NOTE May 13, 2013

So here's my crack at the problem with Q and P. The "correct" way to do everything is to only use symmetric states, in other words to only include physical states in the Hilbert space. I will write this as

$$P_{\rm sym} + Q_{\rm sym} = \mathbb{1}_{\rm sym} \tag{1}$$

These operators have all the nice properties we expect. In particular, let's assume we augment the Hilbert space with HO states of mixed symmetry, adding an "identity"

$$P_{\rm mix} + Q_{\rm mix} = \mathbb{1}_{\rm mix} \tag{2}$$

All physical operators should respect the symmetry of the states. So for any physical operator $\mathcal{O}$ we would have a sort of orthogonality relation,

$$P_{\rm mix}\mathcal{O}P_{\rm sym} = Q_{\rm mix}\mathcal{O}Q_{\rm sym} = Q_{\rm mix}\mathcal{O}P_{\rm sym} = P_{\rm mix}\mathcal{O}Q_{\rm sym} = 0 \tag{3}$$

This follows from the commutation relations indicating that physical operators respect the particle statistics,

$$\left[\mathbb{1}_{\rm sym}, \mathcal{O}\right] = \left[\mathbb{1}_{\rm mix}, \mathcal{O}\right] = 0 \tag{4}$$

and thinking of these identity operators as projectors so that $\mathbb{1}_{\rm sym}\mathbb{1}_{\rm mix} = 0$.

In our code we do something weird. We subtract off a set of states, in my case specifically all of the Q states with $l, L > 0$ or an energy greater than some cutoff. Generally we could write this space as $Q_{\rm cut}$ so that we have

$$Q' = Q_{\rm sym} + Q_{\rm mix} - Q_{\rm cut} \tag{5}$$

All of the trouble comes from $Q_{\rm cut}$. It has some "terrible" properties,

$$Q_{\rm cut}\mathbb{1}_{\rm sym} \neq 0 \qquad \text{and} \qquad Q_{\rm cut}\mathbb{1}_{\rm mix} \neq 0 \tag{6}$$

When dealing with three or fewer projection operators, a cavalier approach causes no problems. We run into trouble because of operators like

$$P_{\rm sym}\mathcal{O}_1 Q_{\rm cut}\mathcal{O}_2 P_{\rm mix}\mathcal{O}_3 Q_{\rm cut}\mathcal{O}_4 P_{\rm sym} \neq 0 \tag{7}$$

Physically, we scatter from the symmetric states into the cutoff states, which mix with the states of different symmetry. The next scattering respects the symmetry, but the $Q_{\rm cut}$ operator again mixes symmetries and we get a non-zero matrix element.

This explains why inclusion of the mixed symmetry states below the model space cutoff leads to different results. Of course, there will always be errors from the $Q_{\rm cut}$ terms (i.e. not including the full HO basis). I think it's sensible to get better results from the inversion of $\mathbb{1}' - V_{12,\rm eff}G_0 Q'\Pi$ by defining $\mathbb{1}' = P_{\rm sym} + Q'$ (your definition), rather than $\mathbb{1}' = P_{\rm sym} + P_{\rm mix} + Q'$ (as I originally did).