University of St Andrews

MAY 2021 EXAMINATION DIET

SCHOOL OF MATHEMATICS & STATISTICS

MODULE CODE:	**MT4512**
MODULE TITLE:	Automata, Languages and Complexity
EXAM DURATION:	2 hours
EXAM INSTRUCTIONS:	Attempt ALL questions.
	The number in square brackets shows the maximum marks obtainable for that question or part-question.
	Your answers should contain the full working required to justify your solutions.

INSTRUCTIONS FOR ONLINE EXAMS:

Each page of your solution must have the page number, module code, and your student ID number at the top of the page. You must make sure all pages of your solutions are clearly legible.

1. (a) Consider the DFA D_1 given by $(\{q_0, q_1, q_2\}, \{a, b\}, \delta_1, q_0, \{q_1\})$, where δ_1 is

	a	b
q_0	q_1	q_0
q_1	q_2	q_1
q_2	q_0	q_2

(i) Draw the diagram of D_1. **[2]**

(ii) What is $\hat{\delta}_1(q_1, aba)$? **[1]**

(iii) Prove that $L(D_1)$ is the set of words over the alphabet $\{a, b\}$ that contain a number of as that is congruent to 1 modulo 3. **[4]**

(iv) Give a regular expression for $L(D_1)$. **[2]**

(b) Consider the regular expression $R = a^*(aab + c^2)^*$. Give the diagram of an NFA N such that $L(N) = L(R)$. (You do not need to prove that N accepts the right language). **[3]**

(c) Let $D_2 = (Q, \Sigma, \delta_2, s_0, T)$ be a DFA, and let $a \in \Sigma$. Assume that $\delta_2(q, a) = q$ for all $q \in Q$. Prove that either $\{a\}^* \subseteq L(D_2)$ or $\{a\}^* \cap L(D_2) = \emptyset$. **[2]**

2. (a) Consider the language

$$L_1 = \{w \in \{a, b\}^* \;:\; \text{no initial subword of } w \text{ has more } a\text{s than } b\text{s}\}.$$

Design a PDA P such that $L(P) = L_1$. (You do not need to prove that P accepts the right language). **[3]**

(b) Consider the context-free grammar G given by $(\{S\}, \{a, b\}, \mathcal{R}, S)$, where $\mathcal{R}$ is

$$S \to aSb \mid bSa \mid a \mid b \mid \epsilon.$$

For each of the following two words, either draw a parse tree or explain why they are not in $L(G)$: $abab$, $ababb$. **[3]**

(c) Let L_2 and L_3 be context free languages. Prove that $L_2 \cup L_3$ is context-free. **[2]**

(d) Prove that the language $L_4 = \{a^i b^{i^2} \ : \ i = 0, 1, 2 \ldots\}$ is not context free. **[3]**

3. (a) Consider the nondeterministic Turing Machine

$$M = (\{q_0, q_1, q_a, q_r\}, \{a, b\}, \{a, b, \sqcup\}, \delta, q_0, q_a, q_r)$$

where δ is given by the following table.

δ	a	b	$\sqcup$
q_0	$\{(q_0, a, R)\}$	$\{(q_1, a, R)\}$	$\emptyset$
q_1	$\{(q_1, a, R), (q_0, a, L)\}$	$\{(q_1, b, R), (q_0, b, L)\}$	$\{(q_a, \sqcup, R)\}$

(i) Write down the configurations that can be reached by M on input ab, and on input abb. **[3]**

(ii) Does M halt on all input? Justify your answer. **[4]**

(b) For each of the following languages, say whether it is Turing decidable. Justify your answer in each case.

(i) $L_1 = \{\langle M \rangle \ : \ M \text{ accepts at least two words}\}$.

(ii) $L_2 = \{\langle M \rangle \ : \ M\text{'s read/write head never moves past the first 30 tape cells, for every input word } w\}$.

[4]

4. (a) Show that if L_1 and L_2 are languages in NP then

$$L_1L_2 = \{uv \ : \ u \in L_1, v \in L_2\}$$

is in NP. **[4]**

(b) The language HugeClique is

$$\{\langle G\rangle \ : \ G \text{ is a finite graph with a clique of size at least } |V(G)|/2\}.$$

You may assume that graphs are encoded as in lectures, so that the length of the encoding is proportional to the square of the number of vertices.

(i) Draw an example of a graph with 4 vertices that is in HugeClique. **[1]**

(ii) Draw an example of a graph with 5 vertices and 5 edges that is not in HugeClique **[1]**

(iii) Prove that HugeClique is in NP. **[2]**

(iv) Fix an integer k. Construct a map f from the set of all finite graphs to the set of all finite graphs with at least $2k$ vertices such that $\langle G, k\rangle \in$Clique if and only if $\langle f(G)\rangle \in$HugeClique. **[2]**

(v) Prove that Clique is polynomial-time reducible to HugeClique. **[3]**

(vi) Is HugeClique NP-complete? Justify your answer. **[1]**

END OF PAPER
