Zusatzaufgabe 14.1

$$f_{XY_1}(x,y) = \frac{1}{4}\left[\delta(x-1)\delta(y-1) + \delta(x-1)\delta(y+1) + \delta(x+1)\delta(y+1) + \delta(x+1)\delta(y-1)\right]$$

$$f_{XY_2}(x,y) = \frac{1}{4}\left[\delta(x-1)\delta(y) + \delta(y-1)\delta(x) + \delta(x+1)\delta(y) + \delta(y+1)\delta(x)\right]$$

a) $$f_{X_1}(x) = \int_{-\infty}^{+\infty} f_{XY_1}(x,y)\,dy = \frac{1}{4}\left[\delta(x-1) + \delta(x-1) + \delta(x+1) + \delta(x+1)\right]$$

$$f_{Y_1}(y) = \int_{-\infty}^{+\infty} f_{XY_1}(x,y)\,dx = \frac{1}{4}\left[\delta(y-1) + \delta(y+1) + \delta(y+1) + \delta(y-1)\right]$$

$$f_{X_1}(x) = \frac{1}{2}\delta(x-1) + \frac{1}{2}\delta(y+1)$$

$$f_{Y_1}(y) = \frac{1}{2}\delta(y-1) + \frac{1}{2}\delta(y+1)$$

$$f_{X_1}(x)\cdot f_{Y_1}(y) = \frac{1}{4}\delta(x-1)\delta(y-1) + \frac{1}{4}\delta(x-1)\delta(y+1) + \frac{1}{4}\delta(y+1)\delta(y-1) + \frac{1}{4}\delta(y+1)\delta(y+1)$$

$$= f_{XY_1}(x,y) \quad \square$$

$$f_{X_2}(x) = \int_{-\infty}^{+\infty} f_{XY_2}(x,y)\,dy = \frac{1}{4}\left[\delta(x-1) + \delta(x) + \delta(x+1) + \delta(x)\right]$$

$$= \frac{1}{4}\delta(x-1) + \frac{1}{4}\delta(x+1) + \frac{1}{2}\delta(x)$$

$$f_{Y_2}(y) = \int_{-\infty}^{+\infty} f_{XY_2}(x,y)\,dx = \frac{1}{4}\left[\delta(y) + \delta(y-1) + \delta(y) + \delta(y+1)\right]$$

$$= \frac{1}{4}\delta(y+1) + \frac{1}{4}\delta(y-1) + \frac{1}{2}\delta(y)$$

$$f_{X_2}(x)\cdot f_{Y_2}(y) = \underline{\frac{1}{16}\delta(x-1)\delta(y+1)} + \frac{1}{16}\delta(x-1)\delta(y-1) + \dots$$

$$f_{X_2}(x)\cdot f_{Y_2}(y) \neq f_{XY_2}(x,y)$$

b) $C_{xy} = E(XY) - \mu_x \mu_y$ $\qquad \rho_{xy} = \frac{C_{xy}}{\sigma_x \sigma_y}$

$\underline{\underline{\rho_{XY_1} = 0}}$ stochastische Unabhängigkeit

$\rho_{XY_2} = \frac{C_{XY2}}{\sigma_{X2}\sigma_{Y_2}}$ $\qquad C_{XY2} = E(X_2 Y_2) - \mu_{X_2}\mu_{Y_2}$

unkorreliert

$\mu_{X2} = \mu_{Y2} = 0$ (Symmetrie)

$E(X_2 Y_2) = 0$ (mindestens eine ZV immer 0)

$\underline{\underline{\rho_{XY_2} = 0}}$ $\qquad \Rightarrow C_{XY_2} = 0$

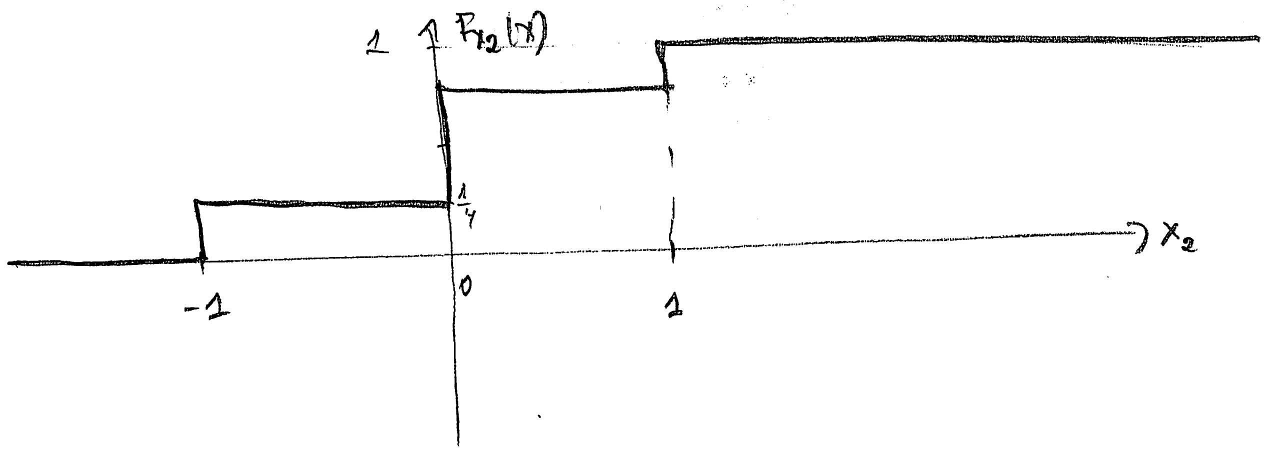
c) $F_{XY_1}(0,-0{,}5) = P(X \leq 0, Y \leq -0{,}5)$
$= P(X=-1, Y=-1) = \underline{\underline{\frac{1}{4}}}$

$F_{XY_1}(0{,}5\ 0{,}5) = P(X \leq 0{,}5, Y \leq 0{,}5)$
$= P(X=-1, Y=-1) = \underline{\underline{\frac{1}{4}}}$

$F_{XY_2}(0, 0{,}5) = P(X \leq 0, Y \leq -0{,}5)$
$= P(X=0, Y=-1) = \underline{\underline{\frac{1}{4}}}$

$F_{XY_2}(0{,}5\ 0{,}5) = P(X \leq 0{,}5, Y \leq 0{,}5)$
$= P(X=0, Y=-1) + P(X=-1, Y=0)$
$= \frac{1}{4} + \frac{1}{4} = \underline{\underline{\frac{1}{2}}}$

$F_{X_2}(x) = \int_{-\infty}^{x} f_{X_2}(z)\,dz = \int_{-\infty}^{x} \frac{1}{4}\left[\delta(z-1) + \delta(z+1) + 2\delta(z)\right]\,dz$
$= \int_{-\infty}^{-1} \frac{1}{4}\delta(z+1)\,dx + \int_{-1}^{0} \frac{2}{4}\delta(z)\,dz + \int_{0}^{1} \frac{1}{4}\delta(z-1)\,dz$


1
$F_{X_2}(x)$
$\frac{1}{4}$
X_2
-1
0
1