Mathematical Foundations of Data Science
Week 1 Workshop

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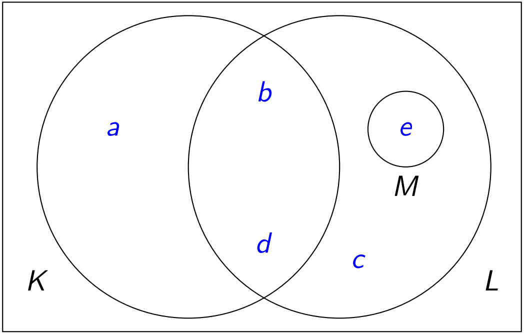
Set Notation

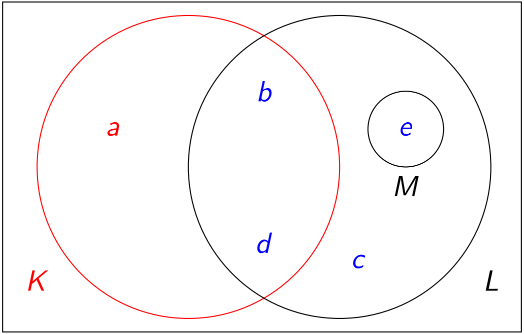
Venn Diagram



Elements: a, b, c, d, e Sets: K, L, M

Set Notation

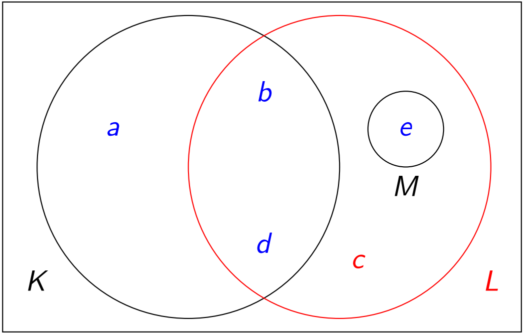
Venn Diagram



Which set is a in? $a \in K$

Set Notation

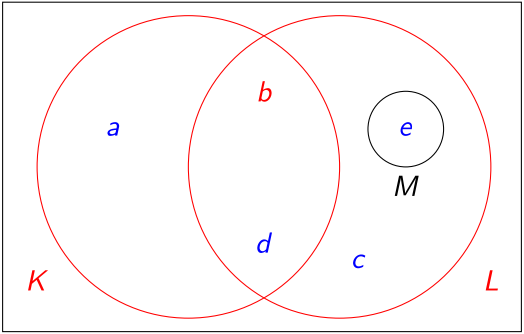
Venn Diagram



Which set is c in? $c \in L$

Set Notation

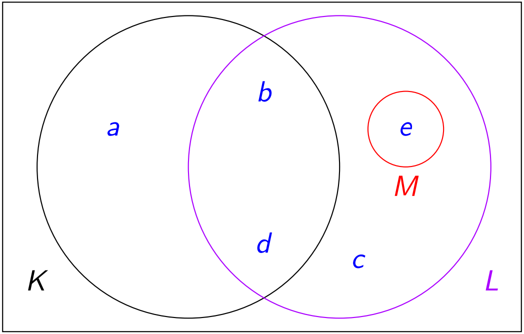
Venn Diagram



Which set is b in? $b \in K$, $b \in L$

Set Notation

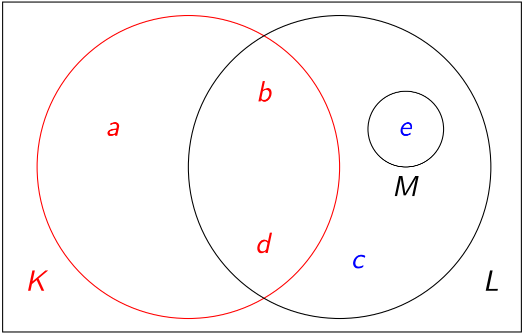
Venn Diagram



Describe the relationship between sets L and M? $M \subset L$

Set Notation

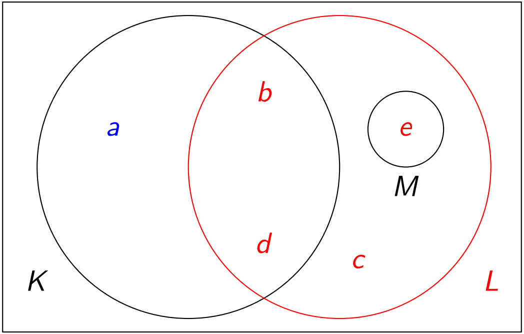
Venn Diagram



Which elements are in the set K? $\{a, b, d\}$

Set Notation

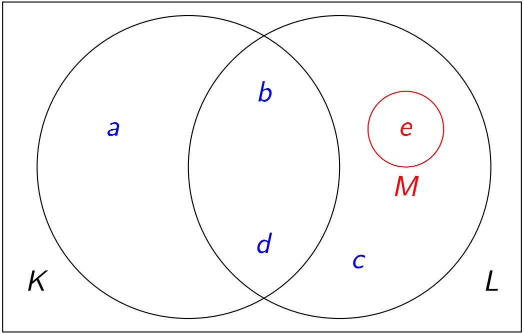
Venn Diagram



Which elements are in the set L? $\{b, c, d, e\}$

Set Notation

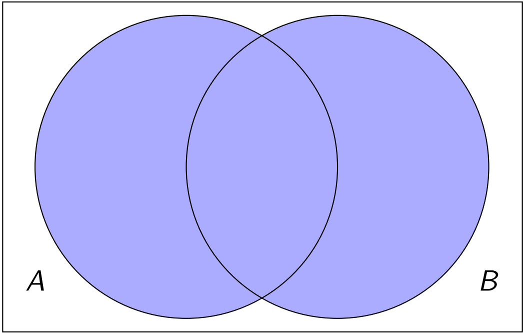
Venn Diagram



Which elements are in the set M? $\{e\}$

Set Notation

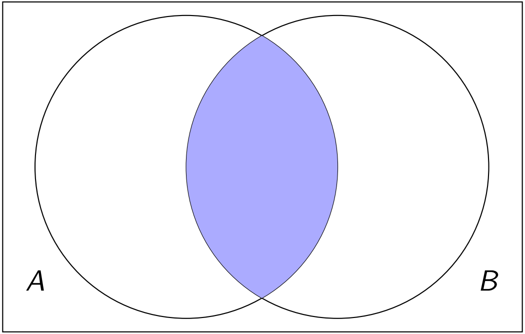
Unions ($\cup$)



What is $A \cup B$? Shaded in blue.

Set Notation

Intersections ($\cap$)



What is $A \cap B$? Shaded in blue.

Set Notation

Suppose $A = \{1, 2\}$ and $B = \{1\}$. What is $A \cup B$?

$A \cup B = \{1, 1, 2\} \times$

Only mention each element of a set once.

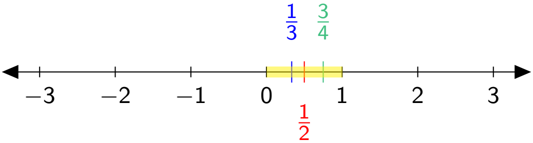
$A \cup B = \{1, 2\} \checkmark$

Sets of Numbers

$\mathbb{N}$	Natural numbers	$1, 2, 3, \ldots$
$\mathbb{Z}$	Integers	$1, 2, 3, \ldots;\ -1, -2, -3, \ldots;\ 0$
$\mathbb{Q}$	Rational numbers	all of $\mathbb{Z}$, and also $\frac{1}{2}, \frac{1}{3}, \frac{-2}{3}$, etc.
$\mathbb{R}$	Real numbers	all of $\mathbb{Q}$, and also π, e, $\sqrt{2}$, etc.

Sets of Numbers

The real number line



Sets of Numbers

Why do we need different sets of numbers?

Suppose $2x = 1$.

The solution to this depends on the type of number x can be.

What if we need $x \in \mathbb{Z}$?

If $2x = 1$, then $x = \frac{1}{2} \in \mathbb{R}, \mathbb{Q}$, but $\frac{1}{2} \notin \mathbb{Z}, \mathbb{N}$.

Thus, if $x \in \mathbb{Z}$, then $2x = 1$ has no solution.

Quantifiers

$\exists$: there exists $\qquad$ $\forall$: for all

We can use these to write statements, which we can then evaluate as true or false.

$\exists x \in \mathbb{Q}$ s.t. $2x = 1$ TRUE e.g., $x = \frac{1}{2}$

$\exists x \in \mathbb{Z}$ s.t. $3x = 1$ FALSE $\frac{1}{3} \notin \mathbb{Z}$

$\forall x \in \mathbb{Q}, \frac{x}{2} \in \mathbb{Q}$ TRUE if $x = \frac{p}{q}$, then $\frac{x}{2} = \frac{p}{2q}$

$\forall x \in \mathbb{Z}, \frac{x}{2} \in \mathbb{Z}$ FALSE e.g., if $x = 3$, then $\frac{x}{2} = \frac{3}{2} \notin \mathbb{Z}$

$\exists$: find one example $\qquad$ $\forall$: must hold in all cases

Quantifiers

Consider $\exists x \in \mathbb{Q}$ s.t. $2x \in \mathbb{Z}$.

Let $x = \frac{1}{3}$, so $2x = \frac{2}{3}$. Then $x \in \mathbb{Q}$ but $2x \notin \mathbb{Z}$.

Instead, try $x = \frac{3}{2}$, so $2x = 3$. Then $x \in \mathbb{Q}$ and $2x \in \mathbb{Z}$, so the statement is TRUE.

Consider $\forall x \in \mathbb{Q}, 2x \in \mathbb{Z}$.

Let $x = \frac{1}{2}$, so $2x = 1$. Then $x \in \mathbb{Q}$ and $2x \in \mathbb{Z}$.

Instead, try $x = \frac{5}{7}$, so $2x = \frac{10}{7}$. Then $x \in \mathbb{Q}$ but $2x \notin \mathbb{Z}$, so the statement is FALSE.

Fermi Estimation

How do you know if a long or complicated calculation is correct?

When you write computer code to solve a problem, how do you know if your answer is correct or reasonable?

What if your code is 100/1,000/10,000 lines long?

What if your code takes a very long time to run?

Estimation can be helpful for figuring out what sort of answers or values to expect.

Fermi Estimation

Estimate π^3.

π is close to 3, so π^3 should be close to $3^3 = 27$.

OR, $\pi > 3$, so $\pi^3 > 3^3$.

How much bigger? $\pi^3 \approx 28, 30, 31, 35$?

π^3 is actually very close to 31. $\pi^3 = 31.00628\ldots$

Fermi Estimation

Estimation is about the process, not about getting the exact answer.

There are no wrong answers, but there are wrong or bad processes.

e.g., saying "$\pi \approx 10$ so $\pi^3 \approx 1,000$" gives you a poor estimate because it uses a poor process.

Fermi Estimation

Estimate the number of people in your country born on the same date (same day and same year) as you.

Guess that the population of Australia is about 24 million.

Guess that there are about 400 days in a year (we know it is 365.25, but this is a difficult number to work with).

Thus, there are about ${}^{24 \text{ million}}/_{400} = 60,000$ people born on each day.

Fermi Estimation

How long do people live?

Guess that people live to be about 80 years old.

We could try $^{60,000}/_{80}$, but even this might be a bit difficult.

We know $^{60,000}/_{60} = 1,000$, and $^{60,000}/_{100} = 600$.

Since 80 is halfway between 60 and 100, guess that $^{60,000}/_{80}$ is halfway between 600 and $1,000$.

Thus, we estimate that there are about 800 people in Australia born on the same day as me.