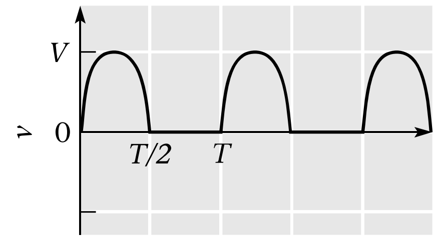
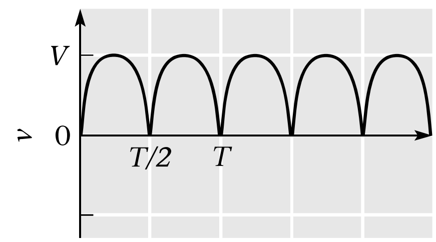
TABLE 1.1 Fourier Series for Common Repetitive Waveforms

1. Half-wave rectified sine wave



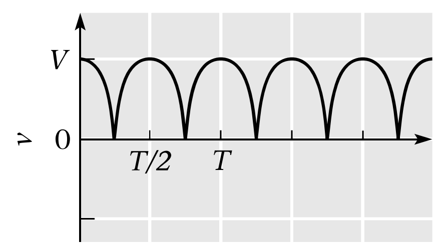
$$v(t) = \frac{V}{\pi} + \frac{V}{2}\ \sin \omega t - \frac{2V}{\pi}\left(\frac{\cos\ 2\omega t}{1 \times 3} + \frac{\cos\ 4\omega t}{3 \times 5} + \frac{\cos\ 6\omega t}{5 \times 7} + \cdots\right)$$

2. Full-wave rectified sine wave
   (a) With time zero at voltage zero



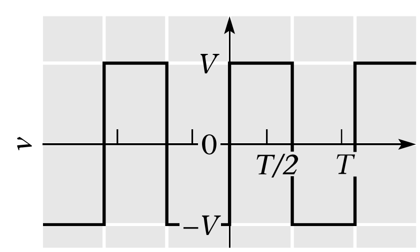
$$v(t) = \frac{2V}{\pi} - \frac{4V}{\pi}\left(\frac{\cos\ 2\omega t}{1 \times 3} + \frac{\cos\ 4\omega t}{3 \times 5} + \frac{\cos\ 6\omega t}{5 \times 7} + \cdots\right)$$

   (b) With time zero at voltage peak



$$v(t) = \frac{2V}{\pi}\left(1 + \frac{2\cos\ 2\omega t}{1 \times 3} - \frac{2\cos\ 4\omega t}{3 \times 5} + \frac{2\cos\ 6\omega t}{5 \times 7} - \cdots\right)$$

3. Square wave
   (a) Odd function

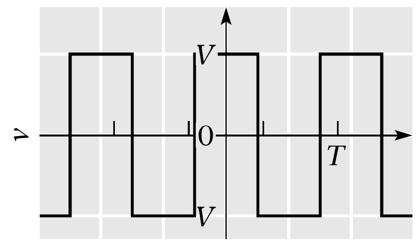


$$v(t) = \frac{4V}{\pi}\left(\sin\ \omega t + \frac{1}{3}\sin\ 3\omega t + \frac{1}{5}\sin\ 5\omega t + \cdots\right)$$

(*continues*)

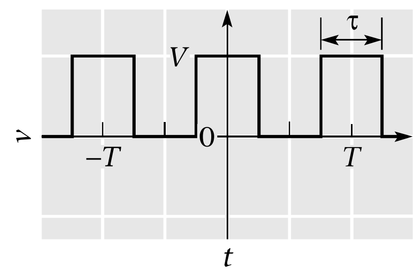
TABLE 1.1 (*continued*)

(b) Even function



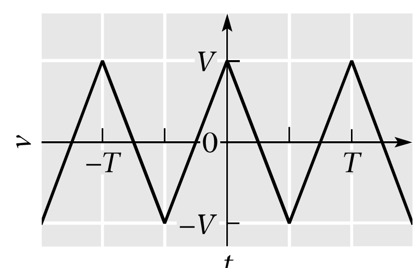
$$v(t) = \frac{4V}{\pi}\left(\cos\,\omega t - \frac{1}{3}\cos\,3\omega t + \frac{1}{5}\cos\,5\omega t - \cdots\right)$$

4. Pulse train



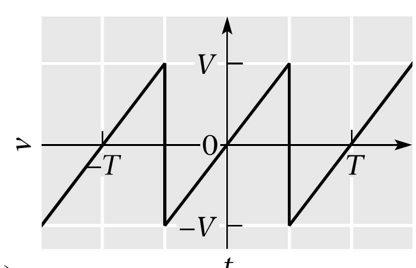
$$v(t) = \frac{V\tau}{T} + \frac{2V\tau}{T}\left(\frac{\sin\,\pi\tau/T}{\pi\tau/T}\cos\,\omega t + \frac{\sin\,2\pi\tau/T}{2\pi\tau/T}\cos\,2\omega t + \frac{\sin\,3\pi\tau/T}{3\pi\tau/T}\cos\,3\omega t + \cdots\right)$$

5. Triangle wave



$$v(t) = \frac{8V}{\pi^2}\left(\cos\,\omega t + \frac{1}{3^2}\cos\,3\omega t + \frac{1}{5^2}\cos\,5\omega t + \cdots\right)$$

6. Sawtooth wave
   (a) With no dc offset

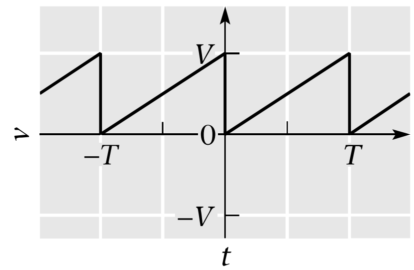


$$v(t) = \frac{2V}{\pi}\left(\sin\,\omega t - \frac{1}{2}\sin\,2\omega t + \frac{1}{3}\sin\,3\omega t - \cdots\right)$$

(*continues*)

TABLE 1.1 (*continued*)

(b) Positive-going



$$v(t) = \frac{V}{2} - \frac{V}{\pi}\left(\sin\,\omega t + \frac{1}{2}\sin\,2\omega t + \frac{1}{3}\sin\,3\omega t + \cdots\right)$$

SOLUTION

A square wave is another signal with a simple Fourier representation, although not quite as simple as for a sine wave. For the signal shown in Figure 1.6(a), the frequency is 1 kHz, as before, and the peak voltage is 1 V.

According to Table 1.1, this signal has components at an infinite number of frequencies: all odd multiples of the fundamental frequency of 1 kHz. However, the amplitude decreases with frequency, so that the third harmonic has an amplitude one-third that of the fundamental, the fifth harmonic an amplitude of one-fifth that of the fundamental, and so on. Mathematically, a square wave of voltage with a rising edge at $t = 0$ and no dc offset can be expressed as follows (see Table 1.1):

$$v(t) = \frac{4V}{\pi}\left(\sin\,\omega t + \frac{1}{3}\sin\,3\omega t + \frac{1}{5}\sin\,5\omega t + \cdots\right)$$

where

V = peak amplitude of the square wave
ω = radian frequency of the square wave
t = time in seconds

For this example, the above equation becomes:

$$v(t) = \frac{4}{\pi}\left(\sin(2\pi \times 10^3 t) + \frac{1}{3}\sin(6\pi \times 10^3 t) + \frac{1}{5}\sin(10\pi \times 10^3 t) + \cdots\right)\mathrm{V}$$

This equation shows that the signal has frequency components at odd multiples of 1 kHz, that is, at 1 kHz, 3 kHz, 5 kHz, and so on. The 1-kHz component has a peak amplitude of

$$V_1 = \frac{4}{\pi} = 1.27\ \mathrm{V}$$