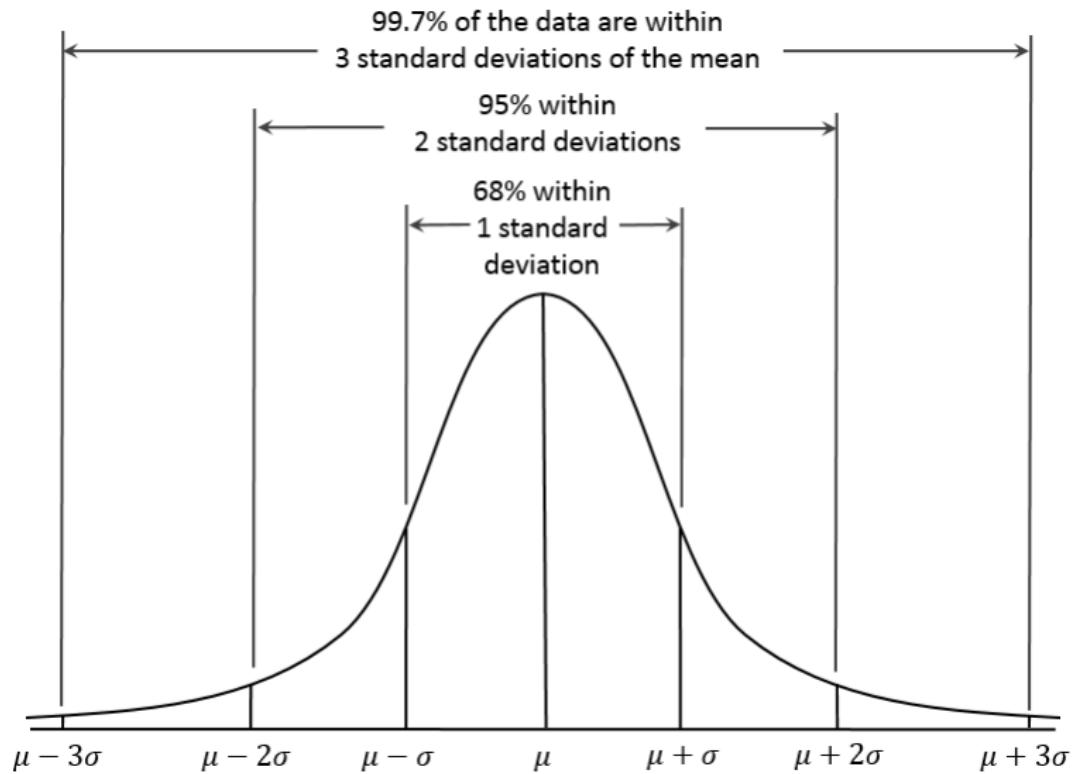
# IQ - 3 Standard Deviation

## Normal Distribution

$$P( 55 < \text{IQ} < 145 ) = 0.997$$



# Characteristics of the Normal distribution



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$$z = \frac{x-\mu}{\sigma}$$

# SAT

The SAT is an aptitude test that high school students take. It is one of the criteria that is used in a college's decision to admit a student. It is composed of a math and a verbal section. Each has a mean of 500 and a standard deviation of 110 and is normally distributed.

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  - ▶ How many people got scores between 390 and 610?
  - ▶ If Sigga got a 350 on the math section, how many people scored below her?
  - ▶ If Einar was in the 98% percentile in math, what was Einar's score?

## Other standard scores

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  - ▶ What percent of the people are between the 3rd and the 6th stanines?

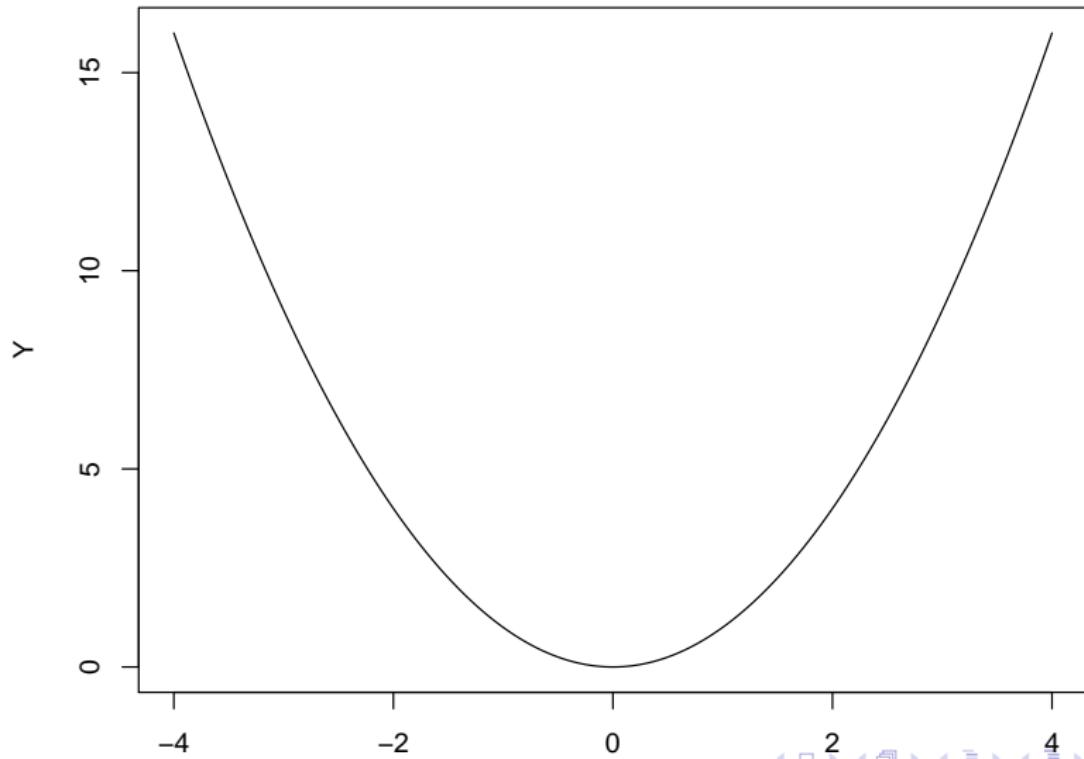
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  - ▶ What percent of the people are between the 3rd and the 6th stanines?
- ▶ Various linear and non-linear transformations are done to create scores and scores may be normalized.

# What is a correlation?

- ▶ Is it an association?
- ▶ Does it imply causation?
- ▶ Is a correlation necessary for causation?
- ▶ Does it need linearity?
- ▶ Is it affected by variability?
- ▶ Is it affected by outliers?
- ▶ Is it related to the simple linear regression?

# What is the Pearson correlation coefficient?



# Pearson correlation coefficient

$$\frac{\sum(X - \bar{X})(Y - \bar{Y})}{\sqrt{\sum(X - \bar{X})^2} \sqrt{\sum((Y - \bar{Y})^2)}}$$

# Calculating Pearson correlation coefficient

X	Y
5	6
3	0
1	0
Mean	3 2

```
x <- c(5, 3, 1)
y <- c(6, 0, 0)
cor(x, y)
```

# R correlation applet

1. Open RStudio
2. Open correlation\_applet.R
3. Click the "Source" button

## Spearman's rho

- ▶ Non-parametric measure of association
- ▶ Appropriate when at least one of your variables is ordinal variables
- ▶ Don't use Pearson's correlation with ordinal variables!

# Simple Linear Regression

- ▶ If you are interested in predicting height given someone's weight, what would you do?

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- ▶ We could consider a regression model.

# Simple Linear Regression

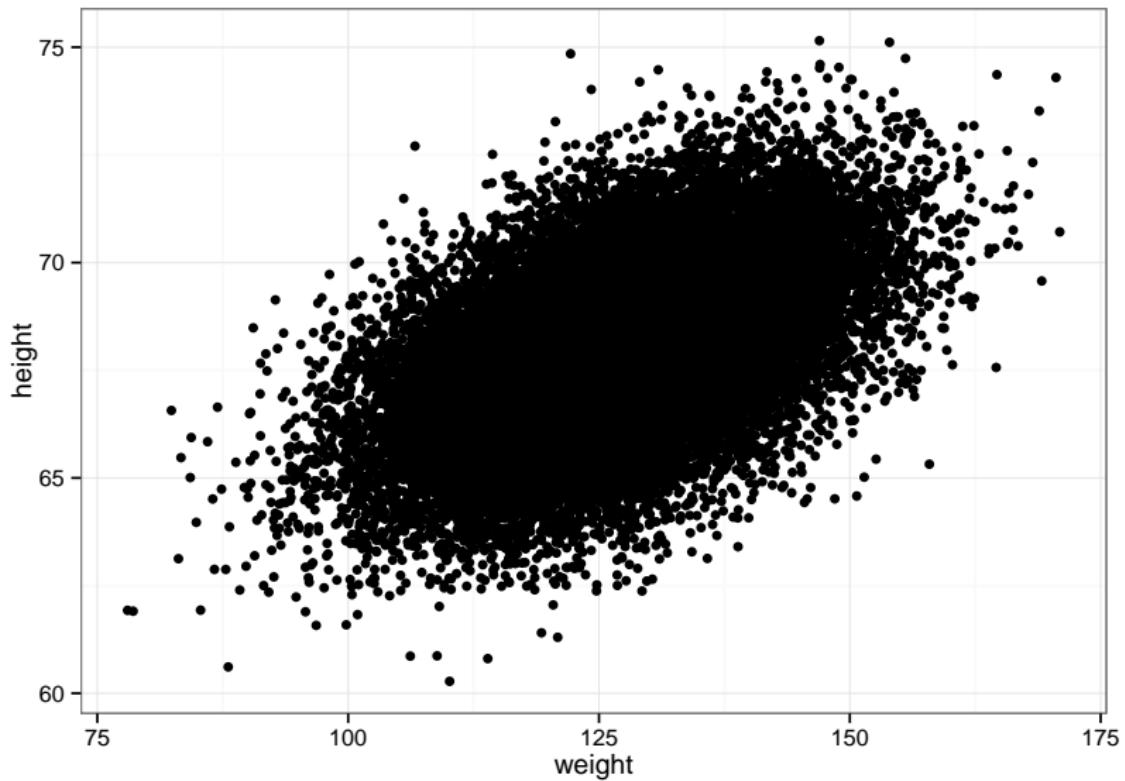
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- ▶  $Y_i = \beta_0 + \beta_1 * X_i$

# Simple Linear Regression

- ▶ If you are interested in predicting height given someone's weight, what would you do?
- ▶ We could consider a regression model.
- ▶  $Y_i = \beta_0 + \beta_1 * X_i$
- ▶ How could we assess if this is appropriate?

# 1993 Growth Survey of 25,000 Hong Kongese children

source: [http://wiki.stat.ucla.edu/socr/index.php/SOCR\\_Data\\_Dinov\\_020108\\_HeightsWeights](http://wiki.stat.ucla.edu/socr/index.php/SOCR_Data_Dinov_020108_HeightsWeights)



## Model Summary

Parameter	Estimate	SE	t-value	p-value
$\beta_0$	57.57	0.11	506.01	< .001
$\beta_1$	0.08	0.001	91.98	< .001

How does this relate to correlation?

## Slope and the correlation

- ▶ There is a relationship between the estimated slope and the correlation between two variables in a simple linear regression.

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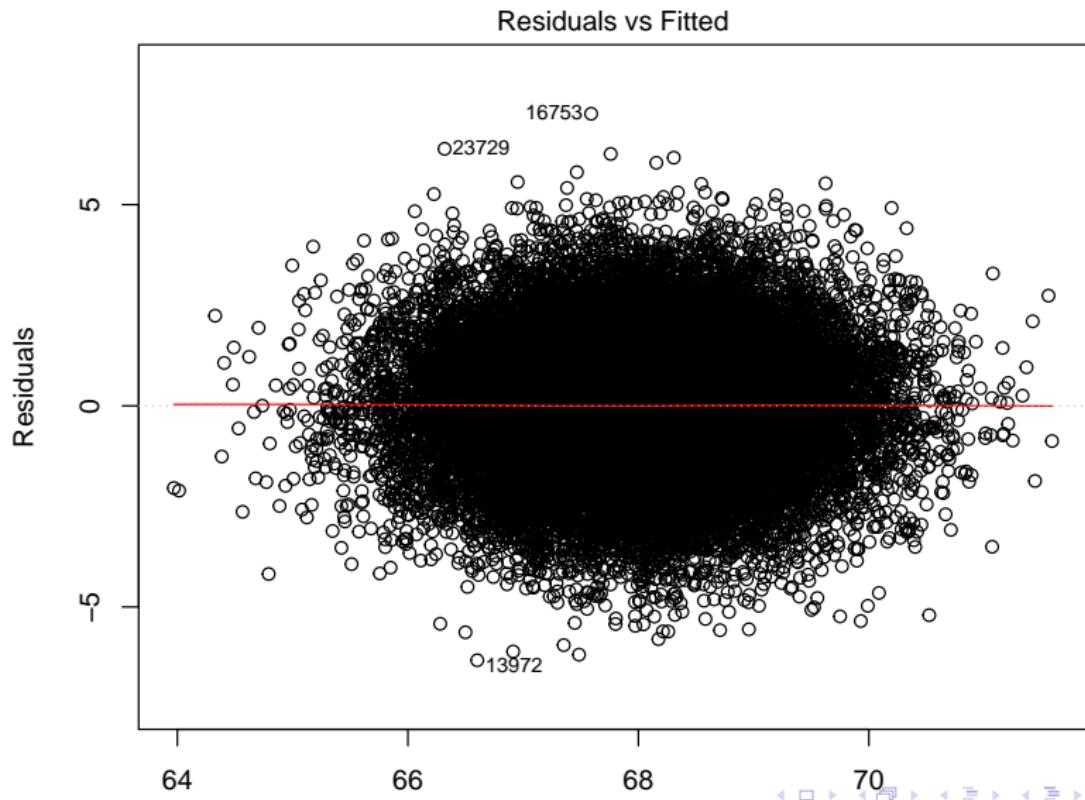
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- ▶ 0.5028585

# Always look at the residuals



# Brief history of testing

- ▶ 2200 BCE, Chinese believed to use testing for determining who would get governmental jobs
- ▶ Greek and Romans categorized individuals based on personality type ("blood" or "phlegm")
- ▶ Francis Galton's classification based on "natural gift" (i.e. eugenics)
  - ▶ Contributed to development of questionnaires, rating scales, and self-report inventories
- ▶ Wilhelm Wundt's laboratory and his focus on "standardization"
  - ▶ James Cattell's mental tests
  - ▶ Charles Spearman - reliability and factor analysis

## Testing in the 20th century

- ▶ 1905, Binet and Simon publish a test measuring intelligence in mental retarded school children in Paris
- ▶ 1939, Wechsler publishes a test to measure intelligence in adults (would become WAIS)
- ▶ Group intelligence test administered by the US military during WWI and WWII
- ▶ WWI personality tests used to screen recruits

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- ▶ Test can be fair
- ▶ Test can benefit society

What makes a good test?

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- ▶ Understanding the normative sample is very important, why?

# Sampling

- ▶ Simple random sample
- ▶ Stratified random sample
- ▶ Cluster random sample
- ▶ Purposive sample
- ▶ Convenience sample

# Different Norms

- ▶ Percentiles
- ▶ Developmental Norms
  - ▶ Age Norms
    - ▶ A 6 year old performs at the level of a 10 year old
    - ▶ This is on this material only though!
  - ▶ Grade Norms
    - ▶ School year typically 10 months in the US
    - ▶ A 4th grader is performing at the level of a 5th grader in third month
    - ▶ This is on this material only though!
- ▶ National Norms, nationally representative
  - ▶ Anchor norms enable two tests to be compared
  - ▶ In USA, students could take SAT or ACT for admission to college

## Fixed Reference and Criterion-Related

- ▶ Fixed reference group scores are used as the basis for calculation of future administrations of the test
- ▶ Raw scores are scaled relative to the performance of the fixed reference group
  - ▶ Answering 50 items correctly one year and 50 on the following year doesn't mean you'll have the same score
- ▶ SAT does this through using anchor items and equating
- ▶ Criterion-referenced, evaluate a score with reference to a set criteria or standard NOT other test takers
- ▶ **What is the fairest way to score grades in a class room?**