Zusatzaufgabe 9.1

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X : Gesamtniederschlagsmenge $[\, l/m^2 \,]$

$$f(x) = \begin{cases} c \sin\left(\frac{\pi}{100} x\right) & , \quad 0 \le x \le 100 \\ 0 & , \quad \text{sonst} \end{cases}$$

$F_X(x) = P(X \le x)$ DZX

$f(x) = \frac{dF(x)}{dx}$

$\Rightarrow F(x) = \int_{-\infty}^{+\infty} f(x)\, dx \overset{!}{=} 1$

a) $$\int_{-\infty}^{+\infty} f(x)\, dx \overset{!}{=} 1$$

$\Rightarrow \int_0^{100} c \sin\left(\frac{\pi}{100} x\right) \overset{!}{=} 1$

$\Rightarrow \left[-c \cdot \frac{100}{\pi} \cos\left(\frac{\pi}{100} x\right) \right]_0^{100} \overset{!}{=} 1$

$\Rightarrow -c \cdot \frac{100}{\pi} \left[\underbrace{\cos \pi}_{-1} - \underbrace{\cos 0}_{1} \right] \overset{!}{=} 1$

$\Rightarrow -c \frac{100}{\pi} (-2) \overset{!}{=} 1$

$\Rightarrow 200c \overset{!}{=} \pi$

$\Rightarrow \underline{\underline{c = \frac{\pi}{200}}}$

b) μ : Erwartungswert

$$\mu = \int_{-\infty}^{+\infty} x \cdot f_X(x)\, dx$$

$f(-x) = c \sin\left(-\frac{\pi}{100} x\right)$

$= -c \sin\left(\frac{\pi}{100} x\right)$

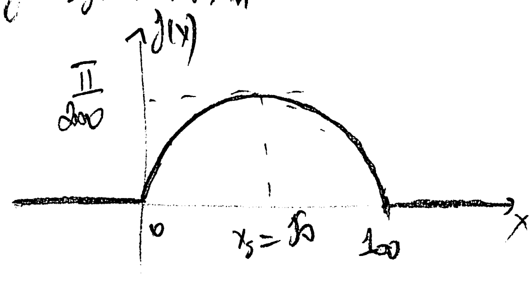
$= -f(x)$

$\Rightarrow f$ symmetrisch

$\Rightarrow \underline{\mu = \cancel{X_s} = 0}$

$f(x) = \frac{\pi}{200} \sin\left(+\frac{\pi}{100} x\right)$

$f(0) = 0 \qquad f(50) = 1 \qquad f(100) = 0$



$\mu = x_5 = 50$

c) $$\sigma_x^2 = E\left((x-\mu)^2\right) = \int_{-\infty}^{+\infty} (x-\mu)^2 \cdot f_x(x) \cdot dx$$

$$= \int_0^{100} (x-\mu)^2 \cdot \frac{\pi}{200} \sin\left(\frac{\pi}{100}x\right) dx = \frac{\pi}{200} \int_0^{100} (x-\mu)^2 \sin\left(\frac{\pi}{100}x\right)$$

$$= \left[2 \cdot \frac{x-\mu}{\left(\frac{\pi}{100}\right)^2} \sin\left(\frac{\pi}{100}x\right) - \left(\frac{(x-\mu)^2}{\frac{\pi}{100}} - \frac{2}{\left(\frac{\pi}{100}\right)^3} \right) \cos\left(\frac{\pi}{100}x\right) \right]_0^{100}$$

Hinweis

$$= \frac{\pi}{200} \left[\frac{2x}{\frac{\pi^2}{10000}} \sin\left(\frac{\pi}{100}x\right) - \left(\frac{x^2}{\frac{\pi}{100}} - \frac{2}{\frac{\pi^3}{1000000}} \right) \cos\left(\frac{\pi}{100}x\right) \right]_0^{100} - \mu^2$$

Tschebyscheff Ungleichung

$$= \frac{\pi}{200} \left[\underbrace{\frac{20000}{\pi^2} \cdot 100 \sin(\pi)}_{0} - \left(\frac{100}{\pi} \cdot 100^2 - \frac{2000000}{\pi^3} \right) \underbrace{\cos(\pi)}_{-1} - \frac{2000000}{\pi^3} \cos 0 \right] - 50^2$$

$$= + \left(\frac{1000000}{\pi} - \frac{2000000}{\pi^3} \right) - \frac{2000000}{\pi^3} - 50^2$$

$$= \frac{1000000}{\pi} - \frac{4000000}{\pi^3} - 50^2 = \frac{1000000\,\pi^2 - 4000000}{\pi^3} - 50^2$$

$$= \frac{1000000}{\pi^3} \left[\pi^2 - 4 \right] - 50^2 = \cancel{1000000 \left[\frac{1}{\pi} - \frac{4}{\pi^3} \right]} - 50^2$$

$$= \frac{\pi}{200} \frac{1000000}{\pi^3} \left[\pi^2 - 4 \right] - 2500$$

$$\sigma^2 = 5000 \left[1 - \frac{4}{\pi^2} \right] - 2500$$

$$\approx 473{,}6$$

$$\boxed{\sigma \approx 21{,}76}$$

$$= \frac{50000}{2\pi^2} \left[\pi^2 - 4 \right] - 2500 = 5000 \cdot \left[1 - \frac{4}{\pi^2} \right] - 2500$$