Zusatzaufgabe 9.1

Gesamtniederschlagsmenge $X \; [\; 1/m^2 \;]$

$$f(x) = \begin{cases} c \sin\left(\frac{\pi}{100} x\right) & \text{für } 0 \leq x \leq 100 \\ 0 & \text{sonst} \end{cases}$$

a.) Gleichung (7.15) Seite 86 : $\int_{-\infty}^{+\infty} f(x)\, dx \overset{!}{=} 1$

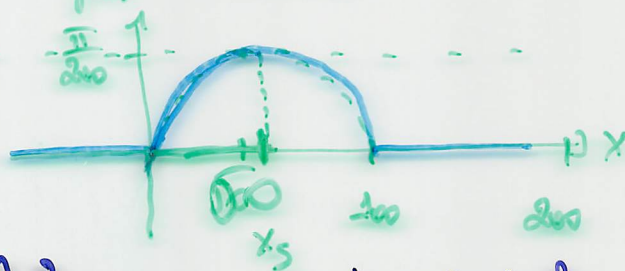$$\Rightarrow \int_0^{100} c \sin\left(\frac{\pi}{100} x\right) dx \overset{!}{=} 1$$

$$\Rightarrow c \left[-\frac{100}{\pi} \cos\left(\frac{\pi}{100} x\right) \right]_0^{100} \overset{!}{=} 1$$

$$\Rightarrow -\frac{c}{\pi} \cdot 100 \underbrace{\left[\cos(\pi) - \cos(0) \right]}_{\substack{1 \quad 1 \\ -2}} \overset{!}{=} 1 \quad \Rightarrow \quad \frac{200 \cdot c}{\pi} \overset{!}{=} 1$$

$$\Rightarrow \underline{\underline{c = \frac{\pi}{200}}}$$

$$\Rightarrow f(x) = \begin{cases} \frac{\pi}{200} \cdot \sin\left(\frac{\pi}{100} \cdot x\right) & 0 \leq x \leq 100 \\ 0 & \text{sonst} \end{cases}$$

b.) $E(X) = \mu_X = \int_{-\infty}^{+\infty} x \cdot f_X(x)\, dx$

(9.7) Seite 101

$$E(X) = \mu_X = \int_0^{100} x \cdot \frac{\pi}{200} \sin\left(\frac{\pi}{100} \cdot x\right) dx \quad \cdots \text{ langwierig ?}$$

Gleichung (9.11) Seite 103 $\Longrightarrow$ $\mu_X = x_S = 50$

c.) $\sigma^2 = E(X^2) - \mu^2$

$$= \int_0^{100} x^2 \cdot \frac{\pi}{200} \sin\left(\frac{\pi}{100} \cdot x\right) dx \overset{\text{Hinweis}}{=} \frac{\pi}{200} \int_0^{100} x^2 \cdot \sin(a \cdot x)\, dx$$

neu Berechnen !

$$= \frac{\pi}{200} \left[\frac{2x}{\left(\frac{\pi}{200}\right)^2} \sin\left(\frac{\pi x}{100}\right) - \left(\frac{x^2}{\frac{\pi}{100}} - \frac{2}{\frac{\pi^3}{100}} \right) \cos\left(\frac{\pi}{100} x\right) \right]_0^{100}$$

$$\cdots = \frac{10^4}{2} \left[1 - \frac{4}{\pi^2} \right] \overset{-\mu^2 = 2500}{\Rightarrow} \sigma = \frac{10^2}{\sqrt{2}} \cdot \frac{1}{\pi} \sqrt{\pi^2 - 4\pi^2} = 21{,}76$$