9.1 a) $\int_{-\infty}^{\infty} f(x)dx = 1$

$\int_0^{100} c \cdot sin(\frac{\pi}{100}x)dx = 1$
$= [-c \cdot \frac{100}{\pi} \cdot cos(\frac{\pi}{100}x)]$
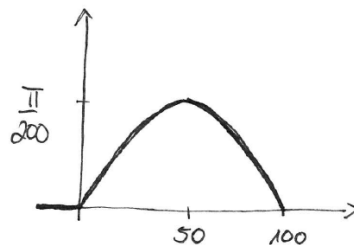$= -c \cdot \frac{100}{\pi}[-1 - 1]$
$\Rightarrow c = \frac{\pi}{200}$

$$f(x) = \begin{cases} \frac{\pi}{200} \cdot sin(\frac{\pi}{100}x) & 0 \leq x \leq 100 \\ 0 & sonst \end{cases}$$

b) partielle Itegration:

$\int_{-\infty}^{\infty} x \cdot f_\times(x)dx$

$\int_0^{100} x \cdot \frac{\pi}{200} \cdot sin(\frac{\pi}{100}x)dx$
$= [-\frac{100}{\pi} \cdot cos(\frac{\pi}{100}x) \cdot x \cdot \frac{\pi}{200}] - \int_0^{100} \frac{\pi}{200} \cdot (\frac{-100}{\pi}) \cdot cos(\frac{\pi}{100}x)dx$
$= -\frac{100}{\pi} \cdot (-1) \cdot \frac{\pi}{2} - \frac{-100}{\pi} \cdot 1 - \int_0^{100} \frac{\pi}{200} \cdot (\frac{-100}{\pi}) \cdot cos(\frac{\pi}{100}x)dx$
$= \frac{100}{\pi} \cdot \frac{\pi}{2} + 0 - \int_0^{100} \frac{\pi}{200} \cdot (\frac{-100}{\pi}) \cdot cos(\frac{\pi}{100}x)dx$
$= 50 - \int_0^{100}(-\frac{1}{2}) \cdot cos(\frac{\pi}{100}x)dx$
$= 50 + \frac{1}{2} \int_0^{100} cos(\frac{\pi}{100}x)dx$
$= 50 + \frac{1}{2} \cdot [\frac{100}{\pi} \cdot sin(\frac{\pi}{100}x)]$
$= 50 + \frac{1}{2} \cdot (\frac{100}{\pi} \cdot 0 - \frac{100}{\pi} \cdot 0) = 50$

oder geometrische Überlegung:

$\int_{-\infty}^{\infty} x \cdot f_\times(x)dx$

$\int_0^{100} x \cdot \frac{\pi}{200} \cdot sin(\frac{\pi}{100}x)dx$
$f(0) = 0$
$f(50) = \frac{\pi}{200}$
$f(100) = 0$
$\Rightarrow \mu = 50$

c) $\sigma_x{}^2 = E(x^2) - \mu_x{}^2)$

$\quad = \int_{-\infty}^{\infty} x^2 f_x(x) - \mu_x{}^2$

$\quad = \int_0^{100} x^2 \frac{\pi}{200} \cdot sin(\frac{\pi}{100}x)dx - \mu_x{}^2$

$\quad = \frac{\pi}{200} [\frac{2x}{\frac{\pi^2}{100^2}} \cdot sin(\frac{\pi}{100}x) - (\frac{x^2}{\frac{\pi}{100}} - \frac{2}{\frac{\pi^3}{100^3}}) \cdot cos(\frac{\pi}{100}x)]$

$\quad = \frac{\pi}{200} ((\frac{100\cdot100^2}{\pi} - \frac{2\cdot100^3}{\pi^3}) - \frac{2\cdot100^3}{\pi^3}) - 50^2$

$\quad = 2973,58 - 2500 = 473.58$

$\quad \sigma_x = 21.76$