Math 45 - Section —
HW 1
Tuesday, March 8, 2016

2, 3, 4, 5

2 By differentiating each function verify that $y_1(t) = e^{-t}$ and $y_2(t) = \sinh t$ both satisfy the differential equation $y'' - y = 0$. Is $y(t) = Ay_1(t) + By_2(t)$, where A and B are arbitrary constants, also a solution? Why or why not?

Note: If you need a refresher on *sinh* (hyperbolic sine) look it up on Wikipedia.

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3 Suppose that an object is moving in a straight line with constant acceleration $a \in \mathbb{R}$. Use properties of integration to show that the position of the object as a function of time t is given by

$$p(t) = \frac{1}{2}at^2 + v_0 t + p_0,$$

where v_0 and p_0 denote the velocity and position at time $t = 0$. Start by observing that acceleration is the second derivative of position, thus,

$$p''(t) = a.$$

Be careful in your solution to rigorously justify each step.

$\blacksquare$

4 Verify that

$$y(t) = e^{t^2} \int_0^t e^{-s^2} ds + e^{t^2}$$

is a solution to the differential equation $y' - 2ty = 1$ with initial condition $y(0) = 1$.

Note: If you're having trouble differentiating that integral, visit
http://mathworld.wolfram.com/SecondFundamentalTheoremofCalculus.html

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5 Examine Student W's work on the following problem. What did the student do correctly? What mistake(s) did the student make? What is a more correct response to the problem, and what would you say to help the student understand how to correctly complete the problem?

Determine if $y(x) = x^2 + 1$ is a solution to the initial value problem consisting of $4yy' = (y')^3 - 3y''x$ and the initial condition $y(0) = 1$.

If $y(x) = x^2+1$ then $y'(x) = 2x$ and $y''(x) = 2$.

Plug these into the DE:

$$4yy' = (y')^3 - 3y''x$$

$$4(x^2+1)\cdot 2x = (2x)^3 - 3\cdot 2\cdot x$$

$$\cancel{8x^3} + 8x = \cancel{8x^3} - 6x$$

$14x = 0$ means $x = 0$ and that matches $x_0 = 0$ in the initial condition. ✓

Also $y(x_0) = y(0) = 1$ so that checks out too. ✓

So $y(x) = x^2+1$ is a solution to the IVP.

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