Notes on Bosonization[*]

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I. 1D ELECTRON GAS

A. Particularity of 1D Electron Gas

Hamiltonian for *compactified*[1] 1DEG is $H = H_0 + H_{\text{int}}$ where

$$H_0 = \sum_\sigma \int_0^L \mathrm{d}\boldsymbol{x}\, \psi_\sigma^\dagger(\boldsymbol{x}) \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - \mu\right) \psi_\sigma(\boldsymbol{x}) \tag{1}$$

and

$$H_{\text{int}} = \sum_{\sigma,\sigma'} \int \mathrm{d}\boldsymbol{x} \tag{2}$$

B. Linearization

We are interested in the low energy effective theory of 1DES, in which fermions around FS has the energy

$$\varepsilon(p) \sim (|p| - p_F)v_F.$$

In momentum space, this means that only the Fourier compponent of

$$\psi_\sigma(\boldsymbol{x}) \equiv \sum \frac{\mathrm{d}p}{2\pi}\psi_\sigma(p)e^{ipx/\hbar}$$

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[*] A footnote to the article title
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[1] To circle S^1.

near $\pm p_F$ contributes to the description of low-energy states. With the cut off Λ, one can write

$$\begin{aligned}\psi_\sigma(x) &\sim \sum_{-\Lambda}^{\Lambda} \frac{\mathrm{d}p}{2\pi} \psi_\sigma(-p_F + p) e^{ix(-p_F+p)/\hbar} + \sum_{-\Lambda}^{\Lambda} \frac{\mathrm{d}p}{2\pi} \psi_\sigma(p_F + p) e^{ix(p_F+p)/\hbar} \\ &=: e^{-ixp_F/\hbar} L_\sigma(x) + e^{ixp_F/\hbar} R_\sigma(x),\end{aligned} \tag{3}$$

where

$$L_\sigma(x) \equiv \sum_{-\Lambda}^{\Lambda} \frac{\mathrm{d}p}{2\pi} \psi_\sigma(p - p_F) e^{ipx/\hbar}, \quad R_\sigma(x) \equiv \sum_{-\Lambda}^{\Lambda} \frac{\mathrm{d}p}{2\pi} \psi_\sigma(p + p_F) e^{ipx/\hbar},$$

or in momentum space

$$L_\sigma(p) \equiv \psi_\sigma(p - p_F), \quad R_\sigma(p) \equiv \psi_\sigma(p + p_F).$$

Therefore the free Hamiltonian

$$H_0 = \sum_\sigma \sum \frac{\mathrm{d}p}{2\pi} \varepsilon(p) \psi_\sigma^\dagger(p) \psi_\sigma(p) \tag{4}$$

can be approximated to narrow integral around the region $p \pm p_F$, i.e.,

$$\begin{aligned}H_0 &= \sum_\sigma \sum_{-\Lambda}^{\Lambda} \frac{\mathrm{d}p}{2\pi} \Big[\varepsilon(p - p_F) \psi_\sigma^\dagger(p - p_F) \psi_\sigma(p - p_F) + \varepsilon(p + p_F) \psi_\sigma^\dagger(p + p_F) \psi_\sigma(p + p_F) \Big] \\ &= \sum_\sigma \sum_{-\Lambda}^{\Lambda} \frac{\mathrm{d}p}{2\pi} \, p v_F \Big(R^\dagger(p) R(p) - L^\dagger(p) L(p) \Big)\end{aligned} \tag{5}$$

II. BOSONIZATION

A. Kac-Moody Algebra

B. Equivalence of Fermionic and Bosonic Description: Partition Function

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Appendix A: Conformal Field Theory