Visualising High Dimensional Data with t-SNE

Harri Edwards

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  to lower dimensional 'map points'

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- Desideratum for a mapping is to *preserve the neighbourhoods* as much as possible, *i.e.* if x_i, x_j are 'close' then y_i, y_j should be 'close' and vice-versa.
- t-SNE comes from the paper 'Visualizing Data using t-SNE' published in 2008 by Laurens van der Maaten and Geoff Hinton (1127 citations!)

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The basic idea of SNE is very simple: for each data point x_i, we specify a probability distribution over its possible neighbours x_j with $j \neq 0$

$$p_{i|j} = \frac{\exp\left(-\|x_i - x_j\|^2 / 2\sigma_i^2\right)}{\sum_{j \neq i} \exp\left(-\|x_i - x_j\|^2 / 2\sigma_i^2\right)} \tag{1}$$

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Now we do the same for the map points:

$$q_{i|j} = \frac{\exp\left(-\|y_i - y_j\|^2\right)}{\sum_{j \neq i} \exp\left(-\|y_i - y_j\|^2\right)} \tag{2}$$

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$$C = \sum_i \sum_j p_{j|i} \log \frac{p_{j|i}}{q_{j|i}} \tag{4}$$

SNE: Gradient

The gradient of the cost function has a nice and simple form:

$$\frac{\partial C}{\partial y_i} = 2 \sum_j \left(p_{j|i} - q_{j|i} + p_{i|j} - q_{i|j} \right) \left(y_i - y_j \right) \tag{5}$$

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The optimisation of C involves initialising the map points from a spherical Gaussian, using gradient descent with momentum, and adding noise to the map points on each iteration with annealing variance.

The Crowding Problem

'High dimensional space is big. You just won't believe how vastly, hugely, mind-bogglingly big it is. I mean, you may think it's a long way down the road to the chemist's, but that's just peanuts to space.' (Euclid)

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- SNE's solution to this problem? Compress all the map points to a blob so everyone is friends with everyone (in particular nobody is not friends on the map who is friends in x-space.)

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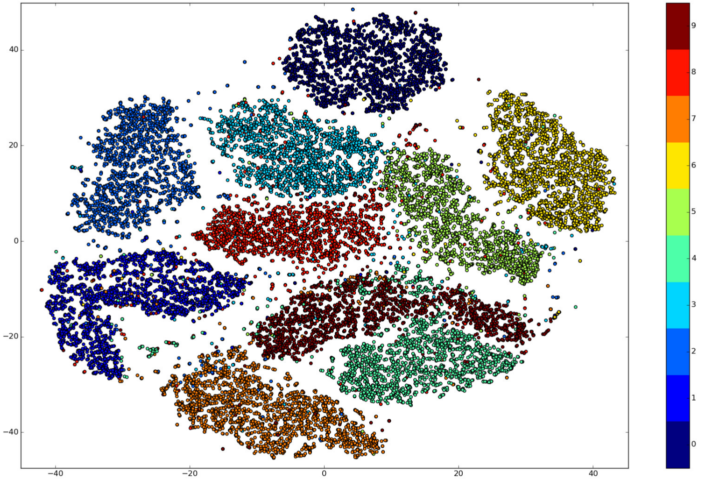
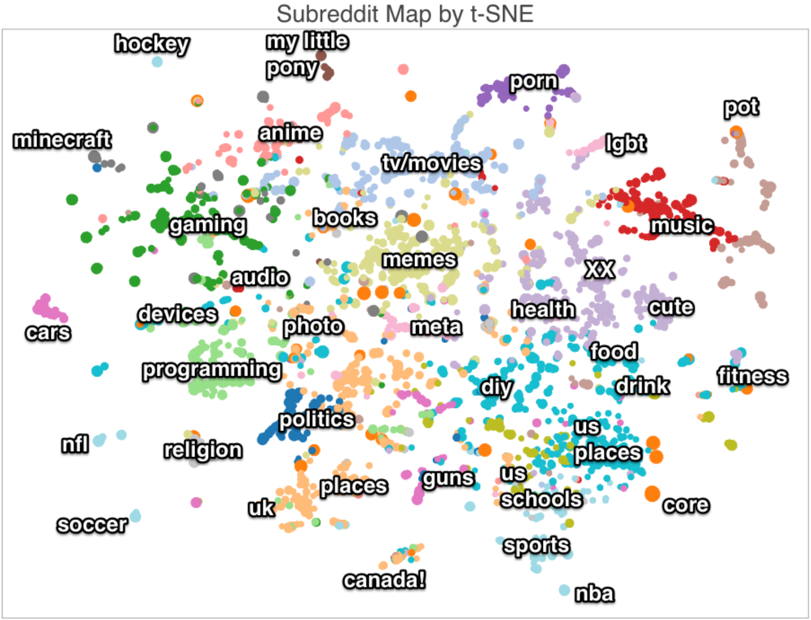
Also note that they use a symmetrised $q_{i,j}$ as opposed to $q_{i|j}$, this makes the computations simpler, but is not conceptually important.

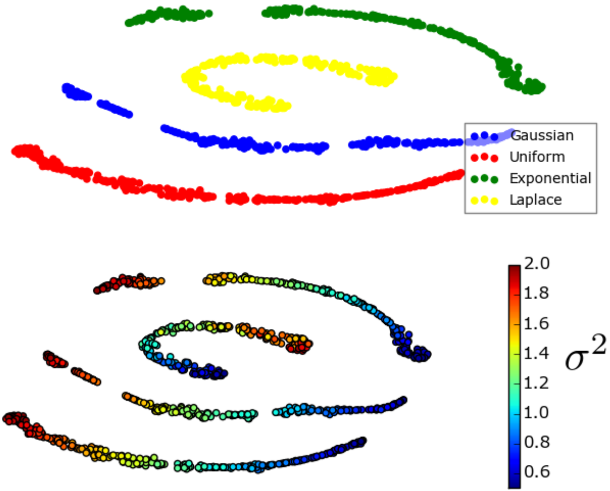
Pictures!!!: MNIST



Pictures!!!: Subreddits



Pictures!!!: Neural Statistician



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