Statistical Analysis Using Structural Equation Models

EPsy 8266

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Modeling strategies for fully latent SR models

- 1. Two-Step
- 2. Four-Step

Two-Step Modeling

- 1. Respecify the fully latent SR model as a CFA and assess whether it fits the the data. Needs to fit well.
- 2. Conditionally on a good fitting CFA, fit the hypothesized SR model
 - The SR model may be nested within the CFA, if so, use a chi-square test of difference
 - ▶ If not, fit is identical to CFA
 - Alternative, nested SR models
- ▶ If the CFA fits well, then the pattern coefficients should change slightly when moving to a SR model.
- If change a lot, then they depend on the structural paths (interpretational confounding).
 - The empirical def'n of the constructs depend on hypotheses about the causal effects.

Four-Step Modeling

Works only when each factor has at least 4 indicators.

- Perform an EFA where you specify the number of factors in your hypothesized fully latent SR model using the same estimation method
- 2. Fit the fully latent SR as a CFA (same as Step #1 from the Two-Step approach)
- 3. Respecifying at least one of the factor covariances as a direct path from Step #2
- Tests of our a priori hypotheses about the relationships between the factors and their parameters
 - ► These last two steps are Step #2 from the Two-Step approach

A Structural Equation Model of Parental Involvement, Motivational and Aptitudinal Characteristics, and Academic Achievement

Gonzalez-Pienda, et al. (2002). Journal of Experimental Education

The authors used the structural equation model (SEM) approach to test a model hypothesizing the influence of parental involvement on students' academic aptitudes, self-concept, and causal attributions, as well as the influence of the 3 variables on academic achievement. The theoretical model was contrasted in a group of 12- to 18-year-old adolescents (N=261) attending various educational centers.

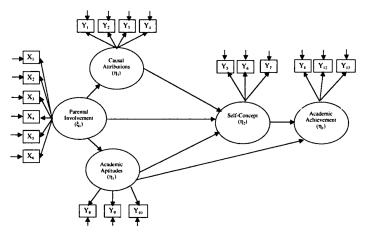
Google Scholar

A structural equation model of parental involvement, motivational and aptitudinal characteristics, and academic achievement

JA Gonzalez-Pienda, JC Nunez... - The Journal of ..., 2002 - Taylor & Francis
The authors used the structural equation model (SEM) approach to test a model
hypothesizing the influence of parental involvement on students' academic aptitudes, selfconcept, and causal attributions, as well as the influence of the 3 variables on academic
achievement. The theoretical model was contrasted in a group of 12-to 18-year-old
adolescents (N= 261) attending various educational centers. The results indicate that (a)
parental involvement had a positive and significant influence on the participant's measured ...

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FIGURE 1. A priori model of causal paths among parental involvement, aptitudinal and motivational characteristics, and academic achievement.



Note. Family variables: X_1 = achievement expectations, X_2 = help, X_3 = interest, X_4 = capacity expectations, X_5 = satisfaction, X_6 = reinforcement. Personal variables: Y_1 = capacity as cause of success in mathematical tasks, Y_2 = effort as cause of success in wribal tasks, Y_3 = capacity as cause of success in verbal tasks, Y_3 = mathematical self-concept, Y_6 = verbal self-concept, Y_7 = self-concept in remaining areas, Y_8 = verbal aptitude, Y_9 = reasoning aptitude, Y_{10} = calculus aptitude. Achievement variables: Y_{11} = mathematical achievement, Y_{12} = verbal achievement, Y_{13} = global achievement in remaining areas.

Methodology

- ▶ In the initial model, no relationship was assumed between the measurement errors of the observed variables or indicators, but these correlations were included in the respecifications carried out on the model. We did this because the indicators of each of the latent variables were obtained from our using the same instrument (e.g., the total score of the subscales).
- Considering that some data support the hypothesis of causal relationship in directions opposite to those established in the model presented in Figure 1 and even reciprocal relations (e.g., between causal attributions and self-concept, or between self-concept and academic achievement) we proceeded to contrast two alternative models involving these other possible causal relations after we contrasted the initial theoretical model.
- ► They presented goodness-of-fit index (GFI) and adjusted version (AGI), CFI, TLI, SRMR, and RMSEA

TABLE 1 Correlation Matrix, Means, and Standard Deviations for Model a

	YI	Y2	Y3	Y4	Y5	Y6	Y 7	Y8	Y9	Y10	Y11	Y12	Y13	ХI	X2	Х3	X4	X5	X6
YI	_																		
Y2	.60																		
Y3	.51	.38	_																
Y4	.27	.56	.48	_															
Y5	.58	.41	.15	.00	_														
Y6	.28	.26	.54	.30	.28	_													
Y 7	.41	.36	.44	.23	.58	.63	_												
Y8	.24	.12	.10	.07	.37	.27	.32												
Y9	.29	.20	.10	.04	.39	.20	.38	.48	_										
Y10	.35	.27	.19	.13	.44	.22	.42	.47	.60	_									
Y11	.44	.31	.25	.05	.55	.34	.60	.41	.46	.46	_								
Y12	.38	.29	.37	.17	.48	.52	.67	.35	.48	.44	.77	_							
Y13	.39	.26	.26	.07	.52	.42	.64	.43	.52	.48	.82	.86							
X1	.32	.35	.30	.16	.43	.32	.57	.18	.30	.38	.50	.50	.47	_					
X2	.16	.30	.25	.32	.07	.19	.25	01	06	04	.10	.13	.05	.23	_				
X3	.15	.24	.25	.28	.12	.23	.31	.07	03	.06	.07	.15	.06	.27	.71				
X4	.40	.37	.37	.20	.53	.45	.73	.20	.30	.31	.54	.57	.56	.65	.43	.41	-		
X5	.30	.27	.24	.13	.42	.37	.65	.12	.31	.23	.49	.50	.51	.47	.41	.42	.75	_	
X6	01	01	00	.02	.02	.07	.08	12	11	12	05	.00	07	.14	.43	.44	.31	.40	
М	2.93	3.38	3.42	3.74	3.45	3.89	4.25	27.92	19.61	18.27	2.82	3.05	3.15	4.32	3.93	4.02	3.59	3.92	3.21
SD	1.17	.95	.93	.87	1.33	.91	1.00	6.56	6.88	6.05	1.04	1.01	1.00	.48	.62	.61	.64	.62	.72

Note. Family variables: X1 = achievement expectations, X2 = help, X3 = interest, X4 = capacity expectations, X5 = satisfaction, X6 = reinforcement. Personal variables: Y1 = capacity as cause of success in mathematical tasks, Y2 = effort as cause of success in mathematical sats, Y3 = capacity as cause of success in verbal tasks, Y5 = mathematical self-concept, Y6 = verbal self-concept, Y7 = self-concept in remaining areas, Y8 = verbal aptitude, Y9 = reasoning aptitude, Y10 = calculus aptitude. Achievement Variables: Y11 = mathematical achievement, Y12 = verbal achievement, Y13 = global achievement in remaining areas.

```
library(lavaan)
lowerTri <- '
.60 1
.51 .38 1
.27 .56 .48 1
.58 .41 .15 .00 1
.28 .26 .54 .30 .28 1
.41 .36 .44 .23 .58 .63 1
.24 .12 .10 .07 .37 .27 .32 1
.29 .20 .10 .04 .39 .20 .38 .48 1
.35 .27 .19 .13 .44 .22 .42 .47 .60 1
.44 .31 .25 .05 .55 .34 .60 .41 .46 .46 1
.38 .29 .37 .17 .48 .52 .67 .35 .48 .44 .77 1
.39 .26 .26 .07 .52 .42 .64 .43 .52 .48 .82 .86 1
.32 .35 .30 .16 .43 .32 .57 .18 .30 .38 .50 .50 .47 1
.16 .30 .25 .32 .07 .19 .25 -.01 -.06 -.04 .10 .13 .05 .23 1
.15 .24 .25 .28 .12 .23 .31 .07 -.03 .06 .07 .15 .06 .27 .71 1
.40 .37 .37 .20 .53 .45 .73 .20 .30 .31 .54 .57 .56 .65 .43 .41 1
.30 .27 .24 .13 .42 .37 .65 .12 .31 .23 .49 .50 .51 .47 .41 .42 .75 1
-.01 -.01 -0 .02 .02 .07 .08 -.12 -.11 -.12 -.05 0 -.07 .14 .43 .44 .31 .4 1
corMat <- getCov(lowerTri,</pre>
                 names = c(paste0("y", 1:13), paste0("x", 1:6)),
                 sd = c(1.17, .95, .93, .87, 1.33, .91, 1.00, 6.56, 6.88, 6.05,
                        1.04, 1.01, 1.00, .48, .62, .61, .64, .62, .72))
```

Four Step Modeling, Step 1: EFA

```
fa.soln <- factanal(covmat = corMat, factors = 5, n.obs = 261, rotation = "promax")</pre>
print(loadings(fa.soln), cutoff = .3)
## Loadings:
##
      Factor1 Factor2 Factor3 Factor4 Factor5
## v1
                      0.574
                                      0.537
## y2
                      0.387
                                     0.700
## y3
                              0.604
                                     0.436
                                      0.630
## y4
## v5
                      0.871
## y6
                              0.854
                      0.331
                              0.604
## y7
## y8 0.376
## y9 0.452
## v10 0.303
                      0.333
## v11 0.885
## y12 0.959
## v13 1.061
## x1
                      0.325
## x2
               0.912
## x3
               0.785
## x4
              0.394
                      0.379
## x5
              0.484
## x6
               0.653
##
##
               Factor1 Factor2 Factor3 Factor4 Factor5
              3.394 2.451 2.052 1.740 1.545
## SS loadings
## Proportion Var 0.179 0.129 0.108 0.092 0.081
## Cumulative Var 0.179 0.308 0.416 0.507
                                                 0.588
```

Four Step Modeling - Step 2

chisq df pvalue cfi tli rmsea

1442.125 152.000 0.000 0.576 0.524 0.180 0.133

srmr

```
meas.mod <- "
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = y1 + y2 + y3 + y4
asc = v5 + v6 + v7
aap = y8 + y9 + y10
ach = v11 + v12 + v13
meas.fit <- cfa(meas.mod, sample.cov = corMat, sample.nobs = 261)
# chi-square test of difference, discriminant validity
anova(meas.fit, one.fit)
## Chi Square Difference Test
##
##
                 AIC BIC Chisq Chisq diff Df diff Pr(>Chisq)
            Df
## meas.fit 142 13154 13325 767.34
## one.fit 152 13808 13944 1442.12 674.78 10 < 2.2e-16 ***
```

Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1

```
fitmeasures(meas.fit, c("chisq", "df", "pvalue", "cfi",
"tli" "rmsea" "srmr" "efi" "ac
```

chisq df pvalue cfi tli rmsea srmr gfi ## 767.343 142.000 0.000 0.795 0.753 0.130 0.106 0.744

agfi ## 0.658

Reported pattern coefficients

TABLE 3
Pattern of Estimated Parameters for Measurement Model of Hypothesized Model

Measure	Error/ Uniqueness		Parental involvement (ξ ₁)		Causal attribution (η ₁)		Self- concept (η ₂)		Academic aptitudes (η ₃)		Academic achievement (η ₄)	
Lambda X (λx)												
PIN1	.51 (.51)	.70	(.70)									
PIN2	.84 (.85(.40*	(.39*)									
PIN3	.81 (.82)	.43*	(.43*)									
PIN4	.16 (.15)	.92**	(.92**)									
PIN5	.33 (.34)	.82**	(.81**)									
PIN6	.98 (.98)	.15	(.15)									
Lambda Y (\(\lambda\)y)	,		, , ,									
CATI	.24 (.22)			.87	(.86)	.000	(.000)	.000	(.000)	.000	(.000)	
CAT2	.52 (.79)			.69**	(.45**)	.000	(.000)	.000	(.000)	.000	(.000)	
CAT3	.63 (.94)			.61**	(.24**)	.000	(.000)	.000	(.000)	.000	(.000)	
CAT4	.91 (.95)			.30**	(.22**)	.000	(.000)	.000	(.000)	.000	(.000)	
ASC1	.53 (.55)			.000	(.000)	.69	(.67)	.000	(.000)	.000	(.000)	
ASC2	.75 (.73)			.000	(.000)	.50**	(.52**)	.000	(.000)	.000	(.000)	
ASC3	.23 (.24)			.000	(.000)	.87**	(.87**)	.000	(.000)	.000	(.000)	
AAP1	.61 (.61)			.000	(.000)	.000	(.000)	.63	(.62)	.000	(.000)	
AAP2	.40 (.40)			.000	(.000)	.000	(.000)	.77**	(.78**)	.000	(.000)	
AAP3	.41 (.41)			.000	(.000)	.000	(.000)	.77**	(.77**)	.000	(.000)	
ACHI	.25 (.25)			.000	(.000)	.000	(.000)	.000	(.000)	.87	(.87)	
ACH2	.19 (.19)			.000	(.000)	.000	(.000)	.000	(.000)	.90**	(.90**)	
ACH3	.11 (.10)			.000	(.000)	.000	(.000)	.000	(.000)	.95**	(.95**)	

Note, Standardized solution. Values without parentheses correspond to the model of attribution of success. Values in parentheses correspond to the model of attribution of failure (see SAS). The first value of each factor is fixed as a reference variable (they were fixed at 1,00), which means that we were unable to estimate their significance (values). The measurement errors of the observed variables (uniqueness) were all at the significance level of p > .001. Significant correlated uniqueness are not presented here because of lack of space. PIN = Parental Involvement. CAT = Causal Attributions. ASC = Academic Self-Concept. AAP = Academic Aptitudes. ACH = Academic Achievement (in Figure 2 the nature of the 19 observed variables is described).

*p < .01. **p < .001.

```
params <- parameterestimates(meas.fit, standardized = TRUE)</pre>
subset(params, op == "=~", select = c(lhs, op, rhs, std.all))
##
    lhs op rhs std.all
## 1 pin =~ x1 0.671
## 2 pin = ^{\sim} x2 0.476
## 3 pin = x3 0.475
## 4 pin =~ x4 0.938
## 5 pin = x5 0.805
## 6 pin = ^{\sim} x6 0.342
## 7 cat =~ y1 0.743
## 8 cat =~ v2 0.768
## 9 cat =~ v3
                 0.641
## 10 cat =~ y4
                 0.573
## 11 asc =~ v5
                 0.643
## 12 asc =~ y6
                 0.638
## 13 asc =~ y7
                 0.928
```

14 aap =~ y8

15 aap =~ y9

16 aap =~ y10 0.765 ## 17 ach =~ v11 0.862 ## 18 ach =~ v12 0.908 ## 19 ach =~ v13

0.626

0.774

0.945

Correlation Residuals

```
round(lavResiduals(meas.fit, type = "cor")$cov, 2)
           x2
              x3 x4 x5 x6
##
      x1
                                    v1
## x1
      0.00
## x2 -0.09 0.00
## x3 -0.05 0.48 0.00
## x4 0.02 -0.02 -0.04 0.00
## x5 -0.07 0.03 0.04 0.00 0.00
## x6 -0.09 0.27 0.28 -0.01 0.12 0.00
## y1 0.06 -0.03 -0.04 0.03 -0.02 -0.14 0.00
## y2 0.08 0.11 0.05 -0.01 -0.06 -0.15 0.03
## v3 0.07 0.09 0.09 0.05 -0.03 -0.12 0.03
## y4 -0.04 0.18 0.14 -0.08 -0.11 -0.08 -0.16
## v5 0.07 -0.18 -0.13 0.03 -0.01 -0.16
                                       0.29
## v6 -0.03 -0.06 -0.02 -0.04 -0.05 -0.11 0.00
## y7 0.06 -0.11 -0.05 0.01 0.04 -0.18 0.00
## v8 0.02 -0.13 -0.05 -0.03 -0.08 -0.20 0.06
## y9 0.10 -0.20 -0.17 0.02 0.07 -0.21 0.07
## v10 0.18 -0.18 -0.08 0.03 -0.01 -0.22 0.13
## v11 0.14 -0.16 -0.19 0.03 0.05 -0.24 0.15
## v12 0.12 -0.14 -0.12 0.03 0.04 -0.20 0.08
## y13 0.07 -0.23 -0.22 0.00 0.03 -0.27 0.08
```

Correlation Residuals

```
round(lavResiduals(meas.fit, type = "cor")$cov, 2)
      y2 y3 y4 y5 y6 y7 y8 y9 y10 y11 y12 y13
##
## y2 0.00
## y3 -0.11 0.00
## v4 0.12 0.11 0.00
## y5 0.11 -0.10 -0.22 0.00
## v6 -0.03 0.29 0.08 -0.13 0.00
## v7 -0.07 0.08 -0.09 -0.02 0.04 0.00
## y8 -0.07 -0.06 -0.07 0.14 0.04 -0.02 0.00 0.00
## v9 -0.03 -0.09 -0.13 0.10 -0.09 -0.03 0.00 0.01 0.00
## v10 0.04 0.00 -0.04 0.16 -0.06 0.01 -0.01 0.00 0.01 0.00
## v11 0.02 0.00 -0.17 0.13 -0.08 -0.01 0.04 0.00 0.01 0.00
## v12 -0.02 0.11 -0.06 0.04 0.08 0.03 -0.04 0.00 -0.04 -0.01
                                                           0.00
## v13 -0.06 -0.01 -0.17 0.06 -0.04 -0.03 0.03 0.02 -0.01 0.00 0.00 0.00
```

Standardized Residuals

```
round(lavResiduals(meas.fit, type = "cor")$cov.z, 2)
           x2 x3 x4 x5 x6 v1
##
      x1
## x1
     0.00
## x2 -2.36 0.00
## x3 -1.29 8.46 0.00
## x4 2.25 -1.53 -3.57 0.00
## x5 -3.26 0.94 1.31 -1.08 0.00
## x6 -2.18 4.98 5.15 -0.91 3.72 0.00
## v1 1.27 -0.53 -0.73 1.08 -0.42 -2.68 0.00
## y2 1.77 2.13 0.94 -0.40 -1.54 -2.76 2.07
## v3 1.54 1.72 1.72 1.44 -0.74 -2.08 1.58
## y4 -0.87 3.18 2.47 -2.18 -2.45 -1.42 -7.40
## v5 1.89 -3.70 -2.73 1.30 -0.17 -3.13 6.50
## v6 -0.82 -1.23 -0.41 -1.75 -1.56 -2.11 -0.11
## y7 2.05 -3.67 -1.73 1.75 1.81 -5.25 -0.14
## v8 0.33 -2.19 -0.80 -0.70 -1.61 -3.37 1.27
## y9 2.12 -3.74 -3.19 0.64 1.77 -3.73 1.65
## v10 3.86 -3.36 -1.54 1.08 -0.23 -3.87 3.20
## v11 3.41 -3.46 -4.04 1.27 1.60 -4.75 4.05
## v12 3.07 -3.33 -2.88 1.74 1.32 -4.24 2.23
## y13 1.85 -5.26 -5.04 0.06 1.07 -5.69 2.27
```

Standardized Residuals

Modification Indices

```
head(modificationindices(meas.fit, sort. = TRUE))
##
     lhs op rhs mi epc sepc.lv sepc.all sepc.nox
## 148 x2 ~~
            x3 107.562
                      0.192 0.192
                                    0.659
                                             0.659
## 226 y1 ~~
            y5 60.071 0.450 0.450 0.566 0.566
## 87 asc = x6 41.706 -0.698 -0.596
                                    -0.829 -0.829
            y6 41.309 0.219 0.219 0.440 0.440
## 248 y3 ~~
## 225 y1 ~~
            y4 41.275 -0.310 -0.310 -0.557 -0.557
## 119 ach = x6 36.341 -0.394 -0.353
                                    -0.491 -0.491
```

What to do/What did they do???

Because of the poor fit of the two initial models, we respecified and reestimated the goodness of fit of the alternative models (sensitivity analysis). An essential criterion at this point was that the critical hypotheses of the initial model not be affected. Given that **most** of the respecifications carried out on the initial models referred to the estimation of the correlation between some measurement errors in the observed variables (measurement model), the final models did not differ significantly from the initial models.

Unfortunately, we don't know what they did and their results are not replicable!

Let's fish our way to a model AKA don't ever do this in your own research!

Number of people who drowned by falling into a pool

correlates with

Films Nicolas Cage appeared in

Correlation: 66.6% (r=0.666004)

Data sources: Centers for Disease Control & Prevention and Internet Movie Database



```
r1.mod <- "
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = y1 + y2 + y3 + y4
asc = v5 + v6 + v7
aap = y8 + y9 + y10
ach = v11 + v12 + v13
# residual correlations
x2 ~~ x3
v1 ~~ v5
r1.fit <- cfa(r1.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r1.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 557.586 140.000 0.000 0.863 0.833 0.107
head(modificationindices(r1.fit, sort. = T, standardized = F))
## lhs op rhs mi epc
## 248 y3 ~~ y6 35.038 0.184
## 235 y2 ~~ y3 34.958 -0.249
## 121 ach = x6 34.677 -0.391
## 89 asc = x6 33.318 -0.622
## 236 y2 ~~ y4 30.800 0.210
## 105 aap =~ x6 30.172 -0.071
```

```
r2.mod <- "
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = v1 + v2 + v3 + v4
asc = y5 + y6 + y7
aap = v8 + v9 + v10
ach = v11 + v12 + v13
# residual correlations
x2 ~~ x3
y1 ~~ y5
y3 ~~ y6
r2.fit <- cfa(r2.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r2.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 519.363 139.000 0.000 0.875 0.846 0.102
head(modificationindices(r2.fit, sort. = T, standardized = F))
## lhs op rhs mi epc
## 90 asc = ^{\sim} x6 34.999 -0.649
## 122 ach =~ x6 34.607 -0.391
## 237 y2 ~~ y4 32.700 0.216
## 106 aap =~ x6 30.384 -0.071
## 236 y2 ~~ y3 28.446 -0.209
```

238 y2 ~~ y5 21.973 0.218

```
r3.mod <- "
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = v1 + v2 + v3 + v4
asc = y5 + y6 + y7
aap = v8 + v9 + v10
ach = v11 + v12 + v13
# residual correlations
x2 ~~ x3
y1 ~~ y5
y2 ~~ y4
y3 ~~ y6
r3.fit <- cfa(r3.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r3.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 487.453 138.000 0.000 0.885 0.858 0.098
head(modificationindices(r3.fit, sort. = T, standardized = F))
## lhs op rhs mi epc
## 91 asc = x6 35.238 -0.624
## 123 ach =~ x6 34.689 -0.391
## 107 aap =~ x6 30.379 -0.071
## 238 y2 ~~ y5 24.022 0.213
```

237 y2 ~~ y3 20.478 -0.158 ## 247 y3 ~~ y4 20.290 0.138

```
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = v1 + v2 + v3 + v4
asc = y5 + y6 + y7
aap = v8 + v9 + v10
ach = v11 + v12 + v13
# residual correlations
x2 ~~ x3
y1 ~~ y5
y2 \sim y4 + y5
y3 ~~ y6
r4.fit <- cfa(r4.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r4.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 458.813 137.000 0.000 0.894 0.868 0.095
head(modificationindices(r4.fit, sort. = T, standardized = F))
## lhs op rhs mi epc
## 124 ach = x6 34.840 -0.392
## 92 asc =~ x6 34.033 -0.593
## 108 aap =~ x6 30.378 -0.071
## 247 y3 ~~ y4 29.168 0.166
## 238 y2 ~~ y3 28.271 -0.195
## 227 y1 ~~ y2 26.859 0.336
```

r4.mod <- "

```
r5.mod <- "
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = v1 + v2 + v3 + v4
asc = y5 + y6 + y7
aap = v8 + v9 + v10
ach = v11 + v12 + v13
# residual correlations
x2 ~~ x3
y1 ~~ y5
y2 \sim y4 + y5
y3 \sim y4 + y6
r5.fit <- cfa(r5.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r5.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 424.057 136.000 0.000 0.905 0.881 0.090
head(modificationindices(r5.fit, sort. = T, standardized = F))
## lhs op rhs mi epc
## 125 ach = ^{\sim} x6 34.889 -0.392
## 93 asc =~ x6 33.540 -0.580
## 109 aap =~ x6 30.317 -0.071
## 201 x5 ~~ x6 18.422 0.073
## 114 aap =~ y5 11.585 0.075
```

278 y6 ~~ y12 11.543 0.069

```
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = v1 + v2 + v3 + v4
asc = v5 + v6 + v7
aap = y8 + y9 + y10
ach = v11 + v12 + v13
# residual correlations
x2 ~~ x3
x5 ~~ x6
v1 ~~ y5
y2 \sim y4 + y5
y3 ~~ y4 + y6
r6.fit <- cfa(r6.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r6.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 405.309 135.000 0.000 0.911 0.888 0.088
head(modificationindices(r6.fit, sort. = T, standardized = F))
## lhs op rhs mi epc
## 126 ach = x6 36.818 -0.397
## 94 asc =~ x6 33.012 -0.580
## 110 aap =~ x6 30.394 -0.069
## 278 y6 ~~ y12 11.630 0.070
## 115 aap = y5 11.615 0.075
```

r6.mod <- "

273 y6 ~~ y7 11.407 0.106

```
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = v1 + v2 + v3 + v4
asc = v5 + v6 + v7
aap = y8 + y9 + y10
ach = v11 + v12 + v13
# residual correlations
x2 ~~ x3
x5 ~~ x6
v1 ~~ y5
y2 \sim y4 + y5
y3 ~~ y4 + y6
y6 ~~ y12
r7.fit <- cfa(r7.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r7.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 392.724 134.000 0.000 0.915 0.892 0.086
head(modificationindices(r7.fit, sort. = T, standardized = F))
## lhs op rhs mi epc
## 127 ach = ^{\sim} x6 36.937 -0.397
## 95 asc =~ x6 33.496 -0.586
## 111 aap =~ x6 30.490 -0.070
## 107 aap =~ x2 11.378 -0.028
## 255 y3 ~~ y12 11.267 0.069
## 116 aap =~ y5 11.241 0.074
```

r7.mod <- "

```
r8.mod <- "
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = y1 + y2 + y3 + y4
asc = v5 + v6 + v7
aap = y8 + y9 + y10
ach = v11 + v12 + v13
# residual correlations
x2 ~~ x3
x5 ~~ x6
y1 ~~ y5
y2 ~~ y4 + y5
y3 \sim y6 + y4 + y12
y6 ~~ y12
r8.fit <- cfa(r8.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r8.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 380.729 133.000 0.000 0.919 0.895 0.084
modind <- modificationindices(r8.fit, sort. = T, standardized = F)
head(subset(modind, op == "~~"), 1)
## lhs op rhs mi epc
## 274 y6 ~~ y7 11.321 0.104
```

```
r9.mod <- "
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = y1 + y2 + y3 + y4
asc = v5 + v6 + v7
aap = y8 + y9 + y10
ach = v11 + v12 + v13
# residual correlations
x2 ~~ x3
x5 ~~ x6
y1 ~~ y5
y2 ~~ y4 + y5
y3 \sim y6 + y4 + y12
v6 ~~ v7 + v12
r9.fit <- cfa(r9.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r9.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 369.014 132.000 0.000 0.922 0.899 0.083
modind <- modificationindices(r9.fit, sort. = T, standardized = F)
head(subset(modind, op == "~~"), 1)
## lhs op rhs mi epc
```

252 y3 ~~ y7 12.793 0.09

```
r10.mod <- "
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = y1 + y2 + y3 + y4
asc = v5 + v6 + v7
aap = y8 + y9 + y10
ach = v11 + v12 + v13
# residual correlations
x2 ~~ x3
x5 ~~ x6
v1 ~~ y5
y2 \sim y4 + y5
v3 \sim v6 + v4 + v12 + v9
v6 ~~ v7 + v12
r10.fit <- cfa(r10.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r10.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 366.344 131.000 0.000 0.923 0.899 0.083
modind <- modificationindices(r10.fit, sort. = T, standardized = F)</pre>
head(subset(modind, op == "~~"), 1)
```

lhs op rhs mi epc ## 192 x4 ~~ x6 11.301 0.055

```
r11.mod <- "
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = v1 + v2 + v3 + v4
asc = y5 + y6 + y7
aap = y8 + y9 + y10
ach = v11 + v12 + v13
# residual correlations
x2 ~~ x3
x4 ~~ x6
x5 ~~ x6
v1 ~~ y5
y2 \sim y4 + y5
y3 ~~ y6 + y4 + y12 + y9
y6 ~~ y7 + y12
r11.fit <- cfa(r11.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r11.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 352.055 130.000 0.000 0.927 0.904 0.081
modind <- modificationindices(r11.fit, sort. = T, standardized = F)
head(subset(modind, op == "~~"), 1)
```

lhs op rhs mi epc ## 145 x1 ~~ x5 12.032 -0.033

```
r12.mod <- "
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = v1 + v2 + v3 + v4
asc = v5 + v6 + v7
aap = v8 + v9 + v10
ach = "y11 + y12 + y13
# residual correlations
x1 ~~ x5
x2 ~~ x3
x4 ~~ x6
x5 ~~ x6
v1 ~~ v5
y2 ~~ y4 + y5
y3 ~~ y6 + y4 + y12 + y9
y6 ~~ y7 + y12
r12.fit <- cfa(r12.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r12.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 337.978 129.000 0.000 0.931 0.909 0.079
modind <- modificationindices(r12.fit, sort. = T, standardized = F)</pre>
head(subset(modind, op == "~~"), 1)
## lhs op rhs mi epc
## 178 x3 ~~ x6 10.183 0.054
```

```
r13.mod <- "
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = v1 + v2 + v3 + v4
asc = v5 + v6 + v7
aap = v8 + v9 + v10
ach = "y11 + y12 + y13
# residual correlations
x1 ~~ x5
x3 ~~x2 + x6
x4 ~~ x6
x5 ~~ x6
v1 ~~ v5
y2 ~~ y4 + y5
y3 ~~ y6 + y4 + y12 + y9
y6 ~~ y7 + y12
r13.fit <- cfa(r13.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r13.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 325.400 128.000 0.000 0.935 0.913 0.077
modind <- modificationindices(r13.fit, sort. = T, standardized = F)</pre>
head(subset(modind, op == "~~"), 1)
## lhs op rhs mi epc
## 163 x2 ~~ x6 30.105 0.128
```

```
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = v1 + v2 + v3 + v4
asc = v5 + v6 + v7
aap = v8 + v9 + v10
ach = v11 + v12 + v13
# residual correlations
x1 ~~ x5
x3 ~~ x2
x6 \sim x2 + x3 + x4 + x5
y1 ~~ y5
v2 \sim v4 + v5
v3 \sim v6 + v4 + v12 + v9
y6 ~~ y7 + y12
r14.fit <- cfa(r14.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r14.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 292,230 127,000 0,000 0,946 0,927 0,071
modind <- modificationindices(r14.fit, sort. = T, standardized = F)
head(subset(modind, op == "~~"), 1)
## lhs op rhs mi epc
```

r14.mod <- "

253 y3 ~~ y7 9.872 0.077

```
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = v1 + v2 + v3 + v4
asc = v5 + v6 + v7
aap = v8 + v9 + v10
ach = v11 + v12 + v13
# residual correlations
x1 ~~ x5
x3 ~~ x2
x6 \sim x2 + x3 + x4 + x5
y1 ~~ y5
v2 \sim v4 + v5
y3 \sim y6 + y4 + y12 + y9 + y7
y6 ~~ y7 + y12
r15.fit <- cfa(r15.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r15.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 281.945 126.000 0.000 0.949 0.931 0.069
modind <- modificationindices(r15.fit, sort. = T, standardized = F)
head(subset(modind, op == "~~"), 1)
## lhs op rhs mi epc
```

r15.mod <- "

275 y6 ~~ y8 9.376 0.672

```
r16.mod <- "
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = v1 + v2 + v3 + v4
asc = v5 + v6 + v7
aap = v8 + v9 + v10
ach = v11 + v12 + v13
# residual correlations
x1 ~~ x5
x3 ~~ x2
x6 \sim x2 + x3 + x4 + x5
v1 ~~ v5
v2 \sim v4 + v5
v3 \sim v6 + v4 + v12 + v9 + v7
v6 \sim v7 + v8 + v12
r16.fit <- cfa(r16.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r16.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 272.131 125.000 0.000 0.952 0.934 0.067
modind <- modificationindices(r16.fit, sort. = T, standardized = F)
```

head(subset(modind, op == "~~"), 1)

lhs op rhs mi epc ## 216 x5 ~~ y9 8.747 0.354

```
r17.mod <- "
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = v1 + v2 + v3 + v4
asc = v5 + v6 + v7
aap = v8 + v9 + v10
ach = v11 + v12 + v13
# residual correlations
x3 ~~ x2
x5 ~~ x1 + v9
x6 \sim x2 + x3 + x4 + x5
y1 ~~ y5
v2 \sim v4 + v5
v3 \sim v6 + v4 + v12 + v9 + v7
v6 \sim v7 + v8 + v12
r17.fit <- cfa(r17.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r17.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 262.912 124.000 0.000 0.954 0.937 0.066
modind <- modificationindices(r17.fit, sort. = T, standardized = F)
```

head(subset(modind, op == "~~"), 1)

lhs op rhs mi epc ## 284 y7 ~~ y12 9.016 0.054

```
r18.mod <- "
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = v1 + v2 + v3 + v4
asc = y5 + y6 + y7
aap = "y8 + y9 + y10"
ach = v11 + v12 + v13
# residual correlations
x3 ~~ x2
x5 \sim x1 + y9
x6 \sim x2 + x3 + x4 + x5
v1 ~~ y5
v2 ~~ v4 + v5
y3 ~~ y6 + y4 + y12 + y9 + y7
y6 ~~ y7 + y8 + y12
y7 ~~ y12
r18.fit <- cfa(r18.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r18.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 253.700 123.000 0.000 0.957 0.940 0.064
modind <- modificationindices(r18.fit, sort. = T, standardized = F)</pre>
head(subset(modind, op == "~~"), 1)
## lhs op rhs mi epc
```

260 y4 ~~ y5 8.149 -0.134

```
r19.mod <- "
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = y1 + y2 + y3 + y4
asc = y5 + y6 + y7
aap = v8 + v9 + v10
ach = v11 + v12 + v13
# residual correlations
x3 ~~ x2
x5 \sim x1 + y9
x6 \sim x2 + x3 + x4 + x5
v1 ~~ y5
v2 \sim v4 + v5
y3 ~~ y6 + y4 + y12 + y9 + y7
y4 ~~ y5
y6 ~~ y7 + y8 + y12
y7 ~~ y12
r19.fit <- cfa(r19.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r19.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 245.343 122.000 0.000 0.960 0.943 0.062
modind <- modificationindices(r19.fit, sort. = T, standardized = F)
head(subset(modind, op == "~~"), 1)
## lhs op rhs mi epc
## 163 x1 ~~ v10 6.706 0.259
```

```
r20.mod <- "
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = y1 + y2 + y3 + y4
asc = v5 + v6 + v7
aap = y8 + y9 + y10
ach = v11 + v12 + v13
# residual correlations
x1 ~~ y10
x3 ~~ x2
x5 ~~ x1 + y9
x6 \sim x2 + x3 + x4 + x5
y1 ~~ y5
y2 ~~ y4 + y5
v3 \sim v6 + v4 + v12 + v9 + v7
y4 ~~ y5
y6 ~~ y7 + y8 + y12
y7 ~~ y12
r20.fit <- cfa(r20.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r20.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 238.438 121.000 0.000 0.961 0.946 0.061
modind <- modificationindices(r20.fit, sort. = T, standardized = F)
head(subset(modind, op == "~~"), 1)
## lhs op rhs mi epc
## 167 x2 ~~ x4 5.708 0.022
```

```
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = v1 + v2 + v3 + v4
asc = v5 + v6 + v7
aap = y8 + y9 + y10
ach = v11 + v12 + v13
# residual correlations
x1 ~~ v10
x2 ~~ x4
x3 ~~ x2
x5 \sim x1 + y9
x6^{-} x2 + x3 + x4 + x5
y1 ~~ y5
y2 \sim y4 + y5
y3 \sim y6 + y4 + y12 + y9 + y7
v4 ~~ v5
y6 ~~ y7 + y8 + y12
y7 ~~ y12
r21.fit <- cfa(r21.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r21.fit, c("chisg", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 232.707 120.000 0.000 0.963 0.947 0.060
modind <- modificationindices(r21.fit, sort. = T, standardized = F)
head(subset(modind, op == "~~"), 1)
## lhs op rhs mi epc
## 266 y4 ~~ y11 5.244 -0.052
```

r21.mod <- "

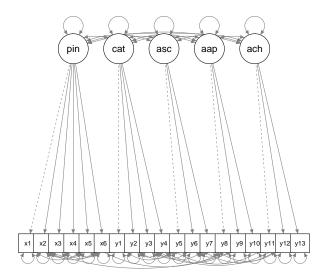
```
r22.mod <- "
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = y1 + y2 + y3 + y4
asc = v5 + v6 + v7
aap = v8 + v9 + v10
ach = v11 + v12 + v13
# residual correlations
x1 ~~ y10
x2 ~~ x4
x3 ~~ x2
x5 \sim x1 + y9
x6^{-} x2 + x3 + x4 + x5
y1 ~~ y5
v2 ~~ v4 + v5
v3 \sim v6 + y4 + y12 + y9 + y7
y4 \sim y5 + y11
y6 ~~ y7 + y8 + y12
y7 ~~ y12
r22.fit <- cfa(r22.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r22.fit, c("chisg", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
## 227.164 119.000 0.000 0.964 0.949 0.059
modind <- modificationindices(r22.fit, sort. = T, standardized = F)
head(subset(modind, op == "~~"), 1)
## lhs op rhs mi epc
## 169 x2 ~~ x5 5.094 0.029
```

```
r23.mod <- "
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = y1 + y2 + y3 + y4
asc = y5 + y6 + y7
aap = v8 + v9 + v10
ach = v11 + v12 + v13
# residual correlations
x1 ~~ y10
x2 ~~x4 + x5
x3 ~~ x2
x5^{~~}x1 + v9
x6 \sim x2 + x3 + x4 + x5
y1 ~~ y5
y2 ~~ y4 + y5
v3 \sim v6 + v4 + v12 + v9 + v7
v4 ~~ y5 + y11
y6 ~~ y7 + y8 + y12
y7 ~~ y12
```

r23.fit <- cfa(r23.mod, sample.cov = corMat, sample.nobs = 261)
fitmeasures(r23.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))

chisq df pvalue cfi tli rmsea ## 221.655 118.000 0.000 0.966 0.951 0.058

Maybe this was there model?



At first glance, the results indicated that the correlation between some measurement errors of certain observed variables of the measurement model lead to a poor fit of the hypothesized model (see Figure 1) and the data obtained in the sample. After the respecifications, the final models (Models a and b with respecifications) were obtained.

Wait ... did they do this on their SR model or their CFA?

I obtained similar RMSE and TLI but probably a different model.

Step 3/4

```
step4.mod <- "
# define measurement models
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = y1 + y2 + y3 + y4
asc = y5 + y6 + y7
aap = v8 + v9 + v10
ach = v11 + v12 + v13
# structural paths
cat ~ pin
aap ~ pin
asc ~ cat + aap + pin
ach ~ asc + aap
# residual correlations
x1 ~~ v10
x2^{-}x4 + x5
x3 ~~ x2
x5 ~~ x1 + y9
x6 \sim x2 + x3 + x4 + x5
v1 ~~ v5
v2 \sim v4 + v5
y3 ~~ y6 + y4 + y12 + y9 + y7
y4 ~~ y5 + y11
y6 ~~ y7 + y8 + y12
y7 ~~ y12
step4.fit <- sem(step4.mod, sample.cov = corMat, sample.nobs = 261)
```

Step 4 - fit

```
fitmeasures(step4.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))

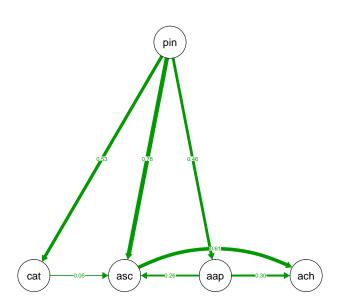
## chisq df pvalue cfi tli rmsea
## 229.957 121.000 0.000 0.964 0.949 0.059

anova(step4.fit, r23.fit)

## Chi Square Difference Test
##

## Df AIC BIC Chisq Chisq diff Df diff Pr(>Chisq)
## r23.fit 118 12656 12913 221.65
## step4.fit 121 12658 12904 229.96 8.3023 3 0.04016 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

But, I used up 3 more df than they did and 261 participants, I estimated 69 parameters or 3.78 participants/parameter estimated.



Alternate paths

```
alt1.mod <- "
# define measurement models
pin = x1 + x2 + x3 + x4 + x5 + x6
cat = v1 + v2 + v3 + v4
asc = y5 + y6 + y7
aap = v8 + v9 + v10
ach = v11 + v12 + v13
# structural paths
cat ~ pin + aap + asc
aap ~ pin
asc ~ cat + aap + pin + ach
ach ~ asc + aap
# residual correlations
x1 ~~ v10
x2^{-}x4 + x5
x3 ~~ x2
x5 ~~ x1 + y9
x6 \sim x2 + x3 + x4 + x5
v1 ~~ v5
v2 \sim v4 + v5
y3 ~~ y6 + y4 + y12 + y9 + y7
y4 ~~ y5 + y11
y6 ~~ y7 + y8 + y12
y7 ~~ y12
alt1.fit <- sem(alt1.mod, sample.cov = corMat, sample.nobs = 261)
```

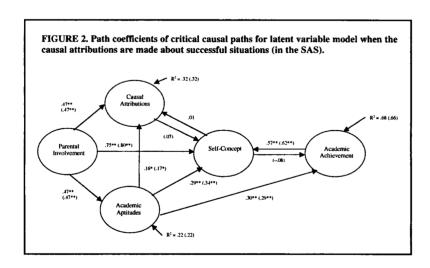
```
fitmeasures(alt1.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))
## chisq df pvalue cfi tli rmsea
```

fitmeasures(r23.fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea"))

221.655 118.000 0.000 0.966 0.951 0.058

chisq df pvalue cfi tli rmsea ## 221.655 118.000 0.000 0.966 0.951 0.058

Alternative model



Their results

TABLE 4 Goodness-of-Fit Indexes for Latent Variable Models

Model	X ²	df	X ² /df	р	GFI	AGFI	CFI	TLI	RMR	RMSEA
		М	odel a (attrib	utions in su	ccessful si	tuations)				
Null model	3,204.87	171	18.70							
Initial theoretical model	849.07	143	5.93	.00	.74	.66	.79	.72	.11	.14
Final model	233.08	124	1.87	.00	.91	.87	.96	.95	.07	.05
Alternative model (a1)	233.08	124	1.87	.00	.91	.87	.96	.95	.07	.05
Alternative model (a2)	233.08	124	1.87	.00	.91	.87	.96	.95	.07	.05