A Study On The Importance Of Gear Theory In The Design And Manufacturing Of Mechanical Sculptures: Blooming Flowers Automata

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This study provides the documentation and methodology necessary for designing, testing, and manufacturing advanced automata using widely available materials while utilizing design engineering principles. The automata have no electrical pieces such as motors or sensors, focusing on simple machines, specifically gears and levers to automate the mechanical sculpture. The theory behind torque, rotational motion, and gear theory is explained in the study even if the recipient has no more than a rudimentary to no grasp of mechanics. A laser cutter is used for the quick prototyping of the automata, however, the design can be manufactured by hand with plywood or MDF and easily accessible tools such as hand saws and wood files, a proper understanding of gear theory is necessary. Making bevel gears is challenging and timely without a machine like a CNC because of their conical-shaped teeth. To compensate for this, cage and peg gears are used. Additionally, spur gears are developed specifically for the sculpture using gear theory. The real-life constraints of mechanical design engineering are dealt with by utilizing gear theory in the study.

Keywords *Gear Theory, Design Engineering, Computer-Aided Design, Computer-Aided Manufacturing*

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1 Introduction

Makers and artists have incorporated movement into sculptures and artwork since the early 20th century. This was done mechanically, statically, or mobile. The use of electricity and motors was classified as mechanic while exploiting natural phenomena such as hydraulic or aerodynamic movement was classified as mobile art. The movement was imitated through static means by utilizing optical illusions in static art, also called op art. [1] Recently, however, achieving kinetic art with simple machines alone has become increasingly popular. The development of the mechanical sculpture in this study relies exclusively on the interactions of simple machines, more specifically the gear.

Gears are "wheels with teeth shaped so as to transmit rotational motion from one wheel to another." [2] They are also commonly referred to as cogwheels where the teeth are called cogs. It is essentially a circular machine with teeth that meshes with the teeth of another gear to transmit torque from one axis to another. By pairing different types of gears it is possible to temper with the speed of a gear, the magnitude, and direction of torque, and the number of revolutions (read: 3, 5). Although they are similar to another simple machine, the pulley, and the teeth of the gear allow for finer control of movement while handling higher loads. This aspect of gears is what makes them so suitable for mechanical art.

2 Terminology and Foundations

Terminology commonly used with gears is important when understanding gear theory as well as doing the calculations necessary in the mechanical sculpture design process. They provide the theory itself as well as increase readability. The following terminology can be used as a foundation for gear theory and while the equations can be applied for some other types of gears, they are specifically for spur gears as they are the most commonly used type of gears.

The pitch circle is an imaginary circle concentric to the center of the wheel. It is the circle where the gear speed is measured. The radius of the pitch circle is the distance between the center of the wheel to the center of one of its teeth. Similarly, the pitch diameter, D, is twice the value of the pitch circle's radius. The pitch circles of the drive and the driven gear must be tangent or maintain a point of intersection to transmit torque which is called the pitch point.

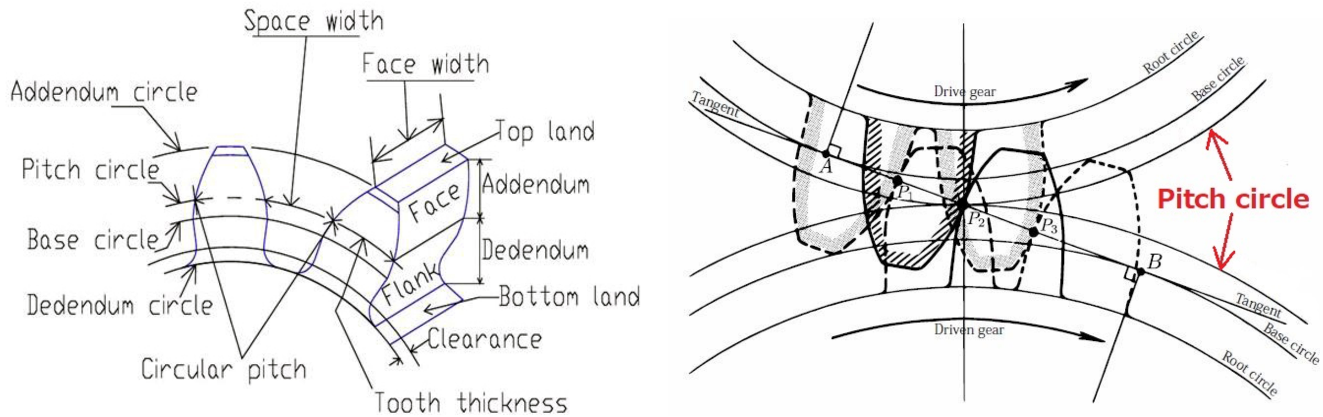


Fig. 1 Gear Nomenclature Diagram

The root circle is the minor diameter of the gear. It is also called the dedendum circle. It is drawn from the point where the flank of a tooth meets the bottom land of the gear, the bottommost part of the teeth. If there is a fillet, p_f, the root circle should be drawn at the point where the fillet ends. The base circle is an imaginary circle used to generate involute profiles of the teeth (read: **3.1**). It has a diameter equal to the product of the pitch circle diameter and the cosine of the pressure angle, α (read: **4**) as shown in the figures above. The addendum circle of the gear is the major circle, drawn from the outermost points of the teeth. The distance between the addendum circle and the root circle of its meshed gear is called clearance, c. The circle drawn from the points at which clearance is calculated is called the clearance circle. The amount of clearance between meshed gears is called backlash, also called lash or play. [4, 9]

Addendum, a, is the height of the tooth above the pitch circle while dedendum, b, is the depth of the tooth from the root circle to the pitch circle. The whole depth, h, is the distance between major and minor circles and can also be calculated by adding up addendum and dedendum values. Working depth is the distance the teeth of one gear extend into the space of its meshed gear. It can be calculated by adding up the addendum of gear, a_G, and the addendum of pinion, a_P.

Gear ratio also called velocity ratio, m_G, is the ratio of the number of teeth on the bigger gear, N_G, by the smaller, N_P. The smaller gear in a meshed gear relationship is called the pinion. It is used to define the basic relationship between meshed gears and can be used to calculate the ratio of revolutions with respect to each other as well as derive equations for other values.

Diametral pitch, P, is the number of teeth, N or z, per unit of pitch diameter. As such, it is found by dividing the number of teeth by the pitch diameter of that gear. It is also used to express the gear size of the teeth.

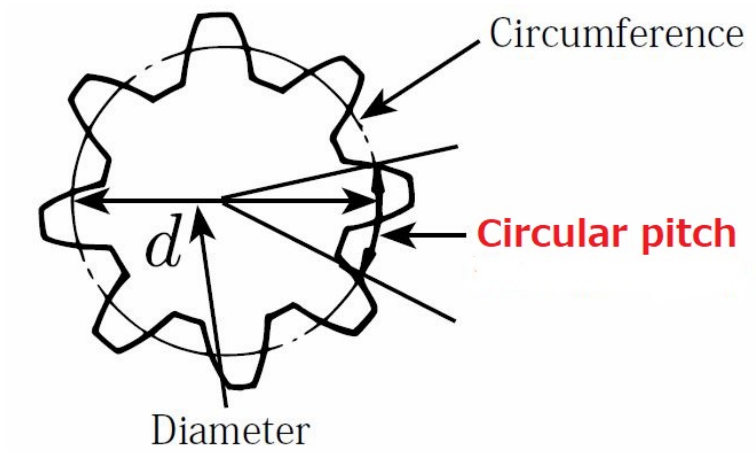


Fig. 2 Circular Pitch Diagram

Circular pitch is an imaginary arc on the pitch circle with the endpoints being the corresponding points of adjacent teeth. In an effectively functioning gear, all the possible values of the circular pitch must be equal to one another, meaning all the teeth should be the same distance away from each other and also have the same size. It is calculated by dividing the product of the pitch diameter and pi by the number of teeth. It can also be found by dividing pi by the diametral pitch. The circular pitch is shown with the letter p. Since pi is a constant it is usually omitted and module, m, is calculated to get a more readable number. The module can also be calculated by dividing the circular pitch by pi.

$$p = \pi \frac{D}{N} \qquad p = \frac{\pi}{P} \qquad m = \frac{D}{N} \qquad m = \frac{P}{\pi}$$

Fig. 3 Circular Pitch And Module Formulas

The pitch diameter is used to determine the center distance, C, which is the distance between the center shafts of two gears, by dividing the sum of the meshed gears' pitch diameters by 2. However, the center distance can be calculated in several other ways as well. For example, dividing the sum of the gear and pinion number of teeth by 2 times the diametral pitch. From these equations, many more can be derived as shown in the figure below. [4]

Center Distance	$C = N_p (m_G + 1) / 2P$ $C = (D_p + D_G) / 2$ $C = (N_G + N_p) / 2P$ $C = (N_G + N_p)\ p / 2P$ $C = (N_G + N_p)\ p / 6.2832$

Fig. 4 Center Distance Derived Formulas

3 Tooth Profile

A tooth profile is the cross-section of a tooth between the addendum circle and the root circle of that gear. The profile is a curve that can be projected from the intersection of the tooth to the transverse, normal, or axial planes which are normal to the plane at the pitch point shown in the figure below. The transverse plane is parallel to the front cross-section of the gear while the normal plane is perpendicular to the curvature of the teeth. In spur gears, the teeth are parallel to each other so there is no curvature on that axis, making the transverse and normal planes at a point coincident. The transverse and normal planes of gears with angled teeth like a helical tooth profile would have that angle of curvature with the point at the origin. In the case of Fig. 5, the pitch point. [5]

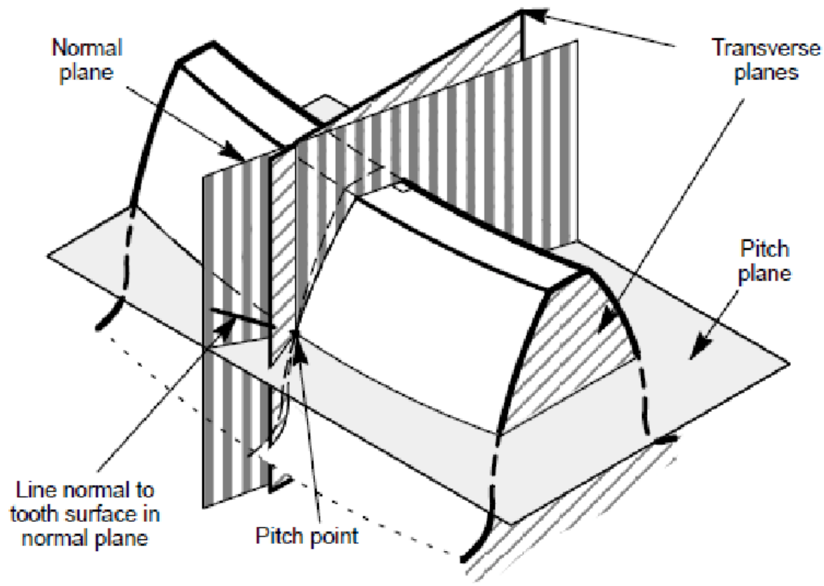


Fig. 5 Planes at Pitch Point

Conjugate tooth profiles are two tooth profiles with their normal lines passing through the pitch point. Sometimes it is stated that the conjugate tooth profiles are any two profiles that satisfy the fundamental law of gear-tooth action. The two tooth profiles that most effectively make conjugate profiles and are easy to manufacture are involute and cycloidal profiles. Since close tolerances aren't necessary with involute profiles the most common curve is the involute curve.

3.1 Involute Curves

Involute tooth profiles are called involute because they're generated from the involute of a circle. An involute curve is "the locus of a point considered as the end of a taut string being unwound from a given curve in the plane of that curve." [7] It can also be generated by a point on a line, as it rolls or rotates on a circle without slipping. The involute curve of the tooth profile is generated from the base circle of that gear. [6]

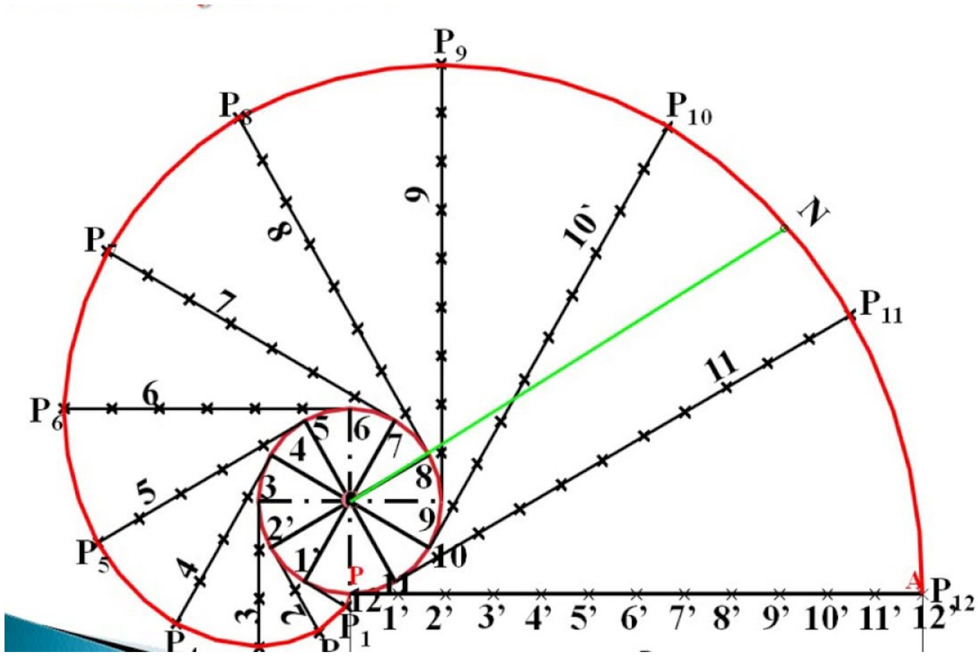


Fig. 6.1 Involute, Unwound String

The first method is shown in Fig. 6.1 above. The red line is unwound from the original circle, eventually becoming a straight line. Here the point that is traced is the very edge of the circumference, drawing from P_1-P_{12}, forming arcs that make up the curve. When generating a curve for gear tooth profiles, the point that is traced is the pitch point.

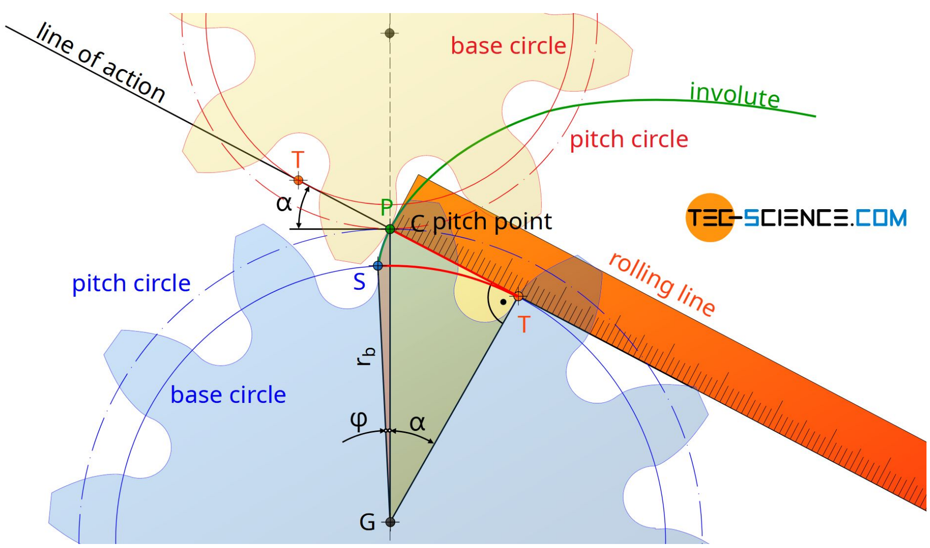


Fig. 6.2 Involute, Rolling Line

The second method is shown in Fig. 6.2 above. A straight line, shown by the edge of a ruler, rolls along the pitch circle and the path taken by the point, the pitch point, is traced to generate the involute curve. As the ruler rolls in the clockwise direction, the pitch point will trace the green line.

3.2 Cycloidal Profiles

Cycloidal tooth profiles are used with cycloidal gears, usually seen in mechanical clocks and watches. This is due to the flat sides of the pinion, making it easier to polish and reduce friction. They further reduce friction as they have 1 to 2 contact point/s at an instance as opposed to involute gears which have 2 to 3 points. Additionally, the hour, minute, and second hands of clocks are achieved by gear trains causing the pinions to have increasingly fewer teeth. Those gears, if they were involute, have to be undercut making them delicate and difficult to manufacture in bulk (read: **3.3**).

The cycloidal tooth profiles are generated by combining epicycloid and hypocycloid curves derived from the pitch circle and pitch point. These curves are formed by tracing a point on a circle rolling on the circumference of another circle external and internal respectively. [8] Epicycloid curves make up the addenda of one gear and the same rotating circle is used to generate the curves of the dedenda of its meshed gear, ensuring that the angular velocities of the gears are locally constant, this is the case for an involute profile as well (read: **4**). The rotating circle is derived from the pitch circles of the gear and pinion both, with the addenda profile being generated from its own pitch circle while the dedenda are from its meshed gear, as shown in Fig. 7 below.

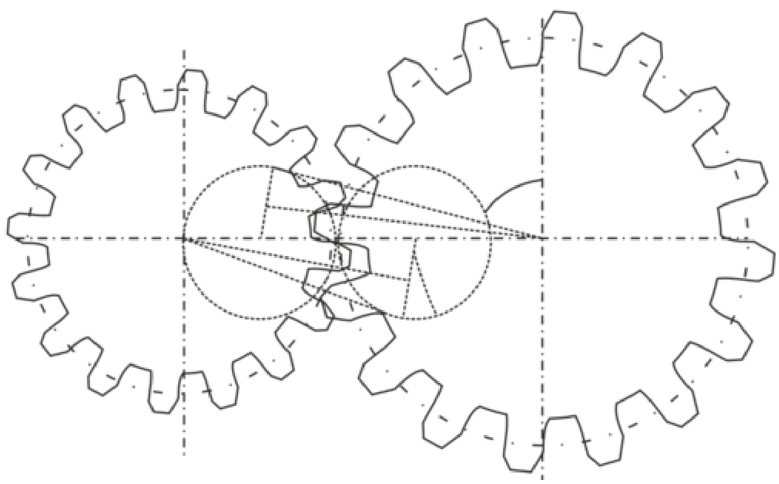


Fig. 7 Cycloidal Gear Construction Diagram

3.3 Profile Shifts

Profile shifts are the modification or correction of the standard gear tooth profiles according to the purpose and circumstances in which the gear is put into use. [12] A positive shift will increase the thickness of the teeth profile while a negative shift will decrease it. However, profile shifting has limits according to the number of teeth and coefficient of the shift without undercut, and a condition to prevent undercut. Undercut, also called deeper cutting, is “the phenomenon of cutting the root of the gear deeper than the involute tooth curve.” [10] This happens when the number of teeth on the gear falls below the minimum. Situations pertaining to the issue can be fixed preemptively via the equations shown in Fig. 9 below. [11]

Condition to Prevent Undercut:

$$m - (x^*m) \leq \frac{z^*m}{2} sin^2\alpha$$

Number of Teeth without Undercut:

$$z = \frac{2^*(1 - x)}{sin^2\alpha}$$

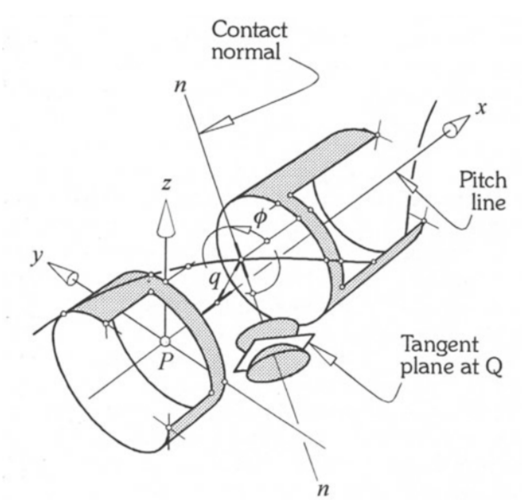
Coefficient without Undercut:

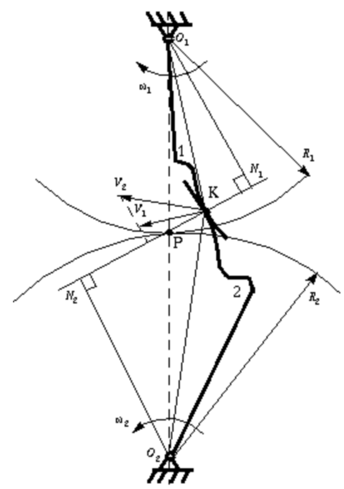
$$x = 1 - \frac{z}{2}^* sin^2\alpha$$

Fig. 8 Profile Shift Equations

4 Fundamental Law of Gearing

While the fundamental of gearing hasn’t been enunciated, most researchers have settled on a principle pertaining to gearing and gear-tooth action. The fundamental law of gear-tooth action may be stated that the pitch point must equally divide the imaginary line that connects the normals from the center of a meshed gear pair. In simple terms, “the common normal at the point of contact between a pair of teeth must always pass through the pitch point”. [13] So the shortest distance, q, and angle, ø, made by the common normal and pitch line must be constant for all stages of meshing as shown in Fig 9.1. Additionally, some state that the fundamental law of gearing is as follows: the ratios of angular velocities of each meshed gear must be equal to the ratios of distance from their respective centers to the pitch point for each respective gear as shown in Fig 9.2 and 9.3.





$$\frac{\omega_1}{\omega_2} = \frac{|O_1P|}{|O_2P|}$$

Fig. 9.1 First Interpretation, Illustration

Fig. 9.2 Second Interpretation, Illustration

Fig. 9.2 Second Interpretation, Equation

The fundamental law of gearing is also used as the condition for correct meshing. It declares that any two meshed gears that do not satisfy the fundamental law/s will not get a correct meshing. An important fact to note is that the modules must be equal to one another. This is critical in getting a constant ratio of angular velocities, also called conjugate action. [14]

5 Types of Gears

5.1 Helical Gear

Helical gears are gears with parallel shafts, similar to that spur gears, but their angled teeth allow continuous contact from one tooth to the next. [15] This ensures they are able to carry more load and work more efficiently, quietly, and with lower vibration levels which makes them suitable for high-speed, high-load applications such as automotive transmissions. One disadvantage is that they are more difficult to manufacture than spur gears and transmit less torque than that spur gears of similar dimensions.

5.2 Bevel Gear

Bevel gears have intersecting axes of rotation and a conical shape unlike the gears mentioned before. This makes them suitable to transmit torque at an angle. Though this is usually at a right angle, any angle that doesn’t interfere with the construction of the gear will work. They mostly have around a pressure angle of 20° and produce a thrust load on both shafts. [16] Multiple teeth are in contact simultaneously, therefore sharing the load, and allowing for a smoother transmission of torque. Similar to helical gears this means they can carry more gear than a spur gear of similar dimensions. [17]

5.3 Rack and Pinion

This type is comprised of a linear gear, called the rack, and a circular gear, called the pinion, and is used to convert rotational motion into translational motion. [18] The rack is essentially another gear, just with an infinite radius, and meshes with the tooth profile of the pinion. Since it converts rotary motion into linear, it is usually compared to ball screws. A ball screw is a long screw-like shaft that is threaded through a hollow cylinder. However, in mechanical sculptures especially, rack and pinions are more suitable because of their mechanical simplicity, ease of manufacturing, and unlimited length of the rack. These are commonly seen in the steering mechanisms of cars and other wheeled vehicles. [19]

5.4 Worm Gear

A worm gear is a cylindrical shaft with a spiral thread, similar to a screw, that meshes with a circular gear. The circular gear has slightly curved and angled teeth, similar to a helical gear. [20] The worm gear, along with its meshed circular pair, is used to transmit torque at a right angle, the gears have perpendicular axes of rotation when projected onto a common plane. They are power components that are used to decrease speed and increase torque.

5.5 Cage and Peg Gear

A cage gear, also called lantern gear and lantern pinion, is a gear with cylindrical dowels or rods arranged in a circular pattern as teeth. [21] Because of their unusual shape, they don't have to follow the law of gearing which makes them incredibly forgiving. They aren't commonly used anymore but are included in the study due to their ease of manufacturing and the aforementioned forgiving aspect, which makes them appropriate for use in rapid prototyping when transmitting torque at an angle or angular reciprocation is necessary.

6 Flower Documentation and Processes

6.1 Flower Design

I sketched out possible designs with estimated measurements, as shown in the figure below, and eliminated them according to feasibility, accessibility to materials, and gear theory. According to these elimination criteria and future design requirements I determined, such as material accessibility, time, budget, etc., I picked the most appropriate CAD engine available to me. I needed to be able to simulate joint relations and solid body interactions so I picked Fusion 360 and started the design.

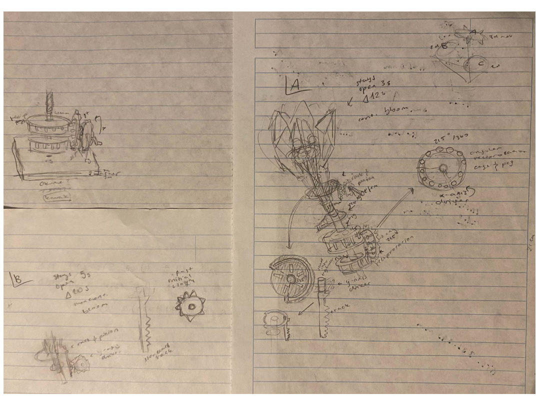


Fig. 10 Initial sketches

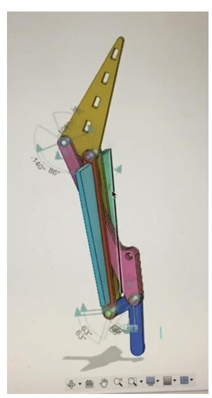


Fig. 11 CAD design

Fig. 12 Versions 1-3 of flower

After completing a high-fidelity model, I treated it as a pre-preliminary design, testing and optimizing the way I would a tangible prototype. After slight adjustments, I made the actual preliminary design and tested it. I settled on a final design for the leaf after one prelim and for the entire flower after three prelims, as shown in the figures above.

The first bloom mechanism was clunky and the joints pulling the top plate bumped into each other, so I switched out the linkage mechanism with wire. That version turned out to be overly mobile as the wires were too thin to fix the plate about its vertical axis, and eventually, I settled on clamping ⌀2mm hinge pieces with M2 screws. This let it be disassembled and reassembled as needed which allowed me to calibrate and make slight adjustments without damaging the petals. I decided to use cast acrylic for the petals and was able to quickly adjust the spacing on the leaf frame by using the thickness parameter that I had set previously.

6.2 Gear Design

When picking a CAD engine I actually picked OnShape, for spur gear simulations, and Fusion 360, for peg gear simulation, as that depends on solid body interactions whereas OnShape works better with visual simulations. I also decided to use acrylic to decrease friction between meshed gears. There are four iterations required for the gears of the automaton: *actuator gear and pair*, *reciprocating cage-peg gears*, *cage-peg gear train*, and *blooming rack and pinion* as shown in the figures below.

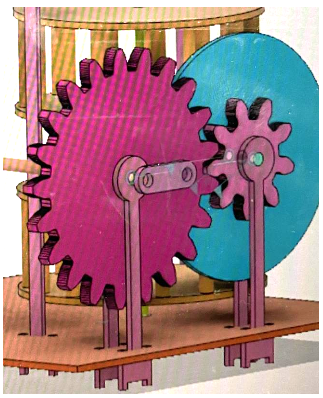


Fig. 13 Actuator and pair

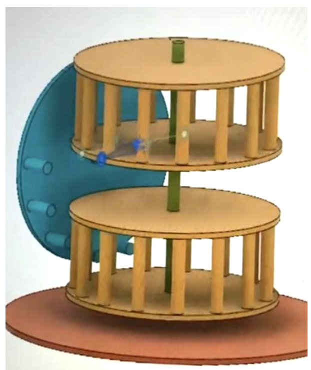


Fig. 14 Reciprocating pegs

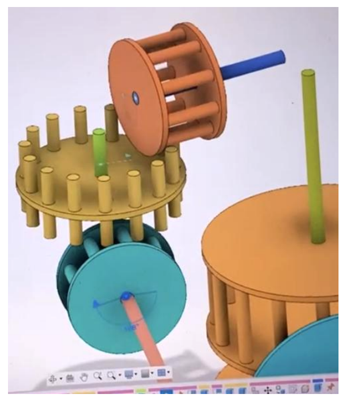


Fig. 15 Gear train

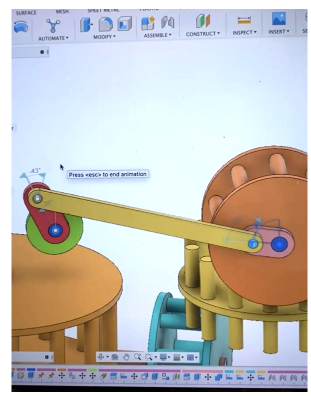


Fig. 16 Pinion linkage

6.2.1 Actuator Gear and Pair

The actuator gear and its meshed pair are a set of ordinary spur gears, with a 2 to 5 velocity ratio. The actuator spur is the pinion which is powered manually by rotating a crank, fixed about its axis of rotation. It provides rotation for the bloom, which is motion in the upwards direction y, as well as torque to rotate the reciprocating gears. The pinion is the actuator and its meshed pair is fixed to the peg driver. The dowel going through the axis of rotation of the pinion is fixed to the start of the gear train with a 1 to 1 velocity ratio and coincident rotational axes. It is spaced via the support beams on the frame.

6.2.2 Reciprocating Peg Gears

The meshed pair of the actuator gear is fixed to the peg with M2 screws. The dowels attached to 270° of the peg rotate the top cage in a clockwise direction and vice versa. The dowel fixed to the top plate of the flower is attached to the cages allowing it to rotate on the horizontal plane with 0° to 270° revolute joint limits. I only needed to make sure that the teeth could get in and out of the cage without fouling, but not much else. Another reason why cage and peg gears are used in this project is that they’re extremely forgiving in their dimensions.

6.2.3 Gear Train

The peg gear train utilizes two small cage gears, one with a parallel rotation axis to the actuator and the other driving the pinion of the rack, and one double-sided peg gear. With this train, I was able to change the direction of torque without having to manufacture bevel gears, which would be impractical as bevel gear teeth are angled and laser cutters trace 2-dimensional paths. The two cages are identical in their dimensions, however, their axes of rotation, even when projected on the same horizontal plane, are intersecting. Since both the peg and the cages can mesh at any given point on the pitch circle, I angled the upper cage to align with the front plane of the pinion in part 6.2.4.

6.2.4 Blooming Rack and Pinion

The blooming pinion is driven with a linkage mechanism where the crank is fixed to the top cage of the gear train and the follower to the pinion, allowing an unchanging rotation direction to the cage while limiting the revolute joint of the pinion to -43° and 34°, abs. 77°. The pinion transmits rotational motion into translational motion via the rack, essentially a gear with an infinite radius, to push the bottom plate upwards closing the petal and vice versa. The rack is affixed to washer-shaped plates at two points. Smaller two plates affixed to the outer casing of the blooming mechanism are suspended but not adhered to the aforementioned washer plates on either side, allowing the rack to push up the bloom case while rotating freely on the horizontal plane.

7 Error Analysis

The errors within this sculpture stem from friction between peg gears, unstable support, and fragile adhesion.

First, the dowels I was able to find to manufacture the cages and pegs of the meshed gears were wooden. Wood-on-wood relations create much more friction, kinetic and static alike, compared to wood-on-acrylic or acrylic-on-acrylic. While some friction would be insignificant since in this sculpture there is only one actuator, and friction increases with each meshed gear; the initial friction increases at an exponential rate. One way to minimize friction, or at least halt its increase, is to use materials whose relations have low friction coefficients. As such, if in the future it is possible to procure acrylic or plastic-derived rods to construct the peg and cage gears, it would decrease the current friction, in turn allowing the sculpture to operate smoothly and require less initial force.

Second, the supports –while able to support the weight of the sculpture– are not stable enough to fix them in place. The gears may push each other instead of rotate on occasion, due to friction as mentioned above, which displaces them out of their correct position. If the supports were more stable, the push wouldn’t cause a displacement and rotation would be achieved. The material is durable but, to a degree, malleable. The force displaces the gears and may bend the support. If I were to switch the material to a more rigid one, it might break. The solution should be to develop locking pieces on either side of the supports to fix them in place and provide more stability.

Third, the glue I used to adhere the spacers to the supports, the dowels to the gear frames, and the dowels to one another (to make them the appropriate length) was industrial-grade adhesive and its activator spray. While this type of glue is suitable for most rapid prototyping, the dowels and gear frames are different materials. It is exceptionally difficult to adhere different materials together without the appropriate adhesive, which in this case would be an epoxy type. The part of the wood that would be exposed to glue would have to be sanded with medium-grit sandpaper (100-150 grit should suffice) to provide a rough surface.

8 Conclusion & Discussion

In this study, a K-12 level gear theory curriculum is presented along with its proof of concept, methodology, and documentation. While the study and proof of concept, the final sculpture shown on the right, are sufficient in forming an ideal case with the curriculum, more data is required; especially from individual, group, classroom, and mentee student settings. The implications of constructivist and student-centric design engineering models must be explored in future work according to the data that is collected.

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