

New York State Mathematics Association of Two-Year Colleges

Math League Contest ~ Spring 2011

Directions: You have one hour to take this test. Scrap paper is allowed. The use of calculators is NOT permitted, as well as computers, books, math tables, and notes of any kind. You are not expected to answer all the questions. However, do not spend too much time on any one problem. Four points are awarded for each correct answer, one point is deducted for each incorrect answer, and no points are awarded/deducted for blank responses. There is no partial credit. Unless otherwise indicated, answers must given in *exact* form, i.e. in terms of fractions, radicals, π , etc.

1. If $f(x) - 2f(2012 - x) = x$, what is the numerical value of $f(2011)$?

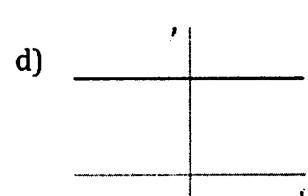
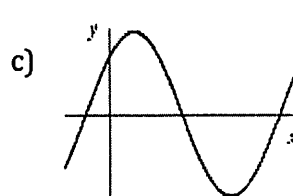
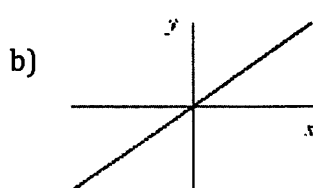
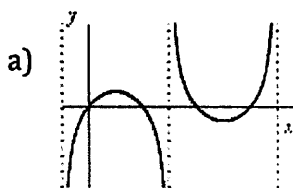
2. How many real values of x satisfy the equation $(x^2 + 7x + 11)^{x^2 + 3x - 10} = 1$?
 a) 1 b) 2 c) 3 d) 4

3. The notation $a|b$ is read as *a divides b* (i.e. b is a multiple of a). What is the largest integer value of n so that $5^n | 200!$? Note: $200!$ is 200 *factorial* and defined as $200! = 200 \cdot 199 \cdot 198 \cdot \dots \cdot 3 \cdot 2 \cdot 1$
 a) 48 b) 49 c) 89 d) 90

4. A group of friends decided to rent a boat for a few hours and share equally in the cost. If they had one more person, the cost per person would decrease by \$5. If they had three more people, the cost per person would decrease by \$12. If the cost of the boat is the same for any number of people, then how many people are in the group?

5. The equation $ax^4 - 7x^3 + 8x^2 - 7x + a = 0$ has two distinct real roots and two complex roots, with one of the real roots being $x = 2011$, for some real value of a . What is the other real root?

6. Which of the following is the graph of $f(x) = \frac{1}{\cos^{-1}(x) + \sin^{-1}(x)}$?

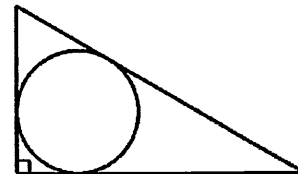


7. What is $\sin(15^\circ) + \sin(75^\circ)$?

- a) 1 b) $\frac{\sqrt{6}}{2}$ c) $\frac{1+\sqrt{3}}{2}$ d) $\frac{\sqrt{2}+\sqrt{3}}{2}$

8. A regular polygon (i.e. a polygon with congruent sides and interior angles) has 54 diagonals. How many sides does this polygon have?

9. A 30° - 60° - 90° (right) triangle has a circle of radius 1 inscribed in it, as shown. In simplest form, what is the area of the triangle?



10. Point A is 1 unit from Point B, while Point B is 1 unit from Point C. What is the probability that Point A is closer to Point C than it is to Point B? Note: All points are on the same plane.

11. How many points with integer coordinates are on the line $y = \pi x - \frac{22}{7}$?

- a) 0 b) 1 c) 2 d) infinitely many

12. $\sqrt{9+4\sqrt{5}} = ?$

- a) $3 + \frac{1}{2}\sqrt{5}$ b) $2 + \sqrt{5}$ c) $3\sqrt[4]{5}$ d) $3 + \sqrt[4]{5}$

13. What value of $x > 1$ satisfies the equation $\log_2 [\log_4 (x)] = \log_8 [\log_2 (x)]$?

14. Three valves A, B, and C, when open, release water into a tank at their own constant rate. When all three valves are open the tank fills in 1 hour, with only valves A and C open the tank fills in 90 minutes, and with only valves B and C open it takes 2 hours. How long would it take to fill the tank with only valves A and B open?

15. If $S(n) = \sum_{k=2}^n \frac{1}{\log_k(n!)}$, what is the value of $S(2011)$?

- a) $\frac{2010}{2011}$ b) 1 c) $\frac{2011}{2010}$ d) $\frac{2011}{\log_{2011}(2011!)}$

16. A check is written in the amount of x dollars and y cents, where x and y are both two-digit numbers. The bank teller mistakenly cashes it for y dollars and x cents, giving the patron more than the actual check amount. Which of the following *cannot* be the amount of over payment?
a) \$1.98 b) \$4.95 c) \$9.72 d) \$10.89
17. Two equally matched teams play a series of five games, the first to win three games is declared the winner. If Team A has won the first game, what is the probability that Team A will win the series?
18. The lines given by $y = x$ and $y = 3x$ form an acute angle in the first quadrant. What is the slope of the line that bisects that angle?
a) $\sqrt{2}$ b) $\frac{1+\sqrt{5}}{2}$ c) $\sqrt{3}$ d) 2
19. In a group of five people, the sums of the ages of each group of four of them are 138, 144, 151, 153, and 158. What is the age of the youngest person?
a) 26 b) 28 c) 29 d) 31
20. Which of the following statements are true?
I. Exactly one of these statements is true.
II. Exactly two of these statements are true.
III. Exactly three of these statements are false.
IV. Exactly two of these statements are false.
a) I only b) II only c) I and III d) II and IV

