Errata and Comments for Geometry, Topology and Physics (Second Edition) by Mikio Nakahara

Christophe De Beule

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Notation and Conventions

It is not clear whether the natural numbers $\mathbb{N}$ are defined with or without zero. From the text, it seems that the definition excludes zero (see page 33 or page 84, for example).

Chapter 1: Quantum Physics

1. Page 5. In the first sentence of *Section 1.1.3*, there is a typo in the word differential.
2. Page 7. In Eqs. (1.23), all derivatives of A on the right-hand side of the equation should be partial derivatives.
3. Page 8. In *Example 1.4*, in the first equation $\dot{\phi}$ should be $\dot{\theta}$.
4. Page 9. In the second to last paragraph, it should be "An element of $\mathcal{H}^*$ ...".
5. Page 17. In Eq. (1.65), it should be 2ε instead of ε. Also, in the sentence above this equation, $\varepsilon = E/\omega$ since $\hbar = 1$ in this section.
6. Page 21. In Eq. (1.92) there is a factor of $\sqrt{m}$ missing on the right-hand side.
7. Page 22. Above the last equation, it should be $p = -\sqrt{\varepsilon/2m}\, k$.
8. Page 33. In Eq. (1.143), it should be $m/2\pi iT$ instead of $1/2\pi iT$. The same holds for the equation on the bottom of the page, where in addition $(1/2\pi i\varepsilon)^{1/2}$ on the right-hand side should be $(m/2\pi i\varepsilon)^{N/2}$.
9. Page 34. In the first equation from the top, as well as in the equation below (1.145), $1/2\pi iT \to m/2\pi iT$, $1/2\pi i\varepsilon \to m/2\pi i\varepsilon$, and $4\pi iT/\pi^2 \to 4\pi iT/m\pi^2$.
10. Page 34. There is a factor of $m^{-1/2}$ missing in the equation below (1.146).

11. Page 34. In Eq. (1.147), $N/2\pi iT \to mN/2\pi iT$, $4iT/\pi \to 4iT/m\pi$ in the first step, and $1/2\pi iT \to m/2\pi iT$ in the second step.

12. Page 35. On the left-hand side of Eq. (1.149), there is a missing factor of $m^{1/2}$ in both steps, as well as a missing factor of m in the first term of the argument of the exponent in the last step. This is also the case for the second step of Eq. (1.151), although m is canceled in the third step and does not appear in the final result.

13. Page 36. In the second equation from the top of the page, there is a factor m missing before the integral in the exponent on each side of the equation.

14. Page 36. The numerical factors in the third equation from the top are $\sqrt{2\pi}$ in the first step (behind the product) and $(2\pi)^{n/2}$ in the last step.

15. Page 36. In Eq. (1.154), there is a factor of m missing in front of the integral in the exponent and the equality sign should be a 'proportional to' sign ($\propto$).

16. Page 38. In Eq. (1.164), $\beta\pi/p\pi$ in the product should be $\beta\omega/p\pi$ and in the second factor, there should not be a factor of $\pi^{1/2}$. Hence, in the result below that equation, the factor of π on the right-hand side should also be absent.

17. Page 39. In Eq. (1.168), $cc^\dagger$ should be replaced with $c^\dagger c$.

18. Page 40. In the sentence above Eq. (1.172), it should be Λ^n instead of λ^n.

19. Page 41. In the second line from the top of the page, it should be Λ^n and not λ^n.

20. Page 43. First equation from the top. In the first line θ_n should be $d\theta_n$. In the third line, the subscript in the partial derivatives should be $\partial/\partial\theta'_1 \cdots \partial/\partial\theta'_n$.

21. Page 43. It should be stated that α is a Grassmann number. Perhaps a better notation would be θ'.

22. Page 44. In the second step of Eq. (1.187), there is a missing prime, $\mathrm{d}\theta \to \mathrm{d}\theta'$. The same holds in the second to last equation, where the $\mathrm{d}\theta_1$ right in front of the exponent should be $\mathrm{d}\theta'_1$.

23. Page 44. At the bottom of the page, $\eta_k = (\theta_k + i\theta'_k)/\sqrt{2}$ and $\eta^*_k = (\theta_k - i\theta'_k)/\sqrt{2}$.

24. Page 45. In the first equation from the top, the right-hand side should be $\eta_i\eta^*_j - \eta_j\eta^*_i$. In the second equation from the top, on the left-hand side $\mathrm{d}\eta_i \to \mathrm{d}\eta_1$ and on the left-hand side, the factor in front should be $(-1)^n$.

25. Page 45. In *Exercise 1.9*(1), the right-hand side vanishes for n odd and for n even the correct result is $\sqrt{\det M}\, e^{-K^t M^{-1} K/2}$.

26. Page 48. In the second line of the third equation from the top, there is a missing factor of $e^{-\theta^*\theta}$ in front of the product.

27. Page 49. In the last step of Eq. (1.200), B in the exponent should be B_N.

28. Page 49. In the first line of the last equation from the top, there is a missing overall factor of ε in the exponent that is in the integrand.

29. Page 50. In the first and second step of the second equation from the top, $n \to k$ in the product. Moreover, the product in the first line should start from $k = 1 - N/4$, which should also be corrected in the text above this equation (where k is defined as $n + 1$). This can be seen by counting the number of k values, which should be $N/2$ and taking into account that $k = 0$ is included in the product.

30. Page 56. The last Maxwell equation should have $\nabla \times \boldsymbol{B}$ and not $\nabla \times \boldsymbol{E}$. Below the Maxwell equations, the four-potential should be defined as $A^\mu = (\phi, \boldsymbol{A})$ or $A_\mu = (-\phi, \boldsymbol{A})$, and $\boldsymbol{E} = -\partial_t \boldsymbol{A} - \nabla\phi$ in Eq. (1.242).

31. ...

32. Page 61. In Eq. (1.279), there is a missing factor $\boldsymbol{r}/r$ in the second term on the right-hand side.

33. Page 66. In *Problem 1.1*(1), $H_1^\lambda[\phi] = \lambda^{2-n} H_1[\phi]$.

Chapter 2: Mathematical Preliminaries

1. Page 68. At the bottom of the page, $f^{-1} : f(x) \to x$ should be $f^{-1} : f(x) \mapsto x$.

2. Page 69. At the bottom of the page, $x \cong y$ should be $X \cong Y$.

3. Page 70. In *Definition 2.2*, requirement (i) is called reflexive instead of reflective.

4. Page 71. In *Example 2.5*(a), the square disc is defined with the wrong inequality signs, i.e., it should be defined as $\{(x, y) \in \mathbb{R}^2 | |x| \leq 1, |y| \leq 1\}$.

5. Page 73. In *Example 2.5*(e), the n-dimensional disc should be defined as $D^n = \{(x_1, \ldots, x_n) \in \mathbb{R}^n | (x_1)^2 + \cdots + (x_n)^2 \leq 1\}$.

6. Page 78. In Eq. (2.15), it should be $g(\boldsymbol{v}_1)$, consistent with previous notation.

7. Page 78. In the sentence below Eq. (2.15), it should read "... the field K is $\mathbb{R}$." or "... the field K are the real numbers $\mathbb{R}$."

8. Page 79. Above Eq. (2.17), it should be "Given a linear map f ...".

9. Page 79. In Eq. (2.17) and at the end of the paragraph below (2.20), it should be $f(\boldsymbol{v})$ and $\tilde{f}(\boldsymbol{w})$, consistent with previous notation.

10. Page 81. In *Definition 2.3*, in requirement (ii), it should "If J is any ...".

11. Page 81. In Eq. (2.25), it should be $U_\varepsilon(x)$ instead of $U_\varepsilon(X)$.

12. Page 83. In *Section 2.3.4*, in the second to last line of the first paragraph it should be "...complement of A° in $\bar{A}$; $b(A) = \bar{A} - A^0$."

13. Page 84. In *Example 2.11*(b), it should be $\bigcup_{n\in\mathbb{N}} U_n = (a, b)$ where $\mathbb{N}$ is the set of natural numbers excluding zero.

14. Page 85. In *Definition 2.8*(a), the sets X_1 and X_2 should both be open, nonempty, and disjoint instead of just open and disjoint.

15. Page 86. In the first line of the second paragraph, it should be "...if there exists continuous maps f ...".

16. Page 88. In the sentence after Eq. (2.30), it should be (2.30) instead of (2.27).

17. Page 90. In the caption of Fig. 2.12, it should be $S^2 \sharp S^2 = S^2$.

18. Page 90. In *Exercise 2.19*(b), the reference to the figure should be 2.7(a).

19. Page 91. In the second line from the top, it should be K_X and K_Y.

20. Page 92. In *Problem 2.3*, it should be figure 2.10(b).

Comments

1. Page 67. The preferred modern terminology for what is called the **range** of a map in the text is the **codomain** of a map.

2. Page 83. In *Section 2.3.4*, in the third line of the first paragraph, the condition that A should be either open or closed is superfluous. One can define the closure, interior, and boundary using the same definition for any set. For example, for the usual topology on $\mathbb{R}$, $A = [a, b)$ is neither open or closed with closure $\bar{A} = [a, b]$, interior $A^0 = (a, b)$, and boundary $b(A) = \{a, b\}$.

3. Page 85. In *Definition 2.8*(b), the definition of **arcwise connected** is nonstandard. The standard definition is more stringent and additionally requires that the map f is a homeomorphism between $[0, 1]$ and its image, in which case f is called an arc. If instead f is only continuous, the definition corresponds to the standard definition of a **path-connected** space. This nonstandard terminology is used throughout the book, especially in Chapter 4. However, if a topological space is a Hausdorff space, path connectedness implies arcwise connectedness.

Chapter 3: Homology Groups

1. Page 98. In Eq. (3.11), it should be σ_r on the left-hand side, consistent with the notation throughout the chapter.

2. Page 99. In the first line of *Section 3.2.2*, it should be "Let K be a set of a finite number ...".

3. Page 109. Above Eq. (3.41), it should be $Z_2(K) = \{0\}$.

4. Page 115. In the third line of the second paragraph, it should be $H_2(\Sigma_g) \cong \mathbb{Z}$.

5. Page 114. Equation (3.48) should be $H_1(K) = \{[z]|[z] + [z] = [0]\} \cong \mathbb{Z}_2$.

6. Page 115. In the second line after Eq. (3.49), it should be figure 2.13.

Comments

On page 117 in the proof of *Theorem 3.6*, there are many equality signs, e.g., in Eq. (3.52), that should strictly speaking be 'isomorphic to' signs ($\cong$) although the isomorphisms involved are trivial.

Chapter 4: Homotopy Groups

1. Page 121. In the sixth sentence from the top of the introduction, it should be " ... the image $f(X)$ can be continuously deformed ...".

2. Page 124. Reflexivity instead of reflectivity.

3. Page 130. In Eq. (4.9) it should be $\frac{1}{2} \leq r \leq \frac{3}{2}$ in order to correspond to Fig. 4.7.

4. Page 130. Below the last equation id_X should be replaced by id_Y.

5. Page 131. In *Example 4.2*, the second line should be: $H : \mathbb{R}^n \times I \to \mathbb{R}^n$ by $H(x,t) = (1-t)x$, we have (i) $H(x,0) = x$ and $H(x,1) = 0$. This is consistent with *Definition 4.8* and in the third line, $H(0,t) = 0$.

6. Page 133. On the third line above *Theorem 4.5*, it should be $\tilde{f} * \tilde{g}(x + 2\pi) = \tilde{f} * \tilde{g}(x) + 2\pi(m+n)$.

7. Page 135. Above Eq. (4.17), it should $g = x^n$ instead of $x^i = x^{n+1}$.

8. Page 136. In *Example 4.4*, in the last sentence $(x, y; xyx^{-1}y^{-1})$ instead of $(x, y : xyx^{-1}y^{-1})$

9. Page 145. In the first paragraph below *Theorem 4.9*, it should be $(x_i; r_m)$, $(x_i; r_m, x_i x_j x_i^{-1} x_j^{-1})$, and $(x, y; \emptyset)$. Note the semicolons.

10. Page 147. Third line from the bottom of the page should be "we simply denote $\pi_0(X)$. Note, however, that".

11. Page 155. Eq. (4.62) should be $\Psi(\boldsymbol{x}) \to e^{i\chi}\Psi(\boldsymbol{x})$.

12. Page 162. In the first line of *Section 4.9.4*, it should be $\pi_3(\mathbb{R}P^2) \cong \mathbb{Z}$.

13. Page 163. The figure corresponds to $\boldsymbol{n}_0 = (0, 0, 1)^t$ while the text, starting from the bottom paragraph of page 162, says $\boldsymbol{n}_0 = (1, 0, 0)^t$.

Comments

1. Page 130. The paragraph below *Definition 4.8* is confusing. First, an example is given of a retract R that is not a deformation retract and then a general statement about a deformation retract is given using the same notation. It would be clearer if the last sentence "Since X and R are of the same homotopy type, ..." is instead placed right below *Definition 4.8*.

2. Page 133. In Section 4.3.1, the notation $\oplus$ is usually reserved for the direct sum of abelian groups. The standard notation for the direct product of two groups G and H is $G \times H$.

3. Page 137 *. A maximal tree

Chapter 5: Manifolds

1. Page 179. figure

2. Page 193.

3. Page 217.

References