Hierarchical Bayesian Continuous Time Dynamic Models

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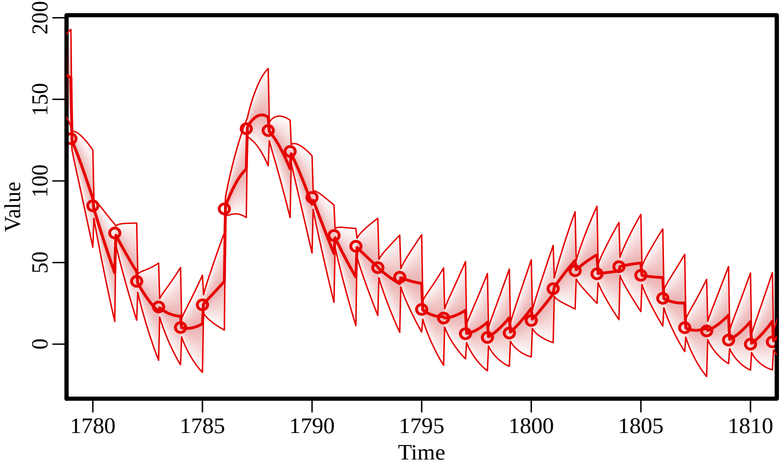
Charles Driver

Max Planck Institute for Human Development

- Model in which the present state depends in some way on the previous state, and an error / uncertainty component



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- Mapping fuzzy theory to dynamic system highlights gaps, helps ensure coherent, testable, incrementally improvable theory.

- Pre register

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What went wrong?

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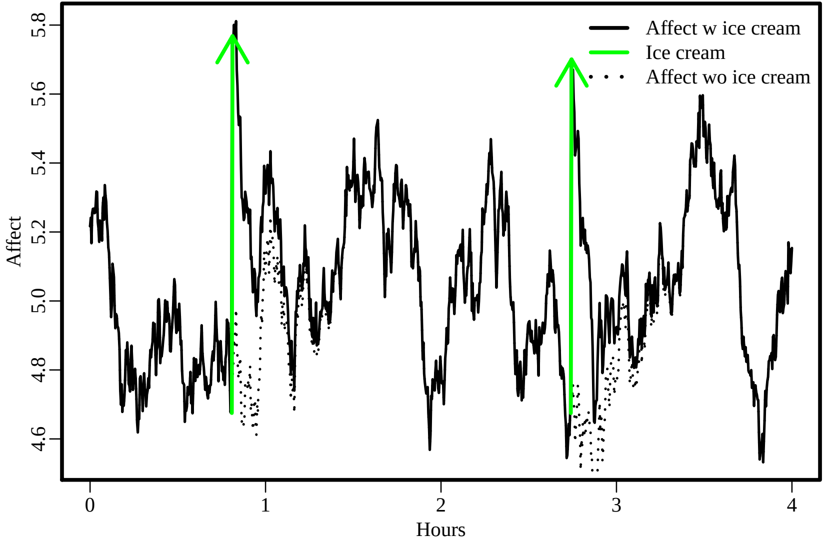
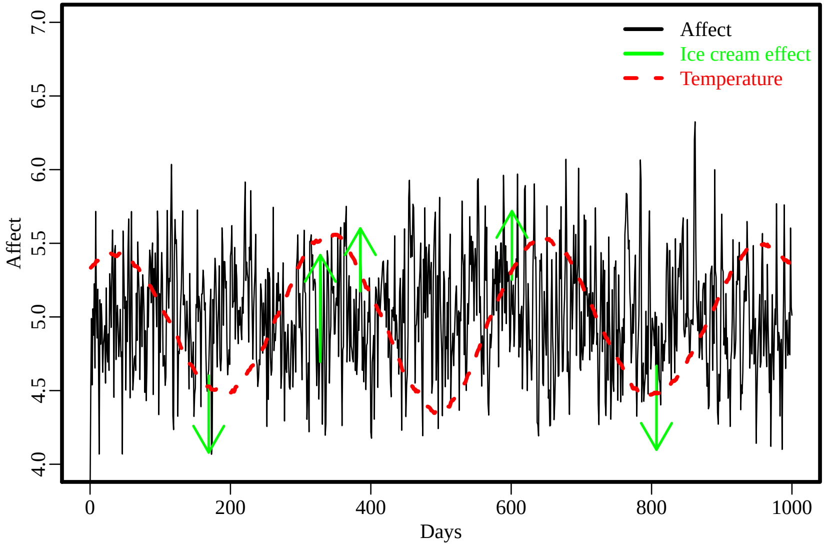
- See Tal Yarkoni, Paul Meehl, no doubt others – long running issues!



Affect w ice cream
Ice cream
Affect wo ice cream
Affect
5.8
5.6
5.4
5.2
5.0
4.8
4.6
Hours
0
1
2
3
4


Affect
Ice cream effect
Temperature
7.0
6.5
6.0
5.5
5.0
4.5
4.0
Affect
0
200
400
600
800
1000
Days

Summary of why:

- Match practice with theory.

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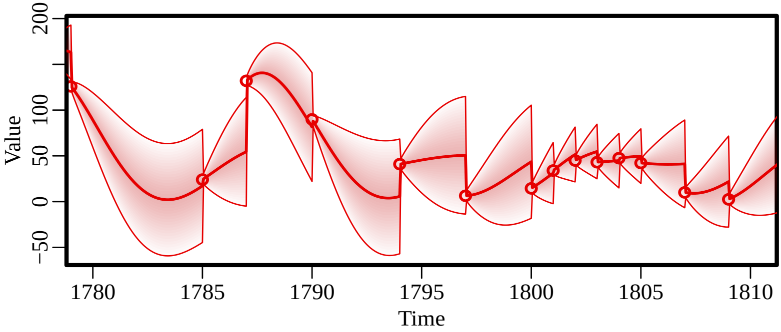
- I'm convinced! How to do it?

- Typical,discrete time approaches to dynamic modelling (e.g. latent change, autoregressive) directly model the dependence of later states on earlier states. $\eta_t = A\eta_{t-1} + b + \zeta(t)$



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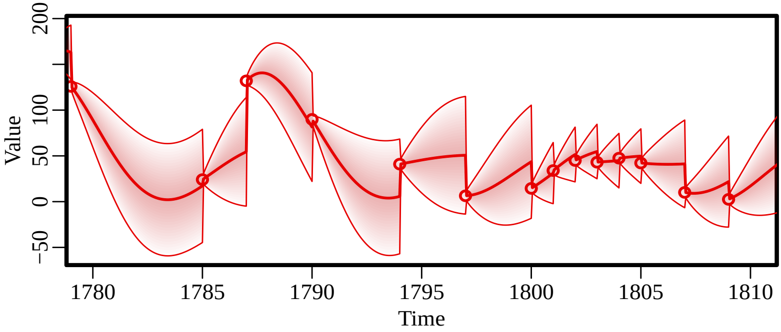
- Continuous time (differential equation) approaches model the *direction of change* given the current state. $\mathrm{d}\eta(t) = A\eta(t) + b + \zeta(t)$



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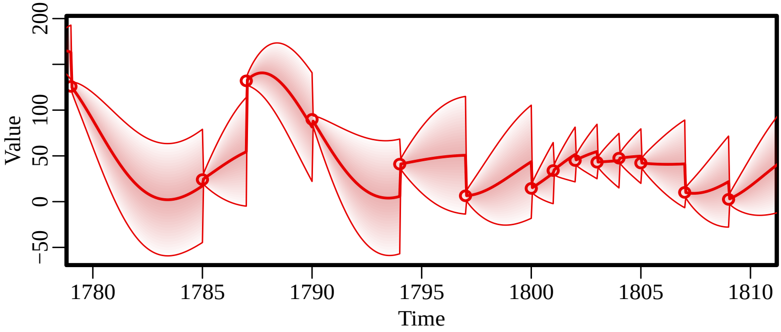
- Continuous time (differential equation) approaches model the *direction of change* given the current state. $\mathrm{d}\eta(t) = A\eta(t) + b + \zeta(t)$
- Expectations regarding future states are then a deterministic function of underlying continuous time parameters and the time interval between measurements.
  $\eta_t = A^*\eta_{t-1} + b^* + \zeta^*(t)$



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- Downsides:
  - More mathematically demanding than autoregressive / latent change approaches, more computationally demanding in general.

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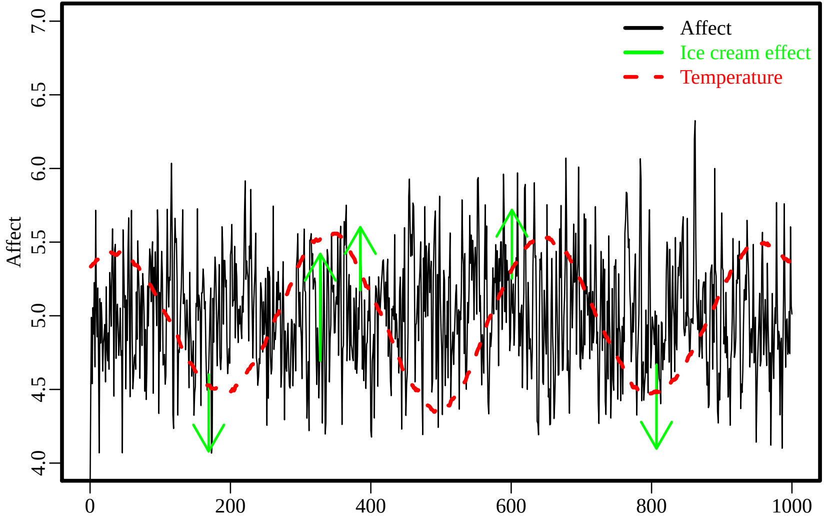

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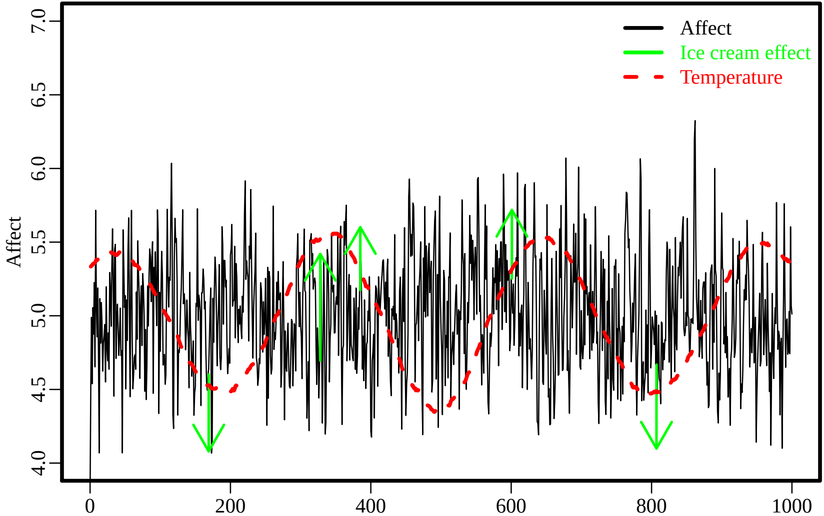
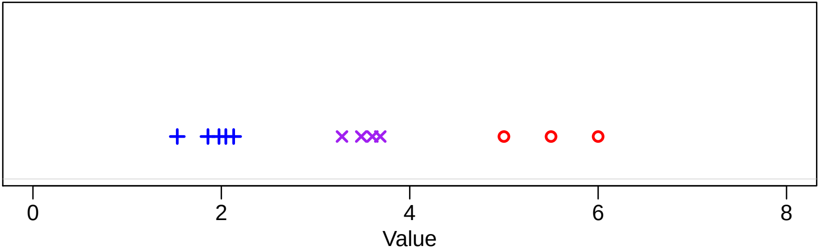
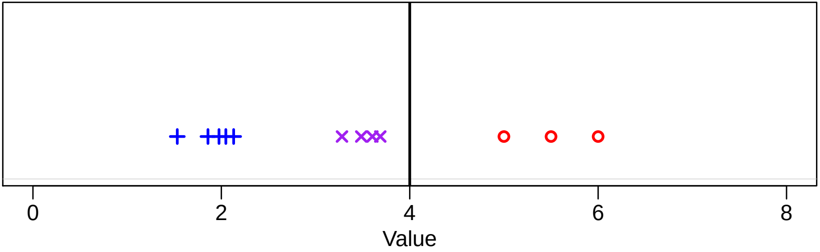
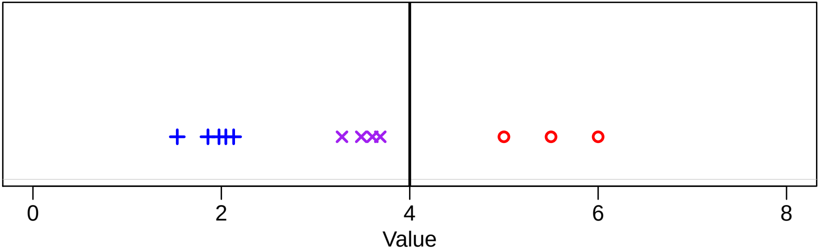
- They can vary depending on time scale.
- They make estimation trickier.




0
2
4
6
8
Value



- Complete pooling - estimate single fixed effect parameter for entire sample.



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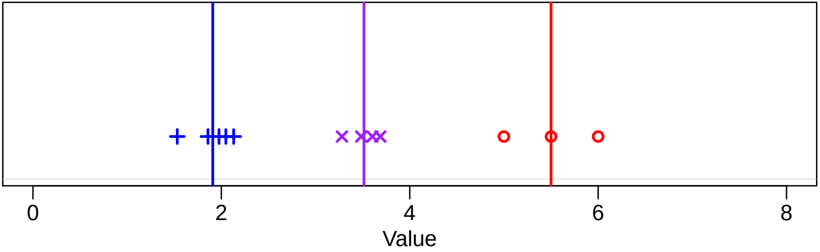

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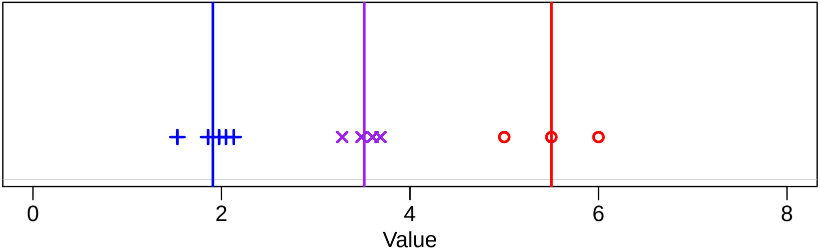

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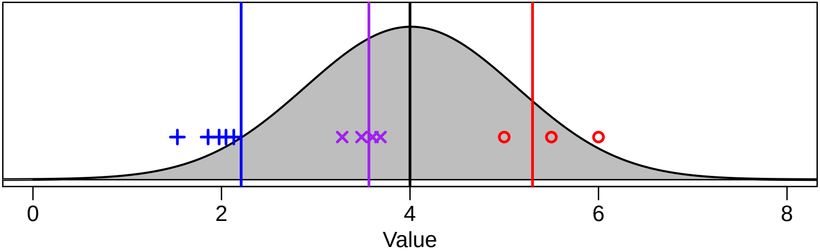

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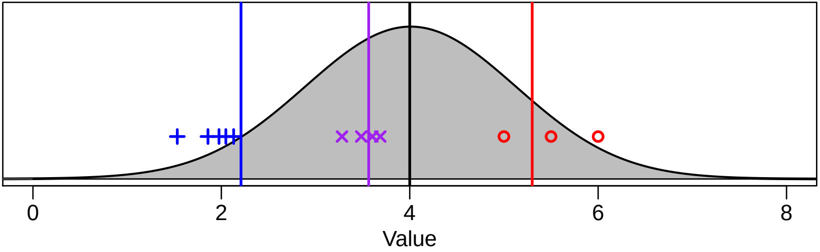

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- Partial pooling - estimate population distribution for individual models.
    - More complex models but most flexible - parameters are not either 'freely varying' or 'not varying at all' but the extent of allowed variation is estimated.

$$p(\boldsymbol{\Phi}, \boldsymbol{\mu}, \mathbf{R}, \beta | \mathbf{Y}, \mathbf{Z}) = \frac{p(\mathbf{Y}|\boldsymbol{\Phi}) p(\boldsymbol{\Phi}|\boldsymbol{\mu}, \mathbf{R}, \boldsymbol{\beta}, \mathbf{Z}) p(\boldsymbol{\mu}, \mathbf{R}, \beta)}{p(\mathbf{Y})} \tag{1}$$

Where subject specific parameters $\boldsymbol{\Phi}_i$ are determined in the following manner:

$$\boldsymbol{\Phi}_i = \text{tform}\left(\boldsymbol{\mu} + \mathbf{R}\mathbf{h}_i + \boldsymbol{\beta}\mathbf{z}_i\right) \tag{2}$$

$$\mathbf{h}_i \sim \mathrm{N}(\mathbf{0}, \mathbf{1}) \tag{3}$$

$$\boldsymbol{\mu} \sim \mathrm{N}(\mathbf{0}, \mathbf{1}) \tag{4}$$

$$\boldsymbol{\beta} \sim \mathrm{N}(\mathbf{0}, \mathbf{1}) \tag{5}$$

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- ctStanFit function constructs a Stan model and calls rstan for estimation, using either Kalman filter for continuous variables or direct sampling of states for other measurement models.

- 200 subjects at random from German socioeconomic panel, 1984-2012. Mean ages 30-70. Maximum of 29 obs. per subject.

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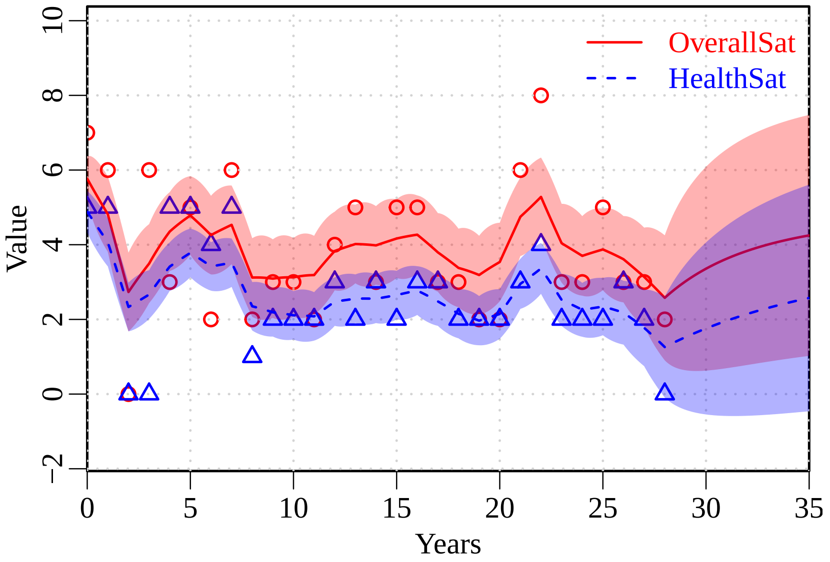
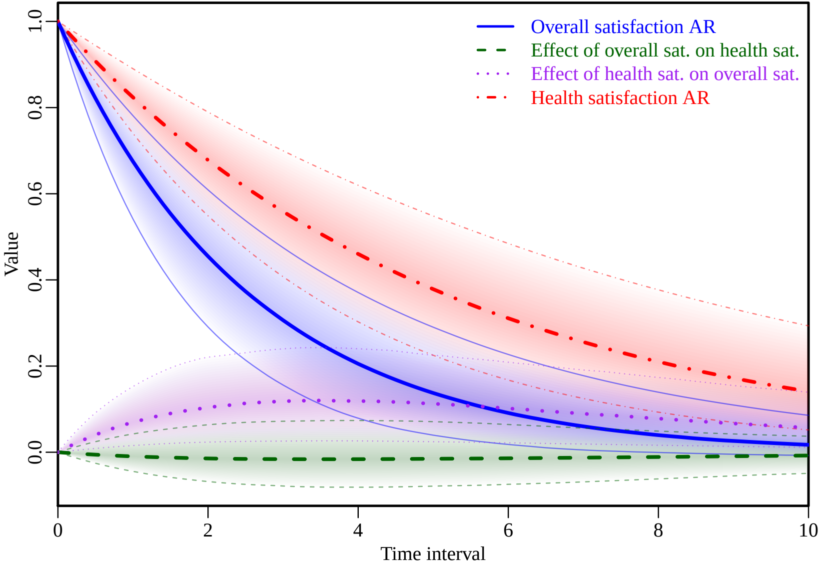
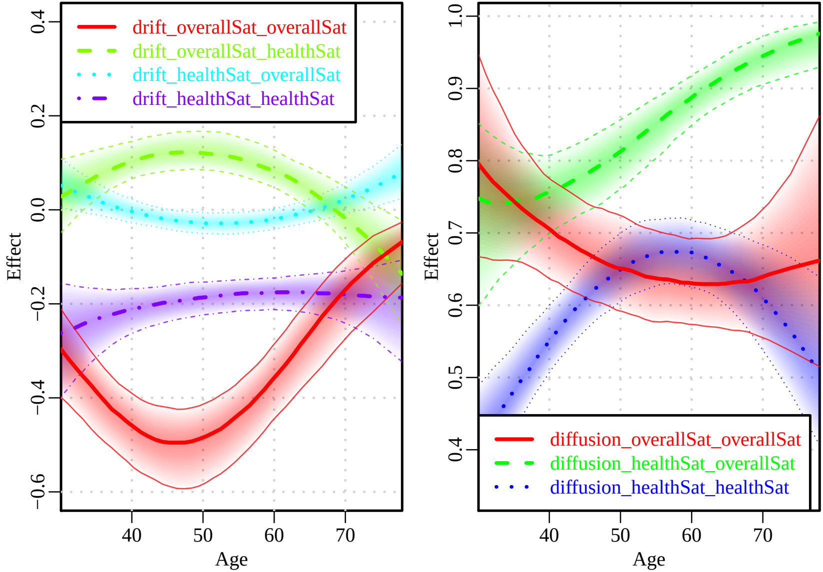
- Scores of overall life satisfaction and satisfaction with health, 0-10 point scale.
- Fit basic bivariate process model to examine individual dynamics, relations between dynamics, and differences in dynamics given age.

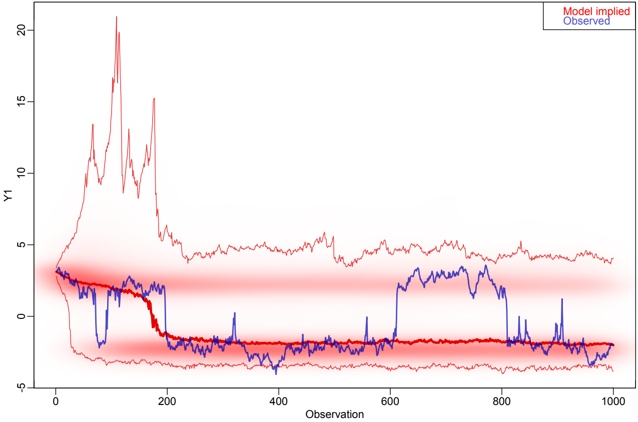


OverallSat
HealthSat
Value
Years
10
8
6
4
2
0
−2
0
5
10
15
20
25
30
35

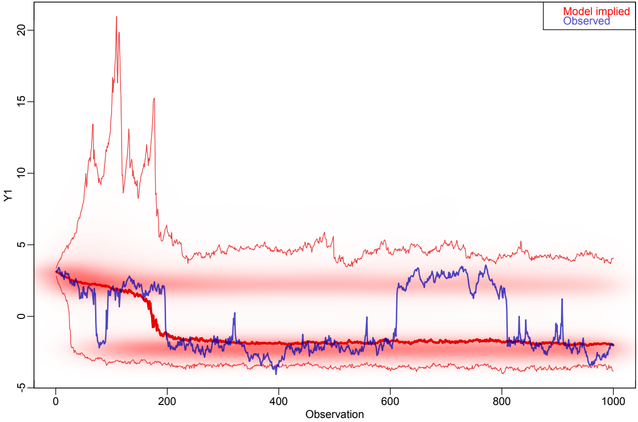

Overall satisfaction AR
Effect of overall sat. on health sat.
Effect of health sat. on overall sat.
Health satisfaction AR
Value
Time interval
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10


drift_overallSat_overallSat
drift_overallSat_healthSat
drift_healthSat_overallSat
drift_healthSat_healthSat
Effect
Age
0.4
0.2
0.0
−0.2
−0.4
−0.6
40
50
60
70
diffusion_overallSat_overallSat
diffusion_healthSat_overallSat
diffusion_healthSat_healthSat
1.0
0.9
0.8
0.7
0.6
0.5
0.4

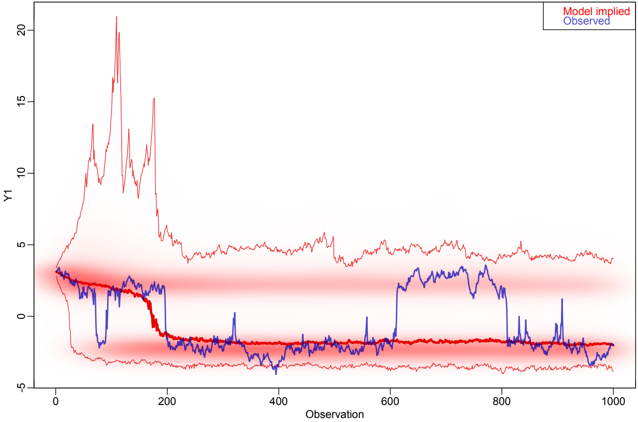
- Nonlinear / time varying models



- Nonlinear / time varying models
- Optimization + importance sampling approach – much quicker than full Bayes



- Nonlinear / time varying models
- Optimization + importance sampling approach – much quicker than full Bayes
- Various posterior predictive plots



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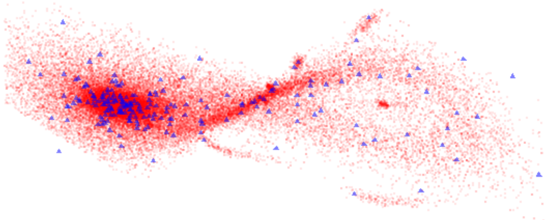

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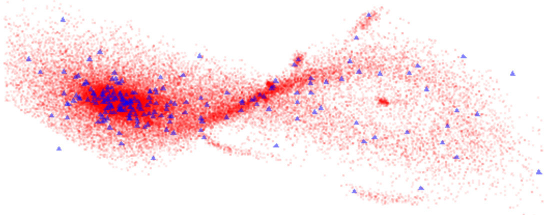
- Relevant articles:
  https://www.researchgate.net/profile/Charles_Driver



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