

SPECULATIVE LOOP PARALLELIZATION

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Bachelor Thesis

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Declaration of Authorship

I, Johannes Doerfert, declare that this thesis titled, ‘SPECULATIVE LOOP PARALLELIZATION’ and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Signed:

Date:

“Debugging is twice as hard as writing the code in the first place. Therefore, if you write the code as cleverly as possible, you are, by definition, not smart enough to debug it.”

Brian Kernighan, professor at Princeton University

SAARLAND UNIVERSITY

Abstract

Compiler Design Lab
Department of Computer Science

Bachelor of Science

by Johannes Doerfert

SPolly, short for speculative Polly, is an attempt to combine two recent research projects in the context of compilers. On the one hand side there is Polly, a LLVM project to increase data locality and parallelism of loop nests. On the other hand there is Sambamba, which pursues a new, adaptive way of compiling and offers features like method versioning, speculation and runtime adaption. As an extension of the former one and with the capabilities offered by the later one, SPolly can perform state-of-the-art loop optimizations on a wide range of loops, even in general purpose benchmarks as the SPEC 2000 benchmark suite. I will explain when speculation is possible, how runtime information is used and how this is integrated into Polly and Sambamba. At the end an evaluation on SPEC 2000 benchmarks and the Polybench 3.2 benchmark suite is presented as well as some discussions on the results.

Acknowledgements

The acknowledgements and the people to thank go here, don't forget to include your project advisor...

CONTENT

Declaration of Authorship	i
Abstract	iii
Acknowledgements	iv
Abbreviations	vii
1 Introduction	1
1.1 Related Work	1
1.2 Overview	2
2 Background	3
2.1 LLVM - The Low Level Virtual Machine	3
2.2 The Polyhedral Model	4
2.3 Polly - A Polyhedral Optimizer For LLVM	6
2.3.1 Static Control Parts	6
2.3.2 SCoP Detection	6
2.3.3 Loop Optimizations	7
2.3.3.1 Parallel Code Generation	7
2.4 Sambamba - A Framework For Adaptive Program Optimization	8
2.3.4 Sambambas Parallelizer	9
2.3.5 Parallel Control Flow Graphs	9
3 Concept	10
3.1 SPolly In A Nutshell	10
3.2 Region Scores	10
4 Implementation	13
4.1 Speculative Polly	14

4.1.1	Speculative SCoPs	15
4.1.2	Region Speculation	15
4.1.2.1	sSCoP extraction	16
4.1.2.2	sSCoP Versions	16
4.1.2.3	Introduced tests	17
4.1.3	Code Generation	20
4.2	Sambama Compile Time Module	20
4.3	Sambama Runtime Module	22
4.3.1	Profiling	22
4.4	Profiling For Sambamba	22
5	Evaluation	23
5.1	The Environment	23
5.2	Compile Time Evaluation	24
5.2.1	Preperation	24
5.2.2	Quantitative Results	24
5.2.2.1	SPEC2000	24
5.2.2.2	Polybench 3.2	25
5.2.2.3	Available tests	25
5.2.3	Sound Transformations	26
5.3	Runtime Evaluation	26
5.4	Problems	26
6	Case Study	27
6.1	Matrix Multiplication	27
6.1.1	Case A	27
6.1.2	Case B	28
6.1.3	Case C	28
7	Conclusion	29
A	Case Study Source Code	32
B	SPolly As Static Optimizer Pass	33
	Bibliography	35

ABBREVIATIONS

AA	A lias A nalysis
LLVM	L ow L evel V irtual M achine
LLVM-IR	L lvm I ntermediate R epresentation
SCoP	S tatic C ontrol P art
SPolly	S peculative P olly
isl	i nteger s et l ibrary
cloog	C hunky L oop G enerator
SE	S calar E volution
SD	S CoP D etection
RS	R egion S peculation (see ...)
Polly	P olyhedral L lvm
CFG	C ontrol F low G raph
LOC	l ines o f c ode
ParCFG	P arallel C FG
OpenMP	O pen M ulti- P rocessing
SIMD	S ingle I nstruction M ultiple D ata

For/Dedicated to/To my...

CHAPTER 1

INTRODUCTION

Multiprocessors became the normal state in everyday computing. Automatic parallelization did not. Programmers still write sequential code which will be translated to sequential binaries and executed by a single thread using only one of many cores. Benefits of modern multiprocessors are still unused because neither programmers nor compilers may utilize their potential to the full. Even if parallelism could employ all threads and cores, it is unclear how to expose a reasonable amount out of sequential code. Apart from the retrieval, parallelism faces the same problems as sequential code does. Slow I/O accesses caused by bad cache locality is a well known one. Heavy research is going on in both fields with various results. As there are promising approaches suffering from poor applicability on general purpose code, the real problem has changed.

One of this techniques is called polyhedral optimization. The underlying polytope model is a mathematical description of loop nests with their data dependencies. Optimal solutions in terms of e.g., locality or parallelism can be derived using this model while it implicitly applies traditional optimizations as loop blocking and unrolling. Various preliminary results reveal the potential but also the limits of this technique.

1.1 Related Work

Research on parallelism and data locality is very popular nowadays, as is the polytope model to tackle these problems. With or without speculation, there are promising attempts all using the polytope model, but the wide range impact on general purpose code is still missing.

Tobias Grosser describes in his thesis[2] a speedup of up to 8 times for matrix multiplication, archived by his polyhedral optimizer Polly[3]. He could also produce similar results for other benchmarks of the Polybench[4] benchmark suite. Other publications on this topic[5–7] show similar results, but they are also limited to the Polybench benchmark suite. Admittedly, Polybench is well suited for comparative studies, between these approaches, but it has less significance for general applicability. Baghdadi et. al.[6] revealed a huge potential for speculative loop optimizations. They state that aggressive

loop nest optimizations (including parallel execution) are profitable and possible, even though data and flow dependencies would statically prevent them. Theirs hand made tests also showed the impact when different kinds of conflict management are used. [TODO use this sentence or loose it]

1.2 Overview

SPolly, short for speculative Polly, is an attempt to combine two recent research projects in the context of compilers. On the one hand there is Polly, a LLVM project to increase data locality and parallelism for loop nests. On the other hand there is Sambamba, which pursues an adaptive way of compiling and offers features like method versioning, speculation and runtime adaption. As an extension of the former one and with the capabilities offered by the later one, SPolly can perform state-of-the-art loop optimizations on a wide range of loops, even in general purpose benchmarks as the SPEC 2000 benchmark suite.

The key idea is to enable more loop optimizations due to speculation. To demarcate this from guessing, static and dynamic information is combined and used by an heuristic to choose promising candidates.

The rest of the thesis will be organised as follows. First Chapter 2 will provide information on the used tools and techniques, especially Polly and Sambamba. Afterwards the concepts and ideas for this work are stated in Chapter 3. Technical details about SPolly are given in Chapter 4 followed by an evaluation on the SPEC 2000 and Polybench 3.2 benchmark suites (Chapter 5). While Chapter 7 concludes the thesis and provides ideas for future work, a detailed case study on different versions of the matrix multiplication example is presented in Chapter 6.

Notes:

For simplicity source code is presented in a C like language only.

TODO more ?

CHAPTER 2

BACKGROUND

This work takes heavy use of different techniques, theories and tools mostly in the context of compiler construction. To simplify the rest of the thesis this chapter explains them as far as necessary, thus I may take them for granted afterwards. As most of the key ideas will suffice, some details will be omitted. Interested readers have to fall back on the further readings instead.

2.1 LLVM - The Low Level Virtual Machine

The Low Level Virtual Machine is a compiler infrastructure designed to optimize during compiletime, linktime and runtime. Originally designed for C and C++, many other frontends for a variety of languages exist by now. The source is translated into an intermediate representation (LLVM-IR), which is available in three different, but equivalent forms. There is the in-memory compiler IR, the on-disk bitcode representation and human readable assembly language. The LLVM-IR is a type-safe, static single assignment based language, designed for low-level operations. It is capable of representing high-level structures in a flexible way. Due to the fact that LLVM is built in a modular way and can be extended easily, most of the state of the art analysis and optimization techniques are implemented and shipped with LLVM. Plenty of other extensions, e.g., Polly, can be added by hand.

Further Reading

- A Compilation Framework for Lifelong Program Analysis & Transformation [8]
- <http://www.llvm.org>

2.2 The Polyhedral Model

The polyhedral model is a mathematical way to describe the iteration space of a subset of all loop nests. Its became popular because it can be used to abstract from the given source and apply loop optimizations in a pure mathematical way.

Definition 2.1 (Polytope). A polytope P in an n -dimensional space restricted by m inequalities is defined as:

$$P := \{x \in \mathbb{Z}^n \mid Ax \leq b \text{ where } A \in \mathbb{Z}^{m \times n} \text{ and } b \in \mathbb{Z}^m \text{ are constant} \}$$

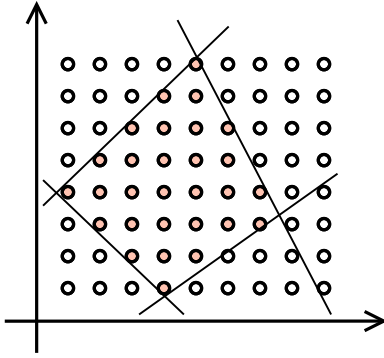


FIGURE 2.1: 2-dimensional polytope

Within the polyhedral model the iteration space is modelt as a \mathbb{Z} -polytope, simply spoken a geometric object with flat surfaces existing in a space of any general dimension. Definition 2.1 restrict all (integer) points within the polytope to be solutions of an affine system of inequalities and figure 2.1 shows an example for dimension $n = 2$ and $m = 4$. If the polytope is bounded we may speak of convex polytopes instead. For clarification it is worth to say that some authors e.g., Benabderrahmane et al. [10] use the term polyhedron instead.

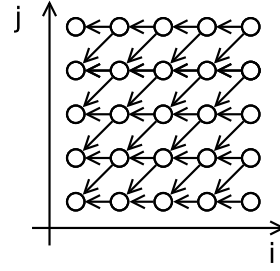
From the compiler writer perspective, the polytope corresponds to the iteration space of a loop nest, where all loops have affine bounds and steppings. Each one corresponds to a dimension of the vector space, thus the dimension of the polytope is determined by the depth of the loop nest. In addition to the points of the iteration space, the polyhedral model is also capable of representing loop carried dependencies between two iterations. Figure 2.2 illustrates this by relating a simple loop nest(A) to its representation within the polyhedral model(B). The main benefit of this representation is that transformations can be done in an optimal manner using an integer linear programming solver which maximizes the loop nest for e.g., parallelism. Such transformations within the model are in fact (composed) affine transformations with the additional advantage that they implicitly apply traditional loop optimizations including tiling, skewing, loop interchange and unrolling. Using the example in listing 2.2a again we may end up with a loop nest as in listing 2.2c. Data-locality is increased by introducing the two inner most loops which also remove the data dependency from the outer most one, thus the whole loop nest may be executed in parallel now.

```

for (i = 1; i < 100; i++) {
  for (j = 1; j < 100; j++) {
    A[i][j] = A[i-1][j-1]
              * B[i-1][j];
  }
}

```

(A) Simple loop nest



(B) Polyhedral representation for listing 2.2a

```

for (c1=-128; c1<=98; c1+=32) {
  for (c2=max(0, -32*floord(c1,32)-32); c2<=min(98, -c1+98); c2+=32) {
    for (c3=max(max(-98, c1), -c2-31); c3<=min(c1+31, -c2+98); c3++) {
      for (c4=max(c2, -c3); c4<=min(min(98, c2+31), -c3+98); c4++) {
        A[c3+c4+1][c4+1] = A[c3+c4][c4] * B[c3+c4][c4+1];
      }
    }
  }
}

```

(c) Optimized version of listing 2.2a

FIGURE 2.2: An example loop nest with its polyhedral representation

Further Reading

- The Polyhedral Model is More Widely Applicable Than You Think [10]
- Loop Parallelization in the Polytope Model [11]
- A practical automatic polyhedral parallelizer and locality optimizer [5]
- PoCC - The Polyhedral Compiler Collection [12]
- Polyhedral parallelization of binary code [7]
- Putting Polyhedral Loop Transformations to Work [13]

2.3 Polly - A Polyhedral Optimizer For LLVM

Exploiting parallelism and data-locality in order to balance the work load and to improve cache utilization are the main goals of the Polly research project. The polytope model is used as abstract mathematical representation to get optimal results for a particular objective. The three-step approach of Polly first detects maximal loop nests suitable for polyhedral representation. These representations are analyzed and transformed before they are converted to LLVM-IR again. This last step of code generation is capable of generating thread level parallelism and vector instructions. The maximal loop nests Polly detects are called static control parts, or short (valid) SCoPs, are the most central entity within Polly and crucial for any kind of argumentation.

2.3.1 Static Control Parts

A Static Control Part is a subgraph of the control flow graph with one entry edge and one exit edge. Inside this subgraph only nested conditionals and loops are allowed (see figure 2.3). All contained loops need affine bounds and a canonical induction variable, thus the lower bound has to be 0 and the stepping 1. While non affine memory accesses are allowed, all branch conditions have to be affine functions, only depending on loop invariant parameters and surrounding induction variables. Aliasing instructions within such a region are not permitted either.

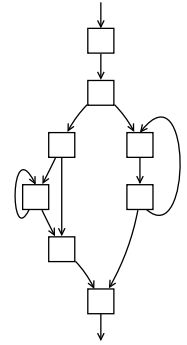


FIGURE 2.3:
SCoP CFG

While some of these conditions (e.g., the canonical induction variables) can be achieved through preprocessing, the rest is still quite restrictive.

Even so, these requirements ensure some very interesting attributes as statically known control flow and computable memory accesses. The later ones are overestimated if a memory access is not affine. If a loop nest fulfills these requirements it implicitly has a polyhedral representation which can be used for further analyses and transformations. Such regions are called valid static control parts, or short SCoP.

SCoPs which do not contain a loop are not interesting, thus we may consider only SCoPs containing at least one loop.

2.3.2 SCoP Detection

Pollys SCoP detection is the gateway to all further analyzsese and transformations. All regions fulfilling the properties described in the last section, or in short, all (valid) SCoPs are detected here. The special interest of this part arises from the fact that all regions

regions declared as valid SCoPs will be considered for polyhedral optimizations. Almost regardless of its content, any region could be given to Polly if the SCoP detection is instrumented to do so. Two important consequences can be derived and summarized as: Utilizing the strength of Polly is possible in much more situations as intentionally implemented but coupled with responsibility of the outcome. [TODO ziemlich melodramatisch oder ?]

2.3.3 Loop Optimizations

Polly uses the integer set library (isl) to compute the scheduling and tiling for a SCoP but it is also possible to use the more matured PoCC optimizer. Once the polyhedral description is computed, an optimized version of the algorithm proposed by Bondhugula et al. [5] will compute a new scheduling and tiling scheme. Traditional loop optimizations such as blocking, interchange, splitting, unrolling or unswitching are implicitly applied during this step. Not only the new scheduling but also the new data dependencies are computed, crucial to exploit parallelism.

2.3.3.1 Parallel Code Generation

While cache locality is implicitly improved by rescheduling and tiling of the loop nest, parallel code needs to be generated explicitly. Polly is capable of generating thread level parallelism using OpenMP annotations and data level parallelism using SIMD instructions. The former one depends on the OpenMP shared library being present while the later one uses the LLVM built-in vector instruction.

Further Reading

- Polly - Polyhedral optimization in LLVM [3]
- Enabling Polyhedral Optimizations in LLVM [2]
- A Framework for Automatic OpenMP Code Generation [14]
- Base algorithm of isl [5]
- <http://polly.llvm.org>
- <http://www.kotnet.org/~skimo/isl/>

2.4 Sambamba

A Framework For Adaptive Program Optimization

The Sambamba project is build on top of the LLVM compiler infrastructure and aims at adaptive and speculative runtime optimizations. Dynamic information about arguments or the global state may allow optimization which could not be applied at compile time or did not seem interesting back then. It is easily extendible with compile time and a runtime parts. While each one is conceptually independent, the compile time parts may store information which can be accessed at runtime to reduce the overhead, even if expensive analysis results are needed. Another fundamental pillar of the framework is the multi versioning system which allows for different, specialized function versions. [TODO should VMAD be mentioned here] A similar approach has been published by Jimborean et al.[16], in fact th In both cases a dispatcher is used to choose one of the available implementations each time the function is called. In this manner exclusive optimizations can be applied on the same function and according to the input a version can be dispatched. Runtime profiling also reveals opportunities to speculatively transform the program and obtain more specialized versions. A main difference between Sambamba and the work of Jimborean is probably how these function versions are constructed. Sambamba does not rely on source code annotations but allows all modules to find and extract the versions fully automatic. A high level view on the Sambama concept is given by figure 2.4 before some of the built-in utilities and modules, used during this work, are explained.

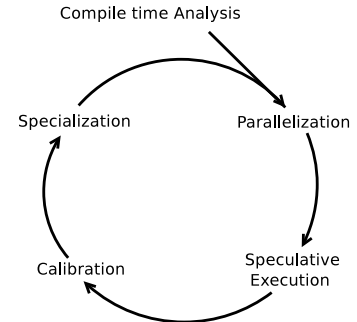


FIGURE 2.4: Sambamba in a nutshell

2.3.4 Sambamba Parallelizer

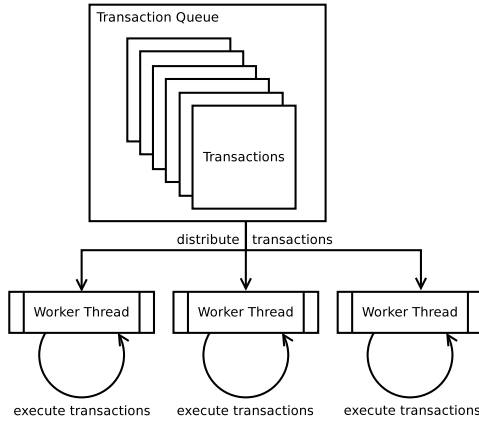


FIGURE 2.5: Symbolized transaction queue with 3 worker threads

The Sambamba parallelizer will become the main interface for any kind of parallelization in the framework. At the moment it is in need of parallel control flow graphs to explicitly state section which should be executed in parallel, but in the future more automatic loop parallelization will be implemented too. The runtime part of the parallelizer instantiates its own worker threads so there is no need for external libraries (as OpenMP) in order to execute tasks in parallel.

2.3.5 Parallel Control Flow Graphs

A parallel control flow graph (ParCFG) is a data structure used by the parallelizer to express parallel sections within an ordinary CFG. Each parallel sections consist of an entry block called **piStart** and an exit block called **piEnd**. The **piStart** is terminated by a symbolic switch statement which may have an arbitrary number of predecessors. Every predecessor denotes a, so called, transaction which ends in the **piEnd** block after some arbitrary computation. Once parallelized each transaction may be executed. Figure 2.6 shows such a parallel section with 3 transactions.

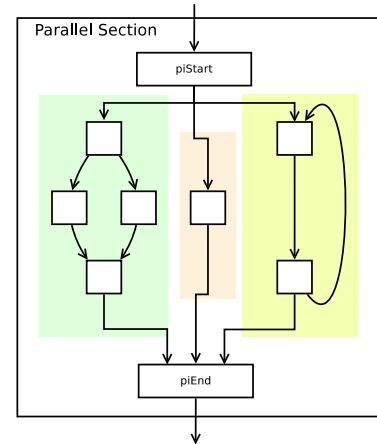


FIGURE 2.6: A parallel section with 3 transactions

Further Reading

- Sambamba: A Runtime System for Online Adaptive Parallelization [17]
- <http://www.sambamba.org>

CHAPTER 3

CONCEPT

From a high point of view, this work tries to weaken the harsh requirements on SCoPs in order to make Pollys loop optimizations applicable on a wider range of programs. Apart from the implementation work, which will be described in the next chapter, immense effort has been made on the concepts and key ideas behind. We believe that these ideas and the knowledge gained during the work is very valuable not only for future work on SPolly or one of its bases but also for other approaches facing similar situations. On the way to a working version many pitfalls have been encountered that should be avoided in the future, perhaps with similar approaches we worked out.

3.1 SPolly In A Nutshell

3.2 Region Scores

Region scores are the heuristic used to decide whether or not a sSCoP is worth to speculate on, thus for which region should profiling and parallel versions be created and used. As the former ones may change the score again it is reasonable to create parallel version later if the collected data suggest to do so. Initial efforts to create these scores did not use any kind of memory, thus every call needed to reconsider the whole region. To avoid this unnecessary computations the current score is a symbolic value which may contain variables for variables not known statically, branch probabilities and the introduced tests. Evaluation of these symbolic values will take all profiling information into account and yield a comparable integer value. Only during the initial score creation region speculation will traverse the region to find parameters

```
for (i = 0; i < 1024; i++) {  
    A[i] = B[i] * C[i];  
}
```

(A) complete static sSCoP

```
for (i = 2; i < 1024; i++) {  
    if (cond(i))  
        A[i] += A[i-1] * A[i-2];  
    else  
        A[i] -= A[i-1];  
}
```

(B) Branch within a sSCoP

```
for (i = 0; i < N; i++) {  
    if (A[i] == 0)  
        printf("Unlikely error!");  
    else  
        A[i] = 1024 / A[i];  
}
```

(C) irreversible call within a sSCoP

FIGURE 3.1: example sSCoPs

and branches for later annotation. All instructions will be scored and the violating ones will be checked for their speculative potential. As memory instructions are guarded by the STM for the case the speculation failed, calls may not be reversible, thus without any speculative potential at all. Such function calls are not checked earlier since the region speculation needs the information about possible other branches within this region. Listing 3.1c provides such an example but these cases will be revisited in the next two chapters too. Table 3.1 lists the scores for the examples in figure 3.1.

TABLE 3.1: Scores for the sSCoPs presented in various listings

listing	score
3.1a	576 (if A,B and C may alias)
3.1b	$63 * (11 + ((7 * \text{@if.then_ex_prob})/100) + ((5 * \text{@if.else_ex_prob})/100))$
3.1c	$((0 \text{ smax } \%N)/16) * (6 + (-1000 * \text{@if.then_ex_prob}/100))$

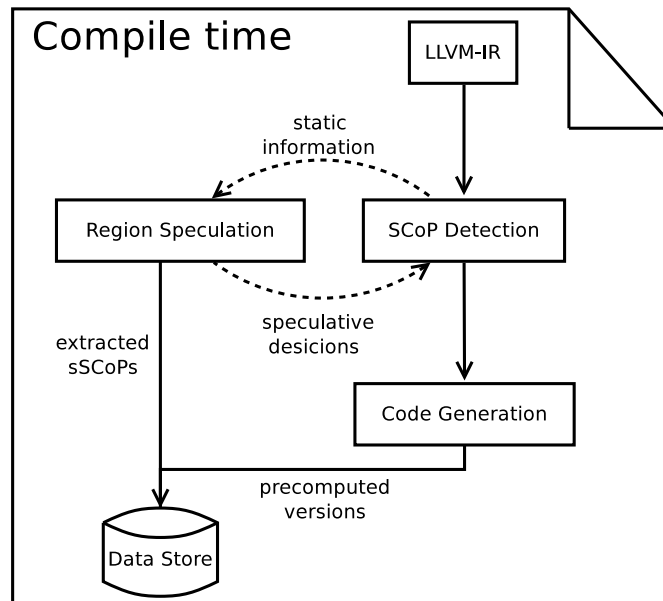


FIGURE 3.2: Draft paper: SPolly at compile time

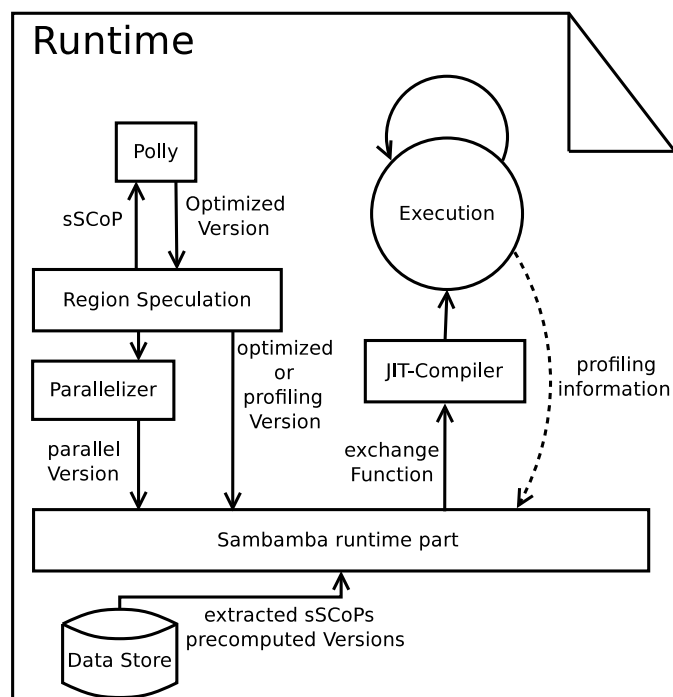


FIGURE 3.3: Draft paper: SPolly at runtime

CHAPTER 4

IMPLEMENTATION

SPolly in its entirety is a compound of three parts. The first one is called region speculation and it is embedded into Polly. The second and the third part are Sambamba modules, one for the compile time and the other one for the runtime. The region speculation part acts like a storage and interface for all discovered sSCoPs, thus it contains most of the transformation code. The Sambamba passes concentrate on the program logic which is at the moment far more evolved in the runtime part. The compile time component plays a minor role but it can be enriched by new functionalities in the future. Beyond the three parts of SPolly I implemented a basic Profiler and a Statistic module for Sambamba which have been very helpful during the development and may become permanent features of Sambamba as there are no comparable counterparts at the moment. During the implementation various bugs occurred and even if most of them arose through my own fault I triggered some in the code base of Polly and Sambamba too. A full table of reported bugs is listed in table ??.

Table 4.1 compares the work in a quantitative manner as it lists the lines of code (LOC) added for SPolly as well as for Sambamba and Polly parts. [TODO one sentence about the table content]

TABLE 4.1: Lines of code for Polly, SPolly and Sambamba components

component	LOC
SPolly	XXX
Region Speculation	XXX
SPolly Sambamba CT	XXX
SPolly Sambamba RT	XXX
Polly	XXX
SCoP Detection	XXX
Code Generation	XXX
Sambamba	XXX
Parallelizer	XXX
TODO MORE	

[TODO rephrase] Table B.1 lists all available command line options added in the context of SPolly. Although all of these options work without the Sambamba modules, yet Sambamba at all, the last three would only produce sequential executable code without any parallelization. [TODO] As for now I am not quite sure if this could be of any practical use, but to my understanding Polly could get a similar option in the near future.

4.1 Speculative Polly

It would be feasible to look at SPolly as extension to Polly, especially designed to interact with Sambamba. As such it was crucial to preserve all functionality of Polly and supplement it with (mostly speculative) new ones. Most of them are implemented in the region speculation, but there are some new options in the code generation too. Apart from these two locations the SCoP detection was the only component which has been touched. It is ideally suited to serve as the bridge between Polly and the speculative part as speculative valid regions would be rejected here. The information currently needed for region speculation is also available at this point and can be directly reused.

As the architecture of Polly is nicely illustrated by figure ??, it has been extended in figure 4.1 to capture the changes introduced by SPolly. In comparison the region speculation, the fork join backend and the sSCoP backend have been added as they can be used without Sambamba.

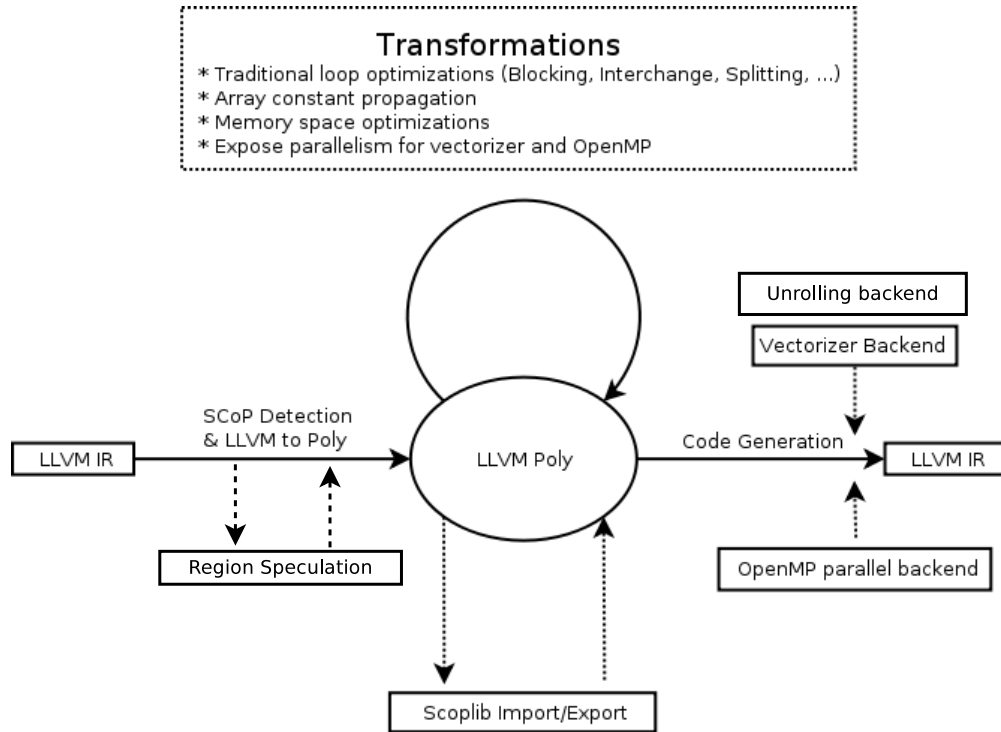


FIGURE 4.1: SPolly architecture

4.1.1 Speculative SCoPs

A speculative valid SCoPs (sSCoPs) is defined similar to a valid SCoPs but with weakened conditions. General SCoPs need to fulfill all constraints listed in ??, while sSCoPs renounce the restrictions on aliasing and partially those on function calls.

4.1.2 Region Speculation

The region speculation (RS) of SPolly has several tasks to fulfill. The first one includes the communication with Polly or more precise, with the SCoP detection. Each region analyzed by the SCoP detection needs to be considered as possibly speculative valid SCoPs, thus all information exposed during the detection are stored. If the region contains a validation not listed in 4.2 or if it is without any validation the information are discarded. Valid SCoPs are handled by Polly while for others a new sSCoP is created which initially validates itself. These validation mainly computes information needed for later transformations but it maybe discards the sSCoP too. This is the case if violating function calls, e.g., a “printf”, occur on every execution path. In the following these computations as well as the creation of profiling and parallel versions for a sSCoP are explained briefly.

TABLE 4.2: Restrictions on sSCoPs

- Only perfectly nested loops and conditionals
- No unsigned iteration variables ¹
- Only canonical PHI nodes and induction variables
- Instructions may not be used in PHI nodes outside the region²
- Only speculatively “non violating” function calls
- No PHI nodes in the region exit block
- Only simple regions not containing the entry block of the function

¹ open for further work of Polly

² open for further work of SPolly

4.1.2.1 sSCoP extraction

The sSCoP extraction was designed to simplify the method versioning for functions containing several speculative valid SCoPs. It creates a new sub function for every sSCoP and inserts a call in the former place. Later on this call could be inlined again but this is not implemented yet. On the one hand side the creation of profiling and parallel versions as well the later function exchanging becomes a lot easier and cheaper this way. On the other hand it is the first step to multiple specialized versions, e.g., with constant instead of variable loop bounds.

4.1.2.2 sSCoP Versions

Method versioning is one of the great benefits of the Sambamba system and allows to adapt the execution at runtime. In the scope of this work two different versions are created, a profiling one and a optimized one. As these two are explained in the following, there are several possible others described in this thesis and summarized in the last chapter.

The Profiling Version

Profiling is a powerful ability for just in time executed program, like the ones produced by Sambamba. SPolly benefits not only from the introduced tests, but also from the monitored branch probabilities and loop trip counts. The retrieved information are in the first place used to compute the region scores, thus to improve the heuristic which chooses sSCoPs for optimization. Later on, specialized versions may arise based on them.

The Optimized Version

Just as valid SCoPs may valid sSCoPs be optimized by Polly, but there are also differences. As Polly normally takes care of possible dependencies it cannot do the same for all kinds of sSCoPs.

4.1.2.3 Introduced tests

To reduce the overhead of misspeculation tests are introduced in front of each sSCoP. At the moment there are two different kinds available, invariants and alias tests. Only the later one is used in optimized versions because the former one needs to check the invariant every iteration, thus it produces a huge overhead. In contrast to profiling versions which uses both tests to refine the score of a sSCoP. Placing the test section was quite easy, since Polly itself introduces a (constant) branch before the SCoP anyway. Figure 4.2a and 4.2b shows the (simplified) CFG introduced by Polly and SPolly, respectively. The dotted edge in the CFG produced by Polly is not taken (constant false), but remains in the CFG. As far as I know, SPolly is the first extension to Polly which uses the untouched original SCoP.

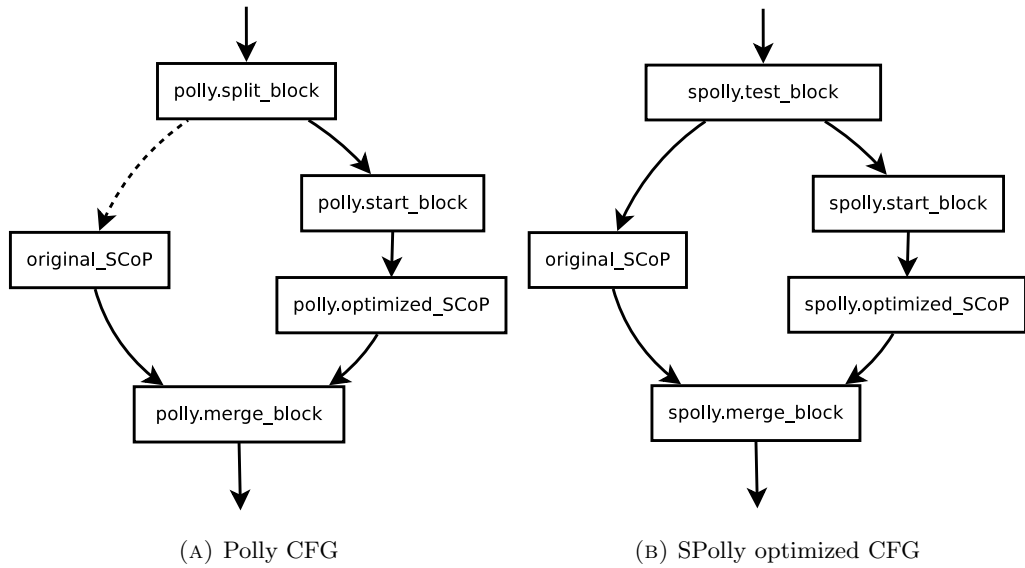
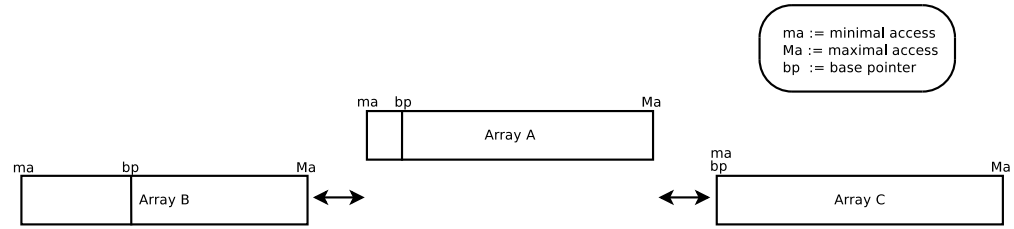


FIGURE 4.2: CFG produced by Polly and SPolly, respectively

Alias tests

Testing for aliasing pointers in general would not be feasible so another way was chosen. Only sSCoP invariant pointers are tested once before the sSCoP is entered. If the test succeeds, thus no aliases are found, the optimized version is executed. At compile time

the accesses for each base pointer are collected and marked either as possible minimal, possible maximal or not interesting access. At runtime all possible minimal and maximal, respectively, accesses are compared and the minimal and maximal access for each base pointer is computed. The alias test as such compares again the minimal access for a base pointer with the maximal accesses for the others and vice versa. At the end of this comparison chain the result replaces the constant guard in the split block right before the original SCoP and the speculative optimized one. If all base pointers are invariant in the SCoP the test is complete, thus aliasing can be ruled out for the sSCoP at runtime. However, non invariant pointers are not tested at all, as it would imply to perform all computation and testing within the loop. Figure 4.3a illustrates the concept of the alias tests while listing 4.3b and figure 4.3c provide an example with the derived accesses. The alias test for this example would look like listing 4.3d.



(A) Alias test from a birds eye view

```

for (i = 0; i < N; i++) {
  for (j = 0; j < N; j++) {
    // I1
    C[i*N+j] = 0;
    for (k = 0; k < N; k++) {
      // I2          I3          I4
      C[i*N+j] += A[i*N+k] * B[k*N+j];
    }
  }
}

```

(B) Aliasing accesses

Acc	bp	ma	Ma
I1	C	0	N*N-1
I2	C	0	N*N-1
I3	A	0	N*N-1
I4	B	0	N*N-1

(C) Statically derived min/maximal accesses

```

bool ab = B[N*N-1] < A[0] || B[0] > A[N*N-1];
bool ac = C[N*N-1] < A[0] || C[0] > A[N*N-1];
bool bc = B[N*N-1] < C[0] || B[0] > C[N*N-1];
bool result = ab && ac && bc;

```

(D) Introduced compare chain

FIGURE 4.3: Alias tests concept

Invariants tests

Apart from alias tests, SPolly may introduce invariants tests if there are possibly invariant variables and a function call within a sSCoP. The key idea is to monitor possible changes in such variables during the execution of the profiling version. As the results may introduce new dependencies between loop iterations, the sSCoP could be discarded. If it does not, the sSCoP may be optimized, depending on its new region score. As this is a disqualification test in the first place, the information gathered about the variables could be used to create specialized sSCoP versions. As mention earlier, the last chapter will discuss specialized versions in more detail. Listing 4.4a gives an example of an sSCoP for which invariant tests can be introduced and 4.4b shows the modified source.

<pre> int c; void f() { int i; for (i = 0; i < 1024; i++) { // function g may change c A[i] = g() + c; } } </pre>	<pre> int c; void f() { int i; int c_tmp = c; for (i = 0; i < 1024; i++) { if (c != c_tmp) signalNonInvariancy(); // function g may change c A[i] = g() + c; } } </pre>
(A) source	(B) source with invariant tests

FIGURE 4.4: Invariant test introduced by SPolly

Complete Checks

Alias tests may rule out aliasing in sSCoPs completely, thus some sSCoPs become valid SCoPs after this tests are introduced. Such non speculative optimizations are done by the compile time part of the Sambamba module and may be included in Polly too. An example for such an sSCoP is given in figure 4.3b. The presented code is the well known matrix multiplication example which is a valid SCoP if the arrays do not alias. For the sake of completeness it is to mention that this code could be rewritten as array of pointers which would also lead to a sSCoP but without complete checks. Chapter 6 will discuss this example and the various implementations in detail.

4.1.3 Code Generation

As part of the extension of Polly a new code generation type was added. Apart from sequential, vectorized and OpenMP annotated code generation SPolly is capable of creating a unrolled and blocked loop, which can be easily translated into an ParCFG, thus parallelized by a Sambama module. Listing 4.5b presents this transformation. The special case where lower and upper bound as well as the stride are statically known constants, the second loop, which computes remaining iterations, is completely unrolled. This kind of loop unrolling and blocking may find its way into Polly in the near future.

Parallelization

In order to secure the speculative executions with Sambambas STM (see ??) the Sambama parallelizer needs to be used. As this parallelizer does not yet support loop parallelization per se, some transformation needed to be done first. The new code generation type was created to do the difficult one without recomputing information provided during by the polytope model (during the code generation) anyway. As these transformation yields code as in listing 4.5b the creation of a ParCFG (see ??) remains. Figures 4.5c and 4.5d visualize these changes.

4.2 Sambama Compile Time Module

The compile time part of the Sambamba module locates all sSCoPs within the given input module and transfers each one afterwards in a separated function. These extracted sSCoPs are stored within the created Sambamba bytecode or respectively executable file. Extracting every single sSCoP decreases the performance but allows to easily change and combine different optimized sSCoPs, even if they originated from the same function. At the moment there is one exception implemented which will be applied on sSCoPs with complete checks, thus valid SCoPs after the tests are passed. All of those are optimized in place without consulting the region scores or any other heuristic. The functionality of the compile time part is minimal but it helps to focus on profiling and execution during runtime, without analysis overhead. Further work could heavily improve this part, beginning by compile time preparation of the sSCoPs, but imaginable is more than just a precomputed profiling or optimized version of a sSCoP. As there are a lot of

```

void f() {
  int i;
  for (i = B; i < E; i += S) {
    LoopNest(i);
  }
}

```

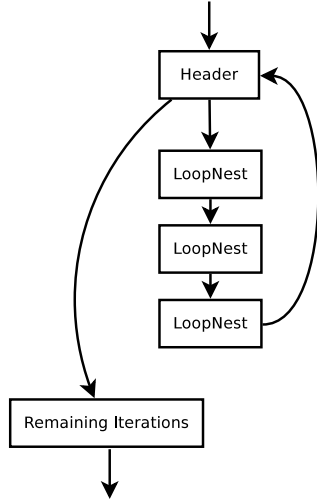
(A) Initial loop nest

```

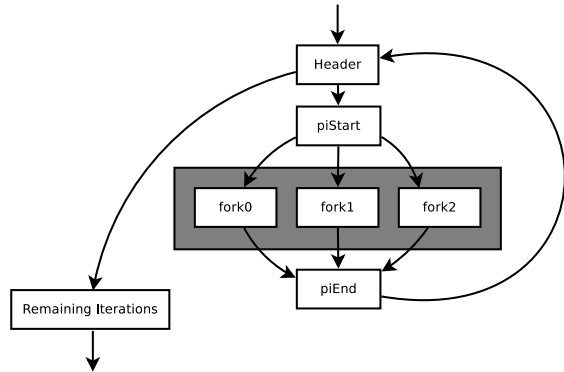
void f() {
  int i;
  for (i = B; i < E - N + 1; i += S * N) {
    LoopNest(i + 0 * S);
    ...
    LoopNest(i + N * S);
  }
  // Remaining iterations
  for (int j = i; j < E; j += S) {
    LoopNest(j);
  }
}

```

(B) Listing 4.5a after N-fork creation



(C) SPolly forked CFG



(D) SPolly ParCFG

FIGURE 4.5: Forked CFG produced by SPolly and resulting ParCFG

parameters which could have significant impact on the performance, several optimized versions of an sSCoP could be created and stored, in order to choose the best depending on the system and the actual run, thus depending on runtime information. While table B.2 gives a brief overview of available options for Polly, it is not clear which ones will fit best for a particular environment and sSCoP. As the method versioning system of Sambamba evolves, the compile time part should, in order to reduce the workload at runtime and increase the ability to adapt.

4.3 Sambama Runtime Module

In addition to the compile time parts, which only rely on static analyses, the runtime part uses different kinds of runtime information to decide. To take advantage of this extra knowledge, most of the decisions, thus most of the program logic, is implemented in the runtime module.

Table ??[TODO] gives an overview of the functionalities.

TABLE 4.3: Functionality of the SPolly Sambama module

4.3.1 Profiling

4.4 Profiling For Sambama

Sambama, as heavily developed research project, was not capable of any kind of profiling when I started my work. By now, there are two profilers available. The first one, implemented by the authors of Sambama, is used for exact time measuring, while I created the second one to profiles executions and data. [TODO if first is used later on, write it here] Both were developed during the same time to fulfill different needs and could be [TODO will be ?] merged anytime soon. As most of SPollys parts are unrelated, to both of them, the profiling versions, as their name indicates, would become useless without. This will definitely increase the number of unnecessary created (parallel) functions, but it would not render SPolly redundant. There a sSCoPs which a guarded by sound checks, thus they can be used with the overhead of only the check. As the use of other sSCoPs could increase the performance too, a heuristic could look for promising candidates, even without any runtime information. This heuristic could be part of future work since even with a profiler by hand, there are cases where the gathered information (see [TODO]) are not helpful at all.

CHAPTER 5

EVALUATION

The evaluation, as essential part of this thesis, allows to compare the this work with others which share the same goals. Since it is hard to put effort in figures, the evaluation may provide information about the results of them. Correctness, applicability and speedup have been the stated goals which were tested using the SPEC 2000 and Polybench 3.2 benchmark suites. While the former one contains general benchmarks used to evaluate most new optimizations, the later one is especially designed for polyhedral optimizations as done by SPolly. These benchmarks might not reflect the reality in everyday optimization but they might be used to compare SPolly and Polly (as presented in the diploma thesis of Tobias Grosser [2]).

5.1 The Environment

(more details in table 5.1).

running Arch linux with an *Intel(R) Core(TM) i5 CPU M 560 @ 2.67GHz* and 6GB RAM. Parallel versions could use up to four simultaneous running threads. The second one ... TODO ...

As the compile time evaluation is machine independent, it was performed on the general purpose machine only. Contrary, the runtime evaluation has been performed twice, once on each machine.

The work and thus the evaluation is based on an LLVM 3.0 build with enabled assertions and disabled optimization. All source files have been converted by clang to LLVM-IR files, optimized by opt and ... TODO linked TODO

TABLE 5.1: The evaluation environment

	A	B
CPU	i5 M560	X5570
clock speed	2.67GHz	2.93GHz
smart cache	3MB	8MB
#cores	2	8
#threads	4	16
RAM	6GB	24 GB
LLVM	3.0 debug	3.0
OS	Arch	Gentoo R7

TODO picture of the chain

5.2 Compile Time Evaluation

The main part of the compile time evaluation aims to get quantitative results about valid and invalid sSCoPs. These results correspond with the applicability of this work, as they both outline how many regions can be taken into account now and which work is needed to increase this number. As mentioned earlier this part is mainly machine independent, since the quantitative results are. There is one case where compile time transformation can improve the program with no need of speculation at all. These cases are explained and evaluated separately in section 5.2.3.

5.2.1 Preperation

TODO -basicaa -indvars -mem2reg -polly-independent -polly-region-simplify -polly-prepare

5.2.2 Quantitative Results

5.2.2.1 SPEC2000

TODO why not all spec2000 benchmarks ?

TODO

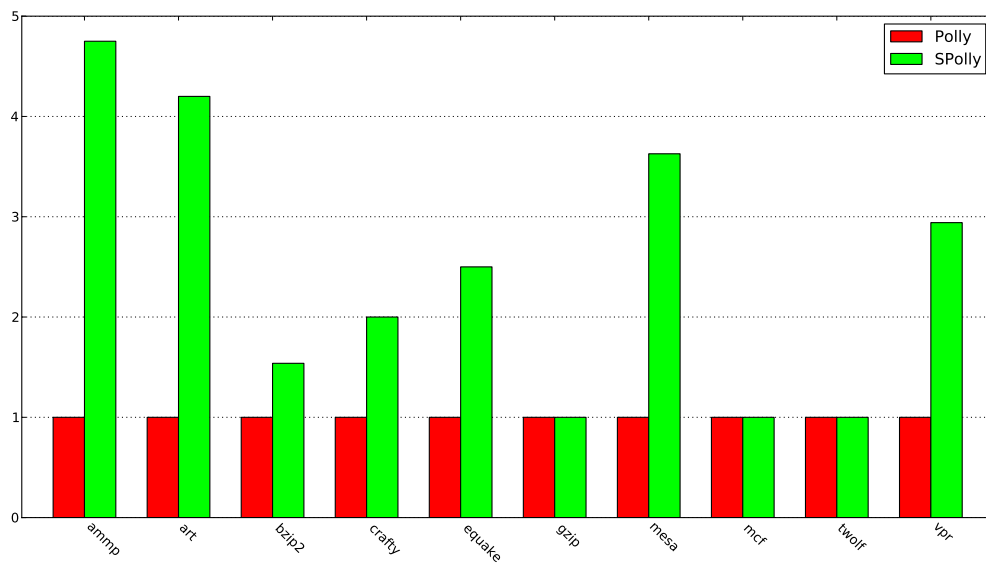


FIGURE 5.1: Numbers of valid and speculative valid SCoPs

5.2.2.2 Polybench 3.2

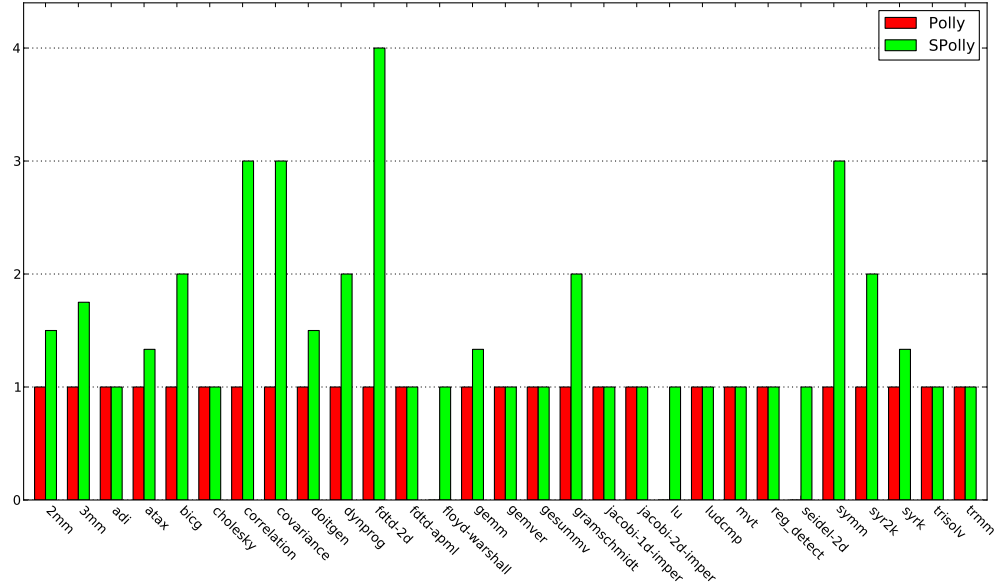


FIGURE 5.2: Numbers of valid and speculative valid SCOPs

TABLE 5.2: Results of running Polly and SPolly on SPEC 2000 benchmarks

Benchmark	#instr	#simple regions		valid SCOPs		Avg detec. time	
		initial	prepared	Polly	SPolly	Polly	SPolly
188.amp	19824	205	208	12	45		
179.art	1667	66	66	5	16		
256.bzip2	3585	114	116	13	7		
186.crafty	25541	305	310	23	23		
183.equake	2585	70	71	10	15		
164.gzip	4773	92	95	6	0		
181.mcf	1663	33	33	0	0		
177.mesa	80952	816	832	94	247		
300.twolf	35796	679	716	6	0		
175.vpr	19547	319	329	17	33		

TODO numbers can be produced via `/home/johannes/git/sambamba/testdata/spec2000/timeAnalysis.`

5.2.2.3 Available tests

Alias tests

Invariant tests

5.2.3 Sound Transformations

As described earlier, region speculation collects violations within a SCoP and can introduce tests for some of them. There are cases when these tests will suffice to get a sound result, thus there is no need for a runtime system at all. Although this hold in respect to the soundness of a program, this does not mean performance will rise when these transformations are used.

TODO scores – heuristic / statistics

5.3 Runtime Evaluation

5.4 Problems

During the work with LLVM 3.0 and a corresponding version of Polly a few problems occurred. Some of them could not be reproduced in newer versions they were just be tackled with tentative fixes, as they will be resolved as soon as Sambamba and SPolly will be ported to a newer version. Others, which could be reproduced in the current trunk versions, have been reported and listed in figure 5.3. All bugs were reported with a minimal test case and a detailed description why they occur.

TABLE 5.3: Reported bugs

ID	Description	Status	Patch provided	Component
12426	Wrong argument mapping in OpenMP subfunctions	RESOLVED FIXED	yes	Polly
12427	Invariant instruction use in OpenMP subfunctions	NEW	yes	Polly
12428	PHI node use in OpenMP subfunctions	NEW	no	Polly
12489	Speed up SCoP detection	NEW	yes	Polly
TODO	add the others to bugzilla			
TODO	PollybenchC2 gemver , 2mm			

CHAPTER 6

CASE STUDY

6.1 Matrix Multiplication

Matrix multiplication is a well known computational problem and part of many algorithms. If the data size grows, the runtime may have crucial impact on the overall performance. Tiling, vectorization and parallel execution yield an enormous speedups as different approaches already showed[2, 16], still the question about applicability remains. A slightly modified source code would not be optimized at all, even if the computation has not been changed.

This section will compare different implementations of a simple 2d matrix multiplication. As sample size 1024*1024 floats are used (N is defined as 1024). Each example is executed 10 times and the geometric mean of the results (without the best and worst one) is computed. All numbers are generated on the server described in table ???. The base algorithm stays the same for each case, so there is no hand made optimization involved. The computed result is checked each time in order to prevent false optimizations.

6.1.1 Case A

Listing 6.1 shows the matrix multiplication as used in many presentations and benchmarks. This case is quite grateful because the global arrays are distinct, the loop nest is perfectly nested and all memory accesses can be computed statically. With this in mind the popularity of this case is hardly surprising, just as the good results of all tested versions are.

```
float A[N][N], B[N][N], C[N][N];

void matmul() {
    int i, j, k;

    for (i = 0; i < N; i++)
        for (j = 0; j < N; j++)
            for (k = 0; k < N; k++)
                C[i][j] += A[k][i]
                           * B[j][k];
}
```

FIGURE 6.1: Matmul case A

6.1.2 Case B

```

void matmul(float A[N][N],
            float B[N][N],
            float C[N][N]) {
    int i, j, k;

    for (i = 0; i < N; i++)
        for (j = 0; j < N; j++)
            for (k = 0; k < N; k++)
                C[i][j] += A[k][i]
                           * B[j][k];
}

```

FIGURE 6.2: Matmul case B

Listing 6.1 shows the matrix multiplication as used in many presentations and benchmarks. This case is quite grateful because the global arrays are distinct, the loop nest is perfectly nested and all memory accesses can be computed statically. With this in mind the popularity of this case is hardly surprising, just as the good results of all tested versions are.

6.1.3 Case C

Listing 6.1 shows the matrix multiplication as used in many presentations and benchmarks. This case is quite grateful because the global arrays are distinct, the loop nest is perfectly nested and all memory accesses can be computed statically. With this in mind the popularity of this case is hardly surprising, just as the good results of all tested versions are.

```

void matmul(float *A,
            float *B, float *C) {
    int i, j, k;

    for (i = 0; i < N; i++)
        for (j = 0; j < N; j++)
            for (k = 0; k < N; k++)
                C[i*N+j] += A[k*N+i]
                           * B[j*N+k];
}

```

FIGURE 6.3: Matmul case C

TABLE 6.1: Case study results

CHAPTER 7

CONCLUSION

LIST OF FIGURES

Figure 2.1	2-dimensional polytope	4
Figure 2.2	An example loop nest with its polyhedral representation	5
Figure 2.3	SCoP CFG	6
Figure 2.4	Sambamba in a nutshell	8
Figure 2.5	Symbolised transaction queue with 3 worker threads	9
Figure 2.6	A parallel section with 3 transactions	9
Figure 3.1	example sSCoPs	10
Figure 3.2	Draft paper: SPolly at compile time	12
Figure 3.3	Draft paper: SPolly at runtime	12
Figure 4.1	SPolly architecture	14
Figure 4.2	CFG produced by Polly and SPolly, respectively	17
Figure 4.3	Alias tests concept	18
Figure 4.4	Invariant test introduced by SPolly	19
Figure 4.5	Forked CFG produced by SPolly and resulting ParCFG	21
Figure 5.1	Numbers of valid and speculative valid SCoPs	24
Figure 5.2	Numbers of valid and speculative valid SCoPs	25
Figure 6.1	Matmul case A	27
Figure 6.2	Matmul case B	28
Figure 6.3	Matmul case C	28
Figure A.1	Matmul case A (full)	32

LIST OF TABLES

Table 3.1	Scores for the sSCoPs presented in various listings	11
-----------	---	----

Table 4.1	Lines of code for Polly, SPolly and Sambamba components	13
Table 4.2	Restrictions on sSCoPs	15
Table 4.3	Functionality of the SPolly Sambamba module	22
Table 5.1	The evaluation environment	23
Table 5.2	Results of running Polly and SPolly on SPEC 2000 benchmarks . .	25
Table 5.3	Reported bugs	26
Table 6.1	Case study results	28
Table B.1	Command line options to interact with SPolly	33
Table B.2	Brief overview of Polly’s optimization options	34

APPENDIX A

CASE STUDY SOURCE CODE

All presented cases of the example matrix multiplication (see section 6.1) are embedded into a small template. Listing A.1 shows the template instanciated with the first matrix multiplication case (A). Apart from the main method and the matmul function time measurement and a correctness check are included.

```
#define N 1024

float A[N][N], B[N][N], C[N][N];

void matmul() {
    int i, j, k;

    for (i = 0; i < N; i++)
        for (j = 0; j < N; j++)
            for (k = 0; k < N; k++)
                C[i][j] += A[k][i] * B[j][k];
}

void init_arrays() {
    int i, j;

    for (i=0; i<N; i++) {
        for (j=0; j<N; j++) {
            A[i][j] = (1+(i*j)%1024)/2.0;
            B[i][j] = (1+(i*j)%1024)/2.0;
            C[i][j] = 0;
        }
    }
}

double sum_array() {
    double sum = 0.0;
    int i, j;

    for (i=0; i<N; i++) {
        for (j=0; j<N; j++) {
            sum += C[i][j];
        }
    }

    return sum;
}

int main() {
    double sum;

    int i;
    for (i = 0; i < 10; i++) {
        init_arrays();
        matmul();
        sum = sum_array();

        if (sum != 69920704991472.0)
            return 1;
    }

    return 0;
}
```

FIGURE A.1: Matmul case A (full)

APPENDIX B

SPOLLY AS STATIC OPTIMIZER PASS

TABLE B.1: Command line options to interact with SPolly

Command line option	Description
-enable-spolly	Enables SPolly during SCoP detection, (options containing spolly will not work without)
-spolly-replace	Replaces all sSCoPs by optimized versions (may not be sound)
-spolly-replace-sound	As spolly-replace, but sound due to runtime checks (iff sound checks can be introduced)
-spolly-extract-regions	Extracts all sSCoPs into their own sub function
-polly-forks=N	Set the block size which is used when polly-fork-join code generation is enabled
-enable-polly-fork-join	Extracts the body of the outermost, parallelizeable loop, performs loop blocking with block size N and unrolls the new introduced loop completely (one loop with N calls in the body remains)
-polly-inline-forks	Inline the call instruction in each fork

TABLE B.2: Brief overview of Polly's optimization options

Short or option name	Description
-polly-no-tiling	Disable tiling in the scheduler
-polly-tile-size=N ¹	Create tiles of size N
-polly-opt-optimize-only=STR	Only a certain kind of dependences (all/raw)
-polly-opt-simplify-deps	Simplify dependences within a SCoP
-polly-opt-max-constant-term	The maximal constant term allowed (in the scheduling)
-polly-opt-max-coefficient	The maximal coefficient allowed (in the scheduling)
-polly-opt-fusion	The fusion strategy to choose (min/max)
-polly-opt-maximize-bands	Maximize the band depth (yes/no)
-polly-vector-width=N ¹	Try to create vector loops with N iterations
-enable-polly-openmp	Enable OpenMP parallelized loop creation
-enable-polly-vector	Enable loop vectorization (SIMD)
-enable-polly-atLeastOnce	Indicates that every loop is at least executed once
-enable-polly-aligned	Always assumed aligned memory accesses
-enable-polly-grouped-unroll	Perform grouped unrolling, but don't generate SIMD

¹ Not available from the command line

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