Lab 4 - Non-negative Matrix Factorization

Submission Instructions

- Submission Deadline: Wednesday (29 September 2021) at 11.59 pm EST.
- Please submit the lab in any one of these forms (we will be strict about students adhering to these formats -
- 1. A completed Lab_4.mlx with the corresponding nmf.m, ssnmf.m, compute_objective.m, compute_objective_ss.m files.
- 2. A completed Lab_4.m file **and** an exported PDF with the corresponding .m files as above. To export your .m file to a PDF, use the Publish Tab in the Editor. It is **necessary** to label all the figures you include, if they have not been labelled in the helper code already.

The ORL Faces Dataset

We use the ORL Face database for this assignment, which consists of 400 images for 40 people, each of size 112×92 . These images were taken at different times, with varying lighting and for different facial expressions. All faces are in an upright position with a frontal view, with a slight left-right rotation. To use this dataset, we perform some pre-processing on them, listed here -

- 1. We use only the train split for this data (the first 9 images per person).
- 2. We construct a data matrix matx of size 10304×360 by flattening all the faces.
- matX is divided by the max value present in the images to normalize the data and avoid overflow issues, giving us the final data matrix v.

```
clear all; close all;
d1 = 112; d2 = 92; d = d1*d2;
num_images = 9; num_people = 40;
images = cell(num_people, num_images);
matX = zeros(d, num_people*num_images);
count = 1;
for i = 1:num_people
    for j = 1:num_images
        % filename = sprintf('/your/local/path/to/orl_faces/Train/s%i/%i.pgm', i, j);
        filename = sprintf('./orl_faces/Train/s%i/%i.pgm', i, j);
        img = double(imread(filename));
        matX(:,count) = reshape(img, d, 1);
        count = count+1;
    end
end
V = matX/max(matX(:));
```

Performing NMF

To perform NMF, we want to decompose the matrix V = BW. To do so, we'll follow the following steps

- 1. Create an NMF function nmf(V, rank, max_iter, lambda)
- 2. Initialize *B* and *W*randomly, and make sure *W*has unit-sum columns (each column should sum to 1).
- 3. Calculate the initial objective (as seen in the lecture for KL Divergence). It will be helpful to define a function compute_objective(V, W, B) that returns the objective value.
- 4. Perform the iterations $B = B \otimes \frac{\left(\frac{V}{BW}\right)W^T}{1W^T}$ and $W = W \otimes \frac{B^T\left(\frac{V}{BW}\right)}{B^T1}$, where \otimes specifies element-wise multiplication and all divisions are element-wise division.
- 5. Calculate the new objective function value.
- 6. Stopping Criteria: Stop when the absolute difference of objective values is smaller than or equal to λ (or) the max number of iterations has been reached.

Notes -

- Boilerplate for nmf(V, rank, max_iter, lambda) has been provided in nmf.m.
- Boilerplate for compute_objective(V, W, B) has been provided in compute_objective.m
- The notation $1W^T$ and B^T1 are another way of writing the sum of each column of W and B. What these denominator terms are doing are normalizing the columns of W and B such that they have unit-sum. You should ensure the columns of your W normalize to 1. What happens if coulmns of B are also normalized to 1?

Validation on the ORL Faces Dataset

NMF: iter 16: obj=95657.718048, nrmB=300.816088, nrmW=3.023851 NMF: iter 17: obj=94222.516526, nrmB=300.816669, nrmW=3.023890 NMF: iter 18: obj=92653.479410, nrmB=300.817459, nrmW=3.023936 NMF: iter 19: obj=90976.860000, nrmB=300.818587, nrmW=3.023988 NMF: iter 20: obj=89227.474154, nrmB=300.820235, nrmW=3.024045

Step 1: Output the new bases and weights and plot Calling your NMF function on the data matrix x with parameters rank = 40, max_iter=500, and lambda=0.001 will look something like

```
[B, W, obj, k] = nmf(V, 40, 500, 0.001);
NMF: init: obj=-153675.601797
NMF: iter 1: obj=104401.810475, nrmB=300.815691, nrmW=3.023651
NMF: iter 2: obj=104141.339094, nrmB=300.815146, nrmW=3.023658
NMF: iter 3: obj=103877.368336, nrmB=300.814708, nrmW=3.023665
NMF: iter 4: obj=103603.027938, nrmB=300.814379, nrmW=3.023671
NMF: iter 5: obj=103310.970429, nrmB=300.814149, nrmW=3.023677
NMF: iter 6: obj=102992.980684, nrmB=300.814009, nrmW=3.023684
NMF: iter 7: obj=102639.671403, nrmB=300.813950, nrmW=3.023691
NMF: iter 8: obj=102240.213345, nrmB=300.813966, nrmW=3.023698
NMF: iter 9: obj=101782.136063, nrmB=300.814050, nrmW=3.023707
NMF: iter 10: obj=101251.261485, nrmB=300.814193, nrmW=3.023718
NMF: iter 11: obj=100631.864487, nrmB=300.814390, nrmW=3.023731
NMF: iter 12: obj=99907.185847, nrmB=300.814635, nrmW=3.023746
NMF: iter 13: obj=99060.439232, nrmB=300.814921, nrmW=3.023765
NMF: iter 14: obj=98076.429504, nrmB=300.815250, nrmW=3.023789
NMF: iter 15: obj=96943.799619, nrmB=300.815631, nrmW=3.023817
```

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NMF: iter 21: obj=87443.970322, nrmB=300.822639, nrmW=3.024105
NMF: iter 22: obj=85662.852170, nrmB=300.826082, nrmW=3.024168
NMF: iter 23: obj=83913.009094, nrmB=300.830872, nrmW=3.024232
NMF: iter 24: obj=82212.424507, nrmB=300.837321, nrmW=3.024295
NMF: iter 25: obj=80567.869424, nrmB=300.845716, nrmW=3.024355
NMF: iter 26: obj=78977.241642, nrmB=300.856295, nrmW=3.024412
NMF: iter 27: obj=77433.397006, nrmB=300.869234, nrmW=3.024465
NMF: iter 28: obj=75928.142369, nrmB=300.884634, nrmW=3.024512
NMF: iter 29: obj=74455.394060, nrmB=300.902515, nrmW=3.024555
NMF: iter 30: obj=73013.015212, nrmB=300.922819, nrmW=3.024593
NMF: iter 31: obj=71603.265384, nrmB=300.945402, nrmW=3.024627
NMF: iter 32: obj=70232.057536, nrmB=300.970044, nrmW=3.024657
NMF: iter 33: obj=68907.384288, nrmB=300.996440, nrmW=3.024685
NMF: iter 34: obj=67637.399983, nrmB=301.024222, nrmW=3.024709
NMF: iter 35: obj=66428.687301, nrmB=301.052965, nrmW=3.024732
NMF: iter 36: obj=65285.125385, nrmB=301.082218, nrmW=3.024753
NMF: iter 37: obj=64207.525787, nrmB=301.111539, nrmW=3.024772
NMF: iter 38: obj=63193.933671, nrmB=301.140522, nrmW=3.024789
NMF: iter 39: obj=62240.330168, nrmB=301.168831, nrmW=3.024804
NMF: iter 40: obj=61341.454717, nrmB=301.196209, nrmW=3.024817
NMF: iter 41: obj=60491.545782, nrmB=301.222482, nrmW=3.024827
NMF: iter 42: obj=59684.902780, nrmB=301.247553, nrmW=3.024835
NMF: iter 43: obj=58916.253335, nrmB=301.271388, nrmW=3.024841
NMF: iter 44: obj=58180.954292, nrmB=301.293999, nrmW=3.024845
NMF: iter 45: obj=57475.069328, nrmB=301.315429, nrmW=3.024846
NMF: iter 46: obj=56795.363684, nrmB=301.335740, nrmW=3.024845
NMF: iter 47: obj=56139.248053, nrmB=301.355004, nrmW=3.024841
NMF: iter 48: obj=55504.694808, nrmB=301.373291, nrmW=3.024836
NMF: iter 49: obj=54890.142581, nrmB=301.390670, nrmW=3.024828
NMF: iter 50: obj=54294.400114, nrmB=301.407202, nrmW=3.024819
NMF: iter 51: obj=53716.556507, nrmB=301.422941, nrmW=3.024808
NMF: iter 52: obj=53155.902279, nrmB=301.437934, nrmW=3.024795
NMF: iter 53: obj=52611.863516, nrmB=301.452219, nrmW=3.024781
NMF: iter 54: obj=52083.949714, nrmB=301.465830, nrmW=3.024766
NMF: iter 55: obj=51571.714753, nrmB=301.478794, nrmW=3.024750
NMF: iter 56: obj=51074.729643, nrmB=301.491134, nrmW=3.024733
NMF: iter 57: obj=50592.565190, nrmB=301.502871, nrmW=3.024715
NMF: iter 58: obj=50124.782623, nrmB=301.514025, nrmW=3.024697
NMF: iter 59: obj=49670.930218, nrmB=301.524614, nrmW=3.024678
NMF: iter 60: obj=49230.544162, nrmB=301.534654, nrmW=3.024659
NMF: iter 61: obj=48803.152152, nrmB=301.544164, nrmW=3.024640
NMF: iter 62: obj=48388.278480, nrmB=301.553162, nrmW=3.024621
NMF: iter 63: obj=47985.449644, nrmB=301.561665, nrmW=3.024602
NMF: iter 64: obj=47594.199756, nrmB=301.569693, nrmW=3.024583
NMF: iter 65: obj=47214.075253, nrmB=301.577266, nrmW=3.024564
NMF: iter 66: obj=46844.638608, nrmB=301.584403, nrmW=3.024546
NMF: iter 67: obj=46485.470958, nrmB=301.591124, nrmW=3.024528
NMF: iter 68: obj=46136.173680, nrmB=301.597451, nrmW=3.024511
NMF: iter 69: obj=45796.369099, nrmB=301.603403, nrmW=3.024493
NMF: iter 70: obj=45465.700518, nrmB=301.609001, nrmW=3.024477
NMF: iter 71: obj=45143.831785, nrmB=301.614265, nrmW=3.024461
NMF: iter 72: obj=44830.446545, nrmB=301.619214, nrmW=3.024445
NMF: iter 73: obj=44525.247289, nrmB=301.623868, nrmW=3.024430
NMF: iter 74: obj=44227.954258, nrmB=301.628245, nrmW=3.024415
NMF: iter 75: obj=43938.304242, nrmB=301.632363, nrmW=3.024401
NMF: iter 76: obj=43656.049328, nrmB=301.636238, nrmW=3.024387
NMF: iter 77: obj=43380.955603, nrmB=301.639887, nrmW=3.024374
NMF: iter 78: obj=43112.801875, nrmB=301.643325, nrmW=3.024361
NMF: iter 79: obj=42851.378437, nrmB=301.646565, nrmW=3.024349
NMF: iter 80: obj=42596.485897, nrmB=301.649622, nrmW=3.024337
NMF: iter 81: obj=42347.934117, nrmB=301.652508, nrmW=3.024326
NMF: iter 82: obj=42105.541254, nrmB=301.655234, nrmW=3.024314
NMF: iter 83: obj=41869.132926, nrmB=301.657811, nrmW=3.024304
NMF: iter 84: obj=41638.541500, nrmB=301.660249, nrmW=3.024293
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NMF: iter 85: obj=41413.605490, nrmB=301.662556, nrmW=3.024283
NMF: iter 86: obj=41194.169060, nrmB=301.664741, nrmW=3.024274
NMF: iter 87: obj=40980.081613, nrmB=301.666812, nrmW=3.024264
NMF: iter 88: obj=40771.197454, nrmB=301.668776, nrmW=3.024255
NMF: iter 89: obj=40567.375502, nrmB=301.670639, nrmW=3.024247
NMF: iter 90: obj=40368.479039, nrmB=301.672407, nrmW=3.024238
NMF: iter 91: obj=40174.375475, nrmB=301.674085, nrmW=3.024230
NMF: iter 92: obj=39984.936130, nrmB=301.675680, nrmW=3.024223
NMF: iter 93: obj=39800.036005, nrmB=301.677194, nrmW=3.024215
NMF: iter 94: obj=39619.553562, nrmB=301.678634, nrmW=3.024208
NMF: iter 95: obj=39443.370497, nrmB=301.680001, nrmW=3.024201
NMF: iter 96: obj=39271.371518, nrmB=301.681302, nrmW=3.024194
NMF: iter 97: obj=39103.444142, nrmB=301.682538, nrmW=3.024188
NMF: iter 98: obj=38939.478506, nrmB=301.683713, nrmW=3.024182
NMF: iter 99: obj=38779.367211, nrmB=301.684830, nrmW=3.024176
NMF: iter 100: obj=38623.005200, nrmB=301.685892, nrmW=3.024170
NMF: iter 101: obj=38470.289672, nrmB=301.686902, nrmW=3.024165
NMF: iter 102: obj=38321.120031, nrmB=301.687862, nrmW=3.024159
NMF: iter 103: obj=38175.397875, nrmB=301.688774, nrmW=3.024154
NMF: iter 104: obj=38033.026997, nrmB=301.689641, nrmW=3.024150
NMF: iter 105: obj=37893.913419, nrmB=301.690465, nrmW=3.024145
NMF: iter 106: obj=37757.965419, nrmB=301.691249, nrmW=3.024141
NMF: iter 107: obj=37625.093576, nrmB=301.691993, nrmW=3.024137
NMF: iter 108: obj=37495.210795, nrmB=301.692700, nrmW=3.024133
NMF: iter 109: obj=37368.232330, nrmB=301.693371, nrmW=3.024129
NMF: iter 110: obj=37244.075790, nrmB=301.694009, nrmW=3.024125
NMF: iter 111: obj=37122.661133, nrmB=301.694615, nrmW=3.024122
NMF: iter 112: obj=37003.910641, nrmB=301.695190, nrmW=3.024118
NMF: iter 113: obj=36887.748885, nrmB=301.695736, nrmW=3.024115
NMF: iter 114: obj=36774.102679, nrmB=301.696254, nrmW=3.024112
NMF: iter 115: obj=36662.901024, nrmB=301.696747, nrmW=3.024109
NMF: iter 116: obj=36554.075053, nrmB=301.697214, nrmW=3.024106
NMF: iter 117: obj=36447.557972, nrmB=301.697657, nrmW=3.024104
NMF: iter 118: obj=36343.285001, nrmB=301.698078, nrmW=3.024101
NMF: iter 119: obj=36241.193324, nrmB=301.698477, nrmW=3.024099
NMF: iter 120: obj=36141.222039, nrmB=301.698856, nrmW=3.024097
NMF: iter 121: obj=36043.312116, nrmB=301.699215, nrmW=3.024095
NMF: iter 122: obj=35947.406357, nrmB=301.699556, nrmW=3.024093
NMF: iter 123: obj=35853.449368, nrmB=301.699879, nrmW=3.024091
NMF: iter 124: obj=35761.387519, nrmB=301.700186, nrmW=3.024089
NMF: iter 125: obj=35671.168918, nrmB=301.700477, nrmW=3.024087
NMF: iter 126: obj=35582.743383, nrmB=301.700753, nrmW=3.024086
NMF: iter 127: obj=35496.062405, nrmB=301.701014, nrmW=3.024084
NMF: iter 128: obj=35411.079114, nrmB=301.701263, nrmW=3.024083
NMF: iter 129: obj=35327.748246, nrmB=301.701498, nrmW=3.024081
NMF: iter 130: obj=35246.026100, nrmB=301.701721, nrmW=3.024080
NMF: iter 131: obj=35165.870495, nrmB=301.701933, nrmW=3.024079
NMF: iter 132: obj=35087.240723, nrmB=301.702135, nrmW=3.024078
NMF: iter 133: obj=35010.097501, nrmB=301.702326, nrmW=3.024077
NMF: iter 134: obj=34934.402919, nrmB=301.702507, nrmW=3.024076
NMF: iter 135: obj=34860.120390, nrmB=301.702680, nrmW=3.024075
NMF: iter 136: obj=34787.214589, nrmB=301.702844, nrmW=3.024074
NMF: iter 137: obj=34715.651406, nrmB=301.703000, nrmW=3.024074
NMF: iter 138: obj=34645.397885, nrmB=301.703148, nrmW=3.024073
NMF: iter 139: obj=34576.422172, nrmB=301.703290, nrmW=3.024072
NMF: iter 140: obj=34508.693463, nrmB=301.703425, nrmW=3.024072
NMF: iter 141: obj=34442.181946, nrmB=301.703555, nrmW=3.024071
NMF: iter 142: obj=34376.858755, nrmB=301.703679, nrmW=3.024071
NMF: iter 143: obj=34312.695921, nrmB=301.703797, nrmW=3.024070
NMF: iter 144: obj=34249.666323, nrmB=301.703912, nrmW=3.024070
NMF: iter 145: obj=34187.743648, nrmB=301.704022, nrmW=3.024070
NMF: iter 146: obj=34126.902347, nrmB=301.704128, nrmW=3.024070
NMF: iter 147: obj=34067.117600, nrmB=301.704231, nrmW=3.024069
NMF: iter 148: obj=34008.365279, nrmB=301.704330, nrmW=3.024069
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NMF: iter 149: obj=33950.621921, nrmB=301.704427, nrmW=3.024069
NMF: iter 150: obj=33893.864695, nrmB=301.704522, nrmW=3.024069
NMF: iter 151: obj=33838.071378, nrmB=301.704615, nrmW=3.024069
NMF: iter 152: obj=33783.220336, nrmB=301.704706, nrmW=3.024069
NMF: iter 153: obj=33729.290499, nrmB=301.704797, nrmW=3.024069
NMF: iter 154: obj=33676.261344, nrmB=301.704886, nrmW=3.024069
NMF: iter 155: obj=33624.112882, nrmB=301.704974, nrmW=3.024069
NMF: iter 156: obj=33572.825643, nrmB=301.705062, nrmW=3.024069
NMF: iter 157: obj=33522.380658, nrmB=301.705150, nrmW=3.024070
{\tt NMF: iter\ 158:\ obj=33472.759453,\ nrmB=301.705238,\ nrmW=3.024070}
NMF: iter 159: obj=33423.944034, nrmB=301.705327, nrmW=3.024070
NMF: iter 160: obj=33375.916879, nrmB=301.705417, nrmW=3.024070
NMF: iter 161: obj=33328.660924, nrmB=301.705507, nrmW=3.024070
NMF: iter 162: obj=33282.159554, nrmB=301.705598, nrmW=3.024071
NMF: iter 163: obj=33236.396592, nrmB=301.705691, nrmW=3.024071
NMF: iter 164: obj=33191.356292, nrmB=301.705786, nrmW=3.024071
NMF: iter 165: obj=33147.023323, nrmB=301.705883, nrmW=3.024071
NMF: iter 166: obj=33103.382764, nrmB=301.705981, nrmW=3.024072
NMF: iter 167: obj=33060.420090, nrmB=301.706082, nrmW=3.024072
NMF: iter 168: obj=33018.121161, nrmB=301.706185, nrmW=3.024072
NMF: iter 169: obj=32976.472215, nrmB=301.706291, nrmW=3.024072
NMF: iter 170: obj=32935.459857, nrmB=301.706400, nrmW=3.024073
NMF: iter 171: obj=32895.071045, nrmB=301.706511, nrmW=3.024073
NMF: iter 172: obj=32855.293085, nrmB=301.706626, nrmW=3.024073
NMF: iter 173: obj=32816.113617, nrmB=301.706743, nrmW=3.024074
NMF: iter 174: obj=32777.520608, nrmB=301.706865, nrmW=3.024074
NMF: iter 175: obj=32739.502342, nrmB=301.706989, nrmW=3.024074
NMF: iter 176: obj=32702.047410, nrmB=301.707117, nrmW=3.024075
NMF: iter 177: obj=32665.144700, nrmB=301.707249, nrmW=3.024075
NMF: iter 178: obj=32628.783392, nrmB=301.707385, nrmW=3.024075
NMF: iter 179: obj=32592.952943, nrmB=301.707525, nrmW=3.024075
NMF: iter 180: obj=32557.643084, nrmB=301.707669, nrmW=3.024076
NMF: iter 181: obj=32522.843810, nrmB=301.707817, nrmW=3.024076
NMF: iter 182: obj=32488.545370, nrmB=301.707969, nrmW=3.024076
NMF: iter 183: obj=32454.738261, nrmB=301.708125, nrmW=3.024076
NMF: iter 184: obj=32421.413217, nrmB=301.708286, nrmW=3.024077
NMF: iter 185: obj=32388.561204, nrmB=301.708451, nrmW=3.024077
NMF: iter 186: obj=32356.173414, nrmB=301.708620, nrmW=3.024077
NMF: iter 187: obj=32324.241250, nrmB=301.708794, nrmW=3.024077
NMF: iter 188: obj=32292.756325, nrmB=301.708973, nrmW=3.024077
NMF: iter 189: obj=32261.710453, nrmB=301.709156, nrmW=3.024078
NMF: iter 190: obj=32231.095641, nrmB=301.709343, nrmW=3.024078
NMF: iter 191: obj=32200.904081, nrmB=301.709536, nrmW=3.024078
NMF: iter 192: obj=32171.128145, nrmB=301.709733, nrmW=3.024078
NMF: iter 193: obj=32141.760379, nrmB=301.709934, nrmW=3.024078
NMF: iter 194: obj=32112.793493, nrmB=301.710141, nrmW=3.024078
NMF: iter 195: obj=32084.220359, nrmB=301.710352, nrmW=3.024078
NMF: iter 196: obj=32056.034001, nrmB=301.710568, nrmW=3.024079
NMF: iter 197: obj=32028.227593, nrmB=301.710788, nrmW=3.024079
NMF: iter 198: obj=32000.794452, nrmB=301.711013, nrmW=3.024079
NMF: iter 199: obj=31973.728033, nrmB=301.711243, nrmW=3.024079
NMF: iter 200: obj=31947.021925, nrmB=301.711478, nrmW=3.024079
NMF: iter 201: obj=31920.669844, nrmB=301.711717, nrmW=3.024079
NMF: iter 202: obj=31894.665632, nrmB=301.711961, nrmW=3.024079
NMF: iter 203: obj=31869.003252, nrmB=301.712210, nrmW=3.024079
NMF: iter 204: obj=31843.676783, nrmB=301.712464, nrmW=3.024079
NMF: iter 205: obj=31818.680419, nrmB=301.712722, nrmW=3.024078
NMF: iter 206: obj=31794.008463, nrmB=301.712985, nrmW=3.024078
NMF: iter 207: obj=31769.655327, nrmB=301.713252, nrmW=3.024078
NMF: iter 208: obj=31745.615527, nrmB=301.713524, nrmW=3.024078
NMF: iter 209: obj=31721.883682, nrmB=301.713800, nrmW=3.024078
NMF: iter 210: obj=31698.454511, nrmB=301.714081, nrmW=3.024078
NMF: iter 211: obj=31675.322832, nrmB=301.714367, nrmW=3.024078
NMF: iter 212: obj=31652.483557, nrmB=301.714657, nrmW=3.024077
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NMF: iter 214: obj=31607.662348, nrmB=301.715251, nrmW=3.024077
NMF: iter 215: obj=31585.670707, nrmB=301.715554, nrmW=3.024077
NMF: iter 216: obj=31563.952055, nrmB=301.715862, nrmW=3.024076
NMF: iter 217: obj=31542.501764, nrmB=301.716175, nrmW=3.024076
NMF: iter 218: obj=31521.315291, nrmB=301.716491, nrmW=3.024076
NMF: iter 219: obj=31500.388183, nrmB=301.716812, nrmW=3.024075
NMF: iter 220: obj=31479.716069, nrmB=301.717137, nrmW=3.024075
NMF: iter 221: obj=31459.294664, nrmB=301.717467, nrmW=3.024074
NMF: iter 222: obj=31439.119765, nrmB=301.717800, nrmW=3.024074
NMF: iter 223: obj=31419.187252, nrmB=301.718138, nrmW=3.024073
NMF: iter 224: obj=31399.493083, nrmB=301.718480, nrmW=3.024073
NMF: iter 225: obj=31380.033297, nrmB=301.718826, nrmW=3.024072
NMF: iter 226: obj=31360.804013, nrmB=301.719177, nrmW=3.024072
NMF: iter 227: obj=31341.801424, nrmB=301.719531, nrmW=3.024071
NMF: iter 228: obj=31323.021800, nrmB=301.719889, nrmW=3.024071
NMF: iter 229: obj=31304.461485, nrmB=301.720251, nrmW=3.024070
NMF: iter 230: obj=31286.116898, nrmB=301.720617, nrmW=3.024070
NMF: iter 231: obj=31267.984526, nrmB=301.720988, nrmW=3.024069
NMF: iter 232: obj=31250.060931, nrmB=301.721361, nrmW=3.024068
NMF: iter 233: obj=31232.342741, nrmB=301.721739, nrmW=3.024068
NMF: iter 234: obj=31214.826653, nrmB=301.722121, nrmW=3.024067
NMF: iter 235: obj=31197.509429, nrmB=301.722506, nrmW=3.024066
NMF: iter 236: obj=31180.387898, nrmB=301.722895, nrmW=3.024065
NMF: iter 237: obj=31163.458952, nrmB=301.723288, nrmW=3.024064
NMF: iter 238: obj=31146.719543, nrmB=301.723684, nrmW=3.024064
NMF: iter 239: obj=31130.166686, nrmB=301.724084, nrmW=3.024063
NMF: iter 240: obj=31113.797456, nrmB=301.724488, nrmW=3.024062
NMF: iter 241: obj=31097.608984, nrmB=301.724895, nrmW=3.024061
NMF: iter 242: obj=31081.598458, nrmB=301.725305, nrmW=3.024060
NMF: iter 243: obj=31065.763122, nrmB=301.725719, nrmW=3.024059
NMF: iter 244: obj=31050.100274, nrmB=301.726136, nrmW=3.024058
NMF: iter 245: obj=31034.607264, nrmB=301.726557, nrmW=3.024057
NMF: iter 246: obj=31019.281493, nrmB=301.726981, nrmW=3.024056
NMF: iter 247: obj=31004.120414, nrmB=301.727408, nrmW=3.024055
NMF: iter 248: obj=30989.121527, nrmB=301.727838, nrmW=3.024054
NMF: iter 249: obj=30974.282380, nrmB=301.728271, nrmW=3.024053
NMF: iter 250: obj=30959.600568, nrmB=301.728708, nrmW=3.024052
NMF: iter 251: obj=30945.073731, nrmB=301.729148, nrmW=3.024051
NMF: iter 252: obj=30930.699554, nrmB=301.729590, nrmW=3.024049
NMF: iter 253: obj=30916.475764, nrmB=301.730036, nrmW=3.024048
NMF: iter 254: obj=30902.400131, nrmB=301.730484, nrmW=3.024047
NMF: iter 255: obj=30888.470466, nrmB=301.730936, nrmW=3.024046
NMF: iter 256: obj=30874.684623, nrmB=301.731390, nrmW=3.024044
NMF: iter 257: obj=30861.040491, nrmB=301.731847, nrmW=3.024043
NMF: iter 258: obj=30847.536001, nrmB=301.732306, nrmW=3.024042
NMF: iter 259: obj=30834.169122, nrmB=301.732768, nrmW=3.024041
NMF: iter 260: obj=30820.937860, nrmB=301.733233, nrmW=3.024039
NMF: iter 261: obj=30807.840257, nrmB=301.733701, nrmW=3.024038
NMF: iter 262: obj=30794.874391, nrmB=301.734171, nrmW=3.024036
NMF: iter 263: obj=30782.038377, nrmB=301.734643, nrmW=3.024035
NMF: iter 264: obj=30769.330364, nrmB=301.735118, nrmW=3.024033
NMF: iter 265: obj=30756.748534, nrmB=301.735595, nrmW=3.024032
NMF: iter 266: obj=30744.291105, nrmB=301.736075, nrmW=3.024030
NMF: iter 267: obj=30731.956327, nrmB=301.736557, nrmW=3.024029
NMF: iter 268: obj=30719.742483, nrmB=301.737041, nrmW=3.024027
NMF: iter 269: obj=30707.647887, nrmB=301.737527, nrmW=3.024026
NMF: iter 270: obj=30695.670887, nrmB=301.738015, nrmW=3.024024
NMF: iter 271: obj=30683.809860, nrmB=301.738506, nrmW=3.024023
NMF: iter 272: obj=30672.063214, nrmB=301.738998, nrmW=3.024021
NMF: iter 273: obj=30660.429388, nrmB=301.739493, nrmW=3.024019
NMF: iter 274: obj=30648.906849, nrmB=301.739989, nrmW=3.024018
NMF: iter 275: obj=30637.494093, nrmB=301.740488, nrmW=3.024016
NMF: iter 276: obj=30626.189646, nrmB=301.740988, nrmW=3.024014
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NMF: iter 278: obj=30603.899910, nrmB=301.741994, nrmW=3.024011
NMF: iter 279: obj=30592.911806, nrmB=301.742499, nrmW=3.024009
NMF: iter 280: obj=30582.026377, nrmB=301.743007, nrmW=3.024007
NMF: iter 281: obj=30571.242279, nrmB=301.743515, nrmW=3.024006
NMF: iter 282: obj=30560.558192, nrmB=301.744026, nrmW=3.024004
NMF: iter 283: obj=30549.972819, nrmB=301.744538, nrmW=3.024002
NMF: iter 284: obj=30539.484886, nrmB=301.745051, nrmW=3.024000
NMF: iter 285: obj=30529.093143, nrmB=301.745566, nrmW=3.023998
NMF: iter 286: obj=30518.796358, nrmB=301.746082, nrmW=3.023996
NMF: iter 287: obj=30508.593324, nrmB=301.746600, nrmW=3.023994
NMF: iter 288: obj=30498.482851, nrmB=301.747119, nrmW=3.023992
NMF: iter 289: obj=30488.463770, nrmB=301.747639, nrmW=3.023991
NMF: iter 290: obj=30478.534932, nrmB=301.748161, nrmW=3.023989
NMF: iter 291: obj=30468.695204, nrmB=301.748683, nrmW=3.023987
NMF: iter 292: obj=30458.943474, nrmB=301.749207, nrmW=3.023985
NMF: iter 293: obj=30449.278646, nrmB=301.749732, nrmW=3.023983
NMF: iter 294: obj=30439.699641, nrmB=301.750258, nrmW=3.023981
NMF: iter 295: obj=30430.205396, nrmB=301.750785, nrmW=3.023979
NMF: iter 296: obj=30420.794865, nrmB=301.751313, nrmW=3.023977
NMF: iter 297: obj=30411.467017, nrmB=301.751842, nrmW=3.023975
NMF: iter 298: obj=30402.220838, nrmB=301.752372, nrmW=3.023973
NMF: iter 299: obj=30393.055327, nrmB=301.752903, nrmW=3.023971
NMF: iter 300: obj=30383.969498, nrmB=301.753434, nrmW=3.023968
NMF: iter 301: obj=30374.962379, nrmB=301.753967, nrmW=3.023966
NMF: iter 302: obj=30366.033014, nrmB=301.754500, nrmW=3.023964
NMF: iter 303: obj=30357.180457, nrmB=301.755034, nrmW=3.023962
NMF: iter 304: obj=30348.403779, nrmB=301.755568, nrmW=3.023960
NMF: iter 305: obj=30339.702061, nrmB=301.756103, nrmW=3.023958
NMF: iter 306: obj=30331.074398, nrmB=301.756639, nrmW=3.023956
NMF: iter 307: obj=30322.519898, nrmB=301.757175, nrmW=3.023954
NMF: iter 308: obj=30314.037680, nrmB=301.757712, nrmW=3.023951
NMF: iter 309: obj=30305.626877, nrmB=301.758249, nrmW=3.023949
NMF: iter 310: obj=30297.286630, nrmB=301.758787, nrmW=3.023947
NMF: iter 311: obj=30289.016096, nrmB=301.759325, nrmW=3.023945
NMF: iter 312: obj=30280.814440, nrmB=301.759864, nrmW=3.023943
NMF: iter 313: obj=30272.680840, nrmB=301.760403, nrmW=3.023940
NMF: iter 314: obj=30264.614484, nrmB=301.760942, nrmW=3.023938
NMF: iter 315: obj=30256.614570, nrmB=301.761481, nrmW=3.023936
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NMF: iter 317: obj=30240.810920, nrmB=301.762561, nrmW=3.023932
NMF: iter 318: obj=30233.005633, nrmB=301.763101, nrmW=3.023929
NMF: iter 319: obj=30225.263690, nrmB=301.763641, nrmW=3.023927
NMF: iter 320: obj=30217.584341, nrmB=301.764182, nrmW=3.023925
NMF: iter 321: obj=30209.966845, nrmB=301.764722, nrmW=3.023922
NMF: iter 322: obj=30202.410474, nrmB=301.765263, nrmW=3.023920
NMF: iter 323: obj=30194.914508, nrmB=301.765803, nrmW=3.023918
NMF: iter 324: obj=30187.478236, nrmB=301.766344, nrmW=3.023916
NMF: iter 325: obj=30180.100957, nrmB=301.766884, nrmW=3.023913
NMF: iter 326: obj=30172.781980, nrmB=301.767425, nrmW=3.023911
NMF: iter 327: obj=30165.520623, nrmB=301.767965, nrmW=3.023909
NMF: iter 328: obj=30158.316214, nrmB=301.768505, nrmW=3.023906
NMF: iter 329: obj=30151.168089, nrmB=301.769045, nrmW=3.023904
NMF: iter 330: obj=30144.075593, nrmB=301.769585, nrmW=3.023902
NMF: iter 331: obj=30137.038081, nrmB=301.770125, nrmW=3.023899
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NMF: iter 334: obj=30116.249131, nrmB=301.771742, nrmW=3.023892
NMF: iter 335: obj=30109.425280, nrmB=301.772281, nrmW=3.023890
NMF: iter 336: obj=30102.653319, nrmB=301.772819, nrmW=3.023888
NMF: iter 337: obj=30095.932655, nrmB=301.773357, nrmW=3.023885
NMF: iter 338: obj=30089.262703, nrmB=301.773894, nrmW=3.023883
NMF: iter 339: obj=30082.642886, nrmB=301.774432, nrmW=3.023881
NMF: iter 340: obj=30076.072637, nrmB=301.774968, nrmW=3.023878
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NMF: iter 342: obj=30063.078609, nrmB=301.776040, nrmW=3.023874
NMF: iter 343: obj=30056.653733, nrmB=301.776576, nrmW=3.023871
NMF: iter 344: obj=30050.276231, nrmB=301.777111, nrmW=3.023869
NMF: iter 345: obj=30043.945574, nrmB=301.777645, nrmW=3.023867
NMF: iter 346: obj=30037.661242, nrmB=301.778179, nrmW=3.023864
NMF: iter 347: obj=30031.422719, nrmB=301.778712, nrmW=3.023862
NMF: iter 348: obj=30025.229499, nrmB=301.779245, nrmW=3.023859
NMF: iter 349: obj=30019.081083, nrmB=301.779777, nrmW=3.023857
NMF: iter 350: obj=30012.976978, nrmB=301.780308, nrmW=3.023855
NMF: iter 351: obj=30006.916700, nrmB=301.780839, nrmW=3.023852
NMF: iter 352: obj=30000.899768, nrmB=301.781369, nrmW=3.023850
NMF: iter 353: obj=29994.925713, nrmB=301.781899, nrmW=3.023848
NMF: iter 354: obj=29988.994069, nrmB=301.782428, nrmW=3.023845
NMF: iter 355: obj=29983.104377, nrmB=301.782956, nrmW=3.023843
NMF: iter 356: obj=29977.256186, nrmB=301.783483, nrmW=3.023840
NMF: iter 357: obj=29971.449049, nrmB=301.784010, nrmW=3.023838
NMF: iter 358: obj=29965.682529, nrmB=301.784536, nrmW=3.023836
NMF: iter 359: obj=29959.956191, nrmB=301.785061, nrmW=3.023833
NMF: iter 360: obj=29954.269608, nrmB=301.785586, nrmW=3.023831
NMF: iter 361: obj=29948.622361, nrmB=301.786110, nrmW=3.023828
NMF: iter 362: obj=29943.014032, nrmB=301.786633, nrmW=3.023826
NMF: iter 363: obj=29937.444214, nrmB=301.787155, nrmW=3.023824
NMF: iter 364: obj=29931.912501, nrmB=301.787677, nrmW=3.023821
NMF: iter 365: obj=29926.418497, nrmB=301.788197, nrmW=3.023819
NMF: iter 366: obj=29920.961807, nrmB=301.788717, nrmW=3.023816
NMF: iter 367: obj=29915.542046, nrmB=301.789236, nrmW=3.023814
NMF: iter 368: obj=29910.158830, nrmB=301.789754, nrmW=3.023812
NMF: iter 369: obj=29904.811784, nrmB=301.790272, nrmW=3.023809
NMF: iter 370: obj=29899.500534, nrmB=301.790788, nrmW=3.023807
NMF: iter 371: obj=29894.224715, nrmB=301.791304, nrmW=3.023805
NMF: iter 372: obj=29888.983964, nrmB=301.791819, nrmW=3.023802
NMF: iter 373: obj=29883.777925, nrmB=301.792333, nrmW=3.023800
NMF: iter 374: obj=29878.606245, nrmB=301.792846, nrmW=3.023797
NMF: iter 375: obj=29873.468578, nrmB=301.793358, nrmW=3.023795
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NMF: iter 383: obj=29833.551501, nrmB=301.797423, nrmW=3.023776
NMF: iter 384: obj=29828.705064, nrmB=301.797926, nrmW=3.023774
NMF: iter 385: obj=29823.889415, nrmB=301.798429, nrmW=3.023771
NMF: iter 386: obj=29819.104256, nrmB=301.798931, nrmW=3.023769
NMF: iter 387: obj=29814.349294, nrmB=301.799432, nrmW=3.023767
NMF: iter 388: obj=29809.624237, nrmB=301.799933, nrmW=3.023764
NMF: iter 389: obj=29804.928802, nrmB=301.800432, nrmW=3.023762
NMF: iter 390: obj=29800.262708, nrmB=301.800930, nrmW=3.023759
NMF: iter 391: obj=29795.625677, nrmB=301.801427, nrmW=3.023757
NMF: iter 392: obj=29791.017438, nrmB=301.801923, nrmW=3.023755
NMF: iter 393: obj=29786.437721, nrmB=301.802418, nrmW=3.023752
NMF: iter 394: obj=29781.886263, nrmB=301.802913, nrmW=3.023750
NMF: iter 395: obj=29777.362804, nrmB=301.803406, nrmW=3.023748
NMF: iter 396: obj=29772.867086, nrmB=301.803898, nrmW=3.023745
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NMF: iter 398: obj=29763.957873, nrmB=301.804879, nrmW=3.023741
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NMF: iter 400: obj=29755.156652, nrmB=301.805857, nrmW=3.023736
NMF: iter 401: obj=29750.795939, nrmB=301.806344, nrmW=3.023734
NMF: iter 402: obj=29746.461510, nrmB=301.806830, nrmW=3.023731
NMF: iter 403: obj=29742.153137, nrmB=301.807315, nrmW=3.023729
NMF: iter 404: obj=29737.870591, nrmB=301.807800, nrmW=3.023727
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NMF: iter 406: obj=29729.382093, nrmB=301.808765, nrmW=3.023722
NMF: iter 407: obj=29725.175702, nrmB=301.809246, nrmW=3.023720
NMF: iter 408: obj=29720.994264, nrmB=301.809726, nrmW=3.023717
NMF: iter 409: obj=29716.837565, nrmB=301.810205, nrmW=3.023715
NMF: iter 410: obj=29712.705399, nrmB=301.810683, nrmW=3.023713
NMF: iter 411: obj=29708.597558, nrmB=301.811159, nrmW=3.023710
NMF: iter 412: obj=29704.513839, nrmB=301.811635, nrmW=3.023708
NMF: iter 413: obj=29700.454041, nrmB=301.812110, nrmW=3.023706
NMF: iter 414: obj=29696.417967, nrmB=301.812584, nrmW=3.023704
NMF: iter 415: obj=29692.405420, nrmB=301.813056, nrmW=3.023701
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NMF: iter 417: obj=29684.450135, nrmB=301.813999, nrmW=3.023697
NMF: iter 418: obj=29680.507016, nrmB=301.814468, nrmW=3.023694
NMF: iter 419: obj=29676.586665, nrmB=301.814936, nrmW=3.023692
NMF: iter 420: obj=29672.688896, nrmB=301.815404, nrmW=3.023690
NMF: iter 421: obj=29668.813526, nrmB=301.815870, nrmW=3.023687
NMF: iter 422: obj=29664.960375, nrmB=301.816335, nrmW=3.023685
NMF: iter 423: obj=29661.129266, nrmB=301.816800, nrmW=3.023683
NMF: iter 424: obj=29657.320021, nrmB=301.817263, nrmW=3.023681
NMF: iter 425: obj=29653.532466, nrmB=301.817725, nrmW=3.023678
NMF: iter 426: obj=29649.766429, nrmB=301.818186, nrmW=3.023676
NMF: iter 427: obj=29646.021739, nrmB=301.818645, nrmW=3.023674
NMF: iter 428: obj=29642.298228, nrmB=301.819104, nrmW=3.023672
NMF: iter 429: obj=29638.595728, nrmB=301.819562, nrmW=3.023669
NMF: iter 430: obj=29634.914076, nrmB=301.820019, nrmW=3.023667
NMF: iter 431: obj=29631.253108, nrmB=301.820474, nrmW=3.023665
NMF: iter 432: obj=29627.612661, nrmB=301.820929, nrmW=3.023663
NMF: iter 433: obj=29623.992577, nrmB=301.821382, nrmW=3.023660
NMF: iter 434: obj=29620.392697, nrmB=301.821834, nrmW=3.023658
NMF: iter 435: obj=29616.812865, nrmB=301.822285, nrmW=3.023656
NMF: iter 436: obj=29613.252925, nrmB=301.822736, nrmW=3.023654
NMF: iter 437: obj=29609.712724, nrmB=301.823185, nrmW=3.023651
NMF: iter 438: obj=29606.192111, nrmB=301.823633, nrmW=3.023649
NMF: iter 439: obj=29602.690934, nrmB=301.824080, nrmW=3.023647
NMF: iter 440: obj=29599.209046, nrmB=301.824525, nrmW=3.023645
NMF: iter 441: obj=29595.746297, nrmB=301.824970, nrmW=3.023643
NMF: iter 442: obj=29592.302542, nrmB=301.825414, nrmW=3.023640
NMF: iter 443: obj=29588.877637, nrmB=301.825856, nrmW=3.023638
NMF: iter 444: obj=29585.471436, nrmB=301.826298, nrmW=3.023636
NMF: iter 445: obj=29582.083799, nrmB=301.826738, nrmW=3.023634
NMF: iter 446: obj=29578.714583, nrmB=301.827178, nrmW=3.023631
NMF: iter 447: obj=29575.363649, nrmB=301.827616, nrmW=3.023629
NMF: iter 448: obj=29572.030858, nrmB=301.828053, nrmW=3.023627
NMF: iter 449: obj=29568.716073, nrmB=301.828489, nrmW=3.023625
NMF: iter 450: obj=29565.419156, nrmB=301.828924, nrmW=3.023623
NMF: iter 451: obj=29562.139973, nrmB=301.829358, nrmW=3.023621
NMF: iter 452: obj=29558.878389, nrmB=301.829791, nrmW=3.023618
NMF: iter 453: obj=29555.634270, nrmB=301.830223, nrmW=3.023616
NMF: iter 454: obj=29552.407486, nrmB=301.830654, nrmW=3.023614
NMF: iter 455: obj=29549.197904, nrmB=301.831084, nrmW=3.023612
NMF: iter 456: obj=29546.005394, nrmB=301.831512, nrmW=3.023610
NMF: iter 457: obj=29542.829828, nrmB=301.831940, nrmW=3.023608
NMF: iter 458: obj=29539.671077, nrmB=301.832366, nrmW=3.023605
NMF: iter 459: obj=29536.529014, nrmB=301.832792, nrmW=3.023603
NMF: iter 460: obj=29533.403512, nrmB=301.833216, nrmW=3.023601
NMF: iter 461: obj=29530.294448, nrmB=301.833639, nrmW=3.023599
NMF: iter 462: obj=29527.201696, nrmB=301.834062, nrmW=3.023597
NMF: iter 463: obj=29524.125133, nrmB=301.834483, nrmW=3.023595
NMF: iter 464: obj=29521.064637, nrmB=301.834903, nrmW=3.023592
NMF: iter 465: obj=29518.020087, nrmB=301.835322, nrmW=3.023590
NMF: iter 466: obj=29514.991363, nrmB=301.835740, nrmW=3.023588
NMF: iter 467: obj=29511.978344, nrmB=301.836158, nrmW=3.023586
NMF: iter 468: obj=29508.980914, nrmB=301.836574, nrmW=3.023584
```

```
NMF: iter 469: obj=29505.998954, nrmB=301.836989, nrmW=3.023582
NMF: iter 470: obj=29503.032348, nrmB=301.837403, nrmW=3.023580
NMF: iter 471: obj=29500.080980, nrmB=301.837816, nrmW=3.023578
NMF: iter 472: obj=29497.144736, nrmB=301.838227, nrmW=3.023575
NMF: iter 473: obj=29494.223503, nrmB=301.838638, nrmW=3.023573
NMF: iter 474: obj=29491.317167, nrmB=301.839048, nrmW=3.023571
NMF: iter 475: obj=29488.425617, nrmB=301.839457, nrmW=3.023569
NMF: iter 476: obj=29485.548742, nrmB=301.839865, nrmW=3.023567
NMF: iter 477: obj=29482.686432, nrmB=301.840272, nrmW=3.023565
NMF: iter 478: obj=29479.838578, nrmB=301.840678, nrmW=3.023563
NMF: iter 479: obj=29477.005072, nrmB=301.841083, nrmW=3.023561
NMF: iter 480: obj=29474.185806, nrmB=301.841487, nrmW=3.023559
NMF: iter 481: obj=29471.380675, nrmB=301.841889, nrmW=3.023557
NMF: iter 482: obj=29468.589573, nrmB=301.842291, nrmW=3.023555
NMF: iter 483: obj=29465.812394, nrmB=301.842692, nrmW=3.023552
NMF: iter 484: obj=29463.049035, nrmB=301.843092, nrmW=3.023550
NMF: iter 485: obj=29460.299393, nrmB=301.843491, nrmW=3.023548
NMF: iter 486: obj=29457.563366, nrmB=301.843889, nrmW=3.023546
NMF: iter 487: obj=29454.840852, nrmB=301.844286, nrmW=3.023544
NMF: iter 488: obj=29452.131749, nrmB=301.844682, nrmW=3.023542
NMF: iter 489: obj=29449.435959, nrmB=301.845077, nrmW=3.023540
NMF: iter 490: obj=29446.753382, nrmB=301.845472, nrmW=3.023538
NMF: iter 491: obj=29444.083918, nrmB=301.845865, nrmW=3.023536
NMF: iter 492: obj=29441.427471, nrmB=301.846257, nrmW=3.023534
NMF: iter 493: obj=29438.783942, nrmB=301.846648, nrmW=3.023532
NMF: iter 494: obj=29436.153235, nrmB=301.847039, nrmW=3.023530
NMF: iter 495: obj=29433.535255, nrmB=301.847428, nrmW=3.023528
NMF: iter 496: obj=29430.929905, nrmB=301.847817, nrmW=3.023526
NMF: iter 497: obj=29428.337092, nrmB=301.848205, nrmW=3.023524
NMF: iter 498: obj=29425.756720, nrmB=301.848591, nrmW=3.023522
NMF: iter 499: obj=29423.188697, nrmB=301.848977, nrmW=3.023520
NMF: iter 500: obj=29420.632930, nrmB=301.849362, nrmW=3.023518
figure;
% suptitle('Basis functions obtained by NMF');
sgtitle('Basis functions obtained by NMF'); % mine is matlab 2021a please
for k = 1:40
  subplot(5, 8, k);
  imagesc(reshape(B(:,k), d1, d2));
```

colormap gray; axis image off;

end

Basis functions obtained by NMF



Step 2: Compare your results with MATLAB's predefined NMF function

Basis functions obtained by MATLAB NMF Function



Performing NMF with added sparsity constraints

The process for performing sparse NMF is the same as above, with a few changes to Step 4 (the update rules).

- 1. Create a sparse NMF function ssnmf(V, rank, max_iter, lambda, alpha, beta)
- 2. Initialize *B* and *W*randomly, and make sure *W*has unit-sum columns (each column should sum to 1).
- 3. Calculate the initial objective. It will be helpful to define a new function for sparse NMF, compute_objective_ss(V, W, B, alpha, beta) that returns the objective value.
- 4. Perform the iterations $B=B\otimes \frac{\left(\frac{V}{BW}\right)W^T}{1W^T+\beta}$ and $W=W\otimes \frac{B^T\left(\frac{V}{BW}\right)}{B^T1+\alpha}$, where \otimes specifies element-wise multiplication and all divisions are element-wise division.
- 5. Calculate the new objective function value.
- 6. Stopping Criteria: Stop when the absolute difference of objective values is smaller than or equal to λ (or) the max number of iterations has been reached.

Notes -

- Boilerplate for ssnmf(V, rank, max_iter, lambda) has been provided in ssnmf.m
- Boilerplate for compute_objective_ss(V, W, B, alpha, beta) has been provided in compute_objective_ss.m

• Here, the notation $1W^T + \beta$ and $B^T 1 + \alpha$ are normalizing the columns of W and B by the sum of column elements plus β , and the sum of column elements plus α respectively. You should ensure you perform this normalization.

Validation on the ORL Faces Dataset

Step 1: As before, output the new bases and weights and plot them by calling your SSNMF function on the data matrix V with parameters rank = 40, max_iter=500, lambda=0.001, alpha=100, and beta=1 will look something like

```
[B, W, obj, k] = ssnmf(V, 40, 500, 0.001, 100, 1);
```

```
ssNMF: init: obj=-157142.965401
ssNMF: iter 1: obj=244654.196798, nrmB=150.404983, nrmW=5.798916
ssNMF: iter 2: obj=585641.885822, nrmB=78.429911, nrmW=9.981026
ssNMF: iter 3: obj=1296266.665919, nrmB=45.484001, nrmW=13.523735
ssNMF: iter 4: obj=1938154.213233, nrmB=33.514666, nrmW=14.450768
ssNMF: iter 5: obj=2101350.161247, nrmB=31.345429, nrmW=14.491294
ssNMF: iter 6: obj=2107847.712766, nrmB=31.260458, nrmW=14.492311
ssNMF: iter 7: obj=2107521.889443, nrmB=31.258844, nrmW=14.494817
ssNMF: iter 8: obj=2107261.480010, nrmB=31.256477, nrmW=14.497360
ssNMF: iter 9: obj=2106906.211183, nrmB=31.254153, nrmW=14.500916
ssNMF: iter 10: obj=2106536.882619, nrmB=31.251175, nrmW=14.505213
ssNMF: iter 11: obj=2106097.163396, nrmB=31.247750, nrmW=14.510652
ssNMF: iter 12: obj=2105594.980049, nrmB=31.243611, nrmW=14.517369
ssNMF: iter 13: obj=2105002.008219, nrmB=31.238690, nrmW=14.525723
ssNMF: iter 14: obj=2104302.948466, nrmB=31.232772, nrmW=14.536086
ssNMF: iter 15: obj=2103470.488378, nrmB=31.225646, nrmW=14.549001
ssNMF: iter 16: obj=2102477.911414, nrmB=31.217015, nrmW=14.565163
ssNMF: iter 17: obj=2101294.023457, nrmB=31.206519, nrmW=14.585529
ssNMF: iter 18: obj=2099888.976097, nrmB=31.193705, nrmW=14.611385
ssNMF: iter 19: obj=2098237.678598, nrmB=31.178053, nrmW=14.644456
ssNMF: iter 20: obj=2096328.174273, nrmB=31.159008, nrmW=14.687003
ssNMF: iter 21: obj=2094170.504690, nrmB=31.136094, nrmW=14.741868
ssNMF: iter 22: obj=2091804.845577, nrmB=31.109074, nrmW=14.812405
ssNMF: iter 23: obj=2089303.221822, nrmB=31.078158, nrmW=14.902225
ssNMF: iter 24: obj=2086761.269987, nrmB=31.044154, nrmW=15.014724
ssNMF: iter 25: obj=2084280.337863, nrmB=31.008471, nrmW=15.152434
ssNMF: iter 26: obj=2081946.356092, nrmB=30.972912, nrmW=15.316354
ssNMF: iter 27: obj=2079814.078187, nrmB=30.939328, nrmW=15.505458
ssNMF: iter 28: obj=2077902.196138, nrmB=30.909255, nrmW=15.716566
\mathtt{ssNMF: iter 29: obj=2076198.911222, nrmB=30.883678, nrmW=15.944647}
{\tt ssNMF: iter 30: obj=2074673.357461, nrmB=30.862961, nrmW=16.183477}
ssNMF: iter 31: obj=2073287.595915, nrmB=30.846926, nrmW=16.426463
ssNMF: iter 32: obj=2072005.708100, nrmB=30.835003, nrmW=16.667421
ssNMF: iter 33: obj=2070798.854303, nrmB=30.826398, nrmW=16.901153
ssNMF: iter 34: obj=2069646.828603, nrmB=30.820239, nrmW=17.123771
ssNMF: iter 35: obj=2068537.354922, nrmB=30.815678, nrmW=17.332775
ssNMF: iter 36: obj=2067464.364792, nrmB=30.811960, nrmW=17.526955
ssNMF: iter 37: obj=2066426.130818, nrmB=30.808451, nrmW=17.706190
ssNMF: iter 38: obj=2065423.692829, nrmB=30.804657, nrmW=17.871196
ssNMF: iter 39: obj=2064459.679241, nrmB=30.800225, nrmW=18.023282
ssNMF: iter 40: obj=2063537.451660, nrmB=30.794943, nrmW=18.164124
ssNMF: iter 41: obj=2062660.461467, nrmB=30.788729, nrmW=18.295580
ssNMF: iter 42: obj=2061831.742132, nrmB=30.781629, nrmW=18.419538
ssNMF: iter 43: obj=2061053.512755, nrmB=30.773793, nrmW=18.537811
ssNMF: iter 44: obj=2060326.902241, nrmB=30.765463, nrmW=18.652056
ssNMF: iter 45: obj=2059651.810495, nrmB=30.756946, nrmW=18.763726
```

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ssNMF: iter 46: obj=2059026.910099, nrmB=30.748588, nrmW=18.874042
ssNMF: iter 47: obj=2058449.771911, nrmB=30.740757, nrmW=18.983986
ssNMF: iter 48: obj=2057917.081556, nrmB=30.733817, nrmW=19.094303
ssNMF: iter 49: obj=2057424.906065, nrmB=30.728123, nrmW=19.205523
ssNMF: iter 50: obj=2056968.971292, nrmB=30.724004, nrmW=19.317980
ssNMF: iter 51: obj=2056544.918435, nrmB=30.721765, nrmW=19.431840
ssNMF: iter 52: obj=2056148.518542, nrmB=30.721685, nrmW=19.547132
ssNMF: iter 53: obj=2055775.834431, nrmB=30.724021, nrmW=19.663773
ssNMF: iter 54: obj=2055423.328197, nrmB=30.729008, nrmW=19.781598
ssNMF: iter 55: obj=2055087.918842, nrmB=30.736866, nrmW=19.900384
ssNMF: iter 56: obj=2054766.998357, nrmB=30.747801, nrmW=20.019867
ssNMF: iter 57: obj=2054458.416242, nrmB=30.762004, nrmW=20.139763
ssNMF: iter 58: obj=2054160.442360, nrmB=30.779652, nrmW=20.259776
ssNMF: iter 59: obj=2053871.716647, nrmB=30.800909, nrmW=20.379615
ssNMF: iter 60: obj=2053591.192243, nrmB=30.825915, nrmW=20.498999
ssNMF: iter 61: obj=2053318.076458, nrmB=30.854792, nrmW=20.617660
ssNMF: iter 62: obj=2053051.772339, nrmB=30.887640, nrmW=20.735350
ssNMF: iter 63: obj=2052791.822641, nrmB=30.924543, nrmW=20.851840
ssNMF: iter 64: obj=2052537.857814, nrmB=30.965568, nrmW=20.966925
ssNMF: iter 65: obj=2052289.549827, nrmB=31.010785, nrmW=21.080420
ssNMF: iter 66: obj=2052046.573879, nrmB=31.060270, nrmW=21.192160
ssNMF: iter 67: obj=2051808.579845, nrmB=31.114118, nrmW=21.302004
ssNMF: iter 68: obj=2051575.174606, nrmB=31.172455, nrmW=21.409828
ssNMF: iter 69: obj=2051345.915346, nrmB=31.235447, nrmW=21.515527
ssNMF: iter 70: obj=2051120.312826, nrmB=31.303301, nrmW=21.619016
ssNMF: iter 71: obj=2050897.842966, nrmB=31.376269, nrmW=21.720224
ssNMF: iter 72: obj=2050677.964824, nrmB=31.454654, nrmW=21.819100
ssNMF: iter 73: obj=2050460.143309, nrmB=31.538798, nrmW=21.915606
ssNMF: iter 74: obj=2050243.875389, nrmB=31.629077, nrmW=22.009720
ssNMF: iter 75: obj=2050028.718781, nrmB=31.725863, nrmW=22.101435
ssNMF: iter 76: obj=2049814.322018, nrmB=31.829453, nrmW=22.190758
ssNMF: iter 77: obj=2049600.454119, nrmB=31.939920, nrmW=22.277708
ssNMF: iter 78: obj=2049387.030911, nrmB=32.056858, nrmW=22.362316
ssNMF: iter 79: obj=2049174.133436, nrmB=32.179030, nrmW=22.444624
ssNMF: iter 80: obj=2048962.012426, nrmB=32.303961, nrmW=22.524686
ssNMF: iter 81: obj=2048751.072963, nrmB=32.427728, nrmW=22.602562
ssNMF: iter 82: obj=2048541.836576, nrmB=32.545211, nrmW=22.678324
ssNMF: iter 83: obj=2048334.884236, nrmB=32.651003, nrmW=22.752046
ssNMF: iter 84: obj=2048130.790679, nrmB=32.740767, nrmW=22.823810
ssNMF: iter 85: obj=2047930.064320, nrmB=32.812409, nrmW=22.893701
ssNMF: iter 86: obj=2047733.105353, nrmB=32.866479, nrmW=22.961808
ssNMF: iter 87: obj=2047540.188821, nrmB=32.905637, nrmW=23.028219
ssNMF: iter 88: obj=2047351.473210, nrmB=32.933563, nrmW=23.093024
ssNMF: iter 89: obj=2047167.031346, nrmB=32.953845, nrmW=23.156312
ssNMF: iter 90: obj=2046986.898052, nrmB=32.969188, nrmW=23.218173
ssNMF: iter 91: obj=2046811.125680, nrmB=32.981084, nrmW=23.278694
ssNMF: iter 92: obj=2046639.833906, nrmB=32.989888, nrmW=23.337963
ssNMF: iter 93: obj=2046473.237890, nrmB=32.995215, nrmW=23.396063
ssNMF: iter 94: obj=2046311.643618, nrmB=32.996478, nrmW=23.453079
ssNMF: iter 95: obj=2046155.410522, nrmB=32.993371, nrmW=23.509089
ssNMF: iter 96: obj=2046004.893281, nrmB=32.986122, nrmW=23.564168
ssNMF: iter 97: obj=2045860.380459, nrmB=32.975489, nrmW=23.618387
ssNMF: iter 98: obj=2045722.045458, nrmB=32.962569, nrmW=23.671809
ssNMF: iter 99: obj=2045589.918333, nrmB=32.948565, nrmW=23.724491
ssNMF: iter 100: obj=2045463.879862, nrmB=32.934598, nrmW=23.776483
ssNMF: iter 101: obj=2045343.674552, nrmB=32.921599, nrmW=23.827827
ssNMF: iter 102: obj=2045228.937098, nrmB=32.910266, nrmW=23.878558
ssNMF: iter 103: obj=2045119.226335, nrmB=32.901077, nrmW=23.928701
ssNMF: iter 104: obj=2045014.061121, nrmB=32.894314, nrmW=23.978277
ssNMF: iter 105: obj=2044912.953588, nrmB=32.890100, nrmW=24.027299
ssNMF: iter 106: obj=2044815.436639, nrmB=32.888441, nrmW=24.075775
ssNMF: iter 107: obj=2044721.084086, nrmB=32.889253, nrmW=24.123706
ssNMF: iter 108: obj=2044629.523199, nrmB=32.892390, nrmW=24.171091
ssNMF: iter 109: obj=2044540.440351, nrmB=32.897671, nrmW=24.217925
```

```
ssNMF: iter 110: obj=2044453.581038, nrmB=32.904890, nrmW=24.264198
ssNMF: iter 111: obj=2044368.745747, nrmB=32.913837, nrmW=24.309902
ssNMF: iter 112: obj=2044285.783142, nrmB=32.924302, nrmW=24.355025
ssNMF: iter 113: obj=2044204.581894, nrmB=32.936084, nrmW=24.399557
ssNMF: iter 114: obj=2044125.062222, nrmB=32.948997, nrmW=24.443487
ssNMF: iter 115: obj=2044047.167954, nrmB=32.962873, nrmW=24.486806
ssNMF: iter 116: obj=2043970.859604, nrmB=32.977558, nrmW=24.529506
ssNMF: iter 117: obj=2043896.108695, nrmB=32.992921, nrmW=24.571581
ssNMF: iter 118: obj=2043822.893342, nrmB=33.008847, nrmW=24.613025
ssNMF: iter 119: obj=2043751.194982, nrmB=33.025236, nrmW=24.653838
ssNMF: iter 120: obj=2043680.996052, nrmB=33.042007, nrmW=24.694018
ssNMF: iter 121: obj=2043612.278446, nrmB=33.059088, nrmW=24.733568
ssNMF: iter 122: obj=2043545.022582, nrmB=33.076423, nrmW=24.772491
ssNMF: iter 123: obj=2043479.206947, nrmB=33.093963, nrmW=24.810792
ssNMF: iter 124: obj=2043414.807992, nrmB=33.111668, nrmW=24.848478
ssNMF: iter 125: obj=2043351.800290, nrmB=33.129505, nrmW=24.885556
ssNMF: iter 126: obj=2043290.156855, nrmB=33.147448, nrmW=24.922037
ssNMF: iter 127: obj=2043229.849546, nrmB=33.165474, nrmW=24.957928
ssNMF: iter 128: obj=2043170.849500, nrmB=33.183564, nrmW=24.993242
ssNMF: iter 129: obj=2043113.127536, nrmB=33.201702, nrmW=25.027987
ssNMF: iter 130: obj=2043056.654480, nrmB=33.219873, nrmW=25.062175
ssNMF: iter 131: obj=2043001.401384, nrmB=33.238066, nrmW=25.095817
ssNMF: iter 132: obj=2042947.339619, nrmB=33.256269, nrmW=25.128923
ssNMF: iter 133: obj=2042894.440878, nrmB=33.274470, nrmW=25.161503
ssNMF: iter 134: obj=2042842.677130, nrmB=33.292660, nrmW=25.193567
ssNMF: iter 135: obj=2042792.020566, nrmB=33.310830, nrmW=25.225126
ssNMF: iter 136: obj=2042742.443574, nrmB=33.328971, nrmW=25.256188
ssNMF: iter 137: obj=2042693.918741, nrmB=33.347076, nrmW=25.286763
ssNMF: iter 138: obj=2042646.418869, nrmB=33.365138, nrmW=25.316858
ssNMF: iter 139: obj=2042599.916989, nrmB=33.383151, nrmW=25.346482
ssNMF: iter 140: obj=2042554.386377, nrmB=33.401108, nrmW=25.375641
ssNMF: iter 141: obj=2042509.800553, nrmB=33.419006, nrmW=25.404343
ssNMF: iter 142: obj=2042466.133302, nrmB=33.436839, nrmW=25.432595
ssNMF: iter 143: obj=2042423.358696, nrmB=33.454605, nrmW=25.460402
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ssNMF: iter 360: obj=2039633.370699, nrmB=35.081364, nrmW=26.912211
ssNMF: iter 361: obj=2039630.053694, nrmB=35.081701, nrmW=26.912373
ssNMF: iter 362: obj=2039626.776073, nrmB=35.082035, nrmW=26.912516
ssNMF: iter 363: obj=2039623.537800, nrmB=35.082369, nrmW=26.912637
ssNMF: iter 364: obj=2039620.338800, nrmB=35.082703, nrmW=26.912739
ssNMF: iter 365: obj=2039617.178947, nrmB=35.083037, nrmW=26.912819
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ssNMF: iter 366: obj=2039614.058051, nrmB=35.083372, nrmW=26.912880
ssNMF: iter 367: obj=2039610.975849, nrmB=35.083709, nrmW=26.912919
ssNMF: iter 368: obj=2039607.931983, nrmB=35.084049, nrmW=26.912939
ssNMF: iter 369: obj=2039604.925991, nrmB=35.084393, nrmW=26.912937
ssNMF: iter 370: obj=2039601.957297, nrmB=35.084742, nrmW=26.912914
ssNMF: iter 371: obj=2039599.025215, nrmB=35.085097, nrmW=26.912871
ssNMF: iter 372: obj=2039596.128951, nrmB=35.085459, nrmW=26.912806
ssNMF: iter 373: obj=2039593.267625, nrmB=35.085830, nrmW=26.912720
ssNMF: iter 374: obj=2039590.440285, nrmB=35.086209, nrmW=26.912612
ssNMF: iter 375: obj=2039587.645940, nrmB=35.086600, nrmW=26.912483
ssNMF: iter 376: obj=2039584.883580, nrmB=35.087001, nrmW=26.912331
ssNMF: iter 377: obj=2039582.152208, nrmB=35.087414, nrmW=26.912156
ssNMF: iter 378: obj=2039579.450859, nrmB=35.087841, nrmW=26.911959
ssNMF: iter 379: obj=2039576.778628, nrmB=35.088280, nrmW=26.911738
ssNMF: iter 380: obj=2039574.134687, nrmB=35.088734, nrmW=26.911494
ssNMF: iter 381: obj=2039571.518301, nrmB=35.089202, nrmW=26.911226
ssNMF: iter 382: obj=2039568.928837, nrmB=35.089684, nrmW=26.910934
ssNMF: iter 383: obj=2039566.365772, nrmB=35.090181, nrmW=26.910617
ssNMF: iter 384: obj=2039563.828686, nrmB=35.090693, nrmW=26.910275
ssNMF: iter 385: obj=2039561.317257, nrmB=35.091221, nrmW=26.909908
ssNMF: iter 386: obj=2039558.831244, nrmB=35.091763, nrmW=26.909516
ssNMF: iter 387: obj=2039556.370467, nrmB=35.092321, nrmW=26.909098
ssNMF: iter 388: obj=2039553.934781, nrmB=35.092895, nrmW=26.908653
ssNMF: iter 389: obj=2039551.524053, nrmB=35.093485, nrmW=26.908183
ssNMF: iter 390: obj=2039549.138140, nrmB=35.094092, nrmW=26.907685
ssNMF: iter 391: obj=2039546.776873, nrmB=35.094715, nrmW=26.907161
ssNMF: iter 392: obj=2039544.440048, nrmB=35.095355, nrmW=26.906610
ssNMF: iter 393: obj=2039542.127410, nrmB=35.096014, nrmW=26.906032
ssNMF: iter 394: obj=2039539.838652, nrmB=35.096690, nrmW=26.905426
ssNMF: iter 395: obj=2039537.573399, nrmB=35.097386, nrmW=26.904793
ssNMF: iter 396: obj=2039535.331203, nrmB=35.098101, nrmW=26.904132
ssNMF: iter 397: obj=2039533.111541, nrmB=35.098837, nrmW=26.903443
ssNMF: iter 398: obj=2039530.913816, nrmB=35.099593, nrmW=26.902726
ssNMF: iter 399: obj=2039528.737371, nrmB=35.100371, nrmW=26.901981
ssNMF: iter 400: obj=2039526.581510, nrmB=35.101170, nrmW=26.901207
ssNMF: iter 401: obj=2039524.445519, nrmB=35.101991, nrmW=26.900405
ssNMF: iter 402: obj=2039522.328697, nrmB=35.102835, nrmW=26.899575
ssNMF: iter 403: obj=2039520.230376, nrmB=35.103701, nrmW=26.898715
ssNMF: iter 404: obj=2039518.149943, nrmB=35.104590, nrmW=26.897828
ssNMF: iter 405: obj=2039516.086855, nrmB=35.105502, nrmW=26.896911
ssNMF: iter 406: obj=2039514.040642, nrmB=35.106438, nrmW=26.895966
ssNMF: iter 407: obj=2039512.010905, nrmB=35.107398, nrmW=26.894992
ssNMF: iter 408: obj=2039509.997296, nrmB=35.108382, nrmW=26.893989
ssNMF: iter 409: obj=2039507.999501, nrmB=35.109389, nrmW=26.892957
ssNMF: iter 410: obj=2039506.017216, nrmB=35.110422, nrmW=26.891897
ssNMF: iter 411: obj=2039504.050127, nrmB=35.111480, nrmW=26.890808
ssNMF: iter 412: obj=2039502.097898, nrmB=35.112563, nrmW=26.889691
ssNMF: iter 413: obj=2039500.160158, nrmB=35.113672, nrmW=26.888545
ssNMF: iter 414: obj=2039498.236505, nrmB=35.114808, nrmW=26.887371
ssNMF: iter 415: obj=2039496.326506, nrmB=35.115971, nrmW=26.886169
ssNMF: iter 416: obj=2039494.429703, nrmB=35.117161, nrmW=26.884939
ssNMF: iter 417: obj=2039492.545634, nrmB=35.118378, nrmW=26.883681
ssNMF: iter 418: obj=2039490.673846, nrmB=35.119624, nrmW=26.882395
ssNMF: iter 419: obj=2039488.813931, nrmB=35.120897, nrmW=26.881081
ssNMF: iter 420: obj=2039486.965548, nrmB=35.122199, nrmW=26.879740
ssNMF: iter 421: obj=2039485.128457, nrmB=35.123530, nrmW=26.878372
ssNMF: iter 422: obj=2039483.302538, nrmB=35.124889, nrmW=26.876977
ssNMF: iter 423: obj=2039481.487798, nrmB=35.126278, nrmW=26.875556
ssNMF: iter 424: obj=2039479.684365, nrmB=35.127696, nrmW=26.874108
ssNMF: iter 425: obj=2039477.892461, nrmB=35.129143, nrmW=26.872634
ssNMF: iter 426: obj=2039476.112376, nrmB=35.130619, nrmW=26.871135
ssNMF: iter 427: obj=2039474.344426, nrmB=35.132125, nrmW=26.869611
ssNMF: iter 428: obj=2039472.588927, nrmB=35.133662, nrmW=26.868062
ssNMF: iter 429: obj=2039470.846169, nrmB=35.135228, nrmW=26.866489
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ssNMF: iter 430: obj=2039469.116402, nrmB=35.136824, nrmW=26.864892
ssNMF: iter 431: obj=2039467.399839, nrmB=35.138450, nrmW=26.863272
ssNMF: iter 432: obj=2039465.696651, nrmB=35.140106, nrmW=26.861630
ssNMF: iter 433: obj=2039464.006983, nrmB=35.141792, nrmW=26.859965
ssNMF: iter 434: obj=2039462.330965, nrmB=35.143508, nrmW=26.858279
ssNMF: iter 435: obj=2039460.668717, nrmB=35.145253, nrmW=26.856572
ssNMF: iter 436: obj=2039459.020366, nrmB=35.147026, nrmW=26.854844
ssNMF: iter 437: obj=2039457.386048, nrmB=35.148828, nrmW=26.853096
ssNMF: iter 438: obj=2039455.765912, nrmB=35.150658, nrmW=26.851330
ssNMF: iter 439: obj=2039454.160111, nrmB=35.152515, nrmW=26.849545
ssNMF: iter 440: obj=2039452.568800, nrmB=35.154399, nrmW=26.847742
ssNMF: iter 441: obj=2039450.992119, nrmB=35.156308, nrmW=26.845921
ssNMF: iter 442: obj=2039449.430190, nrmB=35.158243, nrmW=26.844085
ssNMF: iter 443: obj=2039447.883107, nrmB=35.160202, nrmW=26.842232
ssNMF: iter 444: obj=2039446.350940, nrmB=35.162184, nrmW=26.840364
ssNMF: iter 445: obj=2039444.833734, nrmB=35.164189, nrmW=26.838482
ssNMF: iter 446: obj=2039443.331513, nrmB=35.166216, nrmW=26.836586
ssNMF: iter 447: obj=2039441.844277, nrmB=35.168263, nrmW=26.834678
ssNMF: iter 448: obj=2039440.372004, nrmB=35.170331, nrmW=26.832757
ssNMF: iter 449: obj=2039438.914651, nrmB=35.172418, nrmW=26.830824
ssNMF: iter 450: obj=2039437.472144, nrmB=35.174523, nrmW=26.828881
ssNMF: iter 451: obj=2039436.044389, nrmB=35.176646, nrmW=26.826928
ssNMF: iter 452: obj=2039434.631272, nrmB=35.178785, nrmW=26.824965
ssNMF: iter 453: obj=2039433.232662, nrmB=35.180940, nrmW=26.822994
ssNMF: iter 454: obj=2039431.848424, nrmB=35.183112, nrmW=26.821016
ssNMF: iter 455: obj=2039430.478416, nrmB=35.185298, nrmW=26.819030
ssNMF: iter 456: obj=2039429.122500, nrmB=35.187500, nrmW=26.817039
ssNMF: iter 457: obj=2039427.780542, nrmB=35.189717, nrmW=26.815043
ssNMF: iter 458: obj=2039426.452416, nrmB=35.191949, nrmW=26.813042
ssNMF: iter 459: obj=2039425.138015, nrmB=35.194196, nrmW=26.811038
ssNMF: iter 460: obj=2039423.837254, nrmB=35.196458, nrmW=26.809031
ssNMF: iter 461: obj=2039422.550080, nrmB=35.198736, nrmW=26.807023
ssNMF: iter 462: obj=2039421.276478, nrmB=35.201029, nrmW=26.805014
ssNMF: iter 463: obj=2039420.016475, nrmB=35.203337, nrmW=26.803005
ssNMF: iter 464: obj=2039418.770130, nrmB=35.205662, nrmW=26.800997
ssNMF: iter 465: obj=2039417.537529, nrmB=35.208003, nrmW=26.798991
ssNMF: iter 466: obj=2039416.318772, nrmB=35.210359, nrmW=26.796988
ssNMF: iter 467: obj=2039415.113966, nrmB=35.212732, nrmW=26.794989
ssNMF: iter 468: obj=2039413.923225, nrmB=35.215121, nrmW=26.792995
ssNMF: iter 469: obj=2039412.746676, nrmB=35.217526, nrmW=26.791007
ssNMF: iter 470: obj=2039411.584472, nrmB=35.219947, nrmW=26.789026
ssNMF: iter 471: obj=2039410.436801, nrmB=35.222384, nrmW=26.787052
ssNMF: iter 472: obj=2039409.303910, nrmB=35.224838, nrmW=26.785088
ssNMF: iter 473: obj=2039408.186105, nrmB=35.227306, nrmW=26.783133
ssNMF: iter 474: obj=2039407.083765, nrmB=35.229790, nrmW=26.781189
ssNMF: iter 475: obj=2039405.997329, nrmB=35.232289, nrmW=26.779257
ssNMF: iter 476: obj=2039404.927286, nrmB=35.234802, nrmW=26.777338
ssNMF: iter 477: obj=2039403.874143, nrmB=35.237328, nrmW=26.775433
ssNMF: iter 478: obj=2039402.838404, nrmB=35.239866, nrmW=26.773543
ssNMF: iter 479: obj=2039401.820553, nrmB=35.242416, nrmW=26.771669
ssNMF: iter 480: obj=2039400.821035, nrmB=35.244977, nrmW=26.769813
ssNMF: iter 481: obj=2039399.840259, nrmB=35.247548, nrmW=26.767975
ssNMF: iter 482: obj=2039398.878587, nrmB=35.250128, nrmW=26.766157
ssNMF: iter 483: obj=2039397.936328, nrmB=35.252719, nrmW=26.764359
ssNMF: iter 484: obj=2039397.013720, nrmB=35.255318, nrmW=26.762583
ssNMF: iter 485: obj=2039396.110919, nrmB=35.257927, nrmW=26.760830
ssNMF: iter 486: obj=2039395.227978, nrmB=35.260544, nrmW=26.759101
ssNMF: iter 487: obj=2039394.364851, nrmB=35.263171, nrmW=26.757397
ssNMF: iter 488: obj=2039393.521383, nrmB=35.265806, nrmW=26.755719
ssNMF: iter 489: obj=2039392.697334, nrmB=35.268451, nrmW=26.754068
ssNMF: iter 490: obj=2039391.892392, nrmB=35.271105, nrmW=26.752446
ssNMF: iter 491: obj=2039391.106203, nrmB=35.273769, nrmW=26.750853
ssNMF: iter 492: obj=2039390.338400, nrmB=35.276442, nrmW=26.749291
ssNMF: iter 493: obj=2039389.588619, nrmB=35.279126, nrmW=26.747761
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ssNMF: iter 497: obj=2039386.762517, nrmB=35.289960, nrmW=26.741975
ssNMF: iter 498: obj=2039386.097110, nrmB=35.292695, nrmW=26.740619
ssNMF: iter 499: obj=2039385.447024, nrmB=35.295441, nrmW=26.739300
ssNMF: iter 500: obj=2039384.811564, nrmB=35.298197, nrmW=26.738020

figure;
sgtitle('Basis functions obtained by Sparse NMF');
for k = 1:40
    subplot(5, 8, k);
    imagesc(reshape(B(:,k), d1, d2));
    colormap gray; axis image off;
end
```

Basis functions obtained by Sparse NMF

ssNMF: iter 494: obj=2039388.856511, nrmB=35.281819, nrmW=26.746263 ssNMF: iter 495: obj=2039388.141724, nrmB=35.284522, nrmW=26.744798 ssNMF: iter 496: obj=2039387.443875, nrmB=35.287236, nrmW=26.743369

