New York State Mathematics Association of Two-Year Colleges

Math League Contest ~ Spring 2012

Directions: You have one hour to take this test. Scrap paper is allowed. The use of calculators is NOT permitted, as well as computers, books, math tables, and notes of any kind. You are not expected to answer all the questions. However, do not spend too much time on any one problem. Four points are awarded for each correct answer, one point is deducted for each incorrect answer, and no points are awarded/deducted for blank responses. There is no partial credit. Unless otherwise indicated, answers must given in *exact* form, i.e. in terms of fractions, radicals, π, etc. NOTE: NOTA = None Of These Answers.

1. If the symbol $\otimes$ is defined as $a \otimes b = ab - a$, for *positive* integers a and b, then which of the following *could* equal zero?

   I. $(a^2+1)\otimes(b^2-1)$ II. $(a^2+b^2)\otimes(a^2-b^2)$ III. $(a^2-b^2)\otimes(a^2+b^2)$

   a) I and II only b) II and III only c) III only d) I, II, and III

2. How many 4-digit numbers satisfy the following criteria?
   - It has four different digits.
   - The largest digit is the sum of the other three.
   - The product of the digits is a multiple of 10, but not zero.

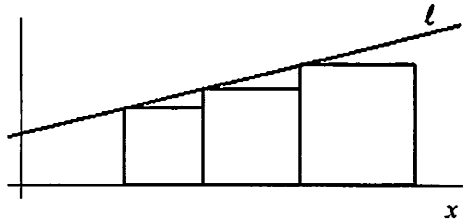
   a) 24 b) 36 c) 48 d) 72

3. The sum of 999 consecutive integers is 999,999. What is the *largest* of these 999 numbers?

4. Fresh apricots have a moisture content of 75%. When left in the sun to dry, they lose 80% of their moisture content. What is the moisture content of dried apricots? Give answer as a percentage.

5. Suppose a piece of rope, of negligible thickness, hangs between two telephone poles that are 40 feet tall. Further suppose that the shape of the rope forms a parabola* when the poles are separated. If the rope is 50 feet long, and the distance from the ground to the lowest point of the hanging rope is 15 feet, then how far apart are the two poles? *The shape is actually called a catenary, described by the hyperbolic cosine function, but for this problem assume the shape is a parabola.

6. The accompanying diagram shows three adjacent squares that have their bases on the x-axis and upper-left corner tangent to line ℓ. The two smaller squares have sides of length 3 and 4, respectively. What is the length of the sides of the largest square? Note: The diagram is *not* drawn to scale.



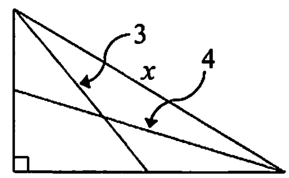
7. If $x=\cfrac{1}{a+\cfrac{1}{b+\cfrac{1}{c+\cfrac{1}{d}}}}$, and a, b, c, and d can only be either 1 or 2, then what is the *minimum* value of x?

a) $\frac{4}{11}$ b) $\frac{12}{29}$ c) $\frac{1}{2}$ d) $\frac{3}{5}$

8. Eastern Standard Time (EST), the time zone that includes New York City (NYC), is 3 hours later than Pacific Standard Time (PST), the time zone that includes Los Angeles (LA). A plane leaves NYC at 10 AM (EST) and arrives in LA at 1 PM (PST). Two hours before that plane landed, a second plane left LA for NYC. If the duration of the flight from LA to NYC is one hour shorter than going from NYC to LA, then at what time (EST) does the second plane land in NYC?

a) 4 PM b) 5 PM c) 6 PM d) 7 PM

9. The medians of a right triangle have lengths 3 and 4, as shown. What is the length of the hypotenuse, x, of the triangle? Note: A median of a triangle is a line segment from a vertex of the triangle to the midpoint of the side opposite that vertex.

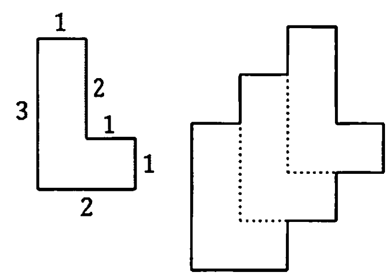


10. The probability that event A occurs is $\frac{3}{4}$, and the probability that event B occurs is $\frac{2}{3}$. What is the *smallest* interval that must contain the probability of A <u>and</u> B?

a) $\left[\frac{5}{12},\frac{1}{2}\right]$ b) $\left[\frac{5}{12},\frac{2}{3}\right]$ c) $\left[\frac{1}{2},\frac{2}{3}\right]$ d) $\left[\frac{1}{2},\frac{3}{4}\right]$

11. If $\dfrac{x^2-2011y^2}{x^2+2012y^2}=\dfrac{1}{2}$, with $xy<0$, then what is $\dfrac{x}{y}$?

12. An L-shaped region is shown along with its dimensions. P_n represents the perimeter of n of the L-shaped regions arranged as shown (for $n=3$). Thus, $P_1=10$, and $P_3=18$. What is P_{2012}?



a) 8048 b) 8054 c) 8056 d) 8058

13. If $\cos(x)+\sin(x)=\frac{1}{2}$, then what is the value of $\cos^3(x)+\sin^3(x)$?

a) $\frac{1}{32}$ b) $\frac{1}{8}$ c) $\frac{13}{32}$ d) $\frac{11}{16}$

14. What is the *smallest* integer $n > 1$ for which $\dfrac{2011n+1}{2012}$ is an integer?

15. At a certain school there are exactly 150 lockers in a long hallway. The lockers are numbered in order, from 1 through 150, and were all closed. One-hundred-fifty students entered the hallway, one at a time, and either opened or closed the lockers as follows. The 1st student opened every locker, the 2nd student closed every even numbered locker, the 3rd changed every third locker, closing those that were opened and opening those that were closed, the 4th student changed every 4th locker, and so on. After all 150 students passed through the hallway, how many lockers were left open?

16. Which of the following is equal to $2012^{\ln(2)}$?
a) $2^{\ln(2012)}$ b) $2000^{\ln(2)} + 12^{\ln(2)}$ c) $2012\ln(2)$ d) NOTA

17. The NYSMATYC Math Contests are graded as follows: 4 points are awarded for each correct answer, 1 point is deducted for each incorrect answer, and no points are awarded or deducted for blank responses. The tests consist of 20 questions, thus the minimum score is –20 for incorrectly answering all 20 questions, the maximum score is 80 when correctly answering all 20 questions. How many scores between -20 and 80 are *not* possible to achieve?

18. A triangle has an area of 30, one side of length 10, and the median to that side of length 9. Let β be the acute angle formed by that side and the median. What is $\sin(\beta)$?
a) $\dfrac{3}{10}$ b) $\dfrac{1}{3}$ c) $\dfrac{9}{20}$ d) $\dfrac{2}{3}$

19. A non-zero digit is chosen in such a way that the probability of choosing the digit d is $\log(d+1) - \log(d)$. The probability that the digit 2 is chosen is exactly $\frac{1}{2}$ the probability that a digit is chosen from which set?
a) $\{3,4\}$ b) $\{4,5,6,7,8\}$ c) $\{5,6,7,8,9\}$ d) $\{4,5,6,7,8,9\}$

20. A leap year is a year that contains an extra day, that day being February 29, giving 366 days rather than 365. Leap years occur only in numbered years that are divisible by 4 but not by 100, or divisible by 400 (e.g. 2012 is a leap year, 2000 was as well, but 2100 will not be). Albert Einstein was born on March 14, 1879. This year, March 14 falls on a Wednesday. On which day of the week was Einstein born?

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