Color

Robert P. Munafo, 2022 Dec 12.

For the purpose of this discussion there are two types of color drawing situations.

"Limited color" refers to the situation where there is some rather small number *N* of colors available (typically 256 or less) and you specify which color to use by giving a single number from 1 to *N*.

"Full color" refers to the situation where you specify the color to use by giving three separate numbers, specifying the color in terms of some kind of three-dimensional color space (like red-green-blue, or hue-saturation-luminance)

Limited Color

When working in limited color it is typical to pick a color from 1 to *N* based on the number of escape iterations of the point. If the number of iterations is less than *N* it can be used directly. If it is bigger than *N* you can pick a fixed "overflow" value, or use (iterations modulo N), which has the effect of cycling through the colors as many times as needed to cover all the possible iteration values.

Full Color

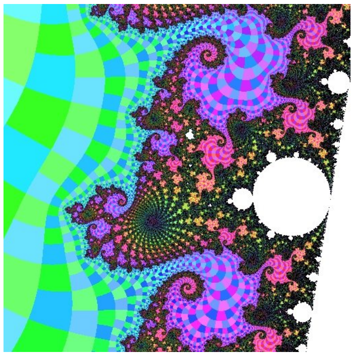
The best approach when working in full color is to treat your available colors as a two-dimensional or three-dimensional space and plot each pixel with a color that combines different attributes of the iterated point, using dimensions in the color space. There are lots of possibilities; here are two examples:

Two dimensions of color, two attributes of point
use hue to show Escape Iterations
use brightness to show Distance Estimate

Three dimensions of color, three attributes of point
use hue to show Continuous Dwell
use saturation to show final point's Angle
use brightness to show final point's Radius

Both of these will produce fairly stunning results and of course there are dozens of other possible combinations.

One of the best ways to color the Mandelbrot Set uses the HSV color space. Use the Distance Estimator function for V. Use Escape-Iterations for H, with a somewhat different hue when the final point's Angle is between 0 and pi. Make S alternate between 1.0 and 0.67, to show odd and even values of Escape-Iterations. The result is stunning:



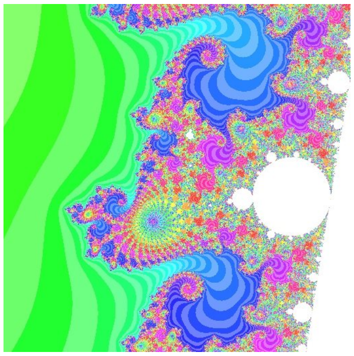
An HSV rendering

(as used in Color MANDELZOOM and its successors)

It is much better to use the hue/saturation/brightness system rather than something like red/green/blue, because of the psychology and physiology of human vision. Most good graphics environments provide a function that converts from HSV to RGB; if your's doesn't, look it up in a book like Graphics Gems or Fundamentals of Interactive Computer Graphics.

For each dimension you want to translate the attribute to the dimension in such a way that you use the whole range, and use it evenly. For example, hue is expressed as an angle on the color wheel. To convert iterations to hue, I would recommend some sort of logarithmic function, so if it takes 20 iterations to go around the wheel the first time, it takes another 40 to go around the second time, then 80, then 160 and so on. Then, to preserve the appearance of stripes at all iterations, you could add 20 degrees to the angle if the iteration is odd. The resulting formula would look like this:

```
hue = (360 * log2(iterations)) modulo 360
if (iterations mod 2 = 1)
  hue = (hue + 20) modulo 360
```



Hue based on iterations

Algorithm Used in the Mu-Ency Illustrations

Most of the JPG and PNG images used to illustrate the Mu-Ency articles are colored using an algorithm similar to the following:

using a fairly large escape radius (such as 1000),
iterate to determine *D* using the continuous dwell method
  (see also escape-iterations)
  set *dwell* = floor(D)
  set *finalrad* = D - floor(D)
  (The floor function gives the integer part of D)

  compute angle of final coordinate of iteration and call it *finalang*
(if the escape radius is big enough, this is also a fractional part
of the external angle)

  compute distance estimate
  set *dscale* = $\log_2$(distance_estimate / pixel_spacing)

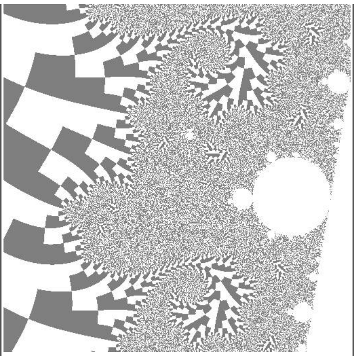
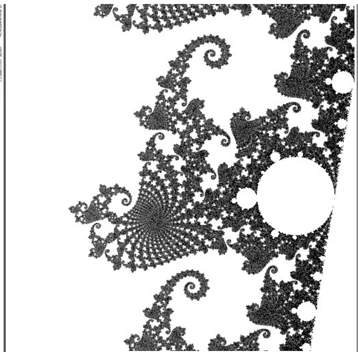
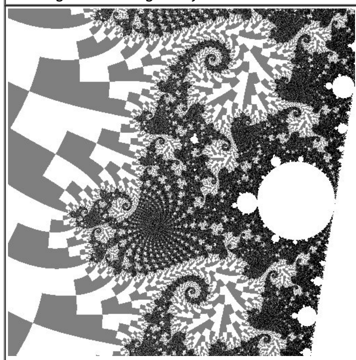
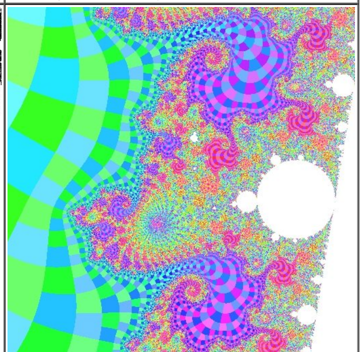
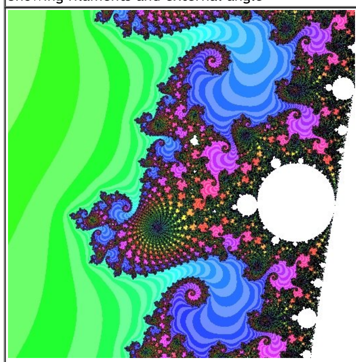
  if *dwell* exceeds iterations limit,

```
    skip all the following and plot a white point

  comment
    Convert the scaled distance estimate to a brightness (called
    "value" in the HSV color space) from 0.0 to 1.0 in 8 intervals.

  if dscale > 0
    value = 1.0
else if dscale > -8
    value = (8 + dscale)/8
else
    value = 0

  comment
    Apply logarithmic scaling to the dwell. The big number "100000"
    should be chosen to be bigger than any iterations limit you'll
    ever use.
P = log(dwell)/log(100000)

  comment
    The following re-maps the range 0.0...1.0 onto an "angle"
    and "radius" on the color wheel. This color wheel has white in the
    center, pastel hues close to the center, and vivid colors around the
    edge of the wheel. In order to gain maximum use of all the available
    colors, we compute an angle and radius in a way that places the
    resulting points equally far apart from one another. The use
    of square root means that the radius increases more slowly as we
    move away from the center. Note also that the angle will be multiplied
    by a constant below, so we'll end up going around the wheel multiple
    times.

  if (P < 0.5) {
    P = 1.0 - 1.5*P
    angle = 1 - P
    radius = sqrt(P)
} else {
    P = 1.5*P - 0.5
    angle = P
    radius = sqrt(P)
}

  comment
    The following makes every other "stripe" a bit lighter
if dwell is odd {
    value = 0.85 * value
    radius = 0.667 * radius
}

  comment
    The following breaks the stripes up into "squares" that make the
    external angles evident. Combined with the previous "stripe"
    operation, this gives the image a checkerboard-like appearance
if finalang > π {
    angle = angle + 0.02
}

  comment
    The following turns each "square" into a rainbow-like gradient
angle = angle + 0.0001 * finalrad

  comment
    The following causes it to use 10 full rainbows to cover the range
    from 1 to 100000 (or whichever big number was used in the first step
    of calculating P)
hue = angle * 10.0
hue = hue - floor(hue)

  saturation = radius - floor(radius)

  comment
    hue is now in the range 0.0...1.0. This represents a full circle
    (normally 0...2π)

  convert (hue, saturation, value) to (red, green, blue)
using standard HSV to RGB mapping
```

The intended benefits of this algorithm are:

- Always show all of the detail in an image, with lots of contrast
- Always show the dwell bands and binary decomposition
- Does not vary with zooming in and out or with changing the iterations (dwell) limit (in other words, an enlarged portion of any image always matches a recomputed version of just that portion)
- Use a color wheel that is rebalanced to match the human psychovisual color primaries (which gives what I consider aesthetically pleasing colors)
- Makes efficient use of the 3-D color space (for example by compensating for less information-carrying ability at low brightness values)
- Shows multiple hues, except when that would preclude any of the above

More Examples

	
Showing external angle only	Showing filaments only
	
Showing filaments and external angle	Showing dwell and external angle
	
Showing dwell and filaments	

See also algorithms, dithering, palette, representation functions.

---

revisions: 20100907 oldest on record; 20101213 fix a couple typos; add "intended benefits" 20221212 Add illustrations

---