Mathematical Foundations of Data Science
Assignment 5

Trimester 1, 2023

1. Consider the following augmented matrix:

$$\left[\begin{array}{ccc|c} x-2 & 0 & 0 & 3 \\ 0 & x^2-4 & 0 & x+2 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

   Determine the number of solutions to this system for all possible values of $x \in \mathbb{R}$, giving justification.

2. Let $K = \begin{bmatrix} 2 & a & 1 \\ 0 & 1 & 1/a \\ 1 & 1 & 0 \end{bmatrix}$ for some $a \neq 0$. Use row reduction to find K^{-1}, showing all details of your row operations.

3. Consider the vectors

$$\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \boldsymbol{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{v}_3 = \begin{bmatrix} -1 \\ 3 \\ k \end{bmatrix}$$

   (a) By solving $c_1\boldsymbol{v}_1 + c_2\boldsymbol{v}_2 + c_3\boldsymbol{v}_3 = \boldsymbol{0}$, determine the unique value k for which $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ is a linearly dependent set.

   (b) For the value of k above, write one of the vectors as a linear combination of the others.

4. Find the eigenvalues and eigenspaces of $A = \begin{bmatrix} 8 & 2 \\ -2 & 3 \end{bmatrix}$, showing all working.