Math 45 - Section —
Homework 5
Friday, April 15, 2016

E.{1,2,3,4,5}

E1 For each of the following differential equations, find its general solution and the solution for the given initial condition:

(a) $y'' + y' - 2y = 2t, \quad y(0) = 0, \quad y'(0) = 1$

(b) $y'' - 2y' + y = te^t + 4, \quad y(0) = 1, \quad y'(0) = 1$

(c) $y'' + 4y = 3\sin(2t), \quad y(0) = 2, \quad y'(0) = -1$

(d) $y'' + 2y' + 5y = 4e^{-t}\cos(2t), \quad y(0) = 1, \quad y'(0) = 0$

■

E2 For each of the following differential equations, find its general solution on the given interval:

(a) $y'' + y = \tan(t)$ with $t \in (0, \frac{\pi}{2})$

(b) $y'' + 4y' + 4y = t^{-2}e^{-2t}$ with $t > 0$

(c) $y'' - 5y' + 6y = g(t)$ with $t \in (-\infty, \infty)$ (you may assume $g(t)$ is continuous for all $t \in \mathbb{R}$)

$\blacksquare$

E3

(a) Find a second order linear homogenous equation with coefficients continuous on $(1, \infty)$, (i.e. of the form $y'' + p(t)y' + q(t)y = 0$,) whose general solution is $C_1 t + C_2 e^t$.

(b) Find a second order linear homogenous equation with coefficients continuous on $(1, \infty)$, (i.e. of the form $y'' + p(t)y' + q(t)y = 0$,) whose general solution is $C_1 t + C_2 t^2$.

■

E4 Reduction of order.

(a) Given that x^{-1} is a solution to $y'' + \frac{3}{x}y' + \frac{1}{x^2}y = 0$ for $x > 0$, find the general solution to this equation on $(0, \infty)$ by looking for a second solution of the form $y(x) = u(x)x^{-1}$.

(b) Given that e^x is a solution to $y'' - \frac{x+2}{x}y' + \frac{2}{x}y = 0$ for $x > 0$, find the general solution on $(0, \infty)$ as in part (a).

■

E5 Determine the general solution of

$$y'' + \lambda^2 y = \sum_{m=1}^{N} a_m \sin(m\pi t),$$

where $\lambda > 0$ and $\lambda \neq m\pi$ for $m = 1, \ldots, N$.

$\blacksquare$