Kuhn Poker and more practice

Wednesday, January 28, 2026 10:12 AM

(J, Q) payoff:

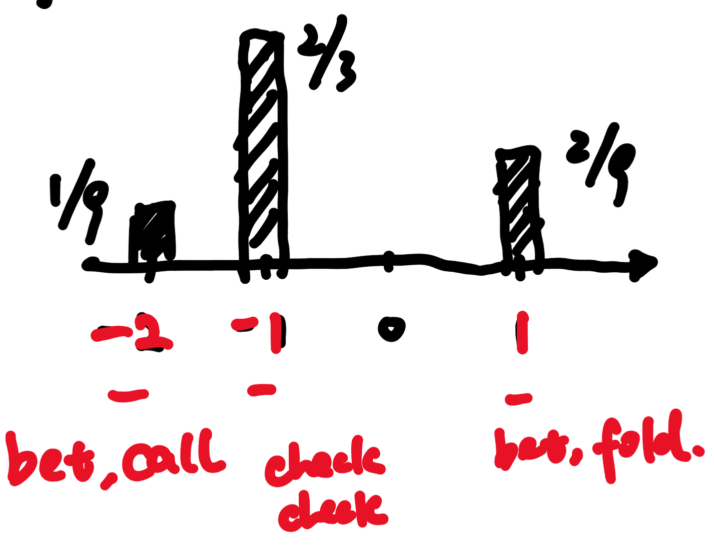
$$E[\text{payoff} \mid (J,Q)] = (-1)\cdot \Pr(P_1 \text{ check}, P_2 \text{ check} \mid (J,Q))$$
$$+ (-2)\cdot \Pr(P_1 \text{ bet}, P_2 \text{ call} \mid (J,Q))$$
$$+ (+1)\cdot \Pr(P_1 \text{ bet}, P_2 \text{ fold} \mid (J,Q))$$

$$\Pr(P_1 \text{ check}, P_2 \text{ check} \mid (J,Q)) = \boxed{\Pr(P_1 \text{ check} \mid (J))} \cdot \Pr(P_2 \text{ check} \mid P_1 \text{ check}, (J,Q))$$
$$= \tfrac{2}{3} \cdot 1$$

$$\Pr(P_1 \text{ bet}, P_2 \text{ call} \mid (J,Q)) = \Pr(P_1 \text{ bet} \mid (J,Q)) \cdot \Pr(P_2 \text{ call} \mid P_1 \text{ bet}, (J,Q))$$
$$= \tfrac{1}{3} \cdot \tfrac{1}{3}$$

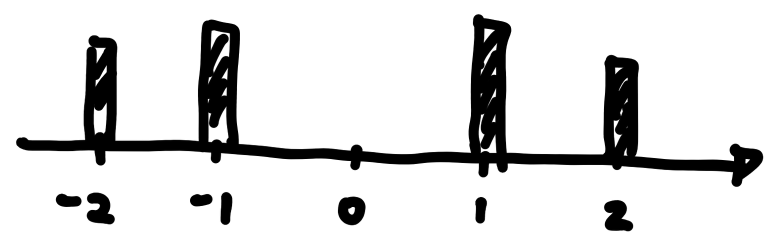
$$\Pr(P_1 \text{ bet}, P_2 \text{ fold} \mid (J,Q)) = \Pr(P_1 \text{ bet} \mid (J,Q)) \cdot \Pr(P_2 \text{ fold} \mid P_1 \text{ bet}, (J,Q))$$
$$= \tfrac{1}{3} \cdot \tfrac{2}{3}$$

Payoff dist. if (J, Q):



$$E[\text{payoff}] = -\tfrac{1}{18}$$

Var[payoff]



Play N times.

Payoff for game i: X_i. $E[X_i] = \tfrac{1}{18}$

Average income over N game:

$$X = \frac{1}{N}\sum_{i=1}^{N} X_i$$

$$E[X] = E\left[\frac{1}{N}\sum_{i=1}^{N} X_i\right] = \frac{1}{N}\sum_{i=1}^{N} E[X_i] = \frac{1}{N}\cdot N \cdot \frac{1}{18} = \frac{1}{18}$$

$$Var(X) = Var\left(\frac{1}{N}\sum_{i=1}^{N} X_i\right) = \frac{1}{N^2} Var\left(\sum_{i=1}^{N} X_i\right) = \frac{1}{N^2}\sum_{i=1}^{N} Var(X_i) = \frac{\sigma^2 \cdot N}{N^2} = \frac{\sigma^2}{N}$$

$1 = w_1 + w_2 + \cdots + w_N$

w_i: investment in asset #i

X_i: change in value.

$E[X_i] = 0$, $Var(X_i) = \sigma^2$, X_i independent.

Profit: $W = w_1 \cdot X_1 + w_2 X_2 + \cdots + w_N \cdot X_N$.

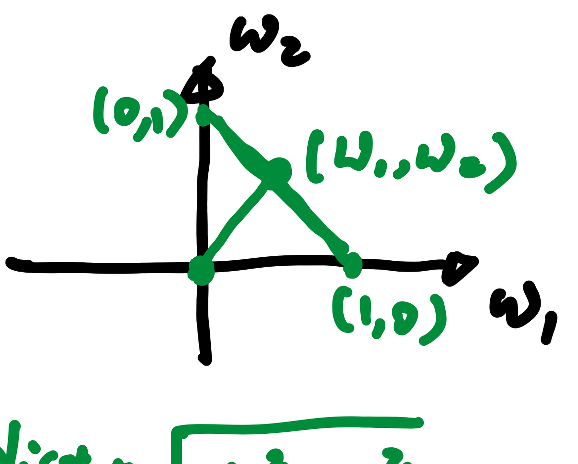
$$E[W] = E[w_1 X_1 + w_2 X_2 + \cdots + w_N X_N] = E[w_1 X_1] + E[w_2 X_2] + \cdots + E[w_N X_N]$$
$$= w_1 \cdot \cancel{E[X_1]}^{0} + w_2 \cdot \cancel{E[X_2]}^{0} + \cdots + w_N \cdot \cancel{E[X_N]}^{0}$$
$$= 0$$

$$Var(W) = Var(w_1 X_1 + \cdots + w_N X_N)$$
$$\overset{(X_i \text{ independent})}{=} Var(w_1 X_1) + \cdots + Var(w_N X_N)$$
$$= w_1^2\, Var(X_1) + \cdots + w_N^2\, Var(X_N)$$
$$= w_1^2 \sigma^2 + \cdots + w_N^2 \sigma^2$$
$$= \sigma^2 (w_1^2 + \cdots + w_N^2)$$

$$\min_{w_1,\dots,w_N} \ \sigma^2 (w_1^2 + \cdots + w_N^2)$$

subject to: $w_1 + w_2 + \cdots + w_N = 1$

$$\boxed{\begin{array}{ll} \min_{w_1, w_2} & w_1^2 + w_2^2 \\ \text{s.t.} & w_1 + w_2 = 1 \end{array}}$$



$\text{dist} = \sqrt{w_1^2 + w_2^2}$