MA 598 Machine Learning Seminar
Summary 2

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2 A Review of Convolutional Neural Networks for Inverse Problems

2.1 Background

The paper begins with a brief introduction on inverse problems and contrasts the traditional approach with the learning-based approach. To keep the mathematics simple, we will talk about the inverse problem as it applies to image-processing. An imaging system is an operator $H\colon \mathcal{X} \to \mathcal{Y}$ that acts on an image $x \in \mathcal{X}$ and creates a vector of measurements $y \in \mathcal{Y}$ with $H(x) = y$. The inverse problem asks given a measurement y, can we recover the original image x? Mathematically, we are looking for reconstruction $R\colon \mathcal{Y} \to \mathcal{X}$ which reverses the sampling done by H.

2.2 Objective function approach

The usual approach for finding R is called the objective function approach, which models H and recovers an estimate of x from y via

$$R_{\mathrm{obj}}(y) = \arg\min_{x \in \mathcal{X}} f(H(x), y), \tag{2.1}$$

where $f\colon \mathcal{Y} \times \mathcal{Y} \to \mathbb{R}^+$ is some appropriate measure of error. The inverse $\bar{H}^{-1}$ is usually found through filtered back projection (FBP) algorithm, or the back projection. However, these direct inverses show significant artifacts when the quality of the measurement decreases.

2.3 Learning-based approach

The proposed alternative is a learning approach, where a training set of ground truth images and their measurements $\{(x_n, y_n)\colon n = 1, ..., N\}$ is know, and a parametric reconstruction algorithm is solved by

$$R_{\mathrm{learn}} = \arg\min_{R_\theta, \theta \in \Theta} \sum_{n=1}^{N} f(x_n, R_\theta(y_n)) + g(\theta), \tag{2.2}$$

where Θ is the set of all possible parameters, $f\colon \mathscr{X} \times \mathscr{X} \to \mathbb{R}^+$ the measure error, and $g\colon \Theta \to \mathbb{R}^+$ a regularizer whose purpose is to avoid parameter overfitting.

The learning-based approach has been successfully employed in CNN. It is mentioned in the paper that a CNN was first used in the 2012 ImageNet Large Scale Visual Recognition Challenge, which achieved an error rate of 15.3% at the object localization and classification task, compared to a 26.2% error rate for the next closest method, and subsequent CNN approaches only kept improving on this error rate in later competitions.

2.4 Designing CNN for inverse problems

The paper goes into some detail as to how to design CNN for the purpose of solving inverse problems. First, we must generate a training set. This can be a very daunting problem; especially for X-ray CT. But the set is typically obtained by corrupting images with noise and feeding both the original image and the corrupted one.

2.5 Preprocessing

Some amount of preprocessing is used in some CNN. This is typically achieved by using a direct inverse operator on the network input. That is, instead of R_{learn}, we have a composition of CNN with a direct inverse $R_g \circ \bar{H}^{-1}$. Examples of applications which use preprocessing are CT, which preprocessed using FBP, and MRI, which uses the inverse Fourier transform.