Math 30b Orrison
Partial Derivatives
Friday, October 9, 2015

1.{a,b,c}, 2.{a,b,c}, 3.{a,b,c}, 4, 5

1.a
Find the matrix Df of partial derivatives for the following function:

$$f(x, y) = \langle e^x, e^y, e^{\cos x} \rangle$$

∎

1.b
Find the matrix Df of partial derivatives for the following function:

$$f(x, y, z) = \left\langle x^2 e^{x+y}, \cos(x-y) \right\rangle$$

∎

1.c
Find the matrix Df of partial derivatives for the following function:

$$f(x, y, z) = \left\langle xy, \frac{x}{1+y^2}, e^{x^y} \right\rangle$$

∎

2.a
Find $\frac{\partial f}{\partial s}$ and $\frac{\partial f}{\partial t}$ for the following function in two ways: first use substitution, then use the chain rule:

$$f(x, y) = \sin(xy) \text{ where } x = s + t \text{ and } y = s^2 + t^2$$

∎

2.b
Find $\frac{\partial f}{\partial s}$ and $\frac{\partial f}{\partial t}$ for the following function in two ways: first use substitution, then use the chain rule:

$$f(x, y) = e^{xy} \text{ where } x = \sin t \text{ and } y = e^{st}$$

■

2.c
Find $\frac{\partial f}{\partial s}$ and $\frac{\partial f}{\partial t}$ for the following function in two ways: first use substitution, then use the chain rule:

$$f(x,y,z) = \cos(xyz) \text{ where } x = st,\ y = st^2 \text{ and } z = st^3$$

■

3.a
Calculate $D(f \circ g)$ in two ways: first use substitution, then use the chain rule:

$$f(x,y) = x^2 - 7y^2 \text{ and } g(s,t) = \left\langle st^2, se^t \right\rangle$$

■

3.b
Calculate $D(f \circ g)$ in two ways: first use substitution, then use the chain rule:

$$f(x,y) = \left\langle xy - y^2, \cos x \right\rangle \text{ and } g(s,t) = \left\langle \frac{s}{t}, st \right\rangle$$

■

3.c
Calculate $D(f \circ g)$ in two ways: first use substitution, then use the chain rule:

$$f(x,y,z) = \left\langle e^x, e^{x+y}, e^{x+y+z} \right\rangle \text{ and } g(s,t) = \left\langle e^s, e^t, e^{s-t} \right\rangle$$

■

4
Suppose an insect flies along a helical curve $x = 2\cos t$, $y = 2\sin t$, $z = 3t$. If the barometric pressure is varying from point to point as $P(x,y,z) = 6x^2z/y$ atm, then use the chain rule to determine how the pressure is changing at the insects location when $t = \pi/4$ min.

■

5
Suppose $w = f\left(\frac{x-y}{xy}, \frac{z-x}{xz}\right)$ is a differentiable function of $u = \frac{y-x}{xy}$ and $v = \frac{z-x}{xz}$. Show that

$$x^2\frac{\partial w}{\partial x} + y^2\frac{\partial w}{\partial y} + z^2\frac{\partial w}{\partial z} = 0.$$

■