

# New York State Mathematics Association of Two-Year Colleges

## Math League Contest ~ Fall 2010

**Directions:** You have one hour to take this test. Scrap paper is allowed. The use of calculators is NOT permitted, as well as computers, books, math tables, and notes of any kind. You are not expected to answer all the questions. However, do not spend too much time on any one problem. Four points are awarded for each correct answer, one point is deducted for each incorrect answer, and no points are awarded/deducted for blank responses. There is no partial credit. Unless otherwise indicated, answers must given in *exact* form, i.e. in terms of fractions, radicals,  $\pi$ , etc.

1. If  $f(x, n) = \sum_{k=0}^n x^k = x^0 + x^1 + x^2 + \dots + x^n$ , what is  $f(i, 2010)$  where  $i = \sqrt{-1}$ ? Express the answer in simplest  $a + bi$  form.

2. What value of  $x$  satisfies:  $\sqrt{2010x+1} - \sqrt[4]{2010x+1} = 2$ ?

3.  $\sqrt{7+\sqrt{48}} + \sqrt{7-\sqrt{48}}$  equals

a)  $\sqrt{14}$

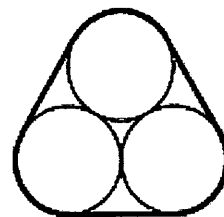
b)  $\sqrt{15}$

c)  $\sqrt{16}$

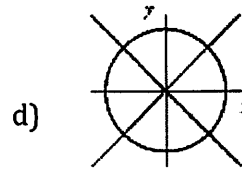
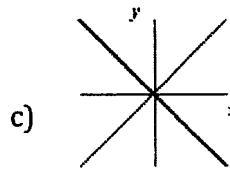
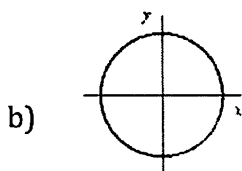
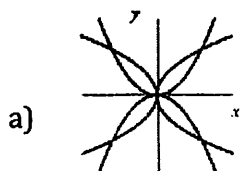
d)  $\sqrt{17}$

4. Rich has an assortment of coins but is unable to give exact change for a dollar, half-dollar, quarter, dime, or nickel. From the set of coins {pennies, nickels, dimes, quarters, half-dollars}, what is the largest amount he can have?
- a) \$0.94      b) \$0.99      c) \$1.19      d) none of those answers

5. Three large pipes are transported on a flatbed truck by stacking the third one on top of the other two and using a tight metal band around all three to hold them together (an end-view is shown). If the outside diameter of each pipe is 2 feet, then what is the length of the metal band, assuming it does not overlap itself?

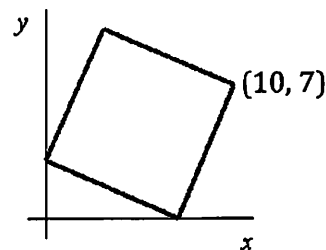


6. Which of the following is the graph of  $x^4 - y^4 = x^2 - y^2$ ?

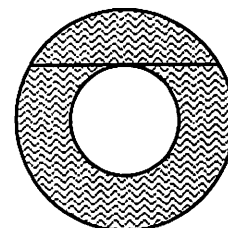


7. If  $a$  and  $b$  are positive integers such that  $a^2 - b^4 = 345$ , then what is the value of  $a + b$ ?

8. A square is constructed so one corner lies on the  $x$ -axis, one on the  $y$ -axis, and one at  $(10, 7)$ , as shown. What is the area of the square?



9. The diagram shows two circles with a common center, and a chord drawn in the larger circle so it is tangent to the smaller circle. The radius of the larger circle is  $R$ , the radius of the smaller circle is  $r$ , while the chord has length 2. What is the area of shaded region (i.e. the area between the two circles)?



- a)  $\pi$       b)  $2\pi$       c)  $2\pi(R-r)$       d)  $\pi(R-r)^2$

10. At a fundraiser 2010 raffle tickets were sold, numbered 1 through 2010. Three prizes were awarded by randomly selecting three of the ticket stubs. What is the probability that the numbers on the three stubs were selected in increasing order?

11. The graph of  $y = \frac{x}{100\pi}$  intersects the graph of  $y = \sin(x)$  at how many points?

12. There is only one number between 2 and  $2^{100}$  whose square root, third root, fourth root, fifth root, and sixth root is a whole number. What is the number?

13. The exact value of  $\log_{2011}(2010) - \frac{1}{\log_{2010}(2011)}$  is

- a)  $-1$       b)  $0$       c)  $1$       d)  $\frac{1}{2010+2011}$

14. Ray can do a job in 9 hours. Ray and his helper Tim start working together, after 4 hours Tim leaves and Ray finishes the job alone in 2 hours. How many hours would it take Tim to do the entire job alone?

15. A coin is so unbalanced that the probability of getting two heads on two tosses is the same as getting tails on one toss. What is the probability of getting heads on a single toss?

16. I wear a clean shirt every day. If I drop off my laundry and pick up the previous week's load every Monday evening, then what is the minimum number of shirts I must own to keep me going?  
 a) 13                      b) 14                      c) 15                      d) 16
17. Ida and Sophia were in a bicycle race and started at the same time. Ida was travelling at a constant speed of 42 km/hr. When Sophia completed half the race, Ida was 3.5 km ahead. Ida completed the race at 11:52 AM. If Sophia also maintained a constant speed, what time did she complete the race?
18. A handball tournament consists of 2010 participants. The format of the event is single-elimination (i.e. a player is out of the tournament after the first loss). How many matches must be played in order to determine a winner?  
 a) 1005                      b) 2009                      c) 2010                      d) 2011
19. The citizens of Taxation Land pay twice as much income tax (percentage-wise) as their hourly wage. For example, someone who earns \$15 per hour would pay 30% income tax. What hourly wage maximizes after taxes income?
20. During a recent mathematics lecture, someone in the audience created a disturbance with loud snoring. In an effort to determine the culprit, the five most likely suspects were interrogated and each offered two statements.
- |          |                                      |
|----------|--------------------------------------|
| Artie:   | It wasn't Edward. It was Barbara.    |
| Barbara: | It wasn't Carmine. It wasn't Edward. |
| Carmine: | It wasn't Artie. It was Edward.      |
| Darci:   | It was Carmine. It was Barbara.      |
| Edward:  | It wasn't Artie. It was Darci.       |
- If each suspect told exactly one lie, then who committed the heinous act?

