Fluid Mechanics – Tutorial Sheet #3

This tutorial sheet is graded. We will cover this tutorial on Monday, 29 August. If you need any help drop by PAP-05-13 and ask for Julien or Milad or send an email to JULIEN001@e.ntu.edu.sg.

You are requested to hand in your work as a hardcopy latest at the beginning of the tutorial class. For more information on the submission rules refer to "Logistics and introduction" PowerPoint by Prof. Ohl.

The steady flow between two parallel plates can be described by this velocity field:

$$\vec{V} = u_1\hat{\boldsymbol{\imath}} + u_2\hat{\boldsymbol{\jmath}}\ ,\ \ u_1 = ay\ ,\ \ u_2 = 0\ \text{, where } a = 1\ s^{-1}.$$

In this question we are going to investigate the deformation of a chosen fluid particle in this flow field. The flow field is described in the Cartesian coordinates, i.e. $\hat{\boldsymbol{\imath}} = (1,0), \hat{\boldsymbol{\jmath}} = (0,1)$, and the fluid element is a 2D box defined by the coordinates $(0.5\,, -0.5), (-0.5\,,\ -0.5), (0.5\,, 0.5), (-0.5\,, 0.5)$.

(a) Find the velocity gradient tensor for this flow field. Decompose the velocity gradient tensor into a symmetric (S_{ij}) and an antisymmetric ($\frac{1}{2}R_{ij}$) tensor. Based on the decomposition of the velocity gradient tensor, can you predict the deformation of the given fluid particle? Does it elongate, shear, rotate, or all of the above?

(b) A given point $\vec{X} = (x_1, x_2) = (x, y)$ in the fluid will move based on the path line expression:

$$\frac{d\vec{X}}{dt} = \vec{V}(\vec{X}) \quad (eq.1)$$

Using a Taylor expansion about the point $\vec{X}_0$, this can be approximated as:

$$\frac{d\vec{X}}{dt} = \vec{V}(\vec{X}) \cong \vec{V}(\vec{X}_0) + (\vec{X} - \vec{X}_0).\nabla\vec{V}(\vec{X}_0) \quad (eq.2)$$

This allows us to find the motion of the fluid particle $\vec{X}$ based on the velocity gradient tensor at the nearby point $\vec{X}_0$, denoted as $\nabla\vec{V}(\vec{X}_0)$.

Write both eq. 1 and eq. 2 for Cartesian coordinates x and y.

Write a python programme to find the deformed shape of the given fluid particle for $dt = 0.2, 0.4, 0.6, 8,$ and 1s.

1. Find the deformed box using the path line expression given in eq. 1.
2. Find the deformed box based on the approximation given in eq. 2.
3. Compare the results for the two methods.

Hint: You can do this by finding the location of every corner of the box after a certain dt.

(c) ***Python programming:*** Find the eigenvalues and eigenvectors of the strain rate tensor both analytically and using the **sympy** python module. There is a code for this in the lecture ipython notebooks, just modify that code. What information do we get from these eigenvectors?

(d) For the given flow field, we see that there seems to be no circulatory motion involved, is it in contradiction with the rotation rate tensor previously calculated?