Statistical Inference and Machine Learning in Earth Science
SIMLES

Module 2
Statistical Inferences

Lecture 1
Maximum Likelihood Estimator

Cristi Proistosescu

Probability Models: Moments

Probability models: allow us to model a random process

Random variables allow us to simulate the outcome of random processes.

Distributions: summarize the outcome of a random process.

- **pmfs, pdfs** describe the underlying *process*.
- **Histograms** summarize a finite sample.

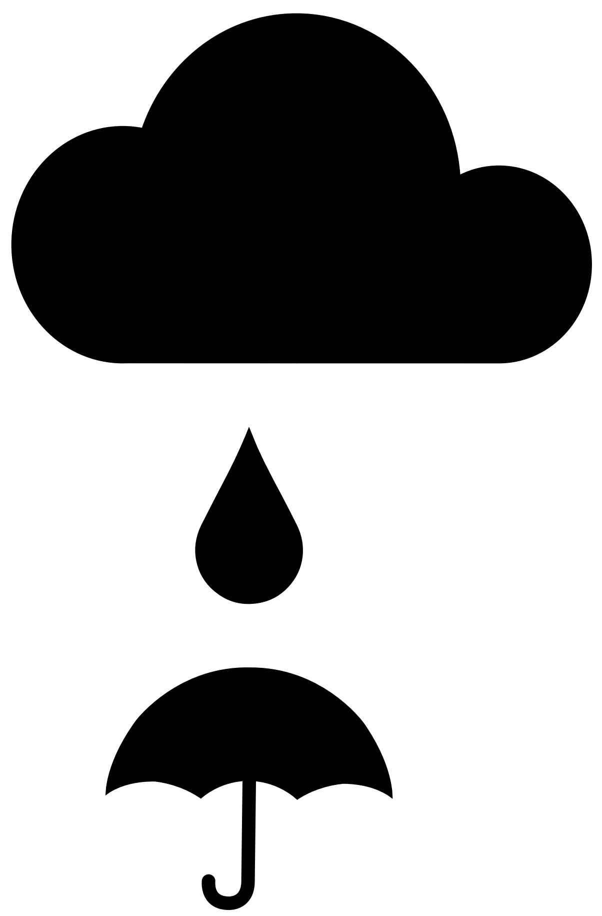
Moments: another useful summary of the outcome of a random process.

Probability Models: Random Variable

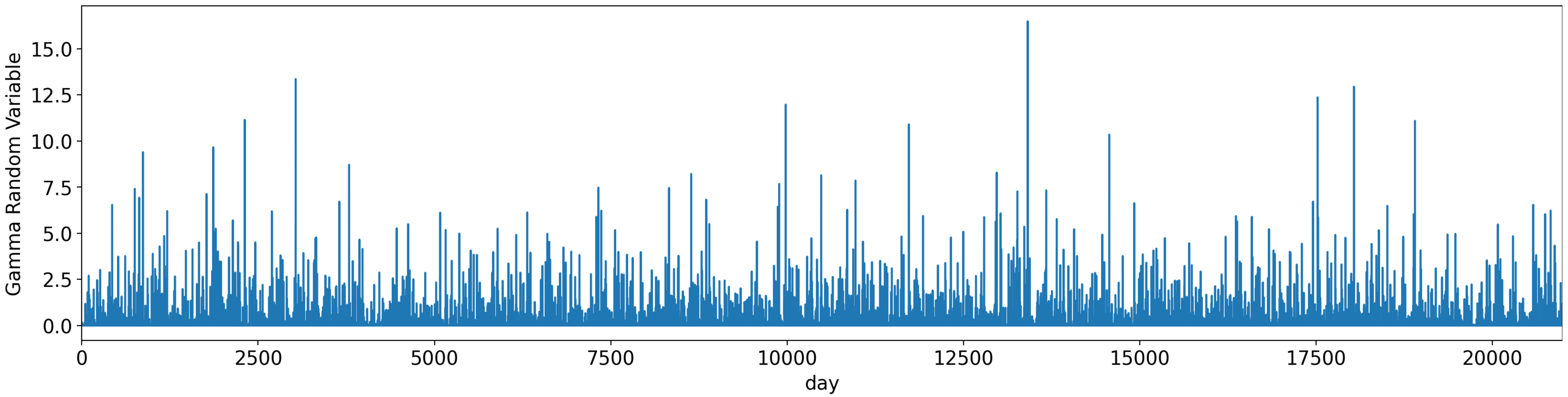
Probability models: allow us to model a random process

Random variables allow us to simulate the outcome of random processes.

- dice roll
- coin flip
- precipitation



$$X \sim \Gamma(\alpha, \beta)$$



Probability ↔ Statistical inference

Probability

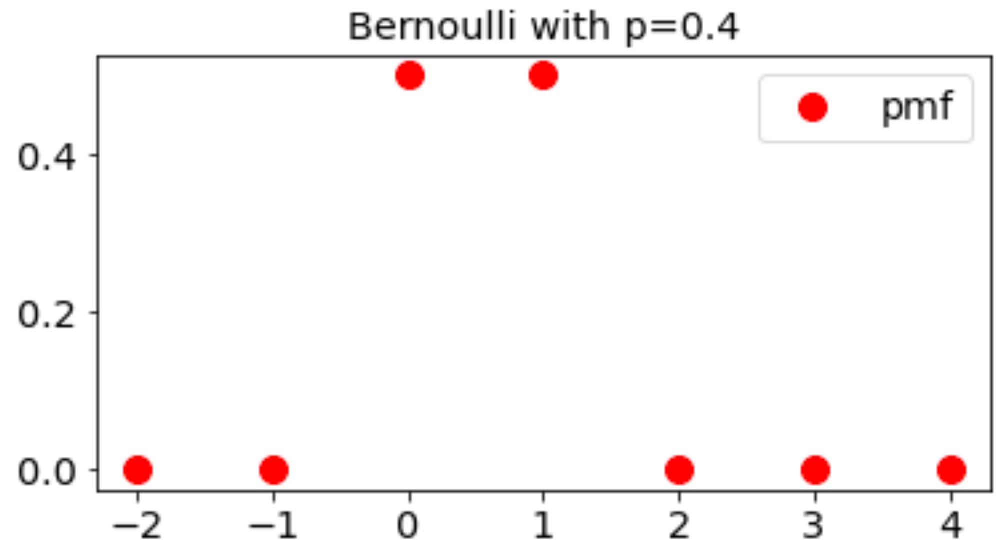
Process

↓

Distribution $p(X = 1) = p$

↓

Data



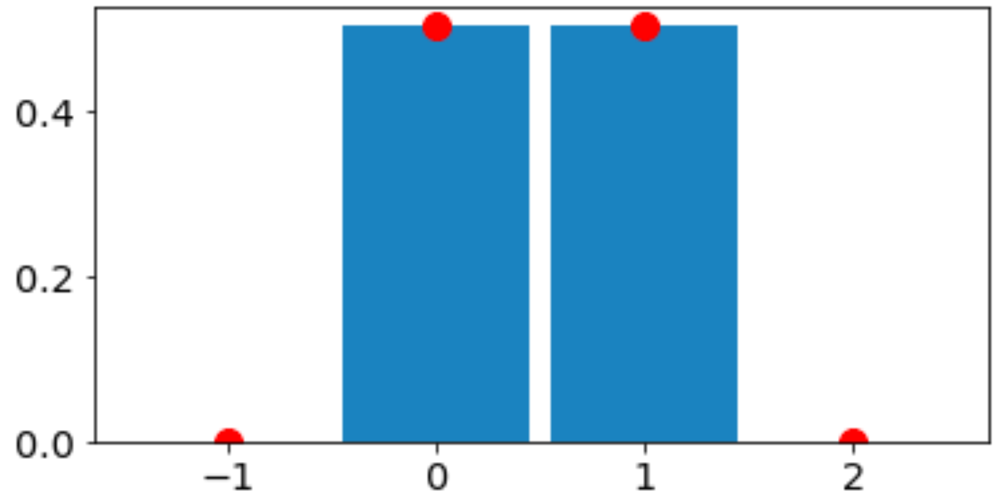
Statistics

[1 1 1 1 0 1 1 1 1 0 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 0 1 1 1 1]

Data

↓

Distribution



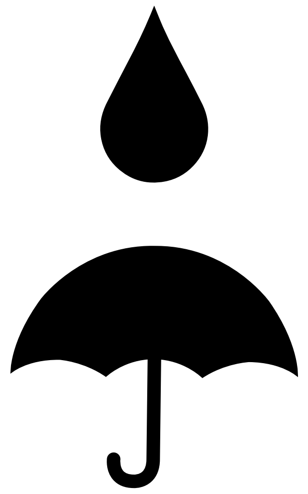
↓

Process

Probability $\leftrightarrow$ Statistical inference

Probability

Process

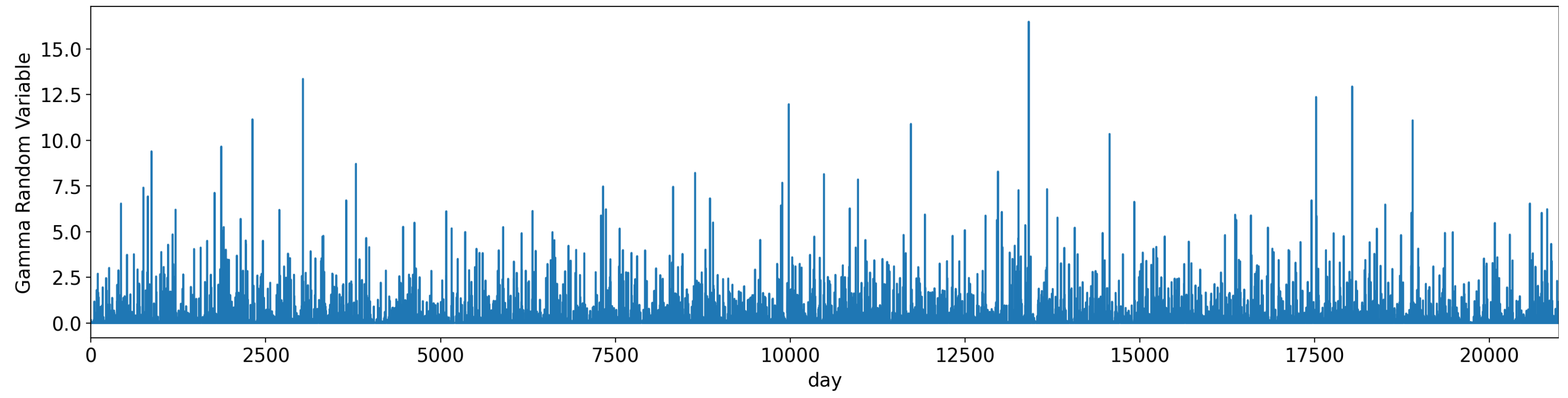


$\downarrow$

Distribution $X \sim \Gamma(\alpha, \beta)$

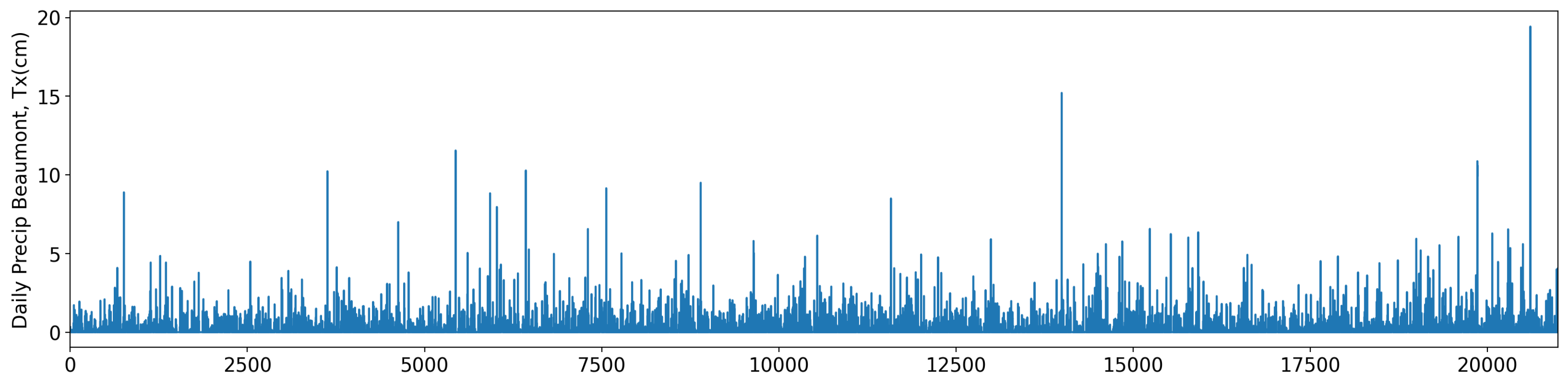
$\downarrow$

Data



Statistics

Data

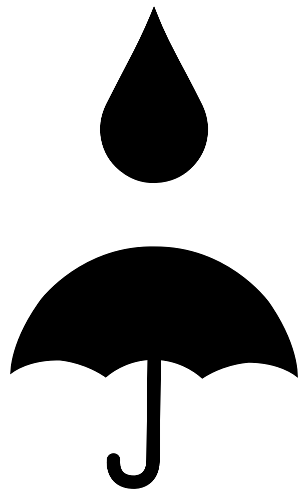


$\downarrow$

Distribution $X \sim \Gamma(\alpha, \beta)$

$\downarrow$

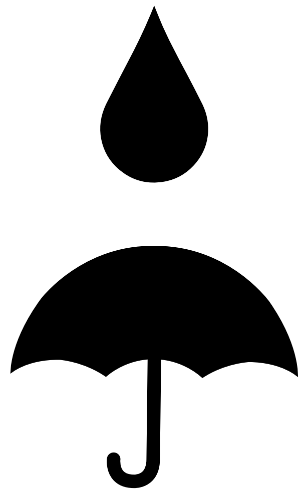
Process



Probability ↔ Statistical inference

Probability

Process



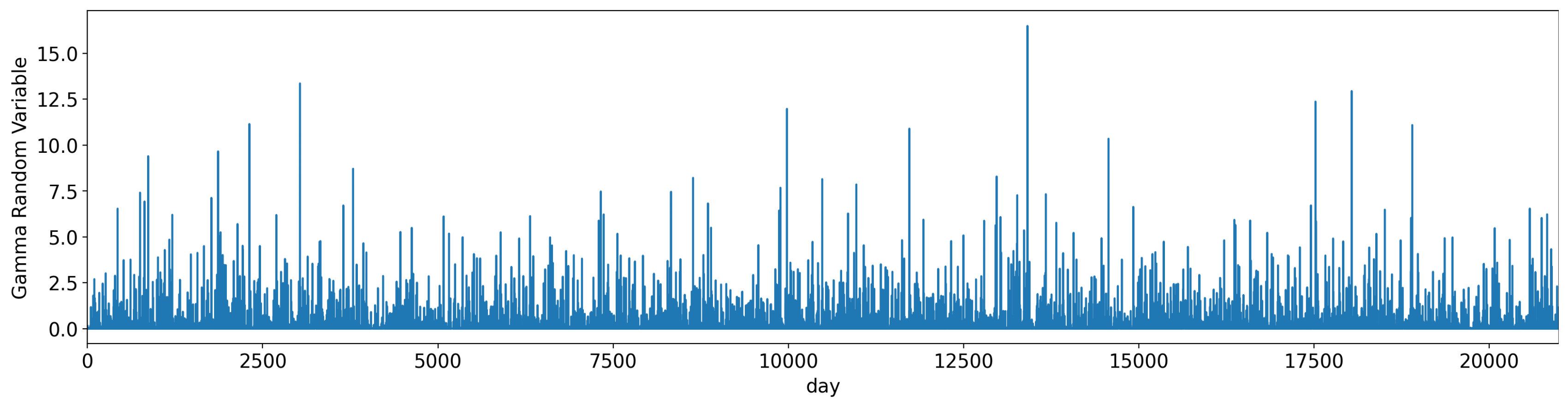
↓

Distribution

$$X \sim \Gamma(\alpha, \beta)$$

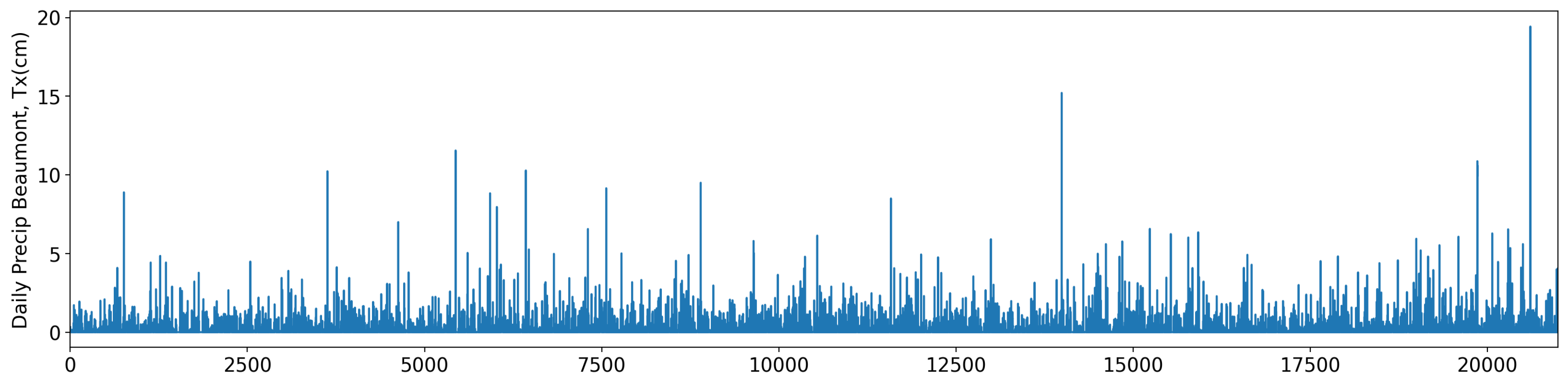
↓

Data



Statistics

Data



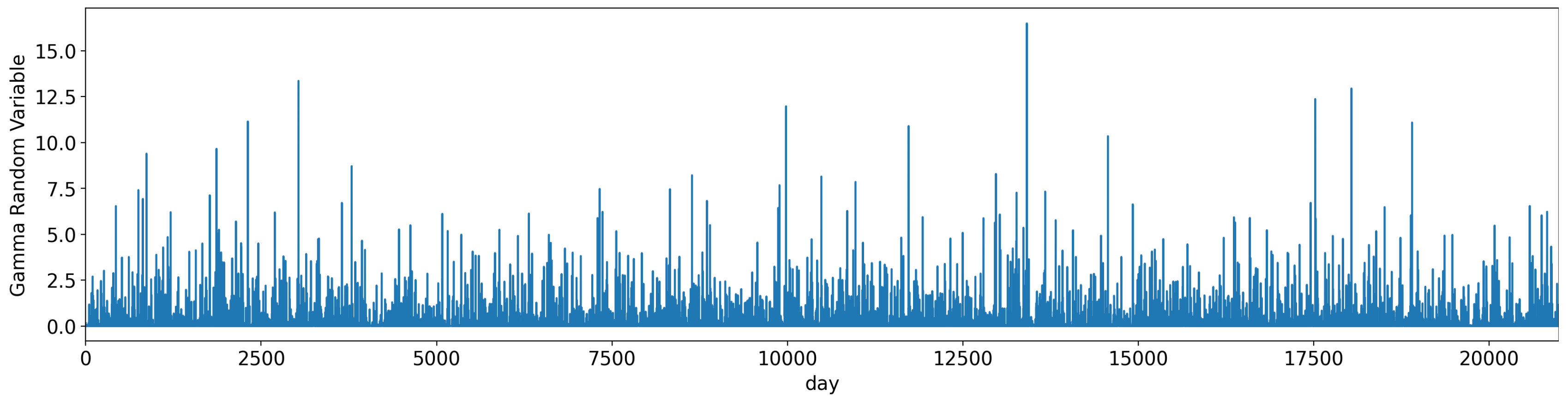
↓

Distribution

$$X \sim \Gamma(\alpha, \beta)$$

↓

Simulation



Statistical Inference: Likelihood

Data

$$\mathbf{x} = \left[x_1, x_2, \ldots, x_n\right]$$

0 1

$$x = [0,1,0,1,1,1,0,0,1,1]$$

Statistical Inference: Likelihood

Data

$$\mathbf{x} = [x_1, x_2, \ldots, x_n]$$

0 1

$$x = [0,1,0,1,1,1,0,0,1,1]$$

Model Parameters

$$\theta = [\theta_1, \theta_2, \ldots, \theta_n]$$

Model Parameters

$$X \sim Bern(p) \Rightarrow \theta = [p]$$

Statistical Inference: Likelihood

Data

$$\mathbf{x} = [x_1, x_2, \ldots, x_n]$$

0 1

$$x = [0,1,0,1,1,1,0,0,1,1]$$

Model Parameters

$$\theta = [\theta_1, \theta_2, \ldots, \theta_n]$$

Model Parameters

$$X \sim Bern(p) \Rightarrow \theta = [p]$$

Likelihood

$$\mathscr{L} = p(x \mid \theta)$$

What is the probability that
a coin with parameter θ
generated this data?

Gaussian

Data

$$\mathbf{x} = \left[x_1, x_2, \ldots, x_n\right]$$

[5.73541792 4.59128669 5.67946042 5.66998442 6.2729418 7.11995521
5.59525591 6.13115623 1.95810532 6.40934121 4.98045955 6.24052625
3.23506281 7.29882568 6.69172193 5.13690828 5.23322762 3.53067353
5.1935833 5.92951912]

Model Parameters

$$\theta = \left[\theta_1, \theta_2, \ldots, \theta_n\right]$$

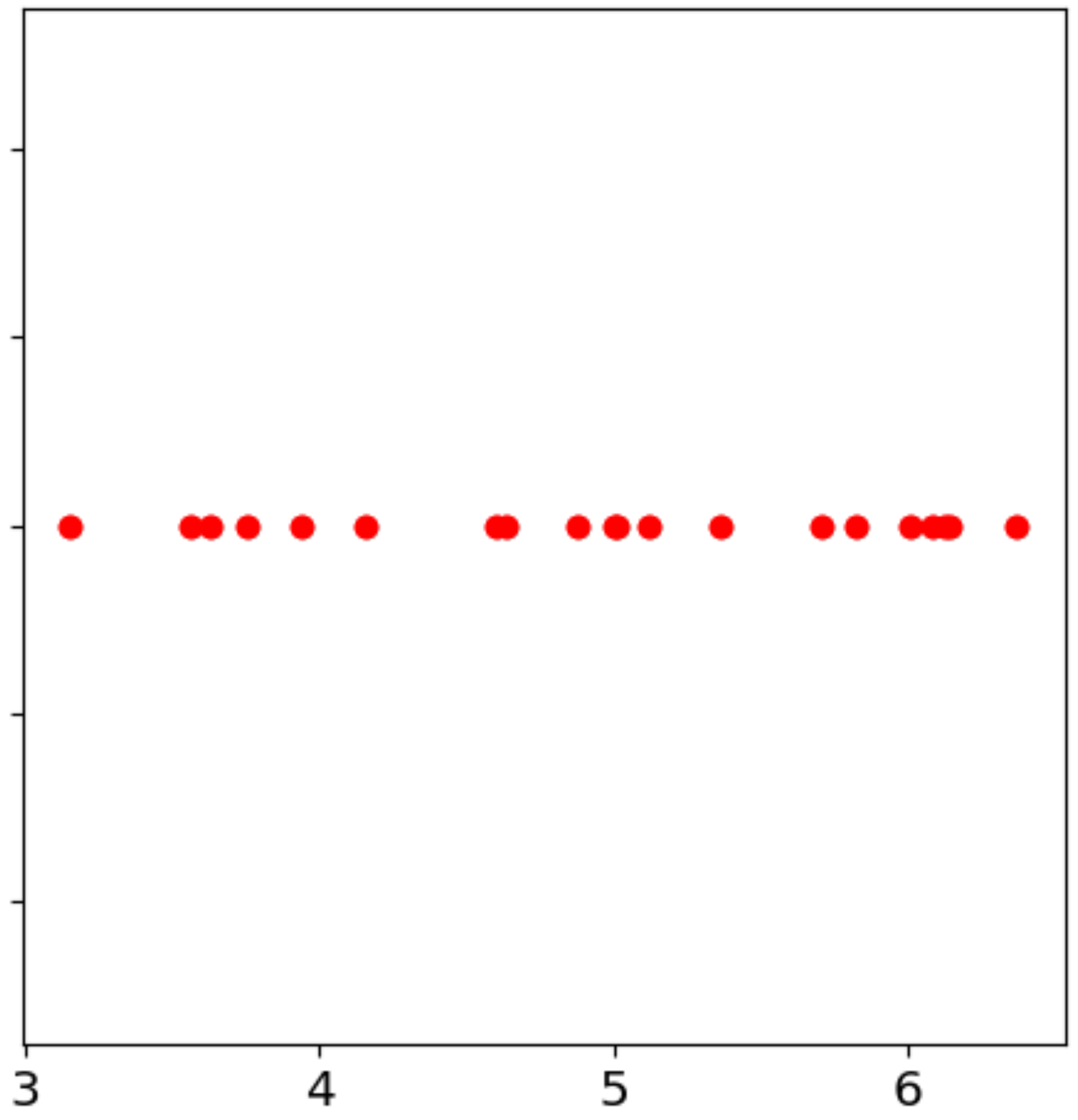
Model Parameters

$$X \sim \mathcal{N}(\mu, \sigma) \Rightarrow \theta = [\mu, \sigma]$$

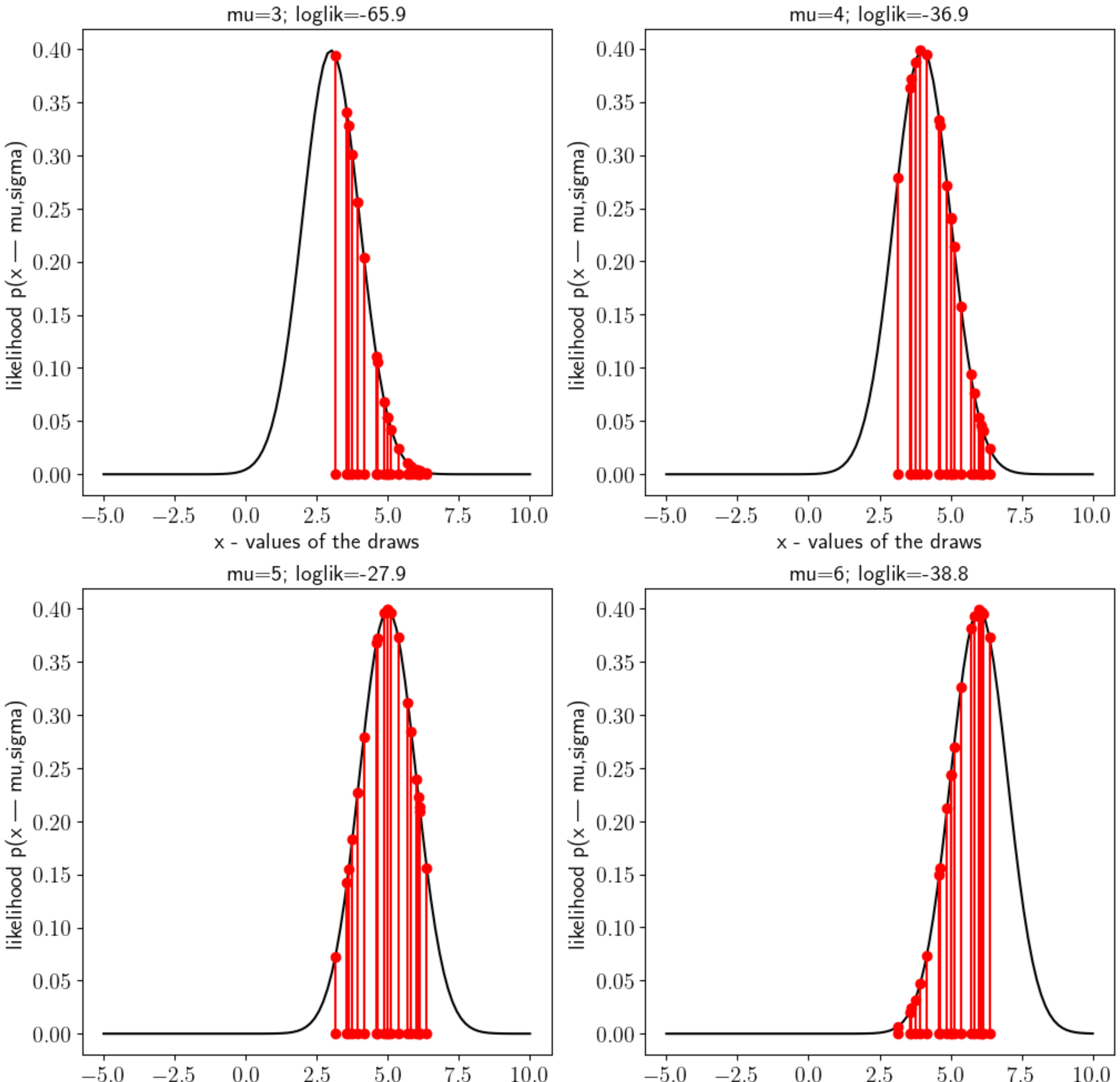
Likelihood

$$\mathcal{L} = p(x \,|\, \theta)$$

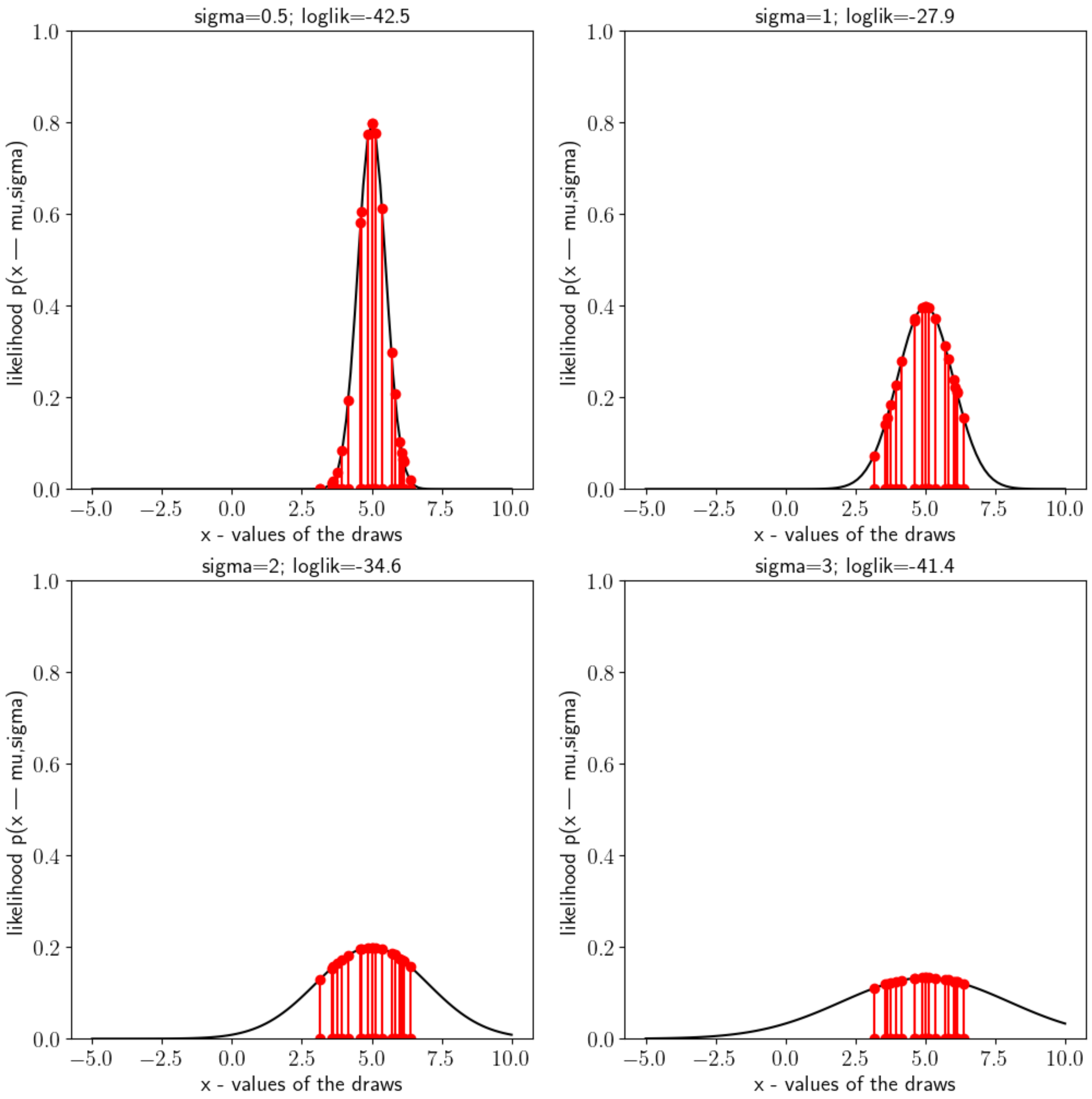
Gaussian



Gaussian


mu=3; loglik=-65.9
mu=4; loglik=-36.9
mu=5; loglik=-27.9
mu=6; loglik=-38.8
likelihood p(x — mu,sigma)
x - values of the draws
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0

Gaussian

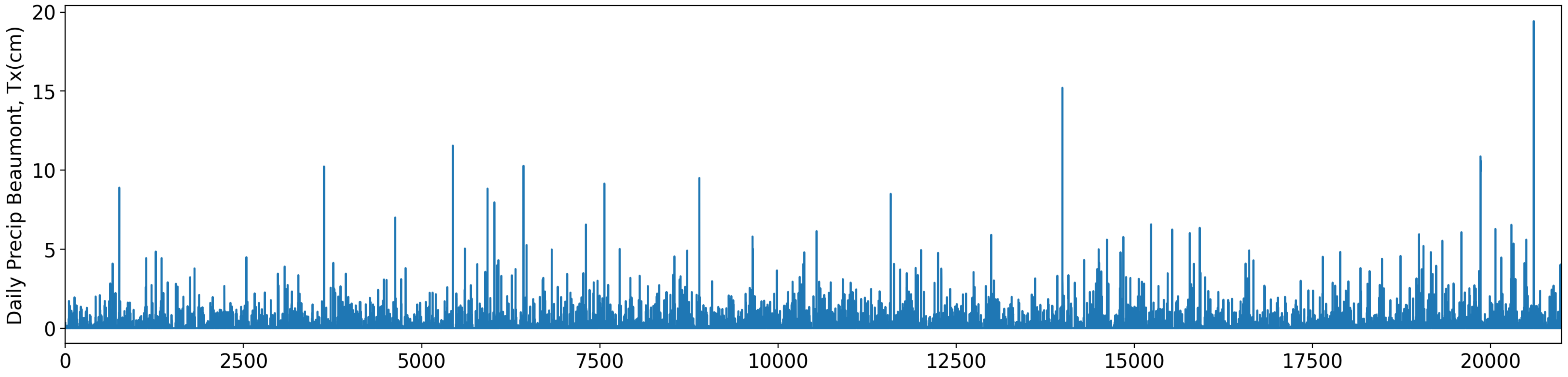


sigma=0.5; loglik=-42.5
sigma=1; loglik=-27.9
sigma=2; loglik=-34.6
sigma=3; loglik=-41.4
likelihood p(x — mu,sigma)
x - values of the draws
1.0
0.8
0.6
0.4
0.2
0.0
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0

Statistical Inference: Likelihood

Data

$$\mathbf{x} = \left[x_1, x_2, \ldots, x_n\right]$$

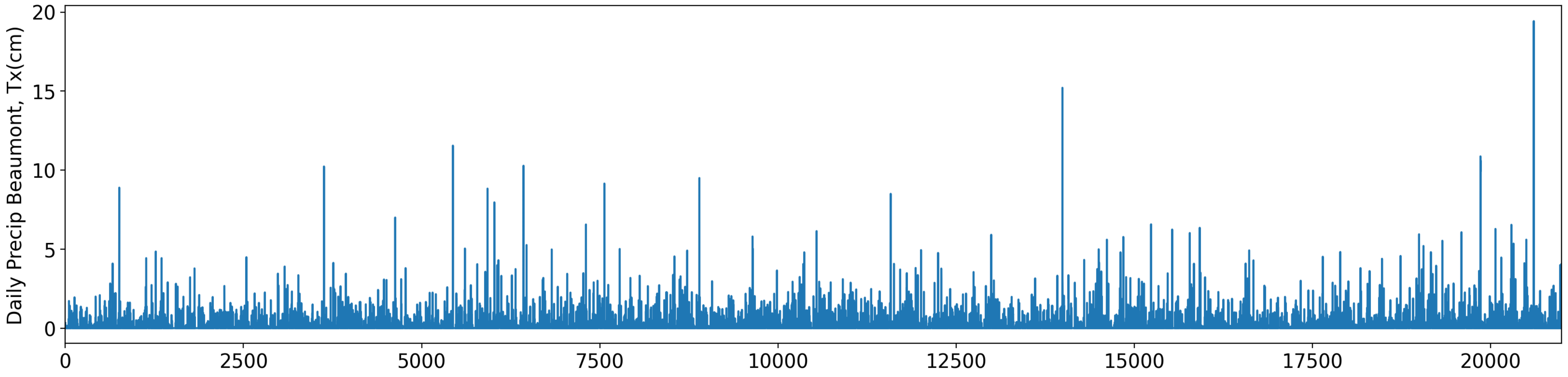


$$\mathbf{x} = \left[P(t_1), P(t_2), \ldots, P(t_n)\right]$$

Statistical Inference: Likelihood

Data

$$\mathbf{x} = \left[x_1, x_2, \ldots, x_n\right]$$



$$\mathbf{x} = \left[P(t_1), P(t_2), \ldots, P(t_n)\right]$$

Model Parameters

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Model Parameters

$$X \sim \Gamma(\alpha, \beta) \Rightarrow \theta = [\alpha, \beta]$$

Statistical Inference: Likelihood

Data

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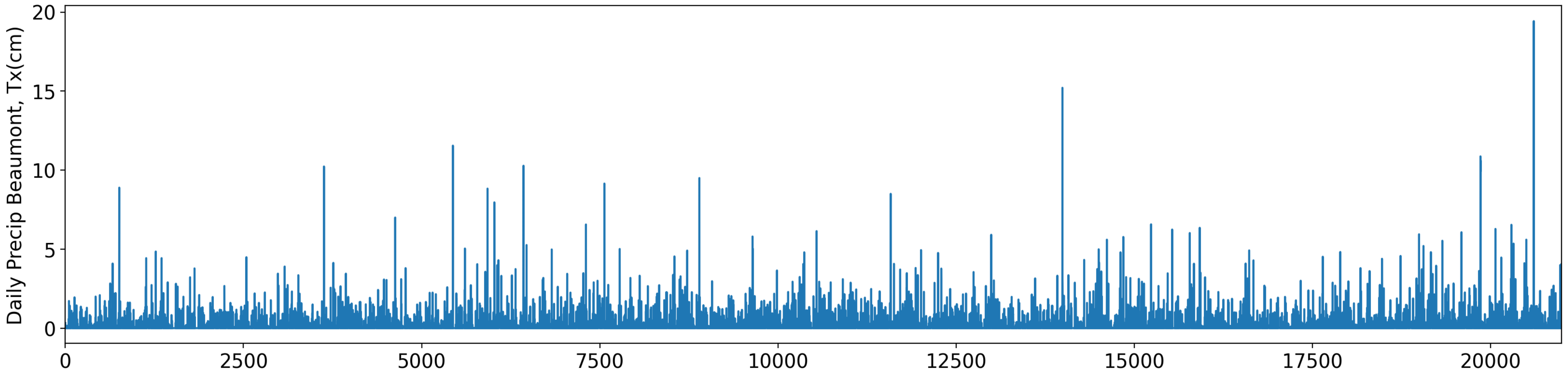

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Model Parameters

$$X \sim \Gamma(\alpha, \beta) \quad \Rightarrow \quad \theta = [\alpha, \beta]$$

Likelihood

$$\mathscr{L} = p(x \,|\, \theta)$$

???