Performance Analysis 2024-25 (IN4-341): L7

1 Disease outbreak

Imagine that there is an outbreak of a disease in a small town.

- Because everybody knows everybody in this town, we can assume that the social network of the town can be represented via a complete graph of $N = 25$ nodes.
- Every person can either be healthy or become sick (via infection of its neighbors).
- The infection rate $\beta = 0.5$ is homogeneous for all links.
- Once you become sick, the curing process with rate $\delta = 1$ starts immediately and you can become healthy again.

Questions

1. Write down the NIMFA-equations for this model.

2. Describe what is the main assumption behind the NIMFA approximation.

3. What is the probability of being sick for a random person of this town in the steady state?

2 Turtle Markov chain

Let us assume that a turtle can only eat (state 1), swim (state 2) or walk (state 3) and its behavior can be modeled as a continuous-time Markov chain.

- The rate of going from eating over to swimming in the next time step is 0.1, from swimming to eating is 1.2.
- Because a turtle only finds food in the water, there is no possibility to go from eating to walking and/or vice versa.
- The rate of going from swimming to walking for a turtle is 0.5 and from walking to swimming is 2.
- The rate unit is per hour.

Questions

1. Draw the Markov chain for the behavior of a turtle.

2. How does the infinitesimal generator matrix Q look like?

3. What is the probability that the turtle will swim continuously for longer than 2 hours?

4. How does the steady state look like for a turtle?

3 At the doctor

- The time spent with the doctor is exponentially distributed with average value of 5 minutes per patient.
- Patients arrive in a doctor's waiting room following a Poisson process with rate λ of one patient per 10 minutes.
- Let us assume that there are no emergency cases, so the patients are called to see the doctor on a first come first serve basis.

Questions

1. Write down the Kendall notation for this queueing system.

2. Calculate the traffic intensity ρ for this system.

3. How much time on average does a patient spend in total at the doctor's?

4. Let us assume that the waiting room has only 2 chairs. My health condition does not allow me to be standing for a long time. What is the probability that I can sit down when I arrive? Assume that everybody sits down after arriving if there is a free chair.

5. Let us assume that the waiting room is under renovation and that people are sent away when the doctor cannot directly welcome them. What is the Kendall notation for this system (if this changed from before)?

6. What is then the probability that an arriving person is sent away?

4 ADSL helpdesk (Ch.7, exercise (viii))

- An ADSL helpdesk treats exclusively customer requests of one of three types: (i) login-problems, (ii) ADSL hardware and (iii) ADSL software problems.
- The opening hours of the helpdesk are from 8:00 until 16:00.
- All requests are arriving at the helpdesk according to a Poisson process with different rates: $\lambda_1 = 8$ requests with login-problems/hour, $\lambda_2 = 6$ requests with hardware problems/hour, and $\lambda_3 = 6$ requests with software problems/hour.
- The Poisson arrival processes for different types of requests are independent.

Questions

1. What is the expected number of requests in one day?

2. What is the probability that in 20 minutes exactly three requests arrive, and that all of them have hardware problems?

3. What is the probability that no requests will arrive in the last 15 minutes of the opening hours?

4. What is the probability that one request arrives between 10:00 and 10:12 and two requests arrive between 10:06 and 10:30?

5. If at time $t+s$ there were $k+m$ requests, what is the probability that at time t there were k requests?

5 Friendship network

Assume that the friendship network among a group of TU Delft students can be modeled as an ER random graph $G_p(N)$ with $N = 10$ nodes and link density of $p = 0.2$.

Questions

1. What is the expected number of links in this network?

2. What is the probability for any node in the network of having more than just one friend?

3. What is the expected reciprocity? Note: reciprocity is the property of observing both directional links between a pair of nodes, over the number of all possible pairs.

4. What is the expected clustering coefficient? Note: CC is the property of having triangles, over all possible connected pairs.