Math 45 - Section —
HW 09
Tuesday, April 19, 2016

1, 2, 3

1

(a) Given that $y = e^x$ is one solution to

$$xy'' - (x+1)y' + y = 0,$$

use reduction of order to find the general solution to this differential equation.

(b) Use variation of parameters to find the general solution to the driven DE

$$xy'' - (x+1)y' + y = x^2 e^x.$$

Important note: If you using Abel's Theorem to reduce the order of the DE or using the formulas for variation of parameters (instead of deriving them from scratch), make sure that the starting point for those formulas matches your DE so that you don't misuse any formulas.

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2 The *Bessel equation* of order one-half is

$$x^2y''(x) + xy'(x) + \left(x^2 - \frac{1}{4}\right) y(x) = 0.$$

(Note that the word "order" here doesn't refer to the highest-derivative of the DE—this is a second-order, linear, homogeneous DE and it involves the constant $1/4$, whose square root is $1/2$.) In this problem, you may assume that $x > 0$.

(a) Show that $y_1(x) = x^{-1/2} \cos x$ is a solution to the Bessel equation.

(b) Find the second solution, $y_2(x) = x^{-1/2} \sin x$, using reduction of order.

(c) Show y_1 and y_2 are linearly independent and therefore form a fundamental solution set.

(d) Find the general solution to the non-homogeneous equation

$$x^2y''(x) + xy'(x) + \left(x^2 - \frac{1}{4}\right) y(x) = x^{5/2}.$$

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3 Meet with your Math 45 final project group and perform the following set of tasks. It is okay for all of you to work on one document and for each person in the group to turn in a copy (print-out) of that document for this problem.

(a) Make a list of the variables and parameters of interest in your problem.

(b) Supply a differential equation or system of differential equations that your variable(s) of interest might satisfy. Give some reasoning behind your choice of differential equation(s).

(c) Perform a cursory mathematical analysis of your differential equation(s). For example, this could be a numerical solution, an analytic (exact) solution, or a computation of long term behavior.

(d) Describe at least one aspect of your mathematical analysis that is consistent with expectations and one aspect that seems unrealistic.

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