ZA 142

(a) Geben Sie die Randdichtefunktion $f_x(x)$ an

$$f_{xy}(x,y) = \frac{1}{16\pi} e^{\frac{-x^2-4y^2-4x+24y-40}{32}} \qquad \left(f_{xy} = f_x \cdot f_y\right)$$

$$= \frac{1}{16\pi} e^{\frac{-(x+2)^2+4-4(y-3)^2+36-40}{32}}$$

$$= \frac{1}{16\pi} e^{-\left[\frac{(x+2)^2}{32} + \frac{4(y-3)^2}{32}\right]} \qquad f_x(x) = \int_{-\infty}^{+\infty} f_{xy}(x,y)\,dy$$

$$= \frac{1}{16\pi} e^{\frac{-(x+2)^2}{32}} \cdot e^{\frac{-(y-3)^2}{8}}$$

$$= \frac{1}{4} \frac{1}{\sqrt{2\pi}} e^{\frac{-(x+2)^2}{2 \cdot 4^2}} \cdot \frac{1}{2} \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-3)^2}{2 \cdot 2^2}}$$

$$= \underbrace{\frac{1}{\sqrt{2\pi}\,\sigma_x} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}}}_{\substack{f_x(x) \text{ mit} \\ \mu_x = -2 \\ \sigma_x = 4}} \cdot \underbrace{\frac{1}{\sqrt{2\pi}\,\sigma_y} e^{-\frac{(y-\mu_y)^2}{2\sigma_y^2}}}_{\substack{f_y(y) \text{ mit} \\ \mu_y = 3 \\ \sigma_y = 2}}$$

$$\underline{\underline{f_x(x) = \frac{1}{4\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x+2)^2}{4^2}}}}$$

$\Rightarrow$ stoch. unabh., da $f_{xy}(x,y) = f_x(x) \cdot f_y(y)$

(b) Berechne Sie $P(-0{,}4 \leq X \leq 2{,}8,\ -0{,}8 \leq Y \leq 5{,}4)$

ZA 14.2

$$P_b(-0{,}4 \le X \le 2{,}8,\ -0{,}8 \le Y \le 5{,}4) \underset{\text{s.u.}}{\overset{!}{=}} P(-0{,}4 \le X \le 2{,}8)\cdot$$

da $f_{XY}(x,y) = f_X(x)\cdot f_Y(y)$

$$P(-0{,}8 \le Y \le 5{,}4)$$

Transformation: $X = \sigma_X x_n + \mu_X = 4x_n - 2$

$$Y = \sigma_Y \cdot y_n + \mu_Y = 2y_n + 3$$

$$P(-0{,}4 \le X \le 2{,}8) = P(-0{,}4 \le 4x_n - 2 \le 2{,}8) =$$

$$P(0{,}4 \le x_n \le 1{,}2) = \Phi(1{,}2) - \Phi(0{,}4) =$$

$$0{,}8849 - 0{,}6554 = 0{,}2295$$

$$P(-0{,}8 \le Y \le 5{,}4) = P(-0{,}8 \le 2y_n + 3 \le 5{,}4) =$$

$$P(-1{,}9 \le y_n \le 1{,}2) = \Phi(1{,}2) - \Phi(-1{,}9) =$$

$$\Phi(1{,}2) - 1 + \Phi(1{,}9) = 0{,}8849 - 1 + 0{,}9713 = 0{,}8562$$

$$P_b = P(-0{,}4 \le X \le 2{,}8)\cdot P(-0{,}8 \le Y \le 5{,}4) = 0{,}2295 \cdot 0{,}8562$$

$$= 0{,}1965$$

$$\underline{\underline{P_b = 19{,}65\ \%}}$$