Newton's Method for Convex Optimization

Conner DiPaolo, Jeffrey Rutledge, Colin Adams

Harvey Mudd College

February, 2016

Finding Roots of Functions

Given a differentiable function $f : \mathbf{R} \mapsto \mathbf{R}$ we want to find the instances when $f(x) = 0$ (not generally solvable in closed form).
Newton's Method:

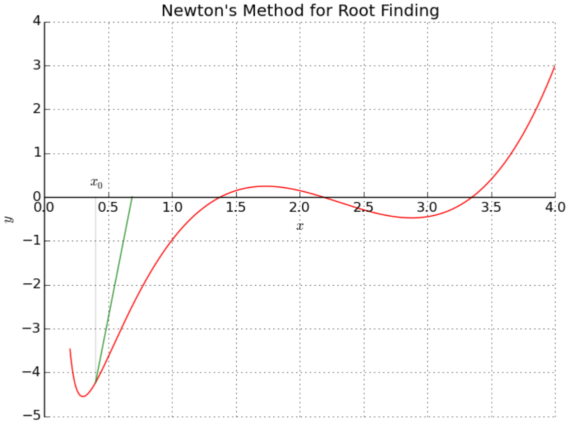
1. Take a starting position x_0
2. Find where the tangent line $y = f(x_n) + (x_{n+1} - x_n)f'(x_n)$ is 0 and iterate:

$$y = f(x_n) + (x_{n+1} - x_n)f'(x_n) = 0$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Finding Roots of Functions

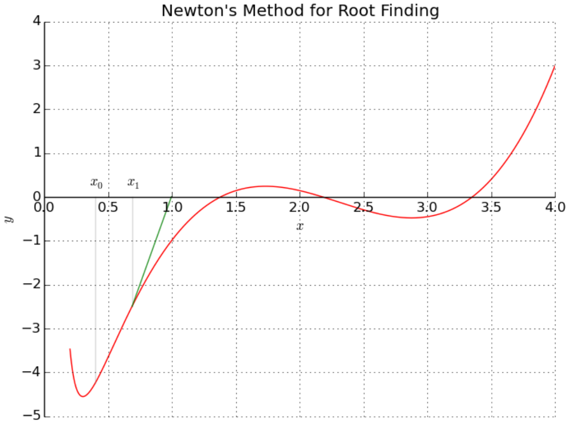
Initial $x_0 = 0.4,\ f(x) = (x-1)(x-3)^2 + e^{\frac{1}{3}/x} - \cos\left(\frac{x}{2}\right) - 1.5$



$$x_0 = 0.4 \quad \Delta = 0.9805$$

Finding Roots of Functions

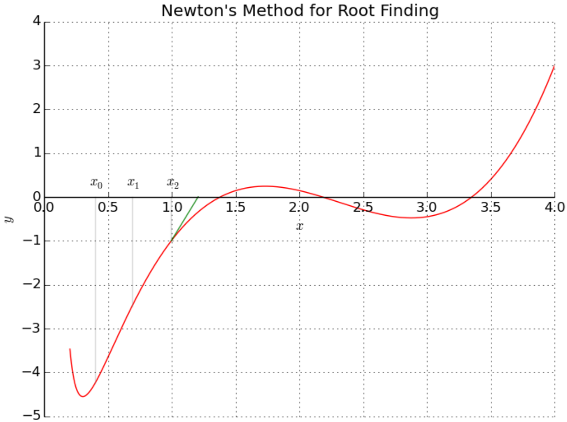
Initial $x_0 = 0.4,\ f(x) = (x-1)(x-3)^2 + e^{\frac{1}{3}/x} - \cos\left(\frac{x}{2}\right) - 1.5$



$x_1 = 0.6866 \quad \Delta = 0.6938$

Finding Roots of Functions

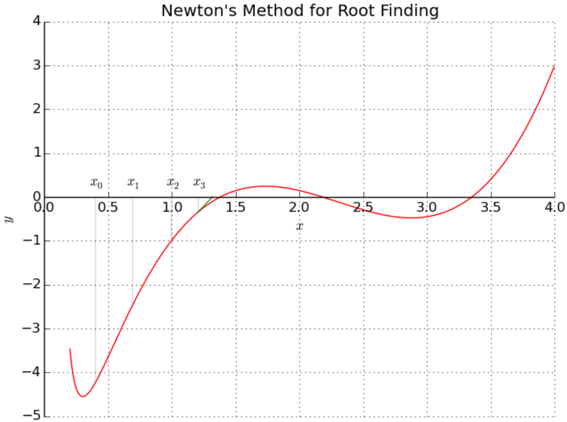
Initial $x_0 = 0.4,\ f(x) = (x-1)(x-3)^2 + e^{\frac{1}{3}/x} - \cos\left(\frac{x}{2}\right) - 1.5$



$$x_2 = 0.9938 \quad \Delta = 0.3866$$

Finding Roots of Functions

Initial $x_0 = 0.4,\ f(x) = (x-1)(x-3)^2 + e^{\frac{1}{3}/x} - \cos\left(\frac{x}{2}\right) - 1.5$



$x_3 = 1.2050 \quad \Delta = 0.1754$

Optimizing Function Using Newton's Method

Instead of finding wheren $f(x) = 0$, how can we find where $f'(x) = 0$? Same method:

$$x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)}$$