

$$\text{Subject parameter distribution: } \underbrace{\begin{bmatrix} \text{T0m\_eta1}_i \\ \text{T0m\_eta2}_i \\ \text{mm\_Stress}_i \\ \text{mm\_Stress2}_i \\ \text{mm\_Quality}_i \end{bmatrix}}_{\phi(i)} \sim \text{tform} \left\{ \text{N} \left( \begin{bmatrix} \text{raw\_T0m\_eta1} \\ \text{raw\_T0m\_eta2} \\ \text{raw\_mm\_Stress} \\ \text{raw\_mm\_Stress2} \\ \text{raw\_mm\_Quality} \end{bmatrix}, \begin{bmatrix} \text{rawPCov\_1\_1} & \text{rawPCov\_2\_1} & \text{rawPCov\_3\_1} & \text{rawPCov\_4\_1} & \text{rawPCov\_5\_1} \\ \text{rawPCov\_2\_2} & \text{rawPCov\_2\_2} & \text{rawPCov\_3\_2} & \text{rawPCov\_4\_2} & \text{rawPCov\_5\_2} \\ \text{rawPCov\_3\_1} & \text{rawPCov\_3\_2} & \text{rawPCov\_3\_3} & \text{rawPCov\_4\_3} & \text{rawPCov\_5\_3} \\ \text{rawPCov\_4\_1} & \text{rawPCov\_4\_2} & \text{rawPCov\_4\_3} & \text{rawPCov\_4\_4} & \text{rawPCov\_5\_4} \\ \text{rawPCov\_5\_1} & \text{rawPCov\_5\_2} & \text{rawPCov\_5\_3} & \text{rawPCov\_5\_4} & \text{rawPCov\_5\_5} \end{bmatrix} \right) \right\}$$

$$\text{Initial latent state: } \underbrace{\begin{bmatrix} \text{eta1} \\ \text{eta2} \end{bmatrix}}_{\eta(t_0)} (t_0) \sim \text{N} \left( \underbrace{\begin{bmatrix} \text{T0m\_eta1} \\ \text{T0m\_eta2} \end{bmatrix}}_{\text{T0MEANS}}, \underbrace{UcorSDtoCov \left\{ \begin{bmatrix} 0.001 & 0 \\ 0 & 0.001 \end{bmatrix} \right\}}_{\underset{\text{T0VAR}}{\mathbf{Q}^*_{t0}}} \right)$$

$$\text{Deterministic change: } \underbrace{\begin{bmatrix} \text{eta1} \\ \text{eta2} \end{bmatrix}}_{\eta(t)} (t) = \left( \underbrace{\begin{bmatrix} \text{drift\_eta1} & \text{drift\_eta1\_eta2} \\ \text{drift\_eta2\_eta1} & \text{drift\_eta2} \end{bmatrix}}_{\underset{\text{DRIFT}}{\mathbf{A}}} \underbrace{\begin{bmatrix} \text{eta1} \\ \text{eta2} \end{bmatrix}}_{\eta(t-1)} (t) + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{\underset{\text{CINT}}{\mathbf{b}}} \right) +$$

$$\text{Random change: } \underbrace{UcorSDtoChol \left\{ \begin{bmatrix} \text{diff\_eta1} & 0 \\ \text{diff\_eta2\_eta1} & \text{diff\_eta2} \end{bmatrix} \right\}}_{\underset{\text{DIFFUSION}}{\mathbf{G}}} \underbrace{\begin{bmatrix} W_1 \\ W_2 \end{bmatrix}}_{\mathbf{w}(t)} (t)$$

$$\text{Observations: } \underbrace{\begin{bmatrix} \text{Stress} \\ \text{Stress2} \\ \text{Quality} \end{bmatrix}}_{\mathbf{Y}(t)} (t) = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ \text{stressLoading} & 1 \end{bmatrix}}_{\underset{\text{LAMBDA}}{\mathbf{\Lambda}}} \underbrace{\begin{bmatrix} \text{eta1} \\ \text{eta2} \end{bmatrix}}_{\eta(t)} (t) + \underbrace{\begin{bmatrix} \text{mm\_Stress} \\ \text{mm\_Stress2} \\ \text{mm\_Quality} \end{bmatrix}}_{\underset{\text{MANIFESTMEANS}}{\boldsymbol{\tau}}}$$

$$\text{Observation noise: } \underbrace{\begin{bmatrix} \text{mvarStress} & 0 & 0 \\ 0 & \text{mvarStress2} & 0 \\ 0 & 0 & \text{mvarQuality} \end{bmatrix}}_{\underset{\text{MANIFESTVAR}}{\boldsymbol{\Theta}}} \underbrace{\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{bmatrix}}_{\boldsymbol{\epsilon}(t)} (t)$$

System noise distribution per time step:  $[W_{j \in [1,2]}](t) \sim \text{N}(0, 1)$       Observation noise distribution:  $[\epsilon_{j \in [1,2]}](t) \sim \text{N}(0, 1)$

Note: *UcorSDtoChol* converts lower tri matrix of standard deviations and unconstrained correlations to Cholesky factor, *UcorSDtoCov* = transposed cross product of *UcorSDtoChol*, to give covariance, See Driver & Voelkle (2018) p11.  
Individual specific notation (subscript i) only shown for subject parameter distribution – pop. means shown elsewhere.