2019 ADA miniHW 8

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(1)

Count	Value	Cost
0	0000000	0
1	1111111	7
2	1010101	3
3	1110001	2
4	1111001	1
5	1101001	1

The counter reads 1101001, and the total cost is $0 + 7 + 3 + 2 + 1 + 1 = 14$.

(2) By aggregation method, the total cost of the n countings is given by:

$$\left\lfloor \frac{n}{1} \right\rfloor + \left\lfloor \frac{n}{2} \right\rfloor + \cdots + \left\lfloor \frac{n}{n} \right\rfloor < n \cdot \int_1^n \frac{1}{x} dx = n \ln n = O(n \log n)$$

The amortized cost of each counting is then:

$$\frac{O(n \log n)}{n} = O(\log n)$$

(3) If there are two numbers x, y such that $x < y$ and the counter reads the same when counting from 0 to x and to y, then the net effect of counting between $x + 1$ to y is that the counters flips no bits at all. This is impossible as we observe that when we count to, say, a, the a-th bit is never changed after this flip. So when we flip the $x + 1$-th bit, there is no way we can cancel this flip when counting from $x + 2$ to y. This shows that the counter cannot reads the same for different numbers x and y, and thus all values will be unique when counting from 0 to n.

Reference

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