Tutorial 1 Microfluidics

Please prepare the answers for the following questions for our tutorial on 15.5.2025

Claus-Dieter Ohl

$$u = a\,x$$
$$v = -ay \quad \text{where } a = \text{const} > 0 \quad .$$

1. **Streamlines**

   Given the following flow field:

   $$u = a\,x$$
   $$v = -ay \quad \text{where } a = \text{const} > 0 \quad .$$

   A. Find an equation for the streamline that passes through a point (x_0, y_0). In which direction moves a fluid particle? If it orginates from a location $y_0 > 0$ does the particle pass through the horizontal line $y = 0$. Where is the stagnation point? What kind of flow could that be?

   B. Plot the flow field (arrow or `plt.quiver` plot for $x \geq 0$). For $x_0 = 1$ and $y_0 = 1$, plot a streamline passing through this point. Then, plot through nearby points a few more streamlines to illustrate the flow field.

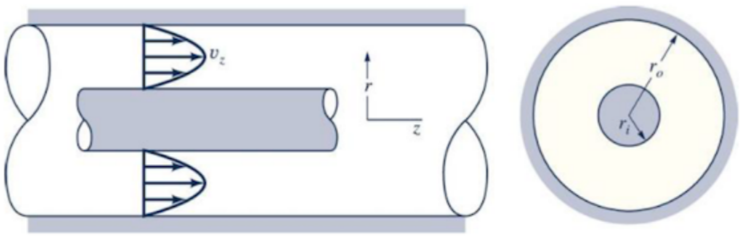
2. **Index Notation** Use the index notation and show that these are equalities:

   A. $$\frac{\partial}{\partial x_i}(p\,\delta_{ij}) = (\nabla p)_j$$

   B. $$\nabla \cdot (\rho \vec{u}) = (\nabla \rho) \cdot \vec{u} + \rho\, \nabla \cdot \vec{u}$$

3. **Navier Stokes Equation**

   Repeat the Navier Stokes derivation for a tube now for 2 concentric tubes with radii r_i and r_o.



   Show that the flow velocity in z-direction is

   $$u_z(r) = \frac{1}{4\mu}\frac{\mathrm{d}p}{\mathrm{d}z}\left(r^2 - r_o^2 + \frac{r_i^2 - r_o^2}{\ln\frac{r_o}{r_i}}\ln\left[\frac{r}{r_i}\right]\right) \quad .$$

4. **Flow Simulations**

   A. Write down the boundary conditions (b.c.) for the velocity $\vec{u}$ and pressure for a flow in a 2d-tube driven by a pressure gradient. You need one b.c. for each boundary, i.e. the walls and the inlet/outlet, and component of the velocity. The pressure at the walls are determined by the incompressibility condition. For help, look into the code.

   B. Do the same for a flow in a 2-tube driven by the upper wall. Again, check your answer with the boundary conditions stated in the code.

   C. Conduct simulations for pressure driven and wall driven flows. Try to obtain a parabolic profile for the pressure driven flow and a linear profile for the wall driven flow. Discuss the results. What is the effect of the time step "dt" of integration.