2019 ADA miniHW 1

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From the definition of $g(n) = \Theta(F(n))$, there exist positive constants c_1, c_2, and n_0, such that $0 \leq c_1F(n) \leq g(n) \leq c_2F(n)$, for all $n \geq n_0$.

And from the definition of $h(n) = o(F(n))$, for every positive constant $c > 0$, there exists a constant $n_1 > 0$, such that $0 \leq h(n) < cF(n)$, for all $n \geq n_1$.

By taking $n_2 = max(n_0, n_1)$, we have $0 \leq h(n) < c_1F(n) \leq g(n) \leq c_2F(n)$, for all $n \geq n_2$. Subtracting the inequalities by $h(n)$, we get $0 < c_1F(n) - h(n) \leq f(n) \leq c_2F(n) - h(n)$, for all $n \geq n_2$.

We now focus on the term $c_1F(n) - h(n)$. Since $h(n) < cF(n)$ for all $c > 0$, we have $c_1F(n) - cF(n) = (c_1 - c)F(n) < c_1F(n) - h(n)$ for all $c > 0$. The term $(c_1 - c)$ can take any value in the interval $(-\infty, c_1)$. Since $c_1 > 0$, we can choose a constant c_3 such that $c_3 \in (-\infty, c_1)$ and $c_3 > 0$.

Now we arrive at $0 < c_3F(n) < f(n)$, which can also be written as $0 \leq c_3F(n) \leq f(n)$, for all $n \geq n_2 = max(n_0, n_1)$ with a positive constant $c_3 > 0$, which is the definition of $f(n) = \Omega(F(n))$. The statement is thus proven.