1. Beaker A contains 500 ml of 20% salt solution, and beaker B contains 800 ml of 50% salt solution. A lab tech pours some of each of these solutions into beakers C and D so that beaker C contains 100 ml of 30% salt solution, and beaker D contains 200 ml of 45% salt solution. How many milliliters remain in beaker B after this is done?

A. 150 B. 200 C. 600 D. 650 E. 700

2. The widths and lengths of two distinct rectangles form a sequence of four consecutive odd integers. The perimeter of the first rectangle is 44 less than twice the perimeter of the second rectangle, and the sum of their areas is less than 150. Find the sum of their areas.

A. 56 B. 94 C. 108 D. 122 E. 148

3. The equation $a^2 + b^2 + c^4 = 2020$ has exactly one solution in the positive integers for which $a > b$. Find $a + b + c$ for this solution.

A. 40 B. 41 C. 42 D. 43 E. 44

4. Consider a balance scale where weights may be placed on either side. We can use this scale to weigh a 3 pound object by placing it on one side and placing a 3 pound weight on the opposite side. Another way would be to place a 4 pound weight on the same side as the object and a 2 pound and a 5 pound weight on the opposite side. Suppose you need to be able to weigh objects with any whole number weight from 1 to 40 pounds. What is the least number of weights that are needed?

A. 4 B. 5 C. 6 D. 7 E. 8

5. The region inside a circle of radius 1 centered at the origin is painted blue. Then, the regions inside two circles of radius 1 centered at (-1,1) and (-1, -1) are painted red. The regions that are painted twice will now be purple. What is the area of the remaining blue region?

A. π - 2 B. 8 - 2π C. 9/5 D. 2 E. $2\pi/3$

6. Let K be an integer that is greater than 1, a perfect square, and equal to $\sum_{i=1}^{D} i$ for some integer D.
Find $\left[1 + 2 + \ldots + \left(\sqrt{K} - 1\right)\right] - \left[\left(\sqrt{K} + 1\right) + \left(\sqrt{K} + 2\right) + \ldots + D\right]$.

A. $-K/2$ B. $-\sqrt{K}/2$ C. 0 D. $\sqrt{K}/2$ E. $K/2$

7. In a track race between Achilles, a tortoise, and a hare, the hare gives the tortoise a head start of 1000 meters and gives Achilles a head start of 100 meters. If Achilles, the tortoise, and the hare move at 1000, 10, and 1050 meters per minute, respectively, for how many minutes will Achilles hold the lead?

A. 10/11 B. 24/25 C. 1 D. 101/100 E. 12/11

8. A collection of 62 coins consists of D dimes, N nickels, and Q quarters. The total value is $8.30. Find the sum of all possible values of N.

A. 42 B. 62 C. 104 D. 146 E. 234

9. Kara looks at a wall clock (with constant velocity hands) sometime between 3 and 4 o'clock and observes that the angle between the hour-hand and the minute-hand is 30°. Ten minutes later, she observes that the angle between the hour-hand and the minute-hand is 85°. Find the time when she first looked at the clock to the nearest second. Write your answer in *hr:min:sec* format on the answer sheet. (For example, 3:11:48 would be 11 minutes and 48 seconds after 3 o'clock).

10. Let # be the binary operation on all real 2 × 2 matrices defined by $A \# B = AB + BA$.
(i) Is # commutative for all real 2 × 2 matrices?
(ii) Is # associative for all real 2 × 2 matrices?

A. (i)Yes (ii)Yes B. (i)Yes (ii)No C. (i)No (ii)Yes D. (i)No (ii)No E. Impossible to determine

11. Three people (X, Y, Z) are in a room with you. One is a knight (knights always tell the truth), one is a knave (knaves always lie), and the other is a spy (spies may either lie or tell the truth). X says, "If you asked me who the spy is, I would say that Z is the spy." Y says, "Z is the spy." Z says, "I am the spy." Which of the following correctly identifies all three people?

A.	B.	C.	D.	E.
X is the spy.	X is the spy.	X is the knave.	X is the knight.	X is the knave.
Y is the knight.	Y is the knave.	Y is the knight.	Y is the spy.	Y is the spy.
Z is the knave.	Z is the knight.	Z is the spy.	Z is the knave.	Z is the knight.

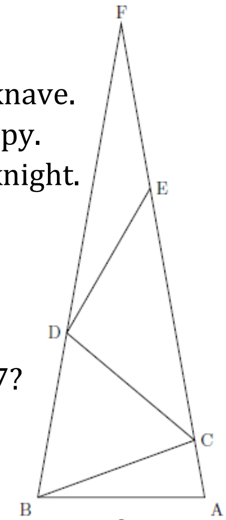

Figure for #12

12. Refer to the figure on the right. Find the measure of $\angle AFB$ if $AB = BC = CD = DE = EF$ and ΔABF is isosceles.

A. 15° B. 20° C. 22° D. 26° E. 30°

13. How many 5-digit palindromic numbers (of the form *abcba*, with $a \neq 0$) are divisible by 37? Note that a, b, c need <u>not</u> be distinct.

A. 0 B. 18 C. 36 D. 45 E. 74

14. Consider the rational function $g(x) = \frac{n(x)}{m(x)}$ where $n(x)$ and $m(x)$ are both polynomials of degree 3 or lower with real coefficients and a leading coefficient of 1. $g(x)$ has a removable discontinuity at $x = 2$, a vertical asymptote of $x = 7$, and is continuous everywhere else. $g(x)$ has exactly two real zeros of $x = 4$ and $x = -3$ and no nonreal complex zeros. $g(x)$ has a slant asymptote. Let N and M be the absolute values of the constant terms of $n(x)$ and $m(x)$. Find $M + N$.

A. 28 B. 38 C. 44 D. 48 E. 52

15. What is the hundreds digit in the product of 5^{94} and 98,777,782,163?

A. 0 B. 2 C. 4 D. 6 E. 8

16. Let M be the unique whole number less than 200 that has exactly 18 whole number factors. Find the sum of all 18 factors of M.

A. 545 B. 546 C. 601 D. 602 E. 646

17. Let r, s, and t be nonnegative integers. How many triples (r, s, t) satisfy the system $\begin{cases} rs + t = 14 \\ r + st = 13 \end{cases}$?

A. 1 B. 2 C. 3 D. 4 E. 5

18. Find the radius of the circle inscribed in an isosceles triangle with two sides of length 20 and a base of length 24.

A. 6 B. 25/4 C. 13/2 D. 27/4 E. 25/2

19. Consider $f(x)\cos\left(\frac{f(x)x}{2}\right) + 2\sin(x) = f(x)$ where $f(x) = \begin{cases} 2\sin(x) & |x| \leq 2 \\ 2 & |x| > 2 \end{cases}$

How many solutions does this equation have in the interval $(-2\pi, 2\pi)$?

A. 1 B. 2 C. 3 D. 4 E. 5 or more

20. Suppose a biased six-sided die is rolled until either two 1s are obtained on successive rolls or until a 1 and then a 2 are obtained on successive rolls. The die will show a 1 with probability 50%, a 2 with probability 20%, and something else with probability 30%. What is the expected number of times the die is rolled?

A. 7/3 B. 3 C. 18/5 D. 4 E. 30/7