ZA 15.1

Berechnen Sie die Dichtefunktion der Summen und skizzieren Sie deren Verläufe.

Stochastisch unabhängige ZV x_i

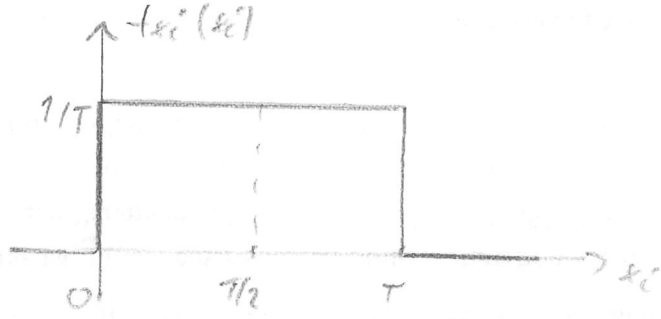
stetige Gleichverteilung im Intervall $[0, T]$

$$f_{x_i}(x_i) = \begin{cases} 1/T : & 0 \le x_i \le T \\ 0, & \text{sonst} \end{cases}$$

$$\mu_{x_i} = \frac{T}{2}$$

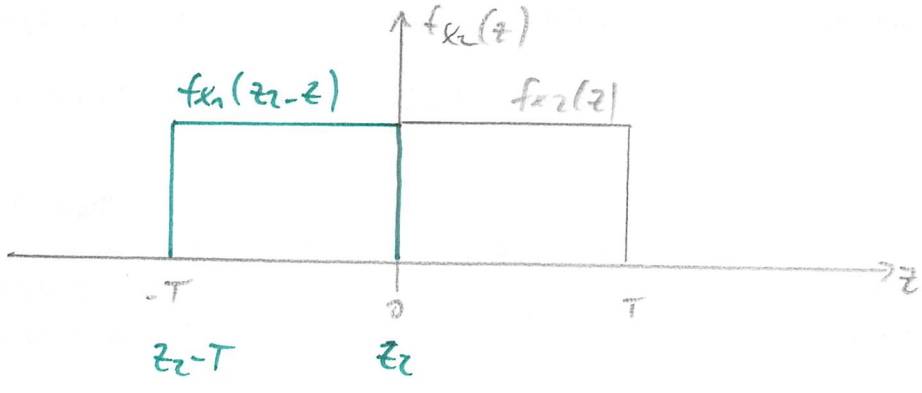
$$\sigma^2_{x_i} = \frac{T^2}{12}$$

$$\Rightarrow \sigma_{x_i} = \frac{T}{2\sqrt{3}}$$



$z_2 = x_1 + x_2$

$$f_{z_2}(z_2) = f_{x_1}(z_2) * f_{x_2}(z_2) = \int_{-\infty}^{\infty} f_{x_1}(z_2 - z)\, f_{x_2}(z)\, dz$$

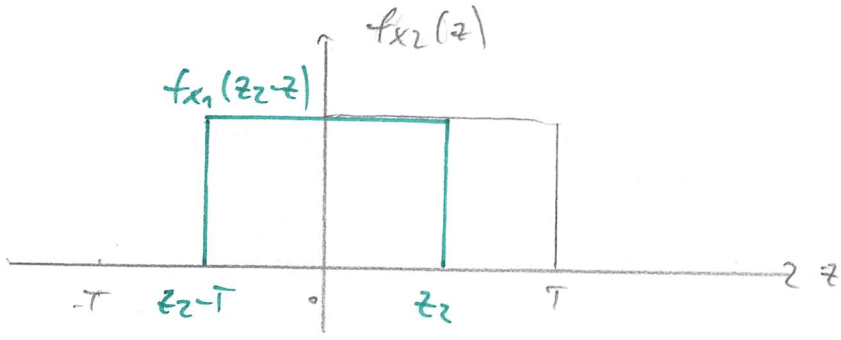


"gespiegelt und um z_2 verschoben"

ZA 15.1

$z_2 \leq 0$: $f_{Z_2}(z_2) = 0$

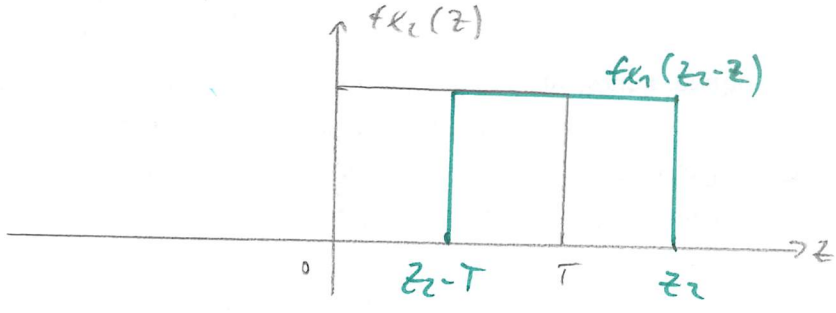
$0 \leq z_2 \leq T$



$$f_{Z_2}(z_2) = \int_0^{z_2} f_{X_1}(z_2 - z)\, f_{X_2}(z)\, dz$$

$$= \int_0^{z_2} \frac{1}{T} \cdot \frac{1}{T}\, dz = \frac{1}{T^2} \int_0^{z_2} 1\, dz = \frac{1}{T^2} z_2$$

$T \leq z_2 \leq 2T$

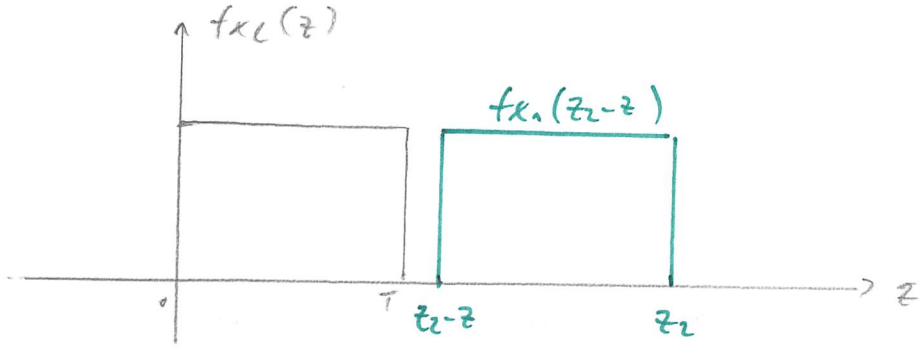


$$f_{Z_2}(z_2) = \int_{z_2 - T}^{T} f_{X_1}(z_2 - z)\, f_{X_2}(z)\, dz$$

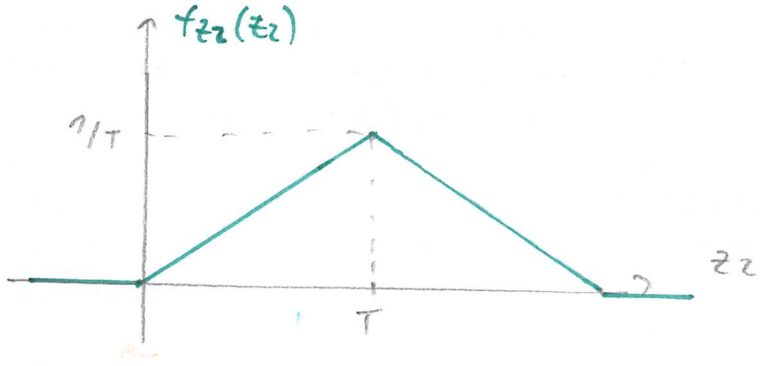
$$= \frac{1}{T^2} \left[T - (z_2 - T) \right] = \frac{2T - z_2}{T^2}$$

ZA 15.1

$z_2 > 2T$



$f_{z_2}(z_2) = 0$



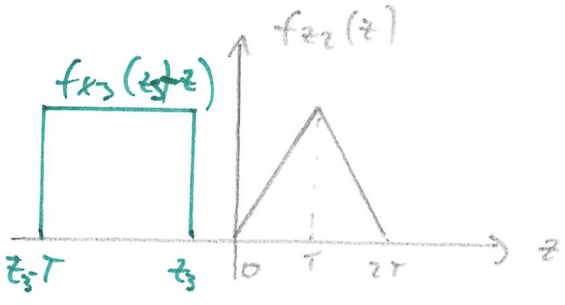
$z_3 = x_1 + x_2 + x_3 = z_2 + x_3$

$$f_{z_3}(z_3) = f_{x_1}(z_3) * f_{x_2}(z_3) * f_{x_3}(z_3)$$

$$= f_{z_2}(z_3) * f_{x_3}(z_3)$$

$$= \int_{-\infty}^{\infty} f_{z_2}(z) \cdot f_{x_3}(z_3 - z)\, dz$$

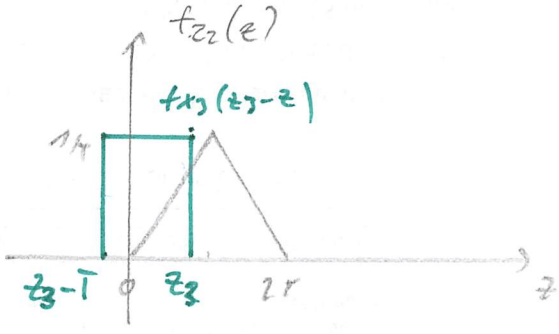
$z_3 \leq 0$



2A15.1

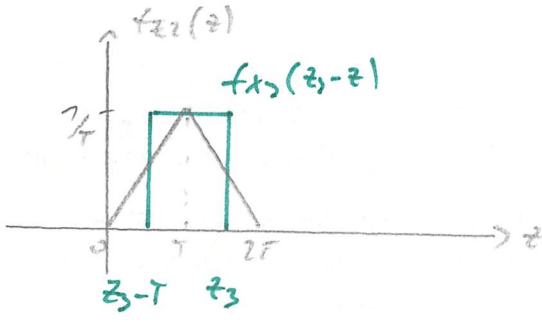
$f_{Z_3}(z_3) = 0$

$0 < z_3 \leq T$:



- $f_{Z_3}(z_3) = \int_0^{z_3} \frac{z}{T^2} \cdot \frac{1}{T}\, dz = \left[\frac{1}{T^3}\,\frac{z^2}{2}\right]_0^{z_3} = \frac{z_3^2}{2T^3}$

$T \leq z_3 \leq 2T$



- $f_{Z_3}(z_3) = \int_{z_3-T}^{T} \frac{z}{T^2}\,\frac{1}{T}\, dz + \int_{T}^{z_3} \frac{2T - z}{T^2}\,\frac{1}{T}\, dz$

$$= \left[\frac{1}{T^3}\cdot\frac{z^2}{2}\right]_{z_3-T}^{T} + \frac{1}{T^3}\left[2T\cdot z - \frac{z^2}{2}\right]_T^{z_3}$$

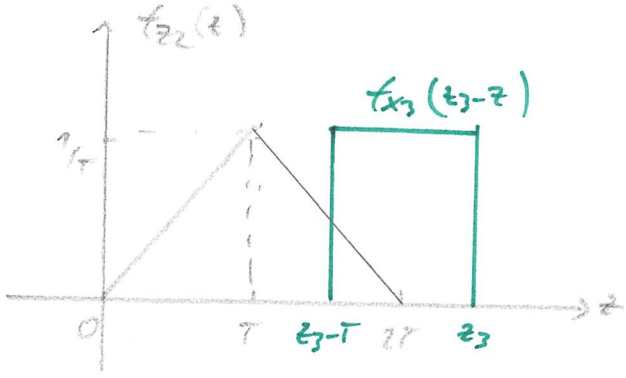
$$= \frac{1}{T^3}\left[\frac{T^2 - (z_3 - T)^2}{2} + 2T z_3 - \frac{z_3^2}{2} - \left(2T^2 - \frac{T^2}{2}\right)\right]$$

$$= \frac{1}{T^3}\left(-T^2 + 2T z_3 - \frac{z_3^2}{2} - \frac{z_3^2}{2} + z_3 \cdot T - \frac{T^2}{2}\right)$$

$$= \frac{1}{T^3}\left(-z_3^2 + 3T\cdot z_3 - \tfrac{3}{2}T^2\right)$$

ZA 15.1

$2T < z_3 \leq 3T$



$$f_{Z_3}(z_3) = \int_{z_3-T}^{2T} \frac{2T-z}{T^2} \cdot \frac{1}{T}\, dz$$

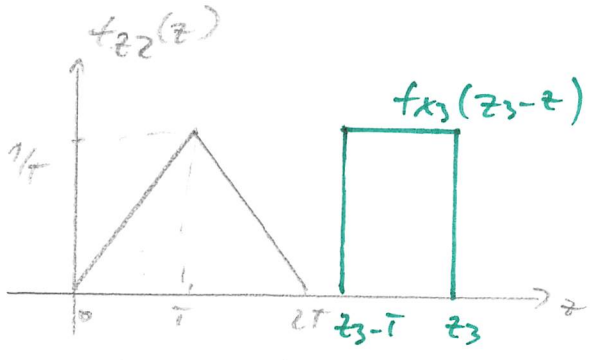
$$= \frac{1}{T^3}\left[2Tz - \frac{z^2}{2}\right]_{z_3-T}^{2T}$$

$$= \frac{1}{T^3}\left[4T^2 - 2T^2 - \left(2Tz_3 - 2T^2 - \frac{z_3^2 - 2Tz_3 + T^2}{2}\right)\right]$$

$$= \frac{1}{T^3}\left[2T^2 - 2Tz_3 + 2T^2 + \frac{z_3^2}{2} - Tz_3 + \frac{T^2}{2}\right]$$

$$= \frac{1}{T^3}\left[\frac{z_3^2}{2} - 3Tz_3 + \frac{9}{2}T^2\right)$$

$z_3 > 3T$



$f_{Z_3}(z_3) = 0$