# Applications of Rational Function Test Review

**1)** $\dfrac{a^2}{8b^3} \cdot \dfrac{3b^5}{8a^3} = \dfrac{3a^2b^5}{64a^3b^3}$

$\dfrac{a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot b}{b \cdot b \cdot b \cdot a \cdot a \cdot a} \Rightarrow \boxed{\dfrac{3b^2}{64a}}$

$\dfrac{b^3}{} = 0 \quad \boxed{b=0} \quad \text{rest.} \quad \dfrac{a^3}{}=0 \quad \boxed{a=0}$

**2)** $\dfrac{a^2}{a+3} \cdot \dfrac{a^2-2a-15}{a^2-1a}$ \qquad warmup 2/8/19

$\dfrac{a(a-5)}{(a-1)}$ \qquad res. $\boxed{\begin{array}{l} a \neq -3 \\ a \neq 0 \\ a \neq 1 \end{array}}$

**3)** $\dfrac{x^2+x-2}{x+3} \cdot \dfrac{x^2-7x-30}{2x^2-x-1}$

$\dfrac{(x+2)(x-1)}{x+3}$ \qquad $\dfrac{(x-10)(x+3)}{x^2-7x}$

$\dfrac{}{(2x+1)(x-1)}$  1) factor  $= \boxed{\dfrac{(x+2)(x-10)}{(2x+1)}}$

rest. $x+3=0 \quad 2x+1=0 \quad x-1=0$

$\boxed{x \neq -3} \quad \boxed{x \neq -\tfrac{1}{2}} \quad \boxed{x \neq 1}$

**4)** $\dfrac{w+2}{w+1} \div \dfrac{w-5}{w^2+3w+2}$ \qquad mult by reciprocal

$\dfrac{w+2}{(w+1)} \cdot \dfrac{(w+1)(w+2)}{(w-5)} = \dfrac{(w+2)(w+2)}{w-5}$

rest. $w+1=0 \quad w-5=0$

$\boxed{w \neq -1} \quad \boxed{w \neq 5}$

**5)** $\dfrac{x^2-9}{x^2+5x+6} \div \dfrac{x^2+2x-15}{x^2+x-20}$

$\dfrac{(x+3)(x-3)}{(x+3)(x+2)} \qquad \dfrac{(x+5)(x-3)}{(x+5)(x-4)}$

$\dfrac{(x+3)(x-3)}{(x+3)(x+2)} \cdot \dfrac{(x+5)(x-4)}{(x+5)(x-3)} = \dfrac{(x-4)}{(x+2)}$

res. $x+3=0 \quad x+2=0 \quad x+5=0 \quad x-3=0$

$\boxed{x \neq -3 \quad x \neq -2 \quad x \neq -5 \quad x \neq 3}$

6. $\dfrac{3}{x^2+x-12}$ and $\dfrac{x}{x^2+2x-15}$ ;

$\dfrac{3}{(x+4)(x-3)}$ ; $\dfrac{x}{(x+5)(x-3)}$

$$\boxed{LCD \ (x+4)(x-3)(x+5)}$$

7) $\dfrac{3x}{x^2-11x+30}$ and $\dfrac{-2}{x^2-10x+24}$ ;

$\dfrac{3x}{(x-6)(x-5)}$ ; $\dfrac{-2}{(x-6)(x-4)}$

$$\boxed{LCD = (x-5)(x-4)(x-6)}$$

8) $\dfrac{7 \cdot (c-4)}{(c+4)(c-4)} + \dfrac{1}{c^2-16}$   1) Factor

$\dfrac{}{(c+4)(c-4)}$   2) LCD   $(c+4)(c-4)$

$\dfrac{7c-28}{(c+4)(c-4)} + \dfrac{1}{(c+4)(c-4)} = \dfrac{7c-28+1}{(c+4)(c-4)}$

$$\boxed{= \dfrac{7c-27}{(c+4)(c-4)}}$$

9) $\dfrac{\overset{(w-8)(w-3)}{w^2-11w+24}}{\underset{(w-3)(w-4)}{w^2-7w+12}} - \dfrac{3 \cdot (w-3)}{(w-4)(w-3)}$   1) Factor

2) LCD

$(w-4)(w-3)$

$\dfrac{(w-8)(w-3)}{(w-3)(w-4)} - \dfrac{3w+9}{(w-3)(w-4)}$

$-3(w-3)$

$-3w+9$

$$\boxed{\dfrac{(w-8)(w-3)-(3w+9)}{(w-3)(w-4)}}$$

$\dfrac{w^2-11w+24-3w+9}{(w-3)(w-4)}$

$$\boxed{\dfrac{w^2-14w+33}{(w-3)(w-4)}}$$

10) $\dfrac{X \cdot 2}{9 \cdot 2} + \dfrac{3y \cdot 6}{3 \cdot 6} - \dfrac{2X \cdot 3}{6 \cdot 3}$  LCD  18

$\dfrac{2X}{18} + \dfrac{18y}{18} - \dfrac{6X}{18}$

| | 3 | 9 | 6 |
|---|---|---|---|
| 1 | 3 | 9 | 6 |
| 2 | 6 | 18 | 12 |
| 3 | 9 | 27 | 18 |
| 4 | 12 | | |
| 5 | 15 | | |
| 6 | 18 | | |

$\dfrac{2X - 6X + 18y}{18}$

$\boxed{\dfrac{-4X + 18y}{18}}$

11) $\dfrac{\frac{1}{X+2}}{\frac{2}{X} - 5}$ $\Rightarrow$ $\dfrac{1}{X+2} \div \dfrac{2}{X} - 5 \cdot \dfrac{X}{X}$

$LCD = 1X$

$\dfrac{1}{X+2} \div \dfrac{2}{X} - \dfrac{5X}{X}$  comb

↙ Like

$\dfrac{1}{X+2} \div \dfrac{-5X+2}{X}$  terms

mult recip  $\dfrac{1}{X+2} \cdot \dfrac{X}{-5X+2} = \boxed{\dfrac{X}{-5X^2 - 8X + 4}}$

$(X+2)(-5X+2)$
$-5X^2 + 2X - 10X + 4$
$-5X^2 - 8X + 4$

12) $\dfrac{\frac{d+5}{d^2 + 11d + 24}}{\frac{d+2}{d+3}}$ $\Rightarrow$ $\dfrac{d+5}{\underset{(X+3)(X+8)}{d^2 + 11d + 24}} \div \dfrac{d+2}{d+3}$

mult by reciprocal  $\dfrac{d+5}{(d+3)(d+8)} \cdot \dfrac{(d+3)}{(d+2)} = \boxed{\dfrac{d+5}{(d+8)(d+2)}}$

13) $\dfrac{X+2 \cdot (X+3)}{X+1 \cdot (X+3)} = \dfrac{X+5 \cdot (X+1)}{X+3 \cdot (X+1)}$  LCD $(X+1)(X+3)$

LCD $\cdot \dfrac{(X+2)(X+3)}{(X+1)(X+3)} = \dfrac{(X+5)(X+1)}{(X+3)(X+1)}$ , LCD

$(X+2)(X+3) = (X+5)(X+1)$

$X^2 + 3X + 2X + 6 = X^2 + X + 5X + 5$

$X^2 + 5X + 6 = X^2 + 6X + 5$
$-X^2 - 6X - 5 \quad -X^2 - 6X - 5$
$\overline{\qquad\qquad}$
$-X + 1 = 0$
$\quad -1 \quad -1$
$\dfrac{-X}{-1} = \dfrac{-1}{-1}$
$\boxed{X = 1}$  solution

$\dfrac{1+2}{1+1} = \dfrac{1+5}{1+3}$

$\dfrac{3}{2} = \dfrac{6}{4}$

$\boxed{\dfrac{3}{2} = \dfrac{3}{2}}$

14) $\dfrac{2}{x+2} \times \dfrac{-3}{x+3}$   LCD or Cross multiply

$$2x + 6 = -3x - 6$$
$$\underline{\phantom{2x}-6 \qquad\quad -6}$$
$$2x = -3x - 12$$
$$\underline{+3x \qquad +3x}$$
$$\dfrac{5x}{5} = \dfrac{-12}{5}$$
$$\boxed{x = \dfrac{-12}{5}}$$
solution

$$\dfrac{2}{\frac{-12}{5}+2} = \dfrac{-3}{\frac{-12}{5}+3}$$
$$\boxed{-5 = -5}$$

15) $\dfrac{x}{x^2-25} + \dfrac{4}{x-5} \cdot \dfrac{(x+5)}{(x+5)} = \dfrac{1}{x+5} \cdot \dfrac{(x-5)}{(x-5)}$   1) Factor
$(x+5)(x-5)$   2) LCD  $(x+5)(x-5)$

$$\dfrac{x}{(x+5)(x-5)} \cdot \text{LCD} + \dfrac{4x+20}{(x+5)(x-5)} \cdot \text{LCD} = \dfrac{(x-5)}{(x+5)(x-5)} \cdot \text{LCD}$$

$$x + 4x + 20 = x - 5 \qquad \text{Plug in}$$

$$5x + 20 = x - 5$$
$$\underline{-x \qquad\quad -x}$$
$$4x + 20 = -5$$
$$\underline{-20 \quad -20}$$
$$\dfrac{4x}{4} = \dfrac{-25}{4}$$
$$x = -\dfrac{25}{4} \quad \text{solution}$$

$$\dfrac{\frac{-25}{4}}{\left(\frac{-25}{4}\right)^2 - 25} + \dfrac{4}{\left(\frac{-25}{4}\right)-5} = \dfrac{1}{\left(\frac{-25}{4}\right)+5}$$

$$\dfrac{-6.25}{39.0625-25} + \dfrac{4}{-11.25} = \dfrac{1}{-1.25}$$

$$\dfrac{-6.25}{14.0625} + \dfrac{4}{-11.25} = \dfrac{1}{-1.25}$$

$$\boxed{-.8 = -.8}$$

16) $\dfrac{1}{x-3} \times \dfrac{3}{x^2+2x-9}$

$$x^2 + 2x - 9 = 3x - 9$$
$$\underline{\qquad\quad +9 \qquad\quad +9}$$
$$x^2 + 2x = 3x$$
$$\underline{-3x \quad -3x}$$

$$\dfrac{x^2}{x} - \dfrac{x}{x} = 0$$

GCF   $x(x-1) = 0$

$\boxed{x = 0}$   $x - 1 = 0$   plugin
Solution   $\boxed{x = 1}$   Solution

$$\dfrac{1}{(0)-3} = \dfrac{3}{0^2+2(0)-9}$$
$$-\dfrac{1}{3} = \dfrac{3}{-9}$$
$$-\dfrac{1}{3} = \dfrac{-1}{3}$$

$$\dfrac{1}{(1)-3} = \dfrac{3}{(1)^2+2(1)-9}$$
$$-\dfrac{1}{2} = \dfrac{3}{1+2-9}$$
$$-\dfrac{1}{2} = \dfrac{3}{-6}$$
$$-\dfrac{1}{2} = \dfrac{-1}{2}$$
Solution

**12)** $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$

$\frac{1}{65} = \frac{1}{r} + \frac{1}{90}$

$-\frac{1}{90} \qquad\qquad -\frac{1}{90}$

$\frac{1 \cdot 18}{65 \cdot 18} - \frac{1 \cdot 13}{90 \cdot 13} = \frac{1}{r}$    LCD 1170

$\frac{18}{1170} - \frac{13}{1170} = \frac{1}{r}$

$\frac{5}{1170} \times \frac{1}{r}$

$\frac{5r}{5} = \frac{1170}{5}$

$\boxed{r = 234}$ ohms

| LCD | 65 | 90 |
|-----|-----|------|
| 1 | 65 | 90 |
| 2 | 130 | 180 |
| ⑬ | 845 | ⑴⑴⑺⑩ |
| ⑱ | ⑴⑴⑺⑩ | 1620 |

**18)** $\frac{1}{t_1} + \frac{1}{t_2} = \frac{1}{t_{total}}$

$\frac{1 \cdot 7}{3 \cdot 7} + \frac{1 \cdot 3}{7 \cdot 3} = \frac{1}{t}$    LCD = 21

$\frac{7}{21} + \frac{3}{21} = \frac{1}{t}$

$\frac{10}{21} \times \frac{1}{t}$

$\frac{10t}{10} = \frac{21}{10}$

$t = \frac{21}{10} = \boxed{\begin{array}{l} 2.1 \text{ hours} \\ 126 \text{ min} \\ 2 \text{ hours } 6 \text{ min} \end{array}}$

19) $\dfrac{1}{t_1} + \dfrac{1}{t_2} = \dfrac{1}{t_{total}}$

$\dfrac{1}{5} + \dfrac{1}{t} = \dfrac{1}{3.5}$

$-\dfrac{1}{5} \qquad\qquad -\dfrac{1}{5}$  LCD = 35

$\dfrac{1}{t} = \dfrac{1 \cdot 10}{3.5 \cdot 10} - \dfrac{1}{5} \cdot \dfrac{7}{7}$

$\dfrac{1}{t} = \dfrac{10}{35} - \dfrac{7}{35}$

$\dfrac{1}{t} \times \dfrac{3}{35}$

$\dfrac{3t}{3} = \dfrac{35}{3}$

$\boxed{t = 11.67 \text{ hours or } 700\text{min}}$

20)  d = r t

| | D | R | t |
|---|---|---|---|
| ~~Rate~~ down | 7 | r + 5 | same |
| up | 3 | r = 3.75 | same |

$\dfrac{7}{r+5} \times \dfrac{3}{r}$

$\boxed{\begin{array}{c} 8.75 \\ \text{down} \end{array}}$

$7r = 3r + 15$
$-3r \quad\; -3r$

$\dfrac{4r}{4} = \dfrac{15}{4}$

$\boxed{r = 3.75}$ up

21)

| | distance | rate | time |
|---|---|---|---|
| You | 8 | r + 15 | same |
| Friend | 2 | r = 2.5 | same |

$\dfrac{8}{r+15} \times \dfrac{2}{r}$   $\dfrac{6r}{6} = \dfrac{15}{6}$   friend

$\boxed{r = 2.5}$   you

$8r = 2r + 15$   $\boxed{r = 17.5}$
$-2r \; -2r$

22) $L = 42/w$

23) $y = \dfrac{1}{x+6} + 2$

St. form of a rational function

24) $y = \dfrac{1}{x-5} + 0$

$$y = \dfrac{a}{x-h} + k$$

25) points of discontinuity
(holes) & (asymptotes)

$y = \dfrac{x-8}{x^2-x-30}$          1) factor
                                     2) cancel
$(x-6)(x+5)$                         3) rewrite

$x-6=0$      $x+5=0$
$\boxed{x \neq 6}$   $\boxed{x \neq -5}$

Vertical asymptotes

26) $\dfrac{(x-4)\cancel{(x+5)}}{\cancel{(x+5)}(x-5)}$     1) cancel (holes)
                                              2) leftovers (asymptotes)

$\boxed{\dfrac{(x-4)}{(x-5)}}$   $x+5=0$          $x-5=0$
                      $\boxed{x \neq -5}$     $\boxed{x \neq 5}$
                        hole               V.A

$(x+1)(x+1)$ hole

27) $\dfrac{x^2+2x+1}{x^2+x-6}$          1) Factor
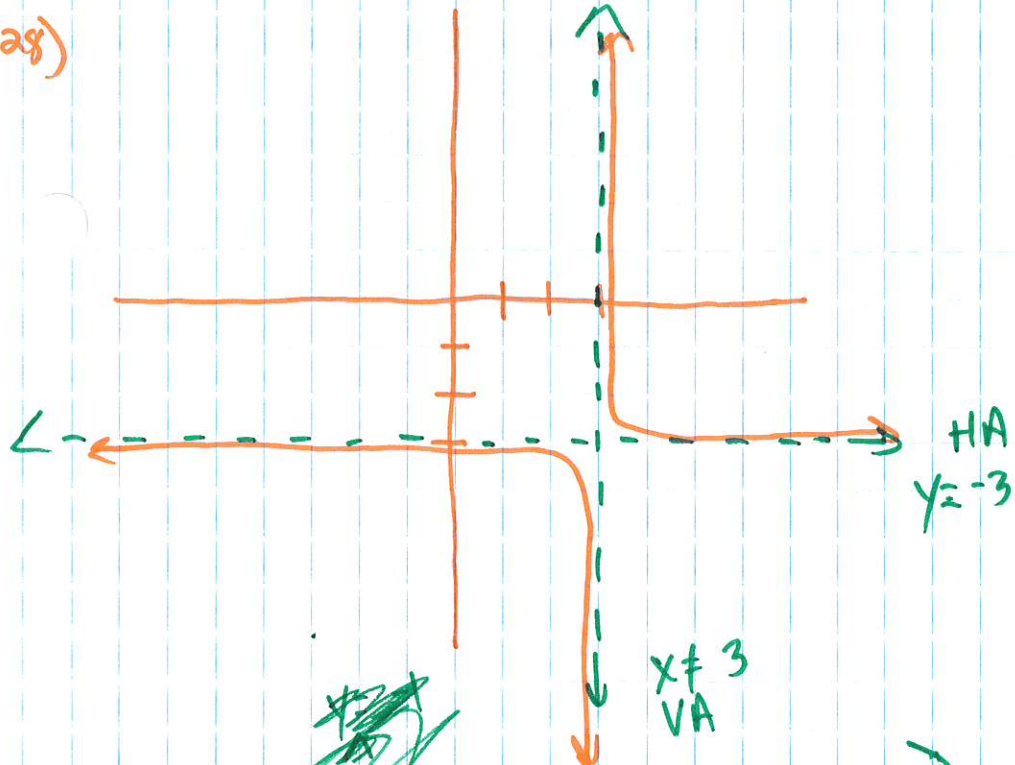                                        2) Cancel
$(x+3)(x-2)$                            3) rewrite

No holes - nothing cancelled

VA's      $x+3=0$          $x-2=0$
          $\boxed{x \neq -3}$     $\boxed{x \neq 2}$

28)



HA
$y = -3$

$x \neq 3$
VA

27)



29)



HA
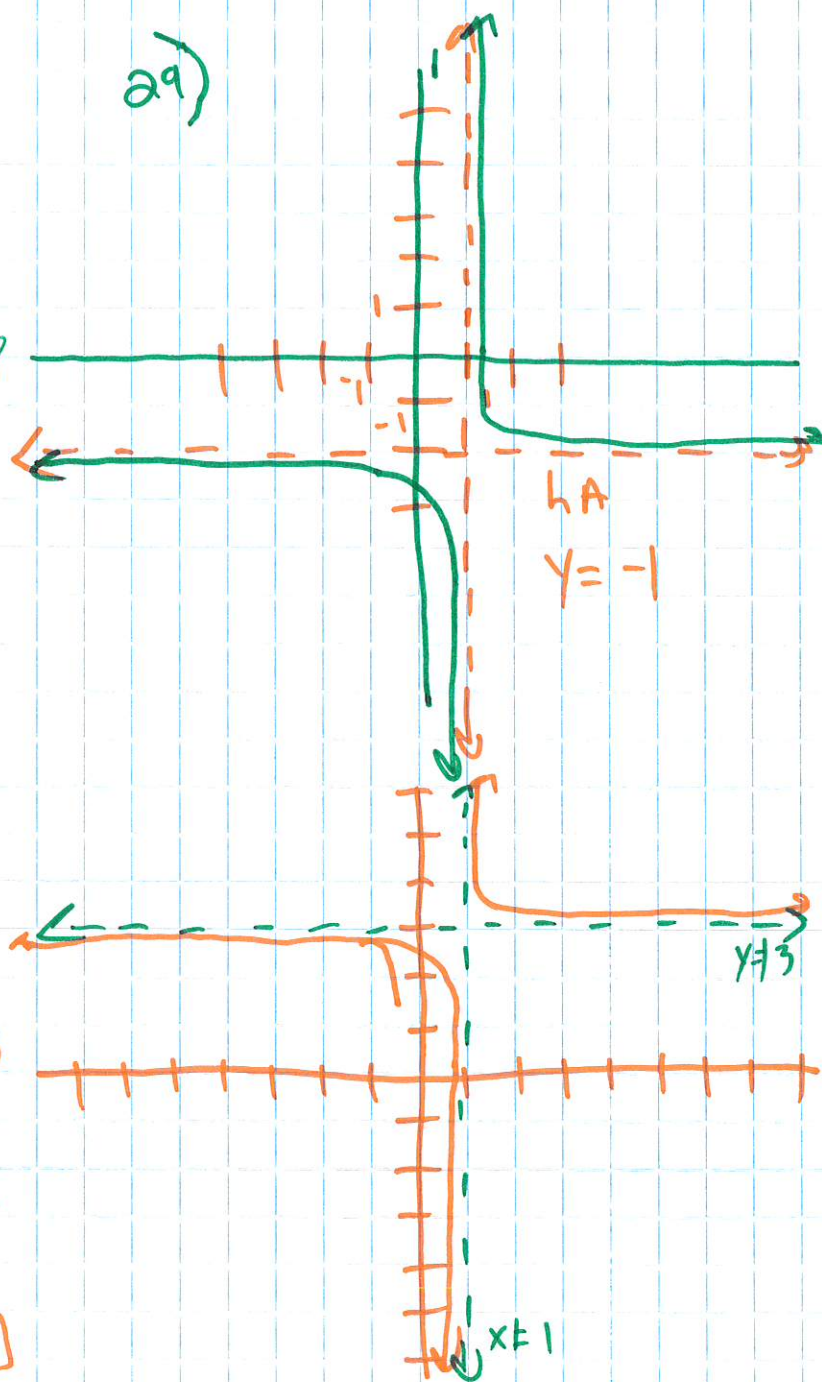$y = -1$

$y \neq 3$

$x \neq 1$

30) $g(x) = \dfrac{2}{x-1} + 3$    from   $\dfrac{1}{x}$

a) Vertical stretch 2
   shift up 3
   shift right 1

b) VA   $x - 1 = 0$
   $\boxed{x \neq 1}$

c) HA   $y \neq 3$

d) domain $R \neq 1$
   Range $R \neq 3$

e) y-int

$g(x) = \dfrac{2}{(0)-1} + 3$

$g(x) = \dfrac{2}{-1} + 3$

$g(x) = -2 + 3$
$\boxed{g(x) = 1}$

31) $x = (x+2)^4 (x-9)^3$

$x+2=0$  $\quad$  $x-9=0$

$\boxed{x = -2}$ $\quad$ $\boxed{x \neq 9}$

mult $\boxed{4}$ $\quad$ mult $\boxed{3}$

7 total

32) end behavior

$f(x)$ $x^5 - x^3 + x - 4$

Left, down $\qquad$ right, up

$f(x) \to \infty$, as $x \to \infty$

$f(x) \to -\infty$, as $x \to -\infty$

33) $x^4 - 25x^2 + 144$

$(x^2 - 16)(x^2 - 9)$

$(x+4)(x-4)(x+3)(x-3)$

$x+4=0$ $\quad$ $x-4=0$ $\quad$ $x+3=0$ $\quad$ $x-3=0$

$-4 \; -4$ $\quad$ $\boxed{x=4}$ $\quad$ $\boxed{x=-3}$ $\quad$ $\boxed{x=4}$

$\boxed{x=-4}$

34) $x^4 - 2x^3 - x^2 - 4x - 6 \div (x+2)$

$-2$ | $\quad$ $-2$ $\quad$ $-1$ $\quad$ $-4$ $\quad$ $-6$

$\quad\quad$ $+-2$ $\quad$ $+8$ $\quad$ $+^-14$ $\quad$ $+36$

$\quad\quad$ $-4$ $\quad$ $7$ $\quad$ $-18$ $\quad$ $\boxed{30 \text{ rem}}$

$x^3$ $\quad$ $x^2$ $\quad$ $x$ $\quad$ $c$

$x^3 - 4x^2 + 7x - 18 \quad$ rem 30

since there is a remainder
it is not a factor

35) $x^3 + 5x^2 + 11x + 10 \div x + 2$

-2 | 1   5     11    10
   |   + -2  + -6   + -10
   _____
     1   3     5     0 rem
     $x^2$  X    C

$x^2 + 3x + 5$

Con't factor