Epistemic Geometry: To Believe or Not To Believe? That is the Question

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Abstract

We construct a novel geometric agent-based model where we define agents to be a pair local sections of associated bundles constructed from a principal bundle $\mathcal{N}$ and fibers representing the agents generative models and beliefs. Agent dynamics is governed by a generalized variational energy principle defined through the gauge connections that mediate interactions between agents. Belief updates occur via the left action of the Lie group G on the associated bundle.

We show that variational energy functional minimization drives agents to coalesce, giving rise to the emergence of meta-agents and new effective gauge connections at multiple scales. We further discuss renormalization and the scale dependence of agent behavior, showing how coarse-graining over belief dynamics leads to effective theories at different levels of organization. Additionally, we demonstrate the existence of a pullback metric from the associated bundle to the base manifold $\mathcal{C}$, which encodes each agent's perception of their environment.

We discuss possible experimental tests of this "cognition-first" approach to physics, and show that assuming Landauer's principle within our model leads to the prediction of a perceived "dark" universe. Finally, we explore the broader philosophical implications of treating physical reality as emerging from interacting cognitive agents.

1 Introduction

The geometric study of the foundations of physics produced breathtaking strides throughout the 20^{th} century and continues to this day. Researchers have discovered that the organizing principles of the natural world are best expressed in the language of symmetry - especially continuous symmetries described by Lie groups.

While physicists were making strides in the foundational understanding of verifiable reality neuroscientists, computer scientists, and cognitive researchers advanced our understanding the nature of perception and cognition. Arguably the singular contribution to the field of cognition is due to the pioneering work of Karl Friston who has developed a variational free energy model that aims to unify the fields of cognition and behavior across many physical scales[1]. Explicit in these formulations is the premise that cognitive systems do not directly perceive the world's "true" states. Instead, cognitive systems build models of their sensory data in such a way as to minimize their variational free energy and ultimately survive, reproduce, and evolve. This echoes Kant's description of "intuitions" in his historic "The Critique of Pure Reason" as well as Helmholtz' early work on cognitive inference[23].

Despite the philosophical challenges in studying the foundational elements of reality (or "noumenon") science progressed steadily developing extremely powerful and accurate theories based upon the assumption that space and time are fundamental aspects of our universe[4]. Einstein and Minkowski (among others) unified space and time into a 3+1 dimensional smooth manifold wherein its geometry is dynamic and malleable. Quantum Field Theory further built upon the scaffold of the Minkowski spacetime manifold injecting quantum theories with relativistic phenomena leading to some of the most elegant, awesome, and precise predictions and verifications humanity has ever known.

Yet, within all these formalisms remain the nagging issue of Kant's perspective versus Einstein's perspective. If space-time is predicated upon our cognitive perceptions then what does that mean for the entire edifice of physical theory? Physics assumes important symmetries such as Lorentz invariance, general coordinate invariance, and gauge invariance and yet subjective experiences—such as those reported under psychedelics, trauma, and mental illness—suggest deviations from standard space-time perception[4]. Such accounts are often dismissed as mere subjective hallucinations, yet modern neuroscience tells us that all perception is, in a sense, a controlled hallucination shaped by evolutionary pressures[5]. We then might be more cautious in our immediate dismissal of such "observations". Indeed we would hope that a consistent theory of physics could just as well describe cognition as a consequence of some underlying physics. When we perform experiments and record our results we are ultimately looking at the dials and knobs via electromagnetic waves and emitting sound waves to correlate to each other and be interpreted as experimental confirmation between our warm, wet brains, locked inside a "dark cavern" extending electric tentacles into the correlations of our universe[5]. Indeed, experiments are only "confirmed" if one agent's perception of space-time corresponds to another's via some lossless mapping (we hope!). What are we to make of physics if our basic notions of reality are themselves subjectively emergent. This harkens to the difficulty in establishing what exactly constitutes an "observer" and "measurement" within quantum theory. Here we adopt the position that any theory of physics should also be cognitively consistent.

In recent decades, physicists have begun to entertain the possibility that space and time are not fundamental, but emergent. Arkani-Hamed and collaborators introduced the amplituhedron[6], a timeless geometric object whose volumes encode scattering amplitudes in supersymmetric field theories. Meanwhile, other researchers have used tensor networks and holographic entanglement to construct emergent geometries resembling Einstein's general relativity[7,8,9]. Still others have found a striking similarity between thermodynamics and general relativity[10]. These results suggest a deeper structure undergirding our reality - Kant's noumenon.

Here we propose a geometric framework aimed towards a cognitively consistent framework of physics (subjective and objective) based upon a potentially timeless, spaceless abstract structure which we call a noumenal manifold with structure group G.

2 Epistemic Geometry

We begin with a noumenal manifold $\mathcal{C}$ representing a timeless, spaceless "fundamental reality" and a Lie group G to define a principal bundle $\mathcal{N}$ over $\mathcal{C}$. We then build associated bundles $\mathcal{B}_q$ and $\mathcal{B}_p$ representing the beliefs and models that can be formed from $\mathcal{C}$. Next we define scale-dependent agents and multi-agents to be pairs of local sections of these associated bundles over $\mathcal{C}$. The associated bundles and multi-agents allow us to construct gauge connections and bundle morphisms connecting agents together in a hierarchical web. Upon defining a variational energy functional of these "epistemic fields", we inject agents with the ability to update their beliefs and models in such a way that the points in $\mathcal{C}$ remain fixed. Instead the beliefs of the agents change under left-action of the Lie group G on the agent sections. We propose that the agents then "perceive" a dynamical patterned and correlated universe (with space and time and causality) yet in the background they are instead interpreted as updating their local beliefs via group action at fixed points in $\mathcal{C}$.

Additionally, we propose two potential mechanisms by which agents might adopt a space-time metric within the noumenal manifold $\mathcal{C}$ by considering the pullback of a metric defined on the associated bundles or by constructing a metric within $\mathcal{C}$. Interestingly the complete pullback metric on $\mathcal{C}$ can be chosen to partition $\mathcal{C}$ into the direct sum of a "space-time" metric and another metric encoding all other correlations (matter fields, gauge fields, etc).

We conclude with reflections on the nature of dark energy/matter, thermodynamics, and philosophical consequences (free will, agency), physical applications (EPR, Bell's theorem, cosmology), and technical aspects (the structure of the pullback metric on $\mathcal{C}$, machine learning and AI, etc).

From this minimal framework we develop a wealth of rich epistemic physical and philosophical consequences. We have, fundamentally, four primary objects $\mathcal{C}$, G, $\mathcal{B}_q$, and $\mathcal{B}_p$ along with a variational energy extremization axiom.

2.1 Definitions

To begin we shall point out the following: in what is to follow we study many Kullback-Liebler divergences. In our theory we may

1. Consider a restricted class of probability distributions (e.g. Laplace approximation
2. Form functional integrals over spaces of distributions

In either case, we expect our results to follow.

Next, we assume that a timeless, spaceless, smooth abstract manifold $\mathcal{C}$ exists. We call this the noumenal manifold which we consider to represent all possible configurations or correlations corresponding to reality. We argue that in either perspective our formalism holds independently of our interpretation. We say the configurational versions of $\mathcal{C}$ are ontological whereas the correlation version of $\mathcal{C}$ is epistemic. Our noumenal manifold may ultimately be more abstract than something configurational or correlational. These notions imply some notion of relationship or causality whereas we will not be considering $\mathcal{C}$ to express such notions. This has the implication that the ordering of events may not necessarily exist at the noumenal level.

Next, we consider the existence of a noumenal Lie Group G with a right G-action ($\curvearrowright_G$) on $\mathcal{C}$

This will represent a fundamental symmetry group of the noumenal "reality". We anticipate such a group exists at some level either completely or approximately. We will take its existence as an axiom of our framework.

From $\mathcal{C}$, G, and $\curvearrowright_G$ we construct a principal bundle $\mathcal{N} \equiv (\mathcal{N}, \pi, \mathcal{C})$ and introduce a connection 1-form $\omega_{\mathcal{N}}$

Next, given this principal bundle we construct two associated bundles $\mathcal{E}_q = \mathcal{N} \times_G \mathcal{B}_q$ and $\mathcal{E}_p = \mathcal{N} \times_G \mathcal{B}_p$ which we will show inherit several connections and two bundle morphisms upon definition of local sections of these bundles.

Next, we define a scale-dependent agent is a pair of local sections $A^i = (\sigma_q^i(c), \sigma_p^i(c))$ over $\mathcal{C}$ where

$$\sigma_q^i : \mathcal{U}_i \subset \mathcal{C} \to \mathcal{B}_q$$

$$\sigma_p^i : \mathcal{U}_i \subset \mathcal{C} \to \mathcal{B}_p$$

We further define a multi-scale agent ($\mathcal{M}^S$) over $\mathcal{C}$ as a tuple of local scale-dependent agents

$$\mathcal{M}^S = \{A^s = (\sigma_q^i(c), \sigma_p^i(c))\}_{i \in \mathcal{I}}$$

Next we have an observation/observable as a local section of $\mathcal{B}_q$

$$\mathcal{O}_q : \mathcal{U} \subset \mathcal{C} \to \mathcal{B}_q$$

This is ultimately identical with the definition of an agent, however for our intuitive considerations we will distinguish the two concepts as we discuss later apropos quantum theory.

Finally we take as a fundamental law that a cost functional or "variational energy" $\mathcal{F}$ on the sections $\Gamma(\mathcal{B}_i)$ satisfies $\delta\mathcal{F} = 0$. Our agents will seek to minimize their variational energies within their associated bundles.

Note that the primary geometric objects used to construct our framework are the data $\mathcal{C}, G, \mathcal{B}_i$ and $\delta\mathcal{F} = 0$. The fibers $\mathcal{B}_i$ shall be taken to be $\mathbb{R}^D$ vector spaces over $\mathcal{C}$. These fibers shall represent the spaces of probability or "belief" (in the Bayesian sense) that an agent can hold about the points in a local subset of $\mathcal{C}$. We take the charts of $\mathcal{B}_i$ to be labeled as θ_q^i and θ_p^j for the q and p manifolds respectively and assume there exists a smooth atlas $\mathscr{A}$.

Therefore we consider that $p(c) \in \mathcal{B}_p$ and $q(c) \in \mathcal{B}_q$ are the theoretical or practical "model" probabilities and the agent's current belief respectively which are points in high dimensional vector spaces labeled by coordinates θ.

By considering our "observation" section we are able to then construct likelihoods $p(\mathcal{O}|c)$ and posteriors $p(c|\mathcal{O})$.

Here we must point out a minor subtlety: constructing $p(o|c)$. We formed an "observable section" on $\mathcal{B}_q$ as

$$\mathcal{O}_q : \mathcal{U} \subset \mathcal{C} \to \mathcal{B}_q$$

Since $o(c)$ lives in $\mathcal{B}_q$ we require the bundle morphism $\Phi : \mathcal{B}_q \to \mathcal{B}_p$ to send observations to p's such that $o \longmapsto p(o|c)$. Explicitly we might model a system via the distribution

$$p(o|c) = \delta(o - \Phi(\sigma_p(c)))$$

or a more general distribution

$$p(o|c) = \rho(o; \Phi(\sigma_p(c)))$$

In either case we need not introduce excess structure upon our minimal geometric framework if we consider observables to be agents in and of themselves. These constructions are standard in modern differential geometry.

Next, given our definitions we will have associated bundle gauge connections and bundle morphisms inherited from $\mathcal{N}$ not only within a single scale but between multiple scales. This web of connections will enable us to compare agent A's beliefs and models with another agent B's beliefs and models within and across many scales. For conceptual and general ease we will give these connections and morphisms descriptive and intuitive names that we adopt via our variational energy considerations. We will generally refer to the collection of connection as "language connections". They are:

1. $\Omega^i : \Gamma(\mathcal{B}_q) \to \Gamma(\mathcal{B}_q)$ is the "agent language" connection
2. $\tilde{\Omega}^i : \Gamma(\mathcal{B}_p) \to \Gamma(\mathcal{B}_p)$ is the "cultural language" connection
3. $\Lambda^i_j : \Gamma(\mathcal{B}_q) \to \Gamma(\mathcal{B}_q)$ is the "cultural comparison" connection
4. $\tilde{\Lambda}^i_j : \Gamma(\mathcal{B}_p) \to \Gamma(\mathcal{B}_p)$ is the "cultural transmission" or "emergence" connection
5. $\Theta^i_j : \Gamma(\mathcal{B}_q) \to \Gamma(\mathcal{B}_p)$ is the "cross-scale inference" or "top-down" connection
6. $\tilde{\Theta}^i_j : \Gamma(\mathcal{B}_p) \to \Gamma(\mathcal{B}_q)$ is the "dogma" connection.
7. $\Phi : \mathcal{E}_p \to \mathcal{E}_q$ is the bundle morphism "agent inference"
8. $\tilde{\Phi} : \mathcal{E}_q \to \mathcal{E}_p$ is the bundle morphism "agent experiment".

Here we have two bundle morphisms $\Phi : \mathcal{E}_p \to \mathcal{E}_q$ and $\tilde{\Phi} : \mathcal{E}_q \to \mathcal{E}_p$. When these diagrams commute (that is when $\pi_q \circ \Phi = \pi_p \circ \tilde{\Phi}$, where π_i are the bundle projection maps) we say we have an "experimentally verified prediction of a model". In general, however, these bundle morphisms are not guaranteed to commute.

2.1.1 Ω^i - Language

The Ω^i language connection is responsible for connecting a belief $q_A(c) \in \mathcal{B}_q$ of agent A to another agent B with belief $q_B(c) \in \mathcal{B}_q$. As a practical analogy this connection corresponds to agent A "discussing/interacting/learning" with agent B thereby allowing them to compare their beliefs about c. We index the language connection with i to signify that this connection will be strictly within the same scale - for example, human to human, culture to culture, atom to atom.

2.1.2 $\tilde{\Omega}^i$ - Cultural Language

The $\tilde{\Omega}^i$ is slightly different. This connection acts on the p's within the same scale. The p's in our framework represent the agent's best "model" of the hidden states. We can think of this as an "institutional language", a "formal system", a "theory", and so forth. For an agent A, updating implies that at scale-i the prior q updates easily but p is much harder to change. We may suspect that q updates "quickly" whereas p changes over longer timescales. Incidentally we could hypothesize that there may systems in which $\Omega^{i+1} \sim \tilde{\Omega}^i$. This connection allows agents to share their models.

2.1.3 Λ^i_j - Cultural Comparison

The Λ^i_j connection manifestly connects the q's of two agents across two different scales i, j. We call this "cultural comparison" since an agent A of the lower scale faces free energy pressure from Agent B at a higher scale to align their beliefs. This is analogous to "peer pressure" or an agent aligning their beliefs with that of their surrounding embedding. This might be analogous to religion, government, etc influencing our beliefs about the world.

2.1.4 $\tilde{\Lambda}^i_j$ - Model Emergence

The $\tilde{\Lambda}^i_j$ connection connects the p's of two agents across two different scales i, j. The p's are the models that each agent uses. This connection than compares the models of a pair of agents across scales. One might view this as institutions shaping sub-agents and vice versa or in our free energy considerations the emergence of higher models. For example, humans do not have an internal model of quantum theory but the multi-agent we call science does. Incidentally, cultures are slowly integrating quantum theory into their models of the universe. Interestingly the pair of Λ's share a striking similarity with renormalization transformations. Whether this similarity is apt will be pursued in future work.

2.1.5 Θ^i_j - Cross-Scale Inference

Θ^i_j connects a q at one scale to a p at another. We call this "cross scale inference" as it manifestly behaves much like our typical Bayesian inference procedures. An agent at small scales will compare their beliefs with respect not only to their own models but to other models of multi-agents that they are embedded within. Typically we expect the agent's model to be nearly identical with its "culture's" model but this needn't be strictly the case. This also might be better viewed as a "top-down" connection: one updates their beliefs according to a higher model. This is similar to the science - if the models accepted by science are strong enough then if a individual human performs an experiment they will use that model to update their belief about their observations.

2.1.6 $\tilde{\Theta}^i_j$ - Dogma Connection

$\tilde{\Theta}^i_j$ connects p's and q's in the other direction. For example, a belief q of a higher scale agent would be compared with the model p of a lower scale agent. we refer to this as "dogma" since a lower level agent compares their model to that of the belief from above. As an amusing example this paper itself is engaging this connection. Our model of reality is being tested against the dogma from above (coordinate invariance, subjectivity, Wigner's friend, etc).

2.1.7 Φ - Agent Inference

This bundle morphism is responsible for an agent updating their beliefs. In this morphism an single scale agent compares their p's and qs at the same scale. Loosely, they turn their p's into q's by updating their beliefs. This is the standard inference term in the variational free energy principle. In traditional formulations the p's and q's are members of the same space. Here, our model, manifestly keeps them separate. Therefore we must connect our

p associated bundle with our q associated bundle. This mapping is given by the left-action of the noumenal group on the associated bundle which we discuss below.

2.1.8 $\tilde{\Phi}$ - Experiment

Here the morphism goes the other way by turning the q's into p's within the same scale. This is very much similar to performing an observational experiment - observational data slowly updates the model.

In general our variational energy functional will be a complicated functional of energies associated with each of these connections and agents. This framework therefore is analogous to field theory from physics where minimizing the action functional introduces equations of motion. We might then loosely call this an "epistemic field theory".

2.2 Variational Energy Functionals

Karl Friston famously proposed his active inference model by introducing a variational free energy (in analogy with thermodynamics) that an agent could minimize by acting on the hidden states (active) or updating their belief (inference).

$$\mathcal{F}[q] = D_{KL}[\ q(c)\ ||\ \Phi p(c)\ ] - \mathbb{E}_{q(c)}[\log p(o|c)]$$

where $p(o|c)$ is the model likelihood of observation o.

This is similar in structure and spirit to the free energy $F = U - TS$ used in physical systems.

If we consider multiple same-scale agents and introduce a language gauge connection we could model a set of communicating/interacting agents by adding the term

$$\mu \sum_B D_{\mathrm{KL}}[q_A(c)||\Omega_{AB} q_B(c)]$$

where μ represents an arbitrary coupling constant. This is to be interpreted as the divergence between two agents' beliefs. This models language as having to represent one's belief as to be interpretable by another agent. Typically this is a lossy process but here we model this as a full comparison for simplicity. The Ω term of the energy functional, if you will, translates between one agent and another and tries to reach a consensus with the whole group.

Interestingly this term has a thermodynamic analogy to the Grand Potential $\Phi = U - TS - \sum_i \mu_i n_i$.

Naturally we can consider adding as many agents and as many scales and connections as we have available. We then have generally a total energy/potential functional as

$$\mathcal{F}_T[q_A] = D_{KL}[\ q_A(c)\ ||\ \Phi p_A(c)\ ] - \mathbb{E}_{q_A}[\log p_A(o|c)] +$$

$$\mathcal{F}_\Lambda + \mathcal{F}_{\tilde{\Lambda}} + \mathcal{F}_\Theta + \mathcal{F}_{\tilde{\Theta}} + \mathcal{F}_\Omega + \mathcal{F}_{\tilde{\Omega}} + \mathcal{F}_{\tilde{\Phi}}$$

For general complex systems this may be computationally intractable to simulate in full detail but avails itself to simplifications. Also note that this will define a set of coupling constants that one may be able to fit phenomenologically.

As an example of a cross-scale connection we would consider higher terms. For example, an agent A at scale i communicating their beliefs with other scale i agents as well as comparing their model with higher culture (scale $(i+1)$) would have a variational energy such as

$$\mathcal{F}_T[q^i_A(c)] = D_{\mathrm{KL}}[(q^i_A(c)||p^i_A(c))] - \mathbb{E}_{q^i_A}[\log p^i_A(o_A|c)]$$

$$+ \sum_B \mu^i D_{\mathrm{KL}}[q^i_A(c)||\Omega_{AB} q^i_B(c)] + \lambda^{i+1}_i D_{\mathrm{KL}}[p^i_A(c)||\tilde{\Lambda}^{(i+1)}_i p^{i+1}(c)]$$

where λ^{i+1}_i is a coupling term.

2.3 Left G-Action

Given a principal bundle $\mathcal{N}$ equipped with a right group action $\curvearrowright_G$ we may induce a left group action $\curvearrowleft_G \equiv \phi^s_{p,q}$ on the associated bundles $\mathcal{E}_{p,q}$. This left group action represents an agent's belief updating step.

The mapping ϕ then ultimately encodes belief dynamics. This action takes, for some $c \in \mathcal{C}$, a belief $q(c) \in \mathcal{B}_q$ into an updated belief $q'(c) = g^{-1}q(c)$ according to $\delta\mathcal{F} = 0$ vis a vis

$$\dot{q} = -\nabla\mathcal{F}(q, p)$$

for an energy/potential functional $\mathcal{F}$.

This has interesting philosophical consequences. On one hand we can envision our Lie Group G shuffling around the points under right action in

$\mathcal{C}$ begging the interpretation of the noumenal universe evolving its hidden states. On the other hand one can view the dynamics and group action as acting on the beliefs about fixed points in $\mathcal{C}$.

This curious duality suggests that one can view the noumenal universe as completely static and instead interpret the beliefs themselves as evolving dynamically: that is, the points $c \in \mathcal{C}$ are fixed while the beliefs $q(c) = q(c, \tau)$ evolve dynamically. In this manner agents, although "composed" from hidden states themselves, have their 'existence' within the associated bundles. This duality potentially presents a novel ontological and epistemological stance. In this manner we can view reality as fundamentally static while our evolving beliefs create the illusion of time, causality, and change under this specific left-action.

We point out that the nature of the belief updating, aside from its relation to a noumenal Lie Group, is unknown. Rather than attempt to pursue questions related to its existence we instead take this as a "proto"-dimension of time as inherently fundamental to our geometry. This updating step may not correspond to physical time, but is instead proposed as a discrete sequence of epistemic updates.

We tentatively define the minimal update step to be such that q changes by a single bit. This appears analogous to the Plank time from physics and defines a lower bound on an epistemic change. We further propose an upper bound on the total amount of information change per step to be related to the Bekenstein bound from physics. This appears as a natural version of Wheeler's "it from bit", "it from qubit" or perhaps even "it from correlate".[11] [12] [13] [14] [15] [16]. Therefore, we assume agents update information in discrete steps such that

$$0 \leq \Delta q \leq I_B$$

We tentatively define the change in information as

$$\Delta q = D_{KL}[q_i(c)|q_f(c)] = D_{KL}[q_i(c)|g^{-1}q_i(c)]$$

2.3.1 Scale-0 Agent

We define a "scale-0 agent" Alice, as an agent which has a one-dimensional belief about $c_0 \in \mathcal{C}$. Namely

$$A_0 : c_0 \in \mathcal{C} \to \mathcal{B}_q$$

$$c_0 \longmapsto A_0^q(c_0) = q(c_0) = 1 = A_0^p(c_0) = p(c_0)$$

This agent has a 1-dimensional belief space over c_0. Upon free energy minimization it's belief sections remain static. Here we could potentially introduce an ignorant scale-0 agent to have a model and belief about c_0 less than 1. For now we shall keep the agent as simple as possible. We could also say that the total bundle is characterized by a spike at only a single coordinate c_0 and zero everywhere else. In this view the dimensionality of $\mathcal{B}$ is extremely large but as far as the agent is concerned she is effectively one-dimensional: the scale-0 agent has no knowledge of anything else.

It becomes quite interesting when we introduce a pair of scale zero agents A_0 and B_0 (Alice and Bob) as

$$A_A^q(c_A) = A_0^p(c_A) = q(c_A) = p(c_A) = 1$$

$$B_0^q(c_0) = B_0^p(c_B) = q(c_B) = p(c_B) = 1$$

We can interpret this as an agent over c_A and a different agent over c_B each of which knows nothing about the other. However, we can connect the two sections with a scale-0 language connection in which case the energy functional for either A or B develops a coupling term:

$$\mu D_{\mathrm{KL}}[q_A(c)||\Omega^0_{AB} q_B(c)]$$

Depending on the sign of μ agents will attempt to align or misalign their beliefs. At each update step their belief fibers begin to develop non-zero beliefs about each other's coordinates. From the perspective of a scale-0 agent their belief manifold begins to grow a second dimension related to their growing knowledge about the other agent's coordinate. Alice is saying to Bob "I'm at c_A" which Bob previously, before the update, had no knowledge of. This effectively combines their belief space into a "Scale-1 Agent". This emergent meta-agent is a local section over $\mathcal{C}$ itself. Alice has strong belief in her θ_i direction whereas agent Bob has strong belief in his θ_j direction of the belief space. The curvature of the fibers can be studied via informational geometry and the Fischer metric.

Marvelously agents evolve in this curved belief space in response to their local Fischer curvature. Yet at the same time as agents coalesce into meta-agents the curvature of the space changes and drives further evolution. This suggests a route towards a "General Relativity" of belief space where we might build an information content tensor $\mathcal{I}_{ij}$ of a meta-agent and compare this to the informational geometry of the belief spaces. In general we have

1. belief curvature tells agents how to evolve
2. meta-agents determine how belief space curves

This establishes interesting parallels with gravity, thermodynamics, and epistemology. A flattening of belief space corresponds to an "epistemic heat death". Agents no longer evolve and explore and their dynamics would effectively end. This harkens to the notion of an entropic heat death of the universe whereby the entire universe thermalizes.

2.3.2 Scale-1 Multi-Agent

We can imagine constructing ever higher scale agents from our scale-0 primal agents. We therefore define a scale-1 multi-agent to be to be the collection of sections

$$\mathcal{M}^1_{AB} = \{q_A(c_A), p_A(c_A), q_A(c_B), p_A(c_B); q_B(c_A), p_B(c_A), q_B(c_B), p_B(c_B)\}$$

Here we have written down the most general "mixed" scale-1 multi-agent. We say a multi-agent is "mixed" if the constituent lower-scale agents have beliefs and/or models about each other. This is a "larger" local section defined across c_A and c_B. For ease we consider these points to be within the same local neighborhood, however, one might consider discrete spaces and generate hypergraphs of agents. Naturally the parameter dimensions θ, through which agents may explore, increases substantially as agents discover one another.

Our framework allows us to study emergent systems under the convexity of our assumed variational energy functional which encourages agent belief alignment upon update - this might be modeled as a full alignment or a mixture of alignment and misalignment terms in the variational energy.

Furthermore, as agents grow and align to higher level agents new gauge connections (e.g. Λ^i_j as new j emerge) also manifestly emerge introducing further free-energy couplings.

We therefore see that renormalization-like operators naturally emerge via the gauge connections $\Lambda, \tilde{\Lambda}$ which couple distinct scales. Fine-grain details (agents) of a multi-agent get integrated out as agents grow and couple to coarse grained, larger scale multi-agents.

Not only do the agents themselves become more complex but the tower of interactions also become ever more complex and entangled carrying the total system to higher complexity.

In this manner we propose that a natural consequence of this Fristonian variational energy minimization with the geometric structure of our formalism comprises a new law that **complexity manifestly increases due to the convexity of the variational energy**. This is in line with other investigators who have speculated on a similar law of the universe in much the same way as the second law of thermodynamics[15] [7]. In our framework it emerges naturally and manifestly from our demand that variational free energy extremizes. From the author's best knowledge this represents the firmest proposition of complexity and doesn't rely on similarly vague definitions of circuit complexity and/or entanglement.

2.4 On Established Physics

2.4.1 Thermodynamics and a Dark Sector of $\mathcal{C}$

How, then, can we make contact between this framework and standard results and experiments in physics? Our framework implies that the thermodynamics we perceive (as humans) is fundamentally a pullback or projection from our belief dynamics over $\mathcal{C}$ down to $\mathcal{C}$. If this is true then we should search for a belief structure in the fibers which maps to the theory of thermodynamics.

The obvious first place to search is the variational energy itself. However, when we pull this back down to the $\mathcal{C}$ manifold it might mix between observable (to humans) reality and an unobservable sector. Tentatively we conjecture

$$\sigma^*(\mathcal{F}) \to E_{obs} + E_{unobs}$$

This naturally suggests the only unobservable energy we know about: dark energy - however we may still anticipate there are unaccounted for properties of $\mathcal{C}$ which wouldn't necessarily show up that which we currently call dark energy.

Furthermore, Landauer gives us a potential tool to connect belief updating to entropy production whereby 1 bit of erasure corresponds to $kT \ln 2$ units of energy.

As an agent learns 1 bit she must dissipate $kT \ln 2$ energy into the environment irreversibly increasing the entropy of her universe.

In the belief fibers she is updating her beliefs by some Δq therefore we might suggest that at minimum

$$\Delta q = 1 \text{ bit} \geq \ln 2 \text{ J/K}$$

or

$$\sigma^*(\Delta q) = S_{obs} + S_{unobs}$$

Therefore, we expect

$$kT\Delta S_{Thermal} \leq \Delta\mathcal{F}$$

or

$$\sigma^*(\Delta\mathcal{F}) = k_B T\Delta S_{obs} + k_D T_D \Delta S_D$$

Where the D's label a "dark" sector of $\mathcal{C}$ which may or may not precisely correspond to the commonly considered "dark energy" of our universe. It rather serves to allow us to specify the equality. It is a profound development for our theory that a dark sector of our perceptions of $\mathcal{C}$ emerges naturally.

We can make further connections with our perceived reality by considering Bekenstein as the upper bound of Δq

$$\sigma^*(\Delta q) = S_{obs} + S_D \leq \frac{4\pi k G M_\sigma^2}{\hbar c}$$

This suggests a connection between the observable mass of an agent and how much it can update per step. This seems quite plausible in our theory - the "larger a multi-agent" the more information that multi-agent can process: where by "large" we mean "however many points over $c \in \mathcal{C}$ the section is given some measure on $\mathcal{C}$"

What does this upper bound suggest then? It appears to connect the observable and dark universes with potentially massive consequences for quantum gravity and blackhole physics.

Now, it may very well be that multi-agents can update past the Bekenstein bound but not without corresponding effects on the dark universe.

2.4.2 The Quantum

We can check what consequences this framework has for other established physical theories. We shall discuss our framework in relation to the principles of quantum theory since experimentally the theory represents a very strong model of our universe. Quantum theory, however, has suffered deeply from being foundationally obscure - namely, its difficulty establishing a clean definition of what constitutes observers, observations, and measurements. Furthermore, Bell, Aspect, and others have convincingly shown that our universe violates local realism. We have two options for our theories of physics then - either abandon our demand of locality or abandon our demand for realism[4]. If we abandon locality then we must allow non-local correlations - such as EPR. If, however, we abandon realism then we cast to the wind the notion that objects of our universe do not have firm properties until they are "observed". However, this is quite vague as the Wigner's friend thought experiment correctly points out[17].

Our framework then appears to abandon the notion of locality in favor of realism. We assume that the noumenal manifold exists representing all possible hidden states - the correlations are baked into the structure. They have a reality but they happen to be completely inaccessible. $\mathcal{C}$ has no such time and space and by our duality enabled by the group G we consider it completely fixed. This then implies a global super-determinism that is manifestly present in our model. This isn't a necessarily damning aspect since we have the epistemic room to allow the belief fibers to be dynamic. This acts as a perceived local freedom of choice, from the agent's point of view, to choose one's perceived path through their perceived universe. We call this phenomenon "local free will" - it is manifestly a perception about the noumenal states that an agent, with a rich enough belief section, would infer about the nature of the hidden states. We view the beliefs fundamentally being updated via left action on the associated bundles (beliefs are dynamic) whereas an agent would perceive a dynamic metric on the noumenal manifold. Importantly our framework may offer experimental classifications of consciousness based on belief fiber curvature if we can establish the relevant renormalization group flows.

In this manner our framework not only satisfies the conditions we expect from our observations of a quantum world but it also potentially resolves the observer/measurement tension in a natural way that emerges purely from our framework's geometry. An observer is an agent - a section of the associated belief bundles - interacting/communicating with an observable - also a section of the associated belief bundle as defined above. Measurement then becomes the updating of an agent's belief from connection with an observable.

One might ask then what even is quantum theory within our framework. Our interpretation is that quantum theory is generally a model about the projections of our beliefs onto the noumenal bundle $\mathcal{N}$- our models exist in

$\mathcal{B}_p$. Therefore, quantum theory is effectively a map

$$\psi : \mathcal{N} \to P(H)$$

$$p_{Born} : P(H) \to \mathcal{B}_p$$

where $P(H)$ is projective Hilbert space and p_{Born} is the Born rule for converting quantum wavefunctions/sections into probabilities.

Barandes however has recently shown a direct mapping from $\mathcal{N}$ to $\mathcal{B}_p$ as

$$\Gamma : \mathcal{N} \to \mathcal{B}_p$$

In Barandes' view quantum states are indivisible non-Markovian stochastic processes[18]. The existence of Barandes' correspondence is evident in our framework naturally implied by our axioms here. It is a direct mapping without the need for projective Hilbert space while retaining all the standard quantum curiosities encoded by its non-Markovian processes.

Quantum theory then is the model that a high-scale multi-agent will use to explain the universe of their perceptions. This multi-agent, in our reality, is composed of many human agents existing over large spatio-temporal scales. At the human-agent scale we individually do not perceive this quantum model of the universe (at least most folks don't observe each other smearing in a superposition of states). This model is a multi-agent model that emerged as a consequence of many human-scale agents interacting (via our language gauge connections across multiple scales). Our formalism even yields a resolution to Wigner's friend. Quantum theory then is an emergent model of the hidden states of our world. It is MUCH more capable of prediction than our individual models of reality. This then implies that if we continue up the chain of multi-agents which themselves are built from multi-agents, and so on, then a consequence might be that as our variational energy functionals are minimized this produces our perceptions of new, unknown layers of our models of reality. Quantum theory was developed across millions of human-scale agents over the course of hundreds of years (many updating steps). It seems this framework we propose ourselves is entirely a continuation of that process built upon ideas from millions of scientists, philosophers, schools, economies, institutions, governments, and so forth - the collection of our entire web of gauge connections.

2.4.3 Λ^i_j , $\tilde{\Lambda}^i_j$, Scales, and Renormalization

Given our construction of multi-scale agents and associated bundles we will have gauge connections between different scale agents. This allows for top-down and bottom-up inferences but perhaps most importantly naturally injects a renormalization flow into our framework. Interestingly, it's not at all clear if a maximal agent exists and furthermore if two agents share incommensurate beliefs then interactions may drive them apart and/or cause agents to explore uncharted vistas of belief.

Regardless, as agents coalesce into multi-agents the higher level beliefs and models become shared among the constituents. In this manner the fine grained beliefs of individual agents within the multi-agent fall away in importance to the coarse grained beliefs and models of the multi-agent. These connections across scales therefore behave as a "scale-parallel transport" of beliefs and models and induce a renormalization group flow under scale changes.

As this renormalization connection couple scales our variational free energy functional will develop scale dependent couplings over the space of possible beliefs analogously to renormalization in physical field theories where all possible interactions are handled at once. Therefore, if we can identify these couplings at, say, the human scale, or microscopic scale, then our energy functional can potentially be studied via the tools of perturbation theory, Wilsonian renormalization, etc to predict possible higher scale beliefs and models and their myriad interactions. Does this then provide a route to discover new theories of scale-dependent agents and/or physics? Are there fixed points of the flows? What might the universality classes of beliefs and models be and how do they related to the noumenal group G?

We conjecture that this renormalization and gauge coupling are a more general manifestation of topics as diverse as Darwinian evolution, complexity, emergence, economics, semantic convergence and language, and much more across many disparate fields of inquiry.

As an example of this dictionary, Darwinian evolution would be mapped to the framework we have presented here as

1. selection $\Longleftrightarrow$ variational energy minimization
2. mutation $\Longleftrightarrow$ belief space trajectories
3. adaptation $\Longleftrightarrow$ flow towards fixed points

This mathematical framework built from a minimal axioms then appears to unify many aspects of reality but a deep category theory review of the general analogies will be saved for future work. We didn't put renormalization and emergence in "by hand" - it emerged as a consequence of the geometry we proposed, convexity of variational energy and its extremization/minimization, along with defining multi-scale agents as sections of an associated bundle.

2.5 On a New Philosophy of Science

Our theoretical framework has many implications for new philosophical stances. As an example we consider the nature of scientific inquiry itself.

A scientist is an agent (themselves a multi-agent) who compares theory with observation. In our model this means there is an variational energy term

$$D_{\mathrm{KL}}[q^H(c)||\tilde{\Theta}p^S(c)]$$

This is a scientist, at the human scale, comparing their observations with respect to the model of a larger multi-agent which we might call the institution of science. This p^S model would necessarily be some scientific theory such as quantum theory, gravitation, and others. Our theoretical models improve as humans and institutions interact, mingle, and explore their belief fibers collectively and minimizing their variational energy at each step. Scientific pursuit becomes an exploration of the space of all possible theories/models/beliefs.

2.5.1 Geometric Perceptions

We also point out the interesting fact that the language gauge connections exists within the fiber bundle as a consequence of the principal bundle connection ω. However, we can push these connections down to $\mathcal{C}$ in the standard geometric way as local connections or Yang-Mills fields on $\mathcal{C}$. This may serve as an "agent-perceived interaction" (as agents may either perceive their beliefs changing or perceive the noumenal manifold's states dynamically evolving).

Realistically, agents, however, cannot perceive the hidden states of $\mathcal{C}$ but instead build dynamic belief and perceptual models in order to describe the $\mathcal{C}$ they are built from. The pushforward from the associated bundle to the base manifold may, in general, allow agents to develop a pullback metric from $\mathcal{B}$ to $\mathcal{C}$. This hypothesis naturally warrants further study. Interestingly, this offers an "epistemic field theory" in resonant analogy with classical field theory: the agents are similar to mass fields whereas the gauge connections we push down are analogous to Yang-Mills fields - we propose an analogy

$$Agents \iff Fermions$$

$$Language\,Connections \iff Gauge\,Bosons$$

This might be a happy coincidence or it may reflect something deeper about the fundamental group G and how agents build perceptual models.

This tentative analogy may allow us to leverage many of the results in classical (or quantum) field theory in attempts to understand cognitive systems.

We conjecture that the pullback metric humans (in particular) apply to describe $\mathcal{C}$, developed via biological evolution to maximize survival and reproduction, is a metric $\mathscr{G}_{ij}$ of the form

$$\mathscr{G}_{ij} = g_{\mu\nu} \oplus \mathcal{G}_{\alpha\beta} \oplus \mathscr{D}_{\xi\zeta}$$

where $g_{\mu\nu}$ represents the perceived Euclidean or Minkowski spacetime metric,$\mathscr{D}_{\xi\zeta}$ represents an unobservable "dark universe metric", and a remaining high dimensional metric $\mathcal{G}_{\alpha\beta}$ on $\mathcal{C}$ potentially encoding every thing else.

These metrics could potentially influence each other in the web of language connections and variational energy as perceived interactions among each other. Namely, $T_{\mu\nu}$, $G_{\mu\nu}$, and Λ as in standard general relativity.

Next, an agent generally has a dynamic belief above a given point $c \in \mathcal{C}$. This belief may initially be zero until the agent "discovers" it through interaction. The agent has a natural Fischer metric on their belief space given by

$$_{ij}(c) = \mathbb{E}_{q(c)} \frac{\partial \log q(c)}{\partial \theta^i} \frac{\partial \log q(c)}{\partial \theta^j}$$

Where we must functionally integrate over the space of distributions or consider the Laplace approximation and consider the space of Gaussians. It may then be possible to pull this metric back to the noumenal manifold $\mathcal{C}$ along with a neighborhood about c relating the perceived metric to the belief space metric.

Furthermore we could propose a second route towards establishing a candidate perceived metric $\mathcal{C}$ if we have a local chart on $\mathcal{C}$

$$\mathscr{G}_{ij}(c) = \frac{\partial^2}{\partial \delta c^i \partial \delta c^j} D_{\mathrm{KL}}[q(c)||q(c+\delta c)]$$

This candidate metric, however, presents challenges such as defining a smooth atlas on such an abstract space we don't necessarily have access to.

Notice that left group action of G on beliefs implies locally this metric is related to the Lie Algebra of our noumenal group

$$D_{\mathrm{KL}}[q(c)||q(c+\delta c)] = D_{\mathrm{KL}}[q(c)||dg^{-1}q(c)]$$

$$\approx D_{\mathrm{KL}}[q(c)||(\mathbb{1} - \epsilon\mathcal{A})q(c)]$$

where $\mathcal{A}$ is the appropriate representation of the Lie Algebra of G on the fibers.

However, notice what this expression implies: it is a metric on (or about) the noumenal manifold but is entirely defined by the beliefs an agent has about the hidden states of $\mathcal{C}$. This, then is the perceptible universe for the agent. More interestingly is the observation that different agents will manifestly perceive different metrics of their universe given their varying models and beliefs. Clearly, the dimensionality of the belief fibers presents enormous challenges in our ability to interpret this framework; however, different agents perceiving different geometries is cognitively consistent.

A natural question is then "why do we, as human agents, perceive a $3+1$ dimensional space-time?" Traditionally, physics assumes that this is the default universe - yet cognitively the perceptions of space-time (and other geometries in, say, the web of particle interactions, etc), a la Kant and modern neuroscience seem to imply that (almost) all humans (and perhaps other animals) evolved so as to perceive these spatial dimensions. It is as though evolution encoded a subset of our beliefs to be fixed with extremely high weights associated with the hidden states corresponding to these directions. This is clearly testable as any user of psychedelics will attest. The interpretation here is that a hallucinogen (or trauma, etc) induces a mixing of the hard-wired weights encoded by evolution.

This smearing of perceptual beliefs in $\mathcal{B}$ then materializes as changes in our perceptual metric $\mathscr{G}_{ij} = g_{\mu\nu} \oplus \mathcal{G}_{\alpha\beta} \oplus \mathscr{D}_{\xi\zeta}$. This comprises a novel, to the best of our knowledge, cognitively and physically consistent picture of our hidden reality at the cost of (potentially) being extremely high dimensional. It even resonates with the computed discrepancy ($\sim 10^{120}$) between vacuum energy and the expansion of the universe! Belief space is HUGE and it's reasonable to expect the dimensions of the belief fibers in our model to be inherited by the dark universe! This absolutely warrants further study.

It appears very natural that we can pick four directions from within the extremely high dimensional belief fiber and rotate them into the standard $3+1$ metric (be it Euclidean, Minkowski, or anything else consistent with the noumenal Lie Group G. Coordinate invariance and Lorentz invariance in physics would then correspond to a statement about how (most) humans (animals?) split this total metric into block diagonal form. $SO(3,1)$ rotations in this sector would be invariant which would percolate up to a statement about multi-agent beliefs and models in the associated bundle. This may be related to grand unified models such as $SO(10)$ and others.

Consequently, our framework may serve to elucidate, in a novel manner, more information about verifiable reality and the hidden manifold by probing informational changes to our beliefs. It appears that if we can pin down the allowed symmetries of our belief fibers, then this would give us enormous information about the noumenal Lie Group G. This would effectively be a new approach in physics, neuroscience, sociology, and other fields that to the best of the author's knowledge, which hasn't been systematically explored.

Clearly the challenge of this approach is defining relevant charts of the belief fibers to establish coordinates θ for each particular agent at a slew of different scales (atoms, molecules, cells, humans, and even more abstract agents such as economies, governments, cultures, and more). Traditionally such abstract agents have been notoriously difficult to study given the lack of objective physical units with which to associate them. Here, we have proposed the natural unit of "belief change per update" - therefore, given some time step one would attempt to track all the information gained and lost for a particular multi-agent at a particular scale by mapping the thermodynamic properties we perceive under our perceptual maps.

3 Conclusion

We have developed a novel mathematical framework not only for the study of cognition, inference, agent-based modeling, and socio-physics but also general theories of physics, philosophy, and more. The primary consequences of our framework is

1. The natural emergence a perceived dark sector of our universe follows as a shocking consequence of our theory. Further the cosmological constant problem may have a resolution in our theory.

2. The suggestion that complexity naturally increases due to convexity of the variational energy functional over an epistemic space
3. Our verifiable reality is an agent-based projection of a belief geometry onto the noumenal manifold
4. There exists a duality between viewing the noumenal manifold as possessing dynamics or the agents' belief space dynamically evolving over a fix noumenal manifold.
5. Human agent perception of, not only space-time but general physical interactions, may emerge from a pullback mapping from dynamic belief fibers onto the noumenal manifold.
6. Renormalization transformations emerge from multi-scale gauge connections on the associated bundles.
7. There exists an unknown Lie Group with right action on the noumenal manifold.
8. Language is a gauge theory

This geometry naturally links many disparate fields of inquiry into a cognitively consistent formalism unifying the epistemic and ontological as well as the subjective and objective. Experience and qualia in this view are pullbacks from the fiber bundles to the noumenal manifold as a agent-specific model of reality. Future pursuits will study the construction of local Yang-Mills fields from the associated gauge connections which themselves ultimately derive from a noumenal gauge connection and use these constructions to study renormalization flows. We also anticipate studies of space-time emergence from perceptual belief models and well as relating the acceleration of the universe to the rate of Δq production as emergent hierarchies are formed.

Our framework possesses many features warranting future study that we are unable to explore here. Machine learning and artificial intelligence, linguistics, sociology, and other researchers may find this framework novel and fruitful. Novel philosophical stances on ontology, epistemology, and the philosophy of science have been proposed that similarly warrant further detailed study.

Most importantly, under our present considerations, a candidate theory of dark energy and dark matter emerges as a pullback of the amount of information an agent updates per step. Our relatively simple geometric model of Kantian and cognitive physics is so rich that we cannot possibly consider all of its implications here.

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