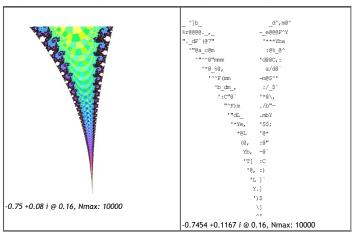
Robert P. Munafo, 2010 Sep 7.

Colloquial name for the cusps between $\frac{R2a}{2}$ and $\frac{R2 \cdot 1/2a}{2}$, particularly the upper one, $R2 \cdot C(1/2-)$. See $\frac{R2 \cdot C(1/2)}{2}$.

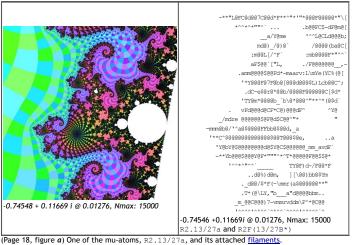


The upper Seahorse valley is shown in the picture. The right side, those parts attached to the continent seed, is part of the Shoulder, and the left side is part of the Head.

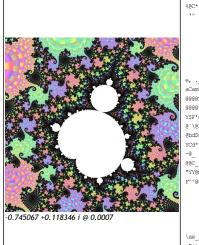
Seahorse Valley zoom sequence

The following zoom sequence is a permanent part of the Mandelbrot set culture. It goes back at least as far as the 1985 Scientific American article, and probably originates with Peitgen and Richter. (These images do not show the same mu-unit and filaments as the article, and the colors and aspect ratios of the frames differ as well.)

Seahorse Valley



The largest complete $\underline{\text{mu-atom}}$ in this image is R2.13/27a. There is one large $\underline{\text{peacock eye}}$, the point at its center is the $\underline{\text{branch point}}$ R2F (13/27B*). On the west side of the valley we find double spirals in place of the peacock eyes. The largest visible island is R2F (13/27B2) S and the branch ending in a spiral is R2F (13/27B2).



```
"@@"
                        d@Fbam ed%/@@*" -(" mec
       .m@@b@@e,, am@@-,__,"@@@Fbd@@Cb/@@@L@L,m_m@@@CL,_
        ^S"^@@*"b,@@@@@@S@"'`T@^^*@ ^b@S@@@F`'`"FY@F@CS@ba /
        CY%00\,^a00F000;0mm,_dem_(FC000*-0m,:bd0*C`dT0*F^^:Ch
     'Y00000bLa"F00b\ '^Tmd000C000F^*00b0$m,mmd)000F0F
        ^^"F)@^` '*"]e_aed%m@@F**^^`--@@d@d@[L@bL%@$,
    _(m@@,^^ '"*d` _ .m@b@F"Y@@F^
                              '*Y0000000' %0__,
aCam@Cdd@,__, :_@@v@bY%"@d%b@@@L
                               .d@@%$@@em md@@~
@@@@%*Y@d%@FYd@:dm Y*@@*Y*`^`` ^^ -.mm@@$@)d@b-F@@*C\^@@F**"e
Y$F'@mm"'@Fem@@@d@@@@F^
                              ^*@@@m@@`.,a*@@bm,
 )\@FC^_, .dC@@@@F`^Y^
                                '*Y@@$@'YC@@@"*
@bd$@@b7\ad@@FY@@bea
                                 "@Y@@@@F`^F,,
 /Cd*^@@` ^$@F"(@$@@F
                                  \_d@\F^TY@@,
        (b(@*@@@*
                                  -00) F^Y00%0F-AT00000
     'Y0000^^0000b_
                                  .0C0F0000*- '* 0F0C
"YY@m7YamdCL$@F`'b@@@me_
F^^@@F ^~\@@dYm=*@@"*(@@@.
                                .a .@@@$h"*@F@dd@h@@
       -**%; ._^@_@bF-@b-,
                             ,.ad@me_mm@@@@@@**/b@@@@[
        _db___@d@FYF@~*@@@Lam__--mmbd@@@TF@@F@@@@%@/bm^` *F"
      ,, :0$0F"~0"00, /Y0000C00mm00FYF"00b0*0bb0*Y0(0^
    -0.745067 +0.118346 i @ 0.0007, nmax: 20000
```

This is the largest visible island in the previous image; its R2-name is R2F(13/27B2) S. It has period 29 and is the 335th largest island in the whole Mandelbrot set.

```
:mmmrmr70000 $00F* )000bm, ee,
                                                                    . F@@@F\/CY@m$(a@b(@ @d^@eb@@C;,
                                                                @@bm@Yb/F/Y$md^^^"* '*^ ^@@'%_Y7Y@*)
                                                                %F0000 m0-*"^"
                                                                                              -GEAFYVGGGGG-
                                                          me.m@@__Y(^Td@@@
                                                                                                ]@m@$@F@,
                                                                                              "*Ybm@^$`
                                                          @@@@@dF^C*-"^
                                                                                   .adm@me,
                                                                               .mFF^.,,Tb __@mm@@@@@m-
-(@d[ """ ,:^T%"@Y@"^`
                                                          %*@@@dbm@@@e
                                                                                ./000_, . _ -YdFe/b:0"
""\b00e0m_000"ba0000bc
                                                          Yr--@r*^^
                                                          TrY000000,
                                                                                   '*"db@b*bd@@F/FF^""^
                                                           "Y@*@^^
                                                                                       00F"**Y0F
                                                          ` dd@c
                                                         asra*
                                                          ^@` .,
                                                         ad@@@@@Cbbb-
-0.744567 + 0.121201 i @ 0.002, Nmax: 1000
                                                         -0.744538 +0.121200 i @ 0.002, Nmax: 1000
                                                         Spiral in R2F (13/27B2)
```

(Page 18, figure b) This spiral feature is called a tail or Shepherd crook. The prominent spiral tip is R2F (13/27B2) {FS[0]}x~

```
100% e e
                                                          '000F**00Y0r
                                         @@@mbd@e,
                                                           a@@@m@._""
                                        T@d@@@@@@$@$@$$e*^ *" ]@@@@@@FC@m@ 'C@F^` ac
                                        ,,"00000000FC\d0.dm00- -d($00C0F%%C (CY00F)
                                       ^^^"*mmmd@b@^@F@@@@@@@L
                                                                 m(000YF
                                        9mm_, '^"^^*F'^` -C/@$@@
                                       900000Lc
                                       30000C)r
                                       ^**@b
-0.7445366 + 0.1217208 i @ 5e-4, Nmax: 1500
                                      -0.744 539 8 +0.121 723 8i @ 0.000 5, Nmax: 1500
```

Here we see several patterns resembling the center of a sunflower or a peacock-feather "eye".

```
0000cY7`'0%000c
                                                                         ./bFC000000000F$F\m00 \ ""*[
                                                   %000 "F0b0:Cv000.
                                                                        abybyy@@@@F@\*@;\$:mree
                                                         /0)000/00bcc$*"0b"L00000[
                                                             '*^Yd@@e
                                                                           '*\@"C$d@@%*"@m,e@FF*F@[
                                                               '^'@Y@e
                                                                             ^`^*"Yb-Yb-**@- '"*
                                                        '-YY$L ^^^ .ed@@"*@*Ta@@__:m_,.
                                                        $@d$ac. ... '^*"\m\@vm@@@*b/@F@YF@m@ .
                                                        ""-b@$/@reY@@F^%T@F*@)m\a@@d^"*^^*"@TYd@
                                                         "^^"@@b(L/@r_%^@*@F*""^` ^ _, 'Yd@mc
                                                            '**'"mY$@@C^\@"YC -Ybd^_ad@F^
                                                                     -%@@FYmaLme
                                                                                    "*Y**"^
                                                                       ^^`Ym@eC*m
                                                                        .000*d"00, _
'^`'Y0007$d0[ .
                                                    -Y$*^)d@...
                                                                           :0*"L00d0^\]00_, adr..,
                                                  -@b"FCe'/Y)/%**emm
                                                                               ^%@@@ (CL@@^%e"@@@@`
-0.74453892 + 0.12172418 i @ 1e-4, Nmax: 2500
                                                                                ^^`"*@@@@*-L`d\dc,
                                                   $mc@@;%beaF@:d@Fe.,,
                                                  -0.744 539 85 +0.121 723 77i @ 0.0001, Nmax: 2500
```

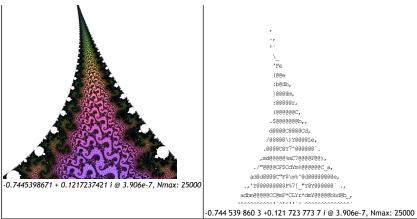
A "double hook" and island.

```
a\@$@TTm_ "***"` ](@@_ a_/@$@@c,a__.a_ee, :@$[
                                                       $00 ^*^""*0me,_ ., -m00dYm_"00)"*0"~000*F*^.~0 _00^
^ ___,^*0Fmm0mm"T0[:-'be_aF"/-**^YC` ^. d"`.,
                                                           :dF:d` ^TLY@*^"(% /@d@d@mL@@bmd^`/`m@_,@@" ([
]@e, , (@@@`:@^,@F@%@@@@@dbddd$F@_^**C@@- '`
                                                           -0F0:0ee d^:000~m0$00C^00Y0Y0000\\"\/*Y_a,
_00/(^..^"00*/\%/00000F"^ '*Y00LFd^'0000Y
                                                                                       1*Y000F/^!0000Y/[
                                                             ^@FY@c'mm_@%/mC$F@@@`
                                                                                         '@@b$"@r-,T@F"
                                                                                          -Y000F$ed) * `,,
                                                          /mm@b_%%L_00]bC0d000000
                                                        nd00FY0;.r~T0~d00000*^^^Y
                                                                                           ^@@@m*F`:@@C,
                                                        cv, ^^) v@(d@@@@@em_,__db,,
                                                                                          .:md@@Cm %Lm@@[
                                                        % ad@'""@*00000000000000000000m, L ,'-0000FCOF`mm,"
                                                        _,, '.$@F_%_,]@*"$Cm(aCL"FL**@"/@*@@GFF@/*"-vd@@@@C
                                                        100b, $00*Y0)]0-L"^^'YF^`Ym0%b%b(0C0$/T0 amm, '`^T[
                                                       -0.744539761 + 0.121724001 i @ 6.25e-6, Nmax: 10000
                                                       -0.744 539 859 +0.121 723 774 i @ 6.25e-6, Nmax: 10000
```

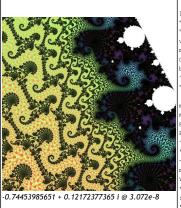
The original Scientific American sequence ended at this point, illustrating the existence of small decorated Mandelbrot sets within the details of the whole.

This island has $\underline{\text{period}}$ 43 and is 7 steps away from the period-29 island shown above (which is R2F (13/27B2) S).

At each step there is a very small island of period N+1 (located in the largest lobe off a peacock eye on the outboard side of the spiral), and a significantly larger island of period N+2 further towards the spiral tip on the inboard side. This is done 7 times starting at the period-29 island, and so the R2-name of this island is R2F(13/27B2) {FS[2]FS[0]}x7S.



The south half of the island's own seahorse valley. The name of this cusp is R2F(13/27B2) {FS[2]FS[0]}x7S.C(1/2)+

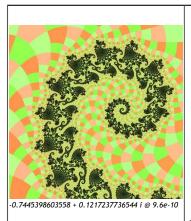


]00b ^**00-._ (000C%/m_dddYF\$0000000000000000 "0" ..,,'@e-(a*^,^'YFme'\$((@/\$@@@@@@@@@@@"^ ` "* -d@@@c'@m m@b.F^'^/@%b@@@@@@@@@@@@@ mmdF0"^ -YC000\adL"-**0:Y000000000000000mm_,_._da_ . /"m,^^` a , /F`'`@(%^FY*@@@@@@@@@@@@@@@@@@ aF@bC^"@@@b@`^ :@bvmFF:(^^d~:@(Y@@@@@@@@@@@@@@@ _ Y@@@F^ _TF aame, 'Ye, ^"^.._, '\$@*@).@\F@@@@@@@@@@@@@@@@ d^^._/00000 'Y0ramY0Y*.F_,aaL*v0000000000000000000F^^, /"0(00C\$; 'Y\$T00Fea0L'^^^*\0\$0000000000000000000 mmF^*"(dC,-r-_@F" ^ @00Y00c]**/00000000000000000000 @F` ___'*b,^^^`.__, 'Y0`,^.@dd^0YF00000000000000000 -0.744 539 860 35 +0.121 723 773 65 i @ 3.072e-8, Nmax: 200000

A couple of the island's own seahorses are on the right.

The filaments on the left are all part of R2F(13/27B2) but are *not* part of the island's mu-unit.

We will now zoom in to the filaments on the left, and find one of its "double-hook" features.

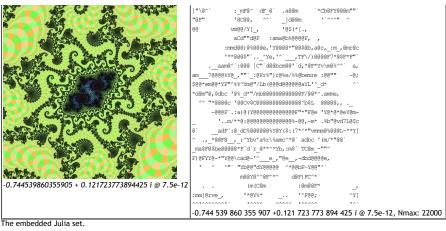


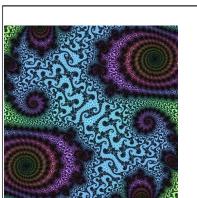
```
@F)
                        /0000F0")0000 .,
                $00md00L^0000"$/"YF00$F",_mmc
               .ad$Y@@\C~*med@@(*Lm@L:@md@@@F"
                                  ] @FadCaL
          ^@d@@@FT&&T.^^ ^`
           d^7Y@*T@@Y"
                                        -bm@@@F+
          ]@)m/bF-"`
                              _mdmb, "Y$ (*Y)
                           -d@^]m/F -m@m@b@m,
        mm@@@@em@@@e
        ^***Y@$""@F
                            -Y@L_ ' , _aed"@F^^
        .:0%_^dFC__,
T0%@bad00*^
                            '*00mvmdd)F000[
       .med@@@b@"*@c
       '^***YF*@.@"_mm_
-@emd^^a@@@^
          'T@b@@F@m**@-
           rT00005m.Cd:d00*
          /F00F(%`^)"*]0000mc .__c
            ' 'bm@*bm/*b^^%Yc:@@@C_., .mdb,
-0.744 539 860 355 8 +0.121 723 773 654 4 i @ 9.6e-10, Nmax: 2500
```



```
'^C@Y@@@@@@@@$("@e$Y"$$@@@b,
                    ^T$C00000F00C$(0(0"`"00FYY0CF
  ) @@@C
  _/0000_
"T0000*emm-
                     '^d@@$@@$d$\@%F"C@d%^` '^"
 C m@@,@7@@@m a
                        *-@^@C$@/)YC^^"^
    "@F'*YbYC@Ymac
         ^` "*"@@Cmm,
               ^^mF000L.
                 ]@@@%b@@_,.a___m_am_,me .
^-@r@YYF^\@@@@%*@$ (@$d@@b@L
-@@@`a-@"*"*@m@;""` ^ ^1*@@m-
       m@@*^^Ye
     _
1000, *c/"
                  ,_._a-vY@@@%@aF"^`
                                         m@/"` d@@
       \000(e,m F0FF/*"Y0YF^%mm,
       '^Y@b@d@d* @@@@*-L~`/@@C^
                                         ^**b@F*"
          ^^ ^"@"'*Y@@"Ld@@@%F^$.__
                    ""^ ^%@@@Fb@@@b...
                           '^^'*Y@@b$[@be._,
                               **^\@@@b@~@F\/@@mm_T@[
                                     "*"\Y@@b"Cd@@_bb~
 Y@@mme_d@L__.m,
                                         'Y*^"*@@@C^]F[
 .@F7F_^/F@*"]@@/ed@_,
                                               (@*`^**
 _)dF(@mF*~@b~"@m_C@^*@@e,__
-0.744 539 860 355 90 +0.121 723 773 894 42 i @ 6e-11, Nmax: 7000
```

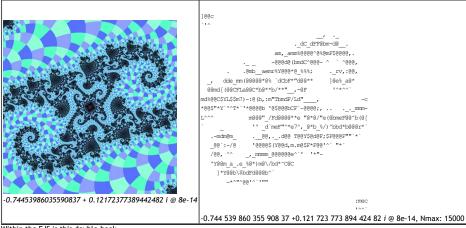
At the center of this double hook, instead of another island, we find an Embedded Julia set.



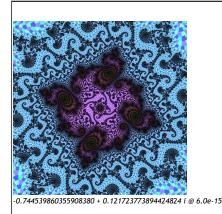


F@d@@@@@@@@@@@@@@b\$@m[d`bd(dd@@@@@@@@@@@@@@@@@@@ peeeeeeeeeeeeeeeee 00000000000000000000TF0000000 (/:mF0m*0^0^T) "0dv000000 [-0.7445398603559061 + 0.1217237738944253 i @ 9.375e-13 -0.744 539 860 355 908 3 +0.121 723 773 894 424 8 i @ 9.375e-13, Nmax: 50000

Inside the embedded Julia set. There are two large features that look like peacock-eyes; on closer inspection they appear to be double spirals.



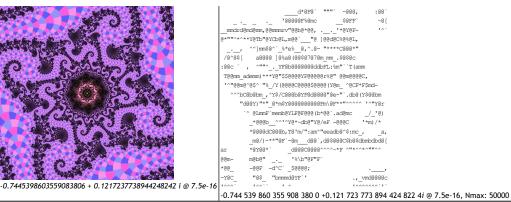
Within the EJS is this double-hook.



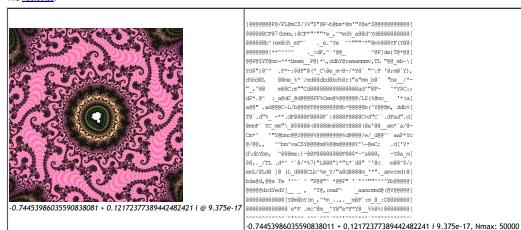
d%")mc mm@@@"*@F:@@Y*@@` ., mm%@L `T@\$"@@@F*m_"*`.@be-@*%)%a@@@@@bF%*ee___,:bO@m _ /@b@`:am'"bmdTFm^)d(@@@@@@@@@@%%Y*`-\d"^ Ym@@ *Y@@" -@m_**`(^*@@@@@@@@@@@@@@"m"(bd@`]@@@GFFY; m_mmme_edY[m^000000000000000000000000db0-\Ybe__,,^00b[YC:-**m\$0000b00000000000000000000bb%80*`".d*` 0^'0_/~C0000000000000x"C00000_^Y000000000000\$CFY0e_'*[C0e_^*C0(*\$Y00000000b*v'0YF**(700000000000000)%e/d_d[/@C *%m@br/F*@@@@@@@@@@@@@@@@@@@Q@Y@YcL@b %La@[-0000***`a*"*(/^\$0000000000000*00Y0YeCdY`.70 .0000 '**`C md`'%a@br\YC@@@@@@@@"F^\$(L@m^^""d^` a@F*c , "@C@e__,,^'@dCFFY@*Y*\ddm@C,dTF d@)'T@@@@b(C@mc '*@YFY@@@@c]F\,\$*"YF\"@^ ^"Ye^^_.@@b/"*^"*`^""*[:/@C%/^^*"`:@_^am@cYd^\ad@be'"Y@@"@@** -0.744 539 860 355 908 379 +0.121 723 773 894 424 824 i @ 6e-15, Nmax: 30000

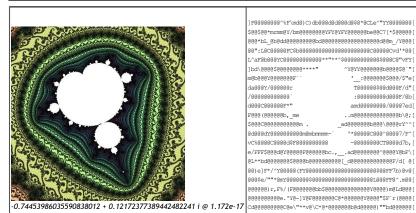
All Embedded Julia sets have paramecia; this is the central nucleus.

___eae_, ^00C, :_a000"^^%Tb -00L .000,



The nucleolus.





\$00\$00*mrmm0Y/bm00000000YFY0YFY0000000be00C7[*\$00000[L^aF@b@@YC@@@@@@@@@@**"**^\$@@@@@@@@@@@@\$@@C@"vFY[]bd\@@@\$@@@@@@****" ^Y@YY@@@@@@@@@@@@\$@`"[__:00000000\$000/\$"e[dagggy/ggggggr TRARRAGARASARARAT/d"[:00000000000d000F/0b[/00000000000000 d000000000F*" amd@@@@@@@/@@@@7ed[F000 (0000000b, me '*00000C000^00007/F^ rC%0000C00000d\$F00000000000 -000000000CT0000d7b, e/FFF\$000d0Y00000F000000bc.,__.ad00000000^0000Y0bF\[0L**bd000000\$0000b0000000000 [d0000000000000F/d[0 [00)e]F*/^Y00000(FY000000000000000000000000\$00FF7b)0v0[000\$e/""*0mY0000000\$00000000000000000000L000FF0^.m00[0000000000.'Y0-]Y0F0000000C0*00000Y0000"\$F`r(0000[-0.74453986035590838011 +0.12172377389442482241 *i* @ 1.172e-17, Nmax: 250000

F@@@@@@@^%F\md@)C)db@@@d@d@@@d@\$@*@CLe^"YY@@@@@@@[

This island has period 1312.

Seahore Valley Islands

Here are the first few islands in the sequence R2F(N/2N+1B2)S:

Rank	R2-Name	<u>period</u>	Area of Island	Coordinates
6	R2F(1/3B2)S	5	1.7621(15)e-5	-0.04332 + 0.98630i @ 0.01259
10	R2F(2/5B2)S	7	9.9155(63)e-6	-0.530101 + 0.668180 <i>i</i> @ 0.009447
16	R2F(3/7B2)S	9	5.5912(42)e-6	-0.650450 + 0.478065i @ 0.007094
18	R2F(4/9B2)S	11	3.1257(30)e-6	-0.694716 + 0.368453 <i>i</i> @ 0.005304
32	R2F(5/11B2)S	13	1.7841(16)e-6	-0.715175 + 0.298825 <i>i</i> @ 0.004007
48	R2F(6/13B2)S	15	1.05522(65)e-6	-0.726129 + 0.251068 <i>i</i> @ 0.003082
72	R2F(7/15B2)S	17	6.4982(45)e-7	-0.732630 + 0.216394i @ 0.002418
102	R2F(8/17B2)S	19	4.1631(34)e-7	-0.736790 + 0.190113 <i>i</i> @ 0.001936
127	R2F(9/19B2)S	21	2.7639(27)e-7	-0.739610 + 0.169522 <i>i</i> @ 0.001577

Computing π in Seahorse Valley

An approximation to the mathematical constant π ("pi") can be calculated in the following way:

```
\pi \cong \epsilon  Dwell(-3/4 + \epsilon) \pm \epsilon
```

where ε ("epsilon") is a very small value, like 0.0001. For example, <u>Dwell(-3/4+0.01i)</u> is 315, giving the approximation π≅3.15. The following results were computed using <u>107-bit precision</u>:

С	Dwell(C)
-3/4 + 1.0	3
-3/4 + 0.1	33
-3/4 + 0.01	315
-3/4 + 0.001	3143
-3/4 + 10 ⁻⁴	31417

-3/4 + 10 ⁻⁵	314160
-3/4 + 10 ⁻⁶	3141593
-3/4 + 10 ⁻⁷	31415927
-3/4 + 10 ⁻⁸	314159266
-3/4 + 10 ⁻⁹	3141592655
-3/4 + 10 ⁻¹⁰	31415926537

Dwell(C) is the number of iterations for the normal Mandelbrot iteration to "escape" with a normal "escape radius" of 2.0; see Escape-Iterations. The numbers (3, 33, 315, 3143, ...) are Sloane's integer sequence 40.07486 (which I have corrected and extended) The sequence is calculated easily in any language that handles floating-point complex values. Here I am using PARI/GP, and the first long line of input should all be typed as a single line:

```
? A097486(n) = local(a, c, z); c = 0.1^n*I - 0.75; z = 0; a = 0; while(abs(z) < 2.0, { z = z^2+c; a = a+1 }); a
? A097486 (0)
%1 = 3
? A097486(1)
%2 = 33
? A097486(2)
%3 = 315
? A097486(3)
%4 = 3143
? A097486(4)
%5 = 31417
? A097486(5)
%6 = 314160
? A097486(6)
%7 = 3141593
? A097486(7)
%8 = 31415927
? A097486(8)
%9 = 314159266
```

revisions: 20080216 oldest on record; 20091105 add list of sequence R2F (N/2N+1B2) S islands; 20091220 add {pi} calculation section; 20091221 add list of {pi} values; 20100107 add PARI/GP code; 20100907 optimize view nmax values; 20100919 add many notes, links and R2-names in descriptions; 20100922 add double spirals link

From the Mandelbrot Set Glossary and Encyclopedia, by Robert Munafo, (c) 1987-2022.

Mu-ency index

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