1 Discrete Distributions

1.1 Discrete Uniform Distribution

Notation	$X \sim \mathcal{U}(a,b)$
Support	$x \in \{a, a+1, \ldots, b-1, b\}$
$p_X(x)$	$\frac{1}{n}$
$F_X(x)$	$\frac{x-a+1}{n}$
$\mathbb{E}[X]$	$\frac{a+b}{2}$
$\mathrm{Var}[X]$	$\frac{(b-a+1)^2-1}{12}$

1.2 Bernoulli Distribution

Notation	$X \sim \mathrm{Be}(p)$
Support	$x \in \{0,1\}$
$p_X(x)$	$\begin{cases} 1-p & x=0 \\ p & x=1 \end{cases}$
$F_X(x)$	$\begin{cases} 0 & x<0 \\ 1-p & 0 \le x \le 1 \\ 1 & x>1 \end{cases}$
$\mathbb{E}[X]$	p
$\mathrm{Var}[X]$	$p(1-p)$

1.3 Binomial Distribution

Notation	$X \sim \mathrm{Bin}(n,p)$
Support	$x \in \{0,1,\ldots,n\}$
$p_X(x)$	$\binom{n}{x} \cdot p^x \cdot (1-p)^{n-x}$
$F_X(x)$	$\sum_{i=1}^{x} p_X(i)$
$\mathbb{E}[X]$	np
$\mathrm{Var}[X]$	$np(1-p)$

1.4 Geometric Distribution

Notation	$X \sim \mathrm{Geo}(p)$
Support	$x \in \{1,2,\ldots\}$
$p_X(x)$	$(1-p)^{x-1} \cdot p$
$F_X(x)$	$1-(1-p)^x$
$\mathbb{E}[X]$	$\frac{1}{p}$
$\mathrm{Var}[X]$	$\frac{1-p}{p^2}$

1.5 Poisson Distribution

Notation	$X \sim \mathrm{Poi}(\lambda)$
Support	$x \in \{0,1,\ldots\} = \mathbb{N}_0$
$p_X(x)$	$e^{-\lambda}\frac{\lambda^x}{x!}$
$F_X(x)$	$e^{-\lambda}\sum_{i=0}^{x}\frac{\lambda^i}{i!}$
$\mathbb{E}[X]$	λ
$\mathrm{Var}[X]$	λ

2 Continuous Distributions

2.1 Uniform Distribution

Notation	$X \sim \mathcal{U}(a,b)$
Support	$x \in [a,b]$
$f_X(x)$	$\begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{else} \end{cases}$
$F_X(x)$	$\begin{cases} 0 & x<a \\ \frac{x-a}{b-a} & a \le x \le b \\ 1 & x>b \end{cases}$
$\mathbb{E}[X]$	$\frac{a+b}{2}$
$\mathrm{Var}[X]$	$\frac{(b-a)^2}{12}$

2.2 Normal Distribution

Notation	$X \sim \mathcal{N}(\mu,\sigma^2)$
Support	$x \in \mathbb{R}$
$f_X(x)$	$\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
$F_X(x)$	$\Phi\left(\frac{x-\mu}{\sigma}\right)$
$\mathbb{E}[X]$	μ
$\mathrm{Var}[X]$	σ^2

2.3 Exponential Distribution

Notation	$X \sim \mathrm{Exp}(\lambda)$
Support	$x \in [0,\infty)$
$f_X(x)$	$\begin{cases} \lambda e^{-\lambda x} & x \ge 0 \\ 0 & x<0 \end{cases}$
$F_X(x)$	$\begin{cases} 1-e^{-\lambda x} & x \ge 0 \\ 0 & x<0 \end{cases}$
$\mathbb{E}[X]$	$\frac{1}{\lambda}$
$\mathrm{Var}[X]$	$\frac{1}{\lambda^2}$

2.4 Gamma Distribution

Notation	$X \sim \mathrm{Ga}(\alpha,\lambda)$
Support	$x \in \mathbb{R}^+$
$f_X(x)$	$\begin{cases} \frac{1}{\Gamma(\alpha)}\lambda^\alpha x^{\alpha-1}e^{-\lambda x} & x \ge 0 \\ 0 & x<0 \end{cases}$
$F_X(x)$	$\int_0^x f_X(t)dt$
$\mathbb{E}[X]$	$\frac{\alpha}{\lambda}$
$\mathrm{Var}[X]$	$\frac{\alpha}{\lambda^2}$

Properties

- If $X = \sum_{i=1}^{\alpha} Y_i$ with Y_i *i.i.d.* $\sim \mathrm{Exp}(\lambda)$ then $X \sim \mathrm{Ga}(\alpha,\lambda)$. This is also called Erlang distribution, where the shape α is a positive integer and the gamma function $\Gamma(\alpha)$ in the denominator is replaced by the factorial: $\Gamma(n) = (n-1)!$ for $n > 0$, see below.
- $Ga(1,\lambda) = Exp(\lambda)$

2.5 Erlang Distribution

Notation	$X \sim \mathrm{Erl}(\alpha,\lambda)$
Support	$x \in \mathbb{R}^+$, α is positive integer
$f_X(x)$	$\begin{cases} \frac{1}{(\alpha-1)!}\lambda^\alpha x^{\alpha-1}e^{-\lambda x} & x \ge 0 \\ 0 & x<0 \end{cases}$
$F_X(x)$	$\int_0^x f_X(t)dt$
$\mathbb{E}[X]$	$\frac{\alpha}{\lambda}$
$\mathrm{Var}[X]$	$\frac{\alpha}{\lambda^2}$