Zusatzaufgabe 14.2

$$f_{XY}(x,y) = \frac{1}{16\pi} e^{\frac{-x^2-4y^2-4x+24y-40}{32}}$$

$$-x^2-4y^2-4x+24y-40 = -(x+2)^2+4 - 4(y-3)^2 + 36 - 40$$

$$= -(x+2)^2 - 4(y-3)^2$$

$$\frac{-(x+2)^2-4(y-3)^2}{32} = -\frac{1}{2}\frac{(x+2)^2}{4^2} - \frac{1}{2}\cdot\frac{(y-3)^2}{2^2}$$

$$\Rightarrow f_{XY}(x,y) = \underbrace{\frac{1}{\sqrt{2\pi}\cdot 4} e^{-\frac{1}{2}\frac{(x+2)^2}{4^2}}}_{f_X(x)} \cdot \underbrace{\frac{1}{\sqrt{2\pi}\cdot 2} e^{-\frac{1}{2}\frac{(y-3)^2}{2^2}}}_{f_Y(y)}$$

$f_X(x)$: $\sigma_X = 4$, $\mu_X = -2$

$f_Y(y)$: $\sigma_Y = 2$, $\mu_Y = 3$

$$x = \sigma_X \cdot x_n + \mu_X = 4x_n - 2$$

$$y = \sigma_Y y_n + \mu_Y = 2y_n + 3$$

b) $P(-0{,}4 \le x \le 2{,}8,\ -0{,}8 \le y \le 5{,}4)$

$= P(-0{,}4 \le 4x_n - 2 \le 2{,}8,\ -0{,}8 \le 2y_n + 3 \le 5{,}4)$ ↓ stoch. unab.

$= P(1{,}6 \le 4x_n \le 4{,}8) \cdot P(-3{,}8 \le 2y_n \le 2{,}4)$

$= P(0{,}4 \le x_n \le 1{,}2) \cdot P(-1{,}9 \le y_n \le 1{,}2)$

$= (\Phi(1{,}2) - \Phi(0{,}4)) \cdot \Phi(1{,}2) - (1 - \Phi(1{,}9))$

$= 0{,}8849 - 0{,}6554 \qquad 0{,}8849 - 1 + 0{,}9713$

$= 0{,}2295 \qquad 0{,}8562$

$= 0{,}1965$