

$$\begin{array}{c} \text{Subject} \\ \text{parameter} \\ \text{distribution:} \end{array} \underbrace{\begin{bmatrix} \text{T0m_eta1}_i \\ \text{T0m_eta2}_i \\ \text{cintStress}_i \\ \text{cintQuality}_i \\ \text{stressLoading}_i \\ \text{drift_eta1}_i \\ \text{drift_eta1_eta2}_i \\ \text{drift_eta2_eta1}_i \\ \text{drift_eta2}_i \\ \text{diff_eta1}_i \\ \text{diff_eta2_eta1}_i \\ \text{diff_eta2}_i \\ \text{mvarStress}_i \\ \text{mvarStress2}_i \\ \text{mvarQuality}_i \\ \text{mintStress2}_i \\ \text{T0var_eta1}_i \\ \text{T0var_eta2_eta1}_i \\ \text{T0var_eta2}_i \end{bmatrix}}_{\phi(i)} \sim \text{tform} \left\{ \text{N} \left(\underbrace{\begin{bmatrix} \text{raw_T0m_eta1} \\ \text{raw_T0m_eta2} \\ \text{raw_cintStress} \\ \text{raw_cintQuality} \\ \text{raw_stressLoading} \\ \text{raw_drift_eta1} \\ \text{raw_drift_eta1_eta2} \\ \text{raw_drift_eta2_eta1} \\ \text{raw_drift_eta2} \\ \text{raw_diff_eta1} \\ \text{raw_diff_eta2_eta1} \\ \text{raw_diff_eta2} \\ \text{raw_mvarStress} \\ \text{raw_mvarStress2} \\ \text{raw_mvarQuality} \\ \text{raw_mintStress2} \\ \text{raw_T0var_eta1} \\ \text{raw_T0var_eta2_eta1} \\ \text{raw_T0var_eta2} \end{bmatrix}}_{\text{T0MEANS}}, \underbrace{\begin{bmatrix} \text{rawPCov_1_1} & \text{rawPCov_2_1} & \text{rawPCov_3_1} & \text{rawPCov_4_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{rawPCov_2_1} & \text{rawPCov_2_2} & \text{rawPCov_3_2} & \text{rawPCov_4_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{rawPCov_3_1} & \text{rawPCov_3_2} & \text{rawPCov_3_3} & \text{rawPCov_4_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{rawPCov_4_1} & \text{rawPCov_4_2} & \text{rawPCov_4_3} & \text{rawPCov_4_4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{\text{TOVAR}}^* \right) + \underbrace{\begin{bmatrix} \text{raw_T0m_eta1_initialStress} & \text{raw_T0m_eta1_meanStress} \\ \text{raw_T0m_eta2_initialStress} & \text{raw_T0m_eta2_meanStress} \\ \text{raw_0} & \text{raw_cintStress_meanStress} \\ \text{raw_cintQuality_initialStress} & \text{raw_cintQuality_meanStress} \\ \text{raw_stressLoading_initialStress} & \text{raw_stressLoading_meanStress} \\ \text{raw_drift_eta1_initialStress} & \text{raw_drift_eta1_meanStress} \\ \text{raw_drift_eta1_eta2_initialStress} & \text{raw_drift_eta1_eta2_meanStress} \\ \text{raw_drift_eta2_eta1_initialStress} & \text{raw_drift_eta2_eta1_meanStress} \\ \text{raw_drift_eta2_initialStress} & \text{raw_drift_eta2_meanStress} \\ \text{raw_diff_eta1_initialStress} & \text{raw_diff_eta1_meanStress} \\ \text{raw_diff_eta2_eta1_initialStress} & \text{raw_diff_eta2_eta1_meanStress} \\ \text{raw_diff_eta2_initialStress} & \text{raw_diff_eta2_meanStress} \\ \text{raw_mvarStress_initialStress} & \text{raw_mvarStress_meanStress} \\ \text{raw_mvarStress2_initialStress} & \text{raw_mvarStress2_meanStress} \\ \text{raw_mvarQuality_initialStress} & \text{raw_mvarQuality_meanStress} \\ \text{raw_mintStress2_initialStress} & \text{raw_mintStress2_meanStress} \\ \text{raw_T0var_eta1_initialStress} & \text{raw_T0var_eta1_meanStress} \\ \text{raw_T0var_eta2_eta1_initialStress} & \text{raw_T0var_eta2_eta1_meanStress} \\ \text{raw_T0var_eta2_initialStress} & \text{raw_T0var_eta2_meanStress} \end{bmatrix}}_{\beta} \underbrace{\begin{bmatrix} \text{initialStress} \\ \text{meanStress} \end{bmatrix}}_{\mathbf{z}}$$

$$\begin{array}{c} \text{Initial} \\ \text{latent} \\ \text{state:} \end{array} \underbrace{\begin{bmatrix} \text{eta1} \\ \text{eta2} \end{bmatrix}}_{\eta(t_0)}(t_0) \sim \text{N} \left(\underbrace{\begin{bmatrix} \text{T0m_eta1} \\ \text{T0m_eta2} \end{bmatrix}}_{\text{T0MEANS}}, \underbrace{UcorSDtoCov \left\{ \begin{bmatrix} 0.001 & 0 \\ 0 & 0.001 \end{bmatrix} \right\}}_{\text{TOVAR}}^* \right)$$

$$\begin{array}{c} \text{Deterministic} \\ \text{change:} \end{array} \underbrace{\begin{bmatrix} \text{eta1} \\ \text{eta2} \end{bmatrix}}_{\eta(t)}(t) = \underbrace{\begin{bmatrix} \text{drift_eta1} & \text{drift_eta1_eta2} \\ \text{drift_eta2_eta1} & \text{drift_eta2} \end{bmatrix}}_{\mathbf{A}_{\text{DRIFT}}} \underbrace{\begin{bmatrix} \text{eta1} \\ \text{eta2} \end{bmatrix}}_{\eta(t-1)}(t) + \underbrace{\begin{bmatrix} \text{cintStress} \\ \text{cintQuality} \end{bmatrix}}_{\mathbf{b}_{\text{CINT}}} +$$

$$\begin{array}{c} \text{Random} \\ \text{change:} \end{array} \underbrace{UcorSDtoChol \left\{ \begin{bmatrix} \text{diff_eta1} & 0 \\ \text{diff_eta2_eta1} & \text{diff_eta2} \end{bmatrix} \right\}}_{\mathbf{G}_{\text{DIFFUSION}}} \underbrace{\begin{bmatrix} W_1 \\ W_2 \end{bmatrix}}_{\mathbf{w}(t)}(t)$$

$$\begin{array}{c} \text{Observations:} \end{array} \underbrace{\begin{bmatrix} \text{Stress} \\ \text{Stress2} \\ \text{Quality} \end{bmatrix}}_{\mathbf{Y}(t)}(t) = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ \text{stressLoading} & 1 \end{bmatrix}}_{\mathbf{\Lambda}_{\text{LAMBDA}}} \underbrace{\begin{bmatrix} \text{eta1} \\ \text{eta2} \end{bmatrix}}_{\eta(t)}(t) + \underbrace{\begin{bmatrix} 0 \\ \text{mintStress2} \\ 0 \end{bmatrix}}_{\mathbf{\tau}_{\text{MANIFESTMEANS}}} +$$

$$\begin{array}{c} \text{Observation} \\ \text{noise:} \end{array} \underbrace{\begin{bmatrix} \text{mvarStress} & 0 & 0 \\ 0 & \text{mvarStress2} & 0 \\ 0 & 0 & \text{mvarQuality} \end{bmatrix}}_{\mathbf{\Theta}_{\text{MANIFESTVAR}}} \underbrace{\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{bmatrix}}_{\epsilon(t)}(t)$$

System noise distribution per time step: $[W_{j \in [1,2]}](t) \sim \text{N}(0, 1)$ Observation noise distribution: $[\epsilon_{j \in [1,2]}](t) \sim \text{N}(0, 1)$

Note: $UcorSDtoChol$ converts lower tri matrix of standard deviations and unconstrained correlations to Cholesky factor, $UcorSDtoCov =$ transposed cross product of $UcorSDtoChol$, to give covariance, See Driver & Voelkle (2018) p11. Individual specific notation (subscript i) only shown for subject parameter distribution – pop. means shown elsewhere.