Statistical Analysis Using Structural Equation Models

EPsy 8266

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Factor fallacies

- **Naming fallacy** - just cause you call it "intelligence" doesn't make it intelligence.
- **Reification** - belief that the factor must be a real thing.
- **Jingle-jangle fallacy** - Two things with the same name don't necessarily mean the same thing (jingle) and having two separate names doesn't make them distinct (jangle)

Problems in CFA

Many problems arise within CFAs (and SEMs):

- Heywood cases (standardized loading > 1 & negative error variance)
- Nonconvergence.
- Nonpositive definite (one or more eigenvalues $<= 0$) factor covariance and error covariance matrices.

Especially likely when the number of observation is small.

Some causes/fixes

- Model overparameterized/fix parameters
- Non-normal distributions & outliers/initial data analysis & transformations
- Empirical underidentification/bring in additional indicators
- Misspecified measurement model/look at residuals & modification indices

Assessing empirical underidentification with lavaan

```r
# Checking empirical underidentification
library(lavaan)
HS.model <- ' visual  =~ x1 + x2 + x3
              textual =~ x4 + x5 + x6
              speed   =~ x7 + x8 + x9 '

# default lavaan starting values
fit.raw <- cfa(HS.model, data = HolzingerSwineford1939)

# change the starting values
fit.altstart <- cfa(HS.model, data = HolzingerSwineford1939, start = "simple")

# verify these are different
inspect(fit.raw, "start")
inspect(fit.altstart, "start")

# extract model-implied covariance-matrix
covMat <- inspect(fit.raw, "implied")$cov[,]
fit.cov <- cfa(HS.model, sample.cov = covMat, sample.nobs = nrow(HolzingerSwineford1939))

# obtain parameter estimates
raw.params <- parameterEstimates(fit.raw)[,"est"]
altstart.params <- parameterEstimates(fit.altstart)[,"est"]
cov.params <- parameterEstimates(fit.cov)[,"est"]

data.frame(params = do.call(paste, parameterEstimates(fit.cov)[,1:3]),
           raw = raw.params,
           alt = altstart.params,
           cov = cov.params)
```

Types of indicators

Indicators may be of certain types ...

- **Congeneric** - Measure the same construct but not equally.
- **Tau-equivalent** - Congeneric and have equal true score variance (fix pattern coefficients to 1.0).
- **Parallel** - Add equal error variance constraint (constrain error variances to be equal).

Can test with chi-square test of difference.

Reliability

Composite reliability (factor rho coefficient) ratio of explained variance over total variance.

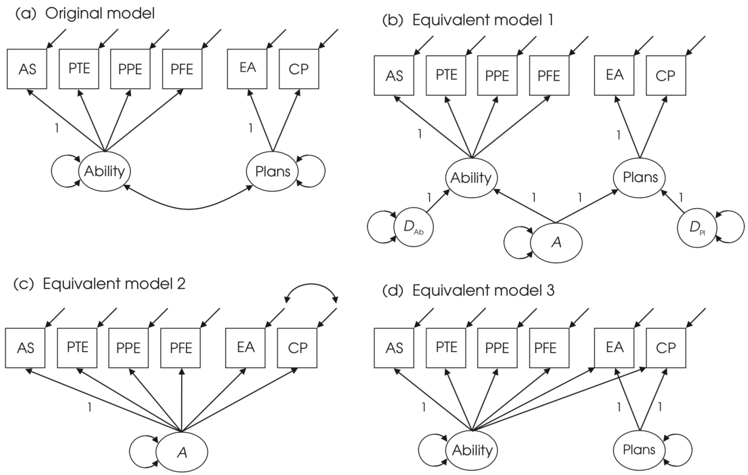
$$CR = \frac{(\sum \hat{\lambda}_i)^2 \hat{\phi}}{(\sum \hat{\lambda}_i)^2 \hat{\phi} + \sum \hat{\theta}_{ii} + 2 \sum \hat{\theta}_{ij}}$$

where

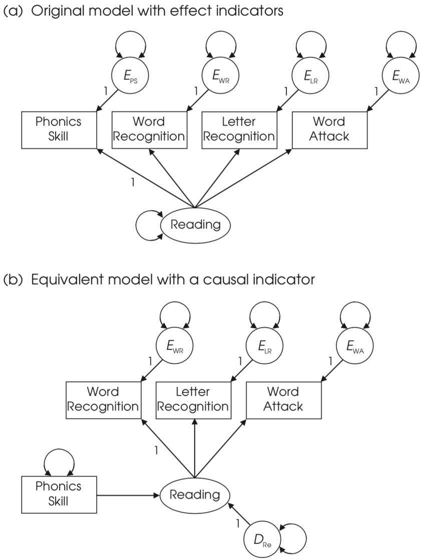
- $\hat{\lambda}_i$ is the unstandardized pattern coefficients among indicators for the same factor
- $\hat{\phi}$ is the factor variance
- $\hat{\theta}_{ii}$ is the unstandardized error variances
- $\hat{\theta}_{ij}$ are the nonzero unstandardized error covariances (often zero)

Alternatively, can take average of the squared standardized pattern coefficients (**average variance extracted**)

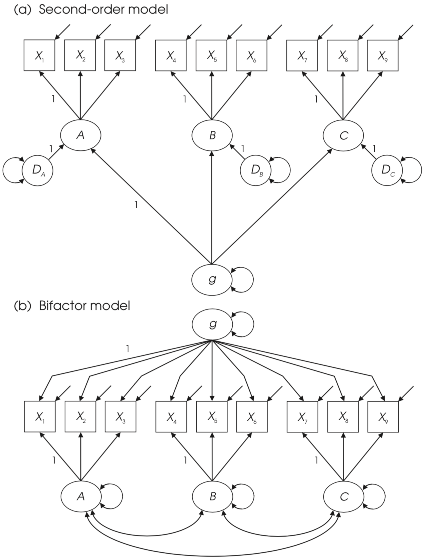
Equivalent CFA models



Equivalent CFA models - 2



Hierarchical & bifactor models



Hierarchical & bifactor models

- Hierarchical
  - A second order factor causes the relationship between the first order factors.
  - Measured indirectly through the first order factors (i.e., no direct indicators).
- Bifactor
  - Indicators directly load onto the general factor and orthogonal to specific factors
  - General factor unrelated to specific factors.
  - Predictive validity of specific factors, partialling out a general factor, can be examined.

Ordinal indicators

- So far, we've assumed our indicators are ratio/interval scale (i.e., continuous)
- This means we can't use full information maximum likelihood
- With ordinal data, requires new parameters, new intermediate, latent variables, and a new estimator.

Latent response variables

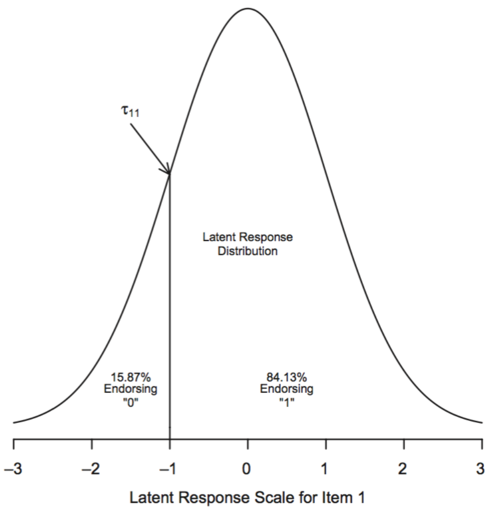


Figure 1. Latent response distribution for a single dichotomous item representing the latent distribution of interest. τ_{11} marks the latent cut-point between observed responses.

Latent response variables

Let X^* be the latent response variable.

If we let $X^* \sim N(0,1)$ then the threshold (τ_1) correspond to z-scores and

$$X = \begin{cases} 0 & \text{if } X^* \leq \tau_{11} \\ 1 & \text{if } X^* > \tau_{11} \end{cases}$$

So, if a respondents score on the latent response variable is $\leq \tau_1$ they will not endorse the item.

Latent response variables have **nonlinear relationships with the indicators** BUT have **linear relationships with the factors**.

Fit an ordinal variable in lavaan

```
lsat6 <- data.frame(psych::lsat6)
library(lavaan)
lsat.mod <- '
  lsat =~ Q1 + Q2 + Q3 + Q4 + Q5
'
lsat.fit <- cfa(lsat.mod, lsat6, ordered = paste0("Q", 1:5))
```

How are thresholds calculated?

```
lsat.params <- parameterEstimates(lsat.fit)
calc_cumprob <- function(x){
  cumsum(prop.table(table(x)))
}
cum_probs <- apply(lsat6, 2, calc_cumprob); cum_probs

##       Q1    Q2    Q3    Q4   Q5
## 0 0.076 0.291 0.447 0.237 0.13
## 1 1.000 1.000 1.000 1.000 1.00

qnorm(cum_probs[1, ])

##         Q1         Q2         Q3         Q4
## -1.4325027 -0.5504657 -0.1332445 -0.7159860
##         Q5
## -1.1263911

subset(lsat.params, rhs == "t1", select = est, drop = TRUE)

## [1] -1.4325027 -0.5504657 -0.1332445 -0.7159860
## [5] -1.1263911
```

Parameterizations

There are two ways to scale latent response variables.

- **Delta scaling**
  - Total variance of latent response variable fixed to 1.
  - For the standardized solution, pattern coefficients represent for a 1 SD increase in the factor, expect an XX SD change for the latent response variable.
  - For the standardized solution, threshold correspond to normal deviates based corresponding to cumulative probabilities
- **Theta scaling**
  - Residual variance of each latent response variable fixed to 1 (like probit regression scaling).
  - For the unstandardized solution, pattern coefficients represent for a 1 unit increase in the factor, expect an XX probit (normal deviates) change for the latent response variable,
  - For the unstandardized solution, threshold correspond to normal deviates for the lowest response category.
- Standardized solution identical between the two

Ordinal model in lavaan

```
summary(lsat.fit, standardized = TRUE)


## lavaan 0.6-3 ended normally after 29 iterations
##
##   Optimization method                           NLMINB
##   Number of free parameters                         10
##   Number of observations                          1000
##   Estimator                                       DWLS      Robust
##   Model Fit Test Statistic                       4.051       4.740
##   Degrees of freedom                                 5           5
##   P-value (Chi-square)                           0.542       0.448
##   Scaling correction factor                                  0.867
##   Shift parameter                                            0.070
##     for simple second-order correction (Mplus variant)
## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model        Unstructured
##   Standard Errors                           Robust.sem
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   lsat =~
##     Q1                1.000                                0.389    0.389
##     Q2                1.020    0.358    2.846    0.004    0.397    0.397
##     Q3                1.210    0.447    2.709    0.007    0.471    0.471
##     Q4                0.968    0.352    2.751    0.006    0.377    0.377
##     Q5                0.879    0.352    2.499    0.012    0.342    0.342
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Q1                0.000                               0.000    0.000
##    .Q2                0.000                               0.000    0.000
##    .Q3                0.000                               0.000    0.000
##    .Q4                0.000                               0.000    0.000
##    .Q5                0.000                               0.000    0.000
##     lsat              0.000                               0.000    0.000
```

Ordinal model in lavaan

```r
summary(lsat.fit, standardized = TRUE)
```

```
## Thresholds:
##                    Estimate  Std.Err  z-value   P(>|z|)   Std.lv  Std.all
##     Q1|t1            -1.433    0.059  -24.431     0.000   -1.433   -1.433
##     Q2|t1            -0.550    0.042  -13.133     0.000   -0.550   -0.550
##     Q3|t1            -0.133    0.040   -3.349     0.001   -0.133   -0.133
##     Q4|t1            -0.716    0.044  -16.430     0.000   -0.716   -0.716
##     Q5|t1            -1.126    0.050  -22.395     0.000   -1.126   -1.126
##
## Variances:
##                    Estimate  Std.Err  z-value   P(>|z|)   Std.lv  Std.all
##    .Q1                0.848                                0.848    0.848
##    .Q2                0.842                                0.842    0.842
##    .Q3                0.778                                0.778    0.778
##    .Q4                0.858                                0.858    0.858
##    .Q5                0.883                                0.883    0.883
##     lsat              0.152    0.087    1.743     0.081    1.000    1.000
##
## Scales y*:
##                    Estimate  Std.Err  z-value   P(>|z|)   Std.lv  Std.all
##     Q1                1.000                                1.000    1.000
##     Q2                1.000                                1.000    1.000
##     Q3                1.000                                1.000    1.000
##     Q4                1.000                                1.000    1.000
##     Q5                1.000                                1.000    1.000
```

Estimating ordinal data

There are two types of robust estimator: mean-adjusted WLS (WLSM) and mean- and variance-adjusted WLS (WLSMV)

Makes different adjustments to the chi-square statistic to better approximate a chi-square distribution

WLSMV is the more favored approach (and is labeled Robust in lavaan)

Other estimators available and will talk more about this later.