Escape Radius

Robert P. Munafo, 1997 Nov 19.

The escape radius is a parameter used in iterating a point under the Mandelbrot Set and Julia Set functions. It is the radius of a circle on the Complex Plane used as a boundary to determine when iteration can stop. The circle is centered at the origin, and has a radius of at least 2.0. As soon as the iteration yields a value that falls outside the circle, iteration can stop and it has been determined that the point that was iterated is not a member of the Mandelbrot Set.

The reason for having an escape radius is to prevent iteration from going on forever. The Mandelbrot iteration is supposed to terminate when the iteration becomes unbounded, i.e. goes off to infinity. It is fairly straightforward to show that if a point ends up outside a radius of 2.0 then the iteration will go off to infinity.

Here is the proof:

%% need to translate into Mu Ency terminology, try to simplify a bit, eliminate terms "induction" and "triangle inequality".

Original USENET article:

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-------R-P-M----N-e-w-s-b-l-i-n-k-----------------------------
Xref: world sci.math:34837
Path: world!eff!sol.ctr.columbia.edu!zaphod.mps.ohio-state.edu!uwm.edu!biosci!agate!usenet.ins.cwru.edu!math26647.math.cwru.edu!user
From: mgh3@po.cwru.edu (mike hurley)
Newsgroups: sci.math
Subject: Re: Mandelbrot question
Message-ID: <mgh3-260493142114@math26647.math.cwru.edu>
Date: 26 Apr 93 18:58:27 GMT
References: <1993Apr26.173119.1002@eurom.rhein-main.de>
Followup-To: sci.math
Organization: case western reserve u
Lines: 43
NNTP-Posting-Host: math26647.math.cwru.edu

In article <1993Apr26.173119.1002@eurom.rhein-main.de>,
hein@eurom.rhein-main.de (Hein Roehrig) wrote:
>
>
> I currently am working on a (high school) report concerning the mandelbrot
> set. In one book (The Emperor's New Mind by Penrose) I learned that the
> series is guaranteed to be unbounded if the absolute value of one of its
> elements surpasses 1+sqrt(2). Can anybody explain me why?
>
> Thanks in advance for any advice, including book tips!
>
> Hein
> ----
> eurom: Free Multiline Unix BBS                    Home of the FSAG
> Frankfurt/Main,Germany                         Data: ++49-69-6312934

The sequence generated by f(z) = z^2 + c beginning with 0 begins
     0,c,c^2 + c, ...
Call these terms z0, z1, z2, ... .

In fact this sequence will be unbounded if any of its terms
has magnitude larger than 2.

The main idea is that if the magnitude |z| is bigger than both
2 and the magnitude |c|, then |f(z)|/|z| > |z| - 1 > 1, so that
by induction the sequence of magnitudes will grow geometrically.

To establish the inequality, note that
  |f(z)|/|z| = |z^2 + c|/|z|
            >= (|z|^2-|c|)/|z|          [ triangle inequality ]
             = |z| - (|c|/|z|)
             > |z| - 1                  [ |z| > |c| ]
             > 1                        [ |z| > 2 ]

If  |c| <= 2 and  |zn| > 2 then certainly
z = zn satisfies these hypotheses, so the sequence is
unbounded.

If |c| > 2 then |c^2+c| >= |c|^2 - |c| = |c|(|c|-1) > |c| > 2
so that z=z2 satisfies the hypotheses of the argument above.

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Acknowledgments

Proof that 2.0 is sufficient: Mike Hurley (mgh3@po.cwru.edu)

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