Statistical Analysis Using Structural Equation Models

EPsy 8266

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Motivation

- You measure executive functioning (EF) in two groups of children: a) children living in homeless shelters and b) children not living in homeless shelters.
- You administer several EF tasks (e.g., Stroop, peg tapping)
- Ultimately, you'd like to be able to test if children not living in homeless shelters have greater EF than children in homeless shelters.
- Can you just sum up the scores on the tasks and compare the children using a t-test? Why? Why not?

Motivation

- You want to measure problem behaviors from childhood to early adulthood.
- You collect data on problem behaviors at 7 years, 10 years, 14 years, 18, and 22 years.
- Ultimately, you'd like to examine how problem behaviors develop from childhood to adulthood (does it decrease? increase? is it linear? nonlinear?)
- Can you just fit a latent growth curve model?

Multi-samples SEM

- In a multi-samples SEM, we want to know do the parameters of interest vary appreciable across samples.
- What are these *parameters*?

Multi-samples SEM

- In a multi-samples SEM, we want to know do the parameters of interest vary appreciable across samples.
- What are these *parameters*?
- This is analogous to whether group (living in a homeless shelter or not) or time (problem behaviors at 7, 10, 14, 18, and 22) **moderate** the relations in our SEM.
- This means that the group/time affects the estimated parameters of our model and we can't collapse over group/time and these parameters must be estimated separately by group/time.

How might you examine this?

One approach

- For each sample (group or time), fit a separate SEM model.
- Compare the unstandardized pattern and structural coefficients.
- If the values are quite different, conclude that these parameters are not the same.

A better approach

- Use a single SEM to estimate the samples simultaneously (i.e., that estimates all the parameters for all the samples).
- Progressively constrain unstandardized parameters systematically across the groups to test if equality reduces model fit (**free baseline approach**.
- Compare the models using chi-square test of difference, RMSEA, and/or CFI
- If constraining doesn't deteriorate fit significantly (or appreciably) conclude that parameters could be equal in the population.

Multiple-samples CFA

Goal: To assess whether the suite indicators are **invariant**.

How:

1. Specify a measurement model (CFA) with a mean structure,
2. Constrain to equality over the samples the unstandardized parameters,
3. Compare a series of models with differing levels of constraints for the samples.

Measurement invariance

- **Measurement invariance (MI)** - scores from the way a construct is operationalized have the same meaning under different conditions.
- Conditions could be time (month 12, month 24, and so on), methods (paper/pencil vs. Internet-delivered), or group memberships (USA sample vs. Russia sample).
- **Longitudinal measurement invariance** - refers specifically to time. The methods are similar.
  - Without this can't ascertain if change over time is because of real change on the factor or the factor changed.
  - Using the same test over multiple occasions doesn't guarantee longitudinal MI.
  - See Little's or Newsom's longitudinal book.

Types of MI

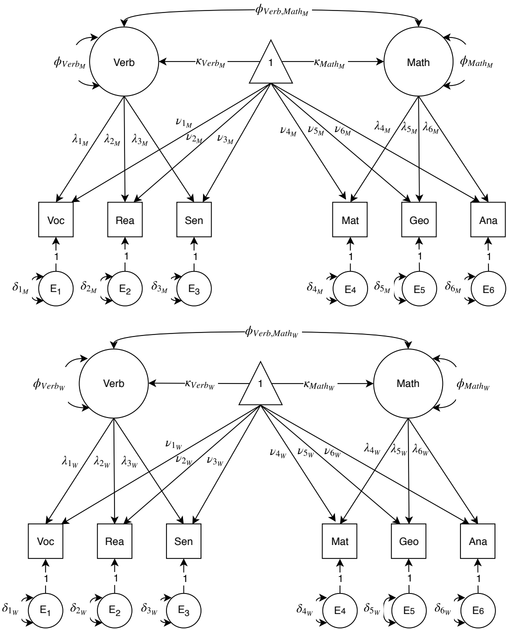
Pre-MI - Does your proposed factor structure actuall fit? If yes, then proceed ...

- Configural - Does the same number of factors and relationship between factors and indicators hold in each group?
  - No parameters are constrained.
- Weak - Assuming configural invariance holds, constrain the unstandardized pattern coefficients across the groups.
- Strong - Assuming weak invariance holds, constrain the unstandardized intercepts.
- Strict - Assuming strong invariance holds, constrain the error variances and covariances across the groups.

Interest inventory

- Data from a cognitive, personality, and vocational interest inventory
- 33 variables on 250 participants.
- `vocab` (Vocabulary test), `reading` (Reading comprehension), `sentcomp` (Sentence completion) - verbal IQ
- `mathmtcs` (Mathematics), `geometry` (Geometry), `analyrea` (Analytical reasoning) - math IQ
- `sex`: coded 1 for male and 2 for female
- `educ`: Years of education
- age: Age, in years

Are there sex differences in verbal and math IQ?



$\phi_{Verb,Math_M}$
Verb
κ_{Verb_M}
1
κ_{Math_M}
Math
ϕ_{Verb_M}
ϕ_{Math_M}
λ_{1_M} λ_{2_M} λ_{3_M}
ν_{1_M} ν_{2_M} ν_{3_M}
ν_{4_M} ν_{5_M} ν_{6_M}
λ_{4_M} λ_{5_M} λ_{6_M}
Voc Rea Sen Mat Geo Ana
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δ_{1_M} E_1 δ_{2_M} E_2 δ_{3_M} E_3 δ_{4_M} E_4 δ_{5_M} E_5 δ_{6_M} E_6
$\phi_{Verb,Math_W}$
Verb
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1
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λ_{1_W} λ_{2_W} λ_{3_W}
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ν_{4_W} ν_{5_W} ν_{6_W}
λ_{4_W} λ_{5_W} λ_{6_W}
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1 1 1 1 1 1
δ_{1_W} E_1 δ_{2_W} E_2 δ_{3_W} E_3 δ_{4_W} E_4 δ_{5_W} E_5 δ_{6_W} E_6

Male Model

$$\begin{aligned}
Voc &= \nu_{1_M} + \lambda_{1_M} Verb + \delta_{1_M} \\
Rea &= \nu_{2_M} + \lambda_{2_M} Verb + \delta_{2_M} \\
Sen &= \nu_{3_M} + \lambda_{3_M} Verb + \delta_{3_M} \\
Mat &= \nu_{4_M} + \lambda_{4_M} Math + \delta_{4_M} \\
Geo &= \nu_{5_M} + \lambda_{5_M} Math + \delta_{5_M} \\
Ana &= \nu_{6_M} + \lambda_{6_M} Math + \delta_{6_M}
\end{aligned}$$

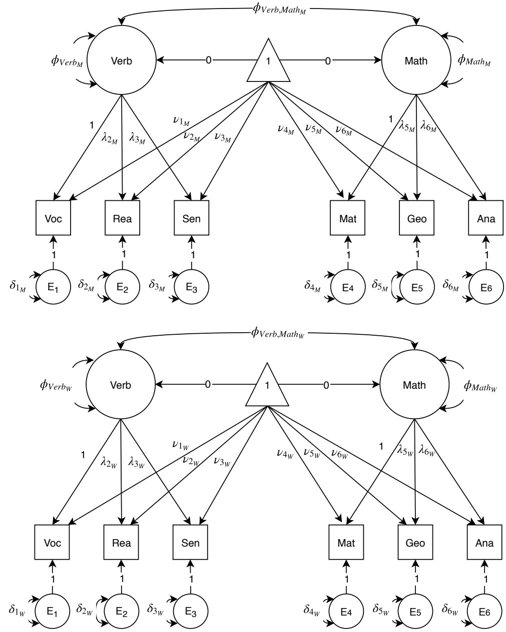
$$\phi_{Verb_M}, \phi_{Math_M}, \phi_{Verb,Math_M}, \kappa_{Verb_M}, \kappa_{Math_M}$$

Female Model

$$\begin{aligned}
Voc &= \nu_{1_W} + \lambda_{1_W} Verb + \delta_{1_W} \\
Rea &= \nu_{2_W} + \lambda_{2_W} Verb + \delta_{2_W} \\
Sen &= \nu_{3_W} + \lambda_{3_W} Verb + \delta_{3_W} \\
Mat &= \nu_{4_W} + \lambda_{4_W} Math + \delta_{4_W} \\
Geo &= \nu_{5_W} + \lambda_{5_W} Math + \delta_{5_W} \\
Ana &= \nu_{6_W} + \lambda_{6_W} Math + \delta_{6_W}
\end{aligned}$$

$$\phi_{Verb_W}, \phi_{Math_W}, \phi_{Verb_W,Math_W}, \kappa_{Verb_W}, \kappa_{Math_W}$$

- Is this model identified?
- How many unique elements do we have in our covariance matrix? In the mean matrix?
- How many by sex?
- Does it have a scale?
- By default for a multi-sample model, lavaan fixes the first indicator per factor to 1 (scale) and the factor mean to 0 (to identify the means).



What is the implicit assumption here?

Step 1: Configural model

```
library(lavaan)
interest <- read.csv("https://bit.ly/2IF3Hcm")
interest$sex <- ifelse(interest$sex == 2, "female", "male")
config.mod <- "
verb  =~ vocab + reading + sentcomp
math =~ mathmtcs + geometry + analyrea
"
config.fit <- cfa(config.mod, data = interest, group = "sex")
fitMeasures(config.fit, c("chisq", "df", "pvalue", "cfi", "rmsea"))
```

```
##  chisq     df pvalue    cfi  rmsea
## 18.413 16.000  0.300  0.998  0.035
```

Step 1: Configural model

```
summary(config.fit, fit.measure = TRUE)

##   Number of free parameters                         38
##
##   Number of observations per group
##   male                                             122
##   female                                           128
##
##   Estimator                                         ML
##   Model Fit Test Statistic                      18.413
##   Degrees of freedom                                16
##   P-value (Chi-square)                           0.300
##
## Chi-square for each group:
##
##   female                                         7.908
##   male                                          10.505
##
## Model test baseline model:
##
##   Minimum Function Test Statistic             1303.205
##   Degrees of freedom                                30
##   P-value                                        0.000
##
##   Comparative Fit Index (CFI)                    0.998
##   RMSEA                                          0.035
##   90 Percent Confidence Interval          0.000  0.093
##
##   SRMR                                           0.014
```

Step 1: Configural model - female

```
## Group 1 [female]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   verb =~
##     vocab             1.000
##     reading           0.942    0.068   13.820    0.000
##     sentcomp          0.951    0.070   13.514    0.000
##   math =~
##     mathmtcs          1.000
##     geometry          0.857    0.069   12.462    0.000
##     analyrea          0.953    0.068   13.936    0.000
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   verb ~~
##     math              0.611    0.096    6.341    0.000
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)
##    .vocab             0.025    0.082    0.301    0.764
##    .reading           0.054    0.086    0.625    0.532
##    .sentcomp         -0.005    0.088   -0.056    0.955
##    .mathmtcs          0.274    0.087    3.131    0.002
##    .geometry          0.219    0.087    2.522    0.012
##    .analyrea          0.275    0.091    3.018    0.003
##     verb              0.000
##     math              0.000
```

Step 1: Configural model - female

```
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)
##    .vocab             0.098    0.031    3.178    0.001
##    .reading           0.269    0.043    6.251    0.000
##    .sentcomp          0.296    0.046    6.402    0.000
##    .mathmtcs          0.121    0.037    3.295    0.001
##    .geometry          0.334    0.050    6.673    0.000
##    .analyrea          0.281    0.048    5.913    0.000
##     verb              0.759    0.110    6.897    0.000
##     math              0.858    0.126    6.813    0.000
```

Step 1: Configural model - male

```
## Group 2 [male]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   verb =~
##     vocab             1.000
##     reading           0.856    0.057   14.994    0.000
##     sentcomp          0.860    0.054   15.942    0.000
##   math =~
##     mathmtcs          1.000
##     geometry          0.870    0.065   13.415    0.000
##     analyrea          0.935    0.059   15.937    0.000
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   verb ~~
##     math              0.913    0.133    6.884    0.000
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)
##    .vocab             0.159    0.096    1.654    0.098
##    .reading           0.220    0.091    2.433    0.015
##    .sentcomp          0.156    0.089    1.755    0.079
##    .mathmtcs         -0.071    0.098   -0.724    0.469
##    .geometry          0.001    0.097    0.008    0.993
##    .analyrea          0.070    0.097    0.725    0.468
##     verb              0.000
##     math              0.000
```

Step 1: Configural model - male

```
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)
##    .vocab             0.119    0.031    3.835    0.000
##    .reading           0.262    0.041    6.454    0.000
##    .sentcomp          0.218    0.036    6.134    0.000
##    .mathmtcs          0.144    0.035    4.105    0.000
##    .geometry          0.360    0.054    6.699    0.000
##    .analyrea          0.240    0.041    5.792    0.000
##     verb              1.007    0.146    6.900    0.000
##     math              1.036    0.153    6.774    0.000
```

Step 1: Configural model - default lavaan method

```
config.mod.lav <- "
# define factor/unstd. pattern coef
verb  =~ c(1, 1)*vocab + reading + sentcomp
math =~ c(1, 1)*mathmtcs + geometry + analyrea

# factor intercepts
verb ~ c(0,0)*1
math ~ c(0,0)*1

# factor variances/covariance
verb ~~ verb + math
math ~~ math

# intercepts
vocab ~ 1
reading ~ 1
sentcomp ~ 1
mathmtcs ~ 1
geometry ~ 1
analyrea ~ 1

# residual variances
vocab ~~ vocab
reading ~~ reading
sentcomp ~~ sentcomp
mathmtcs ~~ mathmtcs
geometry ~~ geometry
analyrea ~~ analyrea
"
```

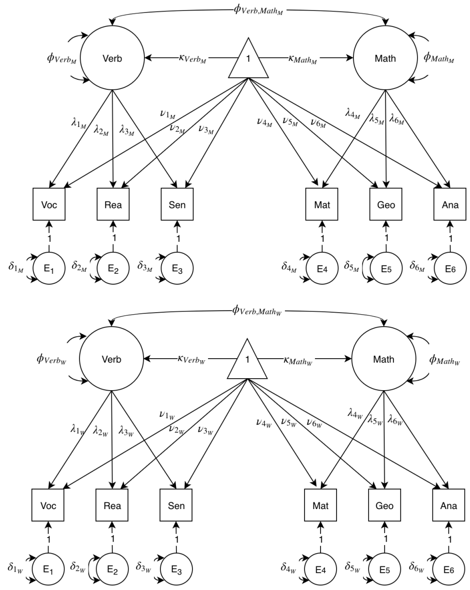
Step 1: Configural model - marker variable approach

```
config.mod.mark <- "
# define factor/unstd. pattern coef
verb  =~ c(1, 1)*vocab + reading + sentcomp
math =~ c(1, 1)*mathmtcs + geometry + analyrea

# factor intercepts
verb ~ 1
math ~ 1

# factor variances/covariance
verb ~~ verb + math
math ~~ math

# intercepts
vocab ~ c(0,0)*1
reading ~ 1
sentcomp ~ 1
mathmtcs ~ c(0,0)*1
geometry ~ 1
analyrea ~ 1

# residual variances
vocab ~~ vocab
reading ~~ reading
sentcomp ~~ sentcomp
mathmtcs ~~ mathmtcs
geometry ~~ geometry
analyrea ~~ analyrea
"
```

A non-arbitrary approach to identifying a multi-samples CFA

Little et al., (2006) proposed the effects coding method.

- Constraining the average unstandardized pattern coefficient to 1.0 for each factor
  - $\sum_{i=1}^{I} \lambda_{if}^{g} = I$. g = group, f = factor, i = indicator, I = # of indicators.
- Constraining the average intercept of the same indicators to 0 for each factor
  - $\sum_{i=1}^{I} \nu_{if}^{g} = 0$.
- This method results in estimates of the **latent variances that are the average of the indicators' variances accounted for by the construct**.
- **Latent means are estimated as optimally weighted averages** of the set of indicator means for a given construct.
- The estimated latent variances and means reflect the observed metric of the indicators, optimally weighted by the degree to which each indicator represents the underlying latent construct.



$\phi_{Verb,Math_M}$
Verb
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ϕ_{Math_M}
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ν_{1_M} ν_{2_M} ν_{3_M}
ν_{4_M} ν_{5_M} ν_{6_M}
λ_{4_M} λ_{5_M} λ_{6_M}
Voc
Rea
Sen
Mat
Geo
Ana
1
δ_{1_M} E_1
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δ_{3_M} E_3
δ_{4_M} E4
δ_{5_M} E5
δ_{6_M} E6
$\phi_{Verb,Math_W}$
Verb
1
Math
ϕ_{Verb_W}
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λ_{1_W} λ_{2_W} λ_{3_W}
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ν_{4_W} ν_{5_W} ν_{6_W}
λ_{4_W} λ_{5_W} λ_{6_W}
Voc
Rea
Sen
Mat
Geo
Ana
1
δ_{1_W} E_1
δ_{2_W} E_2
δ_{3_W} E_3
δ_{4_W} E4
δ_{5_W} E5
δ_{6_W} E6

Step 1: Configural model - effects coding

```
config.mod <- "
# define factor/unstd. pattern coef
verb  =~ c(lam1m, lam1w)*vocab + c(lam2m, lam2w)*reading + c(lam3m, lam3w)*sentcomp
math =~ c(lam4m, lam4w)*mathmtcs + c(lam5m, lam5w)*geometry + c(lam6m, lam6w)*analyrea

# factor intercepts
verb ~ 1
math ~ 1

# factor variances/covariance
verb ~~ verb + math
math ~~ math

# intercepts
vocab ~ c(nu1m, nu1w)*1
reading ~ c(nu2m, nu2w)*1
sentcomp ~ c(nu3m, nu3w)*1
mathmtcs ~ c(nu4m, nu4w)*1
geometry ~ c(nu5m, nu5w)*1
analyrea ~ c(nu6m, nu6w)*1

# residual variances
vocab ~~ c(e1m, e1w)*vocab
reading ~~ c(e2m, e2w)*reading
sentcomp ~~ c(e3m, e3w)*sentcomp
mathmtcs ~~ c(e4m, e4w)*mathmtcs
geometry ~~ c(e5m, e5w)*geometry
analyrea ~~ c(e6m, e6w)*analyrea

# define constraints
lam1m == 3 - lam2m - lam3m
lam1w == 3 - lam2w - lam3w
lam4m == 3 - lam5m - lam6m
lam4w == 3 - lam5w - lam6w
nu1m == 0 - nu2m - nu3m
nu1w == 0 - nu2w - nu3w
nu4m == 0 - nu5m - nu6m
nu4w == 0 - nu5w - nu6w
"
config.fit <- lavaan(config.mod, data = interest, group = "sex")
```

```r
config.fit <- lavaan(config.mod, data = interest, group = "sex")
summary(config.fit)
```

```
## Group 1 [female]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   verb =~
##     vocab   (lm1w)    1.037    0.040   25.627    0.000
##     reading (lm2w)    0.977    0.046   21.341    0.000
##     sentcmp (lm3w)    0.986    0.047   21.055    0.000
##   math =~
##     mthmtcs (lm4w)    1.067    0.042   25.117    0.000
##     geomtry (lm5w)    0.915    0.048   18.878    0.000
##     analyre (lm6w)    1.018    0.046   21.923    0.000
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)
##     verb              0.024    0.078    0.313    0.754
##     math              0.256    0.081    3.168    0.002
##    .vocab   (nu1w)   -0.001    0.029   -0.025    0.980
##    .reading (nu2w)    0.030    0.036    0.832    0.405
##    .sentcmp (nu3w)   -0.029    0.037   -0.790    0.429
##    .mthmtcs (nu4w)    0.001    0.033    0.023    0.982
##    .geomtry (nu5w)   -0.015    0.041   -0.368    0.713
##    .analyre (nu6w)    0.014    0.039    0.371    0.711
```

```r
config.fit <- lavaan(config.mod, data = interest, group = "sex")
summary(config.fit)
```

```
## Group 2 [male]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   verb =~
##     vocab   (lm1m)    1.105    0.037   30.216    0.000
##     reading (lm2m)    0.945    0.041   22.834    0.000
##     sentcmp (lm3m)    0.950    0.039   24.114    0.000
##   math =~
##     mthmtcs (lm4m)    1.070    0.038   27.944    0.000
##     geomtry (lm5m)    0.931    0.046   20.435    0.000
##     analyre (lm6m)    1.000    0.041   24.488    0.000
##
## Intercepts:
##
##    .vocab   (nu1m)   -0.038    0.031   -1.226    0.220
##    .reading (nu2m)    0.052    0.037    1.413    0.158
##    .sentcmp (nu3m)   -0.014    0.035   -0.391    0.696
##    .mthmtcs (nu4m)   -0.071    0.033   -2.132    0.033
##    .geomtry (nu5m)    0.001    0.041    0.021    0.983
##    .analyre (nu6m)    0.070    0.037    1.926    0.054
##
## Constraints:
##                                                |Slack|
##     lam1m - (3-lam2m-lam3m)                      0.000
##     lam1w - (3-lam2w-lam3w)                      0.000
##     lam4m - (3-lam5m-lam6m)                      0.000
##     lam4w - (3-lam5w-lam6w)                      0.000
##     nu1m - (0-nu2m-nu3m)                         0.000
##     nu1w - (0-nu2w-nu3w)                         0.000
##     nu4m - (0-nu5m-nu6m)                         0.000
##     nu4w - (0-nu5w-nu6w)                         0.000
```

Configural model - summary

- Configural model has good fit (based on chi-square and fit statistics).
- Conclude this model is reasonable for both groups.
- Move onto weak invariance.

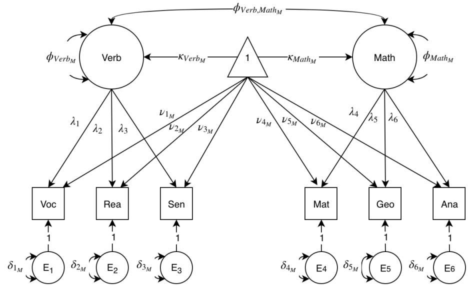
Weak Invariance

- Adds the constraint that unstandardized pattern coefficients are the same across groups.
- Weak invariance model is nested within configural model, can use chi-square test of difference.
  - Also, should consider change in RMSEA and CFI.
- If weak invariance not meaningful reduction in fit, constructs are manifested in the same way in each group
- Factor scores would be calculated using the same weighting scheme in all groups.

What if we fail to obtain weak invariance?

Gregorich (2006) provides two reasons

- The factors, or a subset of the items, have different meaning over the groups.
- **Extreme response style** - Low ERS, avoidance of extremes, high ERS, favoring extremes.
  - Maybe one group favors/discourages extremes or a group changes in how it views extremes over time.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Math
ϕ_{Math_M}
λ_1 λ_2 λ_3
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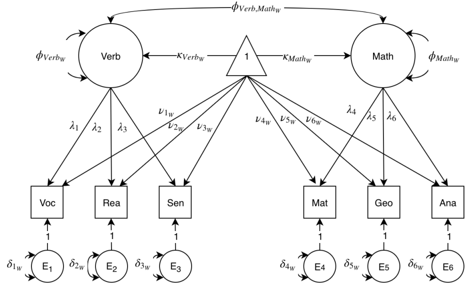
$\phi_{Verb,Math_W}$
ϕ_{Verb_W}
Verb
κ_{Verb_W}
1
κ_{Math_W}
Math
ϕ_{Math_W}
λ_1 λ_2 λ_3
ν_{1_W} ν_{2_W} ν_{3_W}
ν_{4_W} ν_{5_W} ν_{6_W}
λ_4 λ_5 λ_6
Voc
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1
δ_{1_W} E_1
δ_{2_W} E_2
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δ_{4_W} E_4
δ_{5_W} E_5
δ_{6_W} E_6

Male Model

$$\begin{aligned}
Voc &= \nu_{1_M} + \lambda_1 Verb + \delta_{1_M} \\
Rea &= \nu_{2_M} + \lambda_2 Verb + \delta_{2_M} \\
Sen &= \nu_{3_M} + \lambda_3 Verb + \delta_{3_M} \\
Mat &= \nu_{4_M} + \lambda_4 Math + \delta_{4_M} \\
Geo &= \nu_{5_M} + \lambda_5 Math + \delta_{5_M} \\
Ana &= \nu_{6_M} + \lambda_6 Math + \delta_{6_M}
\end{aligned}$$

$$\phi_{Verb_M}, \phi_{Math_M}, \phi_{Verb_M, Math_M}, \kappa_{Verb_M}, \kappa_{Math_M}$$

Female Model

$$\begin{aligned}
Voc &= \nu_{1_W} + \lambda_1 Verb + \delta_{1_W} \\
Rea &= \nu_{2_W} + \lambda_2 Verb + \delta_{2_W} \\
Sen &= \nu_{3_W} + \lambda_3 Verb + \delta_{3_W} \\
Mat &= \nu_{4_W} + \lambda_4 Math + \delta_{4_W} \\
Geo &= \nu_{5_W} + \lambda_5 Math + \delta_{5_W} \\
Ana &= \nu_{6_W} + \lambda_6 Math + \delta_{6_W}
\end{aligned}$$

$$\phi_{Verb_W}, \phi_{Math_W}, \phi_{Verb_W, Math_W}, \kappa_{Verb_W}, \kappa_{Math_W}$$

Step 2: Weak invariance

```
weak.mod <- "
# define factor/unstd. pattern coef
verb  =~ c(lam1, lam1)*vocab + c(lam2, lam2)*reading + c(lam3, lam3)*sentcomp
math =~ c(lam4, lam4)*mathmtcs + c(lam5, lam5)*geometry + c(lam6, lam6)*analyrea

# factor intercepts
verb ~ 1
math ~ 1

# factor variances/covariance
verb ~~ verb + math
math ~~ math

# intercepts
vocab ~ c(nu1m, nu1w)*1
reading ~ c(nu2m, nu2w)*1
sentcomp ~ c(nu3m, nu3w)*1
mathmtcs ~ c(nu4m, nu4w)*1
geometry ~ c(nu5m, nu5w)*1
analyrea ~ c(nu6m, nu6w)*1

# residual variances
vocab ~~ c(e1m, e1w)*vocab
reading ~~ c(e2m, e2w)*reading
sentcomp ~~ c(e3m, e3w)*sentcomp
mathmtcs ~~ c(e4m, e4w)*mathmtcs
geometry ~~ c(e5m, e5w)*geometry
analyrea ~~ c(e6m, e6w)*analyrea

# define constraints
lam1 == 3 - lam2 - lam3
lam4 == 3 - lam5 - lam6

nu1m == 0 - nu2m - nu3m
nu1w == 0 - nu2w - nu3w
nu4m == 0 - nu5m - nu6m
nu4w == 0 - nu5w - nu6w
"
```

```r
weak.fit <- lavaan(weak.mod, data = interest, group = "sex")
anova(weak.fit, config.fit)
```

```
## Chi Square Difference Test
##
##            Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## config.fit 16 3081.8 3215.6 18.413
## weak.fit   20 3075.4 3195.2 20.048     1.6348       4     0.8025
```

```r
fit.stats <- rbind(fitmeasures(config.fit, c("chisq", "df", "pvalue", "cfi", "rmsea", "srmr")),
      fitmeasures(weak.fit, c("chisq", "df", "pvalue", "cfi", "rmsea", "srmr")))
rownames(fit.stats) <- c("Configural", "Weak")
fit.stats
```

```
##               chisq df    pvalue       cfi       rmsea       srmr
## Configural 18.41320 16 0.3002620 0.9981046 0.034736151 0.01433416
## Weak       20.04805 20 0.4549276 0.9999623 0.004384013 0.02119742
```

```r
summary(weak.fit)
```

```
## Group 1 [female]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   verb =~
##     vocab   (lam1)    1.075    0.027   39.261    0.000
##     reading (lam2)    0.961    0.031   31.128    0.000
##     sentcmp (lam3)    0.964    0.030   31.807    0.000
##   math =~
##     mthmtcs (lam4)    1.069    0.028   37.574    0.000
##     geomtry (lam5)    0.924    0.033   27.818    0.000
##     analyre (lam6)    1.008    0.031   32.894    0.000
##
## Group 2 [male]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   verb =~
##     vocab   (lam1)    1.075    0.027   39.261    0.000
##     reading (lam2)    0.961    0.031   31.128    0.000
##     sentcmp (lam3)    0.964    0.030   31.807    0.000
##   math =~
##     mthmtcs (lam4)    1.069    0.028   37.574    0.000
##     geomtry (lam5)    0.924    0.033   27.818    0.000
##     analyre (lam6)    1.008    0.031   32.894    0.000
```

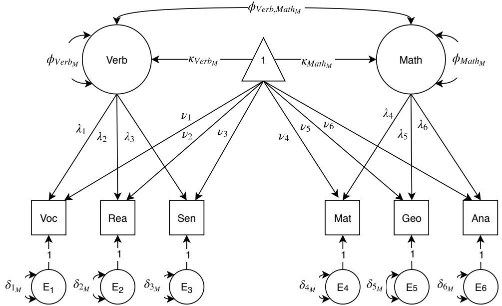
Strong Invariance

- Adds the constraint that unstandardized intercepts are the same across groups.
- Strong invariance model is nested within weak invariance model, can use chi-square test of difference.
- If strong invariance not meaningful reduction in fit, groups use the response scale of the indicator in a similar manner.
- Two people from two different groups with the same score on the factor *expected* to obtain the same observed score on the indicator.
- Ensures that ...
  1. Group differences in estimated factor means will be unbiased
  2. Group differences in indicator means or estimated factor scores will be related to the factor means.
- Minimum level required for group comparisons.

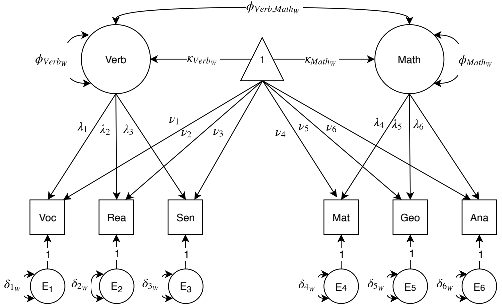
What if we fail to obtain strong invariance?

Systematic influences unrelated to the factor(s) decrease/increase the overall level of responding on an indicator in a given group (**differential additive response style**).

- Could be caused by cultural differences, cohorts effects, differences in how data are collected.
  - Measuring response time to a certain computer task. One sample uses a new computer and records response instantly; another computer is old, and response time adds a couple of deciseconds.
- **Differential item functioning (DIF)** - unequal pattern coefficients or intercepts over groups.
  - Score depends on membership in a group.
  - During instrument development, items flagged for DIF need to be altered or removed, so identifying indicators responsible for violating weak/strong invariance critical.



$\phi_{Verb,Math_M}$
Verb
Math
1
κ_{Verb_M}
κ_{Math_M}
ϕ_{Verb_M}
ϕ_{Math_M}
λ_1 λ_2 λ_3 λ_4 λ_5 λ_6
ν_1 ν_2 ν_3 ν_4 ν_5 ν_6
Voc
Rea
Sen
Mat
Geo
Ana
1
E_1 E_2 E_3 E_4 E_5 E_6
δ_{1_M} δ_{2_M} δ_{3_M} δ_{4_M} δ_{5_M} δ_{6_M}



$\phi_{Verb,Math_W}$
Verb
Math
1
κ_{Verb_W}
κ_{Math_W}
ϕ_{Verb_W}
ϕ_{Math_W}
λ_1 λ_2 λ_3 λ_4 λ_5 λ_6
ν_1 ν_2 ν_3 ν_4 ν_5 ν_6
Voc
Rea
Sen
Mat
Geo
Ana
1
E_1 E_2 E_3 E_4 E_5 E_6
δ_{1_W} δ_{2_W} δ_{3_W} δ_{4_W} δ_{5_W} δ_{6_W}

Male Model

$$\begin{aligned} Voc &= \nu_1 + \lambda_1 Verb + \delta_{1_M} \\ Rea &= \nu_2 + \lambda_2 Verb + \delta_{2_M} \\ Sen &= \nu_3 + \lambda_3 Verb + \delta_{3_M} \\ Mat &= \nu_4 + \lambda_4 Math + \delta_{4_M} \\ Geo &= \nu_5 + \lambda_5 Math + \delta_{5_M} \\ Ana &= \nu_6 + \lambda_6 Math + \delta_{6_M} \end{aligned}$$

$$\phi_{Verb_M}, \phi_{Math_M}, \phi_{Verb_M, Math_M}, \kappa_{Verb_M}, \kappa_{Math_M}$$

Female Model

$$\begin{aligned} Voc &= \nu_1 + \lambda_1 Verb + \delta_{1_W} \\ Rea &= \nu_2 + \lambda_2 Verb + \delta_{2_W} \\ Sen &= \nu_3 + \lambda_3 Verb + \delta_{3_W} \\ Mat &= \nu_4 + \lambda_4 Math + \delta_{4_W} \\ Geo &= \nu_5 + \lambda_5 Math + \delta_{5_W} \\ Ana &= \nu_6 + \lambda_6 Math + \delta_{6_W} \end{aligned}$$

$$\phi_{Verb_W}, \phi_{Math_W}, \phi_{Verb_W, Math_W}, \kappa_{Verb_W}, \kappa_{Math_W}$$

Step 3: Strong invariance

```
strong.mod <- "
# define factor/unstd. pattern coef
verb  =~ c(lam1, lam1)*vocab + c(lam2, lam2)*reading + c(lam3, lam3)*sentcomp
math =~ c(lam4, lam4)*mathmtcs + c(lam5, lam5)*geometry + c(lam6, lam6)*analyrea

# factor intercepts
verb ~ 1
math ~ 1

# factor variances/covariance
verb ~~ verb + math
math ~~ math

# intercepts
vocab ~ c(nu1, nu1)*1
reading ~ c(nu2, nu2)*1
sentcomp ~ c(nu3, nu3)*1
mathmtcs ~ c(nu4, nu4)*1
geometry ~ c(nu5, nu5)*1
analyrea ~ c(nu6, nu6)*1

# residual variances
vocab ~~ c(e1m, e1w)*vocab
reading ~~ c(e2m, e2w)*reading
sentcomp ~~ c(e3m, e3w)*sentcomp
mathmtcs ~~ c(e4m, e4w)*mathmtcs
geometry ~~ c(e5m, e5w)*geometry
analyrea ~~ c(e6m, e6w)*analyrea

# define constraints
lam1 == 3 - lam2 - lam3
lam4 == 3 - lam5 - lam6

nu1 == 0 - nu2 - nu3
nu4 == 0 - nu5 - nu6
"
```

```r
strong.fit <- lavaan(strong.mod, data = interest, group = "sex")
anova(strong.fit, weak.fit)
```

```
## Chi Square Difference Test
##
##            Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## weak.fit   20 3075.4 3195.2 20.048
## strong.fit 24 3070.6 3176.3 23.230     3.1816       4     0.5279
```

```r
fit.stats <- rbind(fit.stats,
                   fitmeasures(strong.fit, c("chisq", "df", "pvalue", "cfi", "rmsea", "srmr")))
rownames(fit.stats)[3] <- c("Strong")
fit.stats
```

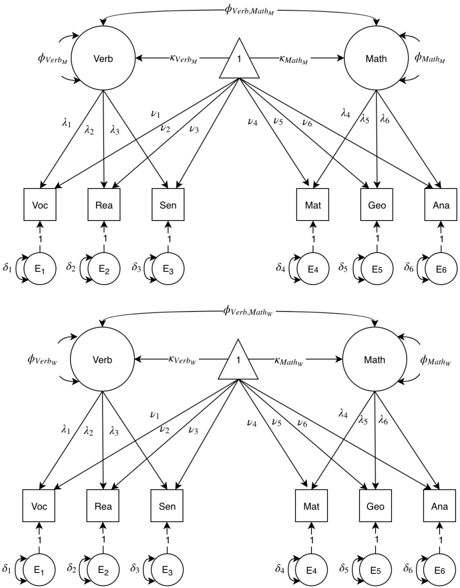
```
##               chisq df    pvalue       cfi       rmsea       srmr
## Configural 18.41320 16 0.3002620 0.9981046 0.034736151 0.01433416
## Weak       20.04805 20 0.4549276 0.9999623 0.004384013 0.02119742
## Strong     23.22963 24 0.5062782 1.0000000 0.000000000 0.02371500
```

```r
summary(strong.fit)
```

```
## Group 1 [female]:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   verb =~
##     vocab   (lam1)    1.073    0.027   39.362    0.000
##     reading (lam2)    0.962    0.031   31.298    0.000
##     sentcmp (lam3)    0.965    0.030   31.957    0.000
##   math =~
##     mthmtcs (lam4)    1.075    0.028   38.100    0.000
##     geomtry (lam5)    0.922    0.033   28.009    0.000
##     analyre (lam6)    1.002    0.030   32.944    0.000
##
##                    Estimate  Std.Err  z-value  P(>|z|)
##     verb              0.174    0.086    2.029    0.042
##     math             -0.009    0.089   -0.105    0.916
##    .vocab    (nu1)   -0.016    0.021   -0.768    0.443
##    .reading  (nu2)    0.038    0.026    1.500    0.133
##    .sentcomp (nu3)   -0.022    0.025   -0.870    0.384
##    .mathmtcs (nu4)   -0.035    0.023   -1.491    0.136
##    .geometry (nu5)   -0.011    0.029   -0.365    0.715
##    .analyrea (nu6)    0.045    0.026    1.718    0.086
##
## Group 2 [male]:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   verb =~
##     vocab   (lam1)    1.073    0.027   39.362    0.000
##     reading (lam2)    0.962    0.031   31.298    0.000
##     sentcmp (lam3)    0.965    0.030   31.957    0.000
##   math =~
##     mthmtcs (lam4)    1.075    0.028   38.100    0.000
##     geomtry (lam5)    0.922    0.033   28.009    0.000
##     analyre (lam6)    1.002    0.030   32.944    0.000
##
##                    Estimate  Std.Err  z-value  P(>|z|)
##     verb              0.031    0.077    0.409    0.683
##     math              0.268    0.080    3.340    0.001
##    .vocab    (nu1)   -0.016    0.021   -0.768    0.443
##    .reading  (nu2)    0.038    0.026    1.500    0.133
##    .sentcomp (nu3)   -0.022    0.025   -0.870    0.384
##    .mathmtcs (nu4)   -0.035    0.023   -1.491    0.136
##    .geometry (nu5)   -0.011    0.029   -0.365    0.715
##    .analyrea (nu6)    0.045    0.026    1.718    0.086
```

Strict invariance

- Adds the constraint that error variances/covariances are the same across groups.
- Strict invariance model is nested within strong invariance model, can use chi-square test of difference.
- If strict invariance not meaningful reduction in fit, indicators measure the same factor in each group with the same degree of error (inversely, precision).
- Disagreement as to whether it is necessary to obtain strict invariance to say factors are measured identically across groups. Why?
  - Recall, the residual (or unique variance) contains both random measurement error and systematic error.
  - Strict invariance means the sum of these components are the same across group.
  - Reasonable to think systematic error could be the same across groups.
  - But would we expect random measurement error to be the same?
- Similar disagreement as to whether it's necessary to have strict invariance to compare observed differences in variances and covariance.


$\phi_{Verb,Math_M}$
ϕ_{Verb_M}
Verb
κ_{Verb_M}
1
κ_{Math_M}
Math
ϕ_{Math_M}
λ_1 λ_2 λ_3 ν_1 ν_2 ν_3 ν_4 ν_5 ν_6 λ_4 λ_5 λ_6
Voc
Rea
Sen
Mat
Geo
Ana
1
δ_1 E_1
δ_2 E_2
δ_3 E_3
δ_4 E_4
δ_5 E_5
δ_6 E_6
$\phi_{Verb,Math_W}$
ϕ_{Verb_W}
Verb
κ_{Verb_W}
1
κ_{Math_W}
Math
ϕ_{Math_W}
λ_1 λ_2 λ_3 ν_1 ν_2 ν_3 ν_4 ν_5 ν_6 λ_4 λ_5 λ_6
Voc
Rea
Sen
Mat
Geo
Ana
1
δ_1 E_1
δ_2 E_2
δ_3 E_3
δ_4 E_4
δ_5 E_5
δ_6 E_6

Male Model

$$\begin{aligned} Voc &= \nu_1 + \lambda_1 Verb + \delta_1 \\ Rea &= \nu_2 + \lambda_2 Verb + \delta_2 \\ Sen &= \nu_3 + \lambda_3 Verb + \delta_3 \\ Mat &= \nu_4 + \lambda_4 Math + \delta_4 \\ Geo &= \nu_5 + \lambda_5 Math + \delta_5 \\ Ana &= \nu_6 + \lambda_6 Math + \delta_6 \end{aligned}$$

$$\phi_{Verb_M}, \phi_{Math_M}, \phi_{Verb_M, Math_M}, \kappa_{Verb_M}, \kappa_{Math_M}$$

Female Model

$$\begin{aligned} Voc &= \nu_1 + \lambda_1 Verb + \delta_1 \\ Rea &= \nu_2 + \lambda_2 Verb + \delta_2 \\ Sen &= \nu_3 + \lambda_3 Verb + \delta_3 \\ Mat &= \nu_4 + \lambda_4 Math + \delta_4 \\ Geo &= \nu_5 + \lambda_5 Math + \delta_5 \\ Ana &= \nu_6 + \lambda_6 Math + \delta_6 \end{aligned}$$

$$\phi_{Verb_W}, \phi_{Math_W}, \phi_{Verb_W, Math_W}, \kappa_{Verb_W}, \kappa_{Math_W}$$

Step 4: Strict invariance

```
strict.mod <- "
# define factor/unstd. pattern coef
verb  =~ c(lam1, lam1)*vocab + c(lam2, lam2)*reading + c(lam3, lam3)*sentcomp
math =~ c(lam4, lam4)*mathmtcs + c(lam5, lam5)*geometry + c(lam6, lam6)*analyrea

# factor intercepts
verb ~ 1
math ~ 1

# factor variances/covariance
verb ~~ verb + math
math ~~ math

# intercepts
vocab ~ c(nu1, nu1)*1
reading ~ c(nu2, nu2)*1
sentcomp ~ c(nu3, nu3)*1
mathmtcs ~ c(nu4, nu4)*1
geometry ~ c(nu5, nu5)*1
analyrea ~ c(nu6, nu6)*1

# residual variances
vocab ~~ c(e1, e1)*vocab
reading ~~ c(e2, e2)*reading
sentcomp ~~ c(e3, e3)*sentcomp
mathmtcs ~~ c(e4, e4)*mathmtcs
geometry ~~ c(e5, e5)*geometry
analyrea ~~ c(e6, e6)*analyrea

# define constraints
lam1 == 3 - lam2 - lam3
lam4 == 3 - lam5 - lam6

nu1 == 0 - nu2 - nu3
nu4 == 0 - nu5 - nu6
"
```

```r
strict.fit <- lavaan(strict.mod, data = interest, group = "sex")
anova(strict.fit, strong.fit)
```

```
## Chi Square Difference Test
##
##            Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## strong.fit 24 3070.6 3176.3 23.230
## strict.fit 30 3062.7 3147.2 27.264     4.0347       6      0.672
```

```r
fit.stats <- rbind(fit.stats,
                   fitmeasures(strict.fit, c("chisq", "df", "pvalue", "cfi", "rmsea", "srmr")))
rownames(fit.stats)[4] <- c("Strict")
fit.stats
```

```
##               chisq df    pvalue       cfi       rmsea       srmr
## Configural 18.41320 16 0.3002620 0.9981046 0.034736151 0.01433416
## Weak       20.04805 20 0.4549276 0.9999623 0.004384013 0.02119742
## Strong     23.22963 24 0.5062782 1.0000000 0.000000000 0.02371500
## Strict     27.26431 30 0.6093608 1.0000000 0.000000000 0.03077196
```

```r
summary(strict.fit)
```

```
## lavaan 0.6-3 ended normally after 33 iterations
##
##   Optimization method                           NLMINB
##   Number of free parameters                         46
##   Number of equality constraints                    22
##
##   Number of observations per group
##   female                                           122
##   male                                             128
##
##   Estimator                                         ML
##   Model Fit Test Statistic                      27.264
##   Degrees of freedom                                30
##   P-value (Chi-square)                           0.609
##
## Chi-square for each group:
##
##   female                                        15.063
##   male                                          12.201
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##   Standard Errors                             Standard
```

```r
summary(strict.fit)
```

```
## Group 1 [female]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   verb =~
##     vocab   (lam1)    1.071    0.027   39.528    0.000
##     reading (lam2)    0.962    0.031   31.395    0.000
##     sentcmp (lam3)    0.968    0.030   31.808    0.000
##   math =~
##     mthmtcs (lam4)    1.076    0.028   38.237    0.000
##     geomtry (lam5)    0.922    0.033   28.018    0.000
##     analyre (lam6)    1.003    0.031   32.805    0.000
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   verb ~~
##     math              0.780    0.111    7.038    0.000
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)
##     verb              0.173    0.086    2.004    0.045
##     math             -0.011    0.090   -0.125    0.900
##    .vocab    (nu1)   -0.016    0.021   -0.769    0.442
##    .reading  (nu2)    0.039    0.026    1.537    0.124
##    .sentcomp (nu3)   -0.023    0.025   -0.901    0.368
##    .mathmtcs (nu4)   -0.035    0.023   -1.519    0.129
##    .geometry (nu5)   -0.008    0.029   -0.284    0.776
##    .analyrea (nu6)    0.044    0.026    1.650    0.099
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)
##     verb              0.846    0.117    7.255    0.000
##     math              0.904    0.125    7.209    0.000
##    .vocab     (e1)    0.110    0.022    5.024    0.000
##    .reading   (e2)    0.265    0.030    8.984    0.000
##    .sentcomp  (e3)    0.258    0.029    8.887    0.000
##    .mathmtcs  (e4)    0.131    0.025    5.161    0.000
##    .geometry  (e5)    0.348    0.037    9.479    0.000
##    .analyrea  (e6)    0.265    0.032    8.383    0.000
```

```r
summary(strict.fit)
```

```
## Group 2 [male]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   verb =~
##     vocab   (lam1)    1.071    0.027   39.528    0.000
##     reading (lam2)    0.962    0.031   31.395    0.000
##     sentcmp (lam3)    0.968    0.030   31.808    0.000
##   math =~
##     mthmtcs (lam4)    1.076    0.028   38.237    0.000
##     geomtry (lam5)    0.922    0.033   28.018    0.000
##     analyre (lam6)    1.003    0.031   32.805    0.000
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   verb ~~
##     math              0.545    0.084    6.462    0.000
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)
##     verb              0.030    0.077    0.386    0.700
##     math              0.267    0.080    3.324    0.001
##    .vocab     (nu1)  -0.016    0.021   -0.769    0.442
##    .reading   (nu2)   0.039    0.026    1.537    0.124
##    .sentcomp (nu3)  -0.023    0.025   -0.901    0.368
##    .mathmtcs (nu4)  -0.035    0.023   -1.519    0.129
##    .geometry (nu5)  -0.008    0.029   -0.284    0.776
##    .analyrea (nu6)   0.044    0.026    1.650    0.099
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)
##     verb              0.688    0.094    7.322    0.000
##     math              0.748    0.103    7.277    0.000
##    .vocab      (e1)   0.110    0.022    5.024    0.000
##    .reading    (e2)   0.265    0.030    8.984    0.000
##    .sentcomp   (e3)   0.258    0.029    8.887    0.000
##    .mathmtcs   (e4)   0.131    0.025    5.161    0.000
##    .geometry   (e5)   0.348    0.037    9.479    0.000
##    .analyrea   (e6)   0.265    0.032    8.383    0.000
```

$$\hat{Voc} = -0.016 + 1.071 Verb$$

$$\hat{Rea} = 0.039 + 0.962 Verb$$

$$\hat{Sen} = -0.023 + 0.968 Verb$$

$$\hat{\sigma}^2_{Voc} = 0.110, \hat{\sigma}^2_{Rea} = 0.265, \hat{\sigma}^2_{Sen} = 0.258$$

$$\hat{Mat} = -0.035 + 1.076 Math$$

$$\hat{Geo} = -0.008 + 0.922 Math$$

$$\hat{Ana} = 0.044 + 1.003 Math$$

$$\hat{\sigma}^2_{Mat} = 0.131, \hat{\sigma}^2_{Geo} = 0.348, \hat{\sigma}^2_{Ana} = 0.265$$

$$\hat{\phi}_{V_W} = 0.846, \hat{\phi}_{M_W} = 0.904, \hat{\phi}_{V_W,M_W} = 0.780, \hat{\kappa}_{V_W} = 0.173, \hat{\kappa}_{M_W} = -0.011$$

$$\hat{\phi}_{V_M} = 0.688, \hat{\phi}_{M_M} = 0.748, \hat{\phi}_{V_M,M_M} = 0.545, \hat{\kappa}_{V_M} = 0.030, \hat{\kappa}_{M_M} = 0.267$$

Now what?

- We concluded we have strict invariance.
  - Extremely rare!
- Compare the groups on IQ and calculate standardized mean differences (i.e., Cohen's d)

Comparing groups

```
params <- parameterestimates(strict.fit)
subset(params, (lhs %in% c("verb", "math") & op == "~1") | (lhs %in% c("verb", "math") & rhs %in% c("verb", "math")))
```

```
##     lhs op  rhs block group label    est    se      z pvalue ci.lower ci.upper
## 7  verb ~1          1     1        0.173 0.086  2.004  0.045    0.004    0.342
## 8  math ~1          1     1       -0.011 0.090 -0.125  0.900   -0.187    0.164
## 9  verb ~~ verb     1     1        0.846 0.117  7.255  0.000    0.617    1.074
## 10 verb ~~ math     1     1        0.780 0.111  7.038  0.000    0.563    0.997
## 11 math ~~ math     1     1        0.904 0.125  7.209  0.000    0.658    1.150
## 30 verb ~1          2     2        0.030 0.077  0.386  0.700   -0.121    0.180
## 31 math ~1          2     2        0.267 0.080  3.324  0.001    0.109    0.424
## 32 verb ~~ verb     2     2        0.688 0.094  7.322  0.000    0.504    0.872
## 33 verb ~~ math     2     2        0.545 0.084  6.462  0.000    0.380    0.710
## 34 math ~~ math     2     2        0.748 0.103  7.277  0.000    0.547    0.949
```

- Do these differ? Perform a t-test.

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}}}, \quad df \approx \frac{\left(\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}\right)^2}{\frac{s_1^4}{N_1^2(N_1-1)} + \frac{s_2^4}{N_2^2(N_2-1)}}$$

use `pt(t = t, df = df, lower.tail = FALSE) * 2` in R

- Calculate effect size.

$$d = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{s_1^2}}$$