

Problem Set 1

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Sets

Let $A = 1, 5, 10$ and $b = 1, 2, \dots, 10$

1. Is $A \subset B$, $B \subset A$, both, or neither?
 2. What is $A \cup B$?
 3. What is $A \cap B$?
 4. Partition B into two sets, A and everything else. Call everything else C . What is C ?
 5. What is $A \cup C$?
 6. What is $A \cap C$?
 7. How many possible committees of 5 could be formed by 100 Senators?
 8. If I gave people a survey with 5 questions (asked in a random order) with 4 response options for each question, how many combinations of responses could I get?
 9. If I gave people a survey with 5 questions (asked in a random order) with 4 response options for each question, how many permutations of responses could I get?
 10. Compute $\frac{12!}{7!}$
 11. Compute $\frac{5!}{6!}$
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Probability

12. In Boston, 30% of the people are conservatives, 50% are liberals, and 20% are independents. In the last election, 65% of conservatives, 82% of liberals, and 50% of independents voted. If a person in Boston is selected at random and we learn that s/he did not vote last election, what is the probability s/he is a liberal?
13. If A, B, C , and D are mutually exclusive and collectively exhaustive, what is the joint probability of A, B, C , AND D ?
14. If A, B, C , and D are mutually exclusive and collectively exhaustive, what is the joint probability of A, B, C , OR D ?
15. Solve what is known as the Monte Hall problem. There are three doors. Behind two of these are goats, while behind the third is a new car. You choose one door. Monte Hall opens one of the other two doors, revealing a goat, and asks if you'd like to stick with the door you have, or switch to the other door he did not open. You get whatever is behind the door you choose. Should you switch doors? Why or why not?
16. In a certain city, 30% of the citizens are conservatives, 30% are liberals, and 40% are independents. In a recent election, 50% of conservatives voted, 40% of liberals voted, and 30% of independents voted.
 - What is the probability that a person voted?
 - If the person voted, what is the probability that the voter is conservative?
 - If the person voted, what is the probability that the voter is liberal?
17. In rolling two dice labeled X and Y , what is the probability that the sum of the up faces is four, given that either X or Y shows a three?

Use this joint probability distribution:

		X		
		0	1	2
Y	0	0.10	0.10	0.01
	1	0.02	0.10	0.20
	2	0.30	0.10	0.07

18. $p(X < 2)$
19. $p(X < 2|Y < 2)$
20. $\Pr(Y = 2|x \leq 1)$
21. $p(X = 1|Y = 1)$
22. $p(Y > 0|X > 0)$

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23. Assume that 2% of the population of the United States are members of some extremist militia group, ($p(M) = 0.02$). However, members may be unwilling to admit their membership on a survey. We develop a survey question that is 95% accurate on positive classification $p(C|M) = 0.95$ and 97% accurate on negative classification, $P(C^C|M^C) = 0.97$. Using Bayes' Law, derive the probability that someone positively classified by the survey as being a militia member really is a militia member.
 24. If A and B are independent events, are the following true or false?
 - a. $\Pr(A \cap B) = \Pr(A) \Pr(B)$
 - b. $\Pr(A|B) = \Pr(A) + \Pr(A) \Pr(B)$
 - c. $\Pr(B|A) = \Pr(B)$
 - d. Let $P(A) = 0.3$ and $P(A \cup B) = .5$. Find $P(B)$, assuming both events are independent?
 - e. What problems do you run into when they are not independent?

Properties of distributions

25. Wasserman exercise 2.9
26. Wasserman exercise 2.17
27. Suppose that $f_{x,y}(x,y) = \exp(-(x+y))$ for $x,y \geq 0$. Find the marginal distribution for X , $f_X(x)$.
28. If X_1, \dots, X_n are iid random variables, find the expected value for

$$\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X}_n)^2$$

29. Prove theorem 3.27 in Wasserman (see hints in Exercis 3.17).
30. Prove lemma 3.31.1 in Wasserman.
31. Wasserman 3.5
32. Find the MGF for the Binomial(n,p) distribution.
33. Wasserman 3.18
34. Wasserman 3.24

Convergence of random variables

34. A recent poll of 698 decided voters in Pennsylvania showed 341 preferred Donald Trump and 357 preferred Hillary Clinton. Let π be the population proportion of decided Pennsylvania voters who prefer Trump. Use the central limit theorem to find the approximate distribution of the sample proportion $\hat{\pi}$.

- 35. In your own way, prove the central limit theorem. (See appendix to Chapter 5)
- 36. Wasserman 5.5