Statistical Analysis Using Structural Equation Models

EPsy 8266

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Interactions

- Interactions in SEM uses the same framework as interactions in regression
- Specifically, we create a product term of the variables involved in the interaction.
- We generally consider one of these variables to be *moderating* the relationship between other variables.
  - Regress Y on X, M, and XM
  - Consider M the *moderator* and X the focal predictor.
  - The relationship between X and Y changes as a function of M.
  - Could X be the moderator?

Do you need significant main effects to consider interactions?

Klein's example

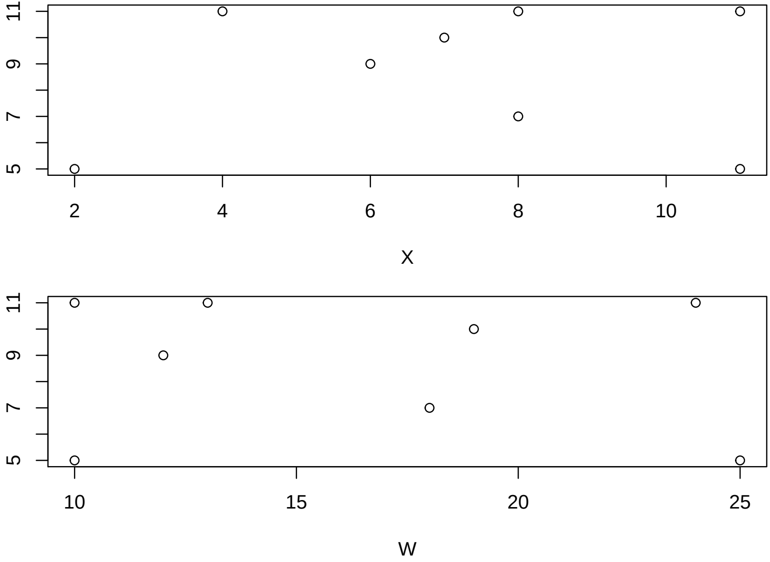
Reading in the data

```
tab17.1 <- data.frame(X = c(2, 6, 8, 11, 4, 7, 8, 11),
                      W = c(10, 12, 13, 10, 24, 19, 18, 25),
                      Y = c(5, 9, 11, 11, 11, 10, 7, 5))

# create interaction
tab17.1$XW <- tab17.1$X * tab17.1$W
```

Code for next slide

```
par(mfrow = c(2, 1),
    mar = c(4, 3, 1, 1) + .1)
plot(Y ~ X, data = tab17.1)
plot(Y ~ W, data = tab17.1)
```



11
9
7
5
2
4
6
8
10
X
11
9
7
5
10
15
20
25
W

Regress Y onto X and W

Our statistical model:

$$Y|X, W \sim N(\beta_0 + \beta_1 X + \beta_2 W, \sigma^2)$$

In R:

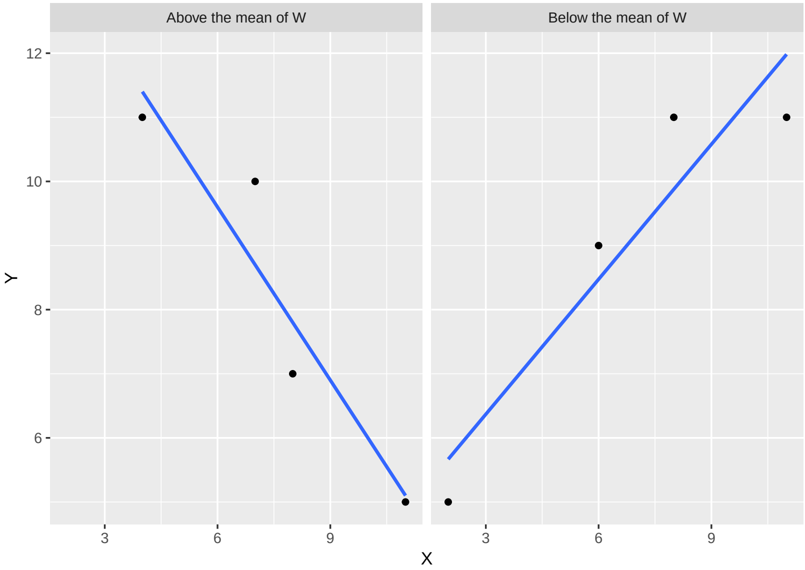
```
mod <- lm(Y ~ X + W, tab17.1)
summary(mod)
```

```
##
## Call:
## lm(formula = Y ~ X + W, data = tab17.1)
##
## Residuals:
##       1       2       3       4       5       6       7       8
## -3.4591  0.2218  2.0622  1.5362  3.2098  1.5562 -1.6191 -3.5080
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  8.87302    3.94435   2.250   0.0743 .
## X            0.11164    0.37113   0.301   0.7757
## W           -0.06372    0.19331  -0.330   0.7550
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.042 on 5 degrees of freedom
## Multiple R-squared:  0.03333,Adjusted R-squared:  -0.3533
## F-statistic: 0.08621 on 2 and 5 DF,  p-value: 0.9187
```

What happens if we mean-center X and W? What changes (anything)? $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2$, and/or $\hat{\sigma}^2$?

Interaction

```
tab17.1$W.split <- ifelse(tab17.1$W > mean(tab17.1$W), "Above the mean of W", "Below the mean of W")
library(ggplot2)
ggplot(tab17.1, aes(x = X, y = Y)) + geom_point() + stat_smooth(method = lm, se = FALSE) + facet_wrap(~ W.split)
```



Interaction model

$$Y = \beta_0 + \beta_1 X + \beta_2 W + \beta_3 XW + e \tag{1}$$
$$E(Y) = \beta_0 + \beta_1 X + \beta_2 W + \beta_3 XW \tag{2}$$
$$E(Y) = (\beta_0 + \beta_2 W) + (\beta_1 + \beta_3 W)X \tag{3}$$
$$E(Y) = (\beta_0 + \beta_1 X) + (\beta_2 + \beta_3 X)W \tag{4}$$

Equation 1: Observed value
Equation 2: Expected (predicted) value
Equation 3: W as moderator
Equation 4: X as moderator

Regress Y onto X, W, and XW

Our statistical model:

```
mod.lm <- lm(Y ~ X + W + XW, tab17.1)
summary(mod.lm)
```

```
##
## Call:
## lm(formula = Y ~ X + W + XW, data = tab17.1)
##
## Residuals:
##        1        2        3        4        5        6        7        8
## -0.60144  0.47366  1.65922 -0.79408 -0.20998  1.15345 -1.69230  0.01148
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -3.11795    3.34653  -0.932   0.4042
## X            1.76810    0.42241   4.186   0.0139 *
## W            0.73433    0.20643   3.557   0.0236 *
## XW          -0.10800    0.02507  -4.307   0.0126 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.432 on 4 degrees of freedom
## Multiple R-squared:  0.8285,Adjusted R-squared:    0.7
## F-statistic: 6.443 on 3 and 4 DF,  p-value: 0.05186
```

What happens if we mean-center X and W? What changes (anything)? $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2$, and/or $\hat{\sigma}^2$?

Must we mean-center X and W when we examine?

What effect does mean-centering have?

Comparing correlation matrices

```
# copy data set and mean-center
tab17.1mc <- subset(tab17.1, select = c(X, W, Y))
tab17.1mc$X <- scale(tab17.1mc$X, scale = FALSE, center = TRUE)
tab17.1mc$W <- scale(tab17.1mc$W, scale = FALSE, center = TRUE)
tab17.1mc$XW <- tab17.1mc$X * tab17.1mc$W

# print the correlation matrices
round(cor(tab17.1), 3)
```

```
##        X      W      Y     XW
## X  1.000  0.156  0.111  0.747
## W  0.156  1.000 -0.126  0.706
## Y  0.111 -0.126  1.000 -0.265
## XW 0.747  0.706 -0.265  1.000
```

```
round(cor(tab17.1mc), 3)
```

```
##         X      W      Y     XW
## X   1.000  0.156  0.111 -0.138
## W   0.156  1.000 -0.126  0.050
## Y   0.111 -0.126  1.000 -0.907
## XW -0.138  0.050 -0.907  1.000
```

Removes **nonessential multicollinearity** between X, W, and XW caused as a function of the underlying scale.

Fitted model

$$\hat{Y} = -3.118 + 1.768X + 0.732W + -0.108XW \tag{5}$$

$$\hat{Y} = \underbrace{(-3.118 + 0.732W)}_{\text{simple intercept}} + \underbrace{(1.768 + -0.108W)X}_{\text{simple slope}} \tag{6}$$

Fitted model

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$$\hat{Y} = \underbrace{(-3.118 + 0.732W)}_{\text{simple intercept}} + \underbrace{(1.768 + -0.108W)X}_{\text{simple slope}} \tag{6}$$

- Simple intercept: Value of the intercept for predicting Y from X given W

Fitted model

$$\hat{Y} = -3.118 + 1.768X + 0.732W + -0.108XW \tag{5}$$

$$\hat{Y} = \underbrace{(-3.118 + 0.732W)}_{\text{simple intercept}} + \underbrace{(1.768 + -0.108W)X}_{\text{simple slope}} \tag{6}$$

- Simple intercept: Value of the intercept for predicting Y from X given W
- Simple slope: Value of the slope for predicting Y from X given W

Fitted model

$$\hat{Y} = -3.118 + 1.768X + 0.732W + -0.108XW \tag{5}$$

$$\hat{Y} = \underbrace{(-3.118 + 0.732W)}_{\text{simple intercept}} + \underbrace{(1.768 + -0.108W)X}_{\text{simple slope}} \tag{6}$$

- Simple intercept: Value of the intercept for predicting Y from X given W
- Simple slope: Value of the slope for predicting Y from X given W
- Simple slope is usually what is of more substantitive interest.

Region of significance plot

- Often interest in knowing for what values of our moderator, W, the relationship between X and Y is significant.
- Could calculate simple regressions substituting in values of the mean of W, $\pm$ 1 SD of W, and $\pm$ 2 SD of W.
- A better approach involves calculating this for all the values of W (region of significance plot) and including a confidence band around it.
- The SE for the simple slope is:

$$SE_{(\hat{\beta}_1+\hat{\beta}_3 W)} = \sqrt{s^2_{\hat{\beta}_1} + 2s_{\hat{\beta}_1\hat{\beta}_3}W + s^2_{\hat{\beta}_3}W^2}$$

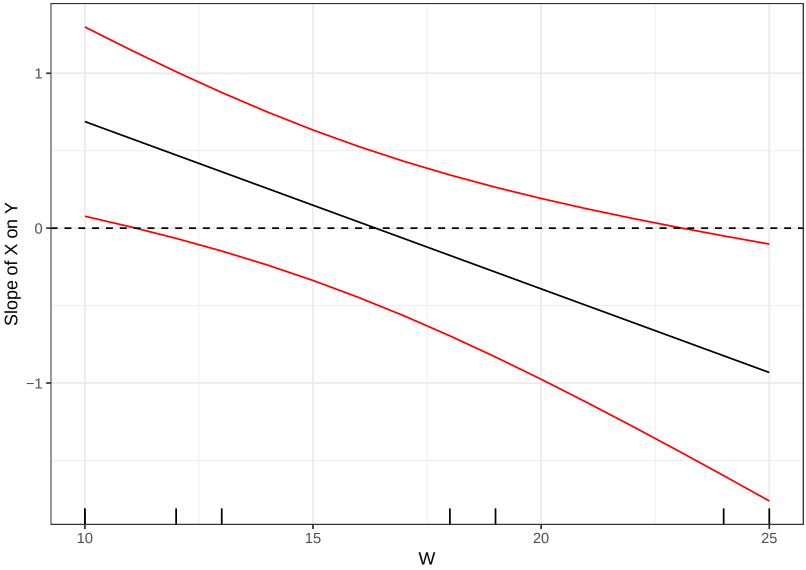
- A 95% CI can be created by $t_{.975,df} * SE_{(\hat{\beta}_1+\hat{\beta}_3 W)}$

Extracting information for regions of significance plot

```
# regression coefficients
par_est <- coef(mod.lm)[c(2, 4)]

# coefficient variances
var_est <- diag(vcov(mod.lm))[c(2, 4)]

# coefficient covariances
cov_est <-  vcov(mod.lm)["X", "XW"]

# residual df
df <- mod.lm$df.residual

# W spans from 10 to 25
W <- 10:25

# calculate SE
SE <- sqrt(var_est[1] + 2*cov_est*W + var_est[2]*W^2)

# Critical values
crit_val <- qt(.975, df = df) * SE

# Calculate simple slopes
simple_slope <- par_est[1] + par_est[2]*W

# Combine into a single data set
reg_sig <- data.frame(W = W,
                      simple_slope = simple_slope,
                      crit_val = crit_val)
```

ggplot2 code

```
ggplot(reg_sig, aes(x = W)) +
  geom_line(aes(y = simple_slope)) +
  geom_line(aes(y = simple_slope - crit_val), col = "red") +
  geom_line(aes(y = simple_slope + crit_val), col = "red") +
  geom_hline(yintercept = 0, lty = 2) +
  geom_rug(data = tab17.1) + # always include a rug
  ylab("Slope of X on Y") +
  xlab("W") +
  theme_bw()
```


1
0
−1
10
15
20
25
W
Slope of X on Y

Residualized product

- An interaction term, XW_{res}, that is uncorrelated with X and W can be created as follows
  1. Regress X and W onto XW, save residuals (this is XW_{res} and is referred to as a **residual centering**)
  2. Regress Y onto X, W, and XW_{res}

```
tab17.1$XW.resid <- resid(lm(XW ~ X + W, tab17.1))
resid.product.mod <- lm(Y ~ X + W + XW.resid, data = tab17.1)
summary(resid.product.mod)
```

```
##
## Call:
## lm(formula = Y ~ X + W + XW.resid, data = tab17.1)
##
## Residuals:
##        1        2        3        4        5        6        7        8
## -0.60144  0.47366  1.65922 -0.79408 -0.20998  1.15345 -1.69230  0.01148
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  8.87302    1.85721   4.778  0.00879 **
## X            0.11164    0.17475   0.639  0.55766
## W           -0.06372    0.09102  -0.700  0.52244
## XW.resid    -0.10800    0.02507  -4.307  0.01257 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.432 on 4 degrees of freedom
## Multiple R-squared:  0.8285,Adjusted R-squared:    0.7
## F-statistic: 6.443 on 3 and 4 DF,  p-value: 0.05186
```

Benefits of residualized product interactions

- XW_{res} uncorrelated with X and W.

```
cor(tab17.1$XW.resid, tab17.1$X)

## [1] 2.592177e-16

cor(tab17.1$XW.resid, tab17.1$W)

## [1] 2.070368e-16
```

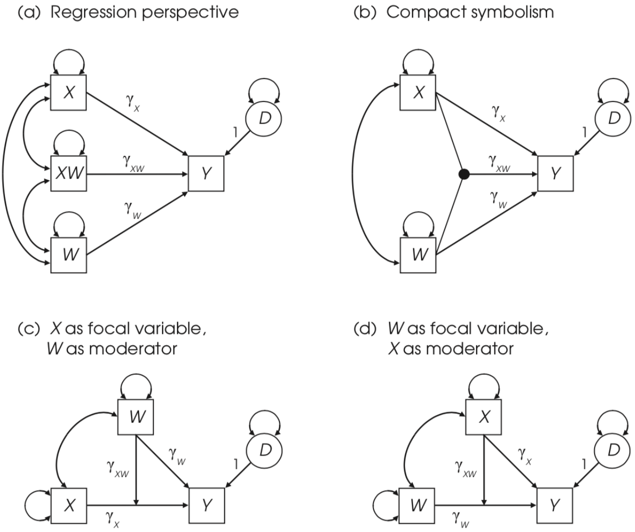
- Main effects have interpretations independent of the interaction (they are unconditional linear relations).
  - Controlling for W, for a one-unit increase in X, except a 0.11 increase in Y
  - **Notice these are the same parameter estimates when the interaction was not included!**

Nonlinear and > 2 interactions

- It is possible to have a non-linear interaction between two variables.
  - XW^2.
  - Means the linear relationship between X and Y changes as a function of W (but quicker at the ends because of the squaring, not as a constant).
  - Means the quadratic relationship between W and Y changes at a constant linear rate across X.
  - Regress Y onto X, W, W^2, XW, and XW^2.
- Note, it is recommended to always investigate/include X^2 and W^2 power terms when investigating XW as curvilinearity and interactive effects may get confounded (see e.g., Edwards (2009)).
- Can also have three, four, n-way interactions.
  - If a variable Z moderates the XW interaction it means the nature of this relationship changes as Z varies.
  - Can get complex quickly

Observed variables interactions in path analysis

Framework same as moderated regression, however, multiple ways to show it!



In lavaan

```
library(lavaan)
mod <- "
Y ~ 1 + X + W + XW
"
fit <- sem(mod, data = tab17.1)
summary(fit)
```

```
## lavaan 0.6-3 ended normally after 23 iterations
##
##   Optimization method                           NLMINB
##   Number of free parameters                          5
##
##   Number of observations                             8
##
##   Estimator                                         ML
##   Model Fit Test Statistic                       0.000
##   Degrees of freedom                                 0
##   Minimum Function Value               0.0000000000000
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##   Standard Errors                             Standard
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   Y ~
##     X                 1.768    0.299    5.919    0.000
##     W                 0.734    0.146    5.031    0.000
##     XW               -0.108    0.018   -6.091    0.000
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)
##    .Y                -3.118    2.366   -1.318    0.188
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)
##    .Y                 1.026    0.513    2.000    0.046
```

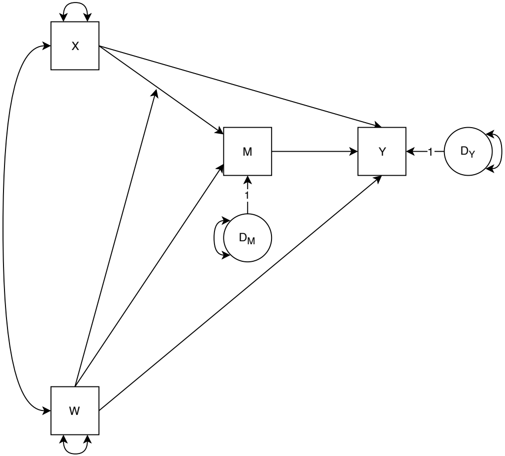
Caveats

- Like mediation and all of SEM, moderation models assume causality and directionality.
  - If your arrows are backwards ...
- Product terms do not actually have causal potency but instead just represents the joint effects of two causal agents.

Conditional process modeling

- Refers to the boundary conditions of direct or indirect effects or the circumstances where causal effects occur.
  - Significant levels in regions of significance plot.
- Klein says mediation needs time precedence, otherwise it's indirect effects.
  - This is debatable and comes from theory, not time precedence.
- **Mediated moderation** - When moderation is transmitted at least in part through a mediator(s).

Mediation moderation



Does brand experience translate into brand commitment?: A mediated-moderation model of brand passion and perceived brand ethicality

- Das et al. (2019) examined the impacts of perceived brand ethicality on brand passion and brand commitment.
- Used 2017 Baylor Religion Survey (n = 1,410)
- A conceptual framework was tested using structural equation modeling with responses from 273 apparel shoppers collected by using a structured questionnaire.
- They found evidence of a mediating-moderation effect in which the moderating power of perceived brand ethicality is eliminated in the presence of full mediator, brand passion.
  - "Brand passion significantly overpowers both the direct role of brand experience (i.e., full mediation) and the moderating role of perceived brand ethicality (i.e., mediated-moderation) in explaining brand commitment"
  - Brand passion fully mediates brand experience and brand commitment linkage.
- Also found dampening effects of perceived brand ethicality at play.
  - "Sensory-related brand experiences desist moral reasoning and tend to succumb to temporal pleasures"

Das et al. (2019) model