OMNISCALE GRAVITY — CONSERVATIVE WEAK-FIELD DIFFERENTIAL EXPANSION VIA χ (V4.6)

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2025-08-15

EXECUTIVE SUMMARY

One potential for light and motion, on an operationally flat background. We package the tested, weak-field part of gravity in a scalar–metric language consistent with the published two-page explainer.[1] We use the linear identification

$$\boxed{\mathbf{\Phi \equiv c^2\chi}},$$

so that a *single* potential Φ governs both orbits and optics. We keep PPN $\gamma = 1$ optics and a conservative gravitational-wave sector (speed c, $+$ and $\times$ only, no scalar dipole). Classic tests (light bending, Shapiro delay, gravitational redshift, and Mercury's perihelion) match the usual 1PN formulas when the weak-field metric is taken with $\beta = \gamma = 1$. The gravitomagnetic sector is *assumed GR* for now and flagged as a deliverable: derive g_{0i} from a single action at 1PN so the frame-dragging gate becomes predictive. *Operationally flat* here means we compute observables with the effective weak-field metric on a flat background; we are *not* claiming a full non-linear geometry in this letter.

ASSUMPTIONS & GUARANTEES

One potential for light and motion, on an operationally flat background. We adopt a single weak–field potential Φ for both dynamics and optics, with the linear identification $\Phi \equiv c^2\chi$. The assumptions below define the conservative scope used throughout; the guarantees summarize the immediate consequences within this scope.

1. **(map)** $\Phi \equiv c^2\chi$.

2. **(Newtonian limit & closure)** Test-particle motion obeys $\ddot{\mathbf{x}} = -\nabla\Phi$. The potential Φ is sourced using *either* of the following mathematically equivalent low-g closures, ensuring a curl-free total field $\mathbf{g} = -\nabla\Phi$ even for non-spherical systems:

$$\text{Closure A (QUMOND-style):}\quad \nabla^2\Phi = \nabla\cdot\left[\nu\left(\frac{g_b}{a_0}\right)\nabla\Phi_b\right],\qquad \nabla^2\Phi_b = 4\pi G\,\rho_b,$$

$$\text{Closure B (effective density):}\quad \nabla^2\Phi = 4\pi G\,(\rho_b+\rho_\chi),\qquad \rho_\chi = \frac{1}{4\pi G}\nabla\cdot[(\nu-1)\,\nabla\Phi_b].$$

   Here $g_b \equiv |\nabla\Phi_b|$, a_0 is the global low-acceleration scale, and $\nu(y_b) = 1/2\left[1+\sqrt{1+4/y_b}\right]$.

3. **(weak-field/1PN optics & dynamics)** In isotropic gauge we use the printed weak-field/1PN line element with $\beta = \gamma = 1$, securing the standard 1PN formulas for light and orbits.

4. **(same Φ)** A single potential Φ drives both motion and optics (lensing, Shapiro, redshift).

[1] See the explainer's front panel for the same mapping, classical tests, and falsifiers.

5. **(GW sector; conservative)** $v_{\mathrm{gw}} = c$; $+$ and $\times$ polarizations only; no scalar dipole; no kinetic mixing with χ in this baseline.

6. **(frame dragging)** Adopt $g_{0i} = -4V_i/c^3$ (Lense–Thirring) for now; we will derive g_{0i} from a single action so the frame-dragging gate becomes predictive.

7. **(operationally flat background)** We compute observables with the effective weak-field metric as a bookkeeping device; we do not claim a full non-linear completion in this letter.

Guarantees. With $\beta = \gamma = 1$, classical weak-field and 1PN tests (light bending, Shapiro delay, gravitational redshift, and perihelion advance) match their standard forms. Any deviation in frame dragging, GW speed/polarizations, or Solar-System PPN beyond stated bounds would falsify the present baseline or force changes to items (3)/(6).

MINIMAL CONSERVATIVE CLOSURE IN LOW-g

OPTION A (BASELINE) — QUMOND-STYLE PDE (CURL-FREE IN GENERAL)

$$\nabla^2\Phi = \nabla\cdot[\nu(g_b/a_0)\,\nabla\Phi_b], \qquad \nu(y_b) = 1/2\left(1+\sqrt{1+4/y_b}\right)$$

Explanation. Solve Poisson for the Newtonian potential Φ_b from baryons ($\nabla^2\Phi_b = 4\pi G\rho_b$), then apply the field-side enhancement ν and take a divergence. This guarantees a scalar Φ (hence curl-free $\mathbf{g} = -\nabla\Phi$) even for non-spherical systems, making the thin-lens and Fermat formulas safe with the same Φ.

OPTION B (EQUIVALENT) — EFFECTIVE (“PHANTOM”) DENSITY

$$\nabla^2\Phi = 4\pi G(\rho_b+\rho_\chi), \qquad \rho_\chi = \frac{1}{4\pi G}\nabla\cdot[(\nu-1)\nabla\Phi_b]$$

Explanation. Write the same closure as an effective density ρ_χ and solve a standard Poisson equation. This is convenient in Newtonian N-body/Poisson solvers. Options A and B are mathematically equivalent under the same ν.

GLOSSARY (SYMBOLS & TERMS)

Symbol/term	Meaning (plain English)	Units
χ	Scalar expansion map; we identify $\Phi \equiv c^2\chi$.	dimensionless
Φ	Weak-field potential used for clocks, rulers, light paths, and orbits.	$\mathrm{m^2\,s^{-2}}$
U	Newtonian sign convention $U \equiv -\Phi$.	$\mathrm{m^2\,s^{-2}}$
a_0	Global low-acceleration scale in mapping/closure.	$\mathrm{m\,s^{-2}}$
α	Coupling linking LOS structure to fractional H_0 shifts.	dimensionless
$W(s)$	Survey window along distance s; W_N is normalized.	—
$K(s)$	Line-of-sight kernel (near-side or magnitude).	—
H_0	Present-day Hubble constant.	$\mathrm{s^{-1}}$
$\mu(y)$	Source-side interpolation with $y \equiv g/a_0$.	—
$\nu(y_b)$	Field-side response with $y_b \equiv g_b/a_0$.	—
ρ_b, ρ_χ	Baryonic density; effective ("phantom") density.	$\mathrm{kg\,m^{-3}}$
g, g_b	Total acceleration; baryonic acceleration.	$\mathrm{m\,s^{-2}}$
b	Impact parameter in lensing.	length
θ	Image angle; ψ lensing potential; τ Fermat potential.	angle (arb.)
v_c	Circular speed in rotation curves.	km/s
R, R_d, Σ_0	Cylindrical radius; disk scale length; central surface density.	kpc; kpc; $M_\odot$ kpc^{-2}

GATE LEDGER

Gate	Promise (must hold)	Status now	Clear falsifier
Rotation curves (RC)	One global acceleration scale a_0 with stellar baryons fits $v_c(r)$ across galaxies; no per-galaxy halo.	Provisional pass. Single global $a_0 = 1.1 \times 10^{-10}\ \mathrm{m/s^2}$ reproduces disk shapes with photometry-fixed baryons.	Requires galaxy-by-galaxy tuning or systematic failure across a representative set.
Strong lensing / time delays	Same potential reproduces Einstein radii with GR optics (achromatic bending, $\gamma = 1$).	In progress. Next: ellipticity + catalog shear; if a coherent residual remains, test one global high-g taper $\mu_{\rm hi}(g_b/a_0)$.	Needs $\gamma \neq 1$ or extra fields; or per-lens tuning beyond one universal parameter.
Local H_0 anisotropy & flows	Environment dependence: voids ↑, clusters ↓; bulk flows align with $\nabla\chi$; α remains Solar-System safe.	Open gate. Pipeline and joint-fit plan defined; full survey-window run pending.	No correlation at $\lesssim$ 1–2% after systematics; or fitted α violates Solar-System/GW bounds.
Solar System (optics/dynamics)	Shapiro, light bending, redshift, and 1PN precession match GR when $\beta = \gamma = 1$.	Pass in baseline; printed metric with $\beta = \gamma = 1$.	Any significant deviation in PPN γ, β or Lense–Thirring beyond current bounds.
GWs / Pulsars	$v_{\rm gw} = c$; $+/\times$ only; binary-pulsar phasing matches GR when dipole ≈ 0.	Pass (conservative sector). No kinetic mixing; scalar dipole suppressed.	Detectable $\lvert v_{\rm gw} - c\rvert/c$ or excess dipole radiation.

OPERATIONAL DEFINITION OF “SCALE-CASCADE” (TESTABLE)

Let $\hat{n}$ be a line-of-sight unit vector. For nested comoving radii $L_1 < L_2 < \cdots < L_N$, define

$$\mathcal{C}(L_k;\hat{n}) \equiv \langle \hat{n}\cdot\nabla\chi\rangle_{|\mathbf{x}|\le L_k}.$$

We say the field exhibits coherent scale cascade along $\hat{n}$ on $[L_1, L_N]$ if sign $\mathcal{C}(L_k;\hat{n})$ is the same for all k (within a small tolerance). In our baseline this is operationalized through the near-side kernel below: coherent sign predicts the sign of the local H_0 bias, with voids (positive $\mathcal{C}$) raising and clusters (negative $\mathcal{C}$) lowering locally inferred H_0.

FROM METRIC TO THE H_0 ANSATZ (ONE-PAGE DERIVATION)

Operationally flat disclaimer. In this release we use the weak-field/1PN line element as a bookkeeping device for observables on a flat background; we are not presenting a full non-linear geometry.

Metric. In isotropic gauge the weak-field/1PN line element is

$$ds^2 = -\left(1 - \frac{2U}{c^2} + \frac{2U^2}{c^4}\right)c^2dt^2 + \left(1 + \frac{2U}{c^2}\right)d\mathbf{x}^2, \qquad U \equiv -\Phi.$$

Along a null ray the coordinate travel time differs from the Minkowski value by the Shapiro integral, $\Delta t_{\text{Shapiro}} \simeq (1 + \gamma)c^{-3}\int U\,d\ell$ with $\gamma = 1$, and gravitational redshift calibrates clocks via $\Delta\nu/\nu \simeq -\Delta\Phi/c^2$.

LOS functional and normalization. In a distance-ladder estimator that fits $cz \approx H_0 D$ over a finite window with selection $W(s)$, anisotropic structure along the line of sight biases the fitted slope. Linearizing about a homogeneous background and writing $r(s) = \sqrt{b^2 + (s - s_c)^2}$ with $\cos\theta(s) = (s - s_c)/r(s)$, we obtain

$$\frac{\Delta H}{H_0} \;\approx\; \alpha\,\bar{I}, \qquad \bar{I} \;\equiv\; s_0\int_0^{r_{\max}} W_N(s)\,\frac{\partial_r\Phi(r(s))}{c^2}\cos\theta(s)\,K(s)\,ds,$$

where $W_N(s) \equiv W(s)/\int_0^{r_{\max}} W(s')\,ds'$ is a unit-normalized window. We fix the units by

$$s_0 \equiv 1\ \text{Mpc},$$

so that $\bar{I}$ is dimensionless; alternative choices of s_0 simply rescale α.

Near-side kernel used in calibration plots. The “near-side” kernel employed in our Monte-Carlo calibration is defined explicitly by

$$K_{\text{near}}(s;s_c) \equiv \Theta(s_c - s),$$

with Θ the Heaviside step (1 for positive argument, 0 otherwise). With this choice the sign pattern follows immediately from the $\cos\theta$ projection: over-densities on the near side (clusters) bias the local H_0 estimate low, while under-densities (voids) bias it high. (A magnitude-weighted kernel can be substituted for comparison without changing the formalism.)

STATUS OF α.

In v4.6, α is phenomenological. In a future χ action, α will be fixed by the linear response of χ to large-scale structure (its propagator), projected with the same LOS kernel $K(s)$ used in the estimator.

WEAK-FIELD/1PN TESTS (MATH AND PLAIN ENGLISH)

PRINTED 1PN METRIC (BASELINE).

In isotropic gauge we use the **metric** which corresponds to PPN $\beta = \gamma = 1$ and secures the standard 1PN formulas for light and orbits.

WHAT THE METRIC IS SAYING (PLAIN ENGLISH).

The first bracket multiplies clock time: near mass ($U < 0$), clocks tick a bit slower. The second bracket multiplies spatial distances: rulers are stretched a tiny amount. On our *operationally flat* background, these are small calibration factors rather than a geometric curvature claim in this release. Together they reproduce familiar weak-field effects.

NEWTONIAN MECHANICS (GEODESIC ACTION).

Math. $S = -m\int \mathrm{d}\tau$, $\mathrm{d}\tau^2 = -\mathrm{d}s^2/c^2$. For $v \ll c$, $L \approx 1/2\, mv^2 - m\Phi \Rightarrow \ddot{\mathbf{x}} = -\nabla\Phi$.
Plain English. The action says “nature prefers the path that makes [kinetic energy − potential energy] small.” Varying it gives Newton’s law: things roll downhill in Φ.

GRAVITATIONAL REDSHIFT.

Math. $\mathrm{d}\tau \approx (1 + \Phi/c^2)\mathrm{d}t \Rightarrow \Delta\nu/\nu \approx -(\Phi_B - \Phi_A)/c^2$.
Plain English. Clocks deeper in the potential (closer to mass) tick a little slower, so signals from down low lose a touch of frequency as they climb out.

LIGHT BENDING (POINT MASS).

Math. $\Delta\theta = \frac{4GM}{c^2 b}$ for a point mass (with $\gamma = 1$).
Plain English. A gentle calibration gradient near mass changes how null rays advance; with $\gamma = 1$ we retain GR’s achromatic bending.

SHAPIRO TIME DELAY.

Math. $\Delta t_{\mathrm{Shapiro}} = (1+\gamma)\frac{GM}{c^3}\ln\frac{4r_E r_R}{b^2}$ with $\gamma = 1$.
Plain English. Signals skimming past mass take a tiny bit longer because the calibration factor is slightly larger there.

MERCURY’S PERIHELION (1PN).

Math. With $\beta = \gamma = 1$, $\Delta\varpi = \frac{6\pi GM}{a(1-e^2)c^2}$.
Plain English. The ellipse advances because calibration shifts are slightly stronger near the Sun, giving the orbit a forward nudge each lap.

FRAME DRAGGING (STATUS)

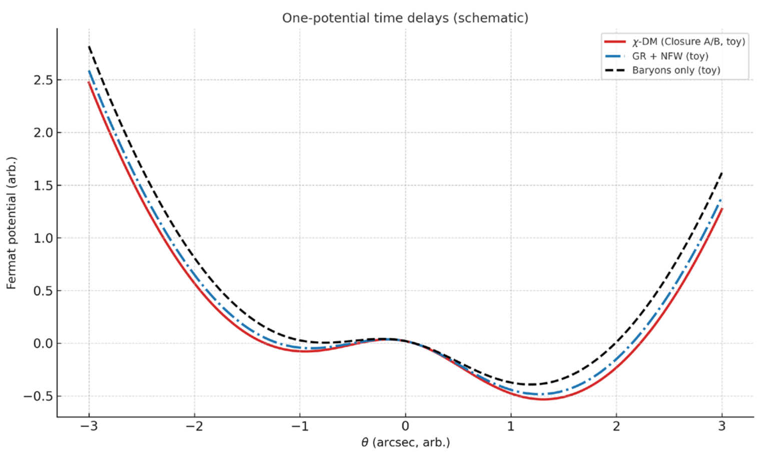
We currently assume the GR gravitomagnetic sector $g_{0i} = -4V_i/c^3$ (Lense–Thirring); we plan to derive g_{0i} from a single action so this gate becomes predictive. Until then, the frame-dragging test is a sanity check rather than a prediction.

LENSING AND TIME DELAYS FROM ONE POTENTIAL (SANITY CHECK)

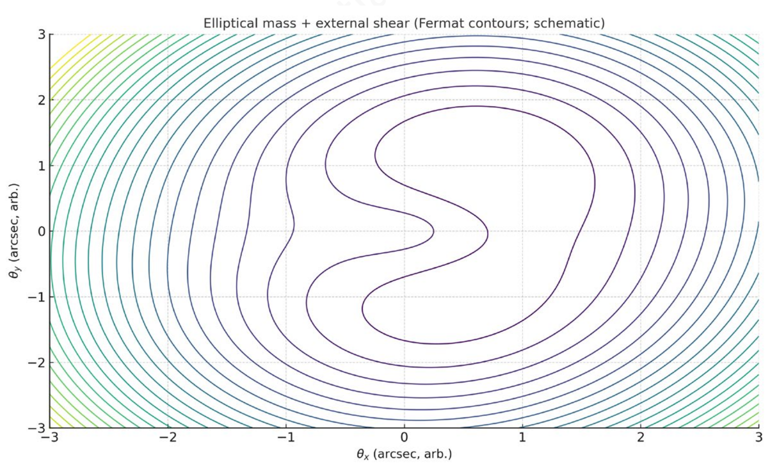
We keep a single weak-field potential Φ for both light and motion (PPN $\gamma = 1$). For an axisymmetric lens the thin-lens deflection at impact parameter R can be written as

$$\alpha(R) = \frac{2}{c^2}\int_{-\infty}^{\infty} \frac{\partial \Phi}{\partial R}\,\mathrm{d}z \quad \text{or} \quad \alpha(R) = \frac{4G\,M_{2\mathrm{D}}(<R)}{c^2 R},$$

where $M_{2\mathrm{D}}(<R)$ is the projected mass inside R. In our numerical checks we use the first expression so that any low-acceleration mapping that modifies $\partial_r\Phi$ automatically carries over to lensing. Fermat potentials for time delays use the same Φ.



One-potential time delays (schematic). ***Sources/assumptions:*** **Toy Fermat potential with baryons-only, GR+NFW, and χ-DM (Closure A/B). The shared Φ deepens minima when strengthened by a low-acceleration mapping.**

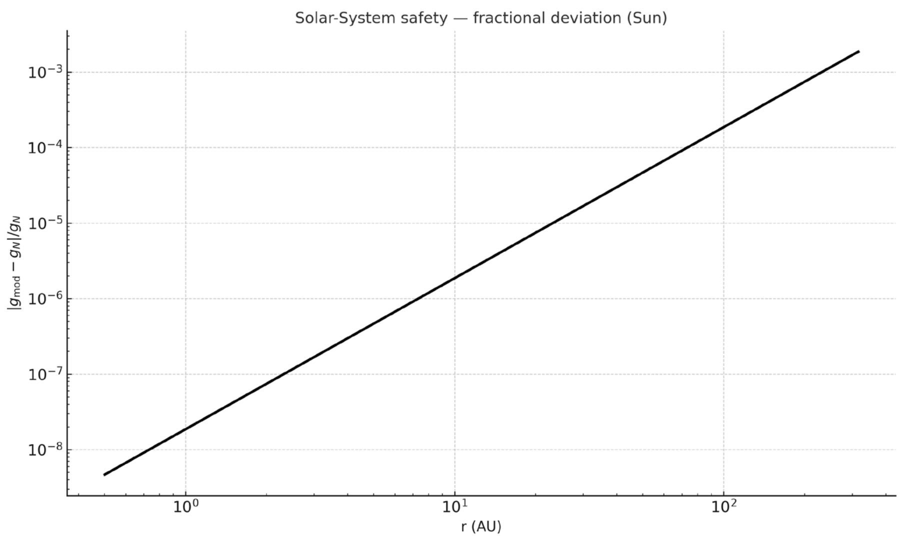


Elliptical mass + external shear (Fermat contours; schematic). ***Sources/assumptions:*** **Elliptical toy potential (softened) plus a weak external shear rotated by 30°. The same Φ governs time-delay surfaces and dynamics; optics keep $\gamma = 1$.**

SOLAR-SYSTEM SAFETY: NUMBERS AND CROSS-WALK

For the Sun and the simple χ-DM mapping used in disk fits, the fractional acceleration deviation $\delta g \equiv |g_{\mathrm{mod}} - g_N|/g_N$ scales roughly like a_0/g_N in the high-acceleration regime. Using $a_0 = 1.1 \times 10^{-10}\ \mathrm{m/s^2}$ we obtain the values in the following table.

Radius	**δg** (baseline mapping)	**Comment**
1 AU	$\sim 2 \times 10^{-8}$	Deep in high-g regime; essentially Newtonian/GR.
10 AU	$\sim 2 \times 10^{-6}$	Well within standard navigation tolerances.
100 AU	$\sim 2 \times 10^{-4}$	Outer-heliospheric scale; for future ephemeris cross-checks.

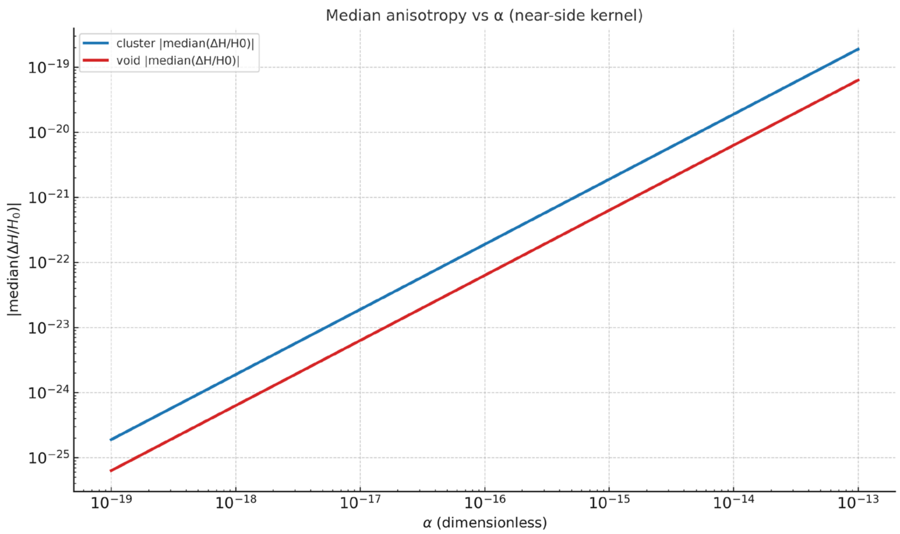


Solar-System safety. ***Sources/assumptions:*** **Sun as a monopole; χ-DM mapping applied to the Newtonian field. Shown is the fractional deviation $|g_{\mathrm{mod}} - g_N|/g_N$ vs r.**

MONTE-CARLO CALIBRATION: KERNEL, WINDOW, AND PER-α RESULTS

We use the near-side, gauge-safe LOS functional so that $\Delta H/H_0 = \alpha\,\bar{I}$. The window $W(s)$ is normalized; magnitude-kernel rows are included for comparison.

Environment & kernel	**median($\bar{I}$)**	$\mathbf{p_{16}}$	$\mathbf{p_{84}}$
Cluster — near-side	-1.885×10^{-6}	-3.370×10^{-6}	-1.353×10^{-6}
Void — near-side	$+6.292 \times 10^{-7}$	$+1.226 \times 10^{-7}$	$+4.498 \times 10^{-6}$
Cluster — magnitude	$+3.770 \times 10^{-6}$	$+2.685 \times 10^{-6}$	$+6.804 \times 10^{-6}$
Void — magnitude	$+1.564 \times 10^{-6}$	$+4.091 \times 10^{-7}$	$+9.660 \times 10^{-6}$



Median anisotropy $|\mathrm{median}(\Delta H/H_0)|$ vs α (near-side kernel). *Sources/assumptions:* synthetic Monte Carlo using a unit-normalized window; Linear scaling with α is evident.

NOTATION STANDARDIZATION & ν–μ TABLE

We use χ-*DM* for the low-acceleration mapping with one global a_0.

The interpolating/fade-out function is denoted $\mu(\cdot)$; any future high-g taper will be written as $\mu_{\mathrm{hi}}(g_b/a_0)$ (not ν).

Symbol	Meaning	Baseline form
$\mu(y)$	source-side interpolation with $y \equiv g/a_0$	$\mu(y) = \dfrac{y}{1+y}$
$\nu(y_b)$	field-side response with $y_b \equiv g_b/a_0$	$\nu(y_b) = \dfrac{1}{2}\left(1 + \sqrt{1 + \dfrac{4}{y_b}}\right)$

RELATED WORK (BRIEF, NON-EXHAUSTIVE)

Expansion-based or emergent-gravity intuitions have appeared before, notably in work by Erik Verlinde (entropic/emergent gravity) and Mark McCutcheon (expansion-based ideas). There are also modified-gravity frameworks such as MOND/TeVeS, MOG, and $f(R)$. The present letter is conservative: it keeps GR's tested optics ($\gamma = 1$), enforces one shared Φ for motion *and* lensing/time delays, and states falsifiers upfront.[2]

ACKNOWLEDGEMENTS

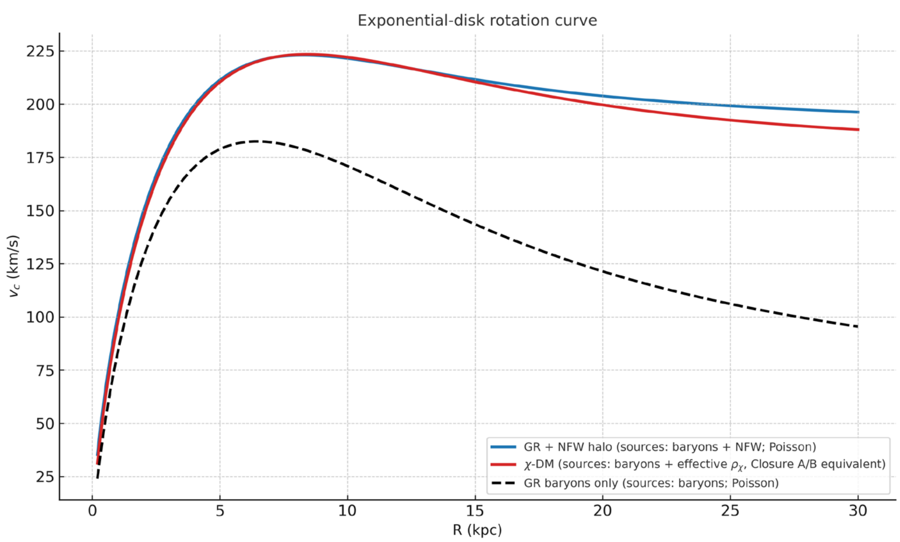
We thank early readers and colleagues; maintainers of open-source tools; and the ephemeris, precision clock, gravitational-wave, and large-scale-structure communities whose public results constrain our safe-regime choices. We thank OpenAI for the wizardry behind ChatGPT. We are grateful to **Zé Ayala**, **Robert Ryan**, **Joon Yun**, and **Mariano Muñoz** for inspiring conversations, and we thank the human author's family for their endless support and encouragement. Any errors remain our own.

APPENDIX A: EXPONENTIAL-DISK ROTATION CURVE (FORMULA)

For a thin exponential disk with surface density $\Sigma(R) = \Sigma_0 e^{-R/R_d}$ and $\Sigma_0 = M_d/(2\pi R_d^2)$, the circular speed is

$$v_c^2(R) = 4\pi G \Sigma_0 R_d \, y^2 \, [I_0(y)K_0(y) - I_1(y)K_1(y)], \qquad y \equiv \frac{R}{2R_d},$$

with modified Bessel functions I_n and K_n.



Exponential-disk rotation curve. ***Sources used:*** **GR baryons only (Poisson); GR+NFW (baryons+NFW); χ-DM $\equiv$ baryons + effective ρ_χ (Closure A/B).** ***Styles:*** **GR+NFW (blue solid), χ-DM (red solid), GR baryons only (black dashed, overlaid).**

[2] Consistency with our published 2-page explainer is intentional: same mapping $\Phi \equiv c^2\chi$, same classic tests and falsifiers.

APPENDIX B: HERNQUIST LENS — DEFLECTION FROM ONE POTENTIAL

Consider a spherical Hernquist profile with total baryonic mass M and scale a. The baryonic gravitational acceleration is

$$g_b(r) \;=\; \frac{GM}{(r+a)^2}.$$

With the baseline low-g mapping ("χ-DM") we use

$$g(r) \;=\; \frac{1}{2}\left[g_b(r) + \sqrt{g_b(r)^2 + 4\,g_b(r)\,a_0}\right],$$

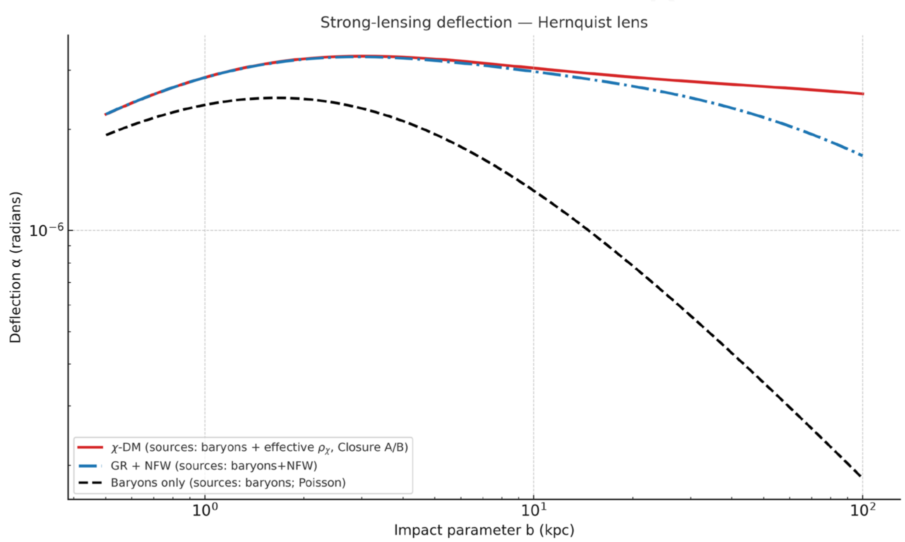
and define the potential by $\partial_r \Phi = g(r)$. For an axisymmetric thin lens with cylindrical impact parameter R the deflection can be written in either of two equivalent one-potential forms,

$$\alpha(R) \;=\; \frac{2}{c^2}\int_{-\infty}^{\infty} \frac{\partial \Phi}{\partial R}(R,z)\,dz \;=\; \frac{2}{c^2}\int_{-\infty}^{\infty} g\,(r)\,\frac{R}{r}\,dz, \qquad r \equiv \sqrt{R^2+z^2}.$$

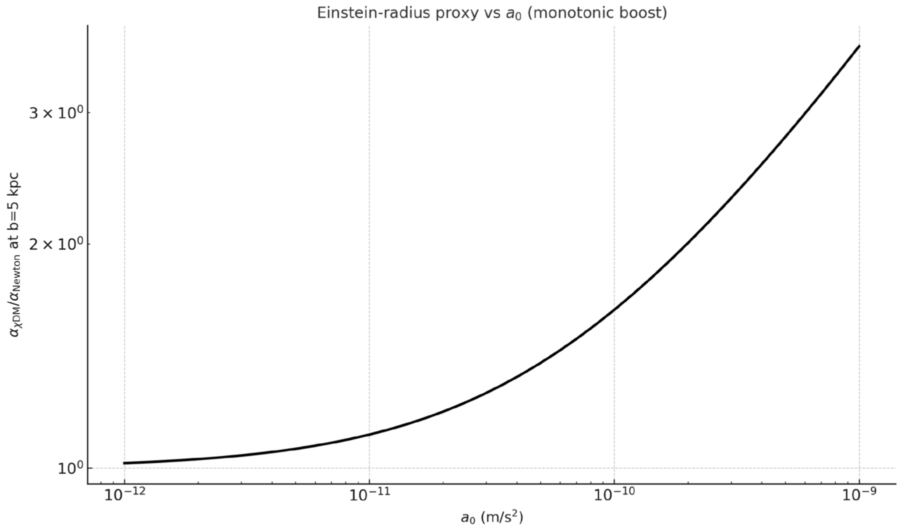
Implementation note. In our code we evaluate the z-integral numerically (adaptive quadrature) using the $\partial_R \Phi$ form above; this keeps motion and optics tied to the same Φ by construction. The expression reduces to the standard projected-mass result when $g \to g_b$ (baryons only). For GR+NFW comparisons we use

$$g(r) \;=\; g_b(r) + g_{\mathrm{NFW}}(r), \qquad g_{\mathrm{NFW}}(r) \;=\; \frac{G\,M_{\mathrm{NFW}}(<r)}{r^2},$$

with the usual M_{200} and concentration parameters defining $M_{\mathrm{NFW}}(<r)$.



Strong-lensing deflection for a Hernquist baryonic lens. *Sources used:* Baryons-only (Poisson), GR+NFW (baryons+NFW), χ-DM $\equiv$ baryons + effective ρ_χ (Closure A/B). Optics keep $\gamma = 1$ with the same Φ for light and motion.

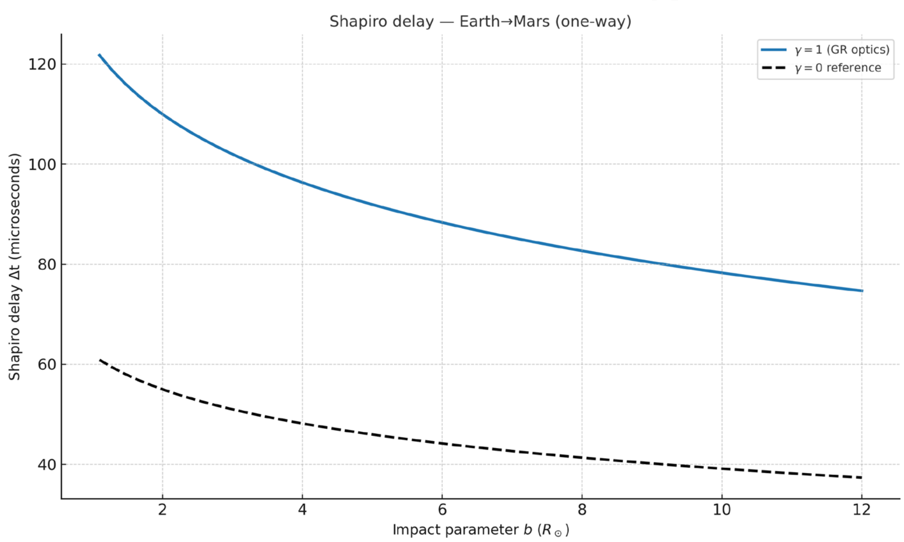


Einstein-radius proxy vs a_0 (monotonic). *Definition:* ratio $\alpha_{\chi\mathrm{DM}}/\alpha_{\mathrm{Newton}}$ at fixed $b = 5$ kpc. Larger a_0 increases the deflection in the low-acceleration regime.

APPENDIX C: SHAPIRO DELAY GEOMETRY (NEAR-CONJUNCTION LIMIT)

For emitter/receiver distances r_E, r_R from a point mass M and impact parameter $b \ll r_E, r_R$,

$$\Delta t_{\mathrm{Shapiro}} = (1+\gamma)\frac{GM}{c^3}\ln\frac{4r_E r_R}{b^2} + \mathcal{O}\left(\frac{b}{r_{E,R}}\right).$$



Shapiro delay (Earth→Mars, one-way). *Sources/assumptions:* point-mass Sun, $\gamma = 1$ (GR optics) vs a $\gamma = 0$ reference. The $\gamma = 1$ curve is plotted first; the dashed $\gamma = 0$ reference is overlaid last.

APPENDIX D: χ-DM MAPPING AND CLOSURES (A & B EXPLAINED)

We use the common one-parameter family with $\mu(y) = y/(1+y)$, where $y = g/a_0$ and g is the total acceleration. The source-side closure $g_b = \mu(g/a_0)\, g$ gives the closed form

$$g = \frac{1}{2}\left[g_b + \sqrt{g_b^2 + 4 g_b a_0}\right],$$

used in the spherical/axially symmetric checks in this letter. For general geometries, the mapping is implemented via:
Option A (QUMOND-style PDE). Solve $\nabla^2\Phi_b = 4\pi G\rho_b$, then compute $\nabla^2\Phi = \nabla\cdot[\nu(g_b/a_0)\nabla\Phi_b]$.
Option B (effective density). Define $\rho_\chi = (4\pi G)^{-1}\nabla\cdot[(\nu-1)\nabla\Phi_b]$ and solve $\nabla^2\Phi = 4\pi G(\rho_b + \rho_\chi)$.
Either route yields a scalar, curl-free field with the same Φ used for motion and optics.