Math 30b - Orrison
Induction
Friday, September 4

2.{1.ii, 2.ii, 10, 20, 26}

2.1.ii Prove the following formula by induction:

$$1^3 + ... + n^3 = (1 + ... + n)^2$$

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2.2.ii Find a formula for:

$$\sum_{i=1}^{n} (2i-1)^2 = 1^2 + 3^2 + 5^2 + ... + (2n-1)^2$$

■

2.10 Prove the Principal of Mathematical Induction from the Well Ordering Principal

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2.20 The Fibonacci sequence $a_1, a_2, a_3, ...$ is defined as follows:

$$\begin{aligned} a_1 &= 1 \\ a_2 &= 1 \\ a_n &= a_{n-1} + a_{n-2} \quad \text{for } n \geq 3 \end{aligned}$$

(some irrelevant history mentioned here...)

Prove that:

$$a_n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}}$$

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2.26 There is a puzzle consisting of three spindles, with n concentric rings of decreasing diameter stacked on the first (Figure 1). A ring at the top of a stack may be moved from one spindle to another spindle, provided that it is not placed on top of a smaller ring. For example, if the smallest ring is moved to spindle 2 and the next-smallest ring is moved to spindle 3, then the smallest ring may be moved to spindle 3 also, on top of the next-smallest. Prove that the entire stack of n rings can be moved onto spindle 3 in $2^n - 1$ moves, and that this cannot be done in fewer than $2^n - 1$ moves.

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