NYS $\int_{1967}^{\infty}$ MATYC

New York State Mathematics Association of Two-Year Colleges

Math League Contest ~ Fall 2014

Directions: You have one hour to take this test. Scrap paper is allowed. The use of calculators is NOT permitted, as well as computers, books, math tables, and notes of any kind. You are not expected to answer all the questions. However, do not spend too much time on any one problem. Four points are awarded for each correct answer, one point is deducted for each incorrect answer, and no points are awarded/deducted for blank responses. There is no partial credit. Unless otherwise indicated, answers must given in *exact* form, i.e. in terms of fractions, radicals, π, etc. NOTE: NOTA = None Of These Answers.

1. The Gamma function, $\Gamma(x)$, satisfies the recurrence relation $\Gamma(x+1)=x\Gamma(x)$ for all positive real values of x. If $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$, then what is the value of $\Gamma\left(\frac{7}{2}\right)$?

   A) $\frac{3}{4}\sqrt{\pi}$ B) $\frac{15}{16}\sqrt{\pi}$ C) $\frac{15}{8}\sqrt{\pi}$ D) $\frac{105}{16}\sqrt{\pi}$

2. I have a $5\times5\times5$ solid cube that I paint on all six sides. I then cut it up into 125 $1\times1\times1$ cubes. How many of the $1\times1\times1$ cubes have *no* paint?

3. Which one of the following numbers is *not* an integer?

   A) $\log_2(2014)\cdot\log_{2014}(2)$ B) $\sin^{-1}(\sin(2))$ C) $\left(2+\sqrt{2}\right)^{-2}+\left(2-\sqrt{2}\right)^{-2}$ D) $4^{\log_2(2014)}$

4. The magic square shown uses each integer from 1 through 9, exactly once, so that the sum along any row, column, and both diagonals is 15. What is the value of x?

	9	4
x		

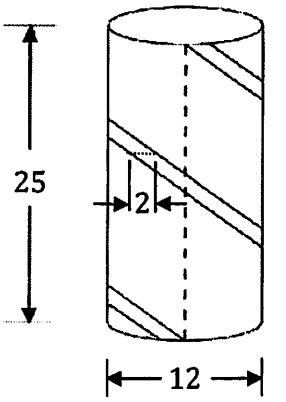
5. When the fraction $\frac{1}{7}$ is expressed in decimal form, what is the digit in the 2014th decimal place?
   (Note: The 2014th decimal place is the digit that is 2014 places to the right of the decimal point.)

6. One solution to the equation $(x-a)(x-b)(x-c)(x-d)=25$ is $x=-2$. If a, b, c, and d are four different integers, then what is the numerical value of $a+b+c+d$?

7. Which of the following numbers is a perfect square?
   Note: $n!=n(n-1)(n-2)\cdots2\cdot1$, e.g. $5!=5\cdot4\cdot3\cdot2\cdot1$

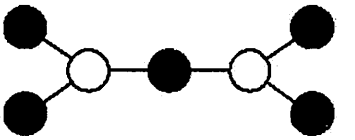
   A) 81!100! B) 99!100! C) 99!101! D) 100!101!

8. A cylindrical storage unit has a diameter of 12 feet and a height of 25 feet. A red stripe with a horizontal width of 2 feet is painted on it, as shown, making two complete revolutions around it. What is the area of the stripe in square feet?



A) 50 B) 24π C) 48π D) $48\sqrt{\pi^2+1}$

9. Suppose a fly lands on one of the seven circles and then moves, exactly one position, along a path to a neighboring circle. What is the probability it will end up on shaded circle? Assume all moves by the fly are random.



A) $\frac{2}{7}$ B) $\frac{1}{3}$ C) $\frac{2}{3}$ D) $\frac{5}{7}$

10. The sum of two real numbers is 3 and the product is -1. What is the numerical value of the sum of their cubes?

11. $444,444,444,445^2 - 444,444,444,444^2 = ?$

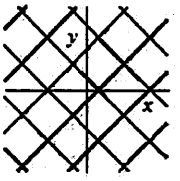
A) 888,888,888,789 B) 888,888,888,889 C) 888,888,889,889 D) 898,989,898,989

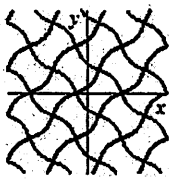
12. How many different triangles with integer length sides have a perimeter of 15?

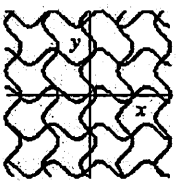
13. If $\cos(x)+\sin(x)=\cos(x)\sin(x)$, then what is $\cos(x)\sin(x)$?

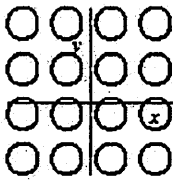
A) $-\frac{\sqrt{2}}{2}$ B) $1-\sqrt{2}$ C) $\frac{1}{2}$ D) $\frac{\sqrt{2}}{2}$

14. The graph of $\cos(y)=\sin(x)$ is best represented by which of the following?

A) 

B) 

C) 

D) 

15. A customer put \$1000 in a bank account and kept it there for 10 years. The interest earned was 3% per year for 6 years, and 2% per year for 4 years. Assuming no other deposits or withdrawals, the total interest earned will be maximized when the 3% interest accrues

A) the first 6 years. B) the middle 6 years. C) the last 6 years.

D) It does not matter, the total interest will be the same in any case.

16. A positive integer, n, is said to be *rare* if $n+\hat{n}$ and $n-\hat{n}$ are both perfect squares, where $\hat{n}$ is the reversal of n. For example, 621770 is a rare number (the second one in fact), since $621770+077126=698896=836^2$ and $621770-077126=544644=738^2$. The first rare number is a 2-digit number. What is it?

17. $\tan(1°)\tan(2°)\tan(3°)\cdots\tan(87°)\tan(88°)\tan(89°)=?$
(i.e. The product of the tangents of 1 degree, 2 degrees, through 89 degrees.)
A) $\frac{1}{2}$ B) 1 C) $\sqrt{2}$ D) $\frac{\pi}{2}$

18. Suppose the graph of the parabola given by $y=ax^2+bx+c$, $a\neq 0$, has its vertex at (h,k). This parabola is now reflected about the line $y=k$, and the resulting parabola has equation $y=dx^2+ex+f$. What is $a+b+c+d+e+f$?
A) $-2ah^2$ B) 0 C) $2ah^2$ D) $2k$

19. A drawer contains a mix of red and blue socks. When two socks are selected at random, the probability that both are blue is $\frac{2}{5}$. What is the *minimum* number of socks in the drawer?

20. If the problem you solved before you solved the problem you solved after you solved the problem you solved before you solved this one, was harder than the problem you solved after you solved the problem you solved before you solved this one, was the problem you solved before you solved this one harder than this one? Assume you solved all problems referenced.
A) No B) Yes C) It is impossible to determine. D) NOTA

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