Math 45 - Section —
Homework 1
Friday, March 11, 2016

1,2,3,4,5,6,7

1 Find the solution to $y' = (1+x)y^2$, $y(0) = 1$. Determine the interval in which the solution is defined. Sketch the graph of the solution.

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2 Find the solution to $xdx + ye^{-x}dy = 0, y(0) = 1$. Determine the interval in which the solution is defined. Sketch the graph of the solution.

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3 Find the solution to $y^2(1-x^2)^{1/2}dy = \arcsin(x)dx$, $y(0) = 0$. Determine the interval in which the solution is defined. Sketch the graph of the solution.

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4 Find the general solution to

$$\text{a) } \frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}, \text{ and b) } \frac{dy}{dx} = \frac{x + 3y}{x - y}.$$

Hint: Use the substitution $y(x) = xv(x)$ to solve for $v(x)$ first.

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5 Consider the equation

$$y' + g(t)y = f(t) \tag{1}$$

where $f(t)$ and $g(t)$ are continuous functions on $\mathbb{R}$. Let

$$I(t) = \int_0^t g(s)ds.$$

Verify that

$$y(t) = Ce^{-I(t)} + \int_0^t e^{I(s)-I(t)} f(s)ds$$

is a solution to (1). What is $y(0)$?

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6 - Bernoulli's Differential Equation Let $n > 1$ be an integer. Show that the change of variables $z = y^{1-n}$ transforms the equation $y' + a(x)y = b(x)y^n$ into a linear differential equation.

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7 An object (skydiver?) falls off a plane from an altitude of 3,000 meters. The gravitational force of the earth increases its vertical velocity at a rate of 10 m/sec^2. After 10 seconds a parachute opens causing drag that reduces the vertical velocity at a rate of twice the vertical velocity.

a) Find the differential equation for the velocity after ten seconds.

b) Solve the equation in part a)

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