2019 ADA miniHW 7

b07902064 資工二 蔡銘軒

December 4, 2019

It is clear that in the final configuration, the farmlands form one or more connected components. We claim that for each connected component, it has only one farmland that is built a resevoir, and the connected component forms a tree structure with the farmland with resevoir being the root. The first property is obvious, as if there are two or more resevoirs in a connected component, we could simply remove the additional resevoirs without affecting irrigation. As for the second property, it is also obvious that for each farmland in the connected component, except for the root, it only needs exactly one connection with one of the other farmlands to gain water, and thus there are exactly (#farmland - 1) edges in the component, which implies a tree structrue.

At this point, the remaining problem is how we can build the trees. We view each farmland as a vertex, and each connection between the vertices as an edge with cost P_{ij}. We further create a dummy point such that when a vertex is connected to the dummy point, we build a resevoir in the vertex and the cost is W_i. The creation of the dummy point connect all the roots of the connected components to form a single connected component, which is a single tree structure.

So the problem is to find the MST on the graph with $N + 1$ vertices and $C_2^N + N$ edges. We sort the edges according to their costs and run Kruskal's algorithm. The time complexity is $O(N^2 logN)$

Reference

none