PH4606 - Lecture 1

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1 Energy & Intensity & Impedance

1.1 Energy

When a wave travels energy is transported, just consider someone speaking to you. For the sound to reach you energy travels from the speaker and some is needed to create the perception of sound, e.g. perturb your ear drum.

We can use our Physics 101 definition of kinetic and potential energy. The kinetic energy of $1/2mu^2$; we can write this using ρ_0 being the density of the fluid particle and V_0 the volume of the unperturbed particle as

$$E_k = \frac{1}{2}\rho_0 V_0 u^2 \quad . \tag{3.1}$$

The potential energy (that which is stored in a fluid particle) is its compression by the acoustic pressure p. This is

$$E_p = -\int_{V_0}^{V} p dV \quad . \tag{3.2}$$

You may note the minus sign; this comes from the fact that compressing the fluid particle (positive pressure) decreases the volume but increases the potential energy. To integrate Eq. (3.2) we need to include the dependency of $V(p)$, i.e.

$$dV = -\frac{V}{\rho}d\rho = \frac{V}{\rho c^2}dp \quad ; \tag{3.3}$$

the first expression comes from mass conservation $\rho V = \rho_0 V_0$ and the second one from the speed of sound, $c^2 = \frac{\mathrm{d}p}{\mathrm{d}\rho}$.

Using expression Eq. (3.3) in Eq. (3.2) and integrating the kernel with the acoustic pressure from 0 to p we obtain the potential energy of the acoustic wave see detailed derivation within the linear approximation.

$$E_p = -\frac{1}{2}\frac{p^2 V_0}{\rho_0 c^2} \quad . \tag{3.4}$$

Thus the total acoustic energy is

$$E = E_k + E_p = \frac{1}{2}\rho_0 V_0 \left[u^2 + \left(\frac{p}{\rho_0 c}\right)^2\right] \quad . \tag{3.5}$$

In continuous media it is more practical to use the specific energy or energy density, the energy per unit volume $\mathcal{E}_i = E/V_0$ which is

$$\mathcal{E}_i = \frac{1}{2}\rho_0 \left[u^2 + \left(\frac{p}{\rho_0 c} \right)^2 \right] \quad . \tag{3.6}$$

The index i stands for instantenous energy density as compared to the time averaged energy density $\mathcal{E}$.

$$\mathcal{E} = \frac{1}{T} \int_0^T \mathcal{E}_i \, dt \tag{3.7}$$

Expressions (3.6) and (3.7) apply to any linear wave, to do the integration one needs to know the relationship between the particle velocity and the speed. For plane waves we have derived this relationship in Eq. (2.6), $p = \pm u\rho_0 c$. Inserting this equation into Eq.(3.6) we obtain

$$\mathcal{E}_i = \rho_0 u^2 = \frac{p^2}{\rho_0 c^2} \quad . \tag{3.8}$$

If the amplitudes of the wave are U for the velocity and P for the pressure, we can express the averaged energy as a function of these amplitudes, namely

$$\mathcal{E} = \frac{PU}{2c} = \frac{P^2}{2\rho_0 c^2} = \frac{\rho_0 U^2}{2} \quad . \tag{3.9}$$

This expression is only valid for plane waves, however it is approximately valid also for propagating spherical waves as long as the radius of curvature of the wave front is much larger than the wavelength.

1.2 Intensity

The product of pressure, p, and particle velocity $\mathbf{u}$ is the intensity of the sound field I. If we consider only the direction of propagation we can ignore the vectorial quantity and write

$$I = \langle I(t) \rangle_T = \langle p\, u \rangle_T = \frac{1}{T} \int_0^T p\, u \, dt \quad . \tag{3.10}$$

The unit is Watt/m^2. It is the work done on a fluid element per unit area and time, thus a force times length per area and time, or in other words a pressure times a velocity, $I = p\, u$.

The period T in Eq.(3.10) only applies for monofrequent waves. If the wave is also a planar wave we can integrate with $p = \pm \rho_0 c u$ and obtain

$$I = \pm \frac{P^2}{2\rho_0 c} \quad . \tag{3.11}$$

For a harmonic waves/oscillations we can use an effect amplitude (or root-mean-squared amplitude) which is defined as

$$F_e = \sqrt{\frac{1}{T} \int_0^T f^2(t) dt} \tag{3.12}$$

Thus for a harmonic pressure change we obtain $P_e = \frac{P}{\sqrt{2}}$ and $U_e = \frac{U}{\sqrt{2}}$. With this we can write the acoustic intensities as

$$I_\pm = \pm P_e U_e = \pm \frac{P_e^2}{\rho_0 c} \quad . \tag{3.13}$$

1.3 Impedance

The ratio between acoustic pressure and particle velocity is termed acoustic impedance

$$Z = \frac{p}{u} \quad , \tag{3.14}$$

which for planar waves is $Z = \pm\rho_0 c$. The sign is for positive and negative travelling wave. The SI unit of Z is Pa·s/m, sometimes the unit rayl or Mrayl (10^6 Pa·s/m) are used. This unit is named after the Lord Rayleigh, one of the founders of modern acoustics.

The product $\rho_0 c$ has great importance, even more than the pressure and velocity. It is named the characteristic impedance.

The acoustic impedance is not necessarily a real valued quantity (which it is for planar waves). In general the acoustic impedance is

$$Z = r + ix \quad , \tag{3.15}$$

where r is the acoustic resistance and x the acoustic reactance.

For water and air $\rho_0 c$ at 20°C and atmospheric pressure $\rho_0 c$ is 1.48×10^6Pa·s/m and 415 Pa·s/m, respectively.

1.4 Decibels

Sound pressures and intensities are described on a logarithmic scale. One reason is that bigh range of sound pressures and intensities found in nature and experiments. Already the ear can cover the intensity range from $10^{-12}\,\mathrm{Wm}^{-2}$ to $10\,\mathrm{Wm}^{-2}$. Most commonly is the use of the dexibel (dB) scale. The intensity level IL for sound for intensity I is

$$IL = 10 \log_{10} \frac{I}{I_{ref}} \quad , \tag{3.16}$$

where I_{ref} is the reference intensity (see below) and the number IL is give in "units" of decibel referenced to I_{ref} or in short dB re I_{ref}. The deci-part of the decibel is from the factor 10 in Eq. (3.16).

For the pressure the formula is slightly different; we have seen that $I = P_e^2/(\rho c)$. Thus it is natural state the sound pressure level as

$$SPL = 20 \log_{10} \frac{P_e}{P_{ref}} \quad , \tag{3.15}$$

where SPL is expressed in dB re P_{ref} with P_e the measured effective pressure. If we choose such a reference Intensity and reference pressure that $I_{ref} = P_{ref}^2/(\rho_0 c)$ the $IL\ re\ I_{ref} = SPL\ re\ P_{ref}$.

1.5 Reference intensities and pressures

In air the reference intensity used is $I_{ref} = 10^{-12}\,\mathrm{W/m^2}$ which is about the threshold at a frequency of 1 kHz for humans with unimpaired hearing to notice. This intensity relates through acoustic impedance of air with the effective pressure of about $P_e = P_{ref} = 20\,\mu\mathrm{Pa}$.

In water there different reference pressures are used, here we present here only one common, which is $P_{ref} = 1\mu\mathrm{Pa}$.

1.6 Sensitivity

When measursing the with microphone (gas) or hydrphone (liquids) one needs to relate the measured signal with the pressure of the sound field. The manufacturer does the calibration and specifies the sensitivity of the microphone as open circuit sensitivity $\{\mathcal{M}_\prime\}$, which means no current is provided by the microphone:

$$\mathcal{M}_\prime = \frac{V}{P_e} \tag{3.16}$$

The sensitivity $\mathcal{M}$ of a microphone is also expressed as sensitivity levels $\mathcal{ML}$ in decibels, thus

$$\mathcal{ML}(re\ \mathcal{M}_{\nabla\rceil\{}) = 20\log_{10}\frac{\mathcal{M}}{\mathcal{M}_{\nabla\rceil\{}} \quad . \tag{3.17}$$

Let's do an example, the common hydrophone 8106 from Bruel and Kjaer has a specified sensitivty level of $\mathcal{ML}$ -173 dB re 1V/μPa. Now let us convert this into a sensityivity $\mathcal{M}$ given in SI units V/Pa:

$$-173\mathrm{dB}\ re\frac{1\mathrm{V}}{\mu\mathrm{Pa}} = 20\log_{10}\left(\frac{\mathcal{M}}{\mathcal{M}_{\nabla\rceil\{}}\right)$$

This can solved for $\mathcal{M}$ and gives

$$10^{-8.65}\frac{\mathrm{V}}{\mu\mathrm{Pa}} = \mathcal{M}$$

and thus

$$\mathcal{M} = 10^{-2.65}\frac{\mathrm{V}}{\mathrm{Pa}} \approx 3\,\mathrm{mV/Pa} \quad .$$

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