Epistemic Geometry: Modeling Qualia

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Abstract

The quantitative modeling of agent qualia remains an open challenge. Here we describe the construction of a multi-agent model which manifestly takes into account cognition and agent communication at the outset. Agents are constructed as pairs of local sections of associated bundles $\mathcal{E}_i$ (where i= belief, model) to a principal G-bundle $\mathcal{N}$ composed of a base manifold $\mathcal{C}$ and Lie group G. Agents interact via induced connections and evolve according to a generalized variational energy. We demonstrate that a $SL(2,\mathbb{R})$ gauge frame acts as an agent "center" where communication between agents is mediated by gauge connections. We treat agent evolution under a dynamical variational generalized energy proffered by a "free energy" and "connection energies". This structure allows agents to maintain coherence in perception and inference, despite fundamentally local and subjective perspectives. Language, in this view, is a gauge theory and agentic perspectives can be both unique and communicable. Importantly, we describe how agent "qualia" can be studied as the pullback of informational geometric quantities from an agent's gauge frame to the base manifold. We further describe, then, a cognition-first approach towards a unified physics.

1 Introduction - From Observation to Ontology

1.1 The Geometric Legacy

"Ubi materia, ibi geometria" - Kepler

Geometry has proven to be a powerful tool with which humans have leveraged to develop myriad theories and models of the natural world throughout our history. As exemplified by Kepler, above, geometry is thought to play a central role in the structure, composition, and dynamics of our perceptual reality. From Euclid to Newton, Maxwell to Einstein, Grothendieck to Witten, and many more, geometry has proven reliable in advancing new ideas and illuminating new paths of scientific inquiry.

The geometric study of the foundations of physics, in particular, produced great strides throughout the 20^{th} century and continues to this day. Researchers have discovered that the organizing principles of the natural world are best expressed in the language of geometry and symmetry - especially continuous symmetries described by Lie groups, and geometry (fiber bundles, manifolds, etc).

Despite the philosophical challenges in studying the foundational elements of reality science has progressed steadily by developing extremely powerful and accurate theories based upon the assumption that space and time (among other dimensionful quantities) are fundamental aspects of our universe[ˆ4]. Einstein and Minkowski (among others) unified space and time into a 3+1 dimensional smooth manifold wherein its geometry is dynamic and malleable. Quantum Field Theory (QFT) further built upon this scaffold of Minkowski spacetime by extending quantum theories to compatibility with Poincare/Lorentz symmetry leading to some of the most elegant, awesome, and precise predictions and verifications humanity has ever known.

Still, deep challenges remain. The fundamental quantities of energy, mass, space, time, and more remain stubbornly devoid of firm definition. Furthermore, the "unreasonable effectiveness of mathematics in the natural sciences" remains a profound but poorly understood property of our universe. We currently do not have a proto-geometry of physics and the natural world aside from candidate theories such as string theory, loop-quantum gravity, geometric unity, and others. Still, these frameworks lack firm ontological definitions of the dimensionful quantities they seek to describe.

1.2 The Cognitive Revolution

Neuroscientists, computer scientists, and cognitive researchers have likewise leveraged information geometry to advance our understanding of the nature of perception and cognition. Arguably, the focal point of the field of cognition is due to the pioneering work of Karl Friston who has developed a variational free energy model that aims to unify the fields of cognition and behavior across many physical scales[ˆ1]. Cognitive systems perform Bayesian model selection continuously, updating beliefs in light of incoming sensory data —

a process often described as predictive coding. The free energy principle unifies action and perception under the same variational imperative: agents act to minimize expected free energy — they sample the world to reduce uncertainty. This reframes action as epistemic foraging, and embodiment as a condition for inference. In this view, perception is not bottom-up sensation, but top-down model confirmation.

Explicit in these formulations is the premise that cognitive systems do not directly perceive the world's "true" states. Instead, cognitive systems build models of their sensory data in such a way as to minimize their variational free energy and ultimately survive, reproduce, and evolve. Central to these studies is the geometry of information as pioneered by Amari, Rao, and Fischer among others and the Bayesean interpretation of probability as inferential logic.

In the cognitive revolution, geometry no longer applies only to physical space, but to abstract spaces of probability distributions. Agents navigate belief manifolds — not just landscapes of atoms — making geometry of thought a rigorous mathematical object. Powerful methods and trends in deep-learning, artificial intelligence, and much more highlight the fruitful path of considering the geometry of beliefs and models as applied to physical systems. Neural networks can be understood as approximate inference engines operating over complex latent spaces — trained via variational autoencoders, energy-based models, or contrastive divergence.

1.3 On The Philosophy of Perception

"Thoughts without content are empty; intuitions without concepts are blind." - Kant

Physics traditionally posits an observer-independent reality governed by invariant laws. Yet, modern cognitive science tells us that all observation is mediated by inference, prediction, and model-building — not direct access to a world "out there." This introduces a fundamental epistemological tension: how can physics describe the world as it is, when all measurements are filtered through subjective models? Physicists recognize these difficulties but rarely frame them as such. The foundational problem of "an observer" is the foremost challenge in physics.

This dilemma echoes the insights of Immanuel Kant, who argued in The Critique of Pure Reason that we never perceive the noumenon — the world-in-itself — but only the phenomenon, shaped by the a priori structures of human cognition. Space and time, for Kant, are not external entities, but forms of intuition — cognitive scaffolds through which experience becomes possible.

Helmholtz elaborated on Kant by considering an early version of the "Bayesean Brain Hypothesis". Helmholtz proposed that perception is an unconscious inference — that the brain constructs its experience of the world through internal hypotheses, constantly updated based on sensory input. Though lacking the formal apparatus of probability theory, Helmholtz's insight prefigured the modern view of the brain as a Bayesian inference machine.

Phenomenology, from Husserl to Merleau-Ponty, emphasized that experience is always perspectival — situated, embodied, and structured by intentionality[x][XX]. If space-time is predicated upon our cognitive perceptions then what does that mean for the entire edifice of physical theory? Physics assumes important symmetries such as Lorentz invariance, general coordinate invariance, and gauge invariance and yet subjective experiences—such as those reported under psychedelics, trauma, and mental illness suggest deviations from standard space-time perception[ˆ4]. Such accounts are often dismissed as mere subjective hallucinations, yet modern neuroscience tells us that all perception is, in a sense, a controlled hallucination shaped by evolutionary pressures[ˆ5]. Valid physical information may well be hidden within the perceptions of the pathological. We then might be more cautious in our immediate dismissal of such "observations" (especially apropos consistent overlaps and descriptions across many individuals).

While Kant saw geometry as a condition of experience, Einstein re-defined geometry as a property of the physical world. The curvature of space-time is not an artifact of perception, but a dynamical field sourced by mass and energy. Yet, this move raises a deep question: how can a theory built upon observer-relative measurement (e.g. Lorentz invariance, coordinate choice) claim objectivity if no notion of an "observer" exists within the framework? This is even more problematic in Quantum Theory where "observers" possess non-trivial abilities to interrupt unitary evolution of quantum wavefunctions. Many famous paradoxes abound! Our most accurate and specific theory of the nature of reality is predicated upon the observer which the theory can say nothing about.

On one hand, physics excels at describing intersubjectively verifiable reality — the world we can agree upon. On the other, it says almost nothing about the cognition that enables this agreement. This disjunction, between shared reality and the private mechanisms of inference, remains unresolved. Here we propose a path to reconciliation: a unified geometric model of agents, grounded in both physical and cognitive principles.

1.4 Towards Unification

Cognitive science and physics remain steadfastly separate. We may observe that when we perform experiments and record our results we are ultimately looking at the dials and knobs via electromagnetic waves and emitting sound waves to correlate to each other and be interpreted as experimental confirmation between our warm, wet brains, locked inside a "dark cavern" extending electric tentacles into the correlations of our universe[^5]. Indeed, experiments are only "confirmed" if one agent's perception corresponds to another's via some lossless mapping (we hope!). What are we to make of physics if our basic notions of reality are themselves subjectively emergent. Here we adopt the position that any theory of physics should also be cognitively consistent and, most importantly, vice versa.

In recent decades, physicists have begun to entertain the possibility that space and time are not fundamental, but emergent. Arkani-Hamed and collaborators introduced the amplituhedron[^6], a timeless geometric object whose volumes encode scattering amplitudes in supersymmetric field theories. Meanwhile, other researchers have used tensor networks and holographic entanglement to construct emergent geometries resembling Einstein's general relativity[15][19][^20]. Still others have found a striking similarity between thermodynamics and general relativity[^17]. These results suggest a deeper structure undergirding our reality - Kant's noumenon.

Here we propose a geometric framework aimed towards a cognitively consistent framework of physics (subjective and objective) based upon a potentially timeless, spaceless abstract structure which we call a noumenal manifold with structure group G.

In this paper, we propose a radical inversion: that cognition is not an emergent property of a physical universe, but rather that physical law is an emergent structure within a web of cognitive inferences. We call this perspective cognitive-first physics. Within this paradigm, the fundamental entities are not particles and fields, but agents, beliefs, and models — connected via the geometric machinery of inference, communication, and gauge connection.

At the heart of this theory lies a novel synthesis of ideas: epistemic geometry and epistemic gauge theory. Every agent maintains a probabilistic belief about an underlying base space — a latent manifold which we interpret as "reality." These beliefs/models are not uniform: they vary across the base manifold, are subject to noise, and potentially differ between agents. To communicate, agents must transport beliefs across distinct representational frames — necessitating a gauge-theoretic structure. From these transformations, a full-fledged field theory emerges. An agent, then, has a particular fixed gauge frame and communication is mediated by gauge transformations between agents.

This is not merely a reinterpretation of physical theories — it is a shift in ontology itself.

Gauge connections represent not electromagnetic or gravitational forces, but epistemic compatibility and communication: how well two agents can align their perspectives/frames. Curvature becomes a measure of semantic holonomy — an obstruction to globally consistent understanding. Belief alignment, Zipf-like scaling, and phase transitions in inference are not byproducts of matter dynamics — they are the very fabric of reality in this view.

Epistemic Pullback Conjecture. Traditional physical quantities (such as mass, charge, and energy) emerge as pullbacks of informational-geometric quantities (e.g. Fisher information, KL divergence) from an agent's belief and model fibers onto the latent base manifold. That is, physical law reflects the local structure of inference.

Trajectories within belief/model fibers encode dynamical physical phenomena. Humans agree on a consistent physics of the universe since, a priori, our generative models agree as the result of evolutionary flow towards our current shared model. The gauge invariance of physics therefore can be ascribed to the gauge invariance of our human-scale generative models: aligned models/beliefs correspond to shared gauge frames. This behaves as a sort of anthropic principle. Shared cognitive experience, and thus shared physics, presupposes aligned generative models. If humans had fundamentally divergent priors, they would not inhabit the same apparent world. The universality of physical law thus reflects — and requires — a kind of epistemic anthropic principle. This naturally accounts for pathological hallucinations but with a stronger claim that these hallucinations (psychedelics, trauma, etc) must themselves be invariant under the gauge group.

This manifesto outlines a formal theory grounded in differential geometry, information theory, cognitive science, and philosophy. It introduces a rigorous variational principle for agent interaction (as an extension of Friston's variational free energy), explores the emergence of large-scale structure via inference, and proposes a set of central conjectures that reframe the laws of physics as consequences of epistemic dynamics.

If successful, this cognitive-geometric approach could offer a unifying principle for physics, complexity, and cognition — one that treats inference not as a tool of physics, but as its very source. In this view, the universe is not a machine to be decoded, but a conversation to be understood — a dialogue between agents, inference, and the hidden structure of the noumenon.

2 Epistemic Geometry and Agent Dynamics

We now introduce the formal model underlying our cognitive-first physics framework.

We model each cognitive agent as a local pair of sections over a principal fiber bundle whose base space $\mathcal{C}$ represents a latent, unobserved "noumenal" manifold — the epistemic base. Beliefs and models are encoded in associated bundles constructed from a structure group G acting on recognition and generative fibers. These structures allow us to formalize both intra-agent inference and inter-agent communication using gauge-theoretic transport.

Let $\pi : \mathcal{N} \to \mathcal{C}$ be a smooth principal G-bundle where $\mathcal{C}$ is a smooth manifold, G is a Lie group acting freely and transitively on the right on $\mathcal{N}$. The projection satisfies $\pi(n \cdot g) = \pi(n)$ for all $g \in G, n \in \mathcal{N}$.

Let $\rho : G \to \mathrm{Aut}(\mathcal{B}_q)$ be a representation of the Lie group G on a smooth manifold $\mathcal{B}_q$, referred to as the recognition or belief fiber. Depending on context, $\mathcal{B}_q$ may be modeled as a K-dimensional probability simplex Δ^K (e.g., for categorical distributions) or as a statistical manifold equipped with a suitable information geometry. Importantly, the relevant geometric structures — such as divergence measures, metrics, and connections — remain well-defined on these fibers even when they lack linear structure (see e.g. Amari's information geometry for examples of dually flat but non-linear statistical manifolds).

Then the associated recognition bundle is defined as:

$$\mathcal{E}_q := \mathcal{N} \times_\rho \mathcal{B}_q = (\mathcal{N} \times \mathcal{B}_q)/\sim,$$

where

$$(n \cdot g, v) \sim (n, \rho(g)v).$$

This gives a fiber bundle $\pi_{\mathcal{E}_q} : \mathcal{E}_q \to \mathcal{C}$ with fiber $\mathcal{B}_q$.

We define a second associated bundle $\mathcal{E}_p$ in the same manner called the model bundle with fiber $\mathcal{B}_p$. $\mathcal{B}_q$ and $\mathcal{B}_p$ are manifolds of probability distributions or model parameters.

Definition: An agent is a pair of local sections $\mathcal{A}^i = (\sigma_q^i(c), \sigma_p^i(c))$ over $\mathcal{C}$

$$\sigma_q^i : \mathcal{U}_i \subset \mathcal{C} \to \mathcal{B}_q,$$

$$\sigma_p^i : \mathcal{U}_i \subset \mathcal{C} \to \mathcal{B}_p.$$

Definition: A multi-agent ($\mathcal{M}$) over $\mathcal{C}$ is a tuple of agents (where $\mathcal{I}$ is an index set)

$$\mathcal{M} = \{A^i = (\sigma_q^i(c), \sigma_p^i(c))\}_{i \in \mathcal{I}}.$$

Definition: A meta-agent is a multi-agent whose agents share the same generative models.

Next, an observation/observable is a local section of $\mathcal{B}_q$

$$\mathcal{O}_q : \mathcal{U} \subset \mathcal{C} \to \mathcal{B}_q.$$

This is similar to an agent but does not necessarily require a section over $\mathcal{B}_p$ although we shall see later that observations can (and must) be described as agents themselves.

Next, in the standard way[^34], via horizontal lifting from $\mathcal{N}$ to $\mathcal{E}_i$ we have a variety of morphisms and induced connections across scales (i, j):

1. $\Omega^i : \Gamma(\mathcal{B}_q) \to \Gamma(\mathcal{B}_q)$
2. $\tilde{\Omega}^i : \Gamma(\mathcal{B}_p) \to \Gamma(\mathcal{B}_p)$
3. $\Lambda_j^i : \Gamma(\mathcal{B}_q) \to \Gamma(\mathcal{B}_q)$
4. $\tilde{\Lambda}_j^i : \Gamma(\mathcal{B}_p) \to \Gamma(\mathcal{B}_p)$
5. $\Theta_j^i : \Gamma(\mathcal{B}_q) \to \Gamma(\mathcal{B}_p)$
6. $\tilde{\Theta}_j^i : \Gamma(\mathcal{B}_p) \to \Gamma(\mathcal{B}_q)$
7. $\Phi : \mathcal{E}_p \to \mathcal{E}_q$

8. $\tilde{\Phi} : \mathcal{E}_q \to \mathcal{E}_p$

where $\Gamma(\mathcal{B}_i)$ denotes the space of smooth sections of $\mathcal{B}_i$ over the relevant open subset of the noumenal manifold $\mathcal{C}$

The two principal bundle morphisms of special interest are $\Phi : \mathcal{E}_p \to \mathcal{E}_q$ and $\tilde{\Phi} : \mathcal{E}_q \to \mathcal{E}_p$. When these diagrams commute (i.e., $\pi_q \circ \Phi = \pi_p \circ \tilde{\Phi}$), we interpret this as a condition for epistemic agreement — an experimentally verified prediction, where an agent's recognition aligns with its generative model across fibers.

In general, however, these bundle morphisms need not commute. In the simulations and examples to follow we will consider these bundle morphisms to be the identity.

2.1 Generalized Variational Energy

Following Friston[ˆ1] we define a variational free energy as

$$\mathcal{F}[q] = D_{KL}[\ q(c) \mid \Phi p(c)\ ] - \mathbb{E}_{q(c)}[\log p(o|c)].$$

In our general framework, agent beliefs and models occupy distinct associated bundles, and the comparison between them is only possible via Φ, $\tilde{\Phi}$ morphisms. This leads to a more flexible and scalable framework, particularly amenable to multi-agent and multi-scale settings with gauge structure.

We extend this principle to agents who share information/beliefs/models via parallel transport. We define a generalized variational energy for an agent A as:

$$\mathscr{V}_A = D_{KL}[\ q_A(c) \mid\mid \Phi p_A(c)\ ] - \mathbb{E}_{q_A}[\log p_A(o|c)] + \sum_i \mathscr{V}_{\Lambda_i},$$

where $\mathscr{V}_{\Lambda_i}$ represents possible interactions between other agents mediated by gauge connections.

These connections lead to terms in the generalized variational energy representing possible agent/agent and agent/multi-agent interactions. In our current considerations we will only focus on the induced connections Ω and $\tilde{\Omega}$ such that agents can compare beliefs and models between different sections of $\mathcal{B}_q$ and $\mathcal{B}_p$ where

$$\Omega : \Gamma(\mathcal{B}_q) \to \Gamma(\mathcal{B}_q).$$

$$\tilde{\Omega} : \Gamma(\mathcal{B}_p) \to \Gamma(\mathcal{B}_p)$$

The induced connection provides a parallel transport of beliefs/models which in turn defines the coordination structure used in our generalized variational energy term. This construction is well defined even when the fibers are not vector spaces so long as they carry a smooth G-action[^34].

In the present work we shall consider the following terms of the generalized variational energy coupling the recognitions and models between multiple agents as

$$\mathscr{V}_T[q_i(c)] = \alpha D_{\text{KL}}[(q_i(c)|\Phi p_i(c))]$$

$$+\beta \sum_j D_{\text{KL}}[q_i(c)|\Omega_{ij} q_j(c)]$$

$$+\tilde{\beta} \sum_j D_{\text{KL}}[p_i(c)|\tilde{\Omega}_{ij} p_j(c)]$$

$$+\tilde{\alpha} D_{\text{KL}}[(p_i(c)|\tilde{\Phi} q_i(c))]$$

where $\alpha, \beta, \tilde{\alpha}, \tilde{\beta} \in \mathbb{R}$ represents general couplings/parameters. We note in passing that this expression bears remarkable similarities to the Grand potential $\Upsilon = U - TS + \mu N$ in standard thermodynamics.

2.2 Gauge Transport via Lie Algebra Frames

The inter-agent transport operators Ω_{ij} and $\tilde{\Omega}_{ij}$ serve to compare beliefs and models, respectively, between agents i and j. These operators are defined pointwise over the base manifold $\mathcal{C}$, and are constructed from local gauge frames associated with each agent.

Let $\phi_i(c) \in \mathfrak{g}$ be a smooth field over $\mathcal{C}$ valued in the Lie algebra $\mathfrak{g} = \text{Lie}(G)$, representing the gauge frame of agent i. Then, for each pair of agents (i, j), we define the induced transport operator as:

$$\Omega_{ij}(c) := \exp(\phi_i(c)) \cdot \exp(-\phi_j(c)),$$

where the exponential map $\exp : \mathfrak{g} \to G$ maps Lie algebra elements to the Lie group G, and the product is taken in the group.

This construction defines a group-valued map $\Omega_{ij}(c) \in G$, which acts on the belief fiber $\mathcal{B}_q$ via the representation ρ. That is, for any section $q_j(c) \in \Gamma(\mathcal{B}_q)$, the transported belief is:

$$\Omega_{ij}(c) \cdot q_j(c) := \rho\left(\Omega_{ij}(c)\right) \cdot q_j(c).$$

Similarly, for the model fibers, we define:

$$\tilde{\Omega}_{ij}(c) := \exp\left(\tilde{\phi}_i(c)\right) \cdot \exp\left(-\tilde{\phi}_j(c)\right),$$

with $\tilde{\phi}_i(c) \in \tilde{\mathfrak{g}} = \mathrm{Lie}(\tilde{G})$, acting on $\mathcal{B}_p$ via the corresponding representation $\tilde{\rho}$.

These induced transport operators implement a form of gauge-covariant parallel transport between agents. The field $\phi_i(c)$ serves as the agent's gauge frame, and differences in these frames determine the epistemic misalignment — or semantic holonomy — between agents.

By construction, $\Omega_{ii} = \mathrm{id}$, and in general $\Omega_{ij} \neq \Omega_{ji}^{-1}$ unless the gauge fields are flat. This asymmetry encodes epistemic curvature, which we will explore in subsequent sections.

In our framework, each agent maintains a fixed gauge frame $\phi_i(c)$, which defines their local epistemic perspective. This frame serves as the reference against which other agents' beliefs are compared. The term $D_{\mathrm{KL}}[\,q_i(c) \,\|\, \Omega_{ij}(c) \cdot q_j(c)\,]$ then quantifies the epistemic misalignment between agent i and agent j, measured in agent i's own frame. Operationally, this can be interpreted as a communicative act: agent i attempts to transform agent j's beliefs into its own representational basis and evaluate their coherence.

Intuitively, the gauge frame $\phi_i(c)$ can be interpreted as a geometric encoding of the agent's cognitive identity — its internally consistent perspective on the latent manifold $\mathcal{C}$. In this sense, the gauge frame acts as a formal proxy for an agent's selfhood or consciousness, organizing all incoming beliefs and observations relative to a fixed internal structure.

This interpretation opens the door to a rich epistemic ontology: communication becomes a gauge-theoretic alignment of perspectives, and disagreement (or confusion) corresponds to curvature in the semantic field. Agents with similar gauge frames can align beliefs easily, while those with strongly divergent frames experience epistemic dissonance.

Additionally, for future exploration, human language itself may be considered as a gauge theory. Each speaker maintains a local linguistic frame —

a grammar, lexicon, and semantic structure — and successful communication requires parallel transport of meaning across these frames. Misunderstandings arise when the linguistic gauge transformations between individuals are ill-defined or insufficient to fully align perspectives. In this view, syntax encodes local structural constraints, semantics provides the fiber content, and discourse becomes a path-dependent operation on shared meaning spaces. The geometric framework developed here may thus offer a natural foundation for modeling language as a structured system of epistemic transport.

To further analyze the geometric structure of inter-agent alignment, we define the gauge field associated with an agent's gauge frame $\phi(c) \in \mathfrak{g}$, where $\mathfrak{g} = \mathrm{Lie}(G)$ is the Lie algebra of the structure group G. In analogy with classical gauge theory, we define the gauge potential (or connection form) as the derivative of the gauge frame:

$$A_\mu(c) := \partial_\mu \phi(c) \in \mathfrak{g},$$

where μ indexes coordinates on the base manifold $\mathcal{C}$, and A_μ is a $\mathfrak{g}$-valued one-form over $\mathcal{C}$. This connection governs infinitesimal parallel transport within an agent's belief fiber.

We then define the curvature — or field strength — of this connection as the standard nonabelian gauge curvature tensor:

$$F_{\mu\nu}(c) := \partial_\mu A_\nu(c) - \partial_\nu A_\mu(c) + [A_\mu(c), A_\nu(c)] \in \mathfrak{g}.$$

The curvature $F_{\mu\nu}$ measures the failure of the gauge field to be locally flat — that is, the extent to which infinitesimal transport around an infinitesimal loop in $\mathcal{C}$ depends on the path taken. In the context of epistemic geometry, nonzero curvature corresponds to **epistemic inconsistency** or **semantic holonomy**: agents attempting to align beliefs through sequences of communication may find that their beliefs do not return to their original form after a closed interaction loop.

This gives rise to a concrete interpretation of gauge curvature as a measure of **cognitive or semantic torsion** — the topological obstruction to global consensus. As such, the norm $\|F_{\mu\nu}(c)\|^2$ might be added to the variational energy to regularize inference processes and penalize incoherent belief cycles across agents.

2.3 Transport, Holonomy, and Path Dependence

In our framework, gauge transport occurs in two distinct forms: intra-agent and inter-agent transport. Though both rely on the structure of the underlying principal bundle and its associated gauge fields, they serve different roles and operate over different domains.

2.3.1 Intra-agent transport.

Each agent A_i defines two local sections over a subset $\mathcal{U}_i \subset \mathcal{C}$: one for beliefs $\sigma_q^i : \mathcal{U}_i \to \mathcal{B}_q$, and one for models $\sigma_p^i : \mathcal{U}_i \to \mathcal{B}_p$.

To compare beliefs or models at two different points $c_1, c_2 \in \mathcal{U}_i$, the agent uses its own local gauge frame $\phi_i(c)$ to define **intra-agent parallel transport**.

This transport is mediated by a spatial gauge field $A_\mu^i = \partial_\mu \phi_i(c) \in \mathfrak{g}$, and the transport of a belief from point c_2 to c_1 within agent i's region is given by a path-ordered exponential:

$$T^{(i)}_{c_2 \to c_1} := \mathcal{P} \exp \left(- \int_{c_2}^{c_1} A_\mu^i \, dc^\mu \right).$$

This defines a gauge-covariant comparison of beliefs within the same agent's field of view, allowing belief gradients, curvature, and local coherence to be assessed.

2.3.2 Inter-agent transport.

By contrast, comparing beliefs between different agents i and j requires **inter-agent gauge transport** — that is, mapping beliefs expressed in agent j's gauge frame into agent i's frame. This is necessary because agents, in general, do not share the same frame $\phi(c)$, even at overlapping points $c \in \mathcal{U}_i \cap \mathcal{U}_j$.

We define the inter-agent transport operator $\Omega_{ij}(c) \in G$ by:

$$\Omega_{ij}(c) := \exp\left(\phi_i(c)\right) \cdot \exp\left(-\phi_j(c)\right),$$

which transports a belief $q_j(c) \in \mathcal{B}_q$ into agent i's frame:

$$q_j^{(i)}(c) := \rho\left(\Omega_{ij}(c)\right) \cdot q_j(c).$$

This transformation is used in the inter-agent alignment term of the variational energy:

$$D_{\mathrm{KL}}\left[\, q_i(c) \,\|\, \Omega_{ij}(c) \cdot q_j(c) \,\right],$$

and encodes the degree to which agent i finds agent j's beliefs compatible with its own gauge frame.

2.3.3 Combined transport.

In more complex scenarios, beliefs may be transported across multiple agents and spatial positions. For instance, a belief at $c_j \in \mathcal{U}_j$ may be transported to $c_i \in \mathcal{U}_i$ via intra-agent transport in j, inter-agent transport via Ω_{ij}, and intra-agent transport in i. Such composite operations yield generalized holonomies and allow semantic curvature to accumulate.

Type	**Domain**	**Operator**
Intra-agent	$c_1, c_2 \in \mathcal{U}_i$	$T^{(i)}_{c_2 \to c_1} = \mathcal{P} \exp\left(-\int A^i_\mu dc^\mu\right)$
Inter-agent	$c \in \mathcal{U}_i \cap \mathcal{U}_j$	$\Omega_{ij}(c) = \exp(\phi_i(c)) \cdot \exp(-\phi_j(c))$

2.3.4 Holonomy in Epistemic Transport

In our epistemic gauge framework, holonomy captures the failure of global semantic consistency under composite belief transport. When a belief is parallel transported along a closed loop — involving both intra-agent motion over space and inter-agent communication across gauge frames — it may fail to return to its original form. This failure defines a nontrivial holonomy, analogous to path dependence in meaning or inference.

Let γ be a closed loop in the extended space of agent identity and base manifold position:

$$\gamma = [(i_1, c_1) \to (i_2, c_2) \to \cdots \to (i_n, c_n) \to (i_1, c_1)],$$

where each pair (i_k, c_k) represents a position $c_k \in \mathcal{U}_{i_k} \subset \mathcal{C}$ within agent i_k's support.

We define the composite transport operator $\mathcal{P}_\gamma$ along the loop as an ordered sequence of:

- **Intra-agent spatial transports** $T^{(i_k)}_{c_{k+1} \to c_k}$ when remaining within the same agent, and
- **Inter-agent frame transports** $\Omega_{i_k i_{k+1}}(c_k)$ when transitioning between agents at shared positions.

The full holonomy operator is thus given by:

$$\mathcal{P}_\gamma := T^{(i_n)}_{c_1 \to c_n} \cdot \Omega_{i_n i_{n-1}}(c_n) \cdot T^{(i_{n-1})}_{c_n \to c_{n-1}} \cdots \Omega_{i_2 i_1}(c_2) \cdot T^{(i_1)}_{c_2 \to c_1}.$$

Given a belief $q \in \mathcal{B}_q$ at the starting point (i_1, c_1), the holonomy is defined as:

$$\mathrm{Hol}_\gamma(q) := \mathcal{P}_\gamma \cdot q.$$

If $\mathrm{Hol}_\gamma(q) \neq q$, the loop is said to have nontrivial holonomy — signaling the presence of **epistemic curvature** in the connection field. This measures the cumulative distortion of belief through a closed sequence of communication and spatial inference. The deviation norm $\|q - \mathrm{Hol}_\gamma(q)\|$ quantifies the degree of semantic dissonance accumulated around the loop.

Holonomy plays a central role in modeling **topological epistemology**: persistent holonomies reflect structural belief misalignments that cannot be resolved by local adjustments alone, analogous to geometric frustration, cognitive bias, or cultural divergence in agent societies.

3 Epistemic Dynamics and Quasistatic Inference

We now define the dynamical evolution of agents within our epistemic gauge framework. Agents are not static observers but active inference systems, continuously updating their beliefs q, models p, and gauge frames ϕ to minimize a generalized variational energy $\mathscr{V}$. These dynamics are inherently local, non-equilibrium, and generally slow — reflecting the constrained, embodied nature of cognitive systems. We refer to this regime as *quasistatic inference*.

The evolution of each field is governed by a variational gradient descent flow, modulated by learning rates and regularization terms. We assume that gauge frames evolve more slowly than beliefs, and that models may be fixed or partially plastic, depending on the context.

3.1 Gradient Dynamics of Beliefs and Models

Beliefs evolve by descending the variational energy gradient with respect to q_i, the agent's recognition field. The basic update rule is:

$$\frac{dq_i(c)}{dt} = -\eta_q \, \frac{\delta \mathscr{V}_i}{\delta q_i(c)}.$$

Here, η_q is the belief update rate (or inference speed), and $\mathscr{V}_i$ is the total variational energy for agent i. This includes both the self-alignment term $D_{\mathrm{KL}}[q_i \| \Phi p_i]$ and the inter-agent alignment term $D_{\mathrm{KL}}[q_i \| \Omega_{ij} q_j]$, as described earlier.

If model plasticity is permitted, the model field $p_i(c)$ also evolves according to:

$$\frac{dp_i(c)}{dt} = -\eta_p \, \frac{\delta \mathscr{V}_i}{\delta p_i(c)},$$

where η_p controls the timescale of model adaptation. In many cases, p_i is held fixed, representing innate priors or evolutionarily hardwired structure.

3.2 Gauge Frame Evolution

Each agent's gauge frame $\phi_i(c) \in \mathfrak{g}$ evolves according to a damped, curvature-regularized gradient flow. This flow is derived from the variational energy $\mathscr{V}_i$, with additional terms penalizing excessive gauge complexity and favoring smoothness:

$$\frac{d\phi_i(c)}{dt} = -\eta_\phi \left(\frac{\delta \mathscr{V}_i}{\delta \phi_i(c)} + \gamma_\phi \, \phi_i(c) - \lambda_\phi \, \Delta \phi_i(c) \right).$$

- η_ϕ: gauge update rate
- γ_ϕ: damping coefficient (a simplicity prior/mass)
- λ_ϕ: curvature regularization (Laplacian term)

This update rule ensures that gauge frames evolve slowly, resist large excursions, and maintain coherence across $\mathcal{C}$. The Laplacian $\Delta\phi_i$ acts as a spatial smoothing term, penalizing rough or disjointed gauge configurations.

3.3 Timescales and Non-Equilibrium Regimes

Our framework reflects the inherently non-equilibrium nature of cognition. The components q_i, ϕ_i, and p_i evolve on distinct timescales, reflecting their respective epistemic roles:

- **Beliefs** $q_i(c)$ evolve rapidly in response to incoming observations and interactions. These encode moment-to-moment inferences about the agent's environment and neighbors.
- **Gauge frames** $\phi_i(c)$ evolve more slowly, capturing the agent's representational stance or semantic orientation. These mediate communication and may adjust via repeated interactions or accumulated dissonance.
- **Models** $p_i(c)$ evolve on the slowest timescale, if at all. These encode the agent's internal generative assumptions — formed developmentally, evolutionarily, or culturally — and represent its most stable epistemic structure.

This timescale separation defines a *quasistatic regime*: beliefs may equilibrate locally given the current ϕ_i and p_i, but the system as a whole remains dynamically evolving, particularly in the presence of epistemic curvature or topological misalignment. Utilizing our regularization terms outlined above our variational energy for agent i becomes

$$\begin{aligned}
\mathscr{V}_i = {} & \alpha\, D_{\mathrm{KL}}\left[q_i(c)\,\|\,\Phi p_i(c)\right] \\
& + \beta \sum_j D_{\mathrm{KL}}\left[q_i(c)\,\|\,\Omega_{ij}(c)\cdot q_j(c)\right] \\
& + \tilde{\beta} \sum_j D_{\mathrm{KL}}\left[p_i(c)\,\|\,\tilde{\Omega}_{ij}(c)\cdot p_j(c)\right] \\
& + \tilde{\alpha}\, D_{\mathrm{KL}}\left[p_i(c)\,\|\,\tilde{\Phi} q_i(c)\right] \\
& + \frac{\gamma_\phi}{2}\left\|\phi_i(c)\right\|^2 + \frac{\lambda_\phi}{2}\left\|\nabla^2\phi_i(c)\right\|^2 \\
& + \frac{\tilde{\gamma}_\phi}{2}\left\|\tilde{\phi}_i(c)\right\|^2 + \frac{\tilde{\lambda}_\phi}{2}\left\|\nabla^2\tilde{\phi}_i(c)\right\|^2
\end{aligned} \tag{1}$$

where we write:

$$\mathscr{R}_\phi = \frac{\gamma_\phi}{2} \left\|\phi_i(c)\right\|^2 + \frac{\lambda_\phi}{2} \left\|\nabla^2 \phi_i(c)\right\|^2, \tag{2}$$

$$\mathscr{R}_{\tilde{\phi}} = \frac{\tilde{\gamma}_\phi}{2} \left\|\tilde{\phi}_i(c)\right\|^2 + \frac{\tilde{\lambda}_\phi}{2} \left\|\nabla^2 \tilde{\phi}_i(c)\right\|^2. \tag{3}$$

and

$$\begin{aligned}
\mathscr{V}_i = {} & \alpha\, D_{\mathrm{KL}} \left[q_i(c) \,\|\, \Phi p_i(c)\right] \\
& + \beta \sum_j D_{\mathrm{KL}} \left[q_i(c) \,\|\, \Omega_{ij}(c) \cdot q_j(c)\right] \\
& + \tilde{\beta} \sum_j D_{\mathrm{KL}} \left[p_i(c) \,\|\, \tilde{\Omega}_{ij}(c) \cdot p_j(c)\right] \\
& + \tilde{\alpha}\, D_{\mathrm{KL}} \left[p_i(c) \,\|\, \tilde{\Phi} q_i(c)\right] \\
& + \mathscr{R}_\phi + \mathscr{R}_{\tilde{\phi}}
\end{aligned} \tag{4}$$

4 Euler–Lagrange Equations for Epistemic Fields

The Euler-Lagrange equations governing the two primary fields are given by

$$\frac{dq_i(c)}{dt} = -\eta_q \cdot \left[\alpha \left(\log \frac{q_i(c)}{\Phi p_i(c)} + 1 \right) + \beta \sum_j \left(\log \frac{q_i(c)}{\Omega_{ij}(c) \cdot q_j(c)} + 1 \right) - \tilde{\alpha}\, \frac{p_i(c)}{q_i(c)} \right]. \tag{5}$$

and

$$\frac{d\phi_i(c)}{dt} = -\eta_\phi \cdot \left[\beta \sum_j \nabla_{\phi_i} D_{\mathrm{KL}} \left(q_i(c) \,\|\, \Omega_{ij}(c) \cdot q_j(c)\right) + \gamma_\phi \cdot \phi_i(c) - \lambda_\phi \cdot \Delta \phi_i(c) \right]. \tag{6}$$

Such that since:

$$\Omega_{ij} = \exp(\phi_i) \cdot \exp(-\phi_j), \quad \Rightarrow \frac{\partial \Omega_{ij}}{\partial \phi_i} = \Omega_{ij} \cdot \mathrm{Ad}_{\exp(-\phi_j)}.$$

Thus,

$$\frac{\partial(\Omega_{ij} \cdot q_j)}{\partial \phi_i} = \Omega_{ij} \cdot (\log q_j) \cdot \mathrm{Ad}_{\exp(-\phi_j)}.$$

Therefore,

$$\nabla_{\phi_i} D_{\mathrm{KL}}[q_i \,\|\, \Omega_{ij} \cdot q_j] = (\log q_i - \log(\Omega_{ij} \cdot q_j))^T \cdot \frac{\partial(\Omega_{ij} \cdot q_j)}{\partial \phi_i}.$$

and similarly for the model fibers.

Importantly, under a variational update $\delta \mathscr{V} = 0$, the belief/model field generally evolves as $q \to q + \Delta q$. The local change in informational content over a single step may be characterized by the self-divergence

$$\Delta \mathcal{I} = D_{\mathrm{KL}}[\, q \,\|\, q + \Delta q \,].$$

In gauge-theoretic terms, we interpret this variation as a local transformation of the form $q \to dg^{-1} \cdot q$, where dg^{-1} is a gauge transformation associated with the Lie group G, acting on the fiber $\mathcal{B}_q$. Thus, we write:

$$\Delta \mathcal{I} = D_{\mathrm{KL}}[\, q \,\|\, dg^{-1} \cdot q \,].$$

This quantity measures the epistemic deviation induced by a local frame change and highlights the informational cost (or tension) of re-expressing a belief in a shifted gauge frame. In the limit $\Delta q \to 0$, this yields a natural information geometry on the belief manifold.

4.1 Agent Pullbacks to $\mathcal{C}$

Agents do not perceive the base manifold $\mathcal{C}$ directly, but rather infer its structure through belief/model fibers $\mathcal{B}_i$ as local sections of the associated bundle $\mathcal{E}_i \to \mathcal{C}$. This is consistent with modern neuroscience, which models perception as inference over latent causes of sensory data[^8]. Accordingly, we define an agent's perceived geometry of $\mathcal{C}$ via the pullback of geometric quantities defined on $\mathcal{B}_i$.

As a concrete example, consider the Fisher information metric defined on the model fiber as

$$\mathscr{F}_{ij}(c) = \mathbb{E}_{p(c)} \left[\frac{\partial \log p(c)}{\partial \theta^i} \frac{\partial \log p(c)}{\partial \theta^j} \right], \tag{7}$$

where θ^i are natural parameters charting the model manifold. Pulling this structure back along the agent's section $\sigma : \mathcal{C} \to \mathcal{E}_p$, we obtain a local induced metric on $\mathcal{C}$:

$$\mathscr{G}_{ij}(c) = \sigma^*(\mathscr{F}_{ij}(c)) = \frac{\partial^2}{\partial \delta c^i \partial \delta c^j} D_{\mathrm{KL}}\left[p(c)||p(c+\delta c)\right]. \tag{8}$$

This metric $\mathscr{G}_{ij}(c)$ governs the agent's perceived geometry of $\mathcal{C}$. Since models/beliefs $p_i(c), q_i(c)$ generally vary between agents, their pullback geometries generally differ unless their fields are locally aligned. In this view, we conject that space-time geometry is not fundamental but emerges through inference dynamics.

Consistent with a cognitive-evolutionary perspective, we propose that the standard Euclidean spacetime geometry (which human scale agents perceive) emerges as an evolutionary attractor in the belief dynamics of human-scale agents. That is, the perceived geometry converges towards a structure compatible with efficient inference and shared coordination. Since the belief/model and base manifolds may generally be high dimensional, we, therefore, assume a decomposition of the pullback metric as:

$$\mathscr{G}_{ij} = g_{\mu\nu} \oplus \mathcal{G}_{\alpha\beta}, \tag{9}$$

where $g_{\mu\nu}$ corresponds to an emergent 3- or 4-dimensional subspace — the agent's perceived physical space — and $\mathcal{G}_{\alpha\beta}$ corresponds to residual or "dark" components not directly accessible via conscious inference. However, in this view, since $\mathscr{F}_{ij}$ is dynamic under belief/model/gauge then dynamics on this pullback can, in principle, "trickle" into an agent's perceived $g_{\mu\nu}$ as perturbations. This is surprisingly consistent with dark matter/energy albeit very loosely connected at this point given that we have not defined what exactly the noumenal manifold is to represent aside from general latent states.

Furthermore, this decomposition is justified by symmetry. If $\mathscr{F}_{ij}(c)$ is invariant under a Lie group G, then any subgroup $H \subset G$ defines an invariant subspace of $\mathcal{B}_i$. Under the adjoint action of H, the Lie algebra splits as:

$$\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{h}^\perp, \tag{10}$$

which implies a block decomposition of the pulled-back metric.

For instance, taking $G = SO(10)$ and $H = SO(3)$, with $\dim \mathcal{B}_p = 10$, we obtain:

$$\mathscr{G}_{ij} = \delta_{\mu\nu} \oplus \mathcal{G}_{\alpha\beta}, \tag{11}$$

where $\delta_{\mu\nu}$ gives the perceived Euclidean metric and $\mathcal{G}_{\alpha\beta}$ spans a hidden 7-dimensional subspace. This "dark geometry" corresponds to degrees of freedom not directly modeled by the agent, though they may still influence belief evolution via curvature or gauge interactions.

Importantly, this framework allows for perceptual deviations from the Euclidean norm — such as those reported under psychedelics, neurological trauma, or extreme states of consciousness[^4]. In our view such altered states correspond to perturbations of the Fisher metric and hence to distortions in the pullback geometry. These subjective distortions are not anomalous under our model — they are expected outcomes of variations in the agent's belief/model field.

More generally, any geometric quantity defined over the fibers may be pulled back to $\mathcal{C}$. For instance, we conject that Planck's constant $\hbar$ emerges as the pullback of the minimum discrete update to the agent's variational energy:

$$\sigma^* \left(\Delta \mathscr{V}_0\right) = \hbar, \tag{12}$$

assuming $\Delta q = 1$ bit. This defines $\hbar$ not as a fundamental physical constant, but as an epistemic threshold: the minimal change in belief/model that produces an irreducible unit of informational action.

In this light, the agent's experience of space, time, energy, and quantization might arise from informational constraints on inference, and their associated pullbacks from fiberwise to basewise geometry. This epistemic geometry offers a path toward unifying observer and observed within a single coherent variational framework.

In general then informational geometric objects pullback to their corresponding agent-specific objects on $\mathcal{C}$. A pair of agents then may pullback precisely the same geometric object from the fibers in manifestly different ways. Furthermore, we can extend this to dynamics and conject that a set of agents with aligned models defined over subset of the base manifold pullback gauge invariant trajectories: gauge invariance in physics as a direct consequence of agent model alignment.

Let us consider two agents A_i and A_j with respective model sections σ_i^p, σ_j^p and gauge frames $\tilde{\phi}_i(c)$, $\tilde{\phi}_j(c)$. Their models are said to be *model aligned* at point $c \in \mathcal{C}$ if there exists a gauge transformation $\tilde{\Omega}_{ij}(c)$ such that

$$p_i(c) \approx \tilde{\Omega}_{ij}(c) \cdot p_j(c), \tag{13}$$

and their models agree under the corresponding $\tilde{\Omega}_{ij}(c)$.

Now consider the induced metric pullbacks from their fibers:

$$\mathscr{G}^{(i)}_{ab}(c) = \sigma_i^* \mathscr{F}^{(i)}_{ab}(c), \qquad \mathscr{G}^{(j)}_{ab}(c) = \sigma_j^* \mathscr{F}^{(j)}_{ab}(c). \tag{14}$$

If p_i and p_j are aligned (up to $\tilde{\Omega}_{ij}$), and the information geometry is G-invariant, then

$$\mathscr{G}^{(i)}_{ab}(c) = \Lambda_a{}^{a'} \Lambda_b{}^{b'} \mathscr{G}^{(j)}_{a'b'}(c), \tag{15}$$

for some local transformation $\Lambda \in \mathrm{Aut}(T_c\mathcal{C})$ induced by the action of $\tilde{\Omega}_{ij}$ on tangent directions. Thus, model alignment implies coordinate invariance of the perceived geometry.

This reframes diffeomorphism invariance and gauge symmetry not as fundamental ontological principles, but as epistemic constraints: conditions for mutual intelligibility between agents.

Gauge Relativity Conjecture — All fundamental symmetries in physics (e.g., Lorentz, diffeomorphism, gauge) arise from the requirement that agent pullback geometries be consistent under epistemic gauge transport. Shared physical laws are not fundamental constraints on reality, but emergent regularities among agents whose beliefs are sufficiently aligned. In this view, a physical law is a maximally symmetric consensus geometry in the space of agent inferences.

While individual human agents do not directly perceive Lorentz symmetry, the collective process of scientific inference — a form of meta-agent epistemology — converges on a shared generative model that is Lorentz invariant. Thus, symmetries like SO(3,1) are not raw percepts but consensus structures in model space arising from epistemic alignment across agents under shared experimental contexts. We claim that a collection of epistemologically interacting agents can condense and align into a meta-agent whose beliefs and models can be manifestly different than that of their lower-agent components.

4.1.1 Meta-Agent Emergence.

We claim that a collection of epistemologically interacting agents can condense and align into a higher-order meta-agent, whose induced beliefs and models may differ qualitatively from those of its constituent agents. That is, alignment of local sections

$$\{\sigma_i^p, \sigma_i^q\}, \quad A_i \in M,$$

over a shared region of $\mathcal{C}$ enables the formation of an effective joint section Σ^p, Σ^q, representing the meta-agent $\mathcal{M}$.

Crucially, the geometric and epistemic structure of this emergent agent need not be reducible to that of any single constituent. In particular, the pullback geometry

$$\mathscr{G}_{\mu\nu}^{(\mathcal{M})}$$

induced by the meta-agent may display symmetries, dimensions, or invariants (e.g., Lorentz structure, conserved quantities) not manifest in any lower-level agent's model space.

In this way, scientific knowledge and physical law may be understood as the result of epistemic condensation: an alignment-driven emergence of invariant structures in model space through multi-agent coordination. The resulting meta-geometries reflect consensus inference across agents, not any one subjective perspective.

4.1.2 Meta-Agent Construction.

Let $\{A_i\}_{i\in I}$ be a collection of agents, each defined as local sections over the model and belief bundles:

$$A_i = \left(\sigma_i^p : U_i \to \mathcal{B}_p, \quad \sigma_i^q : U_i \to \mathcal{B}_q\right),$$

with gauge frames $\tilde{\phi}_i(c), \phi_i(c) \in \mathfrak{g}$.

We say these agents are epistemically coherent on overlaps if there exist gauge transformations $\tilde{\Omega}_{ij}, \Omega_{ij} \in G$ such that

$$\sigma_i^p(c) \approx \tilde{\Omega}_{ij}(c) \cdot \sigma_j^p(c), \quad \sigma_i^q(c) \approx \Omega_{ij}(c) \cdot \sigma_j^q(c), \quad \forall c \in U_i \cap U_j.$$

Given this coherence, we define a meta-agent $\mathcal{M}$ as a pair of global sections

$$\Sigma^p : U \to \mathcal{B}_p, \qquad \Sigma^q : U \to \mathcal{B}_q,$$

where $U = \bigcup_{i \in I} U_i$, and $\Sigma^p|_{U_i} \approx \sigma_i^p$, up to gauge. Similarly for Σ^q. This corresponds to a gluing of local fields into a coherent global epistemic state.

The meta-agent is then endowed with an effective gauge frame

$$\phi^{(\mathcal{M})}(c) := \frac{1}{|N(c)|} \sum_{i \in N(c)} \phi_i(c),$$

where $N(c)$ is the set of agents defined at point $c \in \mathcal{C}$. It also possesses a variational energy functional $V^{(\mathcal{M})}$, constructed from the joint fields $\Sigma^p, \Sigma^q, \phi^{(\mathcal{M})}$ and the same energy principle as in individual agents. In this manner new gauge connections $\Lambda^s_{s'}, \tilde{\Lambda}^s_{s'}$ emerge connecting agents and meta-agents across emergent scales s, s'. These emergent gauge connections between scales act as renormalization operators. These inter-scale transformations encode the relative embedding of local agent models within larger-scale collective structures, and govern the transport of beliefs and models across hierarchical layers of inference.

Furthermore, via the $q \longmapsto p$ and $p \longmapsto q$ bundle morphisms we can connect agent/meta-agent beliefs and modelsx via

$$\Theta^s_{s'} : \Gamma^s(\mathcal{B}_q) \to \Gamma^{s'}(\mathcal{B}_p)$$

$$\tilde{\Theta}^s_{s'} : \Gamma^s(\mathcal{B}_p) \to \Gamma^{s'}(\mathcal{B}_q)$$

This formalism shows how multi-agent epistemic alignment induces emergent geometric and dynamical structure. The resulting meta-agent may exhibit novel symmetries or lower effective curvature due to internal agreement — even if constituent agents are locally noisy or limited in scope.

5 Results