PH4606 - Lecture 5

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1 Reflection and Transmission

We understand that the resistance of a medium, the acoustic impedance, affects the wave propagation. Now, we explore what happens if a planar wave encounters a medium with a different characteristic impedance. Similar to electromagnetic waves some of the pressure wave will be reflected and some transmitted; to quantify this we introduce a pressure transmission, T, and reflection coefficient, R:

$$T = \frac{P_t}{P_i} \qquad R = \frac{P_r}{P_i} \tag{5.1}$$

where P_i is the amplitude of the incoming wave, P_t, the amplitude of the transmitted, and P_r the amplitude of the reflected wave. Note that these coefficient are in general complex. The intensity for planar progressive waves is always real, therefore we can write the intensity transmission and reflection coefficient as

$$T_I = \frac{I_t}{I_i} = \frac{r_1}{r_2} |T|^2 \, R_I = \frac{I_r}{I_i} = |R|^2 \tag{5.2}$$

Fig. 5.1 depicts the geometry for a normal incidence beam. Consider that beams of finite power have a limited diameter. If the beam is not incident under an angle we need to take into account the beam diameter of the incoming and that of the transmitted beam. Thus the transmission coefficient for the power T_Π is

$$T_\Pi = \frac{A_i}{A_t} T_I = \frac{A_i}{A_t} \frac{r_1}{r_2} |T|^2 \quad . \tag{5.3}$$

We show later that the beam diameter of the reflected beam does not change even under an inclinded reflection, thus the reflection ocefficient for the power is

$$R_\Pi = R_I = |R|^2 \quad . \tag{5.4}$$

Conservation of Energy demands

$$R_\Pi + T_\Pi = 1 \tag{5.5}$$

Figure 5.1: Reflection and transmission of a plane wave under normal incidence of a planar boundary between two fluids of different characteristic impedances

For the normal incidence we have to account for three surface normal waves, see Fig. 5.1. These are

$$p_i = P_i e^{i\omega t - k_1 x} \tag{5.6}$$
$$p_r = P_r e^{i\omega t + k_1 x} \tag{5.7}$$

$$p_t = P_t e^{i\omega t - k_2 x} \tag{5.8}$$

for the incoming, the reflected and the transmitted wave, respectively. Both the incoming and the reflected wave have the same wave number $k_1 = \omega/c_1$, while the transmitted wave has a wave number $k_2 = \omega/c_2$. c_1 and c_2 are the speed of sound in medium 1 and 2. The frequency does not change upon reflection.

To formulate the boundary conditions we assume that the boundary is rigid and does not create a force from within. A rigid boundary does not move, demanding that the particle velocity of the interface is 0. The requirement that the boundary creates no force we can obtain a continuity of the force on the left and right side of the boundary. Thus both the pressure and the normal particle velocity on both sides of the boundary are equal:

$$p_i + p_r = p_t \tag{5.9}$$

$$u_i + u_r = u_t \quad . \tag{5.10}$$

at $x = 0$. We can devide Eq. (5.9) by Eq. (5.10), thus

$$\frac{p_i + p_r}{u_i + u_r} = \frac{p_t}{u_t} \quad . \tag{5.11}$$

The ratio on the right hand side of Eq. (5.11) is the specific acoustic impedance, $r = \pm p/u$, while on the left hand side we can replace the particle velocity with the pressure and obtain

$$r_1 \frac{p_i + p_r}{p_i - p_r} = r_2 \quad . \tag{5.12}$$

Equation (5.13) can be rearranged to obtain the reflection coefficient, $R = p_r/p_i$ for normal incidence.

$$R = \frac{r_2 - r_1}{r_2 + r_1} = \frac{\frac{r_2}{r_1} - 1}{\frac{r_2}{r_1} + 1} \tag{5.13}$$

We can obtain the Transmission coefficient from understanding that Eq. (5.9) devided p_i gives

$$1 + R = T \quad , \tag{5.14}$$

and therefore the transmission coefficient

$$T = \frac{2r_2}{r_2 + r_1} = \frac{2\frac{r_2}{r_1}}{\frac{r_2}{r_1} + 1} \quad . \tag{5.15}$$

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The Cython extension is already loaded. To reload it, use:
  %reload_ext Cython
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```
def calcu(un, unn, cc, rho, cconst):
  return 2.*un[1:-1,1:-1]-unn[1:-1,1:-1]+cconst*(cc[1:-1,1:-1]*\
      (un[:-2,1:-1]+un[2:,1:-1]+un[1:-1,:-2]+un[1:-1,2:]-4.*un[1:-1,1:-1])-\
      .25*rho[1:-1,1:-1]*((rho[2:,1:-1]-rho[:-2,1:-1])*(un[2:,1:-1]-un[:-2,1:-1])+\
      (rho[1:-1,2:]-rho[1:-1,:-2])*(un[1:-1,2:]-un[1:-1,:-2])))
```

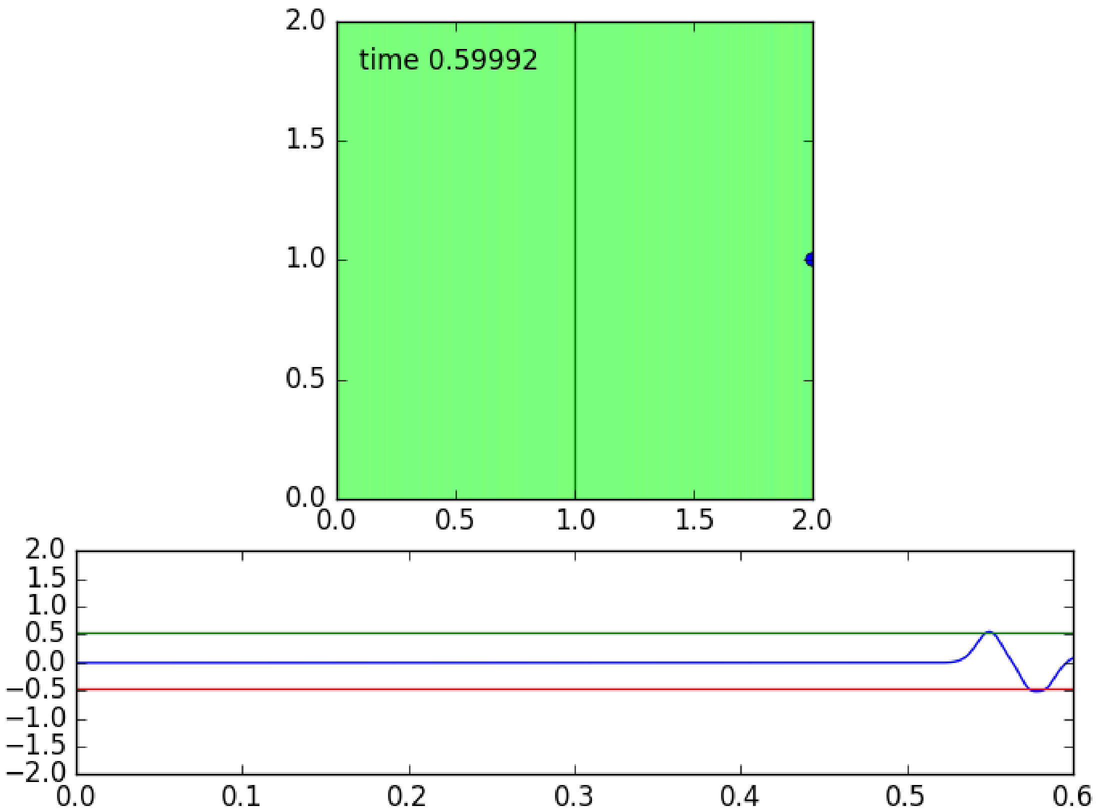
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Figure 1: