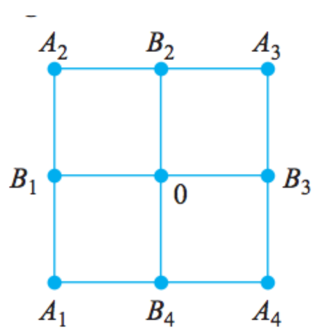
Math 35 - Section
Homework 3
Tuesday, November 10, 2015

3.{1.9, 1.15, 2.15, 2.17, 2.18, 2.32, 2.33}, 4.{1.5, 2.13, 3.28.{abc}, 3.29.{abc}, 3.30.{abc}, 31, 35}

3.1.9 An individual named Claudius is located at the point 0 in the accompanying diagram.



Using an appropriate randomization device (such as a tetrahedral die, one having four sides), Claudius first move to one of the four locations B_1, B_2, B_3, B_4. Once at one of these locations, another randomization device is used to decide whether Claudius next returns to 0 or next visits one of the other two adjacent points. This process then continues; after each moe, another move to one of the (new) adjacent points is determined by tossing an appropriate die or coin.

(a) Let X = the number of moves that Claudius makes before first returning to 0. What are possible values of X? Is X discrete or continuous?

(b) If moves are allowed also along the diagonal paths connecting 0 to A_1, A_2, A_3, and A_4, respectively answer the questions in part (a).

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3.2.15

Many manufacturers have quality control programs that include inspection of incoming materials for defects. Suppose a computer manufacturer receives computer boards in lots of five. Two boards are selected from each lot for inspection. We can represent possible outcomes of the selection process by pairs. For example, the pair (1, 2) represents the selection of boards 1 and 2 for inspection.

(a) List the ten different possible outcomes.

(b) Suppose that boards 1 and 2 are the only defective boards in a lot of five. Two boards are to be chosen at random. Define X to be the number of defective boards observed among those inspected. Find the probability distribution of X.

(c) Let $F(x)$ denote the cdf of X. First determine $F(0) = P(X \leq 0)$, $F(1)$, and $F(2)$; then obtain $F(x)$ for all other x.

■

3.2.17 A new battery?s voltage may be acceptable (A) or unacceptable (U). A certain flashlight requires two batteries, so batteries will be independently selected and tested until two acceptable ones have been found. Suppose that 90% of all batteries have acceptable voltages. Let Y denote the number of batteries that must be tested.

(a) What is $p(2)$, that is, $P(Y = 2)$?

(b) What is $p(3)$? [*Hint*: There are two different outcomes that result in $Y = 3$.]

(c) To have $Y = 5$, what must be true of the fifth battery selected? List the four outcomes for which $Y = 5$ and then determine $p(5)$.

(d) Use the pattern in your answers for parts (a)?(c) to obtain a general formula for $p(y)$.

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3.2.18 8. Two fair six-sided dice are tossed independently. Let M = the maximum of the two tosses (so $M(1,5) = 5$, $M(3,3) = 3$, etc.).

(a) What is the pmf of M? [*Hint*: First determine $p(1)$, then $p(2)$, and so on.]

(b) Determine the cdf of M and graph it.

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3.3.32 An appliance dealer sells three different models of upright freezers having 13.5, 15.9, and 19.1 cubic feet of storage space, respectively. Let X = the amount of storage space purchased by the next customer to buy a freezer. Suppose that X has pmf

x	13.5	15.9	19.1
p(x)	0.2	0.5	0.3

(a) Compute $E(X)$, $E(X^2)$, and $V(X)$.

(b) If the price of a freezer having capacity X cubic feet is$25X - 8.5$, what is the expected price paid by the next customer to buy a freezer?

(c) What is the variance of the price $25X - 8.5$ paid by the next customer?

(d) Suppose that although the rated capacity of a freezer is X, the actual capacity is $h(X) = X - .01X^2$. What is the expected actual capacity of the freezer purchased by the next customer?

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3.3.33 Let X be a Bernoulli rv with pmf as in Example 3.18.

(a) Compute $E(X^2)$.

(b) Show that $V(X) = p(1-p)$.

(c) Compute $E(X^{79})$.

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4.1.5 A college professor never finishes his lecture before the end of the hour and always finishes his lectures within 2 min after the hour. Let X = the time that elapses between the end of the hour and the end of the lecture and suppose the pdf of X is

$$f(x) = \begin{cases} kx^2 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k and draw the corresponding density curve. [*Hint*: Total area under the graph of $f(x)$ is 1.]

(b) What is the probability that the lecture ends within 1 min of the end of the hour?

(c) What is the probability that the lecture continues beyond the hour for between 60 and 90 sec?

(d) What is the probability that the lecture continues for at least 90 sec beyond the end of the hour?

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4.2.13 Example 4.5 introduced the concept of time headway in traffic flow and proposed a particular distribution for X = the headway between two randomly selected consecutive cars (sec). Suppose that in a different traffic environment, the distribution of time headway has the form

$$F(x) = \begin{cases} 0 & x \leq 0 \\ \frac{x}{4}\left[1 + \ln\frac{4}{x}\right] & 0 < x \leq 4 \\ 1 & x > 4 \end{cases}$$

(a) Determine the value of k for which $f(x)$ is a legitimate pdf.

(b) Obtain the cumulative distribution function.

(c) Use the cdf from (b) to determine the probability that headway exceeds 2 sec and also the probability that headway is between 2 and 3 sec.

(d) Obtain the mean value of headway and the standard deviation of headway.

(e) What is the probability that headway is within 1 standard deviation of the mean value?

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4.3.28 Let Z be a standard normal random variable and calculate the following probabilities, drawing pictures wherever appropriate.

(a) $P(0 \leq Z \leq 2.17)$
(b) $P(0 \leq Z \leq 1)$
(c) $P(-2.50 \leq Z \leq 0)$

■

4.3.29 In each case, determine the value of the constant c that makes the probability statement correct.

(a) $\Phi(c) = .9839$
(b) $P(0 \leq Zc) = .291$
(c) $P(c \leq Z) = .121$

4.3.30 Find the following percentiles for the standard normal distribution. Interpolate where appropriate.

(a) 91st
(b) 9th
(c) 75th

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4.3.31 Determine z_α for the following:

(a) $\alpha = 0.0055$
(b) $\alpha = 0.09$
(c) $\alpha = 0.663$

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4.3.35 Suppose the diameter at breast height (in.) of trees of a certain type is normally distributed with $\mu = 8.8$ and $\sigma = 2.8$, as suggested in the article ?Simulating a Harvester-Forwarder Softwood Thinning? (*Forest Products J.*, May 1997: 36?41).

(a) What is the probability that the diameter of a randomly selected tree will be at least 10 in.? Will exceed 10 in.?

(b) What is the probability that the diameter of a randomly selected tree will exceed 20 in.?

(c) What is the probability that the diameter of a randomly selected tree will be between 5 and 10 in.?

(d) What value c is such that the interval includes 98% of all diameter values?

(e) If four trees are independently selected, what is the probability that at least one has a diameter exceeding 10 in.?

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