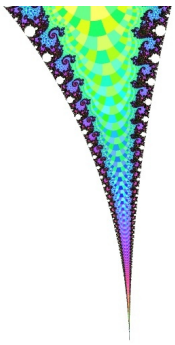
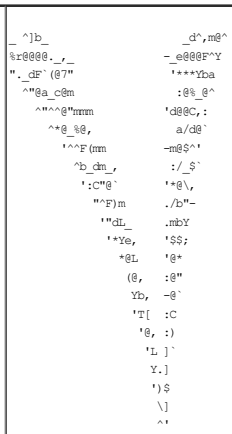
Seahorse Valley

Robert P. Munafo, 2010 Sep 7.

Colloquial name for the cusps between R2a and R2.1/2a, particularly the upper one, R2.C(1/2-). See R2.C(1/2).



-0.75 +0.08 i @ 0.16, Nmax: 10000

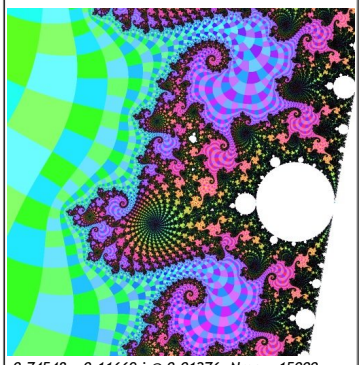


-0.7454 +0.1167 *i* @ 0.16, Nmax: 10000

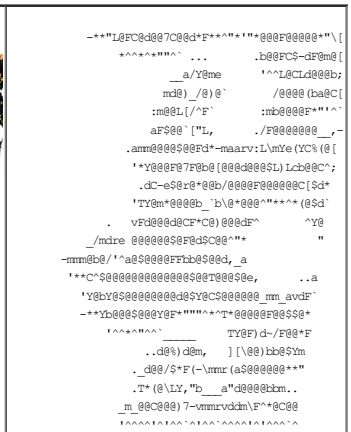
The upper Seahorse valley is shown in the picture. The right side, those parts attached to the continent seed, is part of the Shoulder, and the left side is part of the Head.

Seahorse Valley zoom sequence

The following zoom sequence is a permanent part of the Mandelbrot set culture. It goes back at least as far as the 1985 Scientific American article, and probably originates with Peitgen and Richter. (These images do not show the same mu-unit and filaments as the article, and the colors and aspect ratios of the frames differ as well.)

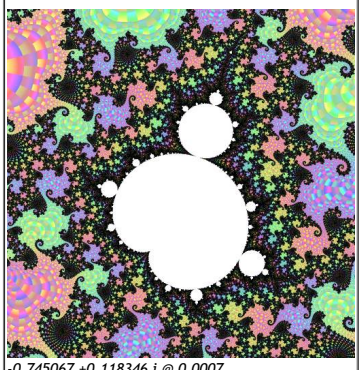


-0.74548 + 0.11669 i @ 0.01276, Nmax: 15000



-0.74546 +0.11669*i* @ 0.01276, Nmax: 15000
R2.13/27a and R2F(13/27B*)

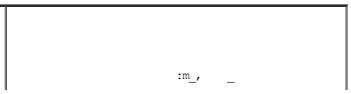
(Page 18, figure ***a***) One of the mu-atoms, R2.13/27a, and its attached filaments.
The largest complete mu-atom in this image is R2.13/27a.
There is one large peacock eye, the point at its center is the branch point R2F(13/27B*).
On the west side of the valley we find double spirals in place of the peacock eyes.
The largest visible island is R2F(13/27B2)S and the branch ending in a spiral is R2F(13/27B2).

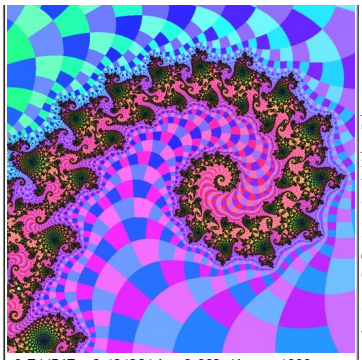


-0.745067 +0.118346 i @ 0.0007

-0.745067 +0.118346 *i* @ 0.0007, nmax: 20000
The island R2F(13/27B2)S

This is the largest visible island in the previous image; its *R2*-name is R2F(13/27B2)S. It has period 29 and is the 335th largest island in the whole Mandelbrot set.

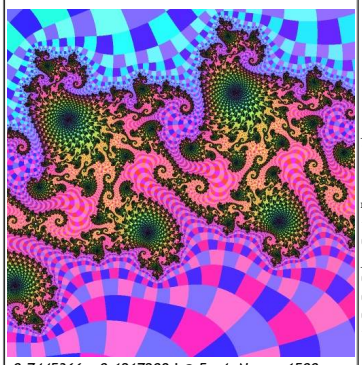




-0.744567 + 0.121201 i @ 0.002, Nmax: 1000

-0.744538 +0.121200 *i* @ 0.002, Nmax: 1000
Spiral in `R2F(13/27B2)`

(Page 18, figure ***b***) This spiral feature is called a tail or Shepherd crook. The prominent spiral tip is `R2F(13/27B2){FS[2]FS[0]}x~`



-0.7445366 + 0.1217208 i @ 5e-4, Nmax: 1500

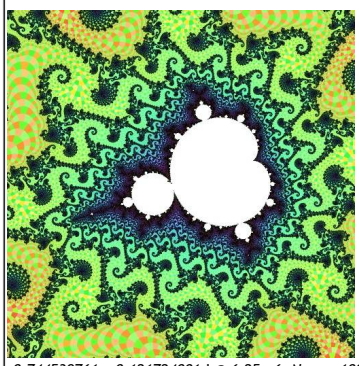
-0.744 539 8 +0.121 723 8*i* @ 0.000 5, Nmax: 1500

Here we see several patterns resembling the center of a sunflower or a peacock-feather "eye".

-0.74453892 + 0.12172418 i @ 1e-4, Nmax: 2500

-0.744 539 85 +0.121 723 77*i* @ 0.0001, Nmax: 2500

A "double hook" and island.



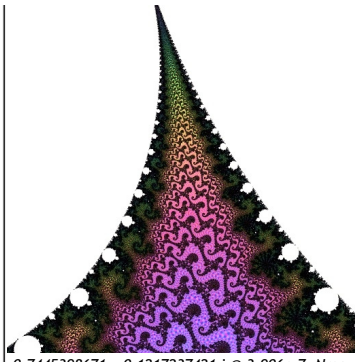
-0.744539761 + 0.121724001 i @ 6.25e-6, Nmax: 10000

-0.744 539 859 +0.121 723 774 *i* @ 6.25e-6, Nmax: 10000

The original Scientific American sequence ended at this point, illustrating the existence of small decorated Mandelbrot sets within the details of the whole.

This island has period 43 and is 7 steps away from the period-29 island shown above (which is `R2F(13/27B2)S`).

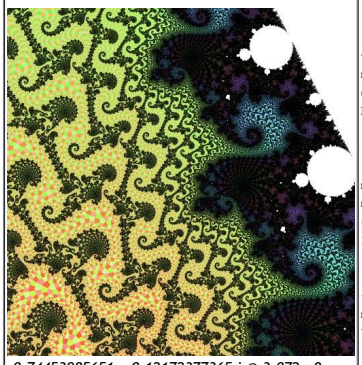
At each step there is a very small island of period *N*+1 (located in the largest lobe off a peacock eye on the outboard side of the spiral), and a significantly larger island of period *N*+2 further towards the spiral tip on the inboard side. This is done 7 times starting at the period-29 island, and so the *R2*-name of this island is `R2F(13/27B2){FS[2]FS[0]}x7S`.



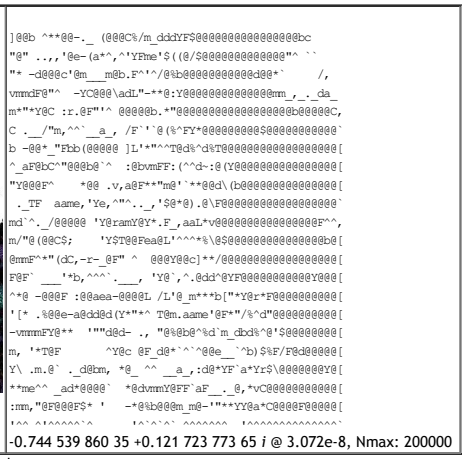
-0.7445398671 + 0.1217237421 i @ 3.906e-7, Nmax: 25000

-0.744 539 860 3 +0.121 723 773 7 *i* @ 3.906e-7, Nmax: 25000

The south half of the island's own seahorse valley. The name of this cusp is `R2F(13/27B2){FS[2]FS[0]}x7S.C(1/2)+`



-0.74453985651 + 0.12172377365 i @ 3.072e-8

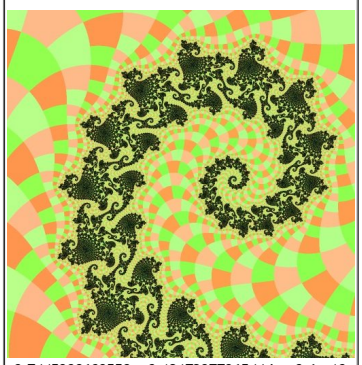


-0.744 539 860 35 +0.121 723 773 65 *i* @ 3.072e-8, Nmax: 200000

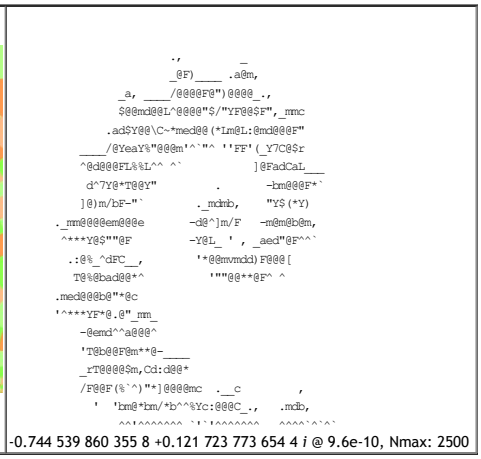
A couple of the island's own seahorses are on the right.

The filaments on the left are all part of `R2F(13/27B2)` but are *not* part of the island's mu-unit.

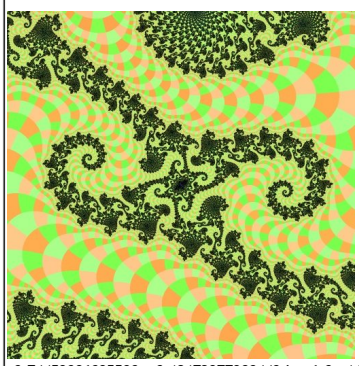
We will now zoom in to the filaments on the left, and find one of its "double-hook" features.



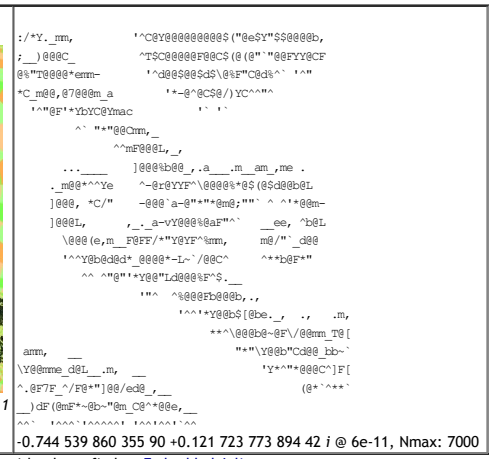
-0.7445398603558 + 0.1217237736544 i @ 9.6e-10



-0.744 539 860 355 8 +0.121 723 773 654 4 *i* @ 9.6e-10, Nmax: 2500

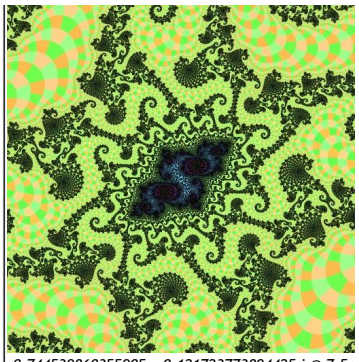


-0.74453986035590 + 0.12172377389443 i @ 6.0e-11



-0.744 539 860 355 90 +0.121 723 773 894 42 *i* @ 6e-11, Nmax: 7000

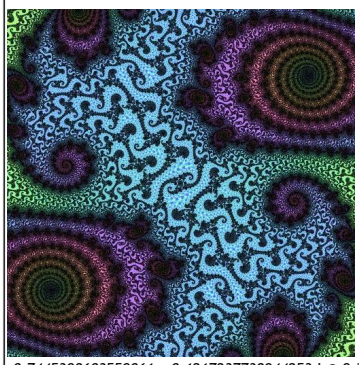
At the center of this double hook, instead of another island, we find an Embedded Julia set.



-0.744539860355905 + 0.121723773894425 i @ 7.5e-12

-0.744 539 860 355 907 +0.121 723 773 894 425 *i* @ 7.5e-12, Nmax: 22000

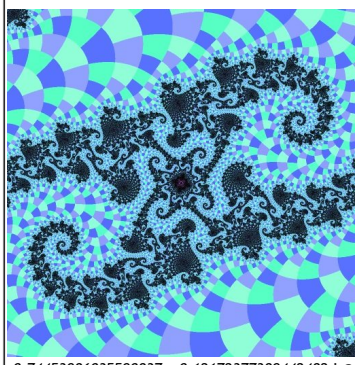
The embedded Julia set.



-0.7445398603559061 + 0.1217237738944253 i @ 9.375e-13

-0.744 539 860 355 908 3 +0.121 723 773 894 424 8 *i* @ 9.375e-13, Nmax: 50000

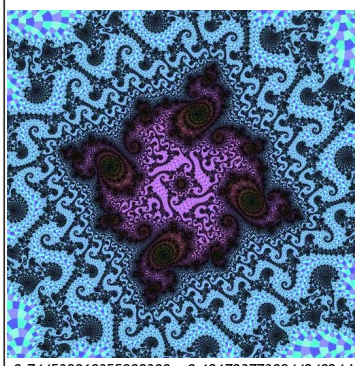
Inside the embedded Julia set. There are two large features that look like peacock-eyes; on closer inspection they appear to be double spirals.



-0.74453986035590837 + 0.12172377389442482 i @ 8e-14

-0.744 539 860 355 908 37 +0.121 723 773 894 424 82 *i* @ 8e-14, Nmax: 15000

Within the EJS is this double-hook.



-0.744539860355908380 + 0.121723773894424824 i @ 6.0e-15

-0.744 539 860 355 908 379 +0.121 723 773 894 424 824 *i* @ 6e-15, Nmax: 30000

All Embedded Julia sets have paramecia; this is the central nucleus.

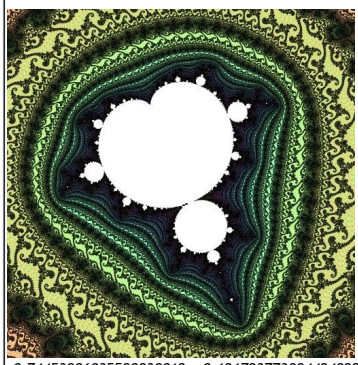
-0.7445398603559083806 + 0.12172377389442482421 i @ 7.5e-16

-0.744 539 860 355 908 380 0 +0.121 723 773 894 424 822 4*i* @ 7.5e-16, Nmax: 50000

The nucleolus.

-0.7445398603559083808 1 + 0.12172377389442482421 i @ 9.375e-17

-0.744539860355908380 11 + 0.12172377389442482241 *i* 9.375e-17, Nmax: 50000



-0.7445398603559083801 2 + 0.12172377389442482241 i @ 1.172e-17

-0.744539860355908380 11 +0.12172377389442482241 *i* @ 1.172e-17, Nmax: 250000

This island has period 1312.

Seahore Valley Islands

Here are the first few islands in the sequence `R2F(`*N*`/2`*N*`+1B2)S`:

Rank	R2-Name	period	Area of Island	Coordinates
6	R2F(1/3B2)S	5	1.7621(15)e-5	-0.04332 + 0.98630*i* @ 0.01259
10	R2F(2/5B2)S	7	9.9155(63)e-6	-0.530101 + 0.668180*i* @ 0.009447
16	R2F(3/7B2)S	9	5.5912(42)e-6	-0.650450 + 0.478065*i* @ 0.007094
18	R2F(4/9B2)S	11	3.1257(30)e-6	-0.694716 + 0.368453*i* @ 0.005304
32	R2F(5/11B2)S	13	1.7841(16)e-6	-0.715175 + 0.298825*i* @ 0.004007
48	R2F(6/13B2)S	15	1.05522(65)e-6	-0.726129 + 0.251068*i* @ 0.003082
72	R2F(7/15B2)S	17	6.4982(45)e-7	-0.732630 + 0.216394*i* @ 0.002418
102	R2F(8/17B2)S	19	4.1631(34)e-7	-0.736790 + 0.190113*i* @ 0.001936
127	R2F(9/19B2)S	21	2.7639(27)e-7	-0.739610 + 0.169522*i* @ 0.001577

Computing π in Seahorse Valley

An approximation to the mathematical constant π ("pi") can be calculated in the following way:

π ≅ ε Dwell(-3/4 + ε) ± ε

where ε ("epsilon") is a very small value, like 0.0001. For example, Dwell(-3/4+0.01*i*) is 315, giving the approximation π≅3.15. The following results were computed using 107-bit precision:

C	Dwell(*C*)
-3/4 + 1.0	3
-3/4 + 0.1	33
-3/4 + 0.01	315
-3/4 + 0.001	3143
-3/4 + 10^{-4}	31417

$-3/4 + 10^{-5}$	314160
$-3/4 + 10^{-6}$	3141593
$-3/4 + 10^{-7}$	31415927
$-3/4 + 10^{-8}$	314159266
$-3/4 + 10^{-9}$	3141592655
$-3/4 + 10^{-10}$	31415926537

Dwell(*C*) is the number of iterations for the normal Mandelbrot iteration to "escape" with a normal "escape radius" of 2.0; see Escape-Iterations. The numbers (3, 33, 315, 3143, ...) are Sloane's integer sequence A097486 (which I have corrected and extended) The sequence is calculated easily in any language that handles floating-point complex values. Here I am using PARI/GP, and the first long line of input should all be typed as a single line:

```
? A097486(n) = local(a, c, z); c = 0.1^n*I - 0.75; z = 0; a = 0; while(abs(z) < 2.0, { z = z^2+c; a = a+1 } ); a
? A097486(0)
%1 = 3
? A097486(1)
%2 = 33
? A097486(2)
%3 = 315
? A097486(3)
%4 = 3143
? A097486(4)
%5 = 31417
? A097486(5)
%6 = 314160
? A097486(6)
%7 = 3141593
? A097486(7)
%8 = 31415927
? A097486(8)
%9 = 314159266
```

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revisions: 20080216 oldest on record; 20091105 add list of sequence `R2F(N/2N+1B2)S` islands; 20091220 add {pi} calculation section; 20091221 add list of {pi} values; 20100107 add PARI/GP code; 20100907 optimize view nmax values; 20100919 add many notes, links and R2-names in descriptions; 20100922 add double spirals link

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