Anderson Higgs Mechanism and Meissner Effect

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Spontaneous Symmetry Breaking

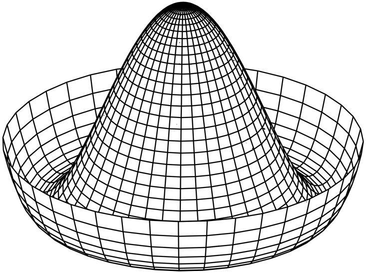

Figure: Mexican Hat potential

Spontaneous Symmetry Breaking

If under a symmetry transformation, the potential of a system is still invariant, but the ground state transforms to another ground state, this process of selecting a ground state from a degenerate set of ground states leads to a "Spontaneous symmetry breakdown".

Goldstone Theorem

Each generator of a spontaneous broken symmetry corresponds to a massless and spinless field, called the Goldstone boson.

Goldstone boson in superfluid

- Lagrangian of the superfluid:
  $\mathcal{L} =$
  $\phi^*(\vec{r},\tau)(\partial_\tau - \frac{\nabla^2}{2m} - \mu)\phi(\vec{r},\tau) + \frac{g}{2}\phi^*(\vec{r},\tau)\phi^*(\vec{r},\tau)\phi(\vec{r},\tau)\phi(\vec{r},\tau)$
  $V = -\mu|\phi|^2 + \frac{g}{2}|\phi|^4$
- $\left.\frac{\partial V}{\partial \phi}\right|_{\phi_0} = 0,$

$$\phi_0 = \begin{cases} 0 & \text{if } \mu < 0, \\ \sqrt{\frac{\mu}{g}}e^{i\theta} = \sqrt{\rho}e^{i\theta} & \text{if } \mu > 0. \end{cases}$$

- The Lagrangian is invariant under the $U(1)$ symmetry transformation $\phi \to \phi e^{i\theta}$. However, when $\mu > 0$, $U(1)$ symmetry is spontaneously broken.

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- Now we choose a ground state $\phi_0 = \sqrt{\rho_0}$ and add a fluctuation around ϕ_0:
  $\phi(\vec{r}, \tau) = \sqrt{\rho_0 + \delta\rho(\vec{r}, \tau)}e^{i\theta(\vec{r},\tau)}, \delta\rho \ll 1.$
- $\mathcal{L} \rightarrow \sqrt{\rho_0 + \delta\rho}e^{-i\theta}(\partial_\tau - \frac{\nabla^2}{2m} - \mu)\sqrt{\rho_0 + \delta\rho}e^{i\theta} + \frac{g}{2}(\rho_0 + \delta\rho)^2$.
  Substitute into the partition function,

$$\mathcal{Z} = \int \mathcal{D}(\delta\rho, \theta)e^{-\int d\tau \int d\vec{r}\mathcal{L}}$$

  Integrate out $\delta\rho$, we get the effective Lagrangian,
  $\mathcal{L}_{eff} = \frac{1}{2g}(\partial_\tau\theta)^2 + \frac{\rho_0}{2m}(\nabla\theta)^2$
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Minimal Coupling

- Write the superfluid lagrangian in a more symmetric form: $\mathcal{L}(\phi) = i\frac{1}{2}(\phi\partial_t\phi^* - \phi^*\partial_t\phi) + \frac{1}{2m}\partial_x\phi^*\partial_x\phi - \mu|\phi|^2 + \frac{g}{2}|\phi|^4$
- By minimal coupling between the bosons and the electromagnetic gauge field A_μ:

$$\mathcal{L}(\phi, A_\mu) = i\frac{1}{2}(\phi(\partial_0 - iA_0)\phi^* - \phi^*(\partial_0 + iA_0)\phi) - \frac{1}{2m}|(\partial_i + iA_i)\phi|^2 + V(\phi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

The lagrangian is therefore gauge invariant under

$$\phi \to e^{if(\vec{x},t)}\phi$$
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Anderson-Higgs Mechanism

- Select the ground state and fix the gauge so that ϕ is real, then $\mathcal{L} = A_0\phi^2 + \frac{1}{2m}(\partial_i\phi)^2 + \frac{\phi^2}{2m}(A_i)^2 + V(\phi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$
- Integrating out the fluctuation $\phi = \phi_0 + \delta\phi$, we get the low energy effective theory,

$$\mathcal{L}_{eff} = -\frac{1}{2g}A_0^2 + \frac{\rho}{2m}A_i^2 + \frac{1}{2}(\mathbf{B}^2 - \mathbf{E}^2)$$

  Since A_0 contain no time derivative terms, it's not dynamical and can be integrated out to give

$$\mathcal{L}_{eff} = -\frac{1}{2}\partial_0 A_j(\delta_{ij} + g\partial_j\partial_i)\partial_0 A_i + \frac{\rho}{2m}A_i^2 + \frac{1}{2}\mathbf{B}^2$$

- The electromagnetic vector potential $\vec{A}$ "eats" the Goldstone boson to acquire mass!

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Meissner Effect

In BCS superconductivity, Cooper pairs can be viewed as bosonic composite particles. So it has the similar Lagrangian as superfluid. Except it assumes that:

1. No electric field, i.e., $A_0 = 0, \partial_0 A_i = 0$.
2. $\nabla \cdot \vec{A} = 0 \Rightarrow \mathbf{q} \cdot \mathbf{A}_q = 0$ in momentum space.

$$\mathcal{S}[\mathbf{A}]_{eff} = \frac{\beta}{2} \sum_q (\frac{\rho}{m} + q^2) \mathbf{A}_q \cdot \mathbf{A}_{-q}$$

Vary the action with respect to $\mathbf{A}$, we get $(\frac{\rho}{m} + q^2)\mathbf{A}_q = 0$, or

London equation

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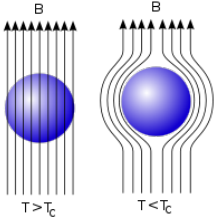
For $\rho \neq 0$, $\mathbf{B}(x) \propto \mathbf{B}_0 exp(-x/\lambda)$, where

$$\lambda = \sqrt{\frac{m}{\rho}}$$

is the **penetration depth**.



Reference

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