Subject parameter distribution:

$$\underbrace{\begin{bmatrix} \text{int1}_i \\ \text{int2}_i \\ \text{slope1}_i \\ \text{slope2}_i \end{bmatrix}}_{\boldsymbol{\phi}(i)} \approx \mathrm{N}\left(\begin{bmatrix} 2.872 \\ 2.599 \\ 0.274 \\ 0.196 \end{bmatrix}, \begin{bmatrix} 2.615 & 2.647 & 0.07 & 0.072 \\ 2.647 & 2.738 & 0.068 & 0.057 \\ 0.07 & 0.068 & 0.037 & 0.037 \\ 0.072 & 0.057 & 0.037 & 0.041 \end{bmatrix} \right)$$

Initial latent state:

$$\underbrace{\begin{bmatrix} \text{eta1} \\ \text{eta2} \end{bmatrix}(t_0)}_{\boldsymbol{\eta}(t_0)} \sim \mathrm{N}\left(\underbrace{\begin{bmatrix} 2.872 \\ 2.599 \end{bmatrix}}_{\underbrace{}_{\text{T0MEANS}}}, \underbrace{\begin{bmatrix} 2.615 & 2.647 \\ 2.647 & 2.738 \end{bmatrix}}_{\underbrace{\mathbf{Q^*}_{t0}}_{\text{T0VAR}}} \right)$$

Deterministic change:

$$\underbrace{\mathrm{d}\begin{bmatrix} \text{eta1} \\ \text{eta2} \end{bmatrix}(t)}_{\mathrm{d}\boldsymbol{\eta}(t)} = \left(\underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{\underbrace{\mathbf{A}}_{\text{DRIFT}}} \underbrace{\begin{bmatrix} \text{eta1} \\ \text{eta2} \end{bmatrix}(t)}_{\boldsymbol{\eta}(t)} + \underbrace{\begin{bmatrix} 0.274 \\ 0.196 \end{bmatrix}}_{\underbrace{\mathbf{b}}_{\text{CINT}}} \right) \mathrm{d}t \quad +$$

Random change:

$$\underbrace{\mathit{cholsdcor}\left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}}_{\underbrace{\mathbf{G}}_{\text{DIFFUSION}}} \underbrace{\mathrm{d}\begin{bmatrix} W_1 \\ W_2 \end{bmatrix}(t)}_{\mathrm{d}\mathbf{W}(t)}$$

Observations:

$$\underbrace{\begin{bmatrix} \text{y1} \\ \text{y2} \\ \text{y3} \end{bmatrix}(t)}_{\mathbf{Y}(t)} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1.029 \end{bmatrix}}_{\underbrace{\boldsymbol{\Lambda}}_{\text{LAMBDA}}} \underbrace{\begin{bmatrix} \text{eta1} \\ \text{eta2} \end{bmatrix}(t)}_{\boldsymbol{\eta}(t)} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 2.956 \end{bmatrix}}_{\underbrace{\boldsymbol{\tau}}_{\text{MANIFESTMEANS}}} + \underbrace{\begin{bmatrix} 0.207 & 0 & 0 \\ 0 & 1.119 & 0 \\ 0 & 0 & 0.487 \end{bmatrix}}_{\underbrace{\boldsymbol{\Theta}}_{\text{MANIFESTVAR}}} \underbrace{\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{bmatrix}(t)}_{\boldsymbol{\epsilon}(t)}$$

Latent noise per time step: $\Delta\left[W_{j\in[1,2]}\right](t-u) \sim \mathrm{N}(0, t-u)$

Observation noise: $\left[\epsilon_{j\in[1,2]}\right](t) \sim \mathrm{N}(0,1)$

Linearised approximation of subject parameter distribution shown.

Indivividual specific notation (subscript i) only shown for subject parameter distribution – pop. means shown elsewhere.

cholsdcor converts lower tri matrix of std dev and unconstrained correlation to Cholesky factor covariance.

covsdcor = transposed cross product of cholsdcor, to give covariance.

See Driver & Voelkle (2018) p11.