5.{3.ii, 3.vi, 3.viii}, 6.{3.a, 3.b, 3.c, 7}

Determine the limit l for the given a, and prove that it is the limit by showing how to find a δ such that $|f(x) - l| < \epsilon$ for all x satisfying $0 < |x - a| < \delta$ where

$$f(x) = x^2 + 5x - 2, \quad a = 2.$$

∎

Determine the limit l for the given a, and prove that it is the limit by showing how to find a δ such that $|f(x) - l| < \epsilon$ for all x satisfying $0 < |x - a| < \delta$ where

$$f(x) = \frac{x}{2 - \sin^2 x}, \quad a = 0.$$

∎

Determine the limit l for the given a, and prove that it is the limit by showing how to find a δ such that $|f(x) - l| < \epsilon$ for all x satisfying $0 < |x - a| < \delta$ where

$$f(x) = \sqrt{x}, \quad a = 1.$$

∎

Suppose that f is a function satisfying $|f(x)| \leq |x|$ for all x. Show that f is continuous at 0. (Notice that $f(0)$ must equal 0.)

∎

Give an example of such a function f which is not continuous at any $a \neq 0$.

∎

Suppose that g is continuous at 0 and $g(0) = 0$, and $|f(x)| \leq |g(x)|$. Prove that f is continuous at 0.

∎

Suppose that f satisfies $f(x + y) = f(x) + f(y)$, and that f is continuous at 0. Prove that f is continuous at a for all a.

∎