Math 45 - Section —
Homework 6
Friday, April 22, 2016

F.{1,2,3,4,5,6,7,8}

F1 For each of the following differential equations find a particular solution using the method of undetermined coefficients. Find also the solution satisfying the given initial condition:

(a) $y'' - y = x + \sin(x), y(0) = 0, y'(0) = 1$

(b) $y'' + y = e^x, y(0) = 0, y'(0) = 1$

(c) $y'' + y' + y = x^2, y(0) = 0, y'(0) = 1$

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F2 For each of the following differential equations find a particular solution using the method of undetermined coefficients. Find also the solution satisfying the given initial condition:

(a) $y'' - y' = x^2, y(0) = 1, y'(0) = 0$

(b) $y'' + 4y = \sin(2x), y(0) = y'(0) = 0$

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F3

(a) Using the method of variations of parameters show that the equation $y'' - y = e^x \sin(x)$ has a solution of the form $e^x(A\sin(x) + B\cos(x))$

(b) Using the method of variations of parameters show that the equation $y'' + y' - y = e^x \sin(x)$ has a solution of the form $e^x(A\sin(x) + B\cos(x))$

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F4 Consider a vibrating system described by the initial value problem

$$y'' + y = \sin(\omega x), \quad y(0) = y'(0) = 0.$$

(a) Find the solution for $\omega \neq 1$.

(b) Draw the graph of the solutions for $\omega = 0.7$, $\omega = 0.8$, $\omega = 0.9$. What can you say about this system as ω tends to 1?

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F5 An RLC circuit has a voltage source given by $E(t) = 20\text{V}$, a resistor 100Ω, an inductor of $4H$, and a capacitor of $0.01F$. If the initial current is zero and the initial charge in the capacitor is $4C$, determing the current in the circuit for $t > 0$.

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F6 An LC circuit has a voltage source given by $E(t) = \sin(50t)$V, an inductor of $2H$, and a capacitor of 0.02F, but no resistor. What is the current in the circuit for $t > 0$ if $I(0) = q(0) = 0$?

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F7 A mass-spring system consists of a 7kg mass, a spring with constant $3N/m$, a frictional component with damping constant $2(N - sec/m)$, and an external force given by $f(t) = 10\cos(10t)N$. Using a 10Ω resistor, construct an RLC circuit that is the analog of this mechanical system in the sense that they are governed by the same differential equation.

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F8 Verify that y_1 and y_2 satisfy the corresponding homogeneous equation; then find a particular solution of the given nonhomogeneous equation.

(a) $y'' - \frac{1+t}{t}y' + \frac{1}{t}y = te^{2t}; \quad t > 0 \quad y_1(t) = 1 + t, \;\; t_2(t) = e^t.$

(b) $y'' + \frac{1}{x}y' + \frac{x^2 - \frac{1}{4}}{x^2}y = 3x^{-1/2}\sin(x) \quad x > 0; \quad y_1(x) = x^{-1/2}\sin(x), \quad y_2(x) = x^{-1/2}\cos(x)$

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