

Subject parameter distribution:

$$\underbrace{\begin{bmatrix} T0m_eta1_i \\ T0m_eta2_i \\ cintStress_i \\ cintQuality_i \end{bmatrix}}_{\phi(i)} \sim \text{tform} \left\{ N \left(\underbrace{\begin{bmatrix} \text{raw_T0m_eta1} \\ \text{raw_T0m_eta2} \\ \text{raw_cintStress} \\ \text{raw_cintQuality} \end{bmatrix}}_{\text{T0MEANS}}, \underbrace{\begin{bmatrix} \text{rawPCov_1.1} & \text{rawPCov_2.1} & \text{rawPCov_3.1} & \text{rawPCov_4.1} \\ \text{rawPCov_2.2} & \text{rawPCov_3.2} & \text{rawPCov_4.2} \\ \text{rawPCov_3.3} & \text{rawPCov_4.3} \\ \text{rawPCov_4.4} \end{bmatrix}}_{\text{TOVAR}} \right) \right\}$$

Initial latent state:

$$\underbrace{\begin{bmatrix} eta1 \\ eta2 \end{bmatrix}}_{\eta(t_0)} (t_0) \sim N \left(\underbrace{\begin{bmatrix} T0m_eta1 \\ T0m_eta2 \end{bmatrix}}_{\text{T0MEANS}}, \underbrace{UcorSDtoCov \left\{ \begin{bmatrix} 0.001 & 0 \\ 0 & 0.001 \end{bmatrix} \right\}}_{\mathbf{Q}_{t_0}^*} \right)$$

Deterministic change:

$$\underbrace{\begin{bmatrix} eta1 \\ eta2 \end{bmatrix}}_{\eta(t)} (t) = \underbrace{\begin{bmatrix} \text{drift_eta1} & \text{drift_eta1_eta2} \\ \text{drift_eta2_eta1} & \text{drift_eta2} \end{bmatrix}}_{\mathbf{A}_{\text{DRIFT}}} \underbrace{\begin{bmatrix} eta1 \\ eta2 \end{bmatrix}}_{\eta(t-1)} (t) + \underbrace{\begin{bmatrix} \text{cintStress} \\ \text{cintQuality} \end{bmatrix}}_{\mathbf{b}_{\text{CINT}}} +$$

Random change:

$$\underbrace{UcorSDtoChol \left\{ \begin{bmatrix} \text{diff_eta1} & 0 \\ \text{diff_eta2_eta1} & \text{diff_eta2} \end{bmatrix} \right\}}_{\mathbf{G}_{\text{DIFFUSION}}} \underbrace{\begin{bmatrix} W_1 \\ W_2 \end{bmatrix}}_{\mathbf{w}(t)} (t)$$

Observations:

$$\underbrace{\begin{bmatrix} \text{Stress} \\ \text{Stress2} \\ \text{Quality} \end{bmatrix}}_{\mathbf{Y}(t)} (t) = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ \text{stressLoading} & 1 \end{bmatrix}}_{\mathbf{\Lambda}_{\text{LAMBDA}}} \underbrace{\begin{bmatrix} eta1 \\ eta2 \end{bmatrix}}_{\eta(t)} (t) + \underbrace{\begin{bmatrix} 0 \\ \text{mintStress2} \\ 0 \end{bmatrix}}_{\mathbf{\tau}_{\text{MANIFESTMEANS}}} +$$

Observation noise:

$$\underbrace{\begin{bmatrix} \text{mvarStress} & 0 & 0 \\ 0 & \text{mvarStress2} & 0 \\ 0 & 0 & \text{mvarQuality} \end{bmatrix}}_{\mathbf{\Theta}_{\text{MANIFESTVAR}}} \underbrace{\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{bmatrix}}_{\boldsymbol{\epsilon}(t)} (t)$$

System noise distribution per time step:

$$[W_{j \in [1,2]}](t) \sim N(0, 1)$$

Observation noise distribution:

$$[\epsilon_{j \in [1,2]}](t) \sim N(0, 1)$$

Note: *UcorSDtoChol* converts lower tri matrix of standard deviations and unconstrained correlations to Cholesky factor, *UcorSDtoCov* = transposed cross product of *UcorSDtoChol*, to give covariance, See Driver & Voelkle (2018) p11.

Individual specific notation (subscript i) only shown for subject parameter distribution – pop. means shown elsewhere.