Math 35, Section ___
Problem Set 4
November 17, 2015

3.4.{48, 49, 58a, 60}, 5.4.{50, 53}

3.4.48 NBC News reported on May 2, 2013, that 1 in 20 children in the United States have a food allergy of some sort. Consider selecting a random sample of 25 children and let X be the number in the sample who have a food allergy. Then $X \sim \text{Bin}(25, .05)$.

(a) Determine both $P(X \leq 3)$ and $P(X < 3)$.

(b) Determine $P(X \geq 4)$.

(c) Determine $P(1 \leq X \leq 3)$.

(d) What are $E(X)$ and σ_X?

(e) In a sample of 50 children, what is the probability that none has a food allergy?

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3.4.49 A company that produces fine crystal knows from experience that 10% of its goblets have cosmetic flaws and must be classified as "seconds".

(a) Among six randomly selected goblets, how likely is it that only one is a second?

(b) Among six randomly selected goblets, what is the probability that at least two are seconds?

(c) If goblets are examined one by one, what is the probability that at most five must be selected to find four that are not seconds?

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3.4.58a A very large batch of components has arrived at a distributor. The batch can be characterized as acceptable only if the proportion of defective components is at most .10. The distributor decides to randomly select 10 components and to accept the batch only if the number of defective components in the sample is at most 2.

(a) What is the probability that the batch will be accepted when the actual proportion of defectives is .01? .05? .10? .20? .25?

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3.4.60 A toll bridge charges \$1.00 for passenger cars and \$2.50 for other vehicles. Suppose that during daytime hours, 60% of all vehicles are passenger cars. If 25 vehicles cross the bridge during a particular daytime period, what is the resulting expected toll revenue? [Hint: Let $X =$ the number of passenger cars; then the toll revenue $h(X)$ is a linear function of X.]

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5.4.50 Let X denote the courtship time for a randomly selected female–male pair of mating scorpion flies (time from the beginning of interaction until mating). Suppose the mean value of X is 120 min and the standard deviation of X is 110 min (suggested by data in the article **"Should I Stay or Should I Go? Condition- and Status-Dependent Courtship Decisions in the Scorpion Fly Panorpa Cognate" (*Animal Behavior*, 2009: 491–497))**.

(a) Is it plausible that X is normally distributed?

(b) For a random sample of 50 such pairs, what is the (approximate) probability that the sample mean courtship time is between 100 min and 125 min?

(c) For a random sample of 50 such pairs, what is the (approximate) probability that the total courtship time exceeds 150 hr?

(d) Could the probability requested in (b) be calculated from the given information if the sample size were 15 rather than 50? Explain.

■

5.4.53 Rockwell hardness of pins of a certain type is known to have a mean value of 50 and a standard deviation of 1.2.

(a) If the distribution is normal, what is the probability that the sample mean hardness for a random sample of 9 pins is at least 51?

(b) Without assuming population normality, what is the (approximate) probability that the sample mean hardness for a random sample of 40 pins is at least 51?

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