Ballistic Transport in a two-dimensional Electron System

Ballistischer Transport: Flippern mit Elektronen

Introduction

Gallium arsenide (GaAs) is not only the material of choice for producing high-frequency electronic semiconductor devices (e.g. the High Electron Mobility Transistor (HEMT)) used in wireless telecommunications, it also plays an important role in basic research on semiconductors. In particular, combining GaAs layers with other materials such as aluminum arsenide (AlAs) or indium arsenide (InAs) opens a wide playground for the investigation of interesting modern physical phenomena.

Growing a layer sequence GaAs – AlGaAs – AlGaAs:Si[1] – AlGaAs with molecular beam epitaxy one ends up with sheet of electrons accumulated in the GaAs nearby the GaAs/AlGaAs interface. This is called a two-dimensional electron system (2DES). These electrons see only a *small* disturbing coulomb potential due to the ionized Si atoms far away from the conducting layer. Consequently they are able to travel typically some μm without being scattered. Therefore, they are highly sensitive to an external perturbing potential such as periodically etched holes or lines along their path. An additonally applied external magnetic field perpendicular to the 2DES forces the electrons on circular orbits with an diameter proportional to $\frac{1}{B}$. Interesting effects occur in the transport properties of such an device if the diameter and the periodicity of the lines or holes are on the same lengthscale. These phenomena will be studied experimentally in detail here.

In this practical exercise you have the opportunity to learn different aspects of measuring and handling delicate samples, e.g. the Lock-In technique to detect low voltages. Furthermore you use superconducting magnets to generate the magnetic fields required, and methods to cool down samples to 4 K and below.

[1] AlGaAs:Si indicates a *doped* AlGaAs layer with only a few silicon atoms per thousand.

1 Some basic considerations

1.1 2D Electron Systems

One possibility to realize a two-dimensional electron system (2DES) is to epitaxially grow an AlGaAs-GaAs heterostructure as shown in figure 1a. Electrons originating from an AlGaAs:Si donor layer move towards the interface between AlGaAs and GaAs due to the lower conduction band energy in GaAs. As a result positively charged atoms are left in the donor layer where they set up an electric field in the space between. This bends the conduction and the valence band such that a triangular shaped potential well located at the heterointerface is formed in the conduction band (see Fig. 1b).

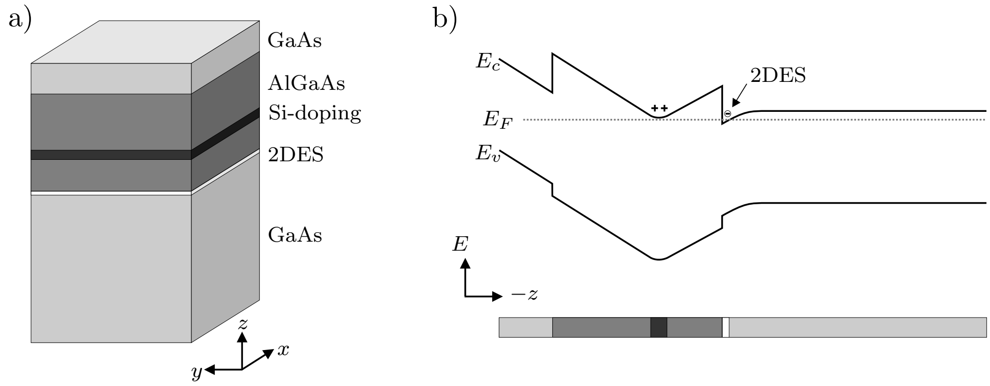

Figure 1: *a) Layer sequence of a GaAs-AlGaAs heterojunction doped with Si. b) The lower edge of the conduction band E_c and the upper edge of the valence band E_v as function of the distance of the surface. For clarity, the layer sequence is repeated below the bandstructure.*

While the electron movement within the xy-plane is free, the movement in growth direction (z-direction) is quantized and the energy of an electron is:

$$E = E_i^z + \left(\frac{\hbar^2 k_x^2}{2m^*} + \frac{\hbar^2 k_y^2}{2m^*} \right) , \tag{1}$$

with the quantized subband energy E_i^z (i=0, 1, 2, ...) in z-direction and the effective[2] electron mass m^*. With sufficiently low temperature and electron density n_s only the lowest of the energy levels E_0^z is occupied[3], so we call our electron system a *two-dimensional* electron system (2DES).

[2]The *effective* mass represents the influence of the periodic crystal potential.
[3]This is the case for the used AlGaAs/GaAs heterostructures at 4.2 K.

The density of states $D(E)$ for such a system is constant within a subband. For the lowest subband E_0^z is:

$$D(E) = \frac{m^*}{\pi\hbar^2} \; . \tag{2}$$

So for the Fermi energy E_F, the Fermi velocity v_F and the Fermi wavevector k_F holds:

$$k_F = \sqrt{2\pi n_s} \; , \tag{3a}$$

$$v_F = \frac{\hbar\sqrt{2\pi n_s}}{m^*} \; , \tag{3b}$$

$$E_F = \frac{\hbar^2 \pi n_s}{m^*} \; . \tag{3c}$$

► EXERCISE 1: Verify the relations given above in equations 3a–3c. ($\rightsquigarrow$ Start with calculating the density of states in $\boldsymbol{k}$-space. Then calculate the area of the Fermi circle and combine both results to obtain n_s for $k = k_F$.)

The simplest way to describe charge transport through such a system is related to the Drude model. Electrons are accelerated in an external electric field until they are stopped after a time τ due to scattering (τ does *not* depend on the magnetic field). Thus they have the drift velocity $\vec{v}_D$ according to:

$$\vec{v}_D = \frac{e\tau}{m^*}\vec{E} = \mu\vec{E} \; , \tag{4}$$

with $\mu = \frac{e\tau}{m^*}$ called mobility. Carrying charges, the current density is

$$\vec{\jmath} = e n_s \vec{v}_D \; . \tag{5}$$

Here another quantity describing the electronic system should be introduced, the mean free path ℓ. Between two scattering events, the electron moves with the Fermi velocity v_F, thus:

$$\ell = \tau v_F = \frac{\hbar}{e}\mu\sqrt{2\pi n_s} \; . \tag{6}$$

The current density $\vec{\jmath}$ and the driving electric field $\vec{E}$ are connected by the conductivity tensor $\boldsymbol{\sigma}$ resp. the resistivity tensor $\boldsymbol{\rho}$ as follows (remember that we have a 2D system!):

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \cdot \begin{pmatrix} E_x \\ E_y \end{pmatrix} \; , \tag{7a}$$

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} \cdot \begin{pmatrix} j_x \\ j_y \end{pmatrix} \; . \tag{7b}$$

In isotropic systems — and the systems we use *are* isotropic — the components of the resistivity tensor are symmetric: $\rho_{xx} = \rho_{yy}$ and $\rho_{xy} = -\rho_{yx}$ and it holds $\boldsymbol{\sigma} = \boldsymbol{\rho}^{-1}$.

1.2 Hall Resistivity and longitudinal Resistivity

Now we apply the Drude model to calculate carrier transport through a Hall bar. A Hall bar is a piece of conducting material — especially in our case a GaAs-AlGaAs heterostructure hosting a 2DES — shaped as shown in figure 2.

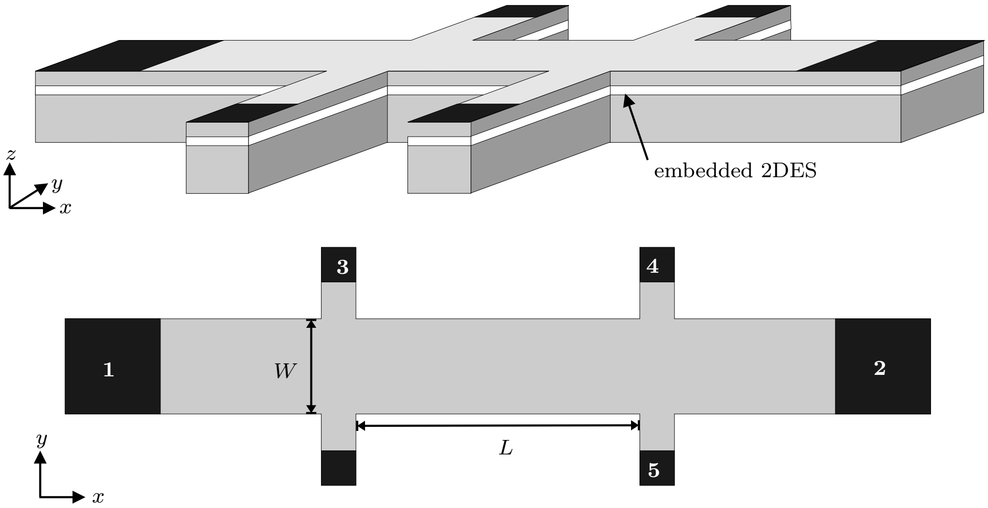

Figure 2: *In the upper part the scetch of a heterostructure hosting the 2DES and shaped as a Hall bar is shown. The lower panel demonstrates the geometry for longitudinal and Hall resistance measurements: The current is driven from contact 1 to 2 while ρ_{xx} is measured between the contacts 3 and 4, and ρ_{xy} is measured between the contacts 4 and 5.*

A current flowing from contact 1 to contact 2 passes through an rectangular area of the width W and the length L. At the corners of this area are the contacts 3, 4 and 5 as voltage probes. If there is a current $\vec{\jmath} = (j_x, 0)$ flowing in x-direction through the sample, a small magnetic field B perpendicular to the 2DES will deflect the electrons due to the Lorentz force. As a consequence, an electric field in the y-direction will be set up, compensating the deflection. This situation could be described by:

$$m^* \frac{d}{dt}\vec{v}_D + \frac{m^*}{\tau}\vec{v}_D = e(\vec{E} + \vec{v}_D \times \vec{B}) \ . \tag{8}$$

► EXERCISE 2: Since we measure currents and voltages we need expressions for the Hall voltage U_{xy} (U_{45} in the geometry given in figure 2) and the longitudinal voltage U_{xx} (U_{34} according to figure 2) as a function of the applied magnetic field B. ($\rightsquigarrow$ Start from equation 8 and assume the stationary case $\frac{d}{dt}\vec{v}_D = 0$. Remember that we are in a two dimensional system, remember that the macroscopic current I flows only in x-direction, and remember further that the magnetic field $\vec{B}$ is perpendicular to the 2DES. Set up an equation $\vec{E} = \mathbf{r} \cdot \vec{\jmath}$ and compare the components of the tensor $\mathbf{r}$ with the components of the resistivity tensor $\boldsymbol{\rho}$ in equation 7b.) Discuss the results and explain how the density n_s and the mobility μ can be extracted from measurements of U_{xy} vs. B and U_{xx} vs. B.

Since the electrons move on cyclotron orbits, the cyclotron radius R_c and the cyclotron frequency ω_c are also of interest:

$$\omega_c = \frac{eB}{m^*} , \tag{9a}$$

$$R_c = \frac{v_F}{\omega_c} = \frac{\hbar\sqrt{2\pi n_s}}{eB} . \tag{9b}$$

The Drude model is only valid for small magnetic fields. At higher magnetic fields the classical model will break down and we have to use quantum mechanics to describe our 2DES properly. A hand-waving argument whether we can calculate classically or not is the following: Assume, that there will be no transport due to *drifting* charge carriers if an electron can turn a lot of cyclotron orbits before it is scattered. Thus, we have to compare the mean free path ℓ with the cyclotron orbit.

► EXERCISE 3: Check, up to which magnetic field you can use the drude model to describe transport through a sample with $\mu = 1 \times 10^6\,\mathrm{cm}^2/\mathrm{Vs}$.

To describe the system more accurately in the case we apply higher magnetic fields, we start with the Schrödinger equation:

$$\left(E_i + \frac{1}{2m^*}(i\hbar\nabla + e\vec{A})^2 + U(y)\right)\Psi(x,y) = E\Psi(x,y) . \tag{10}$$

E_i is the subband energy (quantized in z-direction) and $\vec{A}$ is the magnetic vector potential. The potential $U(y)$ accounts for the geometric restriction due to the Hall bar. Assuming at a first glance $U(y) = 0$ inside the Hall bar we achieve the energies:

$$E_n = E_i + (n + \frac{1}{2})\hbar\omega_c \quad \text{with } n = 0, 1, 2, \ldots \tag{11}$$

as eigenvalues of equation 10. These equidistant energy levels are called Landau levels. So the density of states $D(E)$ is no longer constant, but a series of delta-like peaks. All states condense now on these Landau levels. In real systems these peaks are slightly broadened due to crystal defects and incorporated impurities. As a consequence, the longitudinal resistance ρ_{xx} is no longer constant but drops to zero periodically. These oscillations are called Shubnikov-de Haas (SdH) oscillations. Also the Hall resistance ρ_{xy} shows no longer a linear behaviour, but is a series of plateaus with well defined resistance of $\frac{1}{i} \cdot \frac{h}{2e^2}$, $i = 1, 2, 3, \ldots$ as shown in figure 3.

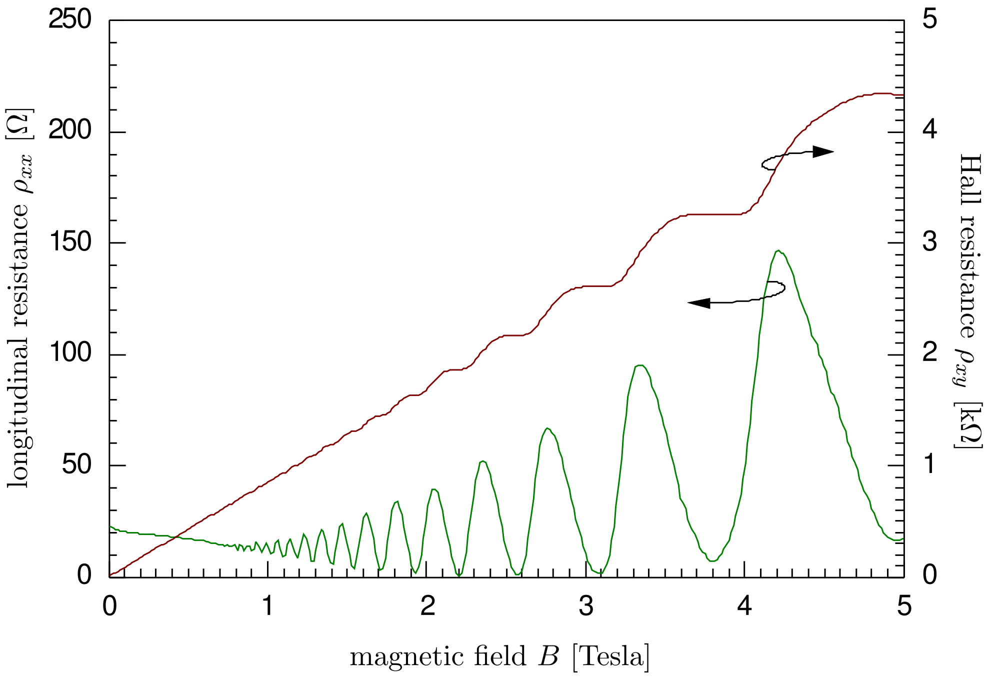

Figure 3: *Longitudinal resistance ρ_{xx} and Hall resistance ρ_{xy} as a function of magnetic field B. At fields greater ~0.7T SdH oscillations in ρ_{xx} and Hall plateaus in ρ_{xy} can clearly be observed.*

This is called the Quantum Hall effect. These and related phenomena are very interesting and until today topics of a lot of research projects. But going into detail on this topic is far beyond this practical exercise and will be omitted here. Nevertheless, the periodicity of the SdH oscillations in $\frac{1}{B}$ is another method to determine the electron density in the 2DEG:

$$n_s = 2\frac{e}{h} \cdot \frac{1}{\frac{1}{B_{i+1}} - \frac{1}{B_i}}\ , \tag{12}$$

with i and $i+1$ are the numbers of two subsequent SdH minima.

1.3 Transport through a structured Hall bar

After this short excursion in the Quantum Hall regime, we come back to our classical ideas of magnetotransport. We can ask ourselves now what happens if we are using not a bare Hall bar as shown in figure 2 but a structured Hall bar with non-conducting barriers[4], as shown in figure 4.

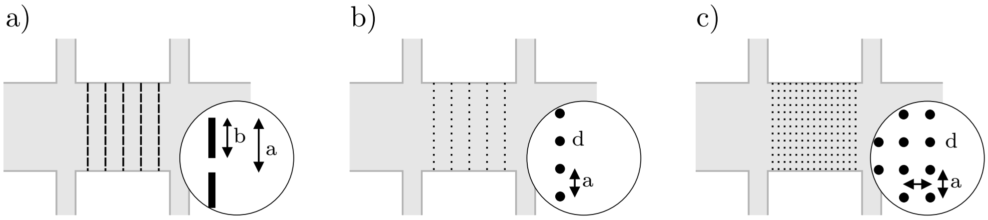

Figure 4: *Hall bar structured with different types of non-conducting barriers: a) Linearly arranged stripes width length* b *and period* a*. b) Linearly arranged antidots with diameter* d *and period* a*. c) Antidots arranged in a square lattice with lattice constant* a *and diameter* d*.*

If we apply a magnetic field B such, that the cyclotron radius R_c equals half the period a, an electron can move around this barrier. Hence, electrons are pinned and do not contribute to transport until they are scattered. As a consequence the longitudinal resistivity $\rho_{xx}(B)$ will show a more or less pronounced peak. A more general condition for seeing these so called commensurability oscillations for the structure shown in figure 4a is:

$$2R_c = \frac{i}{j} \cdot a \; . \tag{13}$$

The peak at B_{ij} corresponding to i=j=1 is called fundamental peak, peaks with i>1, j=1 are called harmonics, while peaks with j>1, i=1 are called subharmonics.

► Exercise 4: Discuss equation 13: how does the pinned electron orbits look like for i=1, j=2, 3, ... , for j=1, i=2, 3,... and for the general case i, j=1, 2, 3,... ? ($\rightsquigarrow$ The electrons are quasi reflected like billardballs when they touch the edge of the sample.) Draw these orbits. For what i and j will the pinning break down? How will the carrier density n_s and the mobility μ influence the commensurability oscillations? How will the exact geometry at a given period a influence $\rho_{xx}(B)$? ($\rightsquigarrow$ Consider the ratio $\frac{c}{b}$ with the barrier length b and the contact width c=a−b.) How does the situation change for the structures shown in figure 4b and 4c?

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[4]In general these barriers are created by etching grooves in the hallbar to remove the electrons.

1.4 Adjusting the carrier density

As we have seen, the electron density n_s and the momility μ are important parameters in magnetotransport measurements. So it might be useful to tune at least n_s. Beside a gate electrode on top of the device[5], an effect related to the doping mechanism can also be used to tune the sheet carrier density. During growth of a small part of the AlGaAs layer Si atoms are incorporated on Ga sites in the lattice. Since Si has 4 valence electrons, and Ga has only 3, the forth valence electron of the Si atom does not contribute to the crystal bindings and is able to transport charge through the sample. But not all Si atoms on Ga sites behave equally: normally electrons from Si atoms have an energy very close to the conduction band and hence can be thermally activated even at low temperatures to the conduction band. But there is a second set of Si atoms on Ga sites, where the Si atoms disturb locally the crystal lattice a little bit and, as a consequence, the corresponding electrons are more stronger bound to the Si atoms. It is *not* possible to thermally activate these electrons at low temperatures, but it is possible to activate them due to photon absorption. Once activated in the conduction band they contribute to transport.

Figure 5: *Two possible configurations for Si atom incorporation in (001)-GaAs. a) 'normal' configuration, Si acts as flat donor. b) asymmetrical configuration, Si acts as DX-center. In this case it is necessary to illuminate the sample to push the electron in the conduction band. (Picture taken from: Chadi et al. Phys. Rev. Lett.* ***61****(7), 873 (1988).)*

This effect is irreversible and is called persistent photo conductivity (PPC). So we have the possibility to tune the electron density n_s of the sample simply by illuminating it. The carriers remain in the conduction band until the sample is heated up to a temperature of ∼150 K. Figure 5 shows schematically the two possible configurations of Si atoms on Ga-sites in AlGaAs.

[5] Our samples are *without* this feature!

2 What we measure and how we do it

2.1 Ingredients

First of all, we need the sample, this means a structured Hall bar. Fortunately this work is done. We used *optical lithography* and *wet chemical etching* to prepare the Hall bar itself, *E-Beam lithography* and *RIE* to structure the Hall bar and indium *alloying* to provide *ohmic contacts* to the 2DES. After this we soldered gold *bond wires* with non-tremulous hands to mount the processed sample at a *DIL8 housing* with eight contacts. The result of all these work is shown in figure 6.

a) b)

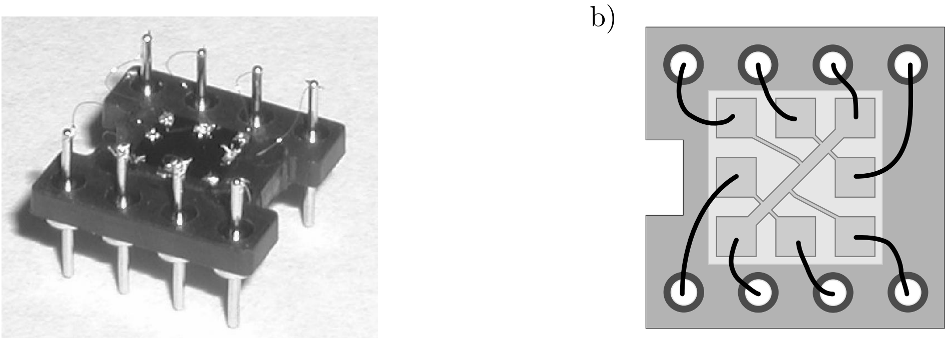

Figure 6: *A Sample prepared for measuring. a) Photograph of the real device completely contacted and mounted. b) Schematic to show connections from the semiconductor to the DIL8 housing. The pins of the DIL8 socket are numbered counter-clockwise from 1 to 8, beginning at the lower left. The left side of the socket is marked.*

► Exercise 5: Explain all expressions in *italic* in the text above. If some of these expressions are unknown, look them up in standard literature on semiconductor processing techniques.

The second most important thing we need is a magnetic field of at least 1 T. The most convenient way to reach these fields is the use of superconducting magnets. In our setup a superconducting NbTi wire is wound up to a coil to generate fields up to 5 T. The point is that these kind of magnet only works at liquid He temperature of 4.2 K. But — and here you should remember footnote 3 — that's o.k. The only restriction arising therefrom is, that we have no possibility to handle the sample directly and that we need a cryoproofed sample holder for this.

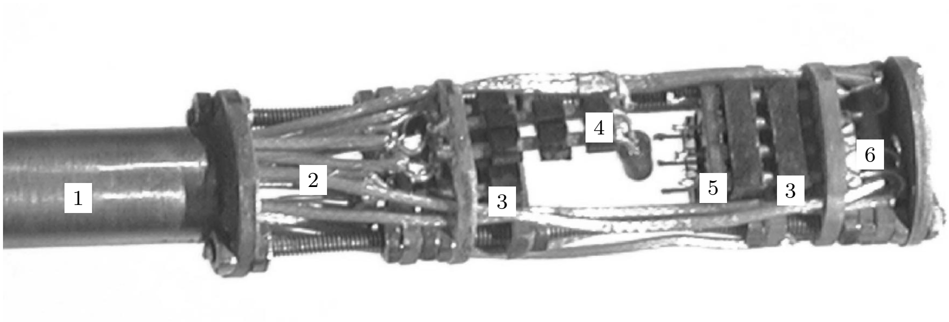

Figure 7: *Sample holder. 1) Supporting rod, 2) Coaxial cables, 3) DIL8 sockets, 4) LED for illumination, 5) Sample with Hall bar, and 6) Si diode for temperature measurements.*

In our case, a long stainless steel rod with two DIL8 sockets mounted at the end as shown in figure 7 is used. It fits exactly in the bore of the magnet such that the sample is centered in the homogenous magnetic field. In addition, a Si diode as temperature sensor and a LED are fitted to the sample holder. The electrical contacts to the DIL8 sockets are coaxial cables: the inner leads are connected directly to the DIL8 socket while the outer shieldings of all coax cables are connected together at the cold side as schematically shown in figure 8.

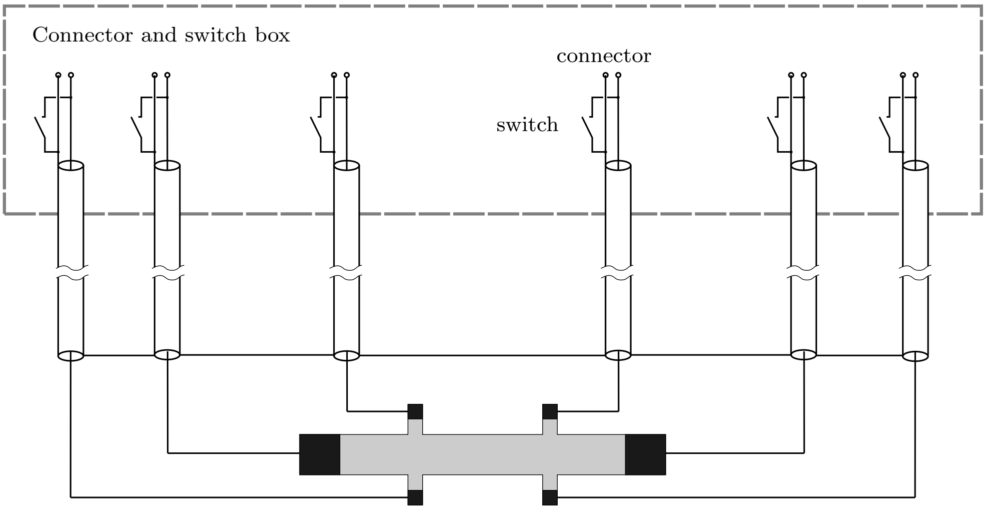

Figure 8: *Connecting scheme. At sample side, all shields of the coax cables are connected together. In the connector box there is a female coaxial connector and a switch to short inner and outer lead for each connection to the sample.*

At room temperature side all coax cable end up in female coax connectors mounted in a switch box, with both inner and outer leads isolated against ground. A switch determines whether a particular coax connector at the front is active or not, another switch short circuits the inner and outer lead. Since we drive only small currents through the sample (typically 200 nA in our case), we expect only small voltages to measure. A standard technique to do this is to use a 'Lock-In amplifier' (short: Lock-In). This is a very sensitive instrument, which amplifies only signals with a certain choosable frequency f_0 and ignores all other signals.

► EXERCISE 6: Calculate the expected longitudinal voltage V_{xx} in a $150\,\mu$m wide Hall bar with a distance between the voltage probing contacts of $750\,\mu$m for $B = 0\,$T and a current $I = 200\,$nA. Assume that the used 2DES has a sheet carrier concentration of $n_s = 2.0 \times 10^{11}\,\mathrm{cm}^{-2}$ and a mobility of $\mu = 1.0 \times 10^6\,\mathrm{cm}^2/\mathrm{Vs}$. Why are we using an Lock-In instead of a 'simple' Micro-Voltmeter? How does a Lock-In work in principle? Since nearly every Lock-In has an built-in oscillator to generate the reference signal needed, can we use the Lock-In not only for measuring but also as current source? How must the sample be connected? ($\rightsquigarrow$ The Lock-In provides the oscillator signal as a *voltage* adjustable in frequency and amplitude (0 V to 2 V!) between the outer and inner lead of a coaxial connector. A Lock-In is also able to measure two voltages between inner and outer leads of two coax connectors and to calculate the *difference* between these two voltages.)

2.2 Behavior in the lab

Before you start your measurements you should keep in mind a few important things which make your life easier and safer (and certainly also your tutor's life):

- Working in a lab means having a maximum of discipline. This includes not to eat or drink, not to smoke, and to keep your fingers away from equipment of other people's setups.
- Do not use any equipment unless you are allowed to do so and unless you have been shown *how* to use it.
- Be extremely careful handling cryogenic liquids, especially He. The most known dangers hereby are frozen skin or limbs, damaged equipment and severe injuries due to exploding vessels, and the risk of suffocation.
- Keep also in mind, that the energy stored in a superconducting coil magnetized to 1 T is enormous. Hence be aware of the breakdown of the superconductivity (quench) since in this case a lot of liquid He will be evaporated at once!

2.3 Tasks to do

Now we have enough background knowledge about a 2DES, a Hall bar and the theory of magnetotransport experiments, as well as we have some basic technical informations about the used equipment. So let us begin with the measurements.

1. Explain in detail the equipment in the lab (e.g. x-y-recorder, LockIn, cryostat, power supply, He handling system and dewar, ...) and the actions to take in the case of emergency.

2. Put one of the mounted samples with a bare Hall bar in the sample holder (do not forget the LED!) and cool down the magnet and the sample to 4.2 K (Since this is a very critical process, do it *only* together with your tutor).

3. When the magnet and the sample are cold, measure n_s and μ of the unilluminated sample. Do this by taking ρ_{xx} and ρ_{xy} as a function of B. Please pay attention: Not all samples we use have the connection scheme shown in figure 6b! In doubt, ask your tutor! Do not use higher fields as 3 T and drive a current of max. 200 nA through the sample. Choose the appropriate settings of the Lock-In (gain, time constant, resolution, etc.)! What can you observe?

4. Repeat the measurement after illuminating the sample (60 sec, 1 mA current through LED). What has changed now?

5. Use the sample with the linearly arranged stripes (see Fig. 4a) and repeat the measurement. Assign the observed features to the fundamental, harmonics or subharmonics and calculate therefrom the period a and the length b and c.

6. If you have to take out the sample holder please ask your tutor to assist you. Do not handle the system alone, until your tutor told you to do so!

7. Perform the same measurements at the samples with the linearly arranged dots and the dot arrays. Does the ρ_{xx} traces change the way you expected? If not, why not?

8. After finishing all measurements you planned to do, pull out the magnet (together with your tutor, for sure), switch off the power supply and put the sample back in the appropriate case.

9. Collect all your personal things (do not forget your empty bottle of coke!) and check if you have all the data necessary for discussing your results.

2.4 Discussion of the results

Discuss your results critically. Do you really observe what you expected? Are the peaks at the correct position according to the given device geometry? If not, what might be the reason therefore? How many peaks can you resolve? What should be changed to achieve higher resolution? What might be the limiting factor? What can you deduce from the linewidth of the peaks? What determines the height of the resistance maxima?

Literature

The following list is surely not complete and is given here only for your convenience. Feel free to search for more literature, if you are interested in more details…

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Useful diagrams

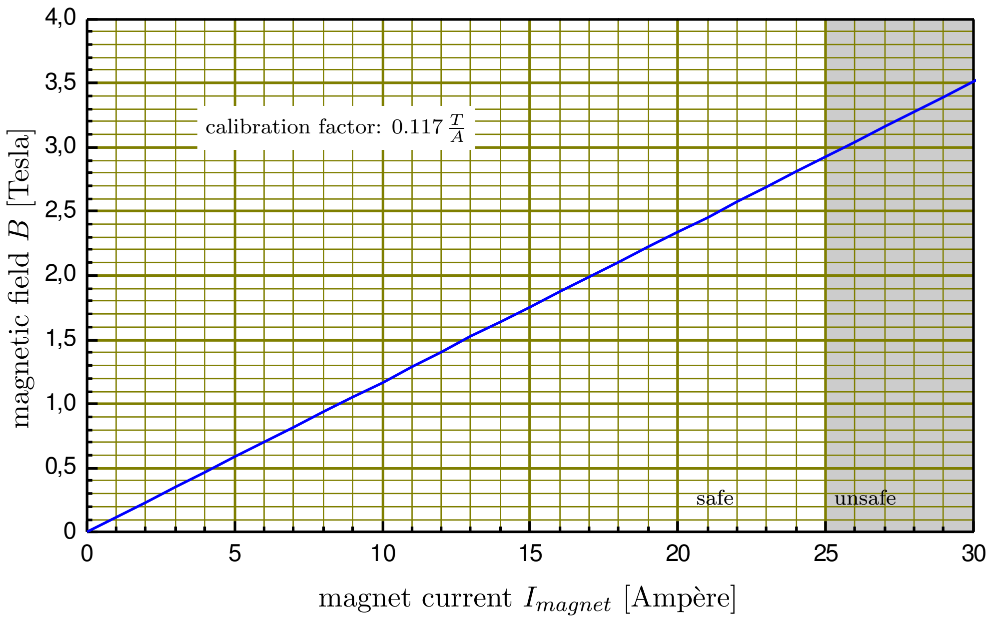

Figure 9: *Magnetic field as function of current through the superconducting magnet. The gray shaded part marks an area of unsafe operation (quench).*

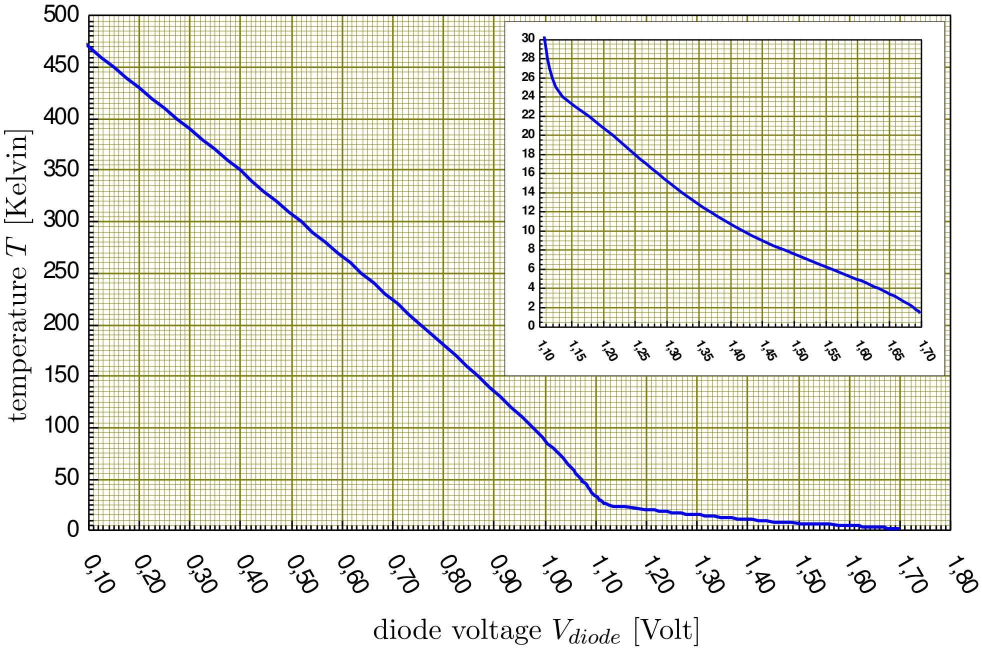

Figure 10: *Calibration of the Si diode as temperature sensor. The inset is a closer look to the relevant temperature range from 1 K to 30 K.*