

Practice Questions (week 1)

1. Decide which of the following statements are true or false for arbitrary sets A, B and C .
- (i) $A \cap A = A$ (ii) $A \subseteq (A \cap B)$ (iii) $(A \cap B) \subseteq (A \cup B)$
 - (iv) $(A \cap B) \subseteq A$ (v) $A \cap B = A$ if and only if $A \subseteq B$
 - (vi) $A \cup B = B$ if and only if $B \subseteq A$ (vii) $(A \cup B) \cap C = A \cup (B \cap C)$
 - (viii) $A \cup A = A$

2. Let

$$U = \{r \in \mathbb{R} | r < 1\}$$

and

$$V = \{q \in \mathbb{Q} | q \geq 2\}.$$

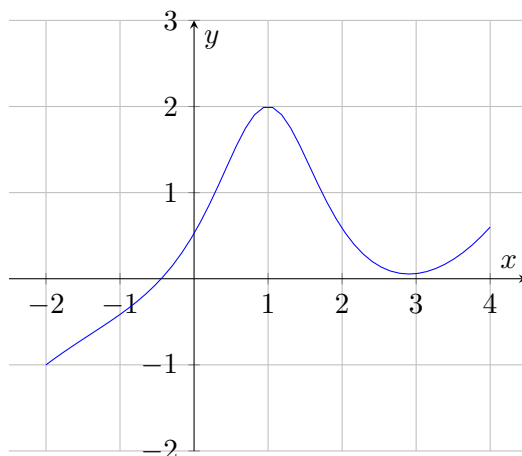
Decide whether each of the following statements is true or false, giving reasons.

- (a) $\forall x \in V, -\sqrt{x} \in U$.
 - (b) $\forall x \in U, x^2 \in V$.
 - (c) $\forall x \in U, \exists y \in V$ such that $100x < y$.
 - (d) $\exists x \in U$ such that $\forall y \in V, x = -y$.
3. Estimate how many Lego bricks it would take to fill a typical house.
4. Sketch a clearly labelled graph of $x(t)$, the distance travelled as a function of time t , for two trips of 70 km given by the following descriptions.
- (a) You start out at the 60 km/hr speed limit in an urban zone for a distance of 45km and then travel the remainder of the trip at the country speed limit of 100 km/hr.
 - (b) You start out at 60 km/hr but run into heavy traffic after 15 minutes. The traffic keeps your average speed to 30 km/hr for the next 10 minutes. Frustrated, you decide to stop for 5 minutes and work out a better route which then allows you to finish the journey at 100 km/hr.

Which trip takes longer, (a) or (b)?

5. Suppose f is a function with domain $(-1, 1)$ and range $(-1, 3]$. Find the domain and range of the following functions:
- (a) $A(x) = 3 - 4f(x)$ (b) $B(t) = f(3t + 2)$.
6. The graph of a function is given below:
- (a) state the value of $f(1)$;

- (b) estimate the value of $f(-1)$;
- (c) estimate for which values of x is $f(x) = 1$?
- (d) estimate the value of x such that $f(x) = 0$;
- (e) state the domain and range of f .



7. Find all numbers $x \in \mathbb{R}$ satisfying:

(a) $|x + 3| = |5 - 2x|$ (b) $|x + 2| = |x^2 - 5x + 1|$.

8. Consider the function

$$f(x) = \begin{cases} |2x - x^2| & \text{if } -1 \leq x < 2 \\ (x - 3)H(x - 3) & \text{if } x \geq 2 \end{cases}$$

on the domain $\mathcal{D} = [-1, 4]$.

Recall that $H(x)$ is the Heaviside function,

$$H(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0. \end{cases}$$

The domain can be broken into four sub-intervals such that on each sub-interval, $f(x)$ can be expressed as a polynomial. Find the sub-intervals and the corresponding polynomial functions. Hence sketch the graph of $f(x)$ and find the range, \mathcal{R} .

9. Find all solutions in $[0, \pi]$ of the equation

$$4 \cos^2 x + 3 \sin^2 x - 2 \sin x = 2,$$

giving your answers in exact form in terms of the arcsin function.

Hint: Use one of the properties of the sin and cos functions to convert the equation into a quadratic in $\sin x$.