Fluid Mechanics – Tutorial Sheet #2

This tutorial sheet is graded. We will cover this tutorial on Monday, 22 August. If you need any help drop by PAP-05-13 and ask for Julien or Milad or send an email to JULIEN001@e.ntu.edu.sg.

You are requested to hand in your work as a hardcopy latest at the beginning of the tutorial class. For more information on the submission rules refer to "Logistics and introduction" PowerPoint by Prof. Ohl.

1 – Streamline, Pathline and Streakline

Given the following velocity field

$$\boldsymbol{V} = ax\hat{\boldsymbol{\imath}} + byt\hat{\boldsymbol{\jmath}} \text{ with } a = 2s^{-1} \text{ and } b = 1s^{-2}.$$

(a) Find an equation for the streamline through the point $\boldsymbol{x}_0 = (x, y) = (1,1)$ at any time, t. Plot these streamlines for the instants $t = 0, 1, 2,$ and $3s$.
(b) Find an equation for the pathline of a fluid particle passing through $\boldsymbol{x}_0 = (x, y) = (1,1)$ at $t = 1s$. Plot this pathline during the interval from $t = 1$ to $t = 4s$.
(c) Find the locus of particles that have passed through $\boldsymbol{x}_0 = (x, y) = (1,1)$ at $t = 4s$. Plot this streakline in the time interval of $1 < \hat{t} < 4s$.
(d) Find the pathline for the particle described in part (b) numerically using a finite-difference scheme. Compare the agreement between the analytical solution plotted in part (b) and the numerical solution. Initially use a time step of 0.1 s and then 0.01 s for the numerical solver. Does using a finer time step improve the agreement?

Hint: This question covers the material from Week 1 and Week 2. For parts (a), (b), and (c), you can use the code from Week 1, Lecture 2 iPython notebook. For part (d), you can use the code from Week 2, Euler Method iPython notebook.

2. Tensors

Given a steady velocity field

$$\boldsymbol{V} = u\hat{\boldsymbol{\imath}} + v\hat{\boldsymbol{\jmath}} + w\widehat{\boldsymbol{k}} \;, u = x^2 - y^2, v = -2xy \;,\; w = 0.$$

(a) Find the velocity gradient tensor for this flow field, $\partial_j u_i$.
(b) Decompose the velocity gradient tensor into a symmetric (S_{ij}) and an antisymmetric ($\frac{1}{2}R_{ij}$) tensor, $\partial_j u_i = S_{ij} + \frac{1}{2}R_{ij}$.
(c) Based on the decomposition in part (b), explain which terms represent linear strain, shear strain, and rotation rate.
(d) Calculate the curl of velocity $\nabla \times V$. Do you find any relation between the curl of velocity and the rotation tensor?

Hint: This question covers the material from Week 2, Lecture #3