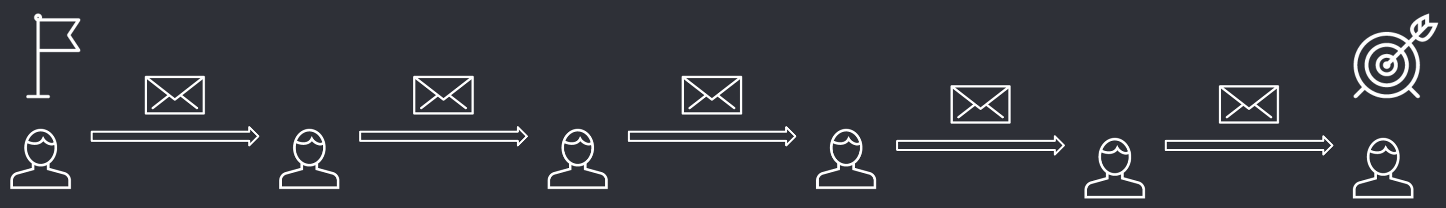
SMALL WORLD PHENOMENON

1

SIX DEGREES OF SEPARATION

"Given any two people in the world, person X & person Z, how many intermediate acquaintance links are needed before X & Z are connected?" - *Milgram (1967)*



Why are we surprised by short paths?

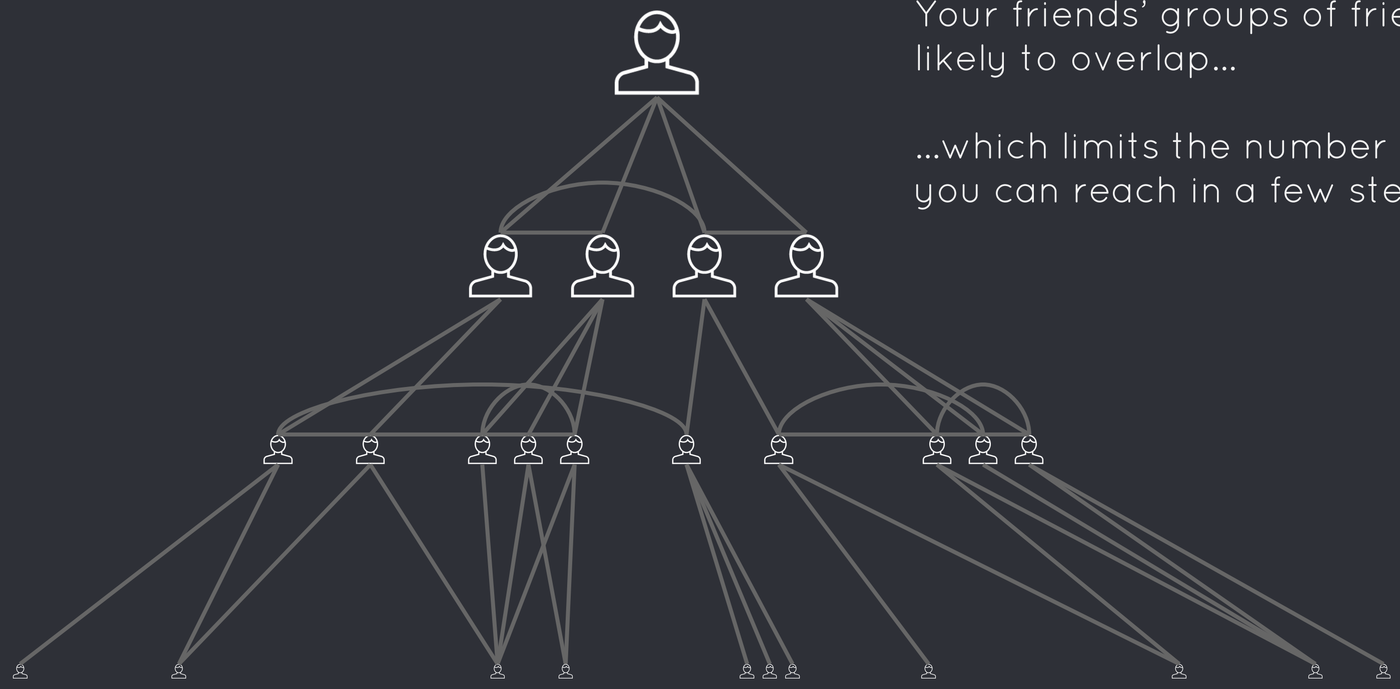
If everyone has at least 100 friends...

... you could in principle be 3 steps away from 1 million people...

...and 5 steps away from 10 billion.

SIX DEGREES OF SEPARATION

Real social networks contain many triangles



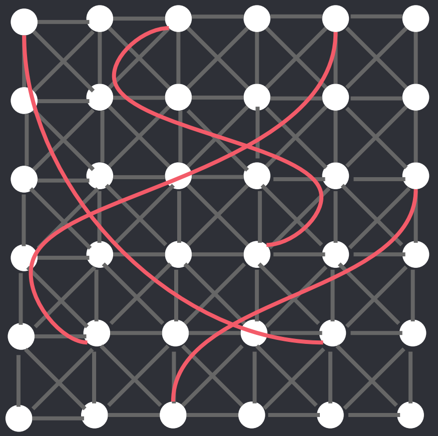
Your friends' groups of friends are likely to overlap...

...which limits the number of people you can reach in a few steps

WATTS-STROGATZ MODEL

Many closed triads, but also very short paths

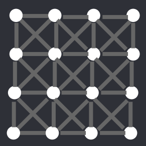
2



WATTS-STROGATZ MODEL

Idea:

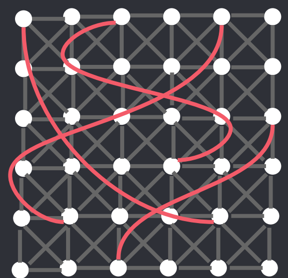
- combine homophily



- with weak ties

Model:

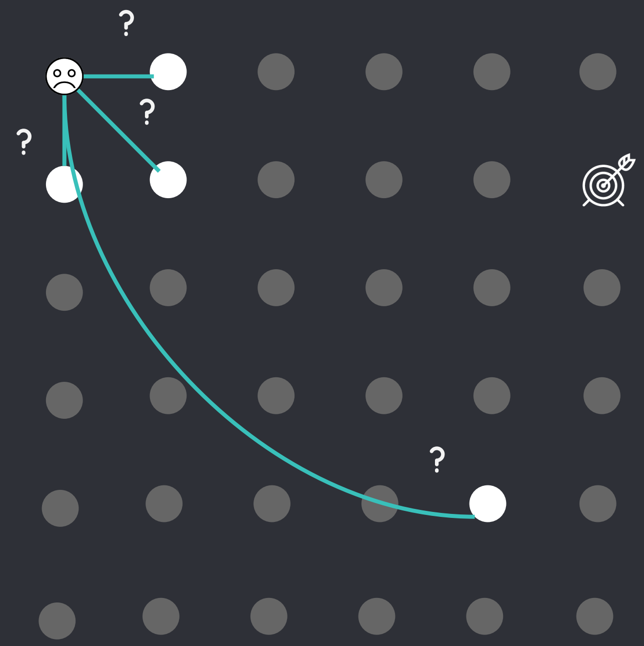
- Each node in lattice connected to all nodes within radius r
- k nodes have an extra random connection



3

FINDING SHORT PATHS

... without a global map



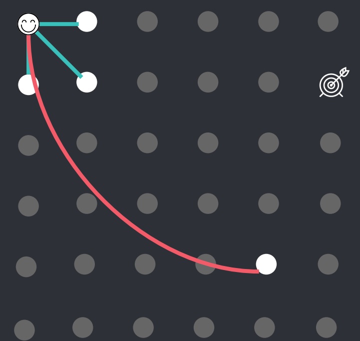
FINDING SHORT PATHS

Decentralised search algorithm

Using knowledge only of:

- layout of the grid
- location of the target
- locations of current node's immediate contacts

Greedily forward message along edge that brings it closest to target



Decentralised search algorithm

Not effective on Watts-Strogatz networks

- Long-range contacts distributed uniformly at random
- For an nxn lattice:
    - Average shortest path length: $O(\log n)$
    - Path length via decentralised search: $O(n^{2/3})$

FINDING SHORT PATHS

Kleinberg model

- Each node in lattice connected to all nodes within radius r
- k extra connections chosen with probability proportional to $\underbrace{d(v,w)}_{\text{lattice distance between nodes}}{}^{-\underbrace{\alpha}_{\text{tunable 'clustering exponent'}}}$

- small α ➡ extra edges span long distances
- large α ➡ extra edges tend to be shorter

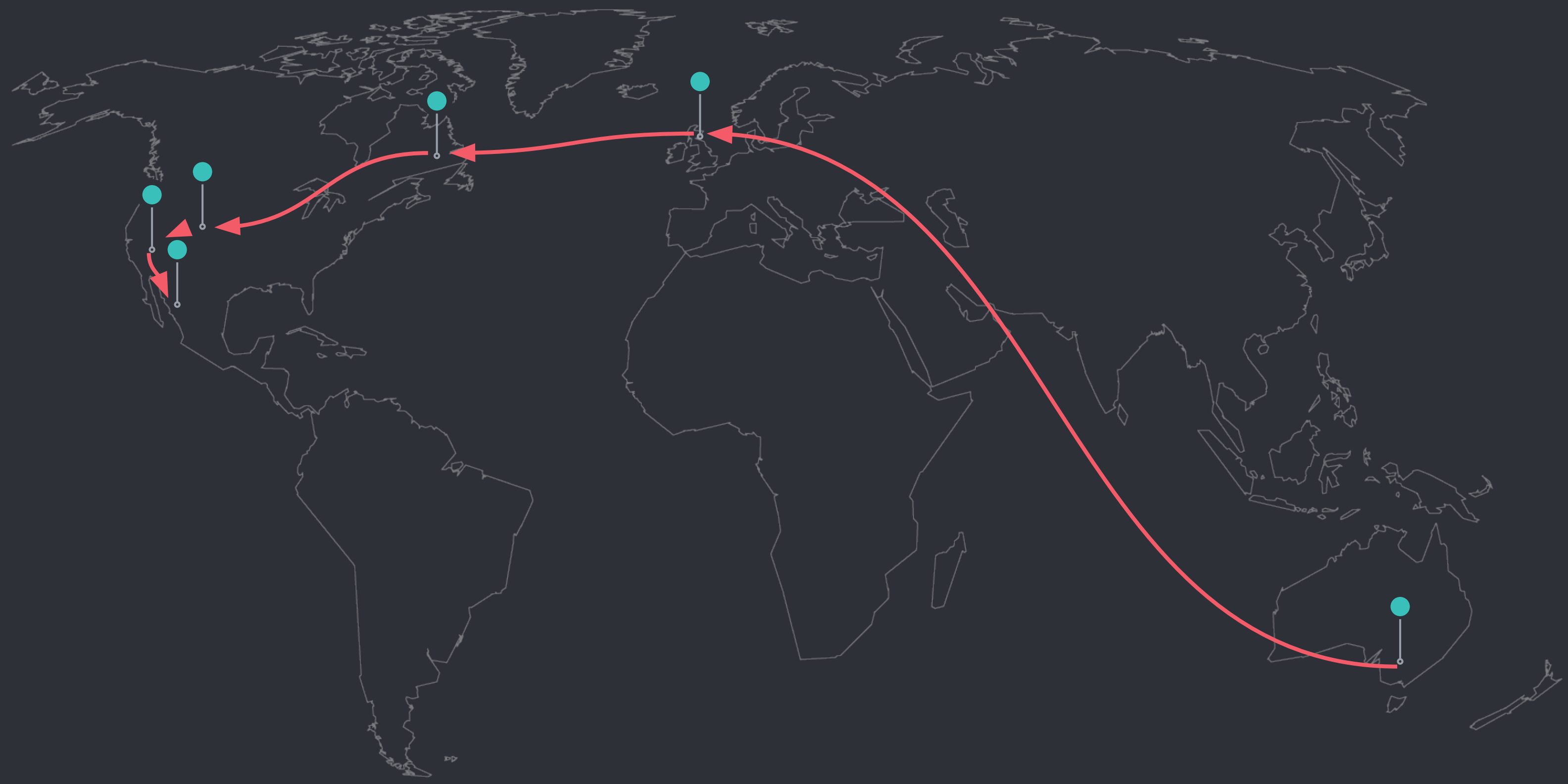
- Watts-Strogatz model: $\alpha = 0$

FINDING SHORT PATHS

Kleinberg model

- Optimal value $\alpha = 2$ (for a 2 dimensional lattice)
- Extra ties distributed uniformly over all “distance scales”
- Path length via decentralised search: $O(\log^2 n)$
- Allows people to consistently find ways of reducing their distance to the target, no matter how near or far they are from it

FINDING SHORT PATHS



"There is a progressive closing in on the target area as each new person is added to the chain" - Milgram (1967)

4

APPLICATIONS

of small-world networks

APPLICATIONS

Freenet

- Secure peer-to-peer communication network
- Nodes can only send data to their direct connections
- Needs efficient routing

APPLICATIONS

Freenet

- Organises nodes by giving them locations (initially random)
- Nodes constantly attempt to reduce distance to their connections
- Network self-organises into small-world

APPLICATIONS

Neuroscience

- Networks of cortical neurons have small-world structure
- Could explain:
  - working memory (Roxin et al, 2004)
  - epileptic seizures (Netoff et al, 2004)

Mirko Monti - Simple line icons
Webalys - Streamline iconset

THANKS

S. Milgram, ``The small world problem," Psychology Today 1, 61 (1967).

D. Watts and S. Strogatz, ``Collective dynamics of small-world networks," Nature 393, 440 (1998).

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T. Netoff, R. Clewley, S. Arno, T. Keck, and J. White, ``Epilepsy in small-world networks", J Neurosci 24, (2004)

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