Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Lecture with Computer Exercises:
Modelling and Simulating Social Systems with MATLAB

Project Report

Phase transition in the Intelligent Driver Model induced by interaction exponent

Authors:
Carlo Del Don
Pol Welter

Supervisors:
Lloyd Sanders
Olivia Woolley

December 11, 2015

Abstract

We introduce a new parameter in the Intelligent Driver Model, which also induces the phase transition from homogeneous to inhomogeneous traffic. By tuning this exponent of the interaction term, the simulated traffic flow can be rendered stable across a wide range of vehicle densities, accelerations, and decelerations.

We propose a semi-quantitative explanation for the observed effects; the change in breaking strategies towards more anticipative behaviour stabilizes the traffic.

Agreement for free-download

We hereby agree to make our source code for this project freely available for download from the web pages of the SOMS chair. Furthermore, we assure that all source code is written by ourselves and is not violating any copyright restrictions.

Carlo Del Don Pol Welter

Individual contributions

The authors have contributed equally to this work.

Contents

1 Introduction and Motivations

Most of the major automotive manufacturers estimate that they will release an autonomous (self-driving) car within the next ten years. Indeed, autonomous vehicles promise to be capable of both safer and smoother traffic than human drivers can currently achieve. Computers will not only steer the vehicles, but most likely also organize traffic in a macroscopic picture. As self-driving cars will be introduced to our streets, we will inevitably have to deal with situations where both human and robotic drivers share the road. For computers to be able to predict and evaluate human driving, we require computer models that accurately reproduce human driving behaviour. In this sense, the understanding of traffic dynamics is as relevant as ever.

Available models (e.g. the *Intelligent Driver Model*, see section 2) are able to predict the transition from smooth, homogeneous traffic to stop-and-go waves, as, for example, the density of the vehicles exceeds a certain threshold. It is well established [3, 4, 6] that the instability may be triggered by a short, localized increase in vehicle density. This cluster scatters at an obstacle (i.e. a region with e.g. a lower speed limit), and creates a traffic jam much larger in size than the original perturbation. Instabilities may also form spontaneously on roads that are populated near (but below) their saturation capacity.

In this work we will investigate how the breaking strategy of the drivers will affect the stability of the flow. Indeed, we will see that one particular parameter in the *Intelligent Driver Model* has a crucial impact on the formation of these spontaneous instabilities. To our knowledge this parameter was not subject to previous studies.

2 Description of the Model

We have implemented the *Intelligent Driver Model* (IDM). The IDM, first introduced by Treiber et al. [3, 4], is deterministic and continuous in both space and time.

This model describes the evolution of the simulated system as a set of differential equations. The behaviour of each vehicle (its acceleration $\dot{v}_\alpha$) is given as a function of its current velocity v_α, the gap to the leading vehicle s_α, etc. The acceleration of vehicle α reads

$$\ddot{x}_\alpha = \dot{v}_\alpha = a\left(\underbrace{1-\left(\frac{v_\alpha}{v_0}\right)^\delta}_{\text{free road term}} - \underbrace{\left(\frac{s^*(v_\alpha,\Delta v_\alpha)}{s_\alpha}\right)^\gamma}_{\text{inteaction term}}\right). \tag{1}$$

This equation contains two parts, the *free road term*, and the *interaction term*. For a low vehicle density (i.e. $s_\alpha \to \infty$) the above equation can be simplified to

$$\dot{v}_\alpha = a\left(1-\left(\frac{v_\alpha}{v_0}\right)^\delta\right). \tag{2}$$

For any $\delta > 0$ this will result a relaxation of the velocity v_α to the speed limit v_0. The value of the exponent δ is typically chosen in between 1 (exponential relaxation) and 6 (the limit $\delta \to \infty$ corresponds to a constant acceleration). Here a denotes the maximum acceleration. Throughout literature the most commonly used value for the exponent is $\delta = 4$. This is the value that has also been adopted in this work.

2.1 The interaction term

The *interaction term* merits further attention. It is designed to limit the cars' velocity on a densely populated road by acting as a 'repulsive potential', which tries to enforce the *desired gap* s^* as the distance between two cars. The desired gap is calculated as

$$s^*(v_\alpha, \Delta v_\alpha) = s_0 + \max\left(v_\alpha T + \frac{v_\alpha \Delta v_\alpha}{2\sqrt{ab}},\ 0\right). \tag{3}$$

It contains

- a minimum distance s_0, which is respected even at stand still,
- a safety distance $v_\alpha T$ based on the drivers' desired time headway[1] T,
- a breaking term $\frac{v_\alpha \Delta v_\alpha}{2\sqrt{ab}}$, with the 'comfortable' deceleration b.

The breaking term inhibits large closing speeds between vehicles. To the best of our knowledge, all of the available literature fixes $\gamma = 2$. Indeed, for the traditional value $\gamma = 2$, and large closing speeds, the interaction term essentially reduces to

$$-a\left(\frac{v\Delta v}{2\sqrt{ab}s}\right)^2 = -\frac{b_k^2}{b}, \tag{4}$$

where b_k denotes the *kinematic deceleration*, i.e. the deceleration necessary to avoid a collision. Since drivers thus underestimate situations where $b_k \ll b$, while they break harder than necessary for $b_k \gg b$, it is easy to see that the resulting acceleration will tend towards $\dot{v} = -b$. [4]

In the present work, particular focus has been paid precisely to $\gamma \neq 2$. We will try to answer the question as to how and why this parameter affects the stability of the traffic flow. In the following we will see that increasing γ, while leaving the other parameters fixed, suppresses any instabilities in the traffic flow.

2.2 Boundary conditions

As for the boundary conditions of the road, there are two obvious choices: either add and remove vehicles during the simulation (as is done e.g. in [3]), or make the road periodic, i.e. ring-like [5]. The results we present in this work are based on simulations of a ring-like periodic road.

The choice of the periodic boundary conditions makes the implementation of the IDM significantly simpler, as no care has to be taken to spawn the vehicles at the right position with an appropriate speed. On the other hand, it provides limited ability to change the number of cars on the road at run-time.

On the subject of periodic boundary conditions, it is worth noting that some experimental data exists on precisely such a setting, namely cars driving on a circular track [1, 2]. Here a spontaneously emerging instability has been observed; we will see that our model can qualitatively reproduce these observations.

[1] The time headway T is the time between the moment the first vehicle's bumper passes a stationary point, and the moment the next vehicle passes it.

Parameter	Typical value
global speed limit v_0	$15\,\mathrm{m/s}$
maximum acceleration a	$0.6\,\mathrm{m/s^2}$
maximum comfortable deceleration b	$1.5\,\mathrm{m/s^2}$
minimum time headway T	$1.5\,\mathrm{s}$
minimum jam distance s_0	$2\,\mathrm{m}$
free road acceleration exponent δ	4
interaction exponent γ	2
car size l_α	$5\,\mathrm{m}$
number of cars N	50
length of road L	$1\,\mathrm{km}$
time step Δt	$0.250\,\mathrm{s}$
end time (duration of simulation) t_{end}	$30\,000\,\mathrm{s}$

Table 1: Overview of the parameters in the simulation.

2.3 Limits of the model

The IDM does not model the finite reaction time of an actual human driver[2]. Indeed, when a realistic reaction time was included, IDM-like simulations show very poor stability when compared to empiric human driving. This suggests that in reality drivers do look ahead further than only the first car in front of them [7].

The road on which the vehicles travel is assumed to be single lane (no overtaking) and straight (constant speed limit). All drivers and vehicles are assumed to be identical.

2.4 Summary of the parameters

In Table 1 the reader may find an overview of all the parameters that appear in the model. The indicated values are those used in the presented simulations unless specified otherwise.

Note that the four last parameters are not directly included in the formulation of the IDM as in equation (1). Instead they are related to the numeric implementation.

3 Implementation

For performance reasons, the simulation engine has been implemented as a C++ program. The executable reads the simulation parameters from a text file, runs the simulation, and dumps the desired values (e.g. vehicle position & velocity as a function of time) in another file.

This back-end is controlled by a suite of tools written in Python, which also perform data analysis and visualization. The Python front-end feeds parameters to the engine, and processes its output.

[2]The finite time step Δt is *not* to be mistaken for a reaction time. Refer to [7] for an extensive explanation of the differences.

The differential equations (1) are integrated via the following Runge-Kutta scheme:

$$v_\alpha(t + \Delta t) = \dot{v}_\alpha(t)\Delta t + v_\alpha(t)$$
$$x_\alpha(t + \Delta t) = \frac{1}{2}\dot{v}_\alpha(t)(\Delta t)^2 + v_\alpha(t)\Delta t + x_\alpha(t)$$

For the case where $\dot{v}_\alpha$ does only depend on x_α, but not on v_α, this scheme is known as the *Verlet method*, and converges with quadratic order. In the present case, these requirements are not met, so second order convergence might not actually be achieved. No tests on this have been carried out, as even first order methods have been proven to be accurate enough [5].

Note that numeric integration discretizes time: while the IDM inherently is continuous in time, our implementation is not. To avoid numerical issues due to the finite time step, we clamp to zero any velocities that would otherwise become negative.

4 Simulation Results

4.1 Stop-and-go waves on a free road

Our implementation successfully reproduces an unstable traffic and apparition of a jam on unobstructed roads (see Figure 1 for a representation of the flow). The vehicles start from stand-still, and reach equilibrium velocity within less than a minute. At about $t = 700\,\mathrm{s}$ two seemingly independent perturbations start to grow in intensity, and develop into full blown stop-and-go waves at $t \approx 1100\,s$.

In the IDM, drivers only look at the vehicle *ahead* of them, i.e. information only spreads upstream. Indeed, we observe that the waves travel in direction opposite to the traffic, which means that the information is spread significantly faster than the speed at which the cars are traveling downstream.

We have verified that the waves visible in Figure 1 are *not* a numerical artefact, as the same pattern can be observed for very different time steps. Indeed, we could reproduce this plot with a time step as small as $\Delta t = 1 \times 10^{-4}\,\mathrm{s}$.

The pattern shown in the simulation is qualitatively very similar to what has recently been experimentally observed by Nakayama et al. [1] and Tadaki et al. [2].

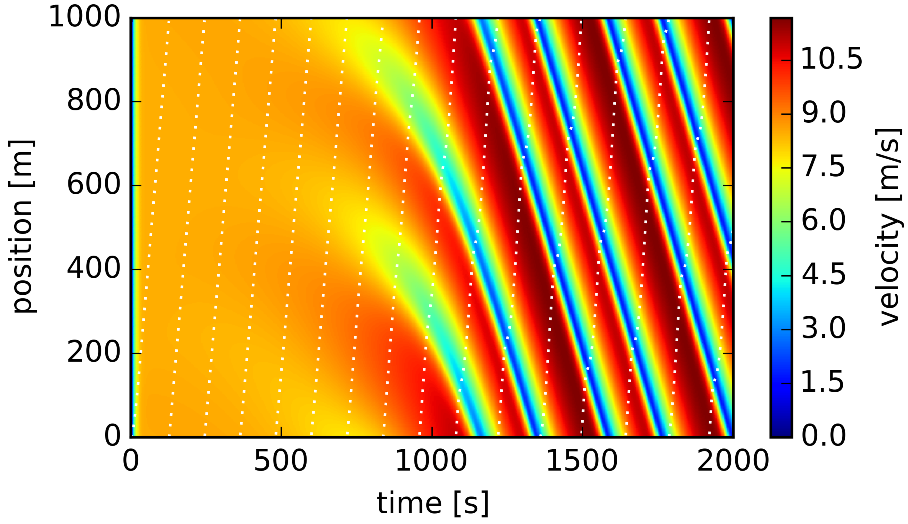

Figure 1: Spatio-temporal plot of the velocity field with stop-and-go waves. The dotted lines trace the trajectory of a selected car. The $N = 50$ vehicles start uniformly distributed along the road. They quickly reach the equilibrium velocity ($\sim 9\,\mathrm{m/s}$). After about $t \approx 700\,\mathrm{s}$ the instabilities start to become noticeable, and continue growing until $t \approx 1100\,\mathrm{s}$. Note that the road is assumed to be periodic, hence the stop-and-go-waves, which leave the plot at the bottom, reappear at the top. Also note that the waves travel upstream, i.e. against the flow. For this simulation a time step $\Delta t = 0.125\,\mathrm{s}$ was used.

4.2 Instability by asymmetry in acceleration/deceleration behaviour

As one can easily imagine, the instabilities are provoked by an asymmetry between the acceleration and deceleration parameters, a and b. Indeed, b is typically chosen significantly larger than a (see Table 1). A car that is moving particularly slow will not get away fast enough as more cars approach it from behind. The cars stack up, leading to a growing perturbation.

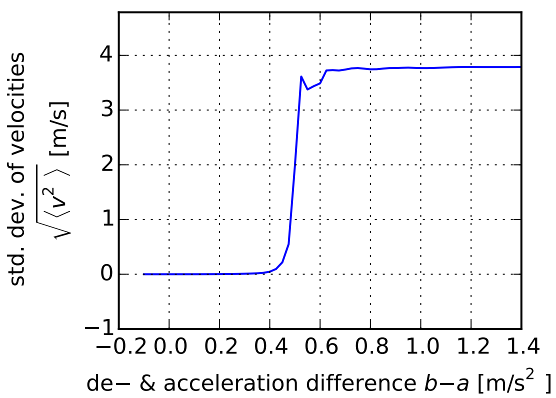

Figure 2: Instability as a function of the deceleration-acceleration asymmetry. See explanatory text.

Indeed, when choosing b close to a, no instabilities can be observed. In Figure 2, the occurrence of instabilities is shown as a function of $b - a$. Clearly, for $b \sim a$ no instabilities are seen, which confirms this conjecture. For this particular run the acceleration was fixed to $a = 0.6\,\mathrm{m/s^2}$, while b was swept over the according range. The simulation was run for a long time ($t_{\mathrm{end}} = 500\,\mathrm{min}$). To measure instabilities, the standard deviation of the final velocity distribution was computed. For all further simulations, we use the values for a and b as listed in Table 1.

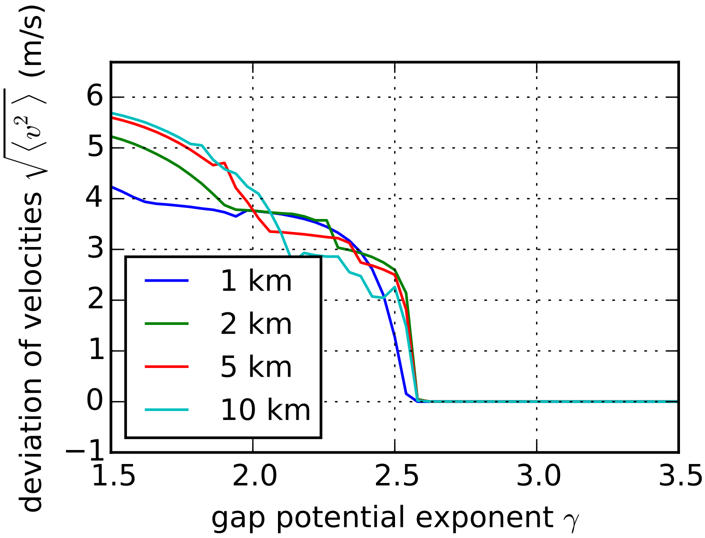

Figure 3: Measure of instabilities as a function of the exponent γ, for different road lengths. If the traffic remains stable, the velocities are equalized, and hence their variance vanishes. Increasing γ beyond a critical value (here $\gamma_c \approx 0.258$) suppresses any instability. The transition appears as a sharp kink in the curve. Although the shape of the curves is slightly different for various road lengths, the critical value γ_c is not. For this plot, the vehicle density was fixed to $N/L = 50\,\mathrm{km}^{-1}$.

4.3 Effects of the interaction exponent γ

We find that by increasing the exponent γ, the instabilities vanish as well. All the vehicles' velocity and spacing remain constant and equal to their equilibrium value. The transition from homogeneous traffic to flow prone to instability as a function of γ is very sharp. Figure 3 shows this relation. Again the standard deviation of the final velocity distribution was computed as measure for inhomogeneity. As the standard deviation of the velocities vanishes in the homogeneous (disordered) phase, and quickly rises in the inhomogeneous (ordered) phase, it qualifies as an order parameter.

The critical value γ_c at which the transition occurs is well defined and mostly independent on the length of the road. Only for $L = 1\,\mathrm{km}$ (blue curve) we see significant finite-size effects.

Having established that tuning γ gives rise to a sharp transition between phases, it stands to reason to map out this phase boundary as a function of another important parameter. Of particular interest is the density of vehicles, which has also been shown to be responsible for such a phase transition. The result can be seen in Figure 4. For this the vehicle density was swept, while the corresponding critical exponent γ_c was then determined via binary search. It is interesting to note that the phase boundary curve is concave, it reaches a maximum for $N/L \approx 70\,\mathrm{km}^{-1}$. This seems to be the density which is most easily destabilized. Investigating higher vehicle densities than shown in Figure 4 does not bring further insight: as every vehicle requires a space of at least $s_0 + l_\alpha = 7\,\mathrm{m}$, the road is completely saturated at ≈ 150 cars per kilometre. We also find that the shape of the phase boundary is perfectly independent of the length of the road (not shown in this graph).

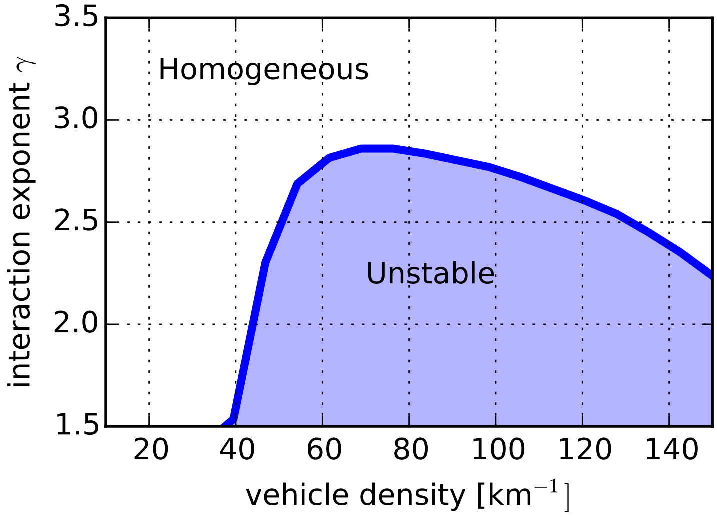

Figure 4: Stability phase diagram with the mean vehicle density on the horizontal axis, and the interaction exponent on the vertical axis. While for the traditional value of $\gamma = 2$ the traffic is unstable for a broad range of densities, a higher value of γ can render the flow stable at high densities. For the chosen acceleration and deceleration parameters (here $a = 0.6\,\mathrm{m/s^2}$ and $b = 1.5\,\mathrm{m/s^2}$) the flow can be made stable unconditional on the vehicle density.

5 Discussion

The obvious question that remains is *why* larger values for γ stabilize the flow. According to subsection 4.2, we know that a sufficient difference in accelerating and breaking behaviour favours instabilities. Might it be that increasing γ limits the deceleration, and consequently, reduces this asymmetry? The answer to this question is a negative.

This can be seen as follows: let $b_{\text{eff}} = a\left(\frac{s^*}{s}\right)^2$ denote the deceleration that the IDM would yield for $\gamma = 2$ (under the approximation of high closing speeds). Following the description in section 2.1, in particular equation (4), for an arbitrary γ the deceleration would then read

$$b'_{\text{eff}} = a\left(\frac{s^*}{s}\right)^{\gamma} = a\left(\frac{b_{\text{eff}}}{a}\right)^{\gamma/2}. \tag{5}$$

We would now like to know when $b_{\text{eff}} > b'_{\text{eff}}$, or equivalently

$$1 > \frac{a}{b_{\text{eff}}}\left(\frac{b_{\text{eff}}}{a}\right)^{\gamma/2} = \left(\frac{b_{\text{eff}}}{a}\right)^{\gamma/2-1} \sim \left(\frac{b}{a}\right)^{\gamma/2-1}. \tag{6}$$

In the last step we used that $b_{\text{eff}} \sim b$. Now, since generally $b > a$, we see that inequality (6) cannot be fulfilled for any $\gamma \geq 2$. We conclude that in fact larger γ encourage *harder* decelerations.

When simulating a single car approaching[3] a standing obstacle, we observe the acceleration behaviour represented in Figure 5. During the accelerating phase (up

[3]The speed limit in this test was increased to $v_0 = 30\,\mathrm{m/s}$.

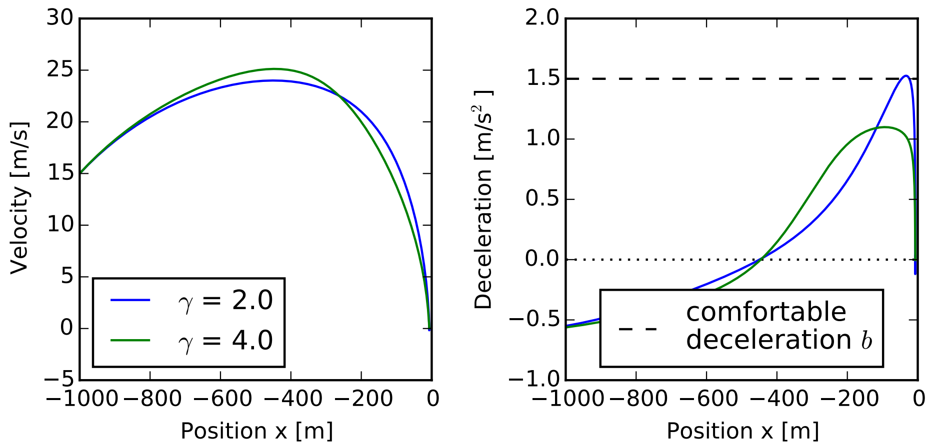

Figure 5: Velocity and deceleration profiles when approaching a stationary obstacle. The car starts at $x = -1000\,\mathrm{m}$, while the obstacle is located at the origin. As expected, the deceleration for $\gamma = 2$ tends towards b. For $\gamma = 4$, the deceleration is *initially* higher, but later the driver does not break as hard.

to $x \approx -400\,\mathrm{m}$), there is little difference between $\gamma = 2$ and $\gamma = 4$. For $\gamma = 2$, the deceleration then tends towards b as expected. The driver with $\gamma = 4$ however breaks harder indeed, but only initially (up to $x \approx -100\,\mathrm{m}$). Later, having reduced the speed already more than required, the breaking is much softer than for the $\gamma = 2$ case.

The driver with $\gamma = 4$ thus breaks earlier, avoiding having to break hard at the last moment. This explains why higher values for γ enhance stability: the low-γ driver rushes towards the obstacle, resulting in the number of cars involved in the perturbation growing quickly. The high-γ drivers break early, avoiding to enter the perturbation themselves. Instead their behaviour signals the drivers further upstream the presence of an obstacle. This way, the perturbation can diffuse upstream.

6 Summary and Outlook

We have demonstrated that instabilities in traffic modelled by the IDM can be suppressed by adapting the breaking behaviour of the drivers. In particular increasing the interaction exponent γ beyond a critical value leads to a complete suppression of any perturbation.

We think this can be understood by realizing that, if the drivers slow down earlier, the *extend* of a perturbation grows and the velocities are equalized again. To further test this hypothesis, additional simulations will need to be carried out. One could e.g. trace the acceleration of vehicles and examine them for evidence whether changing γ does indeed have an effect on the vehicles' behaviour as predicted.

One could also think of *empirically* measuring the value for γ for real drivers. This seems like a difficult task however, as all the parameters in the IDM are tightly coupled. Still a measurement of γ could be used as a test for the validity of the IDM, as in a situation where in reality instabilities can be observed, we should see a value for γ that is in the 'unstable' region of e.g. the phase diagram in Figure 4.

References

[1] Akihiro Nakayama, Minoru Fukui, Macoto Kikuchi, Katsuya Hasebe, Katsuhiro Nishinari, Yuki Sugiyama, Shin-ichi Tadaki, and Satoshi Yukawa. Metastability in the formation of an experimental traffic jam. *New Journal of Physics*, 11(8):083025, 2009.

[2] Shin-ichi Tadaki, Macoto Kikuchi, Minoru Fukui, Akihiro Nakayama, Katsuhiro Nishinari, Akihiro Shibata, Yuki Sugiyama, Taturu Yosida, and Satoshi Yukawa. Phase transition in traffic jam experiment on a circuit. *New Journal of Physics*, 15(10):103034, 2013.

[3] Martin Treiber and Dirk Helbing. Explanation of observed features of self-organization in traffic flow. *arXiv preprint cond-mat/9901239*, 1999.

[4] Martin Treiber, Ansgar Hennecke, and Dirk Helbing. Congested traffic states in empirical observations and microscopic simulations. *Physical Review E*, 62(2):1805, 2000.

[5] Martin Treiber and Venkatesan Kanagaraj. Comparing numerical integration schemes for time-continuous car-following models. *Physica A: Statistical Mechanics and its Applications*, 419:183–195, 2015.

[6] Martin Treiber and Arne Kesting. Traffic flow dynamics. *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013.

[7] Martin Treiber, Arne Kesting, and Dirk Helbing. Delays, inaccuracies and anticipation in microscopic traffic models. *Physica A: Statistical Mechanics and its Applications*, 360(1):71–88, 2006.

A Capabilities of the framework

The framework we implemented for this project has been designed to accommodate an arbitrary number of obstacles. We differentiate two types of obstacles:

- static obstacles, which represent extended regions of the road with local parameters that differ from the general set (e.g. lower speed limit v_0, etc.);
- traffic lights, which are point-like obstacles, periodically modulated in time, and disallow any cars to pass during the red phase.

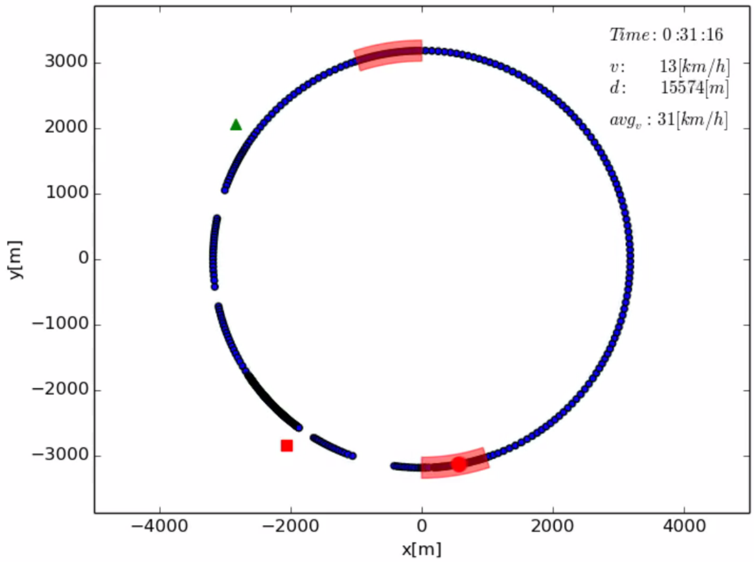

Figure 6: Bird's-eye view of simulated road. The blue markers represent cars (the larger red marker is a reference car for easier visualization). The regions shaded in red are static obstacles, where the speed limit is reduced compared to regular road. The square and the triangle represent traffic lights and their respective state.

The source code of the implementation is available for download from GitHub[4].

[4] `https://github.com/polwel/traffic_simulation`

ETH

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Declaration of originality

The signed declaration of originality is a component of every semester paper, Bachelor's thesis, Master's thesis and any other degree paper undertaken during the course of studies, including the respective electronic versions.

Lecturers may also require a declaration of originality for other written papers compiled for their courses.

I hereby confirm that I am the sole author of the written work here enclosed and that I have compiled it in my own words. Parts excepted are corrections of form and content by the supervisor.

Title of work (in block letters):

Phase transition in the Intelligent Driver Model induced by interaction exponent

Authored by (in block letters):
For papers written by groups the names of all authors are required.

Name(s):	First name(s):
Del Don	Carlo
Welter	Pol

With my signature I confirm that
- I have committed none of the forms of plagiarism described in the 'Citation etiquette' information sheet.
- I have documented all methods, data and processes truthfully.
- I have not manipulated any data.
- I have mentioned all persons who were significant facilitators of the work.

I am aware that the work may be screened electronically for plagiarism.

Place, date

Zurich, 07/12/2015

Signature(s)

Pol Welter

For papers written by groups the names of all authors are required. Their signatures collectively guarantee the entire content of the written paper.