Statistical Analysis Using Structural Equation Models

EPsy 8266

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2/26/19

Factor Analysis

- Originally proposed by Spearman (1904) while studying intelligence.
- Goal is to partition the variance of indicators into common/shared variance (**communality**) and specific/unique variance.
  - Shared variance is caused by the underlying factors.
- The unique variance consists of two parts: random measurement error and specific variance.
  - What might be a source of specific variance?
  - We generally have no control over random measurement error but might be able to reduce/model specific variance.

Exploratory or confirmatory

- In EFA, might not specific the number of factors.
- In EFA, no a priori structure of the indicators is specified (i.e., the measurement model is unrestricted).
- EFA models with more than 1 factor are not identified.
- Specific indicator variance in EFA does not covary.

EFA characteristics

- Every indicator is regressed onto every factor.
  - These direct paths (i.e., factor loadings) are known as **pattern coefficients**.
- **Simple structure** - each indicator loads onto exactly one factor
- Rotate the solution to increase interpretability (**rotation indeterminancy**).
  - Orthogonal or oblique
  - Often, oblique helps to get simple structure
  - Could extract factor scores, but there are multiple, equally valid ways to do this (**factor score indeterminacny**)
- Nothing wrong with doing EFA.

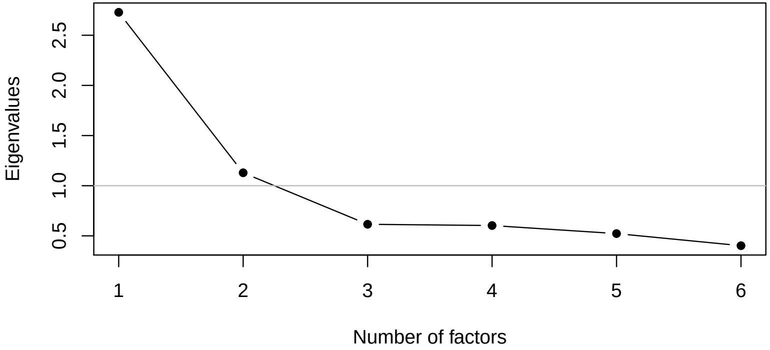
EFA of school subjects

```
subj <- c("gaelic", "english", "history", "math", "algebra", "geometry")
sch.subj <- matrix(c(1, .44, .41, .29, .33, .25,
                     .44, 1, .35, .35, .32, .33,
                     .41, .35, 1, .16, .19, .18,
                     .29, .35, .16, 1, .59, .47,
                     .33, .32, .19, .59, 1, .46,
                     .25, .33, .18, .47, .46, 1),
                   nrow = 6,
                   ncol = 6)
colnames(sch.subj) <- subj
rownames(sch.subj) <- subj
sch.subj
```

```
##          gaelic english history math algebra
## gaelic     1.00    0.44    0.41 0.29    0.33
## english    0.44    1.00    0.35 0.35    0.32
## history    0.41    0.35    1.00 0.16    0.19
## math       0.29    0.35    0.16 1.00    0.59
## algebra    0.33    0.32    0.19 0.59    1.00
## geometry   0.25    0.33    0.18 0.47    0.46
##          geometry
## gaelic       0.25
## english      0.33
## history      0.18
## math         0.47
## algebra      0.46
## geometry     1.00
```

Factors to extract?

```
plot(1:6, eigen(sch.subj)$values, type = "b",
     xlab = "Number of factors", ylab = "Eigenvalues", pch = 16)
abline(h = 1, col = "gray")
```



EFA - 1 factor

```
factanal(covmat = sch.subj, n.obs = 220, factors = 1)
```

```
##
## Call:
## factanal(factors = 1, covmat = sch.subj, n.obs = 220)
##
## Uniquenesses:
##   gaelic  english  history     math  algebra
##    0.750    0.710    0.878    0.473    0.468
## geometry
##    0.621
##
## Loadings:
##          Factor1
## gaelic   0.500
## english  0.539
## history  0.349
## math     0.726
## algebra  0.729
## geometry 0.615
##
##                Factor1
## SS loadings       2.10
## Proportion Var    0.35
##
## Test of the hypothesis that 1 factor is sufficient.
## The chi square statistic is 51.6 on 9 degrees of freedom.
## The p-value is 5.37e-08
```

EFA - 2 factor, orthogonal

```r
factanal(covmat = sch.subj, n.obs = 220, factors = 2)
```

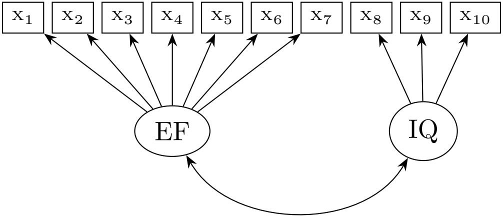
```
##
## Call:
## factanal(factors = 2, covmat = sch.subj, n.obs = 220)
##
## Uniquenesses:
##   gaelic  english  history     math  algebra
##    0.508    0.595    0.644    0.377    0.440
## geometry
##    0.628
##
## Loadings:
##          Factor1 Factor2
## gaelic   0.233   0.661
## english  0.319   0.551
## history          0.591
## math     0.770   0.172
## algebra  0.715   0.220
## geometry 0.570   0.215
##
##                Factor1 Factor2
## SS loadings      1.593   1.215
## Proportion Var   0.265   0.202
## Cumulative Var   0.265   0.468
##
## Test of the hypothesis that 2 factors are sufficient.
## The chi square statistic is 2.18 on 4 degrees of freedom.
## The p-value is 0.703
```

EFA - 2 factor, oblique

```
factanal(covmat = sch.subj, n.obs = 220, factors = 2, rotation = "promax")
```

```
##
## Call:
## factanal(factors = 2, covmat = sch.subj, n.obs = 220, rotation = "promax")
##
## Uniquenesses:
##   gaelic  english  history     math  algebra
##    0.508    0.595    0.644    0.377    0.440
## geometry
##    0.628
##
## Loadings:
##          Factor1 Factor2
## gaelic            0.696
## english   0.150   0.540
## history  -0.138   0.664
## math      0.816
## algebra   0.734
## geometry  0.570
##
##                Factor1 Factor2
## SS loadings      1.571   1.224
## Proportion Var   0.262   0.204
## Cumulative Var   0.262   0.466
##
## Factor Correlations:
##         Factor1 Factor2
## Factor1   1.000   0.565
## Factor2   0.565   1.000
##
## Test of the hypothesis that 2 factors are sufficient.
## The chi square statistic is 2.18 on 4 degrees of freedom.
## The p-value is 0.703
```

CFA from Masten et al. (2012)



EF

Stroop
Simon Says
Peg Tapping
Dimensional Change Card Sort
Gift Delay Part 1
Dinky Toys
Gift Delay Part 2

IQ

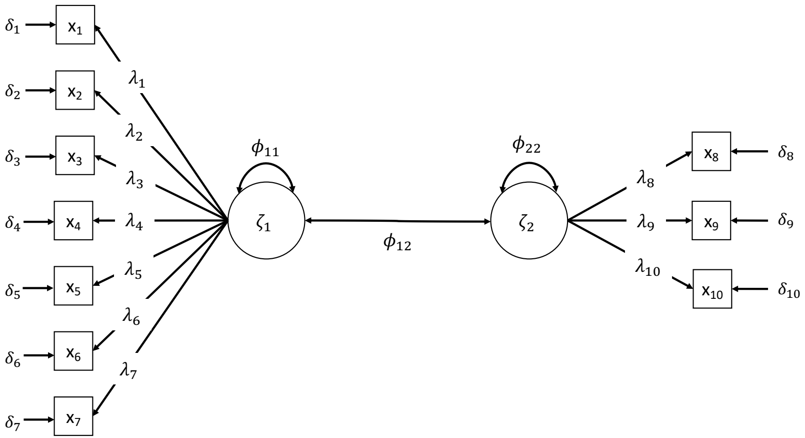
WPPSI-III Block Design scaled score
PPVT-4 standard score
WPPSI-III Matrix Reasoning scaled score

Characteristics of the Masten measurement model

1. Each indicator has two causes: the factor and a unique/specific cause (the error
   - Indicators are **locally-independent**
2. Error terms are all independent of each other and the factors.
3. The associations are linear and the factors are allowed to covary.
4. Some pattern coefficients are set to zero.
   - Don't think these means that IQ doesn't affect X_1. There could be a non-zero structural coefficient.

These are standard characteristics of a CFA model.

Masten measurement model



Is this model identified as it's written?
Can you express this model as a series of equations?

Types of scaling

1. Unit-loading identification - scale of the latent variable is the scale of the marker
2. Unit-variance identification
3. **Effects coding method** - constrains the average pattern coefficients across the indicators (within an factor) to be equal to 1.0 (Little, Slegers, & Card, 2006)
   - Latent variances are the average of the indicators' variances **accounted for by the construct**.
   - Reflects the observed metric of the indictors, optimally weighted by the degree to which each indicator represents the underlying latent construct.
   - In lavaan,

     ```
     # Define constrains for effects-coding
     lam1 == 7 - lam2 - lam3 - lam4 - lam5 - lam6 - lam7
     ```

   - See: https://github.com/cddesja/lavaan-reproducible/blob/master/little2006-scaling.R

Specifying CFAs

Indicator selection

- What should drive selection?
- How many?
- Should we use multiple methods/informants?

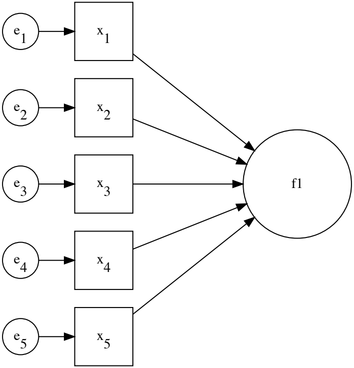
Dimensionality

- Do we have **simple indicators** and **unidimensional measurement**?
- Are one or more indicators associated with multiple factors (**complex**) or do we have correlated error terms? (**multidimensional measurement**)
- What might cause the latter?

Types of measures

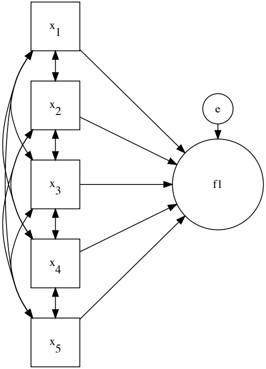
Reflective

Indicators ‘reflect’ the construct



Formative

Indicators ‘form’ the construct



A scale or an index?

A **scale** is formed from a set of items assumed to be *reflective* measures of the latent variable. Scores on items in a scale are theoretically driven by the latent construct (they reflect it).

An **index** is formed from a set of items assumed to be *formative* of the latent variable. Scores on items drive the total score of the index (they form it).

Not always clear cut whether a set of items (i.e. a measure) are formative or reflective.

Scales

Constructs such as personality, attitudes, aptitude, knowledge, intelligence are typically views as constructs that manifest in things we can actually measure.

Berkeley Expressivity Questionnaire: Assesses three facets of emotional expressivity: negative expressivity, positive expressivity, and impulse strength.

For each statement below, please indicate your agreement or disagreement. Do so by filling in the blank in front of each item with the appropriate number from the following rating scale:

1	2	3	4	5	6	7
strongly disagree			**neutral**			**strongly agree**

____ 1. Whenever I feel positive emotions, people can easily see exactly what I am feeling.

____ 2. I sometimes cry during sad movies.

____ 3. People often do not know what I am feeling.

____ 4. I laugh out loud when someone tells me a joke that I think is funny.

____ 5. It is difficult for me to hide my fear.

Indices

Constructs that combine observed behaviors or combine perhaps disjoint observed risk or protective factors are typically viewed as being defined by the observed variables going into it.

Socio-economic status. - Does high SES cause us to be highly educated and wealthy OR does being highly educated and wealthy cause us to have high SES?

Adverse Child Experience (ACE) Questionnaire(?): Assesses different negative childhood experiences. As number of ACEs increases, risk for many negative outcomes increases.

While you were growing up, during your first 18 years of life:

1. Did a parent or other adult in the household **often** …
   Swear at you, insult you, put you down, or humiliate you?
   or
   Act in a way that made you afraid that you might be physically hurt?
   Yes No If yes enter 1 ________

Coltman et al. (2008). Formative versus reflective measurement models: Two applications of formative measurement

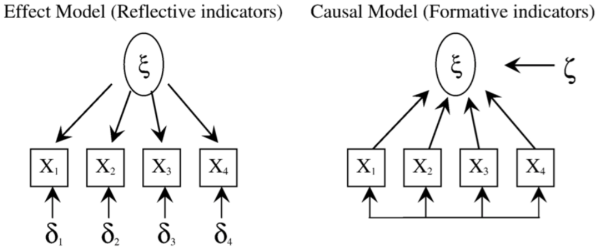
Table 1
A framework for assessing reflective and formative models: theoretical and empirical considerations

Considerations	Reflective model	Formative model	Relevant literature
Theoretical considerations			
1. Nature of construct	Latent construct exists ➢ Latent construct exists independent of the measures used	Latent construct is formed ➢ Latent constructs is a combination of its indicators	Borsboom et al. (2003, 2004)
2. Direction of causality between items and latent construct	Causality from construct to items ➢ Variation in the construct causes variation in the item measures ➢ Variation in item measures does not cause variation in the construct	Causality from items to construct ➢ Variation in the construct does not cause variation in the item measures ➢ Variation in item measures causes variation in the construct	Bollen and Lennox (1991); Edwards and Bagozzi (2000); Rossiter (2002); Jarvis et al. (2003)
3. Characteristics of items used to measure the construct	Items are manifested by the construct ➢ Items share a common theme ➢ Items are interchangeable ➢ Adding or dropping an item does not change the conceptual domain of the construct	Items define the construct ➢ Items need not share a common theme ➢ Items are not interchangeable ➢ Adding or dropping an item may change the conceptual domain of the construct	Rossiter (2002) ; Jarvis et al. (2003)

Coltman et al. (2008). Formative versus reflective measurement models: Two applications of formative measurement

Empirical considerations			
4. Item intercorrelation	Items should have high positive intercorrelations ➢ Empirical tests: assessing internal consistency and reliability by Cronbach alpha, average variance extracted, and factor loadings (e.g., from common or confirmatory factor analysis)	Items can have any pattern of intercorrelation but should possess the same directional relationship ➢ Empirical test: no empirical assessment of indicator reliability possible; various preliminary analyses are useful to check directionality between items and construct	Cronbach (1951); Nunnally and Bernstein (1994); Churchill (1979); Diamantopoulos and Siguaw (2006)
5. Item relationships with construct antecedents and consequences	Items have similar sign and significance of relationships with the antecedents/consequences as the construct ➢ Empirical tests: establishing content validity by theoretical considerations, assessing convergent and discriminant validity empirically	Items may not have similar significance of relationships with the antecedents/consequences as the construct ➢ Empirical tests: assessing nomological validity by using a MIMIC model, and/or structural linkage with another criterion variable	Bollen and Lennox (1991); Diamantopoulos and Winklhofer (2001); Diamantopoulos and Siguaw (2006)
6. Measurement error and collinearity	Identifying the error term in items is possible ➢ Empirical test: identifying and extracting measurement error by common factor analysis	Identifying the error term is not possible if the formative measurement model is estimated in isolation ➢ Empirical test: using the vanishing tetrad test to determine if the formative items behave as predicted ➢ Collinearity should be ruled out by standard diagnostics such as the condition index	Bollen and Ting (2000); Diamantopoulos (2006)

Coltman et al. (2008). Formative versus reflective measurement models: Two applications of formative measurement



$$X_1 = \lambda_1\xi + \delta_1$$
$$X_2 = \lambda_2\xi + \delta_2$$
$$X_3 = \lambda_3\xi + \delta_3$$
$$X_4 = \lambda_4\xi + \delta_4$$

$$\xi = \gamma_1 X_1 + \gamma_2 X_2 + \gamma_3 X_3 + \gamma_4 X_4 + \zeta$$

Focus on scales

Because reflective indicators are thought to be caused by the latent variable(s), we expect the indicators to be correlated with one another (because they share a cause). **This correlation structure we will use to identify our latent variables.**

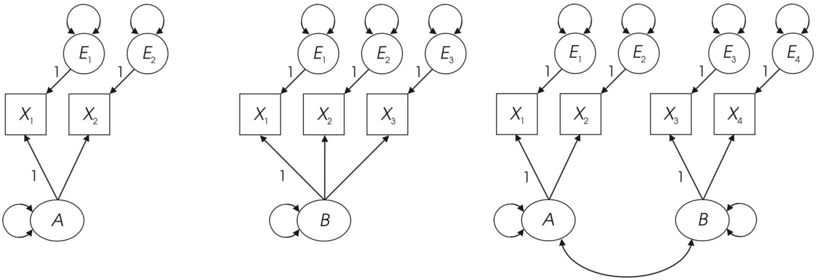
Methods for modeling and examining scales include coefficient alpha, KR-20, exploratory and confirmatory factor analysis.

These methods are NOT appropriate for creating or examining indexes because these methods are based on correlations between the indicators. **A perfectly fine index could be created from indicators that are not correlated at all.**

Think carefully about whether your construct causes your indicators or your indicators cause your construct!

Identification rules for standard CFAs

1. For a single factor has at least 3 indicators OR
2. ≥ 2 factors, where each factor has two or more indicators



E1
E2
1
1
X1
X2
1
A
E1
E2
E3
1
1
1
X1
X2
X3
1
B
E1
E2
E3
E4
1
1
1
1
X1
X2
X3
X4
1
1
A
B