GREEN'S FUNCTIONS

A short introduction

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OUTLINE

Basic idea

Solution methods

Applications

BASIC IDEA

What is a Green's function?

Linear equation to solve:

$$\mathcal{L}u(x) = f(x)$$

Green's function is impulse response:

$$\mathcal{L}G(x, x') = \delta(x - x')$$

- Most EM problems are described by linear (differential) equations with some source function $f(x)$.
- The Green's function is the solution when $f(x)$ is an impulse located at x'.
- Generalization of impulse response from signal processing.

WHY IS IT USEFUL?

Can find the solution directly for any $f(x)$:

$$u(x) = \int G(x, x')f(x')\,\mathrm{d}x$$

because

$$\mathcal{L}u(x) = \int \mathcal{L}G(x, x')f(x')\,\mathrm{d}x = \int \delta(x - x')f(x)\,\mathrm{d}x = f(x)$$

- Once we know the Green's function for a problem, we can find the solution for any source.
- Kind of like splitting the source $f(x)$ into little impulses and then adding up (integrating) the response to each impulse.
- Generalization of convolution from signal processing.
- Can be quicker than adding up infinite series of orthogonal functions.

SUMMARY

Some important thing.

INTRODUCTORY RESOURCES

Balanis (2012), *Advanced engineering electromagnetics*. Less rigorous, but good for getting the key ideas. Good place to start.

Folland (1992), *Fourier analysis and its applications*. Fairly rigorous. Chapter on generalized functions is particularly nice.

Dudley (1994), *Mathematical foundations for electromagnetic theory*. Fairly rigorous. Great introduction to 1D Green's functions: deals with subtleties other books leave out.

ADVANCED RESOURCES

Collin (1990), *Field theory of guided waves*. Huge chapter on Green's functions. Emphasis on dyadics.

Morse and Feshback, *Methods of theoretical physics*. Another big, detailed reference. Emphasis on theory and insights.

Warnick (1996), "Electromagnetic Green functions using differential forms." For the differential forms inclined.

SOLUTION METHODS

SOLUTION METHODS

Boundary condition approaches:

1. Green's function gives particular solution; add homogeneous solution to find boundary conditions. Easier to set up, but requires extra work to deal with BC's.
2. Green's function includes BC's. Harder to set up, but gives full solution including BC's.

SOLUTION METHODS

Solving Green's function approaches:

1. Direct solution. (Great if it's possible.)
2. Eigenvalue expansion. (Works every time.)

APPLICATIONS

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- Born approximation for scattering?
- Perturbation theory?
- Propagator/Huygen's principle?