Math 30B
Integration
September 22, 2015

13.{8.ii, 8.iv, 16, 17, 29}

8.ii Find the areas of the regions bounded by the graphs of $f(x) = x^2$ and $g(x) = -x^2$ and the vertical lines through $(-1, 0)$ and $(1, 0)$.

■

8.iv Find the areas of the regions bounded by the graphs of $f(x) = x^2$ and $g(x) = 1 - x^2$ and $h(x) = 2$.

■

16 Prove that

$$\int_{ca}^{cb} f(t)dt = c\int_{a}^{b} f(ct)dt$$

(Notice that Problem 15 is a special case.)

■

17 Given that the area enclosed by the unit circle, described by the equation $x^2 + y^2 = 1$, is π, use Problem 16 to show that the area enclosed by the ellipse described by the equation $x^2/a^2 + y^2/b^2 = 1$ is πab.

■

29 Suppose that f is integrable on $[a, b]$. Prove that there is a number x in $[a, b]$ such that $\int_a^x f = \int_x^b f$. Show by example that it is not always possible to choose x to be in (a, b).

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