detailed derivation potential energy

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Conservation of mass gives

$$\rho V = \rho_0 V_0 \tag{S1}$$

and

$$V = \rho_0 V_0 \frac{1}{\rho} \tag{S2}$$

Thus we can write the differential dV as

$$dV = -\rho_0 V_0 \frac{1}{\rho^2} d\rho \tag{S3}$$

and inserting the mass conservation Eq. (S1) we obtain

$$dV = -\frac{\rho V}{\rho^2} d\rho = -\frac{V}{\rho} d\rho \tag{S4}$$

which idential to the first part of Eq. (3.3). The second part is just inserting of the definition of the speed of sound and thus $dV = V/(\rho c^2)dp$.

The integration in the acoustic limit follows as such:

$$-\int_{V_0}^{V} p dV = -\int_0^p p \frac{V}{\rho c^2} dp \tag{S5}$$

Now we use Eq. (S1) and solve for ρ and insert it into Eq. (S5) to obtain

$$-\int_0^p p \frac{V^2}{\rho_0 V_0 c^2} dp \approx -\int_0^p p \frac{V_0^2}{\rho_0 V_0 c^2} dp = -\int_0^p p \frac{V_0}{\rho_0 c^2} dp \quad . \tag{S6}$$

Pleas note for the approximation we assumed that $V^2 \approx V_0^2$.

Integrating Eq. (S6) we obtain

$$-\frac{1}{2} \left[p^2 \frac{V_0}{\rho_0 c^2} \right]_0^p = -\frac{1}{2} \frac{p^2 V_0}{\rho_0 c^2} \quad , \tag{S7}$$

which is our Eq. (3.4).

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