ZAS. 1

Gesamtniederschlagmenge X $[l/m^2]$

$$f(t) = \begin{cases} c \sin\left(\frac{\pi}{100} t\right) & \text{für } 0 \le t \le 100 \\ 0 & \text{sonst} \end{cases}$$

a) $\int_{-\infty}^{+\infty} f(t)\, dt \overset{!}{=} 1$

$$\Leftrightarrow \int_0^{100} c \sin\left(\frac{\pi}{100} t\right) dx = 1$$

$$\Leftrightarrow \left[-c \frac{100}{\pi} \cos\left(\frac{\pi}{100} x\right)\right]_0^{100} = 1$$

$$\Leftrightarrow -c \frac{100}{\pi} \left[\cos \pi - \cos 0\right] = 1$$

$$\Leftrightarrow -c \frac{100}{\pi} \left[-1 - 1\right] = 1$$

$$\Leftrightarrow \frac{200\, c}{\pi} = 1$$

$$\Leftrightarrow \underline{c = \frac{\pi}{200}}$$

$$f(x) = \begin{cases} \frac{\pi}{200} \sin\left(\frac{\pi}{100} x\right) & \text{für } 0 \le x \le 100 \\ 0 & \text{sonst} \end{cases}$$

ZA 9.1

(b) Bestimmen Sie den Erwartungswert μ

$$\mu_x = \int_{-\infty}^{\infty} x \, f_X(x) \, dx$$

$$E(X) = \mu_x = \int_0^{100} x \cdot \frac{\pi}{200} \sin\left(\frac{\pi}{100} x\right) dx \quad \ldots \quad \text{langwierig!}$$

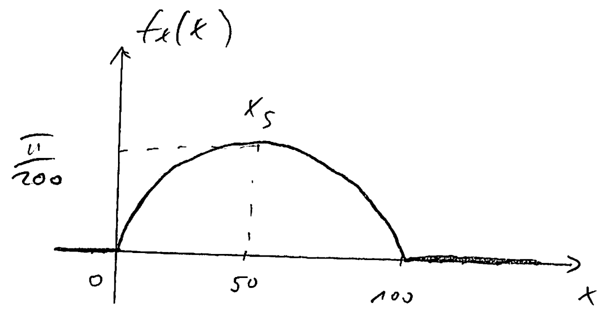
Geometrische Überlegung:

Wenn $f_X(x)$ symmetrische Dichtefunktion, dann $E(X) = x_s$

$f(0) = 0$

$f(50) = \frac{\pi}{200}$

$f(100) = 0$



$\Rightarrow$ $\underline{\mu_x = x_s = 50}$

(c) Bestimmen Sie die Standardabweichung σ!

$$\sigma_x^2 = E\left((X - \mu_x)^2\right) = \int_{-\infty}^{\infty} (x - \mu_x)^2 f_X(x) \, dx$$

$$\sigma_x^2 = E(X^2) - \mu_x^2$$

$$= \int_{-\infty}^{\infty} x^2 f_X(x) \, dx - \mu_x^2$$

ZA 9.1

$$\sigma_x^2 = \int_0^{100} x^2 \frac{\pi}{200} \sin\left(\frac{\pi}{100} x\right) dx - \mu_x^2$$

$$\underset{\uparrow}{=} \frac{\pi}{200} \left[\frac{2x}{\frac{\pi^2}{100^2}} \sin\left(\frac{\pi}{100} x\right) - \left(\frac{x^2}{\frac{\pi}{100}} - \frac{2}{\frac{\pi^3}{100^3}} \right) \cos\left(\frac{\pi}{100} x\right) \right]_0^{100} - \mu_x^2$$

Hinweis

$$= \frac{\pi}{200} \left[\frac{100^2 \cdot 2 \cdot 100}{\pi^2} \underbrace{\sin\left(\frac{\pi}{100} 100\right)}_{0} - \left(\frac{100 \cdot 100^2}{\pi} - \frac{2 \cdot 100^3}{\pi^3} \right) \underbrace{\cos\left(\frac{\pi \cdot 100}{100}\right)}_{-1} - \left(\underbrace{\frac{100^2 \cdot 2 \cdot 0}{\pi^2}}_{0} \sin(0) - \left(\underbrace{\frac{100 \cdot 0}{\pi}}_{0} - \frac{2 \cdot 100^3}{\pi^3} \right) \underbrace{\cos(0)}_{1} \right) \right] - 50^2$$

$$= \frac{\pi}{200} \left(\left(\frac{100 \cdot 100^2}{\pi} - \frac{2 \cdot 100^3}{\pi^3} \right) - \frac{2 \cdot 100^3}{\pi^3} \right) - 50^2$$

$$= \frac{\pi}{200} \left(\frac{100 \cdot 100^2}{\pi} - \frac{4 \cdot 100^3}{\pi^3} \right) - 50^2$$

$$= 2500 - \frac{20\,000}{\pi^2} = 473{,}58$$

$\underline{\sigma_x = 21{,}76}$