Mathematical Foundations of Data Science
Assignment 2

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Chang Dong (a1807402)

1. A pizza restaurant makes 11 standard pizzas, 3 of which contain pepperoni. On 6 different days each week, they discount the price of one pizza. The chef is deciding which pizzas to discount in a given week. Suppose that they only care about which pizzas are discounted, not the specific day that each is discounted.

For each of the following questions, show working to justify your answer. Evaluate all answers (i.e., do not leave them in terms of combination notation or factorials).

How many choices are there if:

(a) there are no restrictions on which pizzas are discounted?

Answer:
This is a problem of combination allows repetition, so according to the equation,
$C_{11+6-1}^{6} = \frac{16!}{10!6!} = 8008$

(b) they want to discount 6 different pizzas?

Answer:
This is a problem of combination with no repetition, so according to the equation, $C_{11}^{6} = \frac{11!}{6!5!} = 462$

(c) they want to discount 6 different pizzas, but the restaurant wants to sell discounted pizzas containing pepperoni on 2 or 3 nights?

Answer:
The same as (b), combination with no repetition, but there are 2 cases, one is choose 3 from 8 pizzas(no pepperoni) and all the 3 pepperoni pizzas, the other is choose 4 from 8 pizzas(no pepperoni) and 2 from 3 pepperoni pizzas. $C_8^3 \times C_3^3 + C_8^4 \times C_3^2 = \frac{8!}{3!5!} \times \frac{3!}{3!0!} + \frac{8!}{4!4!} \times \frac{3!}{2!1!} = 266$

(d) the same pizza can be discounted on multiple nights, but the restaurant wants to sell discounted pizzas containing pepperoni on 2 or 3 nights?

Answer:
This can be converted to another question, two cases:

Case I:
Choose 2 from 3 pepperoni pizzas allows repetition, and choose 4 from 8 pizzas(no pepperoni) allows repetition.

$C^2_{2+3-1} \times C^4_{4+8-1} = \frac{4!}{2!2!} \times \frac{11!}{4!7!} = 1980$

Case II:
Choose 3 from 3 pepperoni pizzas allows repetition, and choose 3 from 8 pizzas(no pepperoni) allows repetition.
$C^3_{3+3-1} \times C^3_{3+8-1} = \frac{5!}{3!2!} \times \frac{10!}{3!7!} = 1200$

All in all, the total choices are Case I + Case II = 1980 + 1200 = 3180

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2. Consider the following infinite sums. In each case, state whether the sum is convergent or divergent, giving justification.

(a)$\sum_{n=1}^{\infty}(\frac{-1}{2})^n$

Answer: Convergent, I can calculated it
$= \lim_{n\to\infty} \frac{-\frac{1}{2}-(\frac{-1}{2})^n}{1+\frac{1}{2}} = -\frac{\frac{1}{2}}{\frac{3}{2}} = -\frac{1}{3}$
on the other hand, we knows this sum of the geometric sequence where $|ratio| < 1$ must converges.

(b)$\sum_{n=1}^{\infty} \sqrt[3]{n}$

Answer: Divergent, firstly, $\sqrt[3]{n+1} > \sqrt[3]{n} > 0 \ \ (\forall n > 0)$, so it is a positive non-decreasing sequence, then, $\lim_{n\to\infty} \sqrt[3]{n}$ doesn't exist because it tends to be $+\infty$, so it can not converge.

(c)$\sum_{n=1}^{\infty} b_n,\ where\ \sum_{n=1}^{N} b_n = 1 + \frac{1}{N}\ for\ all\ N \geq 1$

Answer: Convergent, $\sum_{n=1}^{\infty} b_n = \lim_{N\to\infty} \sum_{n=1}^{N} b_n = \lim_{N\to\infty} 1 + \frac{1}{N} = 1$, so it converges.

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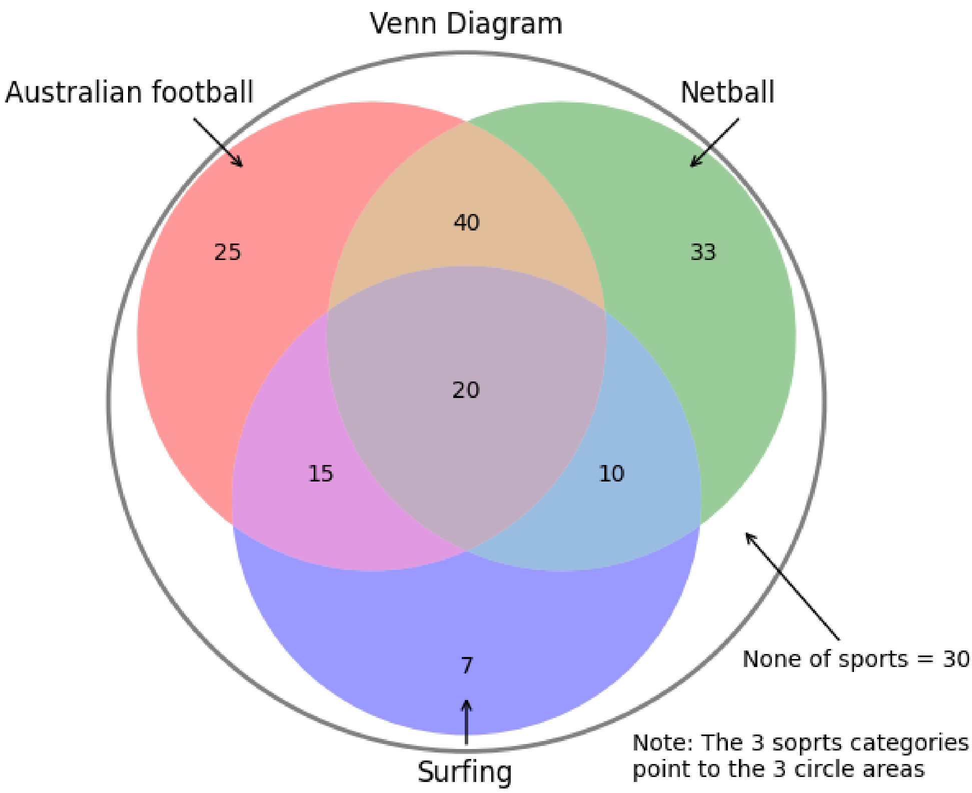
3. 180 students completed a survey about the sports they enjoy. The three sports considered were Australian football, netball, and surfing. The results of the survey are summarised below:

- 30 enjoy none of the sports;
- 25 enjoy Australian football only;
- 33 enjoy netball only;
- 7 enjoy surfing only;
- 40 enjoy Australian football and netball, but not surfing;
- 15 enjoy Australian football and surfing, but not netball;
- 10 enjoy netball and surfing, but not Australian football;
- 20 enjoy Australian football, netball, and surfing.

(a) Draw this information in a Venn diagram.



(b) Find the probability that a randomly selected survey respondent:
Please round your answers to 3 decimal places.

- i. enjoys at least one sport.

  Answer: This problem is equivalent to finding the complement set of "None of sport".
  $P(enjoys\ at\ least\ one\ sport) = 1 - \frac{30}{180} = \frac{5}{6} = 0.833$

- ii. enjoys Australian football.

  Answer: We need to sum all the count in "Fotball"
  $P(enjoys\ Australian\ football) = \frac{25+40+15+20}{180} = \frac{5}{9} = 0.556$

- iii. enjoys netball but does not enjoy surfing.

  Answer: We need to sum all the count in "Netball" not belong to "Surfing"
  $P(enjoys\ netball\ but\ does\ not\ enjoy\ surfing) = \frac{40+33}{180} = \frac{73}{180} = 0.406$

- iv. enjoys Australian football or netball.

  Answer: We need to sum all the count in "Football" and "Netball"
  $P(enjoys\ Australian\ football\ or\ netball) = 1 - \frac{7+30}{180} = \frac{143}{180} = 0.794$

MATHS7027 Mathematical Foundations of Data Science

Assignment 2 - Question 4

In Computer Exercise 2, we saw how to define and plot functions in Python. In this question, we will explore this in more detail.

4(a)

Define the following functions in Python:

(i) $f(x) = \dfrac{1}{1 + x^2}$

(ii) $g(x) = x^2 - x$

In [1]:
```
### Enter your answer to question 4.a here

def f(x):
    """
    Calculate the value of the function 1/(1+x**2) for the given input x.

    Args:
        x (float): The input value for the function.

    Returns:
        float: The output value of the function.
    """

    result = 1/(1+x**2)
    return result

def g(x):
    """
    Calculate the value of the function (x**2 - x) for the given input x.

    Args:
        x (float): The input value for the function.

    Returns:
        float: The output value of the function.
    """

    result = x**2 - x
    return result
```

4(b)

Produce plots for each of the following:

(i) $f \circ g$

(ii) $g \circ f$

In each case, you should choose settings (e.g., domain, number of samples) to produce a smooth plot which displays the key features of each function. You may need to experiment with different values to determine what looks good.

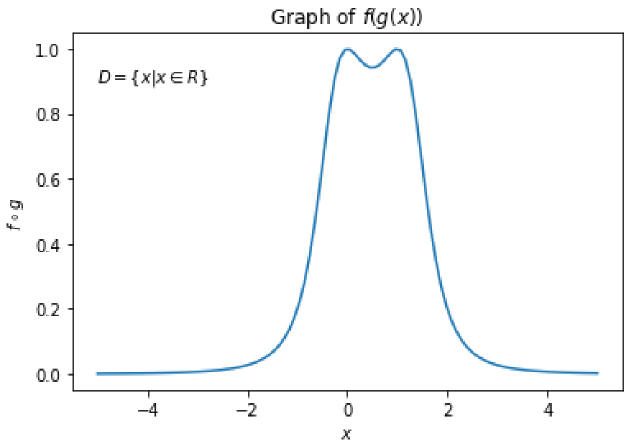
(i) *Answer:* $f \circ g = \dfrac{1}{1 + (x^2 - x)^2}$, so the denominator cannot $= 0$ ($\forall\ x$), so domain is R.

In [2]:
```python
### Enter your answer to question 4.b.i here

import matplotlib.pyplot as plt
import numpy as np

x = np.linspace(-5, 5, 100)
y = f(g(x))

plt.plot(x, y)
plt.xlabel("$x$")
plt.ylabel("$f \circ g$")
plt.title("Graph of $f(g(x))$")
plt.annotate("$D = \{x|x \in R\}$", xy = (-5,0.9))
plt.show()
```

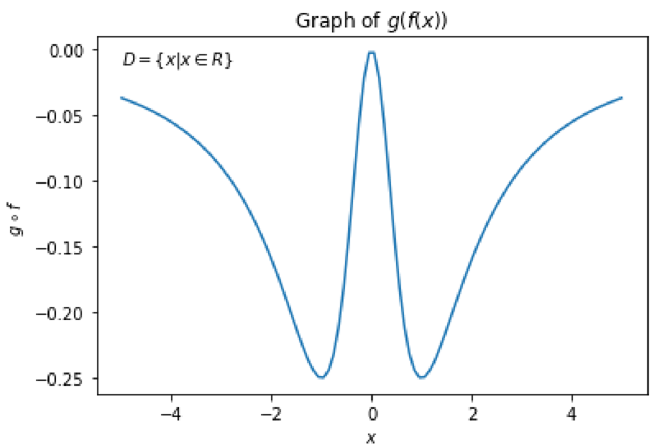


(ii) *Answer:* $g \circ f = \frac{1}{1+x^2}^2 - \frac{1}{1+x^2}$, the same, denominator cannot $= 0$ $(\forall\ x)$, so domain is $\boldsymbol{R}$.

In [3]:
```python
### Enter your answer to question 4.b.ii here

x = np.linspace(-5, 5, 100)
y = g(f(x))

plt.plot(x, y)
plt.xlabel("$x$")
plt.ylabel("$g \circ f$")
plt.title("Graph of $g(f(x))$")
plt.annotate("$D = \{x|x \in R\}$", xy = (-5,-0.01))
plt.show()
```



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