Theory of Nernst Effect Near Quantum Critical Points in Condensed Matter

Cuprates and Relativistic Hydrodynamics

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Outline

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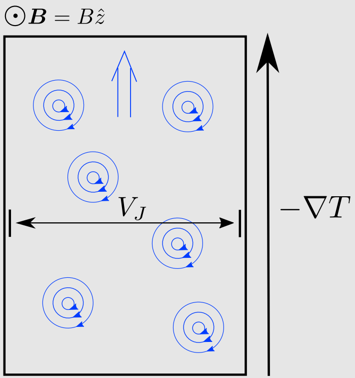
Vortex Nernst Effect

Nernst Effect

Nernst signal e_N is the detection of transverse electric field when a longitudinal thermal gradient $-\nabla T$ is applied

$$e_N := \frac{E_y}{-\partial_x T}.$$



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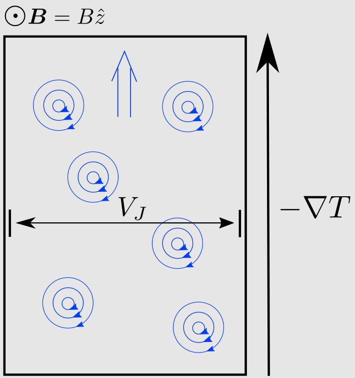


In a vortex-liquid state, longitudinal thermal gradient drives the motion of vortices. So Josphson equation

$$2eV_J = \hbar\partial_t\varphi = 2\pi\hbar\partial_t n_v$$

tells that **the measured transverse voltage (or Nernst signal) is proportional to the density of vortices**.

$La_{2-x}Sr_xCuO_4$

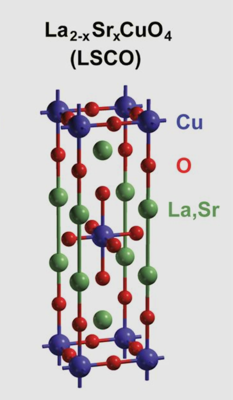


Figure: Extrated from Barišić *et al.* PNAS, **110**, 30 (2013).

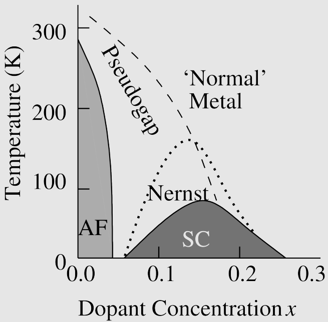


Figure: **Phase Diagram of Dopped Mott Insulator**. Extrated from Wen *et al.* RMP, **28**, 1 (2006).

Nernst Region of LSCO

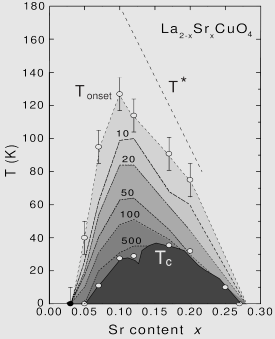
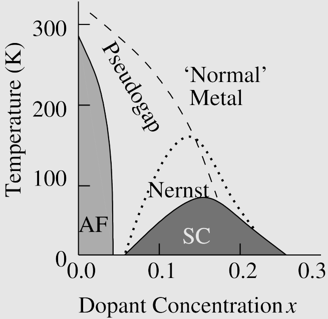


Figure: **Phase diagram of LSCO**. The Nernst coefficient on Contour $\nu \equiv e_N/B$. Extracted from Wang *et al.* PRB, **73**, 024510 (2006).



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Quantum Criticality

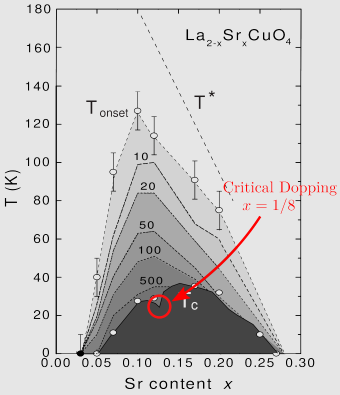

Figure: **QCP**: The dip in T_c near $x = 1/8$ indicates proximity of Insulating Phase.

Contents

Bose-Hubbard Model

The superconductor-insulator phase transition of vortices indicates

Bose-Hubbard Model

$$H = -t\sum_{\langle ij\rangle} b_i^\dagger b_j - \mu\sum_i n_i + \frac{U}{2}\sum_i n_i(n_i-1)$$

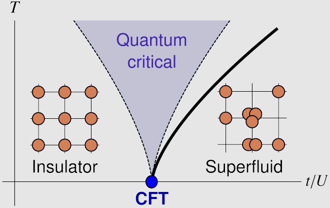

Figure: Extracted from Witczak-Krempa *et al.*, Nat. Phys., **10**, 5 (2014).

And the critical theory is given by (Fisher *et al.*, PRB, **40**, 1 (1989))

(2+1)-D Conformal Field Theory

$$S = \int \mathrm{d}^2r\,\mathrm{d}\tau\left[|\partial_\tau\psi|^2 + v^2|\nabla\psi|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4\right].$$

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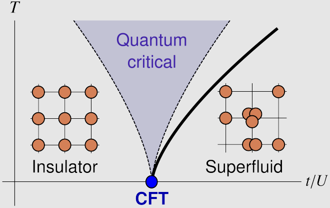

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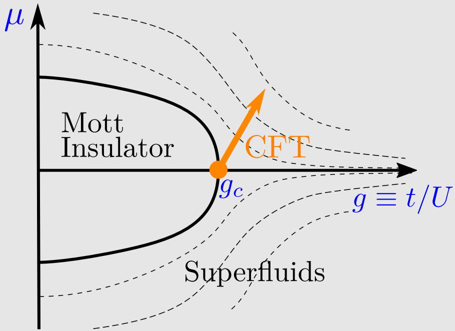
Linear Response Around Critical Point

To study the Nernt effect of LSCO, we are going to perturb such CFT with

- a (non-uniform) chemical potential $-\nabla\mu$
- a magnetic field B
- a (non-uniform) thermal fluctuation $-\nabla T$

and calculate the corresponding linear reponses.



Perturbative CFT

$$S = \int \mathrm{d}^2 r\, \mathrm{d}\tau \left[|\partial_\tau - \mu\psi|^2 + v^2 |(\nabla - i\boldsymbol{A})\psi|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right].$$

Contents

Conservation Law

The Lorentz-invariant microscopic action gives two macroscopic **Ward identities** (Herzog, J. Phys. A, **42**, 34 (2009)) (as **conservation laws**)

Conservation Laws

$$\nabla_\mu \langle T^{\mu\nu} \rangle = F^{\nu\lambda} \langle J_\lambda \rangle, \quad \partial_\mu \langle J^\mu \rangle = 0,$$

where

- (macroscopic) current operator $\langle J^\mu \rangle \equiv -\dfrac{\delta}{\delta A_\mu} W[g, A]$,
- (macroscopic) stress-energy tensor $T^{\mu\nu} \equiv = \dfrac{-2}{\sqrt{-g}} \dfrac{\delta}{\delta g_{\mu\nu}} W[g, A]$,
- $W[g, A]$ is the generating functional (with explicit dependence on metric tensor and vector potential).

What is Hydrodynamics in Condensed Matter Physics?

Landau and Lifshitz (1959) first proposed the modern version of hydrodynanics. The basic assumptions of hydrodynamics are

Assumption

- The long-wave length physics is dominated by very limited number of variables (called hydro-variables), which are nothing but the **conserved ones**. $\Longrightarrow$ Up to the physical length scale, **all physical observables are determined by those hydro-variables**.
- The concrete from of those functionals are contrained merely by the **local version** of the second law of thermodynamics—non-negativity of entropy production.

In our theory, the set of hydro-variables are $\{u^\mu, T, \mu\}$, where velocity vector u^μ is normalized up to the speed of light $u_\mu u^\mu \equiv -1$.

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Constitutive Relation — Zeroth Order

By Eckart, Phys. Rev., **58**, 10 (1940), given a vector u^μ, one can always decompose the two tensors with the projection operator $\mathcal{P}^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$ that

$$J^\mu = \mathcal{N} u^\mu + j^\mu,$$
$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P} P^{\mu\nu} + (q^\mu u^\nu + q^\nu u^\mu) + t^{\mu\nu},$$

where scalar $\mathcal{N}$, $\mathcal{E}$, and $\mathcal{P}$, vector j^μ and q^ν, and tensor $t^{\mu\nu}$ are formally contractions with whether velocity vector or projection operator. Choosing the proper frame of reference, one has

Zeroth Order

$$\mathcal{N} = n(T,\mu), \quad \mathcal{E} = \varepsilon(T,\mu) + P(T,\mu), \quad \mathcal{N} = P(T,\mu).$$

so that **the entropy current is conserved** $\partial_\mu s^\mu \equiv \partial_\mu (s u^\mu) = 0$, or **in equilibrium**.

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Constitutive Relation — First Order

Choosing the **Landau frame** (1959) for the proper definition of **out-of-equilibrium** thermodynamic variables, and using the property of **time-reversal symmetry** and **parity-inversion symmetry**, we can write down the most general combinations of those functionals

First Order

$$
\begin{aligned}
j^\mu &= -\sigma_Q T P^{\mu\nu} \partial_\nu \left(\frac{\mu}{T} \right) - \chi_T P^{\mu\nu} \partial_\nu T, \\
\mathcal{P} &= P - \zeta \partial_\lambda u^\lambda, \\
q^\mu &= 0, \\
t^{\mu\nu} &= -\eta P^{\mu\alpha} P^{\nu\beta} \left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{2} g_{\alpha\beta} \partial_\lambda u^\lambda \right).
\end{aligned}
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The non-trivial constraint of entropy production requires

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\chi_T \equiv 0, \quad \zeta > 0, \quad \sigma_Q > 0, \quad \eta > 0.
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Hydrodynamic EOM

Following Kandanoff&Martin, Ann. Phys., **24**, 419 (1969), the linear response over equilibrium state can be obtained by introducing a purtabative Hamiltonian

$$\mathcal{H} \to \mathcal{H} - \int \mathrm{d}^2 x \left(\frac{\delta T}{T} (\varepsilon - \mu\rho) + \delta\mu\rho + \delta u^\mu T_{\mu 0} \right),$$

where we explicitly split out the perturbative source of energy density, charge density, and momentum densities.

Linearized Hydrodynamic equations of motion are read out from conservation laws

Conservation Laws

$$\partial_t \rho + \nabla \cdot \left\{ \rho \boldsymbol{v} + \sigma_Q \left[-\nabla\mu + \frac{\mu}{T} \nabla T + \boldsymbol{v} \times \boldsymbol{B} \right] \right\} = 0$$

$$\partial_t \varepsilon + \nabla \cdot \Big((\varepsilon + P)\boldsymbol{v} \Big) = 0$$

$$\partial_t \Big((\varepsilon + P)\boldsymbol{v} \Big) + \nabla p - \zeta \nabla(\nabla \cdot \boldsymbol{v}) - \eta \nabla^2 \boldsymbol{v} - \delta \boldsymbol{J} \times B = 0.$$

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Thermoelectric Response

The full thermoelectric response

$$\begin{pmatrix} \boldsymbol{J} \\ \boldsymbol{Q} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\sigma} & \boldsymbol{\alpha} \\ T\boldsymbol{\alpha} & \bar{\boldsymbol{\kappa}} \end{pmatrix} \begin{pmatrix} -\nabla\mu \\ -\nabla T \end{pmatrix}$$

can be obtained after a lengthy calculation, with the poles

Cyclotron Resonance

$$\omega = \pm\omega_c + i\gamma, \quad \omega_c \equiv \frac{v^2}{c^2}\frac{2B}{(\varepsilon+P)/\rho c}, \quad \gamma \equiv \sigma_Q \frac{v^2}{c^2}\frac{B^2}{\varepsilon+P}.$$

We are also interested in

- heat current induced by thermal gradient but in the absence of charge current $\boldsymbol{Q} = -\boldsymbol{\kappa}\nabla T$. So $\boldsymbol{\kappa} \equiv \bar{\boldsymbol{\kappa}} - T\boldsymbol{\alpha}\boldsymbol{\sigma}^{-1}\boldsymbol{\alpha}$,
- electric field induced by thermal gradient but in the absense of charge current $\boldsymbol{E} = -\boldsymbol{\theta}\nabla T$. So $\boldsymbol{\theta} \equiv -\boldsymbol{\sigma}^{-1}\boldsymbol{\alpha}$.

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Thermoelectric Response

On the aspect of "particle",

$$\sigma_{xx} = \sigma_Q \frac{\omega + i\gamma + i\omega_c^2/\gamma}{(\omega + i\gamma)^2 - \omega_c^2}, \qquad \sigma_{xy} = -\frac{\rho}{B}\frac{\gamma^2 + \omega_c^2 - 2i\gamma\omega}{(\omega + i\gamma)^2 - \omega_c^2},$$

$$\alpha_{xx} = \frac{\rho}{T}\frac{i\omega}{(\omega + i\gamma)^2 - \omega_c^2}, \qquad \alpha_{xy} = -\frac{\gamma}{B}\frac{\gamma^2 + \omega_c^2 - i\gamma\omega}{(\omega + i\gamma)^2 - \omega_c},$$

$$\bar{\kappa}_{xx} = s\frac{i\omega - \gamma}{(\omega + i\gamma)^2 - \omega_c^2}, \qquad \bar{\kappa}_{xy} = -s\frac{\omega_c}{(\omega + i\gamma) - \omega_c^2},$$

while on the aspect of "vortex",

$$\rho_{xx} = \frac{1}{\sigma_Q}\frac{\omega(\omega + i\omega_c^2/\gamma + i\gamma)}{(\omega + i\omega_c^2/\gamma) - \omega_c^2}, \qquad \rho_{xy} = \frac{B}{\rho}\frac{(\omega_c^2/\gamma)^2 + \omega_c^2 - 2i(\omega_c^2/\gamma)\gamma\omega}{(\omega + i\omega_c^2/\gamma)^2 - \omega_c^2},$$

$$\theta_{xx} = \frac{s}{\rho}\frac{(\omega_c^2/\gamma)^2 + \omega_c^2 - i(\omega_c^2/\gamma)\omega}{(\omega + i\omega_c^2/\gamma)^2 - \omega_c^2}, \qquad \theta_{xy} = -\frac{B}{T}\frac{i\omega}{(\omega + i\omega_c^2/\gamma) - \omega_c^2},$$

$$\kappa_{xx} = s\frac{i\omega - \omega_c^2/\gamma}{(\omega + i\omega_c^2/\gamma) - \omega_c^2}, \qquad \kappa_{xy} = s\frac{\omega_c}{(\omega + i\omega_c^2/\gamma) - \omega_c^2}.$$

Contents

Byproduct: Nontrivial Self Duality

Under the interchange of $\rho \leftrightarrow B$ and $\sigma_Q \leftrightarrow 1/\sigma_Q$, the cyclotron resonance remain unchanged and $\gamma \leftrightarrow \gamma_v \equiv \omega_c^2/\gamma$. And we observe an amazing self duality that

Particle-Vortex Duality

$$\sigma_{xx}\,,\sigma_{xy}\,,\alpha_{xx}\,,\alpha_{xy}\,,\bar{\kappa}_{xx}\,,\bar{\kappa}_{xy}$$

$$\Updownarrow$$

$$\rho_{xx}\,,-\rho_{xy}\,,-\theta_{xy}\,,-\theta_{xx}\,,\kappa_{xx}\,,-\kappa_{xy}\,,$$

which is believed to have a close relation with M-theory, see Herzog *et al.*, PRD, **75**, 8 (2009).

Nernst Signal

In the presence of momentum relaxation (characterized by τ), the measured Nernst signal corresponds to the transverse component

$$e_N \equiv \theta_{yx} = \frac{k_B}{2e}\frac{\varepsilon + P}{k_B T\rho}\left[\frac{\omega_c/\tau}{(\omega_c^2/\gamma + 1/\tau) + \omega_c^2}\right].$$

Using the previous study of temperature dependence of thermodynamics quantities near the critical point (Chubukov *et al.*, PRB, **49**, 17 (1994) and Sachdev (1994)) that

$$\varepsilon = k_B T\left(\frac{k_B T}{\hbar v}\right)^2 \Phi_\varepsilon, \quad P = k_B T\left(\frac{k_B T}{\hbar v}\right)^2 \Phi_P,$$

where Φ_ε and Φ_P are dimensionless numbers, we can plot e_N as a function of temperature T and magnetic field B and compare that with experiments.

Comparison

- Taking a typical scatter time $\tau \sim 10^{-12}\mathrm{s}$, we can estimate the velocity using simply Drude formula such that $\hbar v = 47\mathrm{meV}\cdot\mathrm{\AA}$. This value is in accordance with experimental ranges (see Balasky *et al.*, Science, **284**, 5417 (1999)).
- We can also calculate Φ_ε and Φ_P with ε-expansion (Damle *et al.*, PRB, **56**, 8714 (1997)). Up to single-loop level $\Phi_s + \Phi_P = 4\pi^2/45$.

Comparison

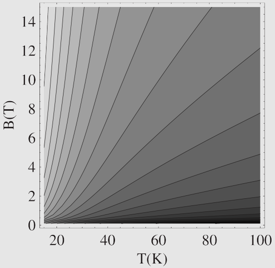

Figure: Calcualted $e_N(B,T)$. Extracted from Hartnoll *et al.*, PRB, **76**, 144502 (2007).

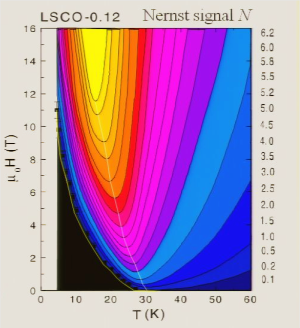

Figure: Measured $e_N(B,T)$. Extracted from Wang *et al.*, PRL, **88**, 25 (2002).

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- We find a realizaion of particle-vortex duality on transport coefficients.
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Thanks!