Statistical Analysis Using Structural Equation Models

EPsy 8266

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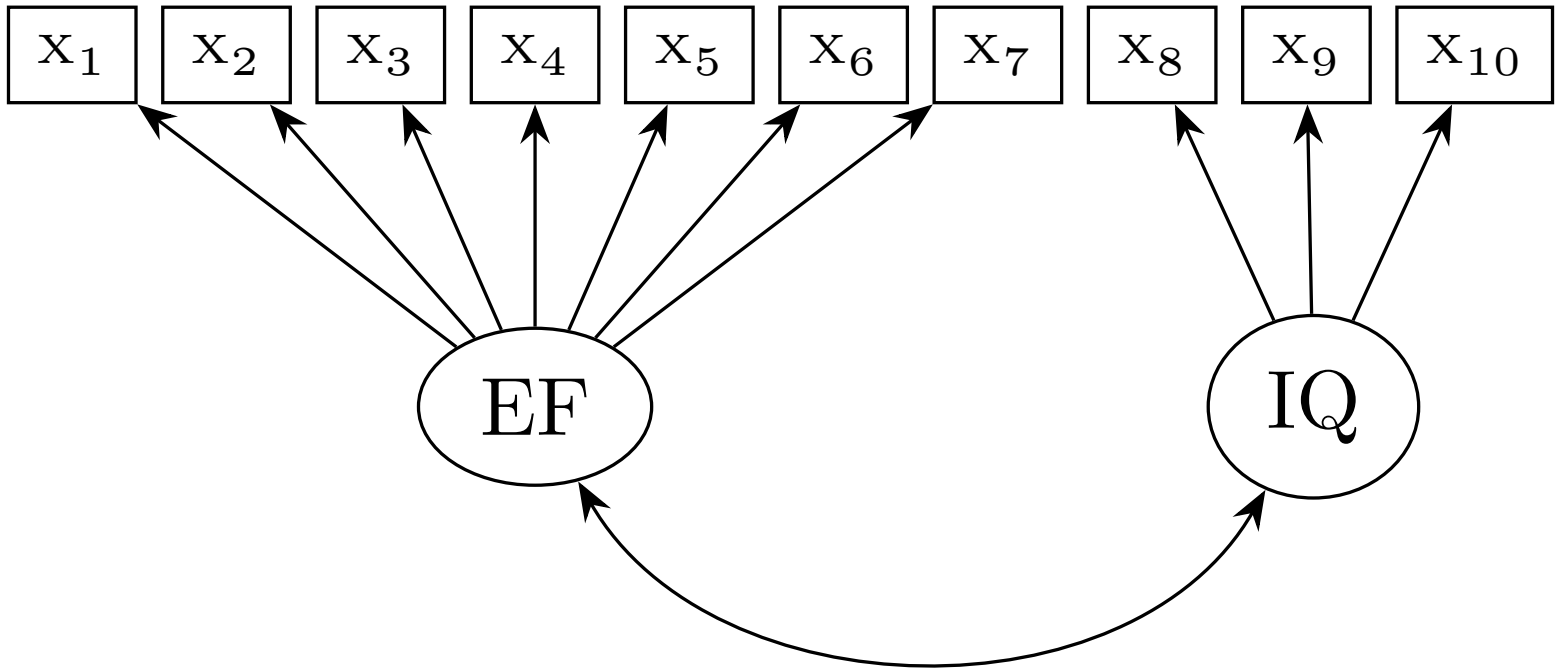
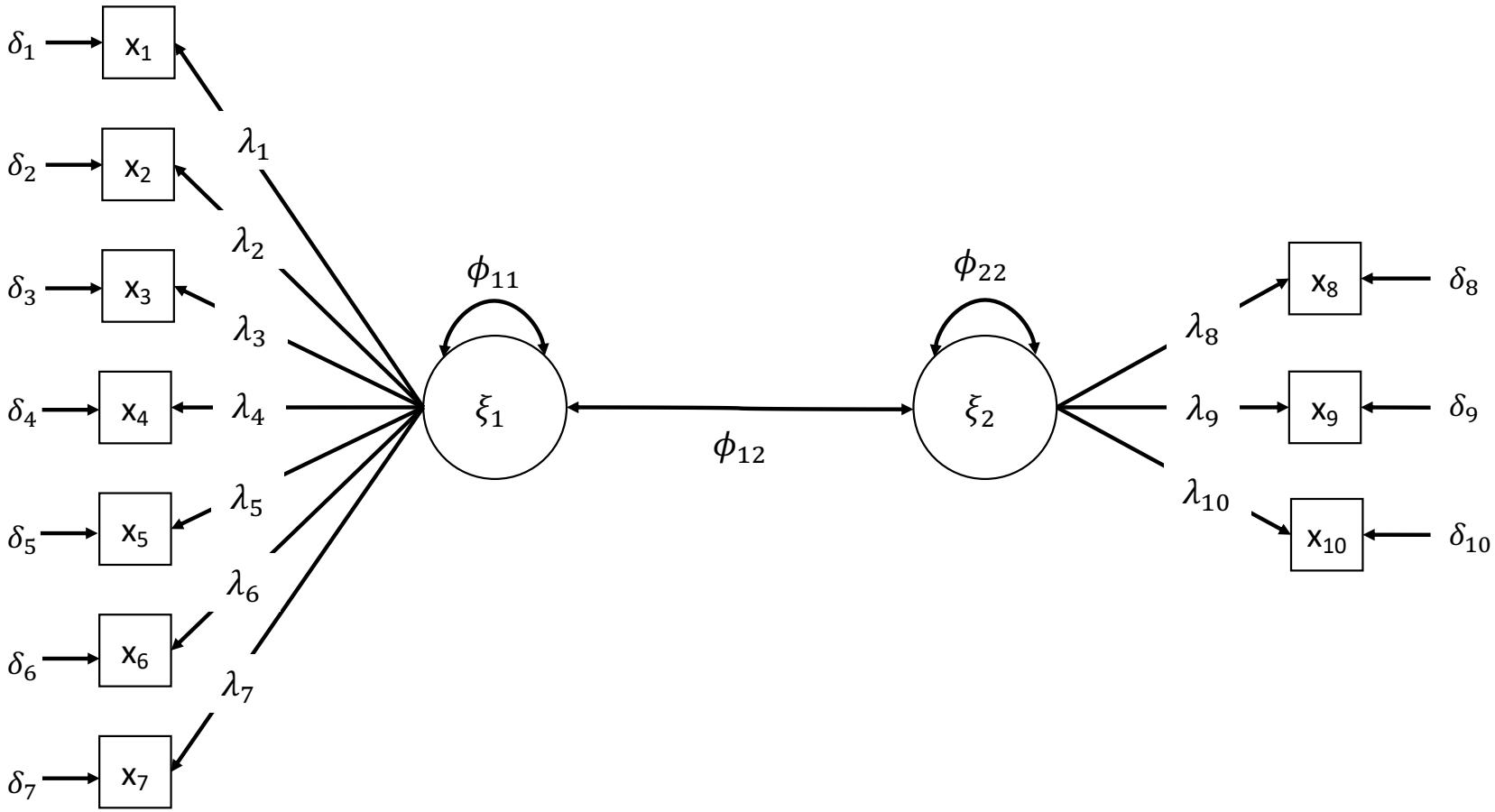
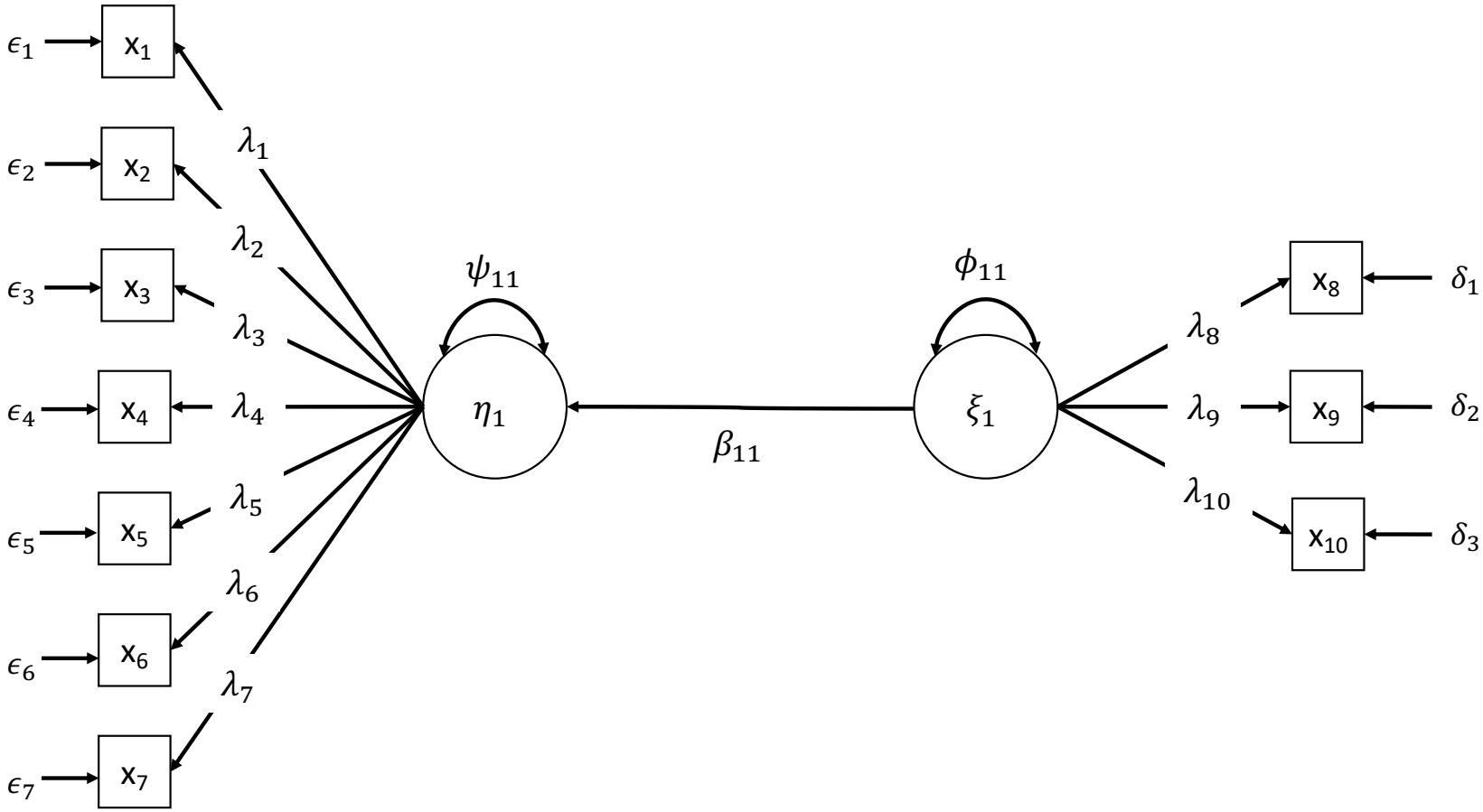
3/13/19


x1
x2
x3
x4
x5
x6
x7
x8
x9
x10
EF
IQ

From CFA ...


δ_1
x_1
δ_2
x_2
δ_3
x_3
δ_4
x_4
δ_5
x_5
δ_6
x_6
δ_7
x_7
λ_1
λ_2
λ_3
λ_4
λ_5
λ_6
λ_7
ϕ_{11}
ξ_1
ϕ_{12}
ξ_2
ϕ_{22}
λ_8
λ_9
λ_{10}
x_8
x_9
x_{10}
δ_8
δ_9
δ_{10}

... to SR model


ϵ_1
x_1
ϵ_2
x_2
ϵ_3
x_3
ϵ_4
x_4
ϵ_5
x_5
ϵ_6
x_6
ϵ_7
x_7
λ_1
λ_2
λ_3
λ_4
λ_5
λ_6
λ_7
ψ_{11}
η_1
β_{11}
ϕ_{11}
ξ_1
λ_8
λ_9
λ_{10}
x_8
x_9
x_{10}
δ_1
δ_2
δ_3

What happened?

Initially, IQ and EF had an unanalyzed (correlational) association

Now, IQ is considered a cause of EF

EF is now endogenous

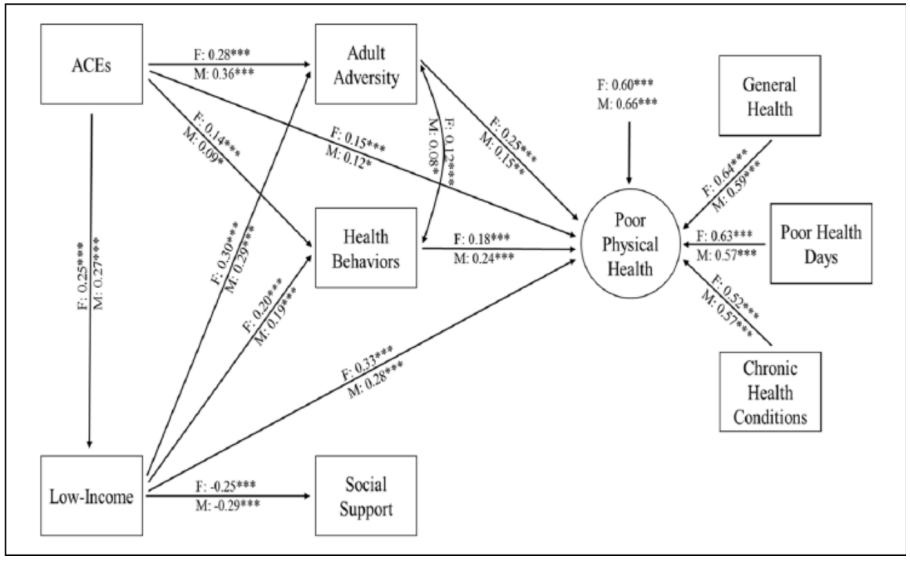
LISREL notation is different now (lavaan syntax is the same)

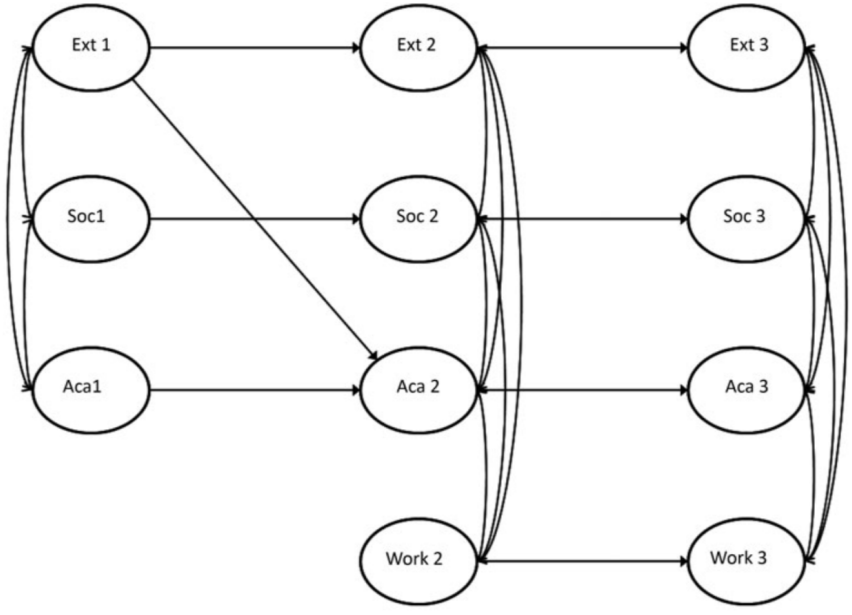
Distinctions from observed variable (OV) path analysis

- In OV path analysis, exogenous variables are assumed to be measured without error
  - In an SR model with exogenous factors, then measurement error is partialed out.
- In OV path analysis, disturbances contain measurement error and omitted causes.
  - in an SR model with endogenous factors, then the disturbances contain only omitted causes.

SR models

Can be either a fully or partially latent SR model

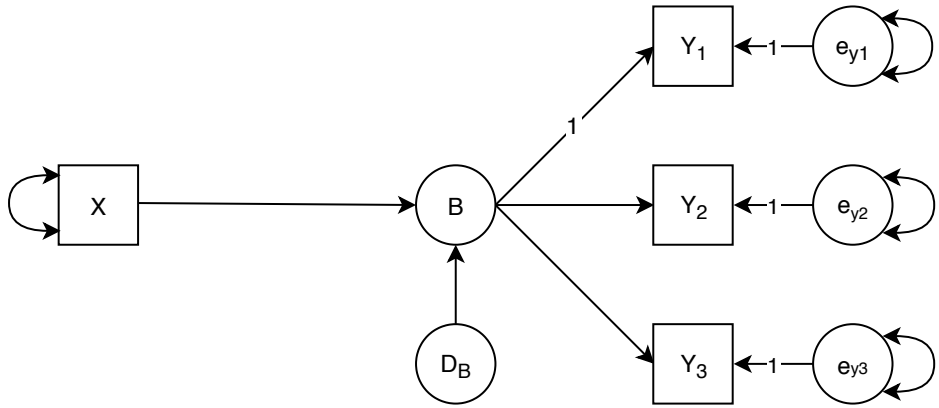




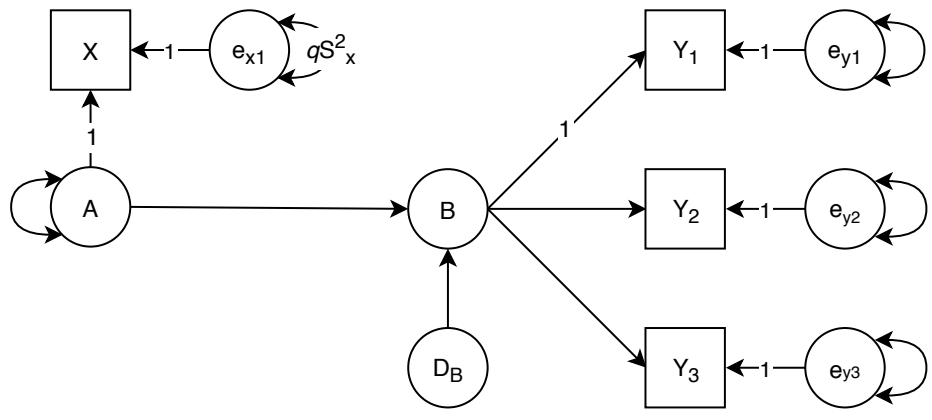
Single indicators

- Single indicators do not have to be treated as observed.
- Factors could be created where the error variance is fixed (based on previous research or theory).
  - It does not affect the df
  - Need to specify a value (or a range of values) because it will likely result in identification problems
  - A path model could be respecified so that every single indicator has measurement error partialled out

Single indicator as observed



Single indicator as latent



where q is $1 - R_{xx}$

Why you might want to do this?

Is anyone variable measured perfectly?

Hayduk & Littvay (2002) mention that demographic variables are likely always reported with some error

Possibly set error variance to 5% to account for this.

Relatedly, is it better to have many okay or potentially unrelated indicators or one really good one?

Identification in SEMs

- If the CFA is identified AND
- The structural paths are (same rules as path analysis) THEN
- The model is identified (this is a **sufficient** condition)
- For a single indicator factor, if pattern coefficient and error variance fixed, it's identified.

CFA rules

- **Standard CFA** is identified if 1) single factor with 3 indicators or 2) two or more factors each with 2 indicators

Indicators with correlated errors for CFAs (Kline, p. 203)

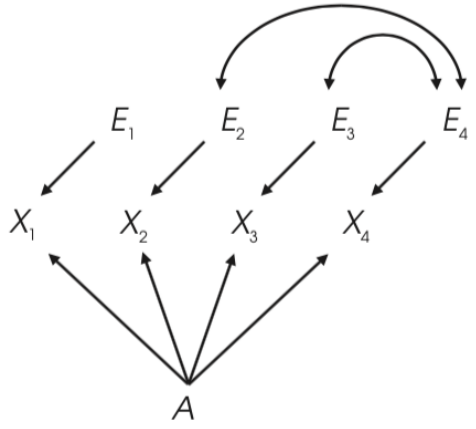
For each factor, at least one of the following must hold: (Rule 9.2a)

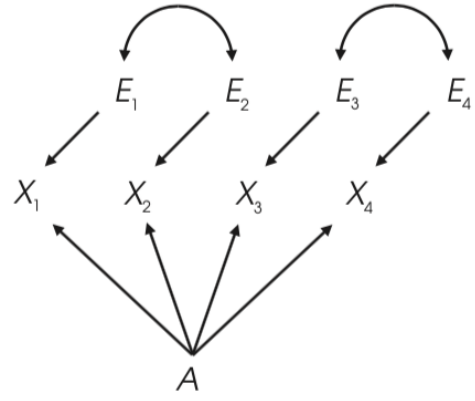
1. There are at least three indicators whose errors are uncorrelated with each other.
2. There are at least two indicators whose errors are uncorrelated and either
   a. the errors of both indicators are not correlated with the error term of a third indicator for a different factor, or
   b. an equality constraint is imposed on the loadings of the two indicators.

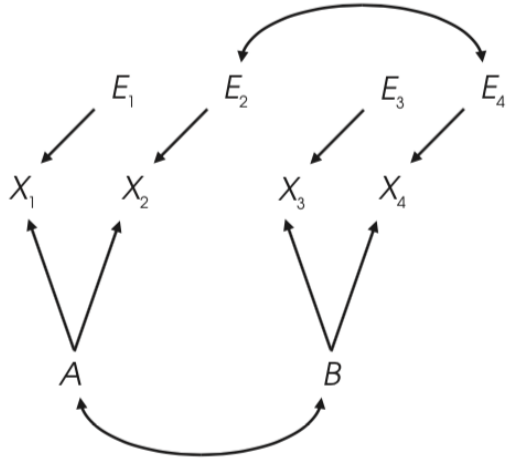
For every pair of factors, there are at least two indicators, one from each factor, whose error terms are uncorrelated. (Rule 9.2b)

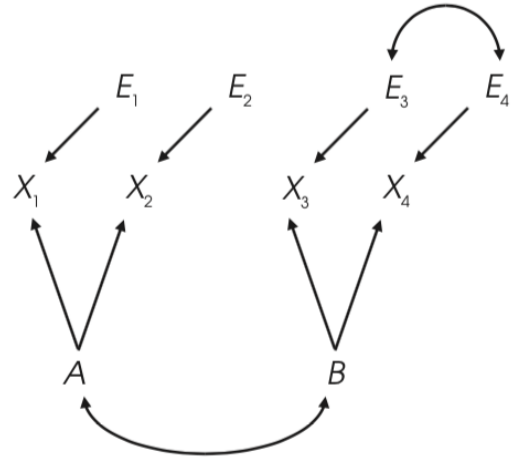
For every indicator, there is at least one other indicator (not necessarily of the same factor) with which its error term is not correlated. (Rule 9.2c)

Identified or not?









Complex CFA structures (Kline, p. 204)

Pattern coefficients

For every complex indicator in a nonstandard CFA model: (Rule 9.3)

In order for the multiple pattern coefficients to be identified, both of the following must hold:

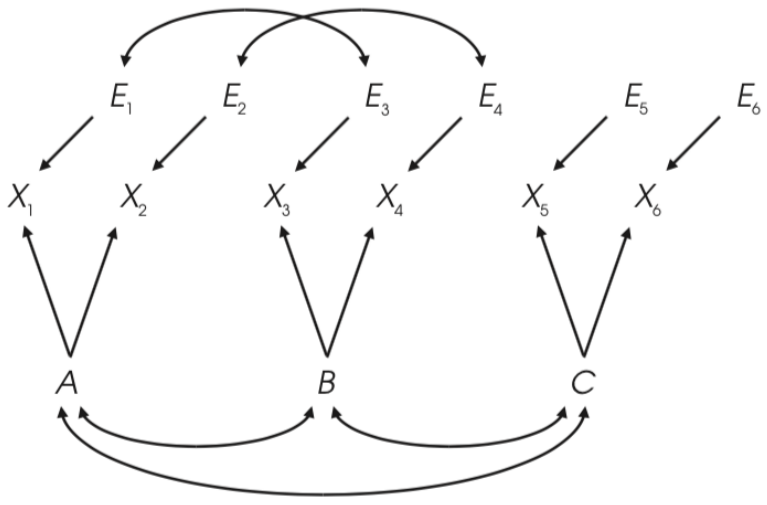
1. *Each factor on which the complex indicator depends* must satisfy Rule 9.2a for a minimum number of indicators.
2. *Every pair of those factors* must satisfy Rule 9.2b that each factor has an indicator that does not have an error correlation with a corresponding indicator on the other factor of that pair.

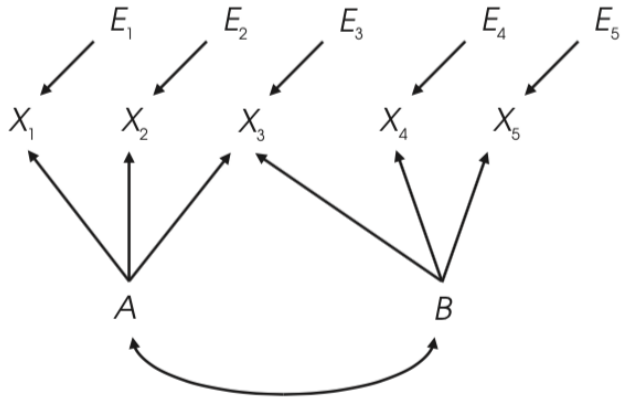
Error correlations

In order for *error correlations* that involve complex indicators to be identified, both of the following must hold: (Rule 9.4)

1. Rule 9.3 is satisfied.
2. For each factor on which a complex indicator depends, there must be at least one indicator with a single loading that does not have an error correlation with the complex indicator.

Identified or not?





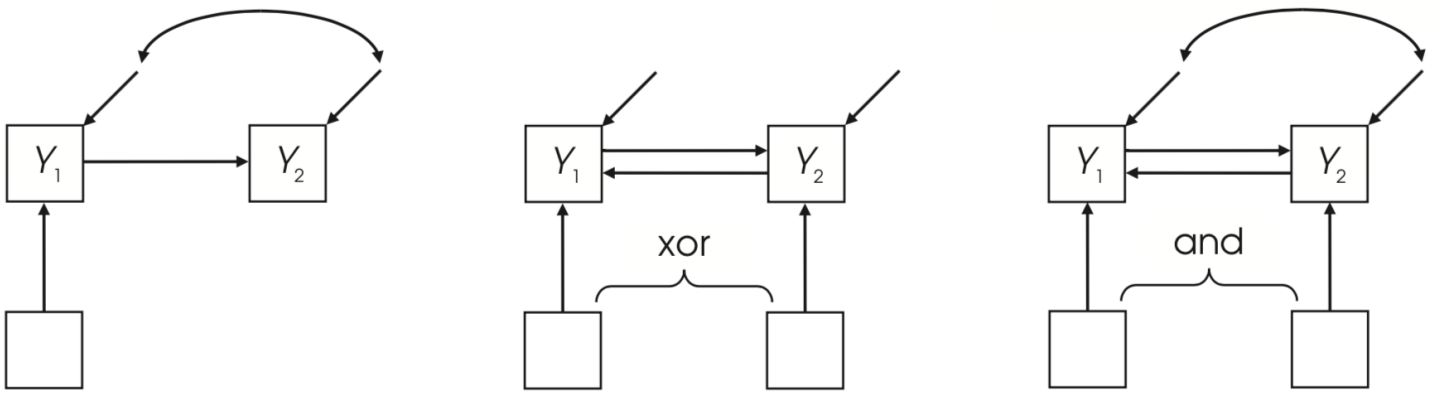
Ridgon's rules for non-recursive models

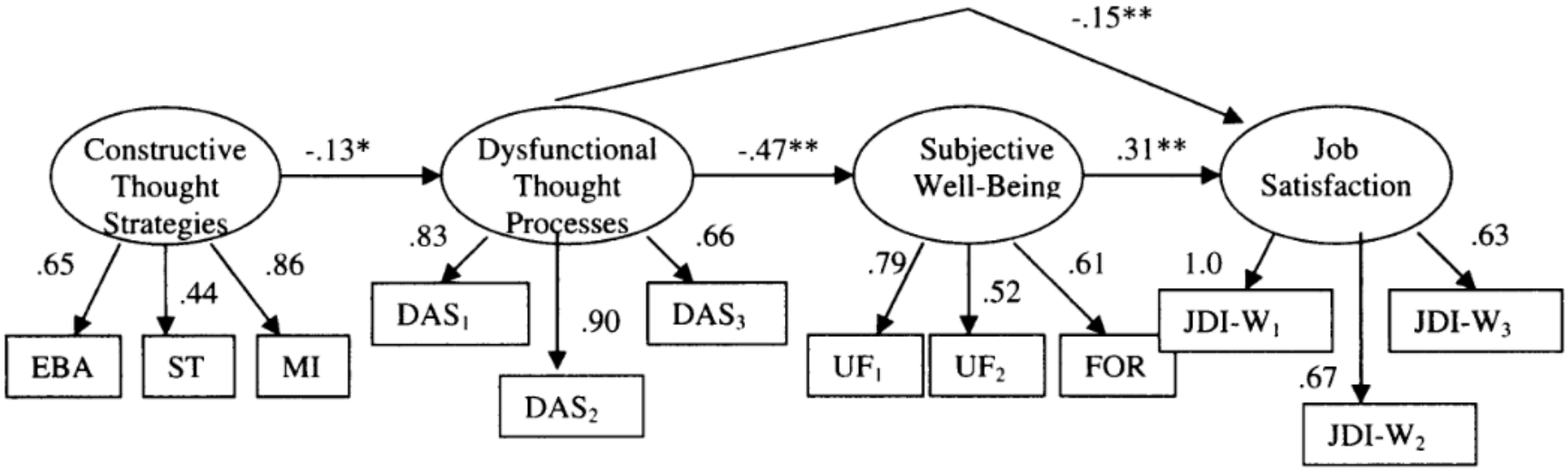
Direct feedback loop

(a) Bow pattern

(b) No disturbance correlation

(c) With disturbance correlation



Recall recursive models are identified.

Constructive Thought Strategies and Job Satisfaction: A Preliminary Examination - Houghton & Jinkerson (2007)