Statistical Analysis Using Structural Equation Models

EPsy 8266

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Research Methodology Consulting Center

1/22/19

Course Website

cddesja.github.io/epsy8266

Course Software

- R: www.r-project.org
- lavaan: http://lavaan.ugent.be/ (version 0.6-3)
  - requires R $\geq$ 3.4

```
R.version

##                _
## platform       x86_64-apple-darwin15.6.0
## arch           x86_64
## os             darwin15.6.0
## system         x86_64, darwin15.6.0
## status
## major          3
## minor          5.2
## year           2018
## month          12
## day            20
## svn rev        75870
## language       R
## version.string R version 3.5.2 (2018-12-20)
## nickname       Eggshell Igloo
```

- bower

R code

```
# -----------------
# This is a code chunk
# ----------------

# Any text/code that's inside a chunk can be copied/pasted into R.

# Any text that begins with a # is a comment
# Text that doesn't begin with a # is valid R code
# Test that begins with a ## is R output

# For example, what is the square root of 9?
sqrt(9)

## [1] 3
```

R day 1 homework

```r
install.packages(c("lavaan", "boot", "devtools"))
```

Questions

What is a structural equation model? What is structural about SEM?

Does structural equation modeling differ from traditional statistical model? If so, how?

Another name for SEM is *analysis of covariance structures*. Is this a good name?

Background

What is the need for SEM in your field?

Do you intend to use SEM? If so, how?

How well do you know matrix algebra?

Do you know covariance algebra?

Google Scholar | educational psychology structural equation modeling

Articles | About 1,400 results (**0.08** sec)

Any time
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The technology acceptance model (TAM): a meta-analytic structural equation modeling approach to explaining teachers' adoption of digital technology in education
R Scherer, F Siddiq, J Tondeur - Computers & **Education**, 2019 - Elsevier
… determining the teacher's intentions toward using new technologies in their **educational** practice … These **models** have emerged from well-established **psychological** theories, including the Theory of … 2009) argued that, for technology integration to occur in **education**, teachers must …
Cited by 1 All 3 versions

Competence and Confidence in Rural and Remote Nursing Practice: A Structural Equation Modeling Analysis of National Data
KL Penz, NJ Stewart, CP Karunanayake… - Journal of clinical …, 2019 - Wiley Online Library
… There are also concerns about the inadequacy of **educational** offerings; specifically that they lack … 4-6, 7-9, and ≥ 10 communities), highest level of nursing **education** attained (bachelor's … centre (from 0-99 km to ≥ 1,000 km), **psychological** sense of community (9 items on a 5 …
All 3 versions

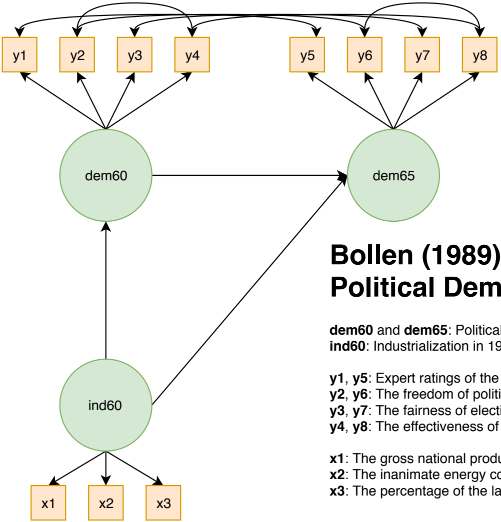
Examining the factor structure of the Self-Compassion Scale in 20 diverse samples: Support for use of a total score and six subscale scores.
KD Neff, I Tóth-Király, LM Yarnell… - **Psychological** …, 2019 - psycnet.apa.org
… Paula Castilho, Faculty of **Psychology** and **Educational** Sciences, University of Coimbra … Hailan Xiaoxia Guo, Beijing Hailan Peer **Education** & Consultation Co., Beijing, China … Michail Mantzios, Department of **Psychology**, Birmingham City University …
Cited by 6 All 6 versions

Analytical assessment of course sequencing: The case of methodological courses in psychology.
L Betancur, BM Rottman, E Votruba-Drzal… - Journal of **Educational** …, 2019 - psycnet.apa.org
… School Universe Survey Data collected by the National Center for **Education** Statistics (NCES … covariates that tend to be associated with student achievement and **educational** attainment to … 2, all **psychology** courses that give letter grades (introduction to **psychology**, core courses …
Related articles All 2 versions

What is SEM?

According to Bollen (1989) an SEM consists of 3 parts:

1. Path Analysis
2. Conceptual synthesis of latent variable and measurement models
3. General estimation procedures



Bollen (1989) Industrialization and Political Democracy Model

dem60 and **dem65**: Political Democracy in 1960 and 1965 in developing countries
ind60: Industrialization in 1960

y1, **y5**: Expert ratings of the freedom of the press in 1960 and 1965
y2, **y6**: The freedom of political opposition in 1960 and 1965
y3, **y7**: The fairness of elections in 1960 and 1965
y4, **y8**: The effectiveness of the elected legislature in 1960 and 1965

x1: The gross national product (GNP) per capita in 1960
x2: The inanimate energy consumption per capita in 1960
x3: The percentage of the labor force in industry in 1960

created using https://www.draw.io/

The Industrialization and Political Democracy Model

- dem60, dem65, and ind60 are **latent variables**.
- y1 - y8 and x1 - x3 are **observed** or **manifest variables**.
- ind60 is an **exogenous** variable, dem60 and dem65 are **endogenous** variables.
- Each arrow represents a regression.
- Absent from this diagram are the errors.

The model represented mathematically

Let ξ_1 (xi 1) represent industrialization in 1960 and η_1 (eta 1) and η_2 (eta 2) represent political democracy in 1960 and 1965, respectively, then:

$$
\begin{aligned}
x_1 &= \lambda_1 \xi_1 + \delta_1 \\
x_2 &= \lambda_2 \xi_1 + \delta_2 \\
x_3 &= \lambda_3 \xi_1 + \delta_3
\end{aligned}
$$

$$
\begin{aligned}
y_1 &= \lambda_4 \eta_1 + \epsilon_1 & y_5 &= \lambda_8 \eta_2 + \epsilon_5 \\
y_2 &= \lambda_5 \eta_1 + \epsilon_2 & y_6 &= \lambda_9 \eta_2 + \epsilon_6 \\
y_3 &= \lambda_6 \eta_1 + \epsilon_3 & y_7 &= \lambda_{10} \eta_2 + \epsilon_7 \\
y_4 &= \lambda_7 \eta_1 + \epsilon_4 & y_8 &= \lambda_{11} \eta_2 + \epsilon_8
\end{aligned}
$$

This is the **measurement model**.

The λs (lambda) are the factor loadings (regression coefficients) and the δs (delta) and ϵs (epsilons) are the measurement errors for the exogenous and endogenous variables, respectively.

In matrix notation

$$\mathbf{x} = \boldsymbol{\Lambda}_x \boldsymbol{\xi} + \boldsymbol{\delta}$$
$$\mathbf{y} = \boldsymbol{\Lambda}_y \boldsymbol{\eta} + \boldsymbol{\epsilon}$$

where

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \boldsymbol{\Lambda}_x = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} \quad \boldsymbol{\xi} = \begin{bmatrix} \xi_1 \end{bmatrix} \quad \boldsymbol{\delta} = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}$$

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \end{bmatrix} \quad \boldsymbol{\Lambda}_y = \begin{bmatrix} \lambda_4 & 0 \\ \lambda_5 & 0 \\ \lambda_6 & 0 \\ \lambda_7 & 0 \\ 0 & \lambda_8 \\ 0 & \lambda_9 \\ 0 & \lambda_{10} \\ 0 & \lambda_{11} \end{bmatrix} \quad \boldsymbol{\eta} = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} \quad \boldsymbol{\epsilon} = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \\ \epsilon_7 \\ \epsilon_8 \end{bmatrix}$$

Mathematical model, part 2

$$\eta_1 = \gamma_{11}\xi_1 + \zeta_1$$
$$\eta_2 = \beta_{21}\eta_1 + \gamma_{21}\xi_1 + \zeta_2$$

This is the **structural component**.

The γs (gamma) and β (beta) are the regression coefficients and the ζs are the disturbances or random errors.

In matrix notation

$$\boldsymbol{\eta} = \boldsymbol{\beta}\boldsymbol{\eta} + \boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{\zeta}$$

where

$$\boldsymbol{\eta} = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} \quad \boldsymbol{\beta} = \begin{bmatrix} 0 & 0 \\ \beta_{21} & 0 \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{11} \\ \gamma_{21} \end{bmatrix} \quad \boldsymbol{\xi} = \begin{bmatrix} \xi_1 \end{bmatrix} \quad \boldsymbol{\zeta} = \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}$$

and

$$E(\boldsymbol{\eta}) = \mathbf{0} \quad E(\boldsymbol{\xi}) = \mathbf{0} \quad E(\boldsymbol{\zeta}) = \mathbf{0}$$

$$cov(\boldsymbol{\xi}) = \begin{bmatrix} \phi_{11} \end{bmatrix} \quad cov(\boldsymbol{\zeta}) = \begin{bmatrix} \psi_{11} & 0 \\ 0 & \psi_{22} \end{bmatrix}$$

Mathematical model, part 3

The only thing missing from our equations, is the covariance between the measurement errors which would need to be estimated. Specifically, we would need to estimate:

$$
\begin{gathered}
cov(\delta_1, \delta_5) \\
cov(\delta_2, \delta_6) \\
cov(\delta_3, \delta_7) \\
cov(\delta_4, \delta_8) \\
cov(\delta_2, \delta_4) \\
cov(\delta_6, \delta_8)
\end{gathered}
$$

Anything missing from the path diagram is assumed to be 0.

In lavaan

```
library(lavaan)
mod <- '
# measurement model
ind60 =~ x1 + x2 + x3
dem60 =~ y1 + y2 + y3 + y4
dem65 =~ y5 + y6 + y7 + y8

# structural paths
dem60 ~ ind60
dem65 ~ ind60 + dem60

# residual correlations
y1 ~~ y5
y2 ~~ y4
y2 ~~ y6 # equivalently y2 ~~ y4 + y6
y3 ~~ y7
y4 ~~ y8
y6 ~~ y8
'
fit <- sem(model = mod, data = PoliticalDemocracy)
```

Assessing fit

```
summary(fit, fit.measures = TRUE)

## lavaan 0.6-3 ended normally after 68 iterations
##
##   Optimization method                           NLMINB
##   Number of free parameters                         31
##
##   Number of observations                            75
##
##   Estimator                                         ML
##   Model Fit Test Statistic                      38.125
##   Degrees of freedom                                35
##   P-value (Chi-square)                           0.329
##
## Model test baseline model:
##
##   Minimum Function Test Statistic              730.654
##   Degrees of freedom                                55
##   P-value                                        0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.995
##   Tucker-Lewis Index (TLI)                       0.993
```

Assessing fit, con't

```
summary(fit, fit.measures = TRUE)

## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)             -1547.791
##   Loglikelihood unrestricted model (H1)     -1528.728
##
##   Number of free parameters                        31
##   Akaike (AIC)                               3157.582
##   Bayesian (BIC)                             3229.424
##   Sample-size adjusted Bayesian (BIC)        3131.720
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                         0.035
##   90 Percent Confidence Interval         0.000  0.092
##   P-value RMSEA <= 0.05                         0.611
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                          0.044
```

Measurement model

```
summary(fit, fit.measures = TRUE)

## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##   Standard Errors                             Standard
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   ind60 =~
##     x1                1.000
##     x2                2.180    0.139   15.742    0.000
##     x3                1.819    0.152   11.967    0.000
##   dem60 =~
##     y1                1.000
##     y2                1.257    0.182    6.889    0.000
##     y3                1.058    0.151    6.987    0.000
##     y4                1.265    0.145    8.722    0.000
##   dem65 =~
##     y5                1.000
##     y6                1.186    0.169    7.024    0.000
##     y7                1.280    0.160    8.002    0.000
##     y8                1.266    0.158    8.007    0.000
```

Structural component

```
summary(fit, fit.measures = TRUE)

## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   dem60 ~
##     ind60             1.483    0.399    3.715    0.000
##   dem65 ~
##     ind60             0.572    0.221    2.586    0.010
##     dem60             0.837    0.098    8.514    0.000
```

Residual covariances

```
summary(fit, fit.measures = TRUE)

## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   .y1 ~~
##     .y5               0.624    0.358    1.741    0.082
##   .y2 ~~
##     .y4               1.313    0.702    1.871    0.061
##     .y6               2.153    0.734    2.934    0.003
##   .y3 ~~
##     .y7               0.795    0.608    1.308    0.191
##   .y4 ~~
##     .y8               0.348    0.442    0.787    0.431
##   .y6 ~~
##     .y8               1.356    0.568    2.386    0.017
```

Variances

```
summary(fit, fit.measures = TRUE)

## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)
##    .x1                0.082    0.019    4.184    0.000
##    .x2                0.120    0.070    1.718    0.000
##    .x3                0.467    0.090    5.177    0.000
##    .y1                1.891    0.444    4.256    0.000
##    .y2                7.373    1.374    5.366    0.000
##    .y3                5.067    0.952    5.324    0.000
##    .y4                3.148    0.739    4.261    0.000
##    .y5                2.351    0.480    4.895    0.000
##    .y6                4.954    0.914    5.419    0.000
##    .y7                3.431    0.713    4.814    0.000
##    .y8                3.254    0.695    4.685    0.000
##     ind60             0.448    0.087    5.173    0.000
##    .dem60             3.956    0.921    4.295    0.000
##    .dem65             0.172    0.215    0.803    0.422
```

100% replicating our model

```
named.mod <- '
ind60 =~ lam1*x1 + lam2*x2 + lam3*x3
dem60 =~ lam4*y1 + lam5*y2 + lam6*y3 + lam7*y4
dem65 =~ lam8*y5 + lam9*y6 + lam10*y7 + lam11*y8

dem60 ~ gam11*ind60
dem65 ~ gam21*ind60 + beta21*dem60

y1 ~~ del1del5*y5
y2 ~~ del2del4*y4
y2 ~~ del2del6*y6
y3 ~~ del3del7*y7
y4 ~~ del4del8*y8
y6 ~~ del6del8*y8

# exogenous latent variable variance
ind60 ~~ phi11*ind0

# residual endogenous latent variable variances
dem60 ~~ psi11*dem60
dem65 ~~ psi22*dem65
'
fit.named <- sem(model = named.mod, data = PoliticalDemocracy)
summary(fit.named)
```

How to prepare for SEM (or statistical modeling, generally)

Kline (2016) suggests the following:

1. Know your content area well.
   - This class won't help with this.
2. Know your measures well.
   - This class won't really help with this. But what makes a good measure?
3. Understand regression well.
   - We'll review MR, logistic, and probit regression starting Thursday.
4. Use your brain during statistical modeling
   - This class will help and teach simulation to use when you're stuck.
5. Learn the software
   - We'll learn R and lavaan
6. Participate online
   - http://www2.gsu.edu/~mkteer/semnet.html
   - https://groups.google.com/forum/#!forum/lavaan

Inputs of SEM

I-1. **A set of qualitative causal** hypotheses based on **theory** or **results** of empirical studies that are represented in a SEM. The hypotheses are typically based on **assumptions**, only **some** of which can actually be **verified** or **tested** with the data.

I-2. A series of **questions about the causal relations** among the variables of interests, which depend on **model specification**.

- For example, what is the direct effect of ind60 on dem65 controlling for the other causes of dem65?
- What are the other causes in that model? What are the other causes outside of that model?

Outputs of SEM

O-1. Numeric estimates of model parameters for hypothesized effects given the data.

- The direct effect (γ_{21}) of ind60 on dem65 is 0.572.
- What is the indirect effect of ind60 on dem65 through dem60?
- What is the total effect of ind60 on dem65?

O-2. A set of implications of the model that may not directly correspond to a specific parameter but that can still be test in the data.

- Is the direct effect significant? (z = 2.661)

O-3. The degree to which the testable implications of the model are supported by the data.

- Any ideas how we do this in SEM?

SEM implications

- Objective is to assess your theory NOT find a model that fits your data well.
  - But ... where do modification indices fit into this?
  - Does everything have to be confirmatory?
  - What might be an alternative?
- Theoretically good models that don't fit well are interesting.
- "Garbage in, garbage"

SEM involves ...

- Observed variables, manifest variables, **indicators**
- Latent constructs, **factors** (*not required*)
- Error (for the measurement model, part of this error is **measurement error**)
- Continuous latent variables NOT categorical ones.
- Large samples (N $>>$ p)
- Covariances and possibly the mean structure

SEM benefits

- Can test your theoretical model and assess the weight of evidence for your theory.
- Because it helps you articulate your theoretical model, you might identify areas that are unclear and need more research.
- Can, and should, help you generate new theoretical questions.

SEM cautions

- Kline writes “Most published SEM studies are probably based on samples that are too small.

  He suggests a ratio of 20:1 number of cases (N) to number of parameters being estimated (p).

  We will use simulation-based power analyses to check this, but this is a good, conservative rule of thumb.

- SEM doesn’t prove causation, but can confirm if the data are consistent with the theoretical model.
  - Equivalent models can explain the data just as well and not be consistent with your theory.
- Very few replication studies
- Likely one of the most abused statistical framework.

  Avoid RMSEA hacking

Equivalent Models

```
mod <- '
  dem60 ~ ind60
  dem65 ~ ind60 + dem60
'
mod.equiv1 <- '
  ind60 ~ dem60
  dem65 ~ ind60 + dem60
'
mod.equiv2 <- '
  ind60 ~ dem60 + dem65
  dem60 ~ dem65
'
```

```
##               chisq df  rmsea    cfi    tli
## Hyp. Model 38.1252 35 0.0345 0.9954 0.9927
## Equiv. 1   38.1252 35 0.0345 0.9954 0.9927
## Equiv. 2   38.1252 35 0.0345 0.9954 0.9927
```

Ugh!

Theory, theory, theory – it is not BS!

Additionally homework

Find a paper in your field that:

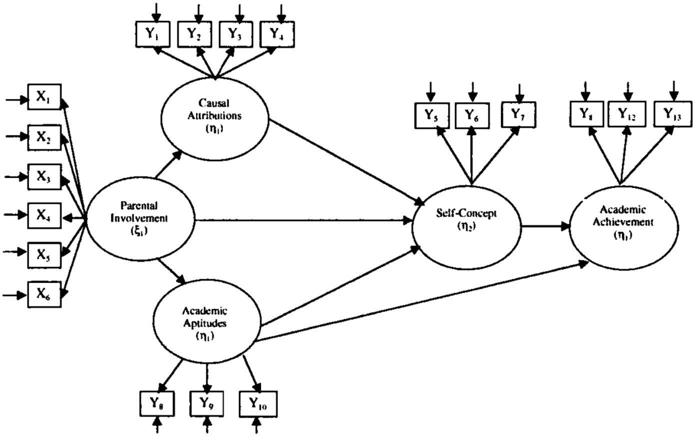
1. Uses a structural equation model
2. Includes a correlation matrix, vector of standard deviations, and vector of means, or covariance matrix (**the latter is preferred**)
3. Read the data into `R` (see below).
4. Be prepared to share this new class.

A Structural Equation Model of Parental Involvement, Motivational and Aptitudinal Characteristics, and Academic Achievement

Gonzalez-Pienda, et al. (2002). Journal of Experimental Education

> *The authors used the structural equation model (SEM) approach to test a model hypothesizing the influence of parental involvement on students' academic aptitudes, self-concept, and causal attributions, as well as the influence of the 3 variables on academic achievement. The theoretical model was contrasted in a group of 12- to 18-year-old adolescents (N = 261) attending various educational centers.*

FIGURE 1. A priori model of causal paths among parental involvement, aptitudinal and motivational characteristics, and academic achievement.



Note. Family variables: X_1 = achievement expectations, X_2 = help, X_3 = interest, X_4 = capacity expectations, X_5 = satisfaction, X_6 = reinforcement. Personal variables: Y_1 = capacity as cause of success in mathematical tasks, Y_2 = effort as cause of success in mathematical tasks, Y_3 = capacity as cause of success in verbal tasks, Y_4 = effort as cause of success in verbal tasks, Y_5 = mathematical self-concept, Y_6 = verbal self-concept, Y_7 = self-concept in remaining areas, Y_8 = verbal aptitude, Y_9 = reasoning aptitude, Y_{10} = calculus aptitude. Achievement variables: Y_{11} = mathematical achievement, Y_{12} = verbal achievement, Y_{13} = global achievement in remaining areas.

TABLE 1
Correlation Matrix, Means, and Standard Deviations for Model a

	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Y10	Y11	Y12	Y13	X1	X2	X3	X4	X5	X6
Y1	—																		
Y2	.60	—																	
Y3	.51	.38	—																
Y4	.27	.56	.48	—															
Y5	.58	.41	.15	.00	—														
Y6	.28	.26	.54	.30	.28	—													
Y7	.41	.36	.44	.23	.58	.63	—												
Y8	.24	.12	.10	.07	.37	.27	.32	—											
Y9	.29	.20	.10	.04	.39	.20	.38	.48	—										
Y10	.35	.27	.19	.13	.44	.22	.42	.47	.60	—									
Y11	.44	.31	.25	.05	.55	.34	.60	.41	.46	.46	—								
Y12	.38	.29	.37	.17	.48	.52	.67	.35	.48	.44	.77	—							
Y13	.39	.26	.26	.07	.52	.42	.64	.43	.52	.48	.82	.86	—						
X1	.32	.35	.30	.16	.43	.32	.57	.18	.30	.38	.50	.50	.47	—					
X2	.16	.30	.25	.32	.07	.19	.25	−.01	−.06	−.04	.10	.13	.05	.23	—				
X3	.15	.24	.25	.28	.12	.23	.31	.07	−.03	.06	.07	.15	.06	.27	.71	—			
X4	.40	.37	.37	.20	.53	.45	.73	.20	.30	.31	.54	.57	.56	.65	.43	.41	—		
X5	.30	.27	.24	.13	.42	.37	.65	.12	.31	.23	.49	.50	.51	.47	.41	.42	.75	—	
X6	−.01	−.01	−.00	.02	.02	.07	.08	−.12	−.11	−.12	−.05	.00	−.07	.14	.43	.44	.31	.40	—
M	2.93	3.38	3.42	3.74	3.45	3.89	4.25	27.92	19.61	18.27	2.82	3.05	3.15	4.32	3.93	4.02	3.59	3.92	3.21
SD	1.17	.95	.93	.87	1.33	.91	1.00	6.56	6.88	6.05	1.04	1.01	1.00	.48	.62	.61	.64	.62	.72

Note. Family variables: X1 = achievement expectations, X2 = help, X3 = interest, X4 = capacity expectations, X5 = satisfaction, X6 = reinforcement. Personal variables: Y1 = capacity as cause of success in mathematical tasks, Y2 = effort as cause of success in mathematical tasks, Y3 = capacity as cause of success in verbal tasks, Y4 = effort as cause of success in verbal tasks, Y5 = mathematical self-concept, Y6 = verbal self-concept, Y7 = self-concept in remaining areas, Y8 = verbal aptitude, Y9 = reasoning aptitude, Y10 = calculus aptitude. Achievement Variables: Y11 = mathematical achievement, Y12 = verbal achievement, Y13 = global achievement in remaining areas.

```
lowerTri <- '
1
.60 1
.51 .38 1
.27 .56 .48 1
.58 .41 .15 .00 1
.28 .26 .54 .30 .28 1
.41 .36 .44 .23 .58 .63 1
.24 .12 .10 .07 .37 .27 .32 1
.29 .20 .10 .04 .39 .20 .38 .48 1
.35 .27 .19 .13 .44 .22 .42 .47 .60 1
.44 .31 .25 .05 .55 .34 .60 .41 .46 .46 1
.38 .29 .37 .17 .48 .52 .67 .35 .48 .44 .77 1
.39 .26 .26 .07 .52 .42 .64 .43 .52 .48 .82 .86 1
.32 .35 .30 .16 .43 .32 .57 .18 .30 .38 .50 .50 .47 1
.16 .30 .25 .32 .07 .19 .25 -.01 -.06 -.04 .10 .13 .05 .23 1
.15 .24 .25 .28 .12 .23 .31 .07 -.03 .06 .07 .15 .06 .27 .71 1
.40 .37 .37 .20 .53 .45 .73 .20 .30 .31 .54 .57 .56 .65 .43 .41 1
.30 .27 .24 .13 .42 .37 .65 .12 .31 .23 .49 .50 .51 .47 .41 .42 .75 1
-.01 -.01 -0 .02 .02 .07 .08 -.12 -.11 -.12 -.05 0 -.07 .14 .43 .44 .31 .4 1
'
corMat <- getCov(lowerTri,
                 names = c(paste0("Y", 1:13), paste0("X", 1:6)))
sdVec <- c(1.17, .95, .93, .87, 1.33, .91, 1.00, 6.56, 6.88, 6.05,
           1.04, 1.01, 1.00, .48, .62, .61, .64, .62, .72)
covMat <- (sdVec %*% t(sdVec)) * corMat # convert cor matrix to a cov matrix
meanVec <- c(2.93, 3.38, 3.42, 3.74, 3.45, 3.89, 4.25, 27.92, 19.61,
             18.27, 2.82, 3.05, 3.15, 4.32, 3.93, 4.02, 3.59, 3.92, 3.21)
```