

$$y' = 5 - ty$$

Integrate both sides with respect to t :

$$\int y' dt = \int (5 - ty) dt$$

$$y = 5t - \frac{1}{2}t^2 y + C$$

Use the initial condition $y(0) = 1$

$$1 = 5 \cdot 0 - 0 + C \Rightarrow C = 1$$

So

$$y = 5t - \frac{1}{2}t^2 y + 1$$

$$y + \frac{1}{2}t^2 y = 5t + 1$$

$$\therefore y(t) = \frac{5t + 1}{1 + \frac{1}{2}t^2}$$