2019 ADA miniHW 9

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December 26, 2019

- **Hamiltonian path problem:** Given a graph G, check whether there exists a path such that it passes through all vertices exactly once.
- **Traveling salesman problem:** Given a wighted graph G such that each edge is assigned a non-negative weight, and an integer k, check whether there exists a simple cycle that passes through all the vertices exactly once with length $\leq k$.

We first reduce the Hamiltonian path problem to the Hamiltonian cycle problem. The reduction can be done by adding an additional vertex v that is connected with all the other vertices in the given graph G, which is clearly feasible in polynomial time. Let the new graph be G', we briefly show that G' contains a Hamiltonian cycle if and only if G contains a Hamiltonian path.
If there is a Hamiltonian cycle $v \to s \to \cdots \to t \to v$ for some vertices s and t in G', we simply remove the additional vertex v, then the remaining part $s \to \cdots \to t$ is a Hamiltonian path in G.
If there is a Hamiltonian path $s \to \cdots \to t$ in for some vertices s and t in G, the cycle $v \to s \to \cdots \to t \to v$ in G' is immediate as v is connected with all the vertices.
Then we reduce the Hamiltonian cycle problem to the traveling salesman problem. Let the graph given in the Hamiltonian cycle problem be G, we derive G' in the traveling salesman problem by making G a complete graph. The weight of each edge is given as follows:

$$weight(u, v) = \begin{cases} 1 \text{ if } (u, v) \in G \\ 0 \text{ if } (u, v) \notin G \end{cases} \quad \text{for all edges } (u, v) \in G'$$

The reduction can also be done in polynomial time. We briefly show that the answer to the traveling salesman for $k = 0$ is positive if and only if there is a Hamiltonian cycle in G.
If there exists a simple cycle such that the length ≤ 0, it can only contain edges that are in G. As the cycle passes through all vertices exactly once, it is a Hamiltonian cycle in G.
If there exists a Hamiltonian cycle in G, then those edges also form a simple cycle in G' with weight 0, so the condition for the traveling salesman problem is satisfied.

Reference

`https://opendsa-server.cs.vt.edu/ODSA/Books/Everything/html/hamiltonianCycle_to_TSP.html`