Superfluidity and Superconductor

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In this note we review the global $U(1)$ symmetry-breaking in superfluidity and Anderson-Higgs mechanism (caused by gauge fixing) in superconductors (BCS theory).

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I. SUPERFLUIDITY — GLOBAL U(1) SYMMETRY-BREAKING

Superfluidity happens for *short-range* interactive bosons

$$H_0 = \int \mathrm{d}\boldsymbol{r}\, a_{\boldsymbol{r}}^\dagger \left(-\frac{\nabla^2}{2m} - \mu\right) a_{\boldsymbol{r}} + \frac{1}{2}\int \mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r'} a_{\boldsymbol{r}}^\dagger a_{\boldsymbol{r'}}^\dagger V(\boldsymbol{r}-\boldsymbol{r'}) a_{\boldsymbol{r'}} a_{\boldsymbol{r}}. \tag{1}$$

where for simplicity we suppose $V(\boldsymbol{r}-\boldsymbol{r'}) = V\delta(\boldsymbol{r}-\boldsymbol{r'})$. To see the necessity of interaction, we start from considering free bosonic theory, where Bose-Einstein condensation is inevitable for *dilute* bosonic gas at *low temperature*.

A. Bose-Einstein Condensation

whose partition function can be directly computed (note that we are working in complex eigenvalue of bosonic coherent states)

$$\mathcal{Z} = \int \mathcal{D}(\phi^*,\phi)\, e^{-\frac{1}{\beta}\sum_{\omega_n}\sum_{\boldsymbol{p}} \phi(-i\omega_n,-\boldsymbol{p})(-i\omega_n+\xi_{\boldsymbol{p}})\phi(i\omega_n,\boldsymbol{p})} = \frac{1}{\det(-i\hat{\omega}_n + \xi_k)} \equiv \prod_{\omega_n}\prod_k \frac{1}{-i\omega_n + \xi_p}.$$

By thermaldynamic relation, particle number can be expressed as

$$N(\mu) = T\frac{\partial}{\partial\mu}\ln\mathcal{Z} = T\sum_{n,k}\frac{1}{i\omega_n - \varepsilon_k + \mu} = \sum_k \frac{1}{e^{\beta(\varepsilon_k-\mu)}-1}, \tag{2}$$

where Matsubara frequency summation technique is utilized in the last equality (and the above expression is obvious if you take bosonic distribution as a priori knowledge).

What's interesting is that no matter our system is interactive or not, the action

$$S[\phi^*,\phi] \equiv \int \mathrm{d}\tau \int \mathrm{d}\boldsymbol{r} \left(\phi^*(\boldsymbol{r},\tau)(\partial_\tau - \frac{1}{2m}\nabla^2 - \mu)\phi(\boldsymbol{r},\tau) + \frac{V}{2}(\phi^*(\boldsymbol{r},\tau)\phi(\boldsymbol{r},\tau))^2\right) \tag{3}$$

written from from (1) always possesses a global $U(1)$ symmetry under $\phi \mapsto e^{i\varphi}\phi$. Thus by Noether theorem, we have a conserved current

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II. SUPERFLUIDITY — ANDERSON-HIGGS MECHANISM