

$$\int y' = \int -x + \int y$$

$$y = -\frac{1}{2}x^2 + \frac{1}{2}y^2 + C$$

$$2y = -x^2 + y^2 + 2C$$

Use the initial condition  $y(\sqrt{2}) = 0$

$$0 = -2 + 0 + 2C \Rightarrow C = 1$$

So,  $x^2 = y^2 - 2y + 2$

$$x^2 - 1 = y^2 - 2y + 1 = (y-1)^2$$

$$\therefore y - 1 = \pm \sqrt{x^2 - 1}$$

Choose the negative sign to get the correct branch of the solution.

$$y = 1 - \sqrt{x^2 - 1}$$

Check:  $y(\sqrt{2}) = 1 - \sqrt{2-1} = 1 - \sqrt{1} = 0 \quad \checkmark$