Zusatzaufgabe 14.1

stochastische Unabhängigkeit $\Leftrightarrow f_{XY}(x,y) = f_X(x)\cdot f_Y(y)$

a) **Bild 1:**

$$f_{XY}(x,y) = \frac{1}{4}\Big[\delta(x-1,y-1) + \delta(x-1,y+1) + \delta(x+1,y+1) + \delta(x+1,y-1)\Big]$$

$$f_X(x) = \int_{-\infty}^{+\infty} f_{XY}(x,y)\,dy$$

$$= \frac{1}{4}\Big[\int_{-\infty}^{\infty}\delta(x-1)\delta(y-1)\,dy + \int_{-\infty}^{+\infty}\delta(x-1)\cdot\delta(y+1)\,dy + \int_{-\infty}^{+\infty}\delta(x+1)\delta(y+1)\,dy + \int_{-\infty}^{+\infty}\delta(x+1)\delta(y-1)\,dy$$

$$= \frac{1}{4}\delta(x-1) + \frac{1}{4}\delta(x-1) + \frac{1}{4}\delta(x+1) + \frac{1}{4}\delta(x+1)$$

$$f_X(x) = \frac{1}{2}\delta(x-1) + \frac{1}{2}\delta(x+1) \qquad (1)$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx = \frac{1}{2}\delta(y-1) + \frac{1}{2}\delta(y+1) \qquad (2)$$

$f_{XY}(x,y) = f_X(x)\cdot f_Y(y) \Rightarrow$ X und Y stoch. unabhängig

$$f_X(x)\cdot f_Y(y) = \left(\frac{1}{2}\delta(x-1) + \frac{1}{2}\delta(x+1)\right)\left(\frac{1}{2}\delta(y-1) + \frac{1}{2}\delta(y+1)\right)$$

$$= \frac{1}{4}\delta(x-1)\delta(y-1) + \frac{1}{4}\delta(x-1)\delta(y+1) + \frac{1}{4}\delta(x+1)\cdot\delta(y-1)$$

$$+ \frac{1}{4}\delta(x+1)\cdot\delta(y+1) \overset{!}{=} f_{XY}(x,y) \qquad \square$$

Bild 2:

$$f_{XY}(x,y) = \frac{1}{4}\delta(x-1)\delta(y) + \frac{1}{4}\delta(x+1)\delta(y) + \frac{1}{4}\delta(x)\delta(y+1) + \frac{1}{4}\delta(x)\delta(y-1)$$

$$f_X(x) = \frac{1}{4}\delta(x-1) + \frac{1}{4}\delta(x+1) + \frac{1}{4}\delta(x) + \frac{1}{4}\delta(x)$$

$$= \frac{1}{4}\delta(x-1) + \frac{1}{4}\delta(x+1) + \frac{1}{2}\delta(x)$$

$$f_Y(y) = \frac{1}{4}\delta(y-1) + \frac{1}{4}\delta(y+1) + \frac{1}{2}\delta(y)$$