Mathematical Foundations of Data Science
Assignment 4

Trimester 1, 2023

1. A sports team plays in a competition that awards 3 points for a win, 1 point for a draw, and 0 points for a loss. For every draw, players are allocated 3 hours of extra training. For every loss, players are allocated 5 hours of extra training. At the end of the season, the team has 73 points, and has won four times as many games as they have lost. Its players were allocated 64 hours of extra training. Formulate a system of equations that would be used to calculate the number of wins, draws, and losses the team had during the season. Express your answer in matrix form $A\boldsymbol{x} = \boldsymbol{b}$, clearly defining A, $\boldsymbol{x}$, and $\boldsymbol{b}$. Note, this question involves formulation only. You do not need to solve the system of equations.

2. Consider the following system of linear equations, in the variables x, y, and z, with some unknown parameter δ:

$$\begin{aligned} x + 2y &= 6 \\ x + \delta y &= 5 \\ y + \delta z &= 4 \end{aligned}$$

   (a) Write this system in matrix form $A\boldsymbol{x} = \boldsymbol{b}$, clearly defining A, $\boldsymbol{x}$, and $\boldsymbol{b}$.

   (b) Using matrix determinants, find the values of δ for which this system of equations has no solution.

   (c) If there is a solution with $z = 0$, what are the possible values of δ?

   (d) Can the system of equations have a solution if $y = 0$? Justify your answer.

3. Consider the following augmented matrix $[A|\boldsymbol{b}]$:

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 2 \\ 1 & 1 & -1 & 2 & -2 \\ 0 & 0 & 1 & 1 & 5 \end{array}\right]$$

   Using elementary row operations, reduce this matrix to reduced row echelon form and determine the set of solutions corresponding to the system of linear equations $A\boldsymbol{x} = \boldsymbol{b}$, where $\boldsymbol{x} = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}$. You must show all working, including stating the row operations that you use.

4. This question is contained in the Jupyter notebook `MFDS_A4_Q4.ipynb`. The notebook asks you to perform some calculations in Python to explore properties of matrix powers, transposes, and inverses. You should follow the instructions in the notebook, filling in code and answers where required.