Math 45 - Section —
Homework 3
Monday, April 4, 2016

C.{1,2,3,4,5,6,7,8}

C1 Find the equilibrium points of the following differential equations and determine their stability:

(a) $y' = y(3 - y) - 1$

(b) $y' = y^3(y^2 - 1)$

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C2 Find the equilibrium points of the following differential equations and determine their stability:

(a) $y' = y^2(y^2 - 1)$

(b) $y' = e^{-y}\sin(y)$

$\blacksquare$

C3 For each of the following find an equation $y' = f(y)$ with each of the following stated properties. If there no examples explain why not. (In all cases assume that f(y) is continuously differentiable.)

(a) Every real number is an equilibrium point.

(b) Every integer is an equilibrium point and there are no others.

(c) There are no equilibrium points.

(d) There are exactly 100 equilibrium points.

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C4 Find the values of $p > 0$ such that the solution to $y' = -y^p$, $y(0) = 1$ satisfies satisfies $y(t) = 0$ for some $t > 0$. Use the existence and uniqueness theorem to prove that if $p \geq 1$ then $y(t) > 0$ for all $t \in [0, +\infty)$.

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C5 Suppose the natural growth of a population is .02 and that there is no competition for resources in the population. If the population is constantly harvested it is modeled by the differential equation

$$P' = .02P - k$$

Discuss the role of the parameter $k > 0$. Prove that if k is very large the population will go extinct while for k small the population will thrive. Find the value of k that separates these regimes in terms of the initial population and the growth rate.

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C6 Let $f(x, y)$ be defined for all $(x, y) \in \mathbb{R}^2$, continuous and continuously differentiable in the variable y. Assuming that $f(x, y)y \leq 0$ for all (x, y), find the solution to $y' = f(x, y)$, $y(-.333) = 0$ (Hint: Compute $f(x, 0)$). Sketch the graphs of the solutions to this differential equation that satisfy $y(0) = 1$, and $y(0) = -1$.

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C7 Justify why the differential equation $y' = \sin(x^2 + y^3)$ is not separable, linear, or exact. Use a computer spreadsheet to approximate the solution given by Eulers method to this differential that satisfies $y(0) = 1$ in the interval $[0, 1]$ with mesh of size $h = .01$, and with mesh size $h = .001$. Find $y(.k)$ for $k = 1, \ldots, 10$ for each approximation. Use the graphing feature of the spreadsheet to obtain in the same caption the graphs of both solutions.

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C8 The solution to $y' = (1 + x^4)y^2$, $y(0) = 1$ blows up in $(0, \infty)$. Use a computer spreadsheet to estimate the blow up time using three Euler approximations to the solution. Find $y(.k)$ for $k = 1, \ldots, 10$ for each approximation. Separate the variables in this equation and find the solution. Compare the estimate for the blow up time obtained from the numerical approximations and the one given by the actual solution.

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