Math 30b Orrison
Sequences and Series
Tuesday, October 6, 2015

(Chp. 22) 1(ii, iv, vi, viii), 2(i, iii, v), 4(a). (Chp. 23) 1(ii, iv, vi, viii), 6(a, b).

22.1.ii Verify the limit:

$$\lim_{n\to\infty} \frac{n+3}{n^3+4} = 0$$

■

22.1.iv Verify the limit:

$$\lim_{n\to\infty} \frac{n!}{n^n} = 0$$

■

22.1.vi
Verify the limit:

$$\lim_{n\to\infty} \sqrt[n]{n} = 1$$

■

22.1.viii
Verify the limit:

$$\lim_{n\to\infty} \sqrt[n]{a^n + b^n} = \max(a, b), \; a, b \geq 0.$$

■

22.2.i
Find the limit:

$$\lim_{n\to\infty} \frac{n}{n+1} - \frac{n+1}{n}$$

■

22.2.iii
Find the limit:

$$\lim_{n\to\infty} \frac{2^n + (-1)^n}{2^{n+1} + (-1)^{n+1}}$$

■

22.2.v
Find the limit:

$$\lim_{n\to\infty} \frac{a^n - b^n}{a^n + b^n}$$

■

22.4.a
Prove that if a subsequence of a Cauchy sequence converges, then so does the original Cauchy sequence.

■

23.1.ii
Decide whether the following infinite series is convergent or divergent:

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

■

23.1.iv
Decide whether the following infinite series is convergent or divergent:

$$\sum_{n=1}^{\infty} (-1)^n \frac{\log n}{n}$$

■

23.1.vi
Decide whether the following infinite series is convergent or divergent:

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2+1}}$$

■

23.1.viii
Decide whether the following infinite series is convergent or divergent:

$$\sum_{n=1}^{\infty} \frac{\log n}{n}$$

■

23.6.a
Let f be a continuous function on an interval around 0, and let $a_n = f(1/n)$ (for large enough n).
Prove that if $\sum_{n=1}^{\infty} a_n$ converges, then $f(0) = 0$.

■

23.6.b
Let f be a continuous function on an interval around 0, and let $a_n = f(1/n)$ (for large enough n).
Prove that if $f'(0)$ exists and $\sum_{n=1}^{\infty} a_n$ converges, then $f'(0) = 0$.

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