Math 30b Orrison Limits and Continuity September 11, 2015

5.3.ii Determine the limit l for the given a, and prove that it is the limit by showing how to find a δ such that $|f(x) - l| < \epsilon$ for all x satisfying $0 < |x - a| < \delta$ where

$$f(x) = x^2 + 5x - 2$$
, $a = 2$.

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5.3.vi Determine the limit l for the given a, and prove that it is the limit by showing how to find a δ such that $|f(x) - l| < \epsilon$ for all x satisfying $0 < |x - a| < \delta$ where

$$f(x) = \frac{x}{2 - \sin^2 x}, \quad a = 0.$$

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5.3.viii Determine the limit l for the given a, and prove that it is the limit by showing how to find a δ such that $|f(x) - l| < \epsilon$ for all x satisfying $0 < |x - a| < \delta$ where

$$f\left(x\right) =\sqrt{x},\quad a=1.$$

6.3.a Suppose that f is a function satisfying $|f(x)| \le |x|$ for all x. Show that f is continuous at 0. (Notice that f(0) must equal 0.)

6.3.b Give an example of such a function f which is not continuous at any $a \neq 0$.

6.3.c Suppose that g is continuous at 0 and g(0) = 0, and $|f(x)| \le |g(x)|$. Prove that f is continuous at 0.

6.7 Suppose that f satisfies f(x + y) = f(x) + f(y), and that f is continuous at 0. Prove that f is continuous at a for all a.