Math 45 - Section —
Homework 2
Friday, March 25, 2016

B.{1,2,3,4,5,6,7,8}

B1 Use integrating factor method to find the general solution to the following differential equations.

(a) $2\frac{dy}{dt} + 3y = e^{4t}$

(b) $\frac{dy}{dt} + ty = 5t$

(c) $\frac{1}{x}\frac{dy}{dx} + \frac{1}{x^2}y = e^x$

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B2 Find the solution to the initial value problem

$$y' + y = \frac{e^{-t}}{t^2}, \qquad y(1) = 0,$$

and explain the behavior of the solution as t tends to $+\infty$.

$\blacksquare$

B3 Determine whether or not each of the following equations is exact. If it is exact, find the general solution.

(a) $\frac{dy}{dx} = -\frac{ax+by}{bx+cy}$

(b) $(e^x \sin y + 3y)dx - (3x - e^x \cos y)dy = 0$

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B4

(a) Find the general solution to the equation

$$\frac{dy}{dt} = \frac{4\sin(2t)}{y}.$$

Your answer should be an explicit function $y(t)$ with two branches.

(b) Now impose the initial condition $y(0) = 1$. What is the solution to the DE that satisfies this initial condition?

(c) What is the largest $t-$interval on which the solution is defined?

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B5 Let $f(x, y) = 3x^2 + 2y^2$. Find an exact first order differential equation whose solutions are the level curves of f.

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B6 For the last few decades, Italy has had its growth rate decline to where soon the country will not even have enough births (or immigration) to replace the number of deaths in the country. Thus, its population may soon start declining. The population of Italy in 1950 was 47.1 million, in 1970 it was 53.7 million, and in 1990 it was 56.8 million. Consider the non-autonomous Malthusian growth model given by the differential equation

$$\frac{dP}{dt} = (at + b)P, \qquad P(1950) = 47.1$$

where the constants a and b are to be determined by the data. Solve this differential equation with the data above. The solution is messy, so you can use a computer to deal with arithmetic or linear algebra (but not the differentiation) as long as you show all the steps of your work and explain what you are doing and why. Retain six places after the decimal. (Thanks to Mahaffy for this problem.)

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B7 You can solve the second order differential equation $y'' = y'$ using a change of variables combined with separation of variables (or inspection). This simple example illustrates a technique that can be applied to harder problems. Heres the idea:

- First define a new variable, $w = y'$.
- Rewrite the original DE in terms of w to get $w' = w$.
- Solve $w' = w$ to find $w(x)$.
- Substitute the answer you get for $w(x)$ into $w = y'$ and solve for y.
- Show that your solution satisfies the original DE.
- Find the solution that satisfies $y(0) = y'(0) = 1$.

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B8 Consider the equation

$$\frac{dy}{dt} = f(t, y) \tag{1}$$

where f and $\frac{\partial f}{\partial y}$ are both continuous functions for all real values of t and y. Let $y(t)$ and $\tilde{y}(t)$ both be two different solutions to (1). Explain why the graphs of y and $\tilde{y}$ cannot intersect.

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