

detailed derivation potential energy

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Conservation of mass gives

$$\rho V = \rho_0 V_0 \quad (\text{S1})$$

and

$$V = \rho_0 V_0 \frac{1}{\rho} \quad (\text{S2})$$

Thus we can write the differential dV as

$$dV = -\rho_0 V_0 \frac{1}{\rho^2} d\rho \quad (\text{S3})$$

and inserting the mass conservation Eq. (S1) we obtain

$$dV = -\frac{\rho V}{\rho^2} d\rho = -\frac{V}{\rho} d\rho \quad (\text{S4})$$

which identical to the first part of Eq. (3.3). The second part is just inserting of the definition of the speed of sound and thus $dV = V/(\rho c^2) d\rho$.

The integration in the acoustic limit follows as such:

$$-\int_{V_0}^V p dV = -\int_0^p p \frac{V}{\rho c^2} dp \quad (\text{S5})$$

Now we use Eq. (S1) and solve for ρ and insert it into Eq. (S5) to obtain

$$-\int_0^p p \frac{V^2}{\rho_0 V_0 c^2} dp \approx -\int_0^p p \frac{V_0^2}{\rho_0 V_0 c^2} dp = -\int_0^p p \frac{V_0}{\rho_0 c^2} dp \quad . \quad (\text{S6})$$

Pleas note for the approximation we assumed that $V^2 \approx V_0^2$.

Integrating Eq. (S6) we obtain

$$-\frac{1}{2} \left[p^2 \frac{V_0}{\rho_0 c^2} \right]_0^p = -\frac{1}{2} \frac{p^2 V_0}{\rho_0 c^2} \quad , \quad (\text{S7})$$

which is our Eq. (3.4).