

Spivak Chp. 9: 14, 15(a), 22(a). Chp. 10: 2(ii), 2(iv)

9.14

Let $f(x) = x^2$ if x is rational, and $f(x) = 0$ if x is irrational. Prove that f is differentiable at 0. (Don't be scared by this function. Just write out the definition of $f'(0)$.)

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9.15.a

Let f be a function such that $|f(x)| \leq x^2$ for all x . Prove that f is differentiable at 0. (If you have done Problem 14 you should be able to do this.)

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9.22.a Suppose that f is differentiable at x . Prove that

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x-h)}{2h}$$

Hint: Remember an old algebraic trick - a number is not changed if the same quantity is added to and then subtracted from it.

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10.2.ii Find $f'(x)$ for

$$f(x) = \sin^3(x^2 + \sin x)$$

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10.2.ii Find $f'(x)$ for

$$f(x) = \sin\left(\frac{x^3}{\cos x^3}\right)$$

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